Credit Cards, Credit Utilization, and Consumption

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Credit Cards, Credit Utilization, and Consumption

Scott L. Fulford† and Scott Schuh‡

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Abstract

We use comprehensive U.S. credit bureau data to document stable consumer utilization of credit card debt over the business cycle, life-cycle, and individually quarter to quarter, despite massive variation in available credit. To explain these facts, we propose a model of life-cycle consumption with heterogeneous discounting and credit cards used for payments and consumption smoothing. Using diary data to identify payment use, the estimated model matches consumption and credit use at every frequency and suggests that around half the population has an endogenously high marginal propensity to consume. The results suggest consumer credit availability and heterogeneity of use could be important for counter-cyclical policy.

Keywords: Credit cards; life cycle; consumption; saving; precaution; buffer-stock; payments
JEL Codes: D14, D15, E21, E27

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1 Introduction

A striking feature of the 2008 financial crisis and Great Recession was a remarkable stability in the average utilization of unsecured revolving credit. Using comprehensive U.S. credit bureau data, Figure 1 shows that the average credit card limit fell by about 40 percent from September 2008 to March 2010, nearly a trillion dollars in aggregate. During this same time period, however, Americans also reduced their aggregate credit card debt by a similar percentage. Consequently, average credit utilization was nearly constant during that tumultuous time, as it was over the longer period from 2000–2015. In aggregate, this debt reduction was approximately double the value of the tax rebates from the Economic Stimulus Act (Parker et al. 2013); individually, average debt fell more than $1,000 dollars per cardholder.

We show that remarkably stable individual credit utilization explains the aggregate stability. Beneath the dramatic cyclical changes in credit and debt, even larger changes occur over the life cycle of individuals: average credit card limits increase more than 700 percent from ages 20–40 and continue to increase after age 40, albeit more slowly (see Figure 2). Because many households hold little or no liquid assets, increases in credit are one of the largest sources of “savings” early in life. These massive increases in credit with age are matched by increases in debt at almost the same rate, so average credit utilization is stable over the life-cycle, declining very slowly with age. Consistent with prior work (Gross and Souleles 2002, Agarwal et al. 2017, Fulford and Schuh 2015, Aydin 2015), we find that increases in credit at the individual level pass through to increases in debt and individuals close to their limits are more sensitive to changes. Our new finding is that utilization is largely fixed at the individual level with shocks dying off quickly so that individual credit utilization is extremely persistent despite credit volatility that is several times greater than income volatility (Fulford 2015). Yet this individually persistent utilization is highly heterogeneous; a large portion of the population is using most of its available credit well into retirement, even as another large portion uses almost none.

Existing models of consumption and saving are not well-suited to explain this individually stable yet heterogeneous utilization of credit. Models of precautionary savings, for example, imply
individuals should move around the savings-debt distribution as they absorb shocks, rather than display persistent and stable utilization. Life-cycle models, whether incorporating precautionary motives or not, typically imply an accumulation of resources for late in life, rather than a large portion of the population borrowing even into retirement age as we document. In addition, no existing models incorporate both the payments use of credit cards and the longer-term revolving credit use. Yet 90 percent of the population with a credit card uses it for payments in a given month and half borrow from month to month, so incorporating both uses is necessary to understand credit card behavior.

To explain the expanded facts about credit card borrowing, we build and estimate a new model that allows for preference heterogeneity, payment choice, and credit cards used for long-term borrowing in a life-cycle model with uninsured shocks. The econometric estimates reveal a clear and significant distinction between two types of consumers. About half the population must have a high discount rate (about 11 percent) and low relative risk aversion to be willing to hold the amount of credit card debt observed. This impatient population has a high marginal propensity to consume, so increases in credit lead directly to increases in debt and a stable—but high—utilization. In contrast, the rest of the population has a “standard” discount rate (about 4 percent) and relative risk aversion and uses their credit cards only for payments. They have low and stable utilization because both their credit limits and their expenditure on credit cards are tied to their income. The econometric results also offer the first estimates of the direct utility value of credit cards as a means of payment. In the model, consumers endogenously decide how much of current consumption to pay for with a credit card. We estimate that consumers would be willing to pay about 0.3 percent of their consumption (around $40 billion a year) to continue using credit cards given the current structure of U.S. credit card payment networks, interchange fees, rewards, and prices.

Identification of the model’s parameters is driven by the heterogeneous uses of credit cards. About half of credit card holders are revolvers who exercise the option to roll over debt indefinitely at 14 percent or higher interest and thus must be discounting the future around the rate of borrowing. Yet because some consumers do not borrow except for payments use on their credit
cards and have liquid savings, there must be a large patient population. We estimate the value of credit cards for payments using new micro data from the Diary of Consumer Payment Choice (DCPC). Credit card purchases are less valuable for revolvers because interest on purchases accumulates immediately, while payments users benefit from a free float for a month. Under simple assumptions, we show that the different payment use of revolvers and non-revolvers identifies the value they put on using credit cards for payments.

Our model is constructed for feasible estimation of all model parameters, rather than calibration, using a rich array of data available to better inform our understanding of heterogeneous consumer choices. We start from the framework of Gourinchas and Parker (2002) and Cagetti (2003), which captures life-cycle variation in income with uninsured income shocks, and add a high-interest liquid borrowing option following Laibson et al. (2003). To this underlying approach, we add a new tractable endogenous consumer payment choice between liquid assets (“cash”) and credit card liabilities. Explicit treatment of payments for expenditures enables the model to distinguish between two distinct uses of credit card debt: 1) convenience use, where consumers pay off all debt each month and incur zero interest; and 2) revolving use, where consumers exercise their option to roll over unpaid debt at high interest (around 14 percent on average). Second, to capture this heterogeneous use we allow our econometric application to include sub-populations with distinct preferences. Allowing this heterogeneity is crucial for capturing both the distinct uses of credit cards and the heterogeneous yet persistent utilization that we document. Third, to capture the unsecured nature of credit card debt, we introduce the ability to default (Livshits et al. 2007, Chatterjee et al. 2007, Athreya 2008), so the interest rate and credit limit faced by consumers is endogenous to past behavior. Finally, we build on Fulford (2015) to incorporate a tractable process for individually varying credit limits over the life-cycle.

The different consumption choices made by revolvers and convenience users is at the heart of our identification of heterogeneous preferences and is empirically necessary because credit bureau data does not distinguish explicitly between revolving and convenience debt. Our parsimonious payment approach captures many different reasons consumers might choose one payment means
over another including: non-pecuniary preferences and rewards (Koulayev et al. 2016, Wakamori and Welte 2017), an ordering of accounts by interest rate cost (Alvarez and Lippi 2017), and the costs of non-acceptance of the preferred method at the point of sale (Telyukova and Wright 2008, Telyukova 2013). Our work thus helps bridge a growing monetary and payment choice literature with the broader consumption literature.

We appear to be the first to study credit limits and utilization over the life cycle, although models with endogenous credit constraints (Lawrence 1995, Cocco et al. 2005, Lopes 2008, Athreya 2008) typically imply increasing credit limits with age as lenders gain more information. In contrast to our focus on the heterogeneity and stability of credit use, Laibson et al. (2003) seek to explain aggregate life-cycle accumulation of both credit card debt and illiquid saving by calibrating a model in which all agents have hyperbolic preferences. While we model the highly impatient consumers as fully time consistent, our results do not preclude other approaches such as hyperbolic discounting (Laibson et al. 2003, Meier and Sprenger 2010) that could explain this population as well or better. In any case, the importance of population level heterogeneity hearkens back to Campbell and Mankiw (1989, 1990), who explained aggregate income and consumption with two representative consumers with similar population shares, one living hand to mouth and the other saving for the future. Heterogeneous preferences also seem necessary to match wealth inequality (Krusell and Smith 1998), the average marginal propensity to consume (Carroll et al. 2017), persistent financial distress (Athreya et al. 2017), simultaneous holdings of liquid assets and credit card debt (Gorbachev and Luengo-Prado forthcoming), and experimentally elicited preferences (see, for example, Andreoni and Sprenger (2012)).

The estimated model explains smooth utilization at the micro and macro levels. In sample, it simultaneously fits the life-cycle paths of debt, consumption, and default. Existing consumption models do not incorporate the life-cycle changes in credit that we document and thus overlook.

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1Our structural model could be used to evaluate credit card use and consumer decision making at the level of daily individual transactions from linked-account data as in related work by Gelman et al. (2014), Baker (2018), and Kuchler and Pagel (2018) among others, although the specialized data to do so is not generally available. Furthermore, with expanded details on individual payment choices, our model can implement the integrated financial accounting framework proposed by Samphantharak et al. (2018) that measures exact cash flows by linking household balance sheets with income statements at the level of individual transactions.
an important form of early-life liquidity. Allowing for heterogeneous uses for credit suggests an explanation for the hump shape of life-cycle consumption (Attanasio et al. 1999) that is subtly different from the combination of precaution and life-cycle savings in the Gourinchas and Parker (2002) framework. Our estimates suggest the hump comes mainly from the average of two populations: one impatient enough that consumption largely follows income over the life-cycle, closely resembling the buffer-stock population in Carroll (1997), and the other patient population with flat or growing consumption. Incorporating increasing credit also reinforces the hypothesis that default must be substantially driven by shocks outside of the consumer’s control rather than strategic default (Livshits et al. 2007). Because credit limits are increasing over the life cycle, the incentive to voluntarily run up a large balance and default is increasing, while default after an expenditure shock is decreasing because credit is more available. Thus, if voluntary default is important, the frequency of default should be increasing over the life cycle rather than decreasing after age 30 as we observe in credit bureau data.

Out of sample, the simulated model can replicate important facets of consumption and credit use. It matches the qualitative smoothness of credit utilization observed during the financial crisis and Great Recession. At the micro-level, our simulations produce estimates of the individual relationship between credit and debt that closely match the reduced form estimates from the credit bureau data. Combining the estimated moments for payment choice and life-cycle consumption with out-of-sample predictions at the micro and macro levels, our estimated model thus fits observed data at every frequency: very short-term (payment choice), quarterly (consumption and savings), life-cycle (accumulation of assets and liabilities), and even the business cycle (aggregate changes during the Great Recession).

This close fit suggests our new facts about credit use, and the model we propose to explain them, may have implications for policy. Our simulated consumption response to a small unexpected cash rebate is about 23 percent within a quarter, very close to estimates based on tax rebates (Parker et al. 2013). Such a strong response is puzzling (Kaplan and Violante 2014), but in our approach it is driven by the impatient population, a result consistent with recent estimates of the
heterogeneity of response by Parker (2017). Yet because so much of the available liquidity of U.S. households comes from credit, the simulated consumption response to an unexpected increase in credit is nearly as large as a cash rebate. In total, the decline in available credit we document over 2008 and 2009 may have been responsible for one quarter of the fall in consumption during the Great Recession. Moreover, we show that the more that declines in credit are concentrated among the high utilization population—who are often the highest risk for banks—the larger the consumption response. Our approach thus further supports and adds micro-empirical support for the emphasis in Guerrieri and Lorenzoni (2017) on the relationship between consumer credit, precaution, and the macroeconomy. Policy makers may benefit from considering the importance of consumer credit supply and the heterogeneity of its use as a complement to existing conventional counter-cyclical policies.

2 Credit card use

This section briefly introduces our main data sources and presents empirical results on credit card use. Fulford and Schuh (2015) provide additional descriptive statistics, including additional evidence on the distribution of credit and on credit card holding by age. Our main data source is the Equifax/Federal Reserve Bank of New York Consumer Credit Panel (CCP) which contains a quarterly 5 percent sample of all accounts reported to the credit-reporting agency Equifax starting in 1999. The data set contains a complete picture of the debt of any individual that is reported to the credit agency: all credit-cards, auto, mortgage, and student-loan debt, as well as some other, smaller categories. While the CCP gives a detailed panel on credit and debt, its coverage of other variables is extremely limited. It contains birth year and geography, but not income, sex, or other demographics. An important advantage of the CCP over other data sources is that it includes all of the credit cards held by an individual. Throughout, we combine all credit cards, giving the com-

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2Lee and van der Klaauw (2010) provide additional details on the sampling methodology and how closely the overall sample corresponds to the demographic characteristics of the overall U.S population, and conclude that the demographics match the overall population very closely. We use only a 0.1 percent sample for analytical tractability for much of the analysis.
plete credit and debt picture. Importantly, the CCP measures total credit card debt and does not directly distinguish new charges from revolving debt from previous months. We limit the sample to include only accounts that have a birth year and that had an open credit card account at some point from 2000–2015. The likelihood of credit card possession increases for people when they are in their 20s, but then it quickly stabilizes. We show the age and year distribution of having a positive limit or debt in Figure A-1 in the appendix.

We use several other data sources: To estimate our payments model, we also use data from the Federal Reserve Bank of Atlanta’s Diary of Consumer Payment Choice, which asks a nationally representative sample of consumers to record all of their expenditures and how they paid for them over a three-day period (Schuh and Stavins 2017, Schuh 2018). In addition, we estimate life-cycle profiles for consumption from from the Consumer Expenditure Survey (CE), for fraction revolving from the Federal Reserve Bank of Atlanta’s Survey of Consumer Payment Choice, and bankruptcy rates from the Consumer Financial Protection Bureau’s Consumer Credit Panel which is derived from a 1 in 48 sample of credit bureau data.

2.1 Credit and debt over the business cycle

Figure 1 shows how the average U.S. consumer’s credit card limit and debt varied significantly from 2000–2014. From 2000–2008, the average credit card limit increased by approximately 40 percent, from around $10,000 to a peak of $14,000. During 2009, overall limits collapsed rapidly before recovering slightly in 2012. Credit card debt shows a similar variation over time. From 2000–2008, the average U.S. consumer’s credit card debt increased from just over $4,000 to just under $5,000 before returning to around $4,000 during 2009 and 2010.4

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3 The CCP reports only the aggregate limit for cards that are updated in a given quarter. Cards with current debt are updated, but accounts with no debt and no new charges may not be. To deal with this problem, we follow Fulford (2015) and create an implied aggregate limit by taking the average limit of reported cards times the total number of open cards. This method is exact if cards that have not been updated have the same limit as updated cards. Estimating the difference based on changes as new cards are reported and the limit changes, Fulford (2015) finds that non-updated cards typically have larger limits, and so the overall limit is an underestimate for some consumers with unused lines. For consumers who use much of their credit and so may actually be bound by the limit, the limit is accurate because all their cards are updated.

4 The fall in debt is not because of charge-offs in which the bank writes off the debt from its books as unrecoverable. The consumer still owes the charged-off debt and it generally still appears on the credit record. Banks may eventually sell charged-off debt to a collection agency, in which case it may no longer appear as credit card debt within credit
Figure 1: Credit card limits, debt, and utilization: 2000–2015

Observed from credit bureau

Model prediction given fall in credit limits

Notes: The left panel shows observed limits, debts, and utilization from credit bureau data (see Section 2 for details). The right panel shows model predictions given an unexpected fall in credit (see section 5 for details). For both panels, the left axis shows the average credit card limits (top line) and debt (bottom line). Note the log scale. The right axis shows mean credit utilization (middle line) defined as the credit card debt/credit card limit if the limit is greater than zero. Source: Authors’ calculations from Equifax/NY Fed CCP.

Utilization is much less volatile than credit or debt. The thick line in the middle of Figure 1 shows credit utilization, the average fraction of available credit used. Because the scale on the left axis of the figure is in logarithms for credit and debt, a 1 percentage point change in utilization on the right axis has the same vertical distance as a 1 percent change in credit or debt. The similar scales mean that we can directly compare the relative changes over time in limits, debt, and credit utilization. Credit and debt vary together in ways that produce extremely stable utilization that has no obvious relationship with the overall business cycle. The next two sections examine how the decisions made by individuals combine to form this aggregate relationship.

2.2 Credit and debt over the life cycle

Figure 2 shows how credit card limits, debt and utilization evolve over the life cycle. In the left panels, each line is for an age cohort that we follow over the entire time possible. The figure

bureau accounts. Charge-offs are not large enough to explain the fall in debt, although they did increase in 2009. The average charge-off rate from 2000–2007 was 4.35, increasing to 5.03 in 2008 and to 6.52 in 2009, before declining again to 4.9 in 2010 and 3.54 in 2011, and averaging 2.41 since then. See https://www.federalreserve.gov/releases/chargeoff/delallsa.htm for charge-off rates for credit cards. Note that our econometric estimation captures defaults.
Therefore makes no assumptions about cohort, age, or time effects. Credit limits increase very rapidly early in life, rising by around 400 percent from age 20–30, and continue to increase after age 30, although less rapidly. Life-cycle variation dominates everything else in Figure 2; while there is clearly some common variation over the business cycle, cohorts move nearly in line with age. We show a more formal decomposition into age and year effects in Figure A-3 in the appendix. Despite the very large variation over the business cycle evident in Figure 1, changes over the life cycle are an order of magnitude greater.
The bottom two panels of Figure 2 show the credit card utilization—credit card debt divided by the credit limit—for each cohort and the distribution of utilization. Consumers with zero debt have zero credit utilization, so they are included in the calculation of utilization but are excluded from mean debt, which includes only positive values. Credit utilization falls slowly from ages 20–80. On average, 20-year-olds are using more than 50 percent of their available credit, and 50-year-olds are still using 40 percent of their credit. Credit utilization does not fall to 20 percent until around age 70. Moreover, there is substantial and persistent heterogeneity of utilization. More than 10 percent of the population is nearly at its credit limit even past age 70.

2.3 The reduced form evolution of individual utilization

This section shows that utilization for an individual rapidly reverts to an individual specific mean. Credit utilization is therefore best characterized by fixed heterogeneity across individuals and relatively small transitory deviations for an individual over time. We present parametric results here and non-parametric results in Appendix A and Appendix Figure A-4. The non-parametric results suggest that the simple linear dynamic reduced-form model we employ is surprisingly accurate. Fulford and Schuh (2015) give additional variations for utilization and show results on how debt and credit co-evolve, rather than fixing the relationship by combining them into utilization. Relatively little is lost by simplifying only to utilization. Moreover, in a Granger Causality sense, the direction of causality moves primarily from changes in credit to change in debt.

Table 1 shows how utilization this quarter relates to utilization in the previous quarter. For simplicity, we estimate AR(1) regressions of the form:

\[ u_{it} = \theta_t + \theta a + \alpha i + \beta u_{i,t-1} + \epsilon_{it}, \]  \hspace{1cm} (1)

where \( u_{it} \) is the credit utilization, conditional on a positive credit limit, and age (\( \theta a \)) and quarter (\( \theta_t \)) effects that allow utilization to vary systematically by age and year. Column 1 does not include fixed effects and so assumes a common intercept. Column 2 includes quarter and age effects, while

\[5\text{The calculations in Figure 2 are the average of log limits and log debts to match later analysis and so exclude zeros except for utilization. Figure A-1 in the appendix shows the fraction in each cohort who have positive credit and debt. Including the zeros would lower the average credit limit and debt, but makes the life-cycle variation larger.}\]
Table 1: Credit utilization

<table>
<thead>
<tr>
<th></th>
<th>Equifax/NY Fed CCP</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Credit utilization</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(t-1)</td>
<td>0.874***</td>
<td>0.699***</td>
</tr>
<tr>
<td></td>
<td>(0.000876)</td>
<td>(0.000492)</td>
</tr>
<tr>
<td>Credit utilization</td>
<td>0.868***</td>
<td></td>
</tr>
<tr>
<td>(t)</td>
<td>(0.000892)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.647***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00131)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.699***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000492)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>347,642</td>
<td>2,168,011</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.741</td>
<td>0.491</td>
</tr>
<tr>
<td>Fixed effects</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Age and year effects</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Number of accounts</td>
<td>10,451</td>
<td>46,607</td>
</tr>
<tr>
<td>Frac. Variance from FE</td>
<td>0.477</td>
<td>0.217</td>
</tr>
</tbody>
</table>

Notes: The sample includes zero credit utilization but excludes individual quarters where the utilization is undefined since the limit is zero and when utilization is greater than five (a very small fraction, see distributions of utilization in Fulford and Schuh (2015)). Source: Authors’ calculations from Equifax/NY Fed CCP.

the other columns include individual fixed effects, quarter effects, and age effects.\(^6\)

Without fixed effects, credit utilization is very persistent in column 1. Including age and year effects in column 2 barely changes the persistence. The third column shows how credit utilization varies around an individual-specific mean. Nearly half of the overall variance in utilization comes from these fixed effects. In other words, about half of the distribution comes from factors that are fixed for an individual, allowing for common age and year trends, and half from relatively short-term deviations from the mean. After a 10 percentage point increase in utilization, 6.47 percentage points remain in one quarter, 1.7 percentage points in a year, and fewer than 0.3 percentage points after two years.

Moreover, this individual persistent utilization is highly heterogeneous. As Figure 2 shows, for most of the life cycle, the 25th percentile is using less than 10 percent of available credit, while the 75th percentile is using more than 80 percent. Following people quarterly for 15 years, people who are using more than 60 percent of their credit on average spend 80 percent of the time using more

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\(^6\)The combined age, year, and individual fixed effects in equation (1) are not fully identified. To implement the additional necessary restriction, we follow Deaton (1997, pp. 123–126) by recasting the age dummies such that \(\hat{I}_a = I_a - [(a - 1)I_{21} - (a - 2)I_{20}]\), where \(I_a\) is 1 if the age of person \(i\) is \(a\) and zero otherwise.
than have 60 percent; people using less than 30 percent, spend more than 90 percent of time using less than 30 percent. In the next section, we describe a model that helps explain this persistent yet heterogeneous utilization.

3 A model of life-cycle consumption and credit card debt

To explain the observations in the previous section, this section describes a life-cycle consumption model that builds on the models in Gourinchas and Parker (2002) and Cagetti (2003) but includes the addition of a payment choice, the ability to borrow at a higher interest rate, the choice to default on debt, expenditure shocks, and changing credit over the life cycle. Although we describe the decision making for a particular consumer, in the estimation we allow for multiple populations of consumers with distinct preferences.

To keep the model numerically tractable and thus able to be estimated, we make a number of modeling decisions that simplify the full richness of the decision environment—particularly of the payment choice and default—but allow us to capture the important dimensions of the problem. We focus on unsecured credit card debt of individual consumers and do not directly model the endogenous decision to take on non-credit card debt or interactions within households. While these other elements likely affect credit card decisions to some extent, data limitations and numerical complexity make them difficult to address directly, although we can deal with some indirectly.\footnote{Most other forms of household debt, such as mortgages, home equity, and auto loans, are secured directly against a household asset, and so their main influence on credit card decisions is how they affect liquidity. The model allows for asset accumulation and income from illiquid assets in late life, but it does not directly model an endogenous liquidity decision as in Kaplan and Violante (2014) or Kaboski and Townsend (2011). Fulford and Schuh (2015) show that the reduced-form relationship between credit card limits and debts explored in Section 2.3 does not seem to change based on whether someone has a mortgage. Households may provide insurance across members (Blundell et al. 2008) and across generations.}

3.1 The decision problem

From any age $t$, a consumer indexed by $i$ seeks to maximize her utility for remaining life given current resources and expected future income. Consumers may belong to a population with distinct preferences which we denote with $j$. With additively separable preferences, the consumer with liquid funds $W_{it}$ and current credit limit $B_{it}$ maximizes the discounted value of expected future
utility:

\[
\max_{\{X_{is}, \pi_{is}, f_{is}\}_{s=t}} \left\{ E \left[ \sum_{s=t}^{T} \beta^{s-t} u(C_{is}) + \beta^{T+1} S(A_{iT}) \right] \right\} \quad \text{subject to}
\]

\[
C_{is} = \nu_{is}(1 - f_{is} \phi_{s}) X_{is} \quad \text{(Consumption from expenditures)}
\]

\[
X_{is} \leq W_{is} \quad \text{(Expenditures limited by liquidity)}
\]

\[
W_{is} = R_{i,s} A_{i,s-1} + Y_{is} + B_{is} - K_{is} \quad \text{(Evolution of liquidity)}
\]

\[
A_{i,s-1} = W_{i,s-1} - B_{is-1} - X_{is-1} \quad \text{(Relationship between liquidity and assets)}
\]

\[
\nu_{is} = \nu(\pi_{is}; A_{i,s-1}) \quad \text{(Payment decision)}
\]

\[
f_{is} = f(F_{is}, W_{is}) \quad \text{(Default decision)}
\]

\[
F_{is} = H(F_{i,s-1}, f_{i,s-1}) \quad \text{(Evolution of default state)}
\]

where she gets period utility \(u(\cdot)\) from consumption \(C_{is}\), which she gets by making expenditures \(X_{is}\) adjusted for the payment choice and default. The decision at \(t\) depends on what she expects her future decisions and utility to be at ages \(s \geq t\). The consumer discounts the future with a fixed discounted factor \(\beta_{j}\) and so has time-consistent preferences. We therefore drop the distinction between age \(t\) and future ages \(s \geq t\) for clarity.

The discount factor is fixed for the individual consumer, but may vary across consumers in different groups \(j\) and we will estimate the importance of this variation. We assume that period utility displays Constant Relative Risk Aversion (CRRA) and allow the risk aversion parameter \(\gamma_{j}\) to vary across types. Appendix B.2 discusses how to rewrite the consumer’s problem recursively in terms of the normalized state variable \(w_{t}\) and thus write the solution of the consumer’s normalized recursive problem as an age-specific expenditure/consumption function \(x_{t}(w_{it}, a_{i,t-1}, F_{it})\).

Beyond expenditures, the consumers faces two additional decisions each period: how to pay for her expenditures and whether to default. Within each period she decides what portion of expenditures to fund using credit versus liquid funds. Making payments from different sources of funds comes at a price that drives a small wedge \(\nu_{it}\) between expenditures and consumption, the evolution of which we explain below. Expenditures are limited by the available liquidity \(W_{it}\), which is the sum of assets left at the end of the previous period \(A_{i,t-1}\) (which may be positive or negative) earning total return \(R_{it}\) which depends on the default status and assets in the previous period, income this period \(Y_{it}\), and the credit limit this period \(B_{it}\), minus an expenditure shock \(K_{it}\). The consumer may choose to default, indicated by the binary variable \(f_{it}\) and enter the default
state $F_{it}$, or be forced to default if the expenditure shock pushes liquidity below zero. Defaulting reduces expenditures in the current period and puts the consumer in the default state which has costs in future periods, but removes all debt. We discuss the consumption and credit implications of default below. Many of the elements in this problem are standard. We focus on the nonstandard ones.

**Rate of return on assets**  Borrowers face a higher interest rate than savers, and those in default face an even higher interest rate. If the assets $A_{i,t-1}$ at the end of the period are positive, her assets grow at the return on savings; if assets are negative, she is revolving debt, and her debt grows at the rate for borrowers or defaulted borrowers if she has a bankruptcy on her credit record:

$$R_{it} = R(A_{i,t-1}, F_{i,t-1}) = \begin{cases} 
R & \text{if } A_{i,t-1} \geq 0 \\
R_B & \text{if } A_{i,t-1} < 0 \\
R_D & \text{if } A_{i,t-1} < 0 \text{ and in default } (F_{i,t-1} = 1), 
\end{cases}$$

with $R_D \geq R_B \geq R$.

**The payments wedge between expenditures and consumption**  Credit card debt includes unpaid revolving debt from a previous period as well as all new charges that may be paid off. To understand credit card debt, we must account for this payment or “convenience” use as well as the revolving-debt use of credit cards. We model the within-period decision of what portion of expenditures to pay for using credit cards in a simple way that allows us to estimate it with observable behavior and embed it in the consumption model.

A consumer has two choices for converting liquid funds into consumption. She can use a credit card or some other option that, for simplicity, we will call cash. The consumer pays a cost or receives some possibly non-pecuniary benefit when using each method. Each fraction of expenditures $\pi \in [0, 1]$ has a value $N(\pi)$ of using a credit card relative to all other payment methods, so that if $N(\pi) > 0$, using a credit card is less costly than other methods. By making the value relative to other means, we effectively normalize the cost of using cash to zero. Thus we ask whether, for that fraction of expenditures, using a credit card is less costly than cash. The
normalization is key to our identification approach, which can identify the value of credit cards only relative to other choices, not in absolute terms. The normalization is innocuous in the consumption model because it affects the marginal value of expenditures in all periods. By indexing the value using the fraction of expenditures, we rule out the possibility that the size of expenditures affects the costs of paying for them. This simplification is important for fitting the within-period payment decision into the consumption decision.

We next put a simple functional form on \( N(\pi) \), which allows us to directly identify willingness-to-pay given observable behavior. We order expenditures so that the value of using a credit card at \( \pi = 0 \) is the largest and \( \pi = 1 \) the smallest. With this order, we assume that the relative value of using a credit card is falling at a linear rate with the fraction of expenditures:

\[
N(\pi) = \nu_0 - \nu_1 \pi.
\]

For the first fraction of expenditures, consumers are willing to pay \( \nu_0 \) to use a credit card instead of cash. For expenditures for which \( N(\pi) \geq 0 \), the consumer prefers using a credit card. When \( N(\pi) < 0 \), she prefers cash because it is less costly. By ordering the costs and assuming a continuous and strictly monotonically decreasing function, we have simplified the consumer’s decision from which option to use for every iota of expenditures to finding the optimal fraction of expenditures \( \pi^* \) to use a credit card for, where \( N(\pi^*) = 0 \). The consumer uses a credit card only for the fraction of expenditures for which she gets positive value, relative to other payment methods.

Consumers who revolved debt the previous period have to immediately pay interest on new payments, while convenience users do not. Revolving makes consumption slightly more costly, and so the payment decision influences the consumption decision. If expenditures are spread evenly over the month, then a revolver will pay additional interest of \((R_B - 1)/24\) on her credit card expenditure that month.\(^8\) Assuming the loss of float is the only factor explaining different usage, the cost function for revolvers shifts down by \( (R_B - 1)/24 \).

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\(^8\)This formula comes from the way that annual credit card rates are reported and interest charged. The interest rate on debt is \( R_B - 1 \). The Annual Percentage Rate, or APR, is not a compound rate, and so it is appropriate to divide it by 12 to find the rate of interest. The financing charge on a credit card is calculated based on the average daily balance within a month, and so the financing charge on consumption spread evenly throughout a month is half the interest rate.
Figure 3: Value or cost of expenditure using a credit card, relative to other means

Notes: This figure shows the value or cost of expenditure on a credit card at each expenditure share \( \pi \) relative to cash. The top line is for convenience users who put an optimal share \( \pi_c \) of consumption on a credit card. The bottom line for revolvers is shifted down by the amount \( -r_B/24 \), because revolvers lose the float on payments made using credit cards and therefore put a smaller optimal share on their credit cards \( \pi_R \).

Figure 3 illustrates these two cost functions and why these simple assumptions help us find the payments wedge. As the fraction spent on a credit card increases, the value of paying for the next bit of expenditures declines. Eventually, expenditures on a credit card are less valuable than expenditures with cash, and so there is an optimum \( \pi^C \). Because revolvers start at a lower initial value, their optimum \( \pi^R \) is lower, a prediction we see in the data and will discuss more when we estimate this model in Section 4. Figure 3 also makes clear the identification strategy. With estimates of \( \pi^C, \pi^R, \) and \( r_B \), it is possible to solve for the two parameters \( \nu_0 \) and \( \nu_1 \) and find the area of the wedge for convenience users, \( \nu^C \), and revolvers, \( \nu^R \). The area is the sum of the benefits of using a credit card to access funds instead of using cash when a credit card is a better choice. Appendix D goes through the algebra of exact expressions for \( \nu^C, \nu^R, \pi^C \) and \( \pi^R \) given \( \nu_0 \) and \( \nu_1 \), and it shows how to calculate standard errors given estimates of \( \pi^C \) and \( \pi^R \) using the delta method.

To understand why we need to model the payments use of credit cards, consider what the model says we will see for convenience use and revolving debt. The observed credit card debt at time \( t \) in the credit bureau data includes both new charges and previous debt for revolvers, but only
convenience debt from charges in the past month for convenience users:

\[ D_{i,t} = \begin{cases} 
\pi^C X_{i,t} & \text{if not revolving so } A_{t-1} \geq 0 \\
\pi^R X_{i,t} + A_{i,t-1} & \text{if revolving so } A_{t-1} < 0.
\end{cases} \]

Debt evolves differently because for revolvers it includes the stock of previous debt, while for convenience users it is only the flow of expenditures.

**The income process and expenditure shocks**  Income or disposable income follows a random walk with drift:

\[
Y_{i,t+1} = P_{i,t+1}(U_{i,t+1} - F_{i,t+1} \phi_{y_{it+1}})
\]

\[
P_{i,t+1} = G_{i+1}^j P_{it} M_{i,t+1},
\]

where \(G_{i+1}^j\) is the known life-cycle income growth rate from period to period for population \(j\). \(F_{i,t+1} \phi_{y_{it+1}}\) is an income cost of being in the default state \(F_{i,t+1} = 1\) discussed more below. The “permanent” or random-walk shocks \(M_{i,t+1}\) are independently and identically distributed as lognormal with mean one: \(\ln M_{i,t+1} \sim N(-\sigma_M^2/2, \sigma_M^2)\). The transitory shocks are similarly distributed lognormally with mean one and variance parameter \(\sigma_U^2\). We allow for a temporary low income \(U_L\) from unemployment or other shocks with probability \(p_L\) each period.\(^9\) The structure of the shocks ensures that the expected income next period is always \(E_t[Y_{i,t+1}] = G_{i+1}^j P_{it}\) when not defaulting, because the mean of both transitory and permanent shocks is one.

A consumer also faces expenditure shocks \(K_{i,t}\) which are either 0 or a multiple of permanent income, \(k P_{i,t}\) with probability \(p^k\). These expenditure shocks represent expenditures the consumer is required to make, but derives no utility from. Thus, while they do not count as consumption for utility purposes, they are expenditures for accounting purposes, and we include them when we compare model expenditures to actual consumer expenditures.

---

\(^9\)Low-income shocks, in addition to lognormal shocks, may matter for precautionary reasons by putting additional probability on very bad outcomes. We introduce low-income shocks in such a way that \(E_t[U_{i,t+1}] = 1\). Formally, the transitory shocks are distributed as: \(U_{i,t+1} = U_L\) with probability \(p_L\) and \(\tilde{U}_i(1 - U_Lp_L)/(1 - p_L)\) with probability \(1 - p_L\), where \(\tilde{U}\) is i.i.d. lognormally distributed with mean one: \(\ln \tilde{U}_{i,t+1} \sim N(-\sigma_U^2/2, \sigma_U^2)\) and \(U_L\) is unemployment income as a fraction of permanent income.
The credit limit  Life-cycle variation in credit limits is proportionally several times larger than life-cycle variation in income (compare Figure 2 to Appendix Figure A-6), and the dispersion of credit limits across individuals of the same age is also large (see Appendix Figure A-2). We allow for life-cycle growth and dispersion across consumers by assuming that the credit limit $B_{it}$ is an age-dependent multiple of permanent income:

$$B_{it} = b_t P_{it} b_f^{F_{it}},$$

where $b_t \geq 0$ is the age-varying fraction of permanent income that can be borrowed, which is set outside the control of the consumer and $b_f$ is the fraction that the consumer can borrow in the default state ($F_{it} = 1$). This approach means that across consumers, $B_{it}$ will be in proportion to income $P_{it}$, but it allows credit to follow an average path over the life cycle that is different from income and affected by consumer decisions.\(^{10}\)

The decision to default  The consumer may voluntarily decide to default ($f_{it} = 1$) and enter the default state ($F_{it} = 1$). Alternatively, if the expenditure shock is sufficient to push $W_{it} \leq 0$, the consumer is forced into involuntary default.

Defaulting has a series of consequences. Involuntary defaulters consume the consumption minimum $c^{\min} P_{it}$. In the period of default for voluntary defaulters, expenditure is all of available liquidity ($X_{it} = W_{it}$), but the consumption value of this expenditure is reduced by $(1 - \phi c)$. We think of this reduction as capturing three costs: a non-pecuniary cost of default; pecuniary default penalties that apply during the period of default; and the possible ability of card issuers to limit default exposure by reducing credit limits proactively. After defaulting, the consumer enters the

\(^{10}\)The consumer’s problem as written, with $W_t$ as a sufficient period budget constraint, implies that a consumer must immediately repay all debt over her limit if her credit limit falls. To see this, consider what happens if $B_{i,t-1} > 0$ and the consumer borrows, leavings negative assets at the end of period $A_{i,t-1} < 0$. If $B_{it} = 0$, then assets at the end of period $t$ must be weakly positive ($A_{it} \geq 0$), and so all debt has been repaid within a single period. A cut in credit limits implies an immediate repayment of debt in excess of the limit. This debt repayment when credit is cut below debt does not match credit card contracts, which do not require immediate and complete payment following a fall in credit (Fulford 2015). Instead, credit card borrowers can pay off their debt under the same terms; they just cannot add to it. However, allowing for such behavior means that there must be an additional continuous state variable, because $W_t$ and $B_t$ no longer fully describe the consumer’s problem. This adds substantially to the numerical complexity of the solution through the curse of dimensionality.
next period with no debt \((A_{i,t+1} = 0)\).

Having entered the default state, the consumer faces a modified consumption problem of someone with a bankruptcy on her credit record. Her credit limits is a fraction \(b_f\) of non-defaulted credit limits. Her cost of borrowing is higher. To reflect possible wage garnishment or the effect default may have on available employment, the income process is reduced by a multiple of the default debt \(\phi^y_{it} = \phi^y(R^B - 1)b_t p_{it}\) in every period. Formulated this way, the cost of default is increasing with the credit limit, so that as credit limits increase with age, so does the cost of default. Because the credit limit is increasing over the life cycle, the consumption value of maxing out credit cards is also increasing, so the incentive to default is increasing. The current period and future costs of default are conceptually distinct, but difficult to distinguish empirically, so we link them and set \(\phi^c = \phi^y\) so that only one parameter governs the total cost of default.

To keep the state space tractable, we model the evolution of the default state \(F_{i,t} = H(F_{i,t-1}, f_{i,t-1})\) as an absorbing Markov process: A consumer in default in the previous period \((F_{i,t-1} = 0)\) stays in default with probability \(p^F\), and exits default with probability \(1 - p^F\). The consumer is in default with certainty if she defaulted in the previous period \((f_{i,t-1} = 1)\).

Given the costs and benefits of default, consumers must decide whether to default. Only consumers not currently in default may decide to default. Because default is a discrete decision, consumers decide whether the value of current and expected future utility from defaulting is greater than defaulting:

\[
f_{it} = f(F_{i,t}, W_{i,t}) = \begin{cases} 1 & \text{if } V^\text{Default}(W_{i,t}) > V^\text{Not Default}(W_{i,t}) \text{ and not in default } (F_{it} = 0) \\ 0 & \text{else.} \end{cases}
\]

Following Chatterjee et al. (2007), we can simplify this decision into finding the crossing point, if it exists, of the two value functions, so characterize the decision as finding the liquidity below which default occurs: \(W_t^\text{Default}\).

The beginning and end of life  Several important decision parameters affect initial distributions and decisions late in life. We assume the initial distribution of the wealth/permanent-income ratio
is lognormal with variance that matches the variance of permanent income shocks and mean $\lambda_0^j$ that may be different for consumers in different populations $j$. The consumer lives for $T$ periods, where T is a random number that we match to actual life tables, and we assume she dies with certainty at age $\tilde{T}$. At death, she receives a final utility $S(\cdot)$ from leftover positive resources. In our base estimations, we set the bequest motive to allow for an annuity to heirs. Appendix B.1 discusses the specific function.\footnote{Recent work has disagreed over the importance of a bequest motive as opposed to other possible motives for keeping assets late in life, such as long-term care and medical needs (De Nardi et al. 2010). Since we focus primarily on debt, our model and estimates are not well situated to distinguish between motives. While the exact form of the bequest motive or another motive for keeping assets late in life is not important, removing it entirely is consequential. Because the likelihood of dying is increasing with age, people with no bequest motive are effectively getting more impatient. Therefore, they should not decrease the amount of debt they hold as much as the data shows they do. We discuss the effects of alternate formulations of the bequest motive more in Section 4.4.}

Late in life, consumers may face income and expenses different from those they face during working years. Labor income may drop, but consumers may start claiming illiquid retirement benefits such as pensions and Social Security, and they may derive income from other illiquid assets such as housing. They may also face an increase in necessary expenses from additional medical care or other needs. We summarize all of these changes by assuming that income starting at $T_{Ret}$ is a fraction $\lambda_1^j$ of pre-retirement permanent income ($\lambda_1^j P_{i,T_{Ret}-1}$). Allowing for a fall in outside disposable income is a flexible way of combining the many late-in-life changes that consumers may want to plan for during working years, including possibly the acquisition of illiquid assets for retirement. Consumers still earn the return on their liquid assets accumulated before $T_{Ret}$, but they face no income volatility and continue to consume optimally given their income and expected longevity.

**Model frequency** We model all decisions as being made quarterly to match the data and adjust the discount rates and interest rates accordingly, although we report the yearly equivalent for straightforward comparison to other work. Quarterly decision-making is approximately four times more computationally intensive than yearly yet helps to capture the within year consequences of hitting a budget constraint. Because of data and computational constraints, much of the structural consumption literature has been limited to examining decisions made at a yearly frequency.
just convenience credit card debt appropriately so that it represents only one month of expenditure when we estimate the model.

### 3.2 The consumer’s decision

For a given set of parameters, we find a numerical approximation of the consumer’s problem by writing the problem recursively and proceed through backward recursion from the end of life. We give a detailed discussion in Appendix B.3. We follow the method of endogenous gridpoints (Carroll 2006), which substantially reduces the computation costs for the expenditure problem. The payments problem can be solved separately from the decision problem in each period, which makes the model numerically tractable.

Figure 4 illustrates some of the complexities of the decision problem. The consumption functions then show how much a consumer at that age with those preferences will consume at each liquidity. Because credit limits also scale with permanent income, only age, default status, previous borrowing, and the current liquidity ratio enter the consumption decision. There are three kinks in the consumption function, which are most visible for the impatient 30-year-olds. First, the consumption function has an inflection point where the consumer goes from leaving nothing for the next to period to leaving some liquidity by not borrowing up to her credit limit as examined by Deaton (1991). The second two inflection points arise because the interest-rate differential means there are two solutions to the Euler equation for leaving zero assets. One, the limit with assets approaching zero from below, uses the borrowing rate $R^B$, and the other uses the savings rate $R$.

Figure 4 is based on the estimates in the next section which suggest a cost of default parameter high enough that voluntary default is never optimal. With a lower cost of default, the decision becomes even more complex as is illustrated by appendix Figure A-5. With a voluntary default, there is a discrete jump in consumption at the optimal default liquidity; below the default point, consumers spend all available liquidity and suffer the costs of default, above the default point consumers leave some liquidity for the next period.\(^{12}\)

\(^{12}\)The standard method of endogenous gridpoints breaks down when there is a discontinuity in the value function at the default point. We therefore use a modified version that forms endogenous gridpoints on either side of the discontinuity and then enters a successive approximation around the discontinuity from above and below to choose
Notes: This figure uses the estimates in Table 3 column 1 at age 30 and age 60 to show the quarterly expenditure function for impatient (A) consumers and patient (B) consumers. Liquidity $w_t$ is a multiple of quarterly permanent income $P_t$ and includes available credit. The densities for liquidity are for age 30 and show where individuals are along their consumption functions. Because the rate of savings is lower than the rate of borrowing, the expenditure function has kink going from borrowing to saving nothing for the next period to actively saving.

4 Estimation

This section describes how we estimate the structural model using life-cycle profiles of consumption, debt, and default. The estimation works in two stages: First, we estimate the payments value of credit cards for revolvers $\nu^R$ and convenience users $\nu^C$ in Section 4.1. The structure of the payments problem means it can be estimated separately. We also estimate other observable parameters at this stage. Second, we estimate the parameters of the model that minimize the difference between the life-cycle profiles the model produces and the life-cycle profiles of debt, consumption, and default we observe in the data.

We allow for preference heterogeneity by introducing two sub-populations with different preferences and overall income. Of course, additional preference heterogeneity is possible, but our results suggest that this is the minimum heterogeneity necessary, and we prefer this parsimonious form because it makes obvious the contribution of different populations while not adding gridpoints that capture the discontinuity correctly.
too much complexity to the computational problem. Moreover, it is not clear that more preference heterogeneity is identified without additional assumptions or data. We estimate differences in the income-generating process between the two populations to allow for arbitrary correlation between preferences and income.

There are thus three forms of heterogeneity in the estimated model: (1) life cycle, as people make different decisions at different ages; (2) heterogeneous agents, as people are hit with different shocks and so have different assets and incomes and make different decisions based on their current wealth; and (3) population-level preference and income heterogeneity, as distinct sub-groups that have different preferences and different income processes react differently to shocks.

To combine groups we estimate the share of group A \( f^A \) and the multiple of the average permanent income earned by group A \( \zeta^A \). We constrain the population average income of the two groups to match the empirical income profile so that if population A has a higher income, then population B must have a lower income.\(^{13}\) For each sub-population, the entire decision is described by four parameters: the discount rate \( \beta \), the coefficient of relative risk aversion \( \gamma \), the initial wealth-to-income ratio \( \lambda_0 \), and the fraction of permanent labor income expected from illiquid assets such as housing, pensions, or Social Security in late life \( \lambda_1 \). Finally, we estimate the probability \( (p^k) \) and cost \( (k) \) of expenditure shocks. We show that the default cost parameter \( \phi \) is identified only up to an inequality, so jointly estimate 12 parameters in the second stage:

\[
\theta = \{\gamma^A, \beta^A, \lambda^A_0, \lambda^A_1, \gamma^B, \beta^B, \lambda^B_0, \lambda^B_1, f^A, \zeta^A, p^k, k\}.
\]

We estimate the parameters of the nonlinear model using the Method of Simulated Moments (MSM) of McFadden (1989). Appendix C gives additional details. Briefly, given set of parameters \( \theta \in \Theta \) and first-stage parameters \( \chi \), we solve the consumer’s problem and then simulate the life-cycle decisions for a large population of consumers. We then minimize the weighted sum of square differences between the empirical and simulated life-cycle moments for the population. Our standard weighting matrix is block proportional to the inverse variance of the empirical moments (the optimal weighting matrix with no first-stage correction). We also show results using the “optimal”

\(^{13}\) Together \( f^A \) and \( \zeta^A \) directly determine \( \zeta^B \). For the average income of the combined populations to equal the average observed income \( f^A \zeta^A + f^B \zeta^B = 1 \), which implies that \( \zeta^B = (1 - f^A \zeta^A)/(1 - f^A) \), since \( f^B = 1 - f^A \).
Table 2: Fraction of expenditure on a credit card and value for payments

<table>
<thead>
<tr>
<th></th>
<th>Fraction on Credit card</th>
<th>Std. error</th>
<th>Std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>All consumers</td>
<td>0.172</td>
<td>0.0082</td>
<td>0.310</td>
</tr>
<tr>
<td>All revolvers</td>
<td>0.156</td>
<td>0.0130</td>
<td>0.283</td>
</tr>
<tr>
<td>All convenience users</td>
<td>0.182</td>
<td>0.0105</td>
<td>0.324</td>
</tr>
</tbody>
</table>

Model Estimates

<table>
<thead>
<tr>
<th></th>
<th>Level $\nu_0$</th>
<th>Slope $\nu_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.035</td>
<td>0.194</td>
</tr>
<tr>
<td></td>
<td>0.0216</td>
<td>0.1259</td>
</tr>
</tbody>
</table>

Implied value of credit card use (percent of consumption)

<table>
<thead>
<tr>
<th></th>
<th>Revolvers</th>
<th>Convenience users</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.235</td>
<td>0.319</td>
</tr>
<tr>
<td></td>
<td>0.1512</td>
<td>0.0962</td>
</tr>
</tbody>
</table>

Notes: Authors’ calculations from the Federal Reserve Bank of Boston Diary of Consumer Payment Choice. The standard errors are calculated by bootstrapping.

weighting matrix, following Laibson et al. (2007), who improve on the work of Gourinchas and Parker (2002) by allowing for the empirical moments to have different numbers of observations.

4.1 Estimation and identification of the payments model

Because of the structure of the consumer’s problem, whether the consumer was revolving as of the previous period is the only way the consumption decision influences the payment decision. We can thus find the solution to the payments problem first and then allow the solution to the payments problem to influence the consumption problem. Table 2 shows the fraction of all expenditures over a three-day period that the nationally representative sample of consumers from the Diary of Consumer Payment Choice puts on a credit card. The average consumer pays for 17.2 percent of expenditure with a credit card. Revolvers pay for slightly less at 15.6 percent, and convenience users pay for slightly more at 18.2 percent.14

The difference between revolvers and convenience users then exactly identifies the payment model, as Figure 3 illustrates. We show the algebra for the identification of the payment parameters

14Credit card use is fairly stable with age, although with wide standard errors (Fulford and Schuh 2015). Interestingly, both revolvers and convenience users over 65 tend to spend more on a credit card.
ν₀ and ν₁ and the delta method to calculate their standard errors in Appendix D. Table 2 shows the estimated coefficients with an interest rate on borrowing of 14.11 percent adjusted for inflation of 2.15 percent (see discussion in Appendix C.2 for sources).

The model directly gives the convenience value of credit cards. For a real borrowing rate of close to 12 percent, the value of using a credit card for payments over other methods is worth 0.319 percent of expenditures to convenience users and 0.235 percent to revolvers, although with fairly wide standard errors. The implied aggregate value of using credit cards for payments is around $40 billion a year.¹⁵ As a comparison, the fees that banks charge merchants for processing credit cards are roughly $60 billion per year.¹⁶ The value of the intercept ν₀ suggests that for the most valuable purchases, using a credit card has a value of 4.1 percent of all expenditures for these purchases. For comparison, if all convenience consumers received the equivalent of 1 percent cash back on their purchases with credit cards, the implied consumer surplus would be 0.182 percent of consumption. This calculation likely overstates the direct value of rewards because not all cards offer rewards, but it suggests that about half of the convenience value from credit cards comes from direct rewards or other card benefits, and the other half comes from their value as a convenient payment mechanism.

4.2 The empirical life-cycle moments and first stage moments

We estimate the model to provide the best fit to three life-cycle profiles: (1) log mean credit card debt over the life cycle from the Equifax/NY Fed CCP described in Section 2, (2) log mean household consumption over the life cycle from the CE from 2000–2014,¹⁷ (3) the fraction of consumers

---

¹⁵Personal consumption expenditures were $12.3 trillion in 2015, according to the BEA. If half of the population is revolving, then 12283 * (0.319/100 + 0.234/100)/2 = 36.6 billion. Note that this calculation is an estimate of the consumer surplus of credit cards as a payment mechanism over other means, given the current payments ecosystem, and so does not directly calculate welfare. For example, the calculation does not take into account the costs of operating the payments system or the producer surplus from additional sales made because some purchases are more convenient, or the gains to the processors, network operators, and banks.

¹⁶The total value of credit card payments was $3.16 trillion in 2015 (see the 2016 Federal Reserve Payments Study https://www.federalreserve.gov/newsevents/press/other/2016-payments-study-20161222.pdf). The percentage charged to merchants varies from approximately 0.75 percent to 4 percent, but appears to average around 2 percent. Fee revenue is therefore around $60 billion, most of which is accounted for by the interchange fees shared by banks after payouts to card networks, processors, and other parties.

¹⁷Because our observed credit data are for individuals rather than households, we adjust household consumption by dividing by the number of adults in the household. We allow for some unobserved taste changes over the life cycle by adjusting consumption for the number of children in the household. Formally, we estimate: ln(Cᵢ,ₜ/Adultsᵢ,ₜ) =
with a credit card line charged off in bankruptcy from the CFPB Consumer Credit Panel which is derived from credit bureau data, and which, unlike the Equifax/NY Fed CCP, shows individual credit card lines.\footnote{We use bankruptcy as the appropriate empirical comparison because we model default as wiping away debts, but there are many forms of default not directly coming from bankruptcy. At any age, the fraction of consumers with a credit card line marked as charged off by the issuer (including for bankruptcy) is approximately double the fraction with a line charged off for bankruptcy. Only in bankruptcy is the debt actually removed for the consumer allowing a clean start, a charge off simple means that the bank has marked the debt on its books as uncollectable for regulatory purposes. The bank may continue to try to collect the debt or sell it to a firm specializing in collection. See Athreya et al. (2017) for a model that allows both non-payment default and bankruptcy.} We show each of these moments in Figure 5 together with their estimates from the model, and have already discussed the debt profile in Section 2.2. Consumption follows the characteristic hump shape (Gourinchas and Parker 2002, Attanasio et al. 1999). Bankruptcy is increasing early in the life-cycle, before declining. Appendix C.3 discusses the construction of the variance-covariance matrix of the combined moments.

We briefly describe the sources and estimates from other data sets that identify the ancillary parameters of the model, providing greater detail in in Appendix C.2. We estimate the average life cycle of income growth \( (G_t) \) using the Consumer Expenditure Survey to match our consumption data, adjusting for aggregate growth. We use the estimates of income shocks from Gourinchas and Parker (2002), which are updates of Carroll and Samwick (1997), calculated from the Panel Study of Income Dynamics. We adjust these volatilities for quarterly dynamics so that four quarterly shocks combine to produce the same variance as one yearly shock and allow for unemployment shocks. We estimate the total credit limit for a consumer from the Equifax/NY Fed CCP to form \( B_t \). For the other parameters and prices, we estimate the interest rate on debt \( R_{b-1} = 14.11 \) percent based on the average revolving interest rate over the period. From the SCF, we estimate that those with a bankruptcy pay 1.92 percentage points more in interest on their credit card debt and have only 42 percent of the credit limit \( (b_f) \). We estimate the return on savings for an all-bond portfolio. We adjust both borrowing and saving interest rates for the geometric average inflation rate from 2000–2015 of 2.15 percent.

\[
\theta_a + \theta_t + \beta \text{Children}_{i,t} + \epsilon_{i,t}, \text{ and then calculate average household consumption per adult at each age after removing the effect of children at the individual level. Removing the implied consumption effect of children has a surprisingly small effect. Figure A-6 in the appendix shows the unadjusted and adjusted consumption. Children slightly raise expenditures per adult household member from ages 35–45, but the adjustment is small.}
\]
4.3 Estimation and identification of the life-cycle model

Using the first-stage estimates of the payments problem and the other parameters, we next estimate the full life-cycle model. Because this is a nonlinear model, all moments are typically used to identify all parameters. Appendix C.5 provides a discussion of how different sources of variation help identify the parameters. Table 3 shows the model estimates, while Figure 5 shows how debt, consumption, and bankruptcy vary over the life cycle in the model and empirical moments. Because the scales of the two top panels of Figure 5 are in logs, the estimation approximately finds the parameters so that the weighted sum of the squared differences between the predicted consumption and debt lines is as small as possible. It is clear that, given the constraints of the life-cycle optimization model, the model estimates can successfully capture the life-cycle profiles of debt, consumption, and default.

To do so, the model suggests that about two thirds of the population \( f^A \) must be fairly impatient \( (\beta^A) \) and not care very much about risks \( (\gamma^A) \). This portion of the population, which the figure and tables call population A, has already acquired some debt \( (\lambda_0^A) \) by age 24 and has substantial revolving debt throughout the life cycle. To match the amount of debt and consumption, the estimates suggest that this population has an income about average \( (\zeta^A) \).\(^{19}\) Because individual credit limits are proportional to income, the members of this group cannot be too poor on average, otherwise they would not be able to hold and make payments on their debts. Because the discount rate is high and risk aversion is low, most of this population lives essentially hand to mouth over the entire life cycle, relying on credit for all of their smoothing. This population’s average utilization is high through much of the life cycle (see the fourth panel in Figure 5).

The estimates suggest that the other portion of the population must be relatively patient and risk averse. Population B is too patient to ever want to hold much debt and has not acquired much debt by age 24 in any case \( (\lambda_0^B) \). So consumers in population B rarely borrow except in their 20s, when some have enough shocks to want to borrow for a brief time. Their credit card debt is thus almost

\(^{19}\)Depending on the particular weights, some estimates suggest an impatient income higher than average. In comparisons using the SCF, we found that the median income of revolvers was larger than the median income of convenience users, while the mean income of convenience users was larger.
<table>
<thead>
<tr>
<th></th>
<th>Standard Weights</th>
<th>Optimal Weights</th>
<th>Endogenous payments</th>
<th>Low bequest</th>
</tr>
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<tbody>
<tr>
<td><strong>Population A</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CRRA $\gamma^A$</td>
<td>0.067</td>
<td>0.121</td>
<td>0.067</td>
<td>0.066</td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>(0.008)</td>
<td>(0.018)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>Discount $\beta^A$</td>
<td>0.892</td>
<td>0.887</td>
<td>0.892</td>
<td>0.893</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.003)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Initial wealth $\lambda_0^A$</td>
<td>0.516</td>
<td>0.481</td>
<td>0.516</td>
<td>0.516</td>
</tr>
<tr>
<td></td>
<td>(0.087)</td>
<td>(0.220)</td>
<td>(0.114)</td>
<td>(0.072)</td>
</tr>
<tr>
<td>Late life inc. $\lambda_1^A$</td>
<td>0.727</td>
<td>0.719</td>
<td>0.727</td>
<td>0.731</td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
<td>(1.766)</td>
<td>(0.049)</td>
<td>(0.033)</td>
</tr>
<tr>
<td><strong>Population B</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CRRA $\gamma^B$</td>
<td>2.023</td>
<td>1.975</td>
<td>2.023</td>
<td>2.023</td>
</tr>
<tr>
<td></td>
<td>(1.007)</td>
<td>(42.111)</td>
<td>(0.946)</td>
<td>(1.134)</td>
</tr>
<tr>
<td>Discount $\beta^B$</td>
<td>0.963</td>
<td>0.963</td>
<td>0.963</td>
<td>0.962</td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>(0.350)</td>
<td>(0.020)</td>
<td>(0.020)</td>
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<tr>
<td>Initial wealth $\lambda_0^B$</td>
<td>1.728</td>
<td>1.658</td>
<td>1.728</td>
<td>1.728</td>
</tr>
<tr>
<td></td>
<td>(2.245)</td>
<td>(90.275)</td>
<td>(2.267)</td>
<td>(1.826)</td>
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<tr>
<td>Late life inc. $\lambda_1^B$</td>
<td>0.212</td>
<td>0.200</td>
<td>0.212</td>
<td>0.212</td>
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<tr>
<td></td>
<td>(0.260)</td>
<td>(8.444)</td>
<td>(0.251)</td>
<td>(0.180)</td>
</tr>
<tr>
<td>Share A $f^A$</td>
<td>0.669</td>
<td>0.648</td>
<td>0.669</td>
<td>0.669</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.005)</td>
<td>(0.008)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>Inc. mult. A $\zeta^A$</td>
<td>0.991</td>
<td>0.971</td>
<td>0.991</td>
<td>0.995</td>
</tr>
<tr>
<td></td>
<td>(0.099)</td>
<td>(0.506)</td>
<td>(0.137)</td>
<td>(0.099)</td>
</tr>
<tr>
<td>Prob. of exp. shock</td>
<td>0.040</td>
<td>0.031</td>
<td>0.040</td>
<td>0.040</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.008)</td>
<td>(0.005)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Size of exp. Shock</td>
<td>0.660</td>
<td>0.532</td>
<td>0.660</td>
<td>0.659</td>
</tr>
<tr>
<td></td>
<td>(0.146)</td>
<td>(0.073)</td>
<td>(0.117)</td>
<td>(0.135)</td>
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<tr>
<td>SSR ($g'g$)</td>
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<td>1.5633</td>
<td>0.3618</td>
<td>0.4558</td>
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<td>J-stat</td>
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<td>1.37E+09</td>
<td>3.62E+08</td>
<td>5.08E+08</td>
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<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Weights</td>
<td>Standard</td>
<td>Optimal</td>
<td>Standard</td>
<td>Standard</td>
</tr>
<tr>
<td>Endogenous payments</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Notes: Standard errors in parentheses. Optimal weights are the inverse of the variance of each individual moment. Endogenous payments makes the consumer’s aware that revolving affects the value of credit cards for payments. The standard default cost parameter $\phi_f = 7$ bequest parameter is 1. Low bequest reduces the bequest motive.
Notes: Life-cycle paths from simulated population using the estimates in column 1 of Table 3.
entirely from convenience use. Because this population expects to receive little income after expenses ($\lambda_B$) in late life and is relatively patient, this population spends early life accumulating savings for late life. Consumption increases early in the life cycle as income and savings increase, but it becomes relatively flat afterward as this population smooths consumption over the rest of the life cycle.

The cost of default is identified only up to an inequality, so it is not estimated directly. As Appendix Figure A-8 shows, holding other parameters fixed at their values in column 1, changing the bankruptcy cost parameter does not improve the fit until it goes below a threshold, after which the model fit rapidly deteriorates. For estimates larger than the threshold, no one voluntarily defaults. To see why the data push this conclusion so strongly, Appendix Figure A-9 shows bankruptcy over the life-cycle with the standard bankruptcy cost parameter and one below the threshold. As credit limits increase and the remaining expected life decreases, the gains from bankruptcy get ever larger. At approximately age 35, much of the impatient population finds it better to default and the bankruptcy rate changes precipitously. Increasing the cost of default increases the age at which it becomes optimal for much of the impatient population to voluntarily default, but does not change the rapid shift to default. Since the observed fraction of the population with a bankruptcy on record is declining over the life-cycle as shown in Figure 5, voluntary default is not useful for explaining the fraction in default. The estimates thus reject cost-of-default parameters in which there is substantial voluntary default. Note that the model ties the default cost to the credit limit, so the costs of default are increasing over the life-cycle. Not doing so makes default even more likely at older ages.

Expenditure shocks, on the other hand, are useful for explaining default over the life cycle. Early in life, consumers hit by an expenditure shock are likely to go bankrupt because they have little credit. Because bankruptcy stays on the record for seven years, the fraction in bankruptcy is increasing. But as credit limits increase, expenditure shocks are less likely to push someone into bankruptcy, and the fraction in bankruptcy starts to decline in mid-life, matching the data.

The added debt from convenience use of credit cards is one month’s worth of consumption (one-third of quarterly consumption) times the estimated rate of consumption on a credit card for a convenience user from Table 2.
The remaining three panels of Figure 5 show model predictions for other life-cycle paths. The model captures the slow fall in credit utilization over the life cycle. The fall comes primarily from revolvers using less of their credit as their limits increase and, secondarily, from incomes decreasing and making debts less affordable. To examine the evolution of wealth, which may be negative, we take the log of wealth after giving everyone $10,000, which allows us to consider the full distribution in a single graph. The model estimates predict less wealth accumulation over the life cycle than estimates from the Survey of Consumer Finances, but it predicts a similar trend increase and flattening after age 55. The model was not estimated to match these profiles, and so its ability to successfully predict something close to their level and evolution suggests that the model is capturing important facets of life-cycle decision-making.

The heterogeneity in preferences is key to the model’s ability to capture, even approximately, more than one life-cycle profile. Gourinchas and Parker (2002) estimate parameters to match the consumption profile and under-predict wealth accumulation, while Cagetti (2003) estimates parameters to match the wealth profile but needs such a high degree of risk aversion that it is difficult to capture the consumption profile.

4.4 Robustness and variations

In this section, we briefly examine the robustness of the estimates to changes in weighting matrices, starting points of the estimation, and model choices. Appendix C.6 offers additional details. The general conclusion is that while particular parameters are sensitive to estimation and model choices, our overall conclusions are not. Our overall conclusions are also robust to alternative starting points for estimation. Table 3 shows the over-identification statistic for each estimation, which always decisively rejects the hypothesis that the model is not over-identified. The choice

We have also estimated the variance of credit card debt and the variance in the change in debt from quarter to quarter, which controls for the permanent income and preference heterogeneity. The model captures the level of the variance of credit card debt reasonably well, although it does not predict the shape very well. Our simulations of the variance of the change are somewhat lower than the empirical counterparts because the only change in credit limits comes from changes in permanent income. Since our estimates do not include credit limit volatility apart from income volatility, and Fulford (2015), using the Equifax/NY Fed data, shows that credit-limit volatility is about four times greater than income volatility, our model has too little credit-limit volatility.

The over-identification statistic is large because the debt and bankruptcy moments are estimated very precisely from the administrative data. The over-identification statistic rejects that the model can simultaneously fit all moments
of weighting matrix is therefore not innocuous; because the model is over-identified, different weighting matrices will give statistically different results, so the best estimate we present should be viewed as one of many possible estimates. Our standard weight matrix gives equal weight to all three blocks of life-cycle moments. The second column of Table 3 shows estimates that use the two-stage “optimal” weighting matrix, which first estimates the parameters using our standard weighting matrix and then uses those estimates to calculate the weights that asymptotically minimize the variance of the estimator. The estimates are broadly similar; the impatient population is more risk averse but less patient, and so it carries more debt and is a smaller share of the population.

The last two columns of Table 3 examine how changing the model changes estimates. Allowing consumers to take into account how their payments decisions will affect their consumption decisions does not appear to be important in column 3. Similarly, substantially increasing the bequest motive in column 4 barely changes the estimates. Appendix C.6 provides additional discussion.

5 Model predictions and policy implications

In this section, we take the estimated model and ask how well it predicts phenomena outside the life cycle. These results provide both an out-of-sample examination of how good the model estimates are and whether the model can successfully explain other phenomena that we did not estimate it explicitly to explain. After showing it has substantial success out of sample, we explore the estimated model’s implications for stimulus policy.

We simulate a large population with an age profile matching the population from age 24–74 and a credit drop of the same size as the one that occurred over 2008–2009. In addition to life-cycle income growth and individual income volatility, aggregate income grows at a constant rate of 1.5 percent per year, just as the consumers in the model assume. We also adjust the dollar values for the average inflation rate. Finally, to mimic the fall in credit limits that started in the final quarter of 2008 and continued through 2009, we introduce a fall in credit of 35 percent for one-sixth of the because the debt and default moments are so precisely estimated that even small departures from exact fit leads to a rejection of the hypothesis.
population over six quarters. This experiment is the simplest way to produce the approximately 35 percent drop in credit limits spread over more than a year that is evident in Figure 1, but it is not a full replication of the changing environment. In particular, it does not include a fall in income or a possible decline in expectations of future income growth.

The individual dynamics of credit utilization from the simulations closely match the dynamics from the credit bureau data. Table 1 shows that once we control for unobserved heterogeneity with fixed effects in the credit bureau data, shocks to utilization disappear quickly, with 64.7 percent of a shock surviving each quarter (the third column). The last column performs exactly the same regression on the simulated data. The simulated consumers experience the large unexpected fall in credit in 2009 and the expected increase over the life cycle, but the only unexpected credit volatility that they face comes because credit is proportional to volatile permanent income. Because volatility in income is much less than volatility in credit (Fulford 2015), the consumers in the model face less credit volatility than actual consumers do over the time period. Nonetheless, their average response to changes in credit limits is very close to that of actual consumers; the estimated model captures the dynamics of credit utilization closely, with 69.9 percent of a shock persisting to the next quarter compared to 64.7 percent in column 3.

The right panel of Figure 1 shows the aggregate response of the simulated consumers to the 35 percent fall in credit introduced over six quarters. Credit continues to increase over the entire period at the same 1.5 percent rate as income, plus 2.1 percent for average inflation, partly counteracting the large fall. Model credit growth is slightly slower than actual credit growth over the period, suggesting that pegging credit to income does not fully capture the aggregate growth. Since consumers expect credit growth, their debt grows at the same time, and credit utilization is stable despite the growth before and after the crisis, just as in the data. In addition, the model successfully predicts about the same credit utilization as in the data.

During the crisis, debt quickly adjusts to the fall in credit, so utilization is much smoother than either credit or debt, although not as smooth as the data. As the individual dynamics show, while shocks at the individual level disappear quickly in both the model and data, it still takes several
Figure 6: Consumption over the business cycle

Notes: This figure shows personal consumption from the BEA and average consumption from simulations with a 35 percent fall in available credit starting in 2008q3. Each series is detrended using the 2000–2008 period. When the credit fall is concentrated among the high utilization, only consumers in this population have a fall in credit, but the aggregate fall in credit is held constant. The BEA consumption series continues to fall after 2012 relative to the 2000-2008 trend. We omit the continuing fall to focus on the impact of consumer credit changes.

quarters for consumers to fully adjust their debt and savings to a 35 percent fall in credit. The excessive smoothness of utilization in the credit bureau data suggests that there must be additional features of the period not captured by the simple simulated shock spread evenly among the population.. Even so, our model produces a notably smoother path than a simple version of the Life Cycle/Permanent Income Hypothesis (LC/PIH) would suggest.23

How important was the fall in credit for consumption? Our model makes clear a causal connection between the fall in credit limits and the fall in debt through a reduction in consumption. Figure 6 shows the relative paths of consumption from our simulations and detrended real personal consumption per person from the BEA. From the second quarter of 2008 to the final quarter of 2009, real consumption per person fell 9.2 percent relative to the trend from 2000–2008. The simulations based on our estimated model suggest that the fall in credit limits over the same period

23Constructing the path of the LCH/PIH is not entirely trivial or without assumptions. By definition, in the PIH, liquidity constraints can never bind, otherwise a precautionary motive arises (Carroll and Kimball 2001). We construct the PIH line in Figure 1 by taking the 2008Q1 debt as the optimal distribution. Since we do not vary the age structure of the population or the growth rate, that amount of debt, adjusted for inflation, is the correct amount of debt for the entire period.
was responsible for a fall in consumption of 2.5 percent relative to trend, or about one-quarter of the fall. The fall in consumption is also quite rapid initially, matching the BEA series well as high utilization consumers are pushed to deleverage and convenience users reduce their consumption to build up their buffer. The fall in consumption from the simulations quickly rebounds, however, as consumers rebuild their liquidity, so a fall in credit does not explain the continuing weakness in consumption after 2009. Other features not captured by our estimated model of consumer decision making must be important.\footnote{Note that consumption from the simulations is actually higher after several years, because debt is lower, so interest payments decline. This initial decrease followed by a higher steady state is a general feature of credit changes in precautionary models (Fulford 2013). Because credit card interest rates were relatively steady over the period (the lack of response of credit card rates was noted in earlier work by Ausubel (1991)), our estimated model captures the consumption response to the fall in credit, but misses production and savings responses which we are not modeling. Guerrieri and Lorenzoni (2017) examine the interaction between credit contractions and precautionary preferences in general equilibrium with plausible, but not estimated, preferences.}

The policy implications are even more striking when we instead assume that the fall in credit was concentrated among the high-utilization consumers. These consumers are often the highest risk, so are the most likely to be targeted for limit cuts when banks want to reduce risk. If the fall in credit had been concentrated among these consumers, it would have explained nearly half of the consumption fall during the Great Recession, yet utilization would have been just as smooth. This heterogeneity in response is thus a central feature in understanding the impact of both monetary and fiscal policy, a topic we turn to next.

5.1 Implications for stimulus policy

The ability to temporarily boost consumption is an important tool for counter-cyclical policy. One way to provide such a boost is with direct cash infusions through tax rebates (Parker et al. 2013). For such a policy to be effective as a stimulus, individuals must increase spending soon after the rebate. Kaplan and Violante (2014) summarize the literature and suggest that the additional non-durable consumption within a quarter is around 25 percent of the rebate. Yet standard models, even with income uncertainty, predict very small responses. Figure 4 illustrates why. Our patient population B has preferences that look similar to standard assumptions based on calibration or estimation that attempts to match the level of wealth. The distribution of liquidity for our patient
Table 4: Effects of temporary cash infusion or permanent credit increase

<table>
<thead>
<tr>
<th></th>
<th>Full pop.</th>
<th>Pop. A</th>
<th>Pop B.</th>
<th>Full pop.</th>
<th>Pop. A</th>
<th>Pop B.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Δ Expenditure from previous quarter</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Transitory income increase</td>
<td>0.226***</td>
<td>0.270***</td>
<td>0.094***</td>
<td>0.296***</td>
<td>0.340***</td>
<td>0.162***</td>
</tr>
<tr>
<td></td>
<td>(0.0250)</td>
<td>(0.0334)</td>
<td>(0.0333)</td>
<td>(0.0248)</td>
<td>(0.0330)</td>
<td>(0.0337)</td>
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<tr>
<td>Permanent credit limit increase</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>533,288</td>
<td>329,560</td>
<td>203,728</td>
<td>533,288</td>
<td>329,560</td>
<td>203,728</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.000</td>
<td>0.001</td>
<td>0.000</td>
<td>0.001</td>
<td>0.001</td>
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</tr>
<tr>
<td>Age effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Notes: This table shows the results of experiments using the estimates from column 1 in Table 3. We give a randomly selected portion of our simulated population a cash gift of 5 percent of permanent income or a 5 percent increase in individual credit limit. The regression is then $\Delta Cons_t = \alpha + f(age) + \beta Cash_t + \epsilon_t$ measuring how much of the increase in cash or credit limit is consumed within one quarter.

Both the reduced-form estimates from the credit bureau data and the structural estimates suggest that changes in consumer credit produce large consumption responses. An alternate way to increase liquidity is to increase credit rather than income. When we increase the credit limits of the population by 5 percent in Table 4, we get consumption effects that are almost as large as direct cash infusions, again driven mostly by our impatient population. While the structural model allows
us to increase credit in a way that is uncorrelated with anything else, our reduced-form estimates from the credit bureau data give nearly the same estimates in response to an increase in credit that reduces utilization (see Table 1).

6 Conclusion

This paper uses the consumer’s decision about how to use credit cards to provide a window into more general savings and consumption decisions. We show that credit changes are very large over the business cycle, the life cycle, and for individuals. Changes in credit are therefore some of the largest changes in liquidity faced by households. On average, people react quickly to these credit changes, so credit utilization is stable over the business cycle, life cycle, and for individuals.

We take the insight this tight link between credit and debt gives and estimate a model of life-cycle consumption, debt, default, and payments. The model has a number of notable successes. It captures the hump shape of debt and consumption. It predicts the slow decline in utilization over the life cycle and the steady increase in wealth. Out of sample, it predicts smooth utilization over the business cycle, and it closely matches the reduced-form relationship at the individual level between credit and debt that we estimate from the credit bureau data.

Many of our results come directly from the insight that not everyone who has a credit card uses it to borrow, while some people are willing to borrow at a high rate of interest. Borrowing implies the consumer places substantial weight on consumption today versus tomorrow. Other people have a credit card and use it only to make payments, suggesting they place more equal weight on today and the future. This heterogeneity of use suggests that preference heterogeneity is an important part of understanding consumption decisions, and that a large fraction of the population must have a relatively high marginal propensity to consume. The preference heterogeneity is key to the estimated model’s ability to match the data on so many dimensions, including the impact of a cash infusion (Kaplan and Violante 2014, Parker et al. 2013). The implications of the heterogeneity of credit use we document for counter-cyclical policy are also important. The more that banks reduce risk or are encouraged to reduce risk by not extending or reducing credit among high-utilization
customers, the larger the consumption impact of a credit crunch. Conversely, credit increases or
other cash infusions targeted to the highest utilization consumers have an especially large impact.

An important unanswered question for future research is the source and nature of the impa-
tience found in the high discount rate population. Certainly, behavioral economic approaches such
as quasi-hyperbolic discounting and present bias could be consistent with our finding of a high
discount rates for some, though not all, consumers. Our results do not rule out such approaches,
but also show they are not necessary to explain credit card use.

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Credit Cards, Credit Utilization, and Consumption

Scott L. Fulford and Scott Schuh

Appendix

For Online Publication

A Changes in credit utilization: non-parametric evidence

Figure A-4 shows conditional mean scatter plots of credit utilization in one quarter against credit utilization in the next quarter, in the next year, and in two years. The top row shows the mean in the future, conditional only on having the utilization shown on the x-axis in that quarter. The bottom row instead takes the within transformation and allows for age and year effects. It therefore shows how far from the individual’s average credit utilization she is in the next quarter, conditional on differing from her average utilization by the amount on the x-axis this quarter. In other words, if an individual is 10 percentage points above her typical utilization in one quarter, how far will she be on average in the next quarter, next year, and in two years? Each dot contains an equal portion of the sample. Figure A-4 thus captures the relationship between utilization today and in the future without imposing any parametric assumptions. Each panel also shows the best fit line for the conditional means and the estimated coefficients.

The top panels show that credit utilization is highly persistent and does not trend to zero on average. Credit utilization this quarter is typically very close to credit utilization next quarter, because the conditional means are typically very close to the 45-degree line. For example, on average, if a person is using 40 percent of her credit this quarter, she will be using about 40 percent of her credit next quarter. On closer examination, average credit utilization is higher next quarter for those using less than 20 percent of their credit, and lower for those using more than 80 percent
of their credit. The best fit line through the conditional means suggests that credit utilization is not
trending to zero. Instead, the long-term steady-state utilization is 0.39.\(^{1}\) The same conclusion is
evident from the conditional changes comparing utilization this quarter to a year from now and to
two years from now. Those consumers using less than approximately 40 percent of their available
credit this quarter are using more of their credit in one year and in two years. Those using more than
40 percent of their credit are using less of their credit on average within one year and two years.
The steady-state credit utilization is around 40 percent (evident by finding where the conditional
expectation function crosses the 45-degree line), although the movement toward the steady state is
fairly slow.

On average, individuals do not trend to zero utilization or to using all their credit. Conditional
on using zero credit this quarter, credit utilization is nearly 5 percent within one quarter and nearly
8 percent in a year. On the other hand, the average person using all her credit in one quarter is
using less than 90 percent of it in a year.

The second row of Figure A-4 allows individuals to return to their own mean and adds sub-
stantial nuance. Credit utilization is so persistent in the top row because individuals have their own
mean to which they actually return quite rapidly. The speed of the return is evident from the slopes
of the lines. Only 67 percent of a shock to utilization remains after one quarter, and 13 percent
remains after two years.

Even if individuals return very rapidly to their own means, it is important to note that those
means are not zero. Credit utilization is persistent in the top row of Figure A-4 because individuals
are typically quite close to their own mean credit utilization. Since credit utilization is the ratio
of debt and credit, the stability of credit utilization implies that an individual with an increase in
credit has increased her debt by 33 percent of the increase in credit within one quarter, and by 87
percent of the increase in credit in two years.

\(^{1}\)Since the conditional expectation of utilization next quarter given this quarter is \(u_{t+1} = 0.041 + 0.896u_t\), the
steady-state utilization is 0.39=0.041/(1-0.896).
B Additional Model Details

This section provides additional discussion of the model’s formulation and numerical solution.

B.1 Bequests

In our base estimation, we give consumers who die with positive resources some extra utility from bequests. Bequests end up being important only because they help explain our impatient group’s decrease in debt after 60, although the exact parameters are not well identified. Without any bequest motive, as the likelihood of dying increases, it becomes optimal to increase debt for impatient nearly risk-neutral consumers because they are effectively becoming more impatient. A bequest motive keeps the increasing probability of death from effectively translating into increased impatience, and so it allows debt to decline with income after age 60. This appendix outlines one flexible approach to including bequests.

When introducing bequests in a model with both debt and savings, it is difficult to value what happens when people die in debt. Unsecured consumer credit is taken out of any estate passed on to heirs, but is not directly passed on. Simply including bequests in the sub-utility function will therefore produce negative infinite value from leaving no bequest, which consumers will counterfactually act to avoid by never being in debt. Instead, we model the bequest motive as the consumer’s considering the marginal utility a bequest to her heirs will bring them on top of the heirs’ own incomes and any non-liquid bequest she may consider leaving. We model these non-liquid bequests and heirs’ income as a multiple $\zeta$ of permanent income on death, adding the annuity value of the assets left at death and taking the present value using the consumer’s preferences:

$$S(A_t) = \left( \sum_{s=0}^{\tilde{T}} \beta^s \frac{(\zeta P_t + r_B A_t)^{1-\gamma}}{1-\gamma} \right).$$

The parameter $\zeta$ determines the marginal utility of bequests and can be thought of as how much more or less income children have compared to their parents, and $\tilde{T}$ is the same finite life as the parents’. The reason to use finite rather than infinite heirs’ lifetimes is to allow for possibly very
patient parents with $\beta$ close to or greater than one.

**B.2 Recursive formulation and normalization**

Rewritten in recursive form and normalized by permanent income $P_t$, the consumer’s problem when not in default and not defaulting is equivalent to:

$$v_t^{NP}(w_t, b_t, a_{t-1}) = \max_{x_t, \pi_t} \left\{ u(\nu_t x_t) + E_t[\beta_t+1(G_t+1N_t+1)^{1-\gamma}v_{t+1}(w_{t+1}, b_{t+1})] \right\} \text{ subject to}$$

$$x_t \leq w_t$$

$$w_{t+1} = R_{t+1}(I_t^R)a_t + U_{t+1} + b_{t+1} - k_{t+1}$$

$$a_t = w_t - b_t - x_t$$

$$\nu_t = \nu(\pi_t; a_{t-1})$$

where $R_{t+1}(I_t^R) = R/(G_t+1N_t+1)$ if $a_t \geq 0$ and $R_{t+1}(I_t^R) = R_B/(G_t+1N_t+1)$ if $a_t < 0$. The expectation at $t$ includes the possibility of death before $T$ and the certainty of death at $\tilde{T}$, leaving a bequest worth $\beta_{t+1}s(a_t)$, where $s(\cdot)$ is the bequest function normalized by $P_t$. Note that if $b_t$ is not stochastic and instead follows the average path of the credit limit to permanent income ratio, then $b_t$, like $G_{t+1}$, is not a part of the state space that differs for individuals and the decision simplifies slightly to $v_t(w_t, a_{t-1})$. Of course, credit limits and income growth still matter, but they do not vary individually and so show up in each consumer’s expenditure function $x_t(w_t, a_{t-1})$. Because of the structure of the payment problem, $\nu(\pi_t; a_{t-1})$ takes on only two values for an optimizing consumer, $\nu^R$ for revolvers and $\nu^C$ for convenience users. The value expenditure function depends only on whether $a_{t-1} \geq 1$, substantially reducing the dimensionality of the problem.

The consumer’s problem when in the default state is nearly analogous. Income and credit limits are lower. In the next period with probability $p^F$ consumer is still in default and uses the value function $v^D$ and with probability $(1 - p^F)$ the non-default value function.
The consumer’s value function if defaulting is then

\[ v^{\text{Defaulting}}_t(w_t, b_t, a_{t-1}) = u(\nu \phi w_t) + E_t[\beta_{t+1}(G_{t+1}N_{t+1})^{1-\gamma}v^{D}_{t+1}(w_{t+1}, b_{t+1})]. \]

If \( v^{\text{Defaulting}}_t(w_t, b_t, a_{t-1}) > v^{ND}(w_t, b_t, a_{t-1}) \), then the consumer defaults.

B.3 Numerical solution

With the problem written recursively, we proceed through backward recursion to find a numerical approximation of the consumer’s problem. Let \( I^R_t \) indicate revolving status: It is 1 if \( a_{t-1} < 0 \) and 0 else. For a given set of parameters, once \( v_{T+1}(a_T, 0, I^R) \) is given, it is possible to find an approximation of \( v_T(w, b, I^R) \) and use the approximation of \( v_T(w, b, I^R) \) to find \( v_{T-1}(w, b, I^R) \).

Note that in each case we find a separate function for revolvers and convenience users. The solution to each period’s value function is an expenditure function \( x_t(w, b, I^R) \). We follow several standard steps (see Carroll (2012) for a more in-depth discussion of many of these approaches). First, we discretize the lognormal shocks using a Gauss-Hermite quadrature, which turns the integration in the expectation function into a summation over discrete states. Because the income process is surely not exactly lognormal, there is no gain or loss in accuracy from doing so; we are simply replacing one approximation of shocks with another.

Second, we follow the method of endogenous gridpoints (Carroll 2006) to find the optimal expenditure that leads to end-of-period assets \( a_t \) at a number of gridpoints for \( a_t \) and \( b_t \). It is then possible to very elegantly find optimal consumption that leaves this amount of assets \( x_t(w, b, I^R) \) at the endogenous gridpoints for \( w \) simply by using the accounting identity \( a_t = w_t - b_t - x_t \). Doing so avoids a computationally costly numerical root-finding approximation entirely. More precisely, if the consumer has not consumed all available liquidity for the next period, and therefore is not strictly constrained by the credit limit, then the standard first-order conditions and the Euler equation imply that:

\[ u'(\nu(I^R_t)x_t) = E_t[\beta_tR_{t+1}(a_t)(G_{t+1}M_{t+1})^{1-\gamma}u'(\nu_{t+1}x_{t+1}(w_{t+1}, b_{t+1}, I^R_{t+1}))]. \]
where, despite its subscript, \( \nu_{t+1} = \nu(I_{t+1}^R) \) is determined entirely by the choice of whether to leave positive or negative assets for the next period. Given the next-period expenditure function, it is straightforward to find the optimal expenditure that leaves end-of-period assets \( a_t \) as:

\[
x_t^a(a, b, I^R) = \frac{1}{\nu_t} \left( E_t \left[ \beta \beta_{t+1} R_{t+1}(a)(G_{t+1}M_{t+1})^{1 - \gamma} (\nu_{t+1} x(R_{t+1}(a)a + U_{t+1} + b_{t+1}, b_{t+1}))^{-\gamma} \right] \right)^{-1/\gamma}.
\]

(1)

For a vector of end-of-period assets \( \vec{a} \), it is nearly costless to find the optimal consumption at a vector of endogenous points for liquidity where \( \vec{w} = \vec{a} + b + x_t^a(\vec{a}, b, I^R) \) is the amount at which consuming \( x_t^a(a, b, I^R) \) and leaving \( a \) for next period is optimal. We linearly interpolate between these points to find an approximation of the expenditure function. Note that the expenditure function is a function of whether the consumer is revolving by having negative assets last period, in addition to the current state of liquidity and the credit limit. While revolving status is not a continuous state, the addition of another state variable complicates the solution because we must find the optimal expenditure for convenience users and revolvers, who find consuming less valuable because they pay for it in a slightly less convenient way. For the most part, someone who is not revolving this period will not be revolving next period, and so \( \nu_t = \nu_{t+1} \), and the payment choice does not affect the expenditure decision directly. It does, however, make revolving somewhat more costly.

Because the consumer’s problem includes an externally imposed credit limit as well as interest rates that differ depending on whether assets are positive or negative, there are several additional complications. The first is that the standard Euler equation does not hold when the consumer is against her credit limit, and so she spends all available resources because she would like to spend more today but cannot (Deaton 1991). This problem is relatively easy to deal with, however, by including the inflection point that is the last point at which the Euler equation holds. At this point the assets left for the next period are \(-b_t\). For any liquidity less than \( w^* = x_t^a(-b, b, I^R) \), the consumer expends all liquidity, so \( x_t(w, b, I^R) = w \) if \( w \leq w^* \). The second problem is that
the interest-rate differential introduces a step in the consumption function, because there are two solutions to equation (1) for \( a = 0 \). One, the limit with assets approaching zero from below, uses the borrowing rate \( R^B \), and the other uses the saving rate \( R \). The economic intuition is that leaving zero assets for the next period is optimal at a high borrowing rate well before it is optimal at a low savings rate. For liquidity between these two points, the consumer has a marginal propensity to consume of one since the return on savings is not high enough to induce her to save, but the cost of borrowing is sufficient to keep her from borrowing, and so additional resources go straight to consumption. To deal with this issue, the endogenous gridpoints include two points where \( a = 0 \): The first, \( x_t^B = x_t^a(0, b, I_R; R^B) \), is the solution to equation (1) when \( a = 0 \) using \( R^B \) and \( w_t^B = 0 + b + x_t^B \); and the second is \( x_t^F = x_t^a(0, b, I_R; R) \) and \( w_t^F \). Between the points \((w_t^B, x_t^B)\) and \((w_t^F, x_t^F)\), the consumer has a marginal propensity to consume of one.

Figure 4 in the main paper illustrates these kink points, labeling \( w_t^B \) as the point where consumers stop borrowing and \( w_t^F \) as the point where they start saving. Several points are worth discussing. First, the consumption function generally falls with age. This occurs as the consumer plans for retirement, when having accumulated a large amount of savings is valuable. Second, for low liquidity below \( w^* \), the marginal propensity to consume is one. Between \( w^* \) and \( w^B \), the consumer is leaving debt for next period and so is paying a high interest rate \( R^B \). Between \( w^B \) and \( w^F \), the consumer does not want to borrow, but the return on savings is not high enough, so she leaves zero assets and has a marginal propensity to consume of one. This kink in the consumption function implies that there can be a positive fraction of consumers who hold exactly zero assets. The distance between \( w^B \) and \( w^F \) depends on the interest-rate differential, with a wider differential implying a larger distance.

C Estimation details

This section provides a more complete discussion of first stage estimates and the construction of the variance covariance matrix of moments from credit bureau data. It then shows the effect of varying the cost of default parameter and characterizes other local minima in the estimation.
C.1 Estimation procedure

For a given set of parameters \( \theta \in \Theta \) and first-stage parameters \( \chi \) such as the interest rates, payments parameters, and income process estimated separately, we numerically find consumption/expenditure functions at each age. These same \( \theta \) and \( \chi \) determine the initial distribution of assets, income, and credit limits across consumers, and how these processes evolve stochastically. For each consumer, we draw from the initial distribution, then for each period we draw from the income-shock distribution. Then the consumer chooses her consumption, whether to default or be forced into default, and her assets or debt accumulates for the next period. This process proceeds until the final period, generating for a large number of simulated consumers their own idiosyncratic paths of expenditure, assets, debt, and default at every quarter over their entire life cycle. Combining the simulated consumers, a given set of model parameters generates a life-cycle distribution of consumption, debt, savings, and default.

The estimation then finds the parameters \( \theta \) that produce a life-cycle evolution of average simulated consumption, debt, and default that best matches their empirical counterparts from ages 24–74. Each profile is annual, so there are \( T = 51 \) years.\(^2\) More formally, for a given \( \theta \in \Theta \), and first stage parameters \( \chi \) estimated below, let \( g_t(\theta; \chi) \) be the difference between an empirical moment and a simulated moment for each of \( 3T \) total moments. The MSM then seeks to minimize the weighted square of these differences:

\[
\min_{\theta \in \Theta} g(\theta; \chi)' W g(\theta; \chi),
\]

where \( g(\theta; \chi) = (g_1(\theta; \chi), \ldots, g_{3T}(\theta; \chi)) \), and \( W \) is a \((3T) \times (3T)\) weighting matrix. Our standard weighting matrix is block proportional to the inverse variance of the empirical moments (the optimal weighting matrix with no first-stage correction). Because our life-cycle moments come from surveys and administrative data, they are estimated with very different levels of precision.

\(^2\)The model is quarterly, so we aggregate appropriately to match the data by taking the average model debt for a given age. The annual nature of the empirical age profiles is driven by the data source. The Equifax/NY Fed CCP, for example, reports only the year of birth from which we calculate age.
and the estimates tend to attempt to fit only the administrative data. We therefore weight each life-cycle moment block so that the each block receives the same weight, but within each life-cycle moment better-estimated moments receive more weight.\(^3\) We also show results using the “optimal” weighting matrix, which takes the estimated \(\hat{\theta}\) using our standard weights and calculates the optimal weights, taking into account the impact of the first-stage estimates. We adjust the variance-covariance matrix of the estimates of \(\theta\) for the first-stage estimates, following Laibson et al. (2007), who improve on the work of Gourinchas and Parker (2002) by allowing for the empirical moments to have different numbers of observations.

C.2 First-stage estimates and observed parameters

This section describes the sources and estimates from other data sets that identify the ancillary parameters of the model. We estimate a fifth-order polynomial of the average life cycle of income to find income growth \((G_s)\) at each age using after-tax income per adult household member from the Consumer Expenditure Survey from 2000–2015. The raw data and the fitted lines are in Appendix Figure A-6. Similarly, we take a fifth-order polynomial estimate of the total credit limit per account from the Equifax/NY Fed CCP to form \(B_s.\)\(^4\)

While average income follows the observed life-cycle path, individual incomes vary based on their idiosyncratic shocks. We use the estimates of the annual income process from Gourinchas and Parker (2002), which are updates of Carroll and Samwick (1997), calculated from the Panel Study of Income Dynamics. We adjust these volatilities for quarterly dynamics so that four quarterly shocks combine to produce the same variance as one yearly shock. The quarterly transitory variance is approximately four times the annual variance because quarterly shocks average out, while

\(^3\)Starting from \(V_M\), the \((3T) \times (3T)\) block-diagonal variance covariance matrix of the moments, we form \(\tilde{W} = V_M^{-1}\) which is also block-diagonal. For each block of \(\tilde{W}\) we take a vector \(\iota\) of ones of size \(T\) and calculate the weight function for each block of life-cycle moments \(w_1 = \iota' W_1 \iota\) which is the impact on the objective function if that all of the moments \((g_1(\theta; \chi), \ldots, g_T(\theta; \chi))\) in that life-cycle block were equal to one. We then form \(W\) by dividing each life-cycle block by its scalar weight.

\(^4\)Not smoothing these two budget constraints makes little difference to the overall estimates, but it introduces distracting jumps in life-cycle consumption and debt as consumers respond to sudden changes in the budget constraint driven by jumps in income or credit that disappear. \(B_s\) is proportional to permanent income for an individual consumer, and the problem is set up so that the average permanent income across all consumers is the average income, allowing us to back out \(b_s.\)
the quarterly permanent variance is approximately one-fourth the yearly variance because perma-
nent shocks stack. We estimate the probability of low income $p_L$ based on the average monthly
unemployment rate from 2000–2015 of 6.3 percent.

We observe three interest rates directly, although there is likely greater heterogeneity in interest
rates than we incorporate in the model. We set the interest rate on debt $R_b - 1 = 14.73$ percent
based on the average revolving interest rate over the period from the Federal Reserve Series G19.
We estimate the increased interest rate when in the default state of 2.1 percent based on the credit
card rate reported by households in the SCF with bankruptcy in the previous seven years.\(^5\) We
would like to capture the returns that people expect to receive on their savings, but the appropriate
rate of return is not obvious because there is only one riskless asset. We therefore set the return
on savings at 5.4 percent, which is the average return on an all-bond portfolio from 1926–2015 as
calculated by the mutual fund company Vanguard. We adjust both borrowing and saving prices
for the geometric average inflation rate from 2000–2015 of 2.15 percent. In the expected growth
over the life cycle, we also include expected real aggregate growth of 1.5 percent, the average
compounded rate from 1947–2015 from the Bureau of Economic Analysis (2009 chained dollars
GDP per capita).\(^6\)

We consider economically active life to last for 51 years (204 quarters) from age 24, when
most people have finished schooling, through age 74, when differential death rates and other end-
of-life concerns dominate. While the Equifax/NY Fed credit data have many observations even for
older ages, the Consumer Expenditure Survey (CE) becomes increasingly sparse and, for privacy
reasons, topcodes ages above 80, with the top age varying by year of the survey. Before age 94,
individuals have a probability of dying and leaving a bequest at each age. We set the probability
of death to match the age structure of the population in 2010.\(^7\)

\(^5\)The Federal Reserve series G19 (Commercial Bank Interest Rate on Credit Card Plans NSA, rate for accounts
assessed interest) average over the period is 14.73 percent. The average credit card interest rate reported in the SCF is
14.22 percent. Based on calculating the risk of default from the PSID, Edelberg (2006) calculates the zero bankruptcy
risk rate would be 0.62 percentage points lower, a smaller adjustment than in Angeletos et al. (2001), who adjust for
default by 2 percentage points.

\(^6\)While each of these parameters is volatile, and different agents may experience different prices, there is no sam-
pling variance about them, and so we do not adjust the MSM variance-covariance matrix for them.

\(^7\)See (Arias 2014) https://www.cdc.gov/nchs/data/nvsr/nvsr63/nvsr63_07.pdf, accessed 8
We set the probability of staying in default from period to period $p^F$ so that the expected duration of being default is 7 years.

C.3 Construction of variance-covariance matrix

The variance-covariance matrix for the combined moments is simply block diagonal, because they are sampled independently from a large population. The Consumer Expenditure Survey block of the variance-covariance matrix is simply diagonal, since the survey does not repeat the same households over multiple years.

Because we observe individuals over time in the Equifax/NY Fed CCP data, the credit bureau portion of the variance-covariance matrix has off-diagonal elements. We populate this matrix by estimating the co-variance of debts in the population at various lags. Since the data is quarterly, the full matrix is 204x204, and because our data agreement limited what we could make public (and the size of the data limited what we could practically calculate), in practice we do not estimate each element separately. Instead we estimate the covariance at each age from 24–74 at quarterly lags 1, 2, 3, 4, 8, 12, 16, 32, and 48 and assume that the co-variance at each age changes smoothly in between them. Since the data cover only 16 years, all covariances beyond 64 quarterly lags are zero. The combination of estimating covariances from a sample, constructing intermediate covariances, and numerical precision leaves the resulting covariance matrix with minimum eigenvalues that are slightly negative, and so the matrix is not positive definite. We make a “ridge adjustment” by adding a small amount to the diagonal until all eigenvalues are greater than or equal to zero. This adjustment effectively increases the variance of our main moments, and so it is generally conservative although not entirely innocuous, since it changes the variances but not the covariances.

C.4 Characterization and discussion of local minima

While Table 3 presents the best estimate after starting the estimation at a grid of points, given the over-identification, it is useful to briefly characterize other possible minima. Table A-1 shows the August 2017.
other minima where the optimization converged to locally. While many starting points converged
to our best estimate, it is clear that the starting point for $\theta_0$ does not determine $\theta^*$ and so one should
be cautious about drawing global conclusions from what may be only local minima.

Among the estimates with objective functions close to that produced by $\theta^*$, the only important
variation is that there is a tradeoff between risk aversion and impatience. Because the coefficient
of relative risk aversion is the inverse of the intertemporal elasticity of substitution, $\gamma$ and $\beta$ have
similar roles in utility. Loosely, $\beta$ governs how much the consumer cares about expected marginal
utility in the future, while $\gamma$ shifts expected marginal utility by making bad states better or worse.
Beyond the best estimate, there are larger differences in the parameters at the local maximum, but
the fit was always substantially worse. The impatient population could be somewhat less patient
and have higher risk aversion. The more patient population B faces a similar tradeoff between $\beta^B$
and $\gamma^B$.

The proportion of population A ($f^A$) changes across the local minima in more substantial ways.
While allowing for preference heterogeneity is a step forward in our work, imposing only two pop-
ulations is still a simplification, albeit one that is useful both expositionally and computationally.
At each minimum, $f^A$ is tightly estimated. However, the range of $f^A$ in the local minima suggests
that different weights would produce different estimates of $f^A$. The overall conclusion holds for
all estimates, however: In order to match the amount of debt we see in the data, half or more of the
population must be fairly impatient and close to risk neutral.

C.5 Discussion of identification of the life-cycle model

Because this is a nonlinear model, all moments are typically used to identify all parameters, but
it is useful to understand how different sources of variation identify the parameters. Both the
consumption and debt that we observe over the life cycle are population averages, so the model is
identified from the average of the two model populations. The share of population A ($f^A$) and its
relative income ($\zeta^A$) change the mix of the two populations. For the model to produce as much
debt as in the data, a large portion of the population ($f^A$) must be relatively impatient and not

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overly concerned about debt. This population’s impatience ($\beta^A$) is mostly pinned down by the borrowing rate ($R^B$) to make its members willing to hold debt. If the population is too patient, it will not accumulate enough debt. If it is too impatient, it will acquire too little debt. It must have enough income to support the amount of debt it holds, helping to identify $\zeta^a$. To get an average consumption profile in which consumption is below income for much of the life cycle therefore requires the other portion of the population to be relatively patient, with its discount rate ($\beta^B$) close to the savings rate $R$.\footnote{Since we include expected aggregate growth and adjust for inflation, $\beta^A$ and $\beta^B$ are more closely pinned down relative to $R^B - \text{Inflation} + \text{Real Aggregate Growth}$. We thank Chris Carroll for pointing out that even if we remove trends from life-cycle profiles, the economic decision of the agent includes expected aggregate growth, and so we need to include it to correctly model their decisions. Aggregate growth implies everyone expects to have more income next period and so should be more impatient.}

While the levels of consumption and debt come from the average, the life-cycle profiles are largely determined by only one of the populations. Because the patient population carries almost no debt—the flow of debt from payments is relatively small compared to the stock of revolving debt—the profile of credit card debt largely identifies the preferences of the impatient population, their initial wealth, and their expected residual income late in life. Given this population’s impatience, consumption must closely follow income. The hump shape of debt comes from increases in credit limits early in life, which allow this population to increase its debts, and the fall in income after age 50, which makes carrying as much debt less affordable. This population’s risk aversion ($\gamma^B$) is identified by how much credit it keeps as a buffer.

The more patient and risk-averse population carries little revolving debt, so all of its debt comes from the convenience use as a share of consumption. The impatient population A has a strong hump in consumption as it follows income. For the average consumption profile to be below average income, the patient population B must have a relatively flat or increasing consumption profile without a downturn late in life. Its preference for risk ($\gamma^B$) and expected late-life income after expenses ($\zeta^B$) are determined by this shape, with its discount rate ($\beta^B$) pinned down by the rate of return on savings. Its risk aversion determines the size of the buffer of savings it builds up early in life, and so the initial level and slope of consumption over the life cycle help identify
\( \gamma^B \). The risk aversion and initial wealth \((\lambda^B)\) are not particularly well identified by the life-cycle moments, and their standard errors are relatively large.

### C.6 Robustness and variations

Our numerical procedure for finding the minimum of equation (2) proceeds by using numerical derivatives calculated from a starting \( \theta_0 \) to move to a local minimum where the derivatives in all dimensions are zero to within a small tolerance. This procedure is only guaranteed to find a local minimum, however. We therefore start the procedure with \( \theta_0 \) at random points in a grid that covers the 12 dimensional parameter space. Not all starting points produce the same estimate of \( \theta \), indicating that the objective function in equation (2) has multiple local minima. The procedure converged to our best \( \theta^* \) from a wide range of starting \( \theta_0 \), and so \( \theta^* \) is a candidate for the global minimum. We discuss other local minima in Appendix C.4. The overall conclusion holds for all local minima: Around half of the population must be fairly impatient and have low risk aversion.

In the last two columns of Table 3 we examine how changing the model changes estimates. Our baseline estimates do not allow consumers to take into account the effect their consumption decisions will have on their payments decisions. In column 3, we allow for this feedback, at the cost of substantial additional computation time. Allowing this feedback leaves the estimates almost exactly the same. Because so few people switch from revolving to convenience use, the value an individual gets from credit card consumption this period is almost always the same as next period. Since the value of consumption on a credit card does not affect the marginal utility tradeoff between today and the future, it does not affect the decision. Including convenience use as part of credit card debt is necessary, however, because the debts we observe in the credit bureau data include both revolving and convenience debts. Allowing consumers to take into account the impact of consumption choices on payment choices in the future does not appear to be particularly important for their consumption decisions.

We do not estimate directly the strength of the bequest motive since it is not well identified. Our bequest function, described in greater detail in Appendix B.1, gives people the discounted
utility from their heirs consuming the annuity value of assets at death as well as the heirs’ own income. The strength of the bequest motive is determined by how much more income the heirs have compared to the individual; as the heirs’ income increases, the marginal value of leaving anything to them diminishes. Our baseline estimates assume heirs have the same permanent income upon death as the individual. The last column assumes heirs instead have five times the permanent income. The estimates are similar, suggesting that our estimates are robust to other assumptions about bequests, and confirming that, given our approach and data, the bequest motive is not well identified.

D Identification of the payments model

This section shows how to identify the payment-model parameters and standard errors from observable moments. It then calculates the consumer surplus and its standard errors. We observe:

\[ \pi_R = \frac{1}{N} \sum_{i=1}^{N} \pi_{i,t} | I_{i,t-1}^R = 1, \]

the average expenditures by revolvers on a credit card, and similarly \( \pi_C \), the average for convenience users. We denote our estimates of the standard errors of these means as \( \sigma_R \) and \( \sigma_C \). Then the intercept for the average convenience user is just \( \pi_C = \nu_0 / \nu_1 \), and for a revolver it is \( \pi_R = (\nu_0 - r^B/24) / \nu_1 \), where \( r^B \) is the APR interest charged on payments, which have an average daily balance of half of the month’s consumption. Solving for \( \nu_0 \) and \( \nu_1 \) gives:

\[
\nu_1 = \frac{r^B/24}{\pi_C - \pi_R},
\]

\[
\nu_0 = \pi_C \nu_1 = \frac{(r^B/24) \pi_C}{\pi_C - \pi_R}.
\]
Solving for the areas under the curves:

\[
\nu_t = \max_{\pi_t} \nu(\pi_t, A_{t-1}) = \begin{cases} 
\nu^C = 1 + (\pi^C \nu_0)/2 & \text{if not revolving} \ (A_{t-1} \geq 0) \\
\nu^R = 1 + (\pi^R(\nu_0 - r_B/24))/2 & \text{if revolving} \ (A_{t-1} < 0),
\end{cases}
\]

where \( \pi^C \) and \( \pi^R \) are the optimum fraction for revolvers and convenience users.

The presence of a difference of two random variables whose supports may overlap in the denominator of the transformed variables makes calculating their variances potentially tricky. Since \( \pi^C - \pi^R \) may be close to zero, then \( \nu_1 \) and \( \nu_0 \) may be very large, which is a different way of saying that the model is not identified if there is not a difference in the average behavior of convenience users and revolvers. We calculate the standard errors of the transformed variables using the delta method, which avoids this issue by examining only small changes around the optimum, and so it does not consider the highly nonlinear increase around \( \pi^C - \pi^R = 0 \). For small changes \( \epsilon^C \) and \( \epsilon^R \) around \( \pi^C \) and \( \pi^R \):

\[
\nu_1 \approx (r_B/24) \left( \frac{1}{\pi^C - \pi^R} - \frac{\epsilon^C - \epsilon^R}{(\pi^C - \pi^R)^2} \right).
\]

Since \( \pi^R \) and \( \pi^C \) are independent, the variance of \( \nu_1 \) is approximately:

\[
Var[\nu_1] \approx \left( \frac{r_B/24}{(\pi^C - \pi^R)^2} \right)^2 (\sigma^2_C + \sigma^2_R).
\]

Taking the same expansion for \( \nu_0 \), including the covariance of the numerator and denominator:

\[
Var[\nu_0] \approx \left( \frac{\pi^C r_B/24}{(\pi^C - \pi^R)^2} \right)^2 \left( \sigma^2_C + \left( \frac{\pi^R}{\pi^C} \right)^2 \sigma^2_R \right).
\]

Finally, the total additional convenience value of using a credit card over the alternatives for a convenience user is just the area under the curve:

\[
\nu^C = \nu(\pi^C ; I_{i,t-1} = 0) = 1 + (\pi^C \nu_0)/2 = 1 + \frac{(r_B/48)(\pi^C)^2}{\pi^C - \pi^R}.
\]
and for revolvers it is:

\[ \nu^R = \nu(\pi^R, i^R_{i-1} = 1) = 1 + \frac{\pi^R}{2} \left( \frac{(i^B/24)\pi^C}{\pi^C - \pi^R} - \frac{i^B}{24} \right). \]

Taking an expansion around \( \pi^C \) and \( \pi^R \) yields:

\[ \text{Var}[\nu^C] \approx \left( \frac{r^B}{48} \right)^2 \left( \frac{2\pi^C}{\pi^C - \pi^R} - \left( \frac{\pi^C}{\pi^C - \pi^R} \right)^2 \right)^2 \sigma_C^2 + \left( \frac{\pi^C}{\pi^C - \pi^R} \right)^2 \sigma_R^2, \]

and

\[ \text{Var}[\nu^R] \approx \left( \frac{r^B}{48} \right)^2 \left( \frac{\pi^C}{\pi^C - \pi^R} - \frac{\pi^C \pi^R}{(\pi^C - \pi^R)^2} \right)^2 \sigma_C^2 + \left( \frac{\pi^C}{\pi^C - \pi^R} - \frac{\pi^C \pi^R}{(\pi^C - \pi^R)^2} - 1 \right)^2 \sigma_R^2. \]
References


Table A-1: Characterization of other local minima

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>CRRA ( \gamma^A )</td>
<td>0.0667</td>
<td>0.0794</td>
<td>0.0822</td>
<td>0.3234</td>
</tr>
<tr>
<td>Discount ( \beta^A )</td>
<td>0.8917</td>
<td>0.8894</td>
<td>0.8894</td>
<td>0.873</td>
</tr>
<tr>
<td>Initial wealth ( \lambda_0^A )</td>
<td>0.5158</td>
<td>0.4998</td>
<td>0.934</td>
<td>0.6322</td>
</tr>
<tr>
<td>Late life inc. ( \lambda_1^A )</td>
<td>0.7271</td>
<td>0.6007</td>
<td>0.5378</td>
<td>0.6175</td>
</tr>
<tr>
<td>CRRA ( \gamma^B )</td>
<td>2.0233</td>
<td>1.7495</td>
<td>1.8937</td>
<td>1.5929</td>
</tr>
<tr>
<td>Discount ( \beta^B )</td>
<td>0.9631</td>
<td>0.92</td>
<td>0.9323</td>
<td>0.8952</td>
</tr>
<tr>
<td>Initial wealth ( \lambda_0^B )</td>
<td>1.7284</td>
<td>3.499</td>
<td>2.1163</td>
<td>2.2534</td>
</tr>
<tr>
<td>Late life inc. ( \lambda_1^B )</td>
<td>0.212</td>
<td>0.5004</td>
<td>0.3111</td>
<td>0.2932</td>
</tr>
<tr>
<td>Share A ( f^A )</td>
<td>0.669</td>
<td>0.62</td>
<td>0.5337</td>
<td>0.6103</td>
</tr>
<tr>
<td>Inc. mult. A ( \zeta^A )</td>
<td>0.9912</td>
<td>0.9001</td>
<td>1.0895</td>
<td>0.8285</td>
</tr>
<tr>
<td>Prob. of exp. shock</td>
<td>0.0402</td>
<td>0.01</td>
<td>0.0175</td>
<td>0.0023</td>
</tr>
<tr>
<td>Size of exp. Shock</td>
<td>0.6603</td>
<td>0.8</td>
<td>0.5159</td>
<td>0.5597</td>
</tr>
</tbody>
</table>

Objective \( (g'Wg) \)   | 0.0032   | 0.0074                 | 0.0085 | 0.0105 |
SSR \( (g'g) \)          | 0.3493   | 0.7131                 | 0.648  | 1.2345 |

Notes: This table shows the top four results from starting the optimization at different points in the parameter space. The objective \( (g'Wg) \) is what the estimation attempts to minimize. The sum of squared residuals (SSR or \( g'g \)) is the sum of the squared difference in moments (which would be the objective function with the identity matrix).

Figure A-1: Fraction with positive credit card limit and debt by cohort and age from CCP
(A) Fraction with positive limit
(B) Fraction with positive debt

Notes: Each line represents the fraction with positive credit card limits or debt of one birth cohort, 1999–2014. Source: Authors’ calculations from Equifax/NY Fed CCP.
Figure A-2: Credit card limit, debt, and credit utilization distributions and standard deviations by age

(A) Credit card limits

(B) Credit card debt

(C) Credit utilization

Notes: Each line is the percentile of credit limit at that age, conditional on having a positive credit limit on a log scale. For example, the 90th percentile line shows that 10 percent of the population (with a positive credit limit) has a limit larger than that line. Source: Authors’ calculations from Equifax/NY Fed CCP.
Figure A-3: Credit card limits, debt, and utilization: age and year effects

Credit card limits

Credit card debt

Credit card utilization

Notes: This figure shows the age and year effects from estimating a simple regression of the form:

$$\ln D_{it} = \theta + \theta_t + \theta_a + \epsilon_{it},$$

where $\ln D_{it}$ is either log debt, log credit limits, or utilization, and allows these to vary between age effects $\theta_a$ and year effects $\theta_t$, but imposes common cohort effects. The excluded group is age 20 and year 2000, so each panel starts at zero at age 20 and year 2000. The estimated effect is in log units, and so the scale of the figure suggests that variation over the life cycle in credit is around nine $(e^{2.5}/e^{0.3})$ times larger than over time, even with a massive credit contraction.

Source: Authors’ calculations from Equifax/NY Fed CCP.
Figure A-4: Changes in credit utilization in one quarter, one year, and two years

Notes: Each point in the top row shows the mean credit utilization in the future, conditional on being in the bin with a mean credit utilization on x-axis today. The bottom row shows the conditional relationship between deviations from the individual mean utilization over the entire sample, adjusting for age and year. Source: Authors’ calculations from Equifax/NY Fed CCP using the program binscatter (Stepner 2013).
Figure A-5: Expenditure functions over the life cycle with borrowing and active default

Notes: This figure uses the estimates in Table 3 column 1, with a lower cost of default to illustrate the decision when voluntary default is important. The incentive to default is increasing with age and, with the assumed cost of default parameter, default is not optimal at 30 but may be at 60 for the impatient population. The point of indifference between defaulting and not is shown by the sharp drop in consumption. Below that point, the consumer defaults, spends all liquidity, but suffers the consumption cost of default in the current period, and the lower credit limit, higher interest rate, and income cost in the future periods. Above the default point, the consumer leaves some liquidity for the next period. The small jumps in the expenditure function just above the default point are caused by the interaction of the discrete income process and the large jump in consumption.
Figure A-6: Consumption and income over the life cycle from the Consumer Expenditure Survey

Notes: This figure shows the average consumption and income at each age from the CE, pooling all surveys from 2000–2014. Consumption is the total household expenditures divided by the number of adults. Adjusted consumption removes the estimated effect of children. Income is after-tax income, and its smoothed version is based on a quintic from ages 24–81. Since the survey pools income and consumption after age 81 (or 83 in later years), ages 81 and older are the average for this group.
Figure A-7: Consumption and debt over the life cycle: model estimates with “optimal” weights

Estimation moments: Debt

Estimation moments: Consumption

Estimation moments: Bankruptcy

Estimation predictions: Utilization

Estimation predictions: Wealth path

Estimation predictions: Fraction revolving

Notes: This figure shows the life-cycle profiles using the “optimal” weighting matrix in column 2 of Table 3.
Figure A-8: Identification of default cost parameter: Sum of squared residuals for different default costs, holding other parameters fixed

![Graph showing the sum of squared residuals (g'Wg) for different default costs](image)

Notes: This figure shows the sensitivity of estimates to the default cost parameter. The default cost parameter is identified only up to an inequality. Holding other parameters fixed at their values in column 1 of Table 3, the figure varies the default cost parameter and plots the weighted sum of squared residuals ($g'Wg$).

Figure A-9: Identification of default cost parameter: Life-cycle in bankruptcy if default costs are lower than the standard cost

![Graph showing the fraction in default over age](image)

Notes: This figure shows the sensitivity of estimates to the default cost parameter. The figure shows the path of the fraction of consumers with a bankruptcy on their record if default costs are lower. The low default cost is 5, while the standard one is 7.