

Fall 2022

Differential Equations

WVU Mathematics Department

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Recommended Citation

WVU Mathematics Department, "Differential Equations" (2022). *M.S. Advanced and Ph.D. Entrance Exams*. 40.

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ODE ENTRANCE EXAM, FALL 2022

August 18 2022

Solve all six problems. Show all work. Explain and justify your answers. All problems carry equal weight. Notation: $\frac{dy}{dt} = y'$.

Name _____

Total Score _____

1. Let $y = f(t)$ be a continuous non constant periodic function on \mathbb{R} with period $P > 0$.

Show that:

a) $y = f(t)$ is bounded on \mathbb{R} ;

b) $\lim_{t \rightarrow \infty} f(t)$ does not exist;

c) no vector solution to any initial value problem of the system

$$y_1' = y_1^2 y_2^4 + 1, \quad y_2' = \sin(y_1^2 y_2^4) + 1 \quad (1)$$

can be periodic.

2. Estimate an interval of existence for the scalar initial value problem

$$z'' = \frac{z+1}{2-z} + tz' + 2, \quad z(0) = 1, \quad z'(0) = 2. \quad (2)$$

3. Consider the linear system

$$y' = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & -2 \end{bmatrix} y - \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}. \quad (3)$$

a) Determine the critical points of (3).

b) Determine all vector solutions of (3) that satisfy $\lim_{t \rightarrow \infty} |y(t)| = \infty$ where $|\cdot|$ is a suitable norm.

4. Let $\phi(t)$ denote a solution of the initial value problem

$$\begin{aligned} y' &= (y-1)(y-2) \dots (y-2022) \\ y(0) &= \pi. \end{aligned}$$

Show that ϕ defined for $t \in \mathbb{R}$ and find $\lim_{t \rightarrow \infty} \phi(t)$ and $\lim_{t \rightarrow -\infty} \phi(t)$.

5. a) State an existence and uniqueness theorem for first-order ODE systems.
b) Find $t_0, y_0 \in \mathbb{R}$ such that the initial value problem

$$y' = y^{2/3}, y(t_0) = y_0$$

does not have unique solution. Why does the theorem you stated in a) fail to apply?

6. Let A be a 2×2 real matrix, and suppose the linear ODE system

$$y' = Ay$$

has solutions ϕ_1, ϕ_2, ϕ_3 satisfying

$$\phi_1(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \phi_2(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \phi_3(0) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

If additionally

$$\phi_1(1) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \phi_2(1) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

find $\phi_3(1)$.