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# Foundations of the WVU Econometric Input-Output Model

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## Foundations of the WVU Econometric Input-Output Model

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# Foundations of the WVU Econometric Input-Output Model

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# 1 Introduction

This document provides an overview of the theoretical foundations and general assumptions of the WVU Econometric Input-Output (ECIO) model. WVU Econometric Input-Output (ECIO) model (hereafter, ECIO model) is a time-series enabled hybrid econometric input-output (IO) model that combines the capabilities of econometric modeling with the strengths of IO modeling. It is designed to facilitate the estimation of economic (specifically, employment and income) impacts of energy technology development, deployment, and operation over a specified forecast period.

The ECIO model integrates a macroeconomic forecasting model of the United States (U.S.) economy with an interindustry accounting framework that characterizes the interdependence of industries, value added sectors, and final demands through sales and purchases.

The model consists of three modules and several sub-modules of interrelated equations that represent the U.S. economy and/or industry level details for 32 industrial sectors. The three major modules are:

1. Fair model: a macroeconomic econometric model of the U.S.
2. Industrial Output Module
3. Employment Module

The national ECIO model can be used to estimate the economic impacts with complete scenario description data sets in place, and the model can be applied to the estimation of economic and employment impacts of new energy technologies.

## 2 Model Design and Assumptions

The ECIO model integrates a macroeconomic econometric forecasting framework that represents the entire U.S. economy with an input-output framework that reflects the interindustry interdependence within the U.S. economy.

The ECIO model has three modules and several submodules of interrelated equations representing the entire U.S. economy, which is modeled as consisting of 32

industrial sectors. The three major modules are the Fair U.S. macroeconomic econometric model, the industrial output module, and the employment module, shown in Figure 1

## 2.1 Macroeconomic Econometric Module

The Fair model, a macroeconomic econometric model developed by Ray Fair (2004), is used to forecast values related to the size of the overall U.S. economy. The Fair model captures the interdependence and interactions among the six major components of the U.S. economy: households ( $h$ ), firms ( $b$ ), financials ( $f$ ), international component ( $r$ ), federal government ( $g$ ), and state governments ( $s$ ). This model provides a theoretical framework for the projections while maintaining a balance among different economic variables. The quarterly model comprises 289 variables in 128 equations that describe the U.S. economy. The model uses stochastic equations that are estimated using the Two Stage Least Squares (2SLS) method. These equations include a lagged dependent variable as explanatory variables to account for both partial adjustment and expectation effects. The key outputs of the model are forecasts of the components of final demand that serve as inputs to the industrial output module (the IO component of the overall model). Gross final demand components of consumption, investment, imports, exports, and government expenditures are all estimated in a consistent manner. A bridge matrix based on the most recent BEA benchmark IO tables is used to transform the forecasted aggregate components of final demand into demand by commodity.

The following subsections describe the final demand components included in the ECIO model and what the determinants are to calculate the forecasts in the Macroeconomic Econometric Module. It will also describe the estimation of the gross domestic product, the total private production and total employment.

### 2.1.1 Final Demand Components

#### 2.1.1.1 Consumption Expenditures

The household consumption component within the Fair model is composed of four consumption categories including consumption of services ( $CS$ ), durable goods ( $CD$ ), nondurable goods ( $CN$ ), and residential investment ( $IHH$ ). This aggregation is

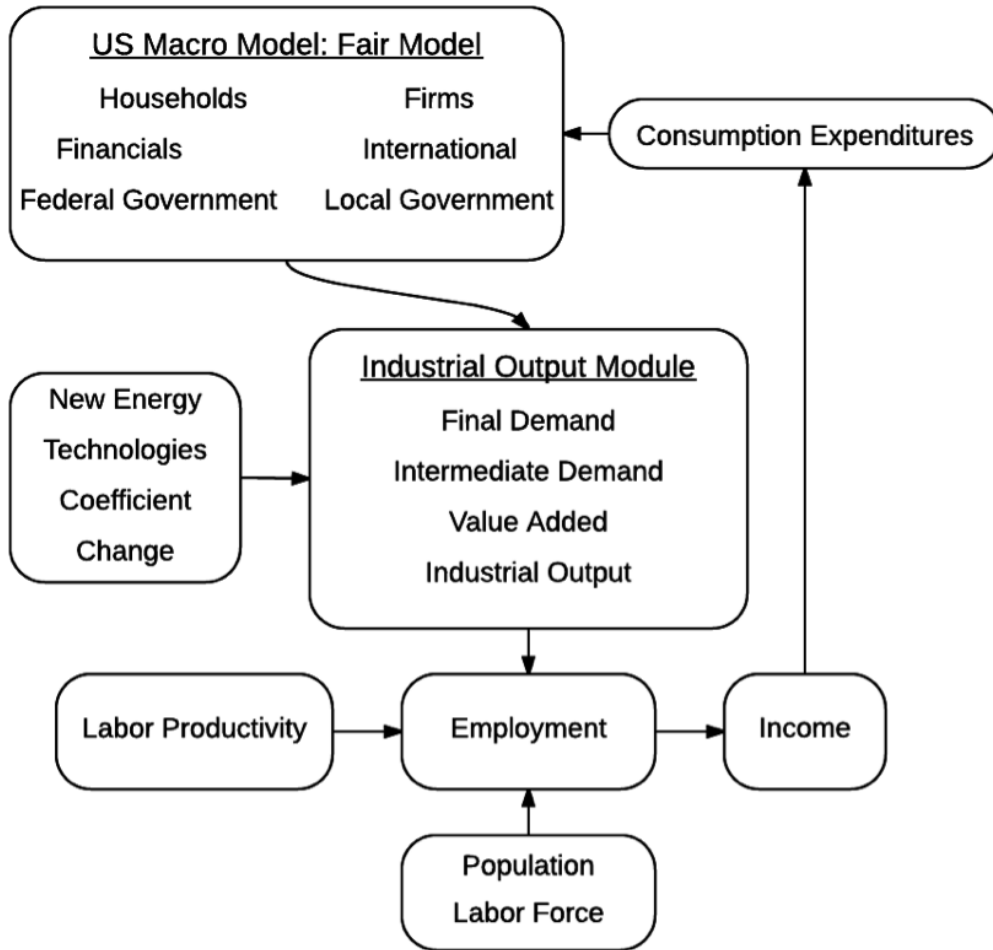


Figure 1: ECIO Model Configuration.

modified in two ways within the ECIO model. The first modification is the disaggregation of the consumption of services, durable goods, and nondurable goods to create four additional consumption categories: gasoline, motor vehicles and parts, natural gas, and electricity. The second modification is moving the residential investment category into the investment component of final demand. The modified seven consumption expenditure categories within the Macroeconomic Econometric Module are:

1. Motor vehicles and parts
2. Gasoline
3. Electricity
4. Other durable goods
5. Other nondurable goods
6. Natural Gas
7. Other services

The key determinants of personal consumption expenditure on commodity  $i$  ( $C_i$ ) are current disposable income ( $YD$ ), past total net wealth ( $AA_{t-1}$ ), past consumption ( $C_{t-1}$ ), the price deflator of consumption ( $PH$ ), the interest rate ( $R$ ), the age distribution of the population ( $AG_i$ ), total population ( $POP$ ), and a time trend ( $t$ ). With the exception of age distribution variables that are exogenous, the remaining variables are endogenously determined within the model. A representative form of the consumption equation is shown in equation ((1)):

$$\begin{aligned}
 \text{Log} \left( \frac{C_i}{POP} \right) = f \left( AG_1, AG_2, AG_3, \text{Log} \left( \frac{C_i}{POP} \right)_{t-1}, \text{Log} \left( \frac{AA}{POP} \right)_{t-1}, \right. \\
 \left. \text{Log} \left( \frac{YD}{POP \times PH} \right), R_{i,t} \right) \tag{1}
 \end{aligned}$$

### 2.1.1.2 Investment

Private investment is composed of seven investment variables, these investment variables are three variables that account for residential investment ( $IHH$ ,  $IHB$ ,  $IHF$ ), three variables that account for nonresidential fixed investment ( $IKH$ ,  $IKB$ ,  $IKF$ ), and inventory investment ( $IVF$ ). The seven investment variables determine the flow of private investment from the households ( $h$ ), firms ( $b$ ), and financial ( $f$ ) sectors to the economy. With the exception of household residential investment ( $IHH$ ), financial sector nonresidential fixed investment ( $IKF$ ), and financial sector inventory investment ( $IVF$ ), the investment variables are exogenously determined outside of the ECIO model. The specifications of the three behavioral equations in the investment sub module are as follows:

$$\Delta \frac{IHH}{POP} = f \left( DELH \left( \frac{KH}{POP} \right)_{t-1} - \left( \frac{IHH}{POP} \right)_{t-1}, \left( \frac{KH}{POP} \right)_{t-1}, \left( \frac{AA}{POP} \right)_{t-1}, \left( \frac{YD}{POP \times PH} \right), RMA_{t-1} \right) \quad (2)$$

$$IKF = KK - (1 - DELK) KK_{t-1} \quad (3)$$

$$IVF = V - V_{t-1} \quad (4)$$

where the variable  $KH$  is the housing stock,  $DELH$  is the depreciation rate of the housing stock,  $KK$  is the stock of capital,  $DELK$  is the physical depreciation rate of the stock of capital,  $IHH$  is the residential investment,  $RMA$  is the mortgage interest rate,  $V$  is the stock of inventory and  $IHH$ ,  $AA$ ,  $YD$  are defined as in the consumption equation.

### 2.1.1.3 Government Expenditure

There are two types of government expenditures in the model: federal and state government ( $COG$  and  $COS$ ) consumption and investment of goods purchased, and nonresidential fixed investment from the government sector ( $IKG$ ). All three of these variables are exogenously determined.



### 2.1.1.4 Net Exports

The exports ( $EX$ ) and export growth rates are exogenous in the Fair model. Imports ( $IM$ ) are determined endogenously as a function of consumption and investment spending, the domestic price level ( $PF$ ), the import price level ( $PIM$ ), and time-dependent dummy variables ( $D_i$ ). The imports function is specified in equation (5).

$$\begin{aligned}
 \text{Log} \left( \frac{IM}{POP} \right) &= f \left( \left( \frac{IM}{POP} \right)_{t-1}, \text{Log} \left( \frac{PF}{PIM} \right), D_i, \right. \\
 \left. \text{Log} \left[ \frac{(CS + CN + CD + IHH + IHB + IHF + IKH + IKB + IKF)}{POP} \right] \right) & \quad (5)
 \end{aligned}$$

### 2.1.2 GDP Determination in the Fair Model

The GDP is equal to consumption plus investment plus government spending plus exports minus imports ( $Y = \text{consumption} + \text{investment} + \text{government spending} + \text{net export}$ ). The Fair model includes the six sectors (households [ $h$ ], firms [ $b$ ], financials [ $f$ ], international [ $r$ ], federal government [ $g$ ], and local government [ $s$ ]) and more than one category of consumption, investment, and government spending; as a result the GDP has a more complex restatement. We define the Real Gross Domestic Product (GDPR) in the ECIO model as the sum of business production, production of the financial sector (capital consumption [ $CCB$ ], + before tax profits, [ $PIEB$ ]), and government sector production (federal civilian [ $JG \times HG$ ], and military [ $JM \times HM$ ], and state [ $JS \times HS$ ]) compensation of civilian and military employees).  $JG$ ,  $JM$ , and  $JS$  are the number of civilian, military, and state jobs respectively.  $HG$ ,  $HM$ , and  $HS$  are the average number of hours paid per civilian, military, and state job respectively. The resulting gross product equation ((6)) is shown below.

$$\begin{aligned}
 GPDR = & Y + PIEB + CCB + \\
 & PSI13 \times (JG \times HG + JM \times HM + JS \times HS) + STATP & \quad (6)
 \end{aligned}$$

And  $PSI13$  is the ratio of gross product of federal and state government to total employee hours in federal and state government.  $STATP$  is a statistical discrepancy pertaining to the use of the chain weighted data in the derivation of the variables.

### 2.1.3 Total Private Production and Total Employment

The production equation is based on the assumption that the firms in the private sector set their prices and know their sales ( $X$ ) for the current period, and the firms will select what and how much to produce for the period. In equation (7),  $V$  is inventory stock and  $D_i$  is a time dummy variable.

$$\text{Log}(Y) = f(Y_{t-1}, \text{Log}(X), \text{Log}(V)_{t-1}, D_i) \tag{7}$$

Total employment is the sum of employment in the private sector ( $JF$ ), public civilian employment in the federal ( $JG$ ) and state ( $JS$ ) governments, and military employment ( $JM$ ) less moonlighters ( $LM$ , persons holding more than one job) as shown in Equation (8).

$$E = JF + JG + JM + JS - LM \tag{8}$$

The variables  $JG$  and  $JM$  in Equation (8) are exogenously determined, and the variables  $JF$  and  $LM$  are endogenously determined in the ECIO model. Employment in the private sector ( $JF$ ) is determined by the total production of the private sector ( $Y$ ), initial employment, the ratio of the actual number of workers on hand at the end of the previous period ( $JF$ ) to the minimum number required to produce the output of that period ( $JHMIN$ ), given an estimate of the desired number of hours worked per worker in the previous period ( $HFS$ ).

$$\Delta \text{Log}(JF) = f \left( \text{Log} \left[ \frac{JF}{\left( \frac{JHMIN}{HFS} \right)} \right]_{t-1}, \Delta \text{Log}(JF)_{t-1}, \Delta \text{Log}(Y) \right) \tag{9}$$

The supply of labor from the household sector is determined by four equations that explain the labor force participation rate for four groups ( $L_i$ ) in the labor force: labor force-men 25-54 ( $L_1$ ), labor force-women 25-54 ( $L_2$ ), labor force-all others 16+ ( $L_3$ ), and the number of moonlighters ( $LM$ ). The key variables that explain the labor force are the unemployment rate and the level of total net wealth ( $AA$ ).

Unemployment ( $U$ ) is explained as the difference between total labor force and number of people employed.

$$U = L_1 + L_2 + L_3 - E \tag{10}$$

$$\text{Log} \left( \frac{L_i}{POP_i} \right) = f \left( \text{Log} \left( \frac{L_i}{POP_i} \right)_{t-1}, \text{Log} \left( \frac{AA}{POP} \right)_{t-1}, UR \right) \tag{11}$$

Total non-institutional population ( $POP$ ) over age 16 and above is the sum of non-institutional population of men ( $POP_1$ ) and women ( $POP_2$ ) 25-54 years of age and all others above 16 ( $POP_3$ ).

## 2.2 Industrial Output Module

The industrial output module takes the final demand projections from the macroeconomic econometric module as inputs to provide projections of sectoral output for 32 sectors of the U.S. economy. There are five energy and 27 non-energy sectors within the industry classification scheme. Table 2.2 displays the aggregation scheme used within the industrial output module.

The key objective of the industrial output module is to create a projection of the industrial output by taking into account the penetration of new energy technologies, changes in the IO coefficients, and changes in the final demands of the economy over time. It also provides an accounting framework that ensures the supply side of the economy is consistent with the demand side.

The general starting point of the industrial output module is the standard input-output model equations shown in ((12)) and ((13)).

$$X = AX + F \tag{12}$$

$$F = B * FD \tag{13}$$

where  $A$  is the matrix of direct coefficients that represents the amounts of inputs required from sector  $i$  per unit of output of sector  $j$ ,  $X$  is a vector ( $n \times 1$ ) of industry

Code	Sector Name	NAICS Codes	Code	Sector Name	NAICS Codes
IND_01	Agriculture, forestry, fishing, and hunting	11	IND_17	Wholesale trade	42
IND_02	Oil and gas extraction	211, 21311	IND_18	Retail trade	441-448, 451-454
IND_03	Coal mining	2121	IND_19	Air, rail and water transportation	481-483
IND_04	Mining, except coal, oil and gas	212X	IND_20	Truck transportation	484
IND_05	Support activities for mining	213X	IND_21	Pipeline transportation	486
IND_06	Electric power generation and distribution	2211, 491	IND_22	Transit and sightseeing transportation and transportation support services	485
IND_07	Natural gas distribution	2212	IND_23	Warehousing and storage	493
IND_08	Water, sewage and other systems	2213	IND_24	Information	51
IND_09	Construction	23	IND_25	Finance, insurance, real estate, rental, and leasing	52-53
IND_10	Primary and fabricated metals	331X, 332X	IND_26	Professional, scientific, and technical services	54
IND_11	Machinery	333X	IND_27	Management of companies and enterprises	55
IND_12	Motor vehicles and other transportation equipment	336X	IND_28	Administrative and support and waste management and remediation services	56
IND_13	Other durable manufacturing	321X, 327X, 334X, 335X, 337X, 339X	IND_29	Educational services, health care and social assistance	6
IND_14	Other nondurable manufacturing	311X, 321X, 314X, 315X, 316X, 322X, 323X	IND_30	Arts, entertainment, recreation, accommodation, and food services	7
IND_15	Petroleum and coal products	324X	IND_31	Other Services (except public administration)	8
IND_16	Chemical, plastics and rubber products	325X, 326X	IND_32	Government and non-NAICS	92

gross outputs, and  $F$  is a vector ( $n \times 1$ ) of industry final demands.  $B$  is the bridge matrix from the BEA benchmark IO tables, and  $FD$  is the estimated aggregate final demand from the macroeconomic econometric module.

Thus, for a given direct coefficient matrix, it is possible to solve the set of simultaneous equations to find the new sector production levels,  $X$ , that are required to satisfy a potential or actual change in the levels of sector final demands  $F$ . By rearranging and converting to differences, this equation can be rewritten as in equation (14):

$$\Delta X = (I - A)^{-1} \Delta F \quad (14)$$

where  $(I - A)^{-1}$ , also defined as the Leontief inverse matrix, describes the direct and indirect changes in the output of each sector in response to a change in the final demand of each sector. Also,  $\Delta F$  includes any element of final demand expenditure from all the sectors considered in the model. The relationships included in the Leontief inverse matrix will result in the gross output or total demand for each sector; these results are used to obtain employment demand. This standard solution is static and does not account for changes in economic structure over time – the interdependence of industries will be the same over the projection. This structural change is modeled using a formulation for integrated input-output econometric models developed by Conway (1990).

The procedure to project the gross output or total demand for each sector follows the next steps. First, we compile a series of historical final demand for the industrial sectors in the model. Next, we calculate the forecasts of the industrial output ( $Z$ ); to calculate these forecasts we pre-multiply historical final demand by the Leontief inverse for the model base year. With the exception of the base year, in which the interindustry and final demand structures are contemporaneous, the predicted output will differ from observed actual output ( $X$ ) as a result of structural change. To correct for these differences in the interindustry structure, we regress the actual output by industry on expected industry output. The regression follows the equation (15):

$$X = f(Z, M) \quad (15)$$

where  $M$ , will include a set of related independent variables that help explain the change in IO coefficients for each sector. The regressions will display that the actual

output grows more slowly or quickly than the predicted output; furthermore, as the model forecasts progresses, the output response to forecasts in the final demand for each industry needs to be adjusted by this relationship between actual and predicted output.

## 2.3 Industrial Employment Module

The industrial employment module uses the industrial output ( $X_i$ ) projections from the industrial output module, average national wage rates ( $AW^{US}$ ), industrial labor productivity ( $INDP_i$ ) trends, and initial industrial employment ( $INDE_i$ ) to project employment for the 32 industries.

$$\text{Log}(INDE_i) = f(AW^{US}, \text{Log}(INDP_i), X_i, \text{Log}(INDE_{i,t-1})) \quad (16)$$

Total industrial employment ( $INDE$ ) at time  $t$  is the sum of all employed persons in all the sectors at time  $t$ .

$$INDE_t = \sum INDE_i \quad (17)$$

The trends of the average wage rates and the productivities by industry are forecasted econometrically. Output forecasts by industry combined with labor productivities determine employment by industry, and we use the employment by industry with wage rate estimates to calculate the production labor income portion of household income. This total labor income together with non-labor income is used to update disposable income ( $YD$ ), which is a major driver of consumption expenditure in the macroeconomic econometric module.

## 2.4 Notable Model Capabilities

### 2.4.1 Power Sector Updates

The ECIO model is able to adapt to and include updates to the aggregated electricity generation, transmission, and distribution sector (IND\_06) based on a specific scenario, such as those that might come from sources like NEMS. Specifically, annual Use tables within the Industrial Output Module can be updated to reflect scenario specific information on how the technology mix within the electricity generation sector changes over the forecast period.

### 2.4.2 Price Flexibility

The model allows taking into account the changes in energy sector prices. Normally, these energy sector price changes are specified exogenously and used as inputs to the ECIO model. The changes in energy prices will affect the production costs of other industries and will result in changes in production costs in all industries. These direct and indirect price effects are incorporated in the ECIO model using a variant of the input-output price model. Although prices of all goods change, there is no input substitution behavior modeled.

## 3 Integration Mechanisms

There are two primary mechanisms that allow integration of the interindustry and the econometric subsystems. The first one relies on the IO requirements solution on econometric estimates; and the second one uses income estimates from the econometric time series forecasts and output estimates.

The first of these is the reliance of the interindustry output requirements solution on econometric final demand estimates. Final demand totals by activity (e.g., consumption, government expenditures, investment expenditures) are transformed using commodity final demand distribution data from the national input-output accounts.

The second integrating mechanism focuses on income estimation. Because income is the source of consumption expenditures and because consumption expenditures are the dominant driving force and determinant of overall economic activity (i.e., GDP), income provides a powerful location for integrating the two model systems. Econometric time series provide the basis for forecast labor and non-labor income estimates, and production based output estimates coupled with productivities and wage rates provide a second source for labor income estimates. The model uses a variable weighting parameter in which full or partial weight can be accorded to either the IO or econometric labor income estimate. By weighting the two equally, the econometric and IO sub-systems exert equal influence on the initial impacts solutions for each year. Because each model year is solved iteratively, the final weighting of econometric and IO labor income estimates may deviate from the equal weighting, but the initial solution starting point for each year allows equal influence of econometric and input-output estimates.

After the weighted combinations of non-labor income estimates are added to econometrically estimated non-labor income, the total is converted to disposable income, which in turn drives consumption.

## References

- Conway, R. S. (1990). The washington projection and simulation model: A regional interindustry econometric model. *International Regional Science Review*, 13(1-2):141–165.
- Fair, R. C. (2004). *Estimating How the Macroeconomy Works*. Harvard University Press.