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ECIO Input-Output Relationships Generating Expected Output Series

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ECIO Input-Output Relationships

Abstract

Generating *expected* output series is a critically important step in building Conway-type hybrid econometric input-output models. Because the RRI most often uses a modeling method that takes advantage of Jackson's Rest-of-World industry mechanism for representing imports in the Supply (Make) table rather than in final demand, we must insure that this method for generating historical expected output series will be consistent with the way in which future expected output is estimated (Jackson, 1998). This brief TechDoc lays out the relevant mathematical foundations. In so doing, we also identify the appropriate corresponding impacts assessment formulation.

We begin with the conventional commodity-by-industry input-output accounting framework, adapted from United Nations (1968) as shown below.

Figure 1: The Commodity-by-Industry Framework

	Commodity	Industries	Final Demand	Totals
Commodity		U	e	q
Industries	V			g
Primary Inputs		va		
Totals	q'	g'		

In matrix notation, we have the following identities:

$$U\mathbf{i} + e \equiv q \tag{1}$$

$$V\mathbf{i} \equiv g \tag{2}$$

$$V'\mathbf{i} \equiv q \tag{3}$$

where U, V, g, q, and e are the Use and Make matrices, industry and commodity output, and a final demand vector, respectively. $\mathbf{i}$ is a summing vector, and $'$ signifies the transpose operation. We define behavioral relationships as follows:

$$B = U\hat{g}^{-1} \tag{4}$$
$$U = B\hat{g} \tag{5}$$
$$D = V\hat{q}^{-1} \tag{6}$$
$$V = D\hat{q} \tag{7}$$

where $\hat{}$ indicates vector diagonalization. Equation 4 defines the production requirements of commodities per industry output dollar, and equation 6 is a statement of the industry-based technology assumption that commodities are produced by industries in fixed proportions.[1] Note that the effect of pre-multiplication of a commodity vector or matrix by D results in a transformation from commodity-space to industry-space, so $V\mathbf{i} = g = Dq$. This system allows us to formulate the following:

$$q = Bg + e \tag{8}$$
$$q = BDq + e \tag{9}$$
$$q = (I - BD)^{-1}e \tag{10}$$
$$\tag{11}$$

Similarly, premultiplying Equation (8) by D yields

$$Dq = DBg + De \tag{12}$$
$$g = DBg + De \tag{13}$$
$$g = (I - DB)^{-1}De \tag{14}$$
$$\tag{15}$$

[1]The alternative is the commodity-based technology assumption, which while not used here, could be developed in parallel fashion.

These relationships have been presented in terms that do not address the openness of an economy. Analysts who use IO for impact assessment need to reformulate the technical requirements matrix to accommodate trade with the rest of the world. In the process, technical coefficients, a_{ij} are bifurcated such that they become equal to the sum of the domestic input per dollar output coefficient, r_{ij}, and the import coefficient, m_{ij}, or $a_{ij} = r_{ij} + m_{ij}$. This bifurcating procedure is commonly called *regionalization* when its goal is the parameterization of a subnational region, but the approach we use can be implemented in similar fashion with national "regions."

Jackson's 1998 method is one of several available to IO analysts. We use it in our research because it offers a number of advantages in terms of transparency in exposition, formal representation and algebraic manipulation. The core of the approach lies in an alternative to Make matrix standardization. In place of total domestic commodity production, we standardize by total commodity supply available to the region, whether it is a nation or a subnational region under study. The effect is analagous to other "rows-only" adjustment methods, where each row is multiplied by a value between zero and 1.0 that reflects the regional supply percentage – the proportion of commodity used by the column entity that is supplied locally, i.e., produced within the region.

Let $\tilde{D}$ denote the Make matrix standardized by $s = q + m$, or $\tilde{D} = V\hat{s}^{-1}$. In Equation (1), commodity final demands were presented as a summary vector, e. Let F denote a commodity-by-activity final demand matrix, such that Equation (1) can be rewritten as Equation (16).

$$U\mathbf{i} + F\mathbf{i} \equiv q \tag{16}$$

Here, U is the Use matrix as before, but F is final demand including a column of negative imports values, $-m$. Note that the imports values in m are positive, but that $-m$ is included in the final demand matrix by convention. Next, we denote the final demand matrix without the imports column as $\tilde{F}$, so that

$$U\mathbf{i} + \tilde{F}\mathbf{i} - m = q \tag{17}$$

Adding m to both sides gives us

$$U\mathbf{i} + \tilde{F}\mathbf{i} = q + m \tag{18}$$

To generate an estimate of industry output required to satisfy a specified distribution final demands by commodity, begin by premultiplying both sides of Equation (18) by $\tilde{D}$, or

$$\tilde{D}U\mathbf{i} + \tilde{D}\tilde{F}\mathbf{i} = \tilde{D}(q + m) \tag{19}$$

Now, by substitution,

$$\tilde{D}(q + m) = \tilde{D}s = V\hat{s}^{-1}s = V\mathbf{i} \equiv g \tag{20}$$

We can also express the relationship between Z, $\tilde{D}$, and U as

$$\tilde{D}U = Z \tag{21}$$
$$\tilde{D}U\hat{g}^{-1} = \tilde{D}B = R \tag{22}$$

where Z is a matrix of *intra*regional interindustry transactions, equivalently defined as $Z = R\hat{g}$, and $Z\mathbf{i} = Rg$, where R is the matrix of trade coefficients discussed earlier. Thus, we can rewrite Equation (20) as

$$Z\mathbf{i} + \tilde{D}\tilde{F}\mathbf{i} = g \tag{23}$$

By substitution and simplification, then

$$Rg + \tilde{D}\tilde{F}\mathbf{i} = g, \tag{24}$$
$$g - Rg = \tilde{D}\tilde{F}\mathbf{i}, \tag{25}$$
$$(I - R)g = \tilde{D}\tilde{F}\mathbf{i}, \tag{26}$$

Because R also can be

so

$$(I - \tilde{D}B)^{-1}\tilde{D}\tilde{F}\mathbf{i} = g = (I - R)^{-1}\tilde{D}\tilde{F}\mathbf{i} \tag{27}$$

or if needed, in change (impact) notation,

$$\Delta g = (I - R)^{-1}\tilde{D}\Delta\tilde{F}\mathbf{i} \tag{28}$$

Finally, we show the form for use in forecasting a time series of *expected* output as used in the construction of hybrid econometric IO models (e.g., Conway (1990)), where the objective is identifying a time series of what would be the output requirements to satisfy a time series of final demand vectors given a fixed interindustry and trade structure. The result is shown below.

$$g_t = (I - R)^{-1} \tilde{D}_t \tilde{F}_t \mathbf{i} \tag{29}$$

Equation (29) is used in the ECIO model data preparation procedure and in each forecast year.

References

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