

Fall 2022

## Topology

WVU Mathematics Department

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NAME (print): \_\_\_\_\_

**Topology Ph.D. Entrance Exam, August, 2022**

Solve **four** from the following five exercises. *We will grade only four of them, so you need to decide which four solutions to submit.* Write a solution of each exercise on a separate page.

In what follows the symbols  $\text{int}(A)$  and  $\text{cl}(A)$  stand, respectively, for the interior and the closure of  $A$ . Any subset of  $\mathbb{R}$  is considered with the standard topology, unless stated otherwise.

**Ex. 1.** Let  $X$  be a normal topological space and let  $F$  and  $K$  be closed disjoint subsets of  $X$ . Show that there exists open sets  $U \supset F$  and  $V \supset K$  with disjoint closures. In your solution include definition of a normal space.

**Ex. 2.**

- (a) Give a definition of Lindelöf space.
- (b) Show, directly from the definition, that if  $A$  is a closed subset of a Lindelöf space  $X$ , then  $A$  is Lindelöf.
- (c) Give a definition of separable space.
- (d) Let  $A$  be a closed subset of a separable space  $X$ . Must  $A$  be separable subspace of  $X$ ? Prove your statement or give a counterexample.

**Ex. 3.** Let  $X$  be a topological space and let  $A$  be a connected subset of  $X$ . Show that if  $A \subseteq B \subseteq \text{cl}(A)$ , then  $B$  is connected.

**Ex. 4.** Let

$$S = \{x \in \mathbb{Q}^{\mathbb{N}} : \text{there is an } n \in \mathbb{N} \text{ so that } x(i) = 0 \text{ for every } i > n\},$$

where  $\mathbb{Q}$  is the set of all rational numbers and  $\mathbb{N} = \{1, 2, 3, \dots\}$ . Find the closure  $\text{cl}_{pr}(S)$  of  $S$  in  $\mathbb{R}^{\mathbb{N}}$  considered with the product topology.

**Ex. 5.** Let  $\langle X, \mathcal{T} \rangle$  be a compact Hausdorff topological space and let  $\mathcal{B} \subset \mathcal{T}$  be such that

- (i)  $U \cap V \in \mathcal{B}$  for all  $U, V \in \mathcal{B}$ ;
- (ii) for every  $x \in X$  we have  $\bigcap \{\text{cl}(U) : x \in U \in \mathcal{B}\} = \{x\}$ .

Show that  $\mathcal{B}$  is a basis for  $\langle X, \mathcal{T} \rangle$ . In your argument include definition of a basis for  $\langle X, \mathcal{T} \rangle$ .