

Spring 2020

Topology

WVU Mathematics Department

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NAME (print): _____

Topology Ph.D. Entrance Exam, April 2020

Solve **four** from the following five exercises. Write a solution of each exercise on a separate page. After you finish, you will need to take pictures of your work and sent it to the examiners according to the separate instructions.

In what follows the symbols $\text{int}(A)$ and $\text{cl}(A)$ stand, respectively, for the interior and the closure of A . Any subset of \mathbb{R} is considered with the standard topology, unless stated otherwise.

Ex. 1. Let A be a connected subset of a topological space X and let $A \subset B \subset \text{cl}(A)$. Show that B is a connected subset of X .

Ex. 2. Let X be an arbitrary set, $\langle Y, \tau \rangle$ a topological space, and $f: X \rightarrow Y$ an arbitrary function. Show that

1. $\mathcal{T}_0 = \{f^{-1}(V): V \in \tau\}$ is a topology on X .
2. If \mathcal{T} is any topology on X such that $f: \langle X, \mathcal{T} \rangle \rightarrow \langle Y, \tau \rangle$ is continuous, then $\mathcal{T}_0 \subseteq \mathcal{T}$.

Ex. 3. Let $\langle X, \rho \rangle$ be a compact metric space.

Show that every sequence in X contains a subsequence converging to an $x \in X$.

Ex. 4. Determine which of the following families of subsets of \mathbb{R} form a basis for some topology on \mathbb{R} . Briefly justify your answers.

- (1) $\mathcal{B}_1 = \{(-\infty, b]: b \in (-\infty, 0]\}$
- (2) $\mathcal{B}_2 = \{(-\infty, b]: b \in \mathbb{R}\}$
- (3) $\mathcal{B}_3 = \{(-\infty, b]: b \in \mathbb{R}\} \cup \{(a, \infty): a \in \mathbb{R}\}$
- (4) $\mathcal{B}_4 = \{(-\infty, b] \cap (a, \infty): a, b \in \mathbb{R}\}$

Ex. 5. For a topological space a family \mathcal{F} of subsets of X (not necessarily open) is a *network* for X provided that for every open subset U of X and $x \in U$ there exists an $F \in \mathcal{F}$ such that $x \in F \subset U$.

Let X be a regular topological space. Prove that if \mathcal{F} is a network for X then so is the family $\mathcal{G} = \{\text{cl}(F): F \in \mathcal{F}\}$, where $\text{cl}(F)$ is the closure of F .