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David E. Boyce

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INTERREGIONAL COMMODITY FLOW, INPUT-OUTPUT AND TRANSPORTATION MODELING: AN ENTROPY FORMULATION

David E. Boyce
Department of Civil Engineering

and

Geoffrey J.D. Hewings
Department of Geography

University of Illinois
Urbana, Illinois, 61801.

1 INTRODUCTION

1.1 Purpose

The open nature of regional economies has always presented a difficult problem for the design and implementation of regional analytical models. In the last two decades, a number of approaches have been developed, ranging from single region economic base models to large-scale multiregional input-output, linear programming and econometric models. Nested within this spectrum is a complex of models which has not received a great deal of attention and has certainly been overlooked in terms of empirical implementation. This complex of models is derived from the general class of entropy maximizing models first proposed by Wilson (1970). This appendix provides some possible directions for the empirical implementation of an entropy constrained formulation within the context of a multiregional commodity flow/input-output model. Particular attention is focused on two sets of links: (i) links with other components of an integrated set of multiregional models; and (ii) links with the problems of modal and route choice in the flows of commodities over a network.
Interest in interregional commodity flow/input-output modelling has been a consistent part of the general literature on regional and interregional analysis. Isard (1951) provided what may be referred to as a full information accounting scheme for an interregional model but the empirical testing of this model has been limited by the dearth of interregional flow data. In Isard's model, the typical direct coefficient matrix, $a$, from the national input-output model was partitioned into regions such that a typical element, $a^L_{hi}$, would represent the flow of goods from industry $h$ in region $J$ per unit of output of industry $i$ in region $L$. Moses (1955) employed an ingenious extension of Isard's framework, utilizing trade coefficients based on US census data and ICC waybill samples. These trade coefficients, $t^L_{hi}$, described the proportion of purchases of good $h$ in region $J$ by industries located in region $L$. In this formulation, it was assumed that all users of good $h$ had similar geographic trading patterns. In the Moses' system, the trade coefficients were represented as a diagonal block matrix. From these entries, Moses was able to estimate the regional input coefficients, $r^L_{hi}$, where:

$$r^L_{hi} = a^L_{hi} t^L_{hi}$$

The coefficient, $a^L_{hi}$, is region $J$'s technical coefficient. As these coefficients were not available locally, 1947 US data were used for all regional $a^L_{hi}$'s. In the empirical test of the model, Moses aggregated spatially into three regions and sectorally into eleven industries.
Later attempts at interregional modelling have employed more sophisticated matrix algebra techniques. These models will be reviewed briefly: they may be grouped into three categories (i) allocation/RAS types, (ii) linear programming models and (iii) gravity type models of the Leontief-Strout/Polenske type.

A popular allocation algorithm which has been used extensively in the preparation of regional input-output tables from national data and the updating of national tables is the RAS technique developed by Stone (1963) and extended by Bacharach (1970). Essentially, the algorithm finds a matrix $B$ from an observed matrix $A$ given only information about the marginal totals of $B$. The matrix $B$ is assumed to be 'as close as possible' to $A$ subject to the marginal control constraints; see Hewings and Janson (1980) for a discussion of this issue. The general form of the model is:

$$B = \hat{R} \hat{A} \hat{S}$$

where $A$ and $B$ are the matrices defined earlier, $\hat{R}$ is a diagonal matrix which ensures that the row constraints are satisfied, and $\hat{S}$ is a diagonal matrix which ensures that the column constraints are satisfied. Hence, in general, with $A$ and $B$ of the order $n\times n$, the constraint set is comprised of $2n$ equations. In input-output terms, the right-hand sides of these equations are usually the vector of intermediate inputs and the vector of intermediate outputs.

One of the first attempts to apply an RAS-type procedure in the interregional context was made by Nevin, Roe and Round (1966) in the development of an input-output model for Wales. Utilizing the existing U.K.
input-output model, the authors used estimates of Welsh final demands and primary inputs to estimate the flows between Wales and the rest of the U.K. A balancing procedure was used with the feature that flows could only move from one region to the other and hence, cross-hauling was precluded. Subsequently, Round (1972, 1979) has suggested several modifications of this procedure.

A more sophisticated allocation procedure has been employed by Corbis and Vallet (1976) in the development of the French interregional model, REGIJA. Given estimates of regional final demand and primary inputs, interregional flows were allocated with reference to various statistics on transportation rates for railroads, trucks and canal traffic, energy flows and agricultural commodity flows.

Hoffman and Kent (1976) have developed an interregional model for Canada based on the commodity-industry accounting model available at the national level. One of the features of this model is the designation of industries as either national or regional (along the lines first proposed by Isard, 1951). In this model, interregional flows are estimated only for those activities which are regarded as national: again, the allocation of flows relied heavily on information of shipments, raw materials and labor availability at the regional level.

Given the need to allocate a set of marginal totals (supplies and demands) among a set of regions, it is obvious that a major candidate for the implementation of a model for these flows would be a linear programming model. Hewings (1970) and Mathur (1972) have proposed various alternative ways in
which linear programming models could be applied to this problem. One of the major difficulties is the small number of flows which would result \((n+m-1)\) in the case of \(n\) origins and \(m\) destinations) unless the constraint set could be expanded to facilitate greater interregional interdependence. The issue of programming models will be addressed in more detail in the sections dealing with the formulation of the entropy model where it will be shown that the programming model is a special case derived from a more general set of models.

The third set of models, gravity-type models, has received the most attention. The earlier conceptual development of Leontief and Strout (1963) has been modified, expanded and applied to a large number of problems by Polenske (1970a, b). Unlike the Nevin, Poe and Round (1966) model in which trade in each commodity can only occur in one direction, Leontief and Strout view regional supply and demand as being comprised of a set of goods whose origins may vary. Producers and consumers are assumed to be indifferent about these origins and, as a result, the model envisages shipments taking place between one region's supply pool and another region's demand pool. No attempt is made to distinguish between local supplies and supplies from other regions. Regional outflows of goods vary directly with the level of output in the producing region, while regional inflows vary directly with regional consumption. Interregional flows are, however, inhibited by the 'friction of distance' effect, namely that interaction varies in an inverse relationship with distance. The Leontief-Strout model can be summarized in the following set of equations:

\[
\begin{align*}
X_{OJ} &= \sum_i a_{ij} X_{iJ} + T_{JO} \\
X_{OL} &= \sum_j X_{JL} \\
X_{OH} &= \sum_J X_{JH}
\end{align*}
\]
\[
X_{JL}^{J0} = \sum_L X_{Jh}^L \\
X_{JL}^{JL} = \left[ X_{Jh}^{J0} / X_{Jh}^{00} \right] \cdot q_{JL}^{JL} \quad J \neq L
\]

where

- \(X_{Jh}^{J0}\) is the total amount of commodity h produced in region J
- \(X_{Jh}^{JL}\) is the total amount of commodity h demanded by all final and intermediate users in region L
- \(X_{Jh}^{00}\) is the total amount of commodity h produced (consumed) in all regions
- \(Y_{Jh}^{J}\) is the amount of commodity h demanded by final users in region J
- \(X_{Jh}^{JL}\) is the amount of commodity h produced in region J which is shipped to region L
- \(q_{JL}^{JL}\) is a trade parameter reflecting the cost of transporting commodity h from region J to region L

While Leontief and Strout performed several tests of their model, by far the greatest use of this model and a number of derivatives has been made by Polenske. Table 1 shows the three versions, one of which is the Leontief-Strout gravity model, which have been extensively employed by Polenske (1972). The multiregional input-output model was first implemented for 1963 and subsequently updated to 1967. The models have been used for transportation impact analysis, energy demand studies, forecasting occupational needs and, most recently, in conjunction with a demographic model, for forecasting activity levels for a small set of states (Evans and Baxter, 1980).
TABLE 1

Three versions of the Gravity Model (After Polenske, 1972)

<table>
<thead>
<tr>
<th>Trade coefficient equation</th>
<th>Row Coefficient Model</th>
<th>Column Coefficient Model</th>
<th>Gravity Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trade equation</td>
<td>( \epsilon_i^h = \epsilon_i^h )</td>
<td>( \epsilon_i^h = \epsilon_i^h )</td>
<td>( \epsilon_i^h )</td>
</tr>
<tr>
<td>Equation system in matrix form</td>
<td>( R \Delta X = A \Delta X + \Delta Y )</td>
<td>( \Delta X = C (A \Delta X + \Delta Y) )</td>
<td>( T^h \Delta X = S (A \Delta X + \Delta Y) )</td>
</tr>
<tr>
<td></td>
<td>( (R' - A) \Delta X = \Delta Y )</td>
<td>( (I - CA) \Delta X = CA Y )</td>
<td>( (T' - SA) \Delta X = SA Y )</td>
</tr>
<tr>
<td></td>
<td>( \Delta X = (R' - A)^{-1} \Delta Y )</td>
<td>( \Delta X = (I - CA)^{-1} CA Y )</td>
<td>( \Delta X = (T' - SA)^{-1} SA Y )</td>
</tr>
</tbody>
</table>

* indicates a block diagonal matrix

\( \Delta \) indicates the change between a base year and the given year

\( n \) the number of regions

\( m \) the number of commodities

**Matrix Notation**

**\( \Delta X \)** column vector \((mn \times 1)\) giving the change in production. Each element describes the change in output of commodity \( i \) produced in region \( g \).

**\( \Delta Y \)** column vector \((mn \times 1)\) giving the change in total final demand. Each element describes the change in the total amount of commodity \( i \) consumed by all final and intermediate consumers in region \( g \) regardless of the place where the good was produced.

**\( A \)** block diagonal matrix \((mn \times mn)\) with \( n \) square matrices \((m \times m)\) of input coefficients along the diagonal describing the structure of production in each region. If separate regional technical coefficients are not available, the matrix of national coefficients customarily is used for each region.

**\( S, T \)** each is a square matrix \((nm \times nm)\) filled with diagonal matrices \((m \times m)\). Each \( s_{gh} \) elements relate outflows from region \( g \) to the production in the region while the \( t_{gh} \) elements relate inflows into region \( h \) to the total consumption in the region.

**\( R \)** square matrix \((nm \times nm)\) filled with diagonal matrices \((m \times m)\). Each \( r_{gh} \) element describes the fraction of total production of commodity \( i \) in region \( g \) that is exported to region \( h \). The sum of each row of this matrix must equal 1, since the coefficients are proportions of total production.

**\( C \)** square matrix \((nm \times nm)\) filled with diagonal matrices \((m \times m)\). Each \( c_{gh} \) element describes the fraction of total consumption of commodity \( i \) in region \( h \) that is imported from region \( g \). The sum of each column of this matrix must equal 1, since the coefficients are proportions of total consumption.

**Element Notation**

\( x_{gh}^i \) the total amount of commodity \( i \) produced in region \( g \).

\( z_{gh}^i \) the total amount of commodity \( i \) demanded by all final and intermediate consumers in region \( h \).

\( y_{gh}^i \) the total amount of commodity \( i \) produced (consumed) in all regions.

\( q_{gh}^i \) a trade parameter which is a function of the cost of transferring commodity \( i \) from region \( g \) to region \( h \) (where the transfer costs reflect various factors, including transportation costs, which determine interregional trade).
The general approaches described above consider interregional trade from an input-output perspective, namely the disaggregation of technical coefficients into a set of local and non-local components. As such, the models may be regarded as models of interregional trade rather than models of interregional commodity flow. The latter models may be thought of as derivatives of transportation models, where the major focus of interest is in the specification of a transportation network over which the commodities may flow and the derivation of the choice of mode to be used over this network. In these models, supply and demand equations are not usually associated with interindustry models although, as will be demonstrated in later sections, the degree of linkage is considerable and of great interest. The transportation system models are important from the perspective of evaluating the impacts of proposed policy changes on the flows of commodities between regions (for example, increasing truck weight limits on interstate freeways, deregulation of railroad rates). Further, increases in energy costs are serving to increase the 'friction of distance' between regions and thus raise the prospect that future patterns of interregional trade may not mirror those of the present decade.

The contribution of this research will be in the development of ways of integrating interregional input-output and commodity flow models in such a way that a broader set of policy issues can be addressed. In the next section, an overview of the proposed research is provided.
1.3 Overview

Section Two of this paper discusses general approaches to commodity flow modelling which are derived from the entropy formulation first proposed by Wilson (1968). This approach has, subsequently, been modified and reformulated by Erlander (1977, 1981). The combined input-output/commodity flow model is presented next: although this linkage can be accomplished in a number of ways, the most useful one is the case in which regional final demands are given. The following sections discuss ways in which the combined model could be expanded to include consideration of modal and route choice. To accomplish this, a representation of the interregional network for the United States will be required as well as information necessary to structure the set of link transportation costs. Information on capacity constraints is also required to avoid unrealistic allocation of flows along a small set of routes.

The final two sections of the paper deal with research needs and alternative approaches to the problem, and with linkages to other multiregional models. These links provide the important empirical estimates of the constraint set in the combined input-output/commodity flow model; in addition, the output from the combined model will provide inputs into the location of industry model.
2 COMMODITY FLOW MODELS: A REVIEW OF RECENT DEVELOPMENTS

That interregional commodity flows are directly proportional to regional production and consumption and inversely proportional to transportation costs (or its surrogate, distance) has been observed by many researchers over several decades (see Isard, 1956, for examples of such 'gravity' models). These relationships were often criticized as only exhibiting empirical regularities in the data and not providing any rigorous theory of spatial interaction. A major step towards such a rigorous derivation of these gravity relationships from more basic precepts was provided by Wilson's (1968) application of entropy maximizing methods to commodity flow analysis. Subsequently, Erlander (1977) showed how the same problem could be given a macrobehavioral interpretation without appealing to the concepts of information theory.

Wilson (1968) also showed how input-output models of a regional economy could be combined with the gravity model to provide a more general model of the interregional economy than any proposed thus far. In this section, these developments are summarized as a basis for the consideration of operational models in subsequent sections.
2.1 A Doubly-Constrained Interregional Commodity Flow Model

Wilson (1958) derived a family of four commodity flow models: the model differ only in what is assumed to be known about regional production and consumption. The four possibilities are:

1. total production of commodity \( h \) for the entire system of regions \( \mathbf{x}^{00} \) is known;

2. region \( J \)'s production of commodity \( h \), \( \mathbf{x}^J \) is known but region \( J' \) consumption of \( h \), \( \mathbf{y}^J \) is not known;

3. converse of 2, i.e., \( \mathbf{y}^J \) is known but \( \mathbf{x}^J \) is unknown;

4. both \( \mathbf{x}^J \) and \( \mathbf{y}^J \) are known for all regions.

For purposes of this review, only the fourth model is described. The other cases follow by analogy. It should also be noted that cases have recently been solved in which \( \mathbf{x}^J \) and \( \mathbf{y}^J \) are upper or lower bounds on production or consumption rather than strict requirements.
Past attempts to model interregional commodity flows have sometimes formulated the problem in terms of cost minimization (e.g., see Harris, 1974). If regional production and consumption are given (case 4 above), then this results in the transportation problem of linear programming as follows. Let:

\[ c_{jL}^h = \text{transportation cost from region } J \text{ to region } L \text{ for commodity } h \]
\[ x_{jL}^h = \text{amount of commodity } h \text{ shipped from region } J \text{ to } L \]
\[ C_h = \text{total cost of transporting commodity } h \text{ in the system} \]

An efficient allocation of production to consumption may be found by

\[ \text{Min } C_h = \sum_j \sum_L c_{jL}^h x_{jL}^h \]  
\[ \text{s.t. } \sum_j x_{jL}^h = y_L \] \text{ for all } L \]
\[ \sum_L x_{jL}^h = y_j^h \] \text{ for all } J \]
\[ x_{jL}^h \geq 0 \]

Solution of this problem results in the identification of, at most, \(2n\)-positive flows, where \(n\) is the number of regions. The cost-minimizing solution also does not allow for shipments between two regions in both directions, since, in principle, one of these shipments would be unnecessary or inefficient.

Observation of commodity flows in the real world clearly indicates that the cost minimizing solution does not describe the actual situation. More than \((2n-1)\) flows occur and often commodities in the same class move in both directions between two regions. This latter phenomenon, known as crosshauling, is observed for two reasons. First, the level of aggregation in such analyses is often high, so that opposing shipments of distinct different goods in the same commodity class may appear as crosshauling. Secondly, actual crosshauling of the same commodity may occur because of
established trading patterns, lack of information, product differentiation (through advertising) and other institutional factors.

Hence, some method is needed to deal with these problems, essentially method to modify the transportation problem formulation to provide a more realistic description of the commodity flow pattern. Erlander (1977) propose that the entropy function:

\[ S_h = -\sum_j \sum_l x_{hl}^J \ln(x_{hl}^J) \]  

(5)

can be used to accomplish this objective. \( S_h \) can be interpreted as a scalar measure of the level of spatial interaction or dispersion in an observed flow matrix \( \Psi^J_h \). This measure accounts for the observed pattern of crosshauling in both of the senses discussed above. By adding constraint (5) to the above transportation problem, the solution is constrained to produce flow matrix which has the same level of spatial interaction as the observed matrix. The model would now become:

\[ \text{Min } \sum_j \sum_l c_{hl} x_{hl}^J \]  

s.t. \((2 - 5)\)

This constrained optimization problem may be solved by the method of Lagrang multiplier. The solution of the above problem is:

\[ x_{hl}^J = A_h^J x_h^L B_h^L y_h^L \exp(-\beta_h c_{hl}^J) \]  

(6)

where:

\[ A_h^J = \sum_j \sum_l x_{hl}^j \exp(-\beta_h c_{hl}^J) \]

\[ B_h^L = \sum_j \sum_l x_{hl}^j \exp(-\beta_h c_{hl}^J) \]

\[ \beta_h = \text{Lagrangian multiplier associated with (5); } \beta_h \text{ may be interpreted as transportation deterrence parameter. The relationship between } \beta_h \text{ and } S_h \text{ is shown in Diagram 1.} \]
Note that when $\beta_h = 0$, (6) simplifies to:

$$x_h^{JL} = x_h^J x_h^L / x_h^{oo}$$  \hspace{1cm} (7)

Equation (7) states that commodity flows are proportional to production and consumption and unrelated to transport costs. As $\beta_h$ approaches infinity ($x_h^{JL}$) tends to the solution of the original transportation problem.

By varying the value of $\beta_h$, one can obtain commodity flow matrices which correspond to various values of $S_h$, and, therefore, total transportation cost. These matrices involve larger or smaller amounts of crosshauling depending on whether $\beta_h$ is small or large. Note that the values of $\beta_h$ depend not only on $S_h$, but also on $x_h^J$ and $y_h^L$ and $(c_h^{JL})$. If any of these values is altered by assumption or by changes in the economic or transportation system, the $\beta_h$ should be revised accordingly. Thus, at least in the authors' view, $\beta_h$ is not a parameter to be calibrated from observed flows and then used as constant in forecasting flows corresponding to various assumptions concerning $x_h^J, y_h^L$ and $(c_h^{JL})$. Rather, should be calibrated for each forecast by assuming the appropriate value of $S_h$ on the basis of assumptions regarding how the level of spatial interaction may change in the future. According to this view, the model is not useful in determining the level of spatial interaction or crosshauling associated with an assumed future condition but only in determining the pattern of flows associated with that condition.

Other forms of the deterrence function may also be derived by making some different assumptions about the total transportation cost. Perhaps the most interesting of these is the case where (1) is replaced as follows:

$$\sum_{J} \sum_{L} x_h^{JL} \ln (c_h^{JL}) = c_h'$$  \hspace{1cm} (1')
This form might be motivated by an assumption that the shipper's disutility of transportation cost is linearly related to the natural logarithm of transportation cost. The corresponding form of the model is then:

\[ x_{IL}^T = A^T X^L B^L y^L (c^T L)^{-\alpha} \]

(5')

with \( A^T \) and \( B^T \) redefined with the power function in place of the exponential function. Equation (6') of course is in the traditional gravity model form.

2.2 Combined Regional Input-Output and Commodity Flow Models

The model reviewed above can be combined with a system of regional input-output models by taking the basic input-output relationship as an additional constraint relating \( x^T_h \) and \( y^L_h \). Wilson (1968) developed a family of these combined models and discussed how they might be used to describe the interregional and interindustry relationships in a system of regions. Here one of these combined models is reviewed for illustrative purposes.

To form the combined model, two basic assumptions are necessary: (i) each region's producers are indifferent as to the final destination of their output and (ii) each region's consumers (both intermediate and final) are indifferent as to the origin of their inputs. In other words, only transportation cost determine interregional flows.
Wilson developed four alternative combined commodity flow/input-output models: the one which is of the greatest interest is the unconstrained model although there exist many possibilities for hybrid models in which the commodity space is divided into weakly separable sets.

Transportation cost is minimized subject to input-output and spatial interaction constraints:

$$\begin{align*}
\text{MIN } C &= \sum_h \sum_J x_{hJ}^L c_{hJ}^L \\
\text{s.t. } &- \sum_J x_{hJ}^L \ln(x_{hJ}^L) \geq \beta_h \quad \text{for all } h \\
&\frac{\partial}{\partial x_{hJ}^L} \sum_j \mu_h \left( \sum_L x_{hJ}^L \right) + y_{hJ}^T \quad \text{for all } h \text{ and } J \\
x_{hJ}^L &\geq 0
\end{align*}$$

where $y_{hJ}^T$ is the final demand in region $J$ for commodity $h$. The Lagrangian is formed to solve (6) as follows:

$$L = C + \sum_j \sum_h y_{hJ}^T \left[ y_{hJ}^T + \sum_i a_{hi}^T \left( \sum_L x_{hJ}^L \right) - \sum_j x_{hJ}^L \right] + \sum_h \mu_h \left[ \beta_h + \sum_L x_{hJ}^L \left( x_{hJ}^L \right) \right].$$

where $y_{hJ}^T$ is a set of Lagrangian multipliers associated with (8) and $\mu_h$ is the set associated with (5). The estimate of $x_{hJ}^L$ is obtained by setting the derivative of $L$ with respect to $x_{hJ}^L$ equal to zero.

$$\frac{\partial L}{\partial x_{hJ}^L} = 0$$

Solving for $x_{hJ}^L$ gives:

$$x_{hJ}^L = \exp \left( \sum_i a_{hi}^T y_{hJ}^T - \mu_h y_{hJ}^L - \mu_h c_{hJ}^L \right)$$

$\mu_h$ is chosen so as to satisfy constraint (5) as an equality, given a observed or assumed value of $\beta_h$, $y_{hJ}^T$, $y_{hJ}^L$ and $\mu_h$ may be determine.
iteratively by applying the Newton-Raphson method or a similar iterative procedure. Wilson (1970) has shown this model reduces to a gravity-type expression of the form:

\[ x_{ij}^{TL} = \delta_{ij} \sum_{h} c_{ij} \exp (-\mu_{i} c_{ij}^{TL}) \]  

(12)

Equation (12) can be called the entropy-constrained version of a spatial interaction model which accounts for interindustry flows. In this formulation, flows of commodity \( h \) between regions \( J \) and \( L \) are proportional to \( \exp (-\mu_{i} c_{ij}^{TL}) \) but not to any other characteristics of the regions \( J \) and \( L \). Equation (12) is analogous to the Leontief-Strout formulation.

Wilson also commented that a hybrid model could be developed in which some commodities would be estimated using the production-consumption constrained model (these might be primary commodities serving as inputs into other primary sectors). Yet other commodities, such as coal, might be estimated using a production constrained model while intermediate goods might be estimated using the consumption constrained version. In summary, the combined model offers a great deal of flexibility. Still to be determined are the possible alternative ways in which the input-output and commodity flow models could be linked under varying assumptions about the structure of production at the regional level. In addition, information on some flows is well known and these activities could be 'blocked out' from the estimation procedures. Further extensions to this work have recently been developed by Schinnar (1978), Macgill (1978) and Snickars (1979); in the case of the former two authors, their work has been conceptualized within the rectangular commodity-industry model accounting framework which has recently been adopted by the U.S. Department of Commerce in the preparation of the 1972 and
subsequent input-output models.

3 EXTENSIONS TO INCLUDE NETWORK REPRESENTATIONS OF TRANSPORTATION COST

Nearly all models of commodity flow, Wilson's included, ignore the question of the source of transportation costs required as one of the major inputs of the model. Except in the simplest situation, data on transportation costs from region J to L as such do not exist. What do exist are transportation rates for individual modes (e.g., rail, motor carrier, pipeline, barge) for specific routings. Unfortunately, even these rates are not readily accessible to the modeller or analyst in most situations. What rates actually are used in commodity shipments only become known in the course of decisions regarding actual mode and route choice.

Much progress has been made in recent years in modelling route and modal choice in urban transportation systems. These basic models provide a suitable basis for commodity flow modelling, and are reviewed in this section. A substantial amount of effort has been invested by the U.S. D.O.T. in developing a network representation of the U.S. transportation system. This representation provides the basic data for the model outlined below. At the end of this section, this data base is reviewed briefly.
3.1 Route Choice on a Congested Network

A useful point of departure for examining route and mode choice models is to consider route choice for a transportation system comprised of one mode, say rail. In the absence of origin-destination rate information for alternative routes, and abstracting from the issues of the originating carrier's control over route choice (see Lansdowne, 1979), assume that each link of the network has a unit user cost, $c_a$, which is an increasing function of the annual flow, $f_a$:

$$c_a = c_a (f_a)$$

The behavior of the shipper in this situation may be generally depicted in terms of cost minimization. If link costs are constant, the shipper will choose the path or sequence of links through the network which has the minimum total cost. Such minimum cost paths may be readily identified, even for very large networks; the path from each origin to destination is independent of all other paths since link costs are fixed. Thus, for this case the problem of route choice is straightforward and manageable.

If congested conditions exist in the network, then the transportation cost on each link are not fixed, but may be regarded as an increasing function of the flow on that link. Since any given link may serve many
origin-destination pairs, the problem is highly interdependent. If, in this situation, shippers continue to seek their minimum cost paths, then a network equilibrium problem results. A network is said to be in equilibrium if (Wardrop, 1952):

1. the travel costs over all paths which are used from each origin to each destination are equal and

2. no unused path has a lower travel cost.

Thus, at network equilibrium, no shipper can lower its costs by switching paths. Note that this is not a supply-demand equilibrium since total shipments between each origin and destination pair are fixed. Unfortunately, the concepts are often confused.

The problem of network equilibrium for a single mode was first formulated mathematically by Beckmann et al. (1956). An efficient computational algorithm was devised by Leblanc et al. (1975). The mathematical formulation of the problem follows from a recognition that at network equilibrium, the total area under the link cost functions is minimized. To see this, imagine a two link network connecting two regions. For a given shipment level, the problem can be solved by finding the intersection of the two link cost functions, as shown in the diagram below. The equilibrium solution A has a smaller area under the two cost functions than any other solution, for
example, by the amount of the shaded area.

This can be readily extended to a large network as follows:

\[
\min \sum_a \int_0^{f_a} c_a(x) \, dx
\]

s.t. \[x_{h}^{JI} = \sum_p x_{h}^{JLP} \]

\[f_a = \sum \sum \sum x_{h}^{JLP} \]

\[x_{h}^{JLP} \geq 0\]

where:

\[x_{h}^{JLP} = \text{amount of commodity } h \text{ shipped from region } J \text{ to region } L \text{ on path } p\]

\[f_a = \text{total flow on link } a\]
\( c_a \) = cost of link \( a \)

\( \delta^p_{JL} = 1 \) if link \( a \) is included in path \( p \) from \( J \) to \( L \);

\( \delta^p_{JL} = 0 \), otherwise.

The solution to this problem is often referred to as a 'user-equilibrium' solution. Unless link costs are constant, this solution does not minimize total transportation costs. The solution which does minimize total cost, referred to as the 'system-optimal' solution, can be found by replacing the above objective function with:

\[
\text{Min } \sum_a f_a c_a (f_a) = \sum_a \int M_a (x) \, dx
\]

s.t. the same constraints

and where \( M_a (x) = d(C_a(f_a)/dx \) = marginal cost of link \( a \).

The computational solution to the user-equilibrium and system-optimal route choice problems consists of computing a sequence of minimum cost path solutions and finding a weighted sum of these solutions so as to minimize the appropriate objective function.

3.2 Modal Choice as an Extension of Route Choice

The above route choice problem can be extended to include modal choice by augmenting the single mode network to include two or more nodes. Appropriate transfer links between nodes need to be provided. If shippers are assumed to choose the minimum cost route and mode combination, then application of a minimum path algorithm will identify the correct mode and route. In this
case, all the entire shipment of a given commodity class between a specified origin-destination pair would be assigned to the minimum mode-route path. Although this simplification of the observed shipment pattern might be acceptable for a highly disaggregated system of regions and commodity groups, it is unlikely to be realistic for more aggregated analysis.

If link costs increase with flow, then the user-equilibrium problem can be extended to a multimodal network. The above Wardrop equilibrium conditions are then extended to mode and route combinations.

3.3 Modal Choice Proportional to Modal Cost Differences

Even with the very detailed coding of the modal transportation networks, the costs of shipping a given commodity by one mode may not be fully comparable with that of shipping by another mode. For this reason, it may be necessary to estimate, from the data, an unidentified or unobserved cost difference or modal bias. In addition, such an approach can allocate proportions of shipments directly to the several modes, rather than building up such proportions iteratively.

A common approach to modelling modal choice is to hypothesize that the proportions shipped by each mode are described by the following logit function:

\[
\hat{\rho}_n^k = \frac{\exp\left(a^k + \lambda c_{nk}^{Tlk}\right)}{\sum_k \exp\left(a^k + \lambda c_{nk}^{Tlk}\right)}
\]
where
\[ p_{Jlk}^h = \text{the proportion of shipments of commodity } h \text{ shipped from region } J \text{ to region } L \text{ by mode } k \]
\[ \sum_k p_{Jlk}^h = 1 \]
\[ a_k = \text{unobserved cost difference between mode } k \text{ and some reference mode } r, \text{ for which } a_r = 0 \]
\[ c_{Jlk}^h = \text{minimum path cost for commodity } h \text{ from region } J \text{ to region } L \text{ on mode } k \]
\[ b = \text{parameter estimated from observed flows.} \]

This type of modal choice model can be applied to the estimated interregional commodity flows, \((x_{Jlk}^h)\). The modal commodity flows, say \((x_{Jlk}^h)\) are then allocated or assigned to individual mode according to the user-equilibrium concepts described above.

Indeed, the combined modal choice/route choice problem can be formulated as an extension of the user-equilibrium route choice problem stated in section 3.1, as follows:

\[
\begin{align*}
\min & \sum_k \sum_h \int_{x_h}^{x_{Jlk}^h} c_{Jlk}^h(x) \, dx \\
\text{s.t.} & \sum_k x_{Jlk}^h = \bar{d} \\
& \sum_k x_{Jlk}^h = \sum_k x_{Jlk}^h \\
& \sum_k x_{Jlk}^h = \sum_k x_{Jlk}^h \\
& \sum_k x_{Jlk}^h \ln(x_{Jlk}^h) = S_h \\
& x_{Jlk}^h \geq 0
\end{align*}
\]

The entropy constraint may be interpreted as the dispersion of shipments across modes. The optimality conditions for this problem are the Wardrop conditions for each mode plus the logit function stated above. This model may
also be stated in the form originally proposed by Beckmann et al. (1956) by selecting a modal demand function corresponding to the logit function. This combined modal choice/route choice problem can be solved by an algorithm similar to the one used for solving the route choice problem. The model has been implemented on urban networks of a size comparable to a large interregional multimodal transportation system.

3.4 Combined Destination, Mode and Route Choice Models

The combined model sketched in section 3.3 can be extended to Wilson's interregional commodity flow model by adding constraints from the development of section 2.1 to the above problem.

\[
- \sum \sum (\sum x_{LK}^h \ln \sum x_{LK}^h) \geq S_h \\
\sum x_{LK}^h = x_{h}^{J} \\
\sum x_{LK}^h = y_{w}^{L}
\]

The solution of this nonlinear programming problem combines the structure of the commodity flow (destination choice) model of section 2.1 and the route/mode choice model of section 3.3. Its solution as combined model insures that at network equilibrium, the transportation costs which determine destination and modal choice are precisely those costs which result from the loading of the shipments onto the respective links of the network. This problem can also be solved by an iterative algorithm similar to the one described in the above sections.
Furthermore, this model can be extended into an input-output model structure by adding constraints of the type described in section 2.2. The convergence properties of this model will require further analysis and computational evaluation.

3.5 Network Representation of the US Transportation System

Attempts to implement and test the modelling framework described to this point depend heavily on three types of data:

1. interregional commodity flows

2. regional input-output tables

3. multimodal transportation networks including models of transportation unit costs

Any attempt to attack this problem, even for a small system of regions, would be formidable to say the least. Fortunately, in the U.S., efforts are well underway in the collection, processing and dissemination of all three types of data. The purpose of this section is to describe briefly the nature of the
Basic sources of commodity flow data in the U.S. include the Census of Transportation (manufactured goods), Army Corps of Engineers (waterway shipments, primarily of bulk commodities), the U.S. I.O.T. one percent waybill sample (all rail traffic) and various surveys of agricultural product shipments. The Transportation System Center of the U.S. Department of Transportation has pursued a program of compilation of a composite commodity flow table from these various sources for 1972; (see Schuessler and Cardellicchio (1976) and Bronzini (1979b). Although these data are not complete for all nodes, they do provide an initial basis for model testing.

The TSC commodity flow table provides annual tons of commodities shipped by 19 commodity classes corresponding to two-digit SIC groups between the 171 economic regions defined for the continental mainland by the Bureau of Economic Analysis of the U.S. Department of Commerce. Weighting factors specifying the trade-off among costs, transit time and energy use have also been developed for each commodity.

The publication of the 1972 U.S. input-output table last year provides a basis for the preparation of a combined commodity flow/input-output model for the same year. The 1972 input-output model has been developed using the accounting scheme recommended by the United Nations, namely a commodity-industry system (instead of an industry-industry system). The new accounting scheme retains all the advantages of the traditional Leontief model while providing somewhat greater flexibility for analysis. In particular,
commodities can be mapped in other than a one-to-one fashion into industries as inputs; furthermore, outputs of similar commodities by different industries are shown directly rather than treated through tedious secondary product transfers for the cases in which the commodity is regarded as a secondary product in an industry. Hence, the linking between the commodity flow model and the input-output model can be achieved in commodity space and then mapped into industry space through the commodity-industry input-output model. It might also be possible to treat commodities produced in different industries as different goods for the purpose of movements over the interregional transportation network.

The Transportation System Center has also undertaken the preparation of a network model for the U.S. based on the 171 BEA economic regions. The network is represented by sets of nodes and links for each mode as follows:

1. nodes - intersections, terminals, link delimiters

2. linehaul links - linehaul transportation facilities

3. access links - local transportation pickup and delivery

4. transfer links - intermodal transfer facilities.
Nodes are characterized by name, location, mode and a performance function class specifying the nodes' operating characteristics; there are 1789 nodes in the network. Links are specified in terms of mode, length and link class. Performance functions specifying operating cost, time and energy consumption as a function of flow are specified for each link and node class. There are 3397 linehaul links with an aggregate length of 239080 miles. In addition, there are 642 access links. In the 1979 network, there are no transfer links because of lack of data on transfer facilities and costs. Computer programs have been developed for manipulating the data, identifying minimum cost paths and assigning commodity flows to the network. These network programs are documented in Bronzini (1979a).

4 RESEARCH NEEDS

This section outlines a research agenda for testing the models sketched above. The general thrust of the needed research is concerned with empirical testing of the models for the United States. Until recently, it was not possible to consider testing such models because of the absence of relevant data and computational facilities. The data and network developed by the Transportation System Center have improved the situation markedly to the point where testing of suitably detailed models can proceed. Although further extensions to the data base are being made, these extensions will certainly benefit from the experiences derived from the use of the existing system for research and policy analysis.
A general remark may be in order about the computational feasibility of these models. Much of the basis for the model formulations, solutions, algorithms and computational requirements for these models is derived from urban transportation modelling. Presently, metropolitan planning organizations are using models of the same general class as the ones described here: some of these are a full order of magnitude larger than the envisaged U.S. commodity flow model. For example, in the Chicago region, travel flows between 1800 zones (regions) over a network of some 30,000 links are routinely computed. Although combined destination, mode and route choice models have not yet been implemented for a system of this size, the computational requirements are similar to those for models presently being used. Hence, while the computational requirements for the research effort needed to test the models described are substantial, it is, nevertheless, appropriate to approach this part of the problem with confidence.

In general, there are two types of research issues involved in testing models of this class. The fundamental issue, of course, is how well do the models reproduce the observed phenomena. In recent years, more data have been made available to researchers but it is unlikely that enough will be forthcoming to facilitate complete testing of the models. For example, while data may be available on the amount of a commodity shipped per year from region J to region L, the actual flow of commodity h or of all commodities on a given link of the transportation network may not be known. Even in the best of circumstances, it is often only possible to verify selected model outputs.
A second research issue concerns the general performance of the computational algorithms and the general reasonableness of the results. The algorithms developed in recent years to solve this class of combined models are 'convergent' as opposed to heuristic. In other words, they are guaranteed to converge to the solution of the formally stated problem rather than rough approximations to the observed flows. What is not known, in advance, is the rate of convergence to the solution - which is, obviously, a key determinant of computational feasibility of increasingly large and complex models.

Also unknown is whether the data inputs taken in combination will result in reasonable outputs. In past research, it has been proven fairly easy to spot a key faulty datum by checking for reasonableness of the output. This often provides an important basis for model evaluation when actual data are unavailable.

Having discussed some of the general research issues associated with these models, we next turn to a discussion of specific research questions. This discussion is organized in three parts: (i) the first deals with the commodity flow model itself, (ii) The second concerns the combined input-output/commodity flow model and, finally, (iii) extensions to the commodity-industry form of the input-output part of the model are considered.
4.1 Tests of Alternative Multiregional Commodity Flow Models

Four research questions were implicitly identified in the development of the models in section 2. These are now discussed here in turn.

First, what is the appropriate form of the cost function in the spatial interaction model? The negative exponential form follows from the simplest formulation of the derivation. Some preliminary model test on both U.S. and Korean data have indicated that the negative power function may fit better empirically. This question has substantial interest in its own right; comparison of the two functional forms also provides a measure of output variance that may be useful in gauging other research results.

Second, what should be the form of the modal choice model? Parsimony suggests that the modal choice be treated simultaneously with route choice. As discussed above, however, such a formulation may be overly simplistic and might need to be replaced with a specific modal choice function. Then, alternative functions need to be considered and tested, and their computational implications examined.

In terms of public policy analysis, the mode choice function is a key research issue in the development of the model. Many policy questions for which the model is potentially useful concern actions to modify modal choice. Thus, this aspect of the research should be emphasized.
Third, there are specific research questions which need to be resolved related to the relationship between the destination choice and modal choice models in general, and the composite cost concept in particular. As with mode and route choice, parsimony suggests that destination and mode choice are determined by the same costs and the same cost functions. While this may be a suitable null hypothesis for model testing, such a formulation is probably simplistic. Different cost function parameters probably are necessary for mode and destination choice. In addition, imputed cost factors may be necessary to reproduce with reasonable accuracy trade flows between certain regions where noncost trade factors prevail. A series of model tests are required to determine to what extent such imputed costs are necessary.

Fourth, the question of the level of spatial aggregation of the model needs to be considered. This is more of a research design question than a research issue and, therefore, was left until last in this discussion. Nevertheless, it is important not only for commodity flow modelling but also for the linkages with other multi-regional modelling efforts.

A major consideration in choosing the level of spatial aggregation in transportation flow models is how much detail is necessary to obtain a reasonable representation of the interregional costs for the network. For the U.S., a system of regions defined on the 50 states would appear to be too coarse a spatial system for network representation. The EEA regional system of 171 regions would appear to be sufficiently detailed. The latter system was chosen by TSC in its data development and modelling efforts - and, as we have noted, these components are key resources for this research.
One problem with the BEA system is that it does not aggregate to the 50 state regional system, the system likely to be used for a great deal of policy analysis. Therefore, it may be necessary to develop some transformation procedure between the two systems, perhaps based on county level data. Although we are unaware of such a system, it may well have been developed already.

4.2 Tests of the Input-output/commodity Flow Model

In our view, several research issues related to commodity flow modelling discussed above need to be considered and solved prior to a full-scale test of a combined input-output/commodity flow model. This does not prevent, however, some of the issues concerning this combined model being considered concurrently. Two principal issues requiring research are discussed here.

The first concerns the specification of algorithms for a combined input-output/commodity flow model. Although the basic derivation of the model has been developed in detail by Wilson (1968), more recent research results with respect to algorithms need to be considered. Since the input-output part of the model structure is straightforward matrix manipulation, this is not expected to result in computational difficulties. What needs to be considered is whether there are computational advantages or efficiencies which result from alternative formulations.
Second, the use of entropy as a suitable measure of spatial interaction needs to be tested in the combined model. Can the observed level of 'cross-hauling' associated with a given level of commodity aggregation be reproduced? How does the level of spatial interaction or entropy differ for different commodities? What questions arise when this concept is juxtaposed with different regional input-output tables?

These are only the initial questions that arise from considering the operationalization of Wilson's model. Others will surely arise as the properties of the model are probed in more detail and the implications of the model structure for computational algorithms are considered. These issues should be the principal long-term focus of any research effort in this area.

4.3 Extensions to commodity-industry input-output commodity flow models

The research that has been developed on combined input-output/commodity flow models has been conceptualized within the framework of the traditional Leontief industry-industry input-output model. In this model, flows are shown as originating from industries and moving to other industries where they are consumed. One of the major problems which this accounting system does not handle very effectively is the problem of commodities being produced in more than one industry - the so called 'secondary product' problem. There are a number of alternative ways in which this problem can be handled, none of which is really very satisfactory. The commodity-industry framework, on the other
hand, obviates an assumption of a one-to-one mapping between industries and commodities. In essence, the input-output structure, as shown in the diagram below contains a decomposition of the production process into (i) production of commodities by industries (the MAKE matrix) and (ii) the absorption of commodities by industries in the production process (the ABSORPTION matrix).

The MAKE matrix has a tendency towards diagonality: off diagonal elements indicate the production of secondary products. The ABSORPTION matrix has no visible structure and, generally, will be a very full matrix. Note that no flows occur between commodities and between industries.
The reduced form equations take on the following characteristics:

\[ q = (I - BD)^{-1} e \]
\[ q = (I - DB)^{-1} De \]

where:

- \( q \) = a vector of commodity outputs
- \( g \) = a vector of industry outputs
- \( B \) = a matrix of coefficients showing the use of commodities per unit of output of industry
- \( D \) = a matrix showing the outputs of commodities per unit of total output by industry
- \( e \) = a vector of final demands for commodities

Although Macgill (1978) has proposed some procedures for coupling entropy maximizing procedures with the commodity-industry structure, no one has yet developed a family of models for the interregional case. The possibilities are greater since one could operate in either the industry or commodity space. Careful attention would have to be paid to the mapping of commodities into industries. Wilson's earlier suggestions about the use of hybrid models for combined input-output/commodity flow models may be extremely useful in this context. Substitution in the commodity space, by an industry, would have a direct link with the interregional commodity flow model - especially in the context of energy inputs.

Hence, the research needs in this area are (i) the formulation of the mathematical and computational relationships between the rectangular input-output model and the commodity flow model, (ii) the specification of the set of exogenous data for the combined model and (iii) alternative
interactions between commodity space in the input-output model and in the
interregional commodity flow model.

5 LINKAGES WITH OTHER NEEDED RESEARCH

If the set of models described earlier is to be successfully implemented,
a number of important exogenous data inputs are required. These will be
derived from the regional econometric models for the most part. On the other
hand, a number of other models will require outputs from the combined
commodity flow/input-output model for their successful operation. Finally,
there will be a considerable degree of interaction with models designed to
predict future location patterns of industry since transportation costs and
the changes in the transportation network are likely to be important variables
in determining future locations of firms. At the same time, new firm
locations will place new demands on the transportation system and, hence,
create changes in flows, in the level of capacity utilization and costs. In
this section of the paper, a review of some of these links is provided.

5.1 Regional Final Demands Derived from Econometric Models

In the specification of the combined input-output/commodity flow models
discussed in earlier sections, it is clear that their estimation required
several sets of data. One of the most important relates to the estimation of
regional production (total outputs) and regional consumption (final demands):
these elements drive the regional consumption economies and, in essence, create the supply and demands for the interregional flow of commodities. Since regional economies are so open, the specification of the final demand vectors is very critical. Changes in household consumption, comparative advantages (and hence, the attractiveness for new investment) and in the pattern of federal, state and local government expenditures will all create important changes in the pattern of regional production and hence interregional trade.

The final demand estimates will be provided for a consistently defined set of industries from a regional econometric model. These estimates will be made, initially, at the state level and then allocated to BEA regions. While this process will not allow consistent balancing of BEA regions with state estimates (since some BEA regions overlap state boundaries), an attempt will be made to provide consistency at multistate levels whenever BEA regions and a set of states containing all of these regions may be found.

5.2 Provision of Transportation Costs for Industrial Location

Industrial location theory has consistently stressed the role of transportation costs in the determination of optimal patterns of location. As energy prices rise, in real terms and in comparison to other inputs, the cost of moving commodities and individuals across space will rise. As yet, there has been very little research (but a great deal of speculation) on the
possible impacts of increasing transportation costs on future patterns of industrial location. One of the important outputs from the combined input-output/commodity flow model will be an estimate of transportation costs for different commodities, the identification of congested links and some analysis of possible alternative transportation investment strategies on the changes in the structure of the transportation cost matrix. These outputs will provide important input components to other models dealing with energy supply-demand relationships and possible changes in industrial location.

5.3 Changing Location Patterns, Changing Regional Input-Output Coefficients and Regional Final Demands

The linkages here are likely to be two-way. For example, changing location patterns will create new demands on the transportation system and hence, on the interregional flow of commodities. These changes, in turn, will affect the future location of new activities and so the process continues.

Structural changes in the economy, for example, the substitution of non-energy inputs for energy inputs, general technological change and so forth are likely to create different demands on the interregional transportation system. In turn, these changes will influence the spatial demand and supply relationships and will hence influence future substitution possibilities. Substitution, in this sense, implies not only the use of a different production function but also the possibility of substitution of place of origin for inputs or markets for outputs as a function of changes in the
structure of the transportation network.

In a similar fashion, changes in final demand created by new household formation, increased labor force attachment by members of the 'secondary' labor force, will create new demands on the economy. These changing demands will, in turn, affect and be affected by the transportation cost structures.

5.4 Links with the Interregional Input-Output Model

In developing this link, it may make sense to separate energy commodities from nonenergy since the linear programming subroutine associated with the energy supply-energy product interface involves a far greater number of constraints. Many of these are inappropriate, or, cannot be implemented because data are unavailable for the nonenergy sectors.

Since the interregional input-output model is currently solved iteratively, the link with the commodity flow model can be handled in a similar fashion. As before, regional final demands will serve as constraints, from which the combined interregional input-output/commodity flow model will generate levels of output of nonenergy products and demands for energy products. The latter will be solved via the cost minimization energy supply-energy product linear program; this solution will, in turn, generate demand for nonenergy products by the energy supply sectors and, hence, trigger a whole new set of interregional commodity flows. The interface of the
input-output/commodity flow model offers no major computational difficulty. Flow costs from the transportation network will be used in developing solutions for the energy supplies-energy product program.

5.5 Links with the Factor Substitution Model

The solution to the combined input-output/commodity flow model will generate a set of outputs by region and industry which correspond to a given level of demand and subject to any constraints imposed by the nature of the transportation system and the availability of energy products. The output levels will then be used as inputs in the generation of the KLEM equation system. The labor outputs from that system, together with population/labor force participation rate estimates will be used as inputs into the input-output model.

Clearly, the linkages are many and will provide a much greater dynamism to the model structure as well as providing greater scope for broader policy analysis.
6 CONCLUSIONS

The research agenda discussed here encompasses a major new approach to the modelling of interregional commodity flows. The approach combines possibilities for linkages between other interregional models as well as opportunities for the calibration and analytical interpretation of a family of entropy constrained models which have not, to date, been subjected to empirical implementation. While major components of the conceptual developments have been undertaken, a considerable degree of further work needs to be done in this area. The data collection problem has been reduced to a tractable level through work already undertaken in the U.S. Department of Transportation. With the development of the associated interregional models, the possibilities for the development of a sophisticated combined input-output/commodity flow model appear to be realistic.
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