

Spring 2022

Analysis

WVU Mathematics Department

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Analysis exam Spring 2022

4/28/2022

1. (1) Write down the definition of the measure and the measure space.
(2) Suppose μ and ν are measures on a measurable space (X, \mathcal{S}) . Prove that $\mu + \nu$ is a measure on (X, \mathcal{S}) .

2. Suppose

$$f_k(x) = \frac{(1-x)^k \cos \frac{k}{x}}{\sqrt{x}}.$$

Prove that

$$\lim_{k \rightarrow \infty} \int_0^1 f_k = 0.$$

3. Suppose \mathcal{S} is the smallest σ -algebra on \mathbb{R} containing $\{(r, n] : r \in \mathbb{Q}, n \in \mathbb{Z}\}$. Prove that \mathcal{S} is the collection of Borel subsets of \mathbb{R} .

4. Suppose $f \in L^1(\mathbb{R})$. Prove that

$$\lim_{t \rightarrow 0^+} \frac{1}{2t} \int_{b-t}^{b+t} |f - f_{[b-t, b+t]}| = 0$$

for almost every $b \in \mathbb{R}$. Here $f_I = \frac{1}{|I|} \int_I f$ is the the mean of the function f on the interval I .

5. Show that $\sqrt{4 + \sqrt{2}}$ is not a rational number.

6. If a sequence $\{a_n\}$ satisfies

$$|a_{n+1} - a_n| < \frac{1}{n^2}.$$

Show that it is a Cauchy sequence.

7. Determine whether the series $\sum [\cos(n\pi/7)]^n$ converge or not. Justify your answers.

8. Prove that $f(x) = 1/x$ is continuous in $(0, 1)$ but not uniformly continuous.