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## Topology

WVU Mathematics Department

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NAME (print): \_\_\_\_\_

**Topology Ph.D. Entrance Exam, April, 2022**

Solve **four** from the following five exercises. *We will grade only four of them, so you need to decide which four solutions to submit.* Write a solution of each exercise on a separate page.

In what follows the symbols  $\text{int}(A)$  and  $\text{cl}(A)$  stand, respectively, for the interior and the closure of  $A$ . Any subset of  $\mathbb{R}$  is considered with the standard topology, unless stated otherwise.

**Ex. 1.** Let  $X$  be a topological space. Prove that  $A \subset X$  is closed if, and only if, for every  $x \in X \setminus A$  there exists closed subset  $F$  of  $X$  such that  $A \subset F$  and  $x \notin F$ .

**Ex. 2.** Show that every infinite connected compact Hausdorff space is uncountable.

**Ex. 3.** Consider  $\mathbb{R}^n$  with the standard topology. Show, that a nonempty  $A \subset \mathbb{R}^n$  is compact if, and only if,  $f[A]$  is bounded for every continuous  $f: A \rightarrow \mathbb{R}$ .

**Ex. 4.** Let  $\mathbb{R}^\omega$  be a countable product of real lines and consider it with the product topology. Show that

$$Z := \{(x_n)_n \in \mathbb{R}^\omega : x_n \text{ is rational for all but finitely many } n\},$$

considered with the subspace topology of  $\mathbb{R}^\omega$ , is connected.

**Ex. 5.** Let  $\langle X, d \rangle$  be a metric space. For any  $x_0 \in X$  and  $\varepsilon > 0$  define

$$B(x_0, \varepsilon) := \{x \in X : d(x, x_0) < \varepsilon\} \quad \& \quad B[x_0, \varepsilon] := \{x \in X : d(x, x_0) \leq \varepsilon\}.$$

(i) Show that  $\text{cl}(B(x_0, \varepsilon)) \subseteq B[x_0, \varepsilon]$ .

(ii) Give an example of a metric space  $X$  where for some  $x_0 \in X$  and  $\varepsilon > 0$  we have  $\text{cl}(B(x_0, \varepsilon)) \neq B[x_0, \varepsilon]$ .