

Spring 2021

## Differential Equations

WVU Mathematics Department

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# ODE ENTRANCE EXAM, SPRING 2021

April 21 2021

*Solve all six problems. Show all work. Explain and justify your answers. All problems carry equal weight.*

Name \_\_\_\_\_ Total Score \_\_\_\_\_

**1** Let  $\alpha \in \mathbb{R}$  and let  $x(t) = [x_1(t), x_2(t), x_3(t)]$  denote the solution of the initial value problem  $dy/dt = Ay$ ,  $x(0) = [1, 1, \alpha]$ , where

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

- Find  $x(t)$ .
- Find  $\lim_{t \rightarrow \infty} \frac{x_1(t)}{x_2(t)}$ .

**2** Show that the initial value problem

$$\frac{dx}{dt} = t^2 + x^2, \quad x(0) = 0 \tag{1}$$

has a unique solution defined on an interval for  $t$  containing  $(-\sqrt{2}/2, \sqrt{2}/2)$ .

**3** a. Find all equilibrium points of the system

$$\begin{aligned} \dot{x} &= x(3 - x - 2y) \\ \dot{y} &= y(2 - x - y) \end{aligned}$$

and study their stability.

b. Sketch a phase diagram with arrows indicating the direction of increasing time.

**4** Consider the ODE system

$$\begin{aligned} \dot{x} &= -x^3 + xy^2 \\ \dot{y} &= -x^2y - y^3 \end{aligned}$$

Show that  $(0, 0)$  is a globally asymptotically stable steady state. *Hint: Look for a quadratic Lyapunov function for the system.*

**5** Show that the system

$$\begin{aligned}\dot{x} &= y \\ \dot{y} &= -x + (1 - x^2 - y^2)y\end{aligned}$$

has a periodic solution.

**6** Find the general solution of

$$y'' + 4y = 8 \tan t, \quad t \in (-\pi/2, \pi/2).$$