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Technical Document for Price Adjustment

Zheng Tian

Mulugeta Kahsai
mkahsai@vsu.edu

Randall Jackson
West Virginia University, randall.jackson@mail.wvu.edu

Regional Research Institute
West Virginia University

Technical Document Series

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Zheng Tian, assistant professor, International School of Economics and Management, Capital University of Economics and Business, Mulugeta Kahsai, Assistant Professor, Department of Technology, Virginia State University, and Randall Jackson

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Zheng Tian[*], Mulugeta Kahsai[†], and Randall Jackson[‡]

Abstract

This document presents the basis for the price adjustment mechanisms in a time series IO model. The essentials of the price adjustment and price change propagation algorithms are presented, along with a matrix permutation algorithm that facilitates the implementation of the price adjustment mechanism. The Matlab function is provided.

1. Theoretical Background

In a time series IO model, the direct requirement matrices (A_t) display various scenarios that are constructed based on the 2008 base year direct requirement matrix by updating only the coefficients in the A matrices without information for nominal transaction values. Although this approach is convenient in computation, this way of updating the A matrix omits other information, such as direct exogenous price changes and the indirect price changes that propagate from them. This document describes how to adjust the coefficients matrices to reflect the exogenously specified changes in price indices.

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[*]Assistant Professor, International School of Economics and Management, Capital University of Economics and Business, 33 Fanyang Rd, Fengtai, Beijing, China, tianzheng@cueb.edu.cn. Zheng Tian was Graduate Research Assistant in RRI at the time the document was first developed.

[†]Assistant Professor, Department of Technology, Virginia State University. Engineering, 200B, 1 Hayden Drive, Petersburg, VA 23806, mkahsai@vsu.edu. Mulugeta Kahsai was Postdoctoral Fellow in RRI at the time the document was first developed.

[‡]Professor, Department of Geology and Geography, Director, Regional Research Institute, 886 Chestnut Ridge Road, P.O. Box 6825, Morgantown, WV 26506-6825, 304.293.8734, randall.jackson@mail.wvu.edu

The theoretical foundation of price adjustment is the price model in the Input-output method, which can be summarized by the following equations

$$p = A'p + v \tag{1}$$

$$(I - A')p = v \tag{2}$$

$$p = (I - A')^{-1}v \tag{3}$$

where p is the vector of price indices of each industry, and v is the vector of ratios of value added to total outputs of each industry. Once the vector of price indices is calculated, the direct requirement matrix can be adjusted by $\hat{p}A\hat{p}^{-1}$ where $\hat{p}$ is the diagonalized matrix of p.

In cases where price indices are unknown except for a few industries, the price model can be applied to calculate these unknown price indices by solving equation (1) in terms of the known price indices. Suppose, for instance, price indices of the energy sectors are known, denoted by p_e, and price indices of the non-energy sectors are unknown, denoted by p_n, using the block matrix notation, equation (1) can be rewritten as follows

$$\begin{pmatrix} p_e \\ p_n \end{pmatrix} = \begin{pmatrix} A'_{ee} & A'_{en} \\ A'_{ne} & A'_{nn} \end{pmatrix} \begin{pmatrix} p_e \\ p_n \end{pmatrix} + \begin{pmatrix} v_e \\ v_n \end{pmatrix} \tag{4}$$

From equation (4), solving p_n in terms of p_e, we get

$$p_n = (I - A'_{nn})^{-1}A'_{ne}p_e + (I - A'_{nn})^{-1}v_n \tag{5}$$

Equations (4) and (5) can also be thought of as the expressions for the change in price indices, that is, the change in price indices of non-energy sectors can be expressed as the function of the change in price indices of energy sectors and the change in the coefficients of value added of non-energy sectors. Suppose the coefficients of value added of non-energy sectors are constant over time, that it, $\Delta v_n = 0$, the change in price indices of non-energy sectors is simply

$$\Delta p_n = (I - A'_{nn})^{-1}A'_{ne}\Delta p_e \tag{6}$$

As such, we can ascertain price indices of non-energy sectors from the information on price indices of energy sectors. After obtaining all price indices, the direct requirement matrix can be adjusted by $\hat{p}A\hat{p}^{-1}$.[1]

Supporting Algorithm(s)/Code

2. The Practical Steps of Price Adjustment

This section explains the steps for using the data files "Direct mat", which is the direct requirement matrices, and "Prices.mat", which is the price indices of known sectors, to write the Matlab code for price adjustment of the direct requirement matrix contained in the m-file "priceAdj.m".

(1) Permutation of Rows and Columns of the A matrix

In the direct requirement matrix being used, the sectors whose prices are known are included in the 3rd and 6th rows and columns of the A matrix. To form a block matrix in equation (4), use the permutation matrix to move rows and columns 3 and 6 to rows and columns 1 and 2. The permutation matrix can perform this task. Suppose there is a 3×3 matrix, A, in which the 2nd row and column need to be moved to the first ones. Then, the permutation matrix for doing so is $D = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, and pre-multiplying A by D changes the sequence of row and post-multiplying A by the transpose of D changes the column, that is $A^* = DAD'$. Since D is nonsingular, to transform back to the original matrix, it is simply done by $A = D^{-1}A^*D'^{-1}$.

(2) Adjust the A matrix using price indices of known sectors

After permuting the original A matrix, the steps for price adjustment are strictly following the theory above. First, define the block matrix in equation (4); second, calculate the price indices of sectors whose prices are not

[1] Based on a similar presentation in Bazzazan and Batey (2003)

known according to equation (6); and finally, use price indices of all sectors to adjust the A matrix by $\hat{p}A\hat{p}^{-1}$.

The Matlab code for price adjustment of one scenario is as follows (the code applies for any number of known sectors and other similar scenarios),

```
function Anew = PriceAjd(A, pe,sec )
% PURPOSE: General function to develop the price adjustment of the direct
% coefficient matrix.
%
%
% This program has been developed to adjust the matrix A, to price changes
% using partial indexes from the sectors prices are known
%
%
% ―――――――――――――――――――――――――――――――――――――――――――――――――――
%  USAGE: results = PriceAdj(U,V)
%  where: A = Direct requirements matrix
%         pe = Prices known
%         sec = Sectors where prices are known
%
%
% ―――――――――――――――――――――――――――――――――――――――――――――――――――
%  RETURNS: a matrix
%         Anew  = price adjusted direct requirements matrix
%  ――――――――――――――――――――――――――――――――――――――――――――――――――
%
% REFERENCES: 1) Bazzazan, F. and Batey, P. (2003). The development and empirical
% testing of extended input—output price models. Economic Systems Research,
% 15(1):69 8 6.
%
%  Function created by J. T. Sayago—Gomez (Summer 2014) to apply the price adjustmen
% to the Direct requirements matrix.
%
% ―――――――――――――――――――――――――――――――――――――――――――――――――――
%
% Calculates the outputs and inputs. Uses the size of the Use and Make matrixes to
% create the D and B matrixes
%
% With D and B calculate A after correcting for scrap
lpe=length(pe);
if length(pe)==length(sec)
```

```
disp('Number of sectors indexes and price list match');
else
errordlg('Number of sectors indexes and price list do not match') ;
end
sA=size(A);
if size(A) == size(A')
disp('Number of sectors in matrix A are the same');
else
errordlg('Number of sectors in matrix A are not the same') ;
end
lA=length(A);
sectors = 1:lA;
nosec = sectors';
nosec(sec)=[];
reorder = [sec;nosec];
I = eye(lA);
D = I(reorder,:);
A1 = D*A*D';


lpe1=lpe+1;

% define the block matrices of A: e--energy sectors, n--non-energy sectors
Aee = A1(1:lpe, 1:lpe);  Ane = A1(1:lpe, lpe1:lA);
Aen = A1(lpe1:lA, 1:lpe); Ann = A1(lpe1:lA, lpe1:lA);


    dpe = (pe-1)';
    dpn = (eye(size(Ann)) - Ann')\(Ane'*dpe'); % solving the change of the unknown p
    p = [dpe'; dpn] + 1;
    Anew = diag(p)*A1/diag(p);
    Anew = D\Anew/D'; % change back to the original order


end
```

References

Bazzazan, F. and Batey, P. (2003). The development and empirical testing of extended input-output price models. *Economic Systems Research*, 15(1):69–86.