

Fall 2020

## Differential Equations

WVU Mathematics Department

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# ODE ENTRANCE EXAM, FALL 2020

September 16 2020

Solve all six problems. Show all work. Explain and justify your answers. All problems carry equal weight.

Notation:  $y' = \frac{dy}{dt}$

Name \_\_\_\_\_

Total Score \_\_\_\_\_

1. Let  $m, n \in \mathbb{N}$  and consider the differential system

$$x' = my^{2m-1}, \quad y' = -nx^{2n-1}. \quad (1)$$

( $x, y$  are scalar functions of  $t$ ). Show that  $(0, 0)$  is a stable equilibrium of (1).

2. Let  $f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be defined by

$$f(y) = \begin{bmatrix} \cos(y_1 y_2 y_3) \\ \sin(y_1 + y_2 + y_3) \\ \exp\left(\frac{2y_1 y_2}{1 + y_1^2 + y_2^2}\right) \end{bmatrix}$$

for all  $y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \in \mathbb{R}^3$ . Show that for the differential system

$$y' = f(y)$$

- a) there are no critical points;
- b) all solutions exist on  $(-\infty, \infty)$ .

3. Let  $n > 1$  be a natural number.

- a) Let  $A$  be an  $n \times n$  constant matrix. Prove that  $Y(t) = \exp(tA)$  is a solution to the matrix differential equation  $Y' = AY$ .
- b) Let  $A(t)$  be an  $n \times n$  time dependent matrix function of  $t$  on  $(-\infty, \infty)$ . Must  $Y(t) = \exp(tA(t))$  be a solution of  $Y' = A(t)Y$ ? Explain.

4. Solve the differential equation

$$y' = t^2 y - 3t^2$$

5. Consider the ODE system

$$x' = y^3 - 4x, \quad y' = y^3 - y - 3x.$$

- a) Find all critical points and classify their stability.
  - b) Show that the line  $x = y$  is invariant (any solution that starts on it stays on it).
  - c) If  $(x(0), y(0)) = (-1, 1)$ , find  $x(t) - y(t)$ .
6. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a differentiable function such that  $f'(t) > f(t)$  for all  $t \in \mathbb{R}$ , and  $f(0) = 0$ . Show that  $f(t) > 0$  for any  $t > 0$ .