Fall 2022

Topology

WVU Mathematics Department

NAME (print): ______________________________

Topology Ph.D. Entrance Exam, August, 2022

Solve ***four*** from the following five exercises. *We will grade only four of them, so you need to decide which four solutions to submit.* Write a solution of each exercise on a separate page.

In what follows the symbols int(A) and cl(A) stand, respectively, for the interior and the closure of A. Any subset of $\mathbb{R}$ is considered with the standard topology, unless stated otherwise.

Ex. 1. Let X be a normal topological space and let F and K be closed disjoint subsets of X. Show that there exists open sets $U \supset F$ and $V \supset K$ with disjoint closures. In your solution include definition of a normal space.

Ex. 2.

(a) Give a definition of Lindelöf space.

(b) Show, directly from the definition, that if A is a closed subset of a Lindelöf space X, than A is Lindelöf.

(c) Give a definition of separable space.

(d) Let A be a closed subset of a separable space X. Must A be separable subspace of X? Prove your statement or give a counterexample.

Ex. 3. Let X be a topological space and let A be a connected subset of X. Show that if $A \subseteq B \subseteq \mathrm{cl}(A)$, then B is connected.

Ex. 4. Let

$$S = \{x \in \mathbb{Q}^{\mathbb{N}} \colon \text{ there is an } n \in \mathbb{N} \text{ so that } x(i) = 0 \text{ for every } i > n\},$$

where $\mathbb{Q}$ is the set of all rational numbers and $\mathbb{N} = \{1, 2, 3, \ldots\}$. Find the closure $\mathrm{cl}_{pr}(S)$ of S in $\mathbb{R}^{\mathbb{N}}$ considered with the product topology.

Ex. 5. Let $\langle X, \mathcal{T} \rangle$ be a compact Hausdorff topological space and let $\mathcal{B} \subset \mathcal{T}$ be such that

(i) $U \cap V \in \mathcal{B}$ for all $U, V \in \mathcal{B}$;

(ii) for every $x \in X$ we have $\bigcap\{\mathrm{cl}(U) \colon x \in U \in \mathcal{B}\} = \{x\}$.

Show that $\mathcal{B}$ is a basis for $\langle X, \mathcal{T} \rangle$. In your argument include definition of a basis for $\langle X, \mathcal{T} \rangle$.