1992

Anharmonic elasticity of smectics A and the Kardar-Parisi-Zhang model

Leonardo Golubović

Zhen-Gang Wang

Anharmonic Elasticity of Smectics *A* and the Kardar-Parisi-Zhang Model

Leonardo Golubović[(1),(2)] and Zhen-Gang Wang[(1)]
[(1)]*Chemical Engineering 210-41, California Institute of Technology, Pasadena, California 91125*
[(2)]*Physics Department, West Virginia University, Morgantown, West Virginia 26506*
(Received 7 August 1992)

We relate anharmonic *equilibrium* thermal fluctuations of smectics *A* to fluctuations of the Kardar-Parisi-Zhang (KPZ) *dynamical* model for a growing interface. The KPZ model in 1+1 dimensions is one to one related to a 2D smectic elastic model whose scaling behavior is then obtained *exactly*. The KPZ model in 2+1 dimensions maps into an elastic critical point of 3D smectics *A* with *broken inversion symmetry* (head-to-tail packing of layers). We discuss the elasticity and fluctuations of these novel smectic-*A* phases.

PACS numbers: 64.70.Md, 05.40.+j, 61.30.−v

One of the most striking phenomena in the statistical physics of elastic media is the breakdown of the harmonic (Hookean) elasticity in smectics *A* [1]. Grinstein and Pelcovits demonstrated that anharmonic effects of thermal fluctuations cause, at long length scales, a nontrivial logarithmic renormalization of 3D smectic elastic constants inducing, in particular, a breakdown of the classical elasticity theory: linear Hooke's law is replaced by a nonlinear strain response to *arbitrarily* weak external stresses. This seminal discovery inspired numerous studies in liquid crystals, spin glasses, exotic magnets, membranes, and nematic polymers [2]. These phenomena exist in seemingly remote physical problems such as Rayleigh-Bénard roll instability [3] (analog of 2D smectics [4]) and pion condensation in neutron stars (analog of 3D smectics [5]).

On the other side, similar breakdown of the classical, harmonic fluctuation theory is the main theme also of the statistical physics of growing interfaces [6]. This field is actively being developed, in particular, due to a direct interest in scaling properties of interfaces growing in the presence of external fluxes [7], as well as its relationship to other physical problems [6,8].

This Letter reveals a deep relationship between *nonequilibrium* statistical physics of growing interfaces and *equilibrium* statistical physics of smectics *A*. We relate the Kardar-Parisi-Zhang model [7] for the dynamics of $(d-1)$-dimensional growing interface to *nonlinear* elasticity theory of *d*-dimensional smectics *A*. For 2D smectics [4], we obtain the *exact* scaling behavior of the nonlinear elasticity theory, with correlations of smectic layer displacements u of the form $\langle[u(x,z)-u(0,0)]^2\rangle^{1/2}=K(x,z)$, with $K(x,0)\sim x^{1/2}$, and $K(0,z)\sim z^{1/3}$ [z axis (x axis) is normal (parallel) to the layers]. Linear Hooke's law breaks down and is replaced by the nonlinear law of strain $\sim$ (stress)$^{2/3}$, for *arbitrarily weak* external stresses normal to layers. These results hold for *stricto sensu* elastic models with no dislocations, which are actually free in a 2D smectic and convert it, at any finite temperature T, into a nematic phase at length scales longer than the sizes of cybotactic groups ξ_x and ξ_z ($\xi_x,\xi_z\to\infty$ as $T\to 0$) [4]. Our theory enables us to obtain the exact form of the *anisotropic scaling* at the zero-T transition from the 2D smectic to the nematic phase: $\xi_z\sim\xi_x^{3/2}$. All these results are of interest not only for 2D smectics, but also for their analogs such as Rayleigh-Bénard systems [3], or stripe domain phases in thin ferromagnetic films in which the anisotropic scaling has also been observed recently [9].

Another important contribution of this work is the elastic model for smectics *A* with *broken spatial inversion symmetry* [10,11], such as ferroelectric smectics *A* with an average dipolar moment normal to the layers, resembling a lamellar phase of surfactant monolayers stacked according to the "head-to-tail" rule [10]. Numerous molecular architectures capable of forming such phases have been proposed in the past [11]. Their experimental realization is, however, of a quite recent date [12]. As discussed here, the elastic model of these phases must contain a new, rotationally invariant term of the form eH, coupling the strain e and the layer curvature H. This term is forbidden in ordinary, inversion symmetric smectics *A* since it changes sign under spatial inversion. We find that the KPZ model in 2+1 dimensions maps into a novel *elastic critical point* of 3D smectics *A* with broken inversion symmetry. At this point, displacement fluctuations strongly diverge as *power laws* of the system sizes L_x and L_z: for $L_z\gg L_x$, $\langle u^2\rangle^{1/2}\sim L_x^\alpha$, $\alpha\approx 0.4$; for $L_x\gg L_z$, $\langle u^2\rangle^{1/2}\sim L_z^\beta$, $\beta\approx 0.25$, in contrast to a much weaker Landau-Peierls logarithmic divergence in inversion symmetric smectics [13].

We first propose a nonlinear elastic model for smectics *A* with broken inversion symmetry and screened long-range dipolar interactions. Their elastic energy can be represented via a sum of layers' energies, $E=\sum_n E_n=\int(dz/l)E_{z/l}$, with l the equilibrium layer thickness. E_n contains a compressional, $E_{n,\mathrm{com}}$, and a curvature contribution, $E_{n,\mathrm{curv}}$. $E_{n,\mathrm{com}}$ is generally of the form $E_{n,\mathrm{com}}=\frac{1}{2}B\int dS\,e^2$, where the integral is over the area of the nth layer, specified by its height $h(\mathbf{x},z)|_{z=nl}$, $dS=d^{d-1}\mathbf{x}[1+(\nabla_\mathbf{x}h)^2]^{1/2}$, and $e=\partial_z h[1+(\nabla_\mathbf{x}h)^2]^{-1/2}-1$ is the Lagrangian strain [the displacement $u(\mathbf{x},z)=h(\mathbf{x},z)-z$ changes layer thickness, measured normal to the layer, by $\delta l=el$]. Layers are fluid, and $E_{n,\mathrm{curv}}$ is, as for fluid membranes [14], $E_{n,\text{curv}}=\frac{1}{2}K\int dS(H^2+2H_0H)$, with H, the layer curvature, $H=\nabla_{\mathbf{x}}\{\nabla_{\mathbf{x}}h/[1+(\nabla_{\mathbf{x}}h)^2]^{1/2}\}$, while the constant H_0 is the so-called *spontaneous curvature.* Its presence *breaks the inversion symmetry* $(h,\mathbf{x})\to(-h,-\mathbf{x})$. $H_0\neq 0$ can arise only if nematogens do not have a center of inversion (head-and-tail molecules) *and* if most of the tails are on one while most of the heads are on the other layer side. Such a layer *alone* would tend to bend towards one, say tail, side. Now, let us stack the layers so that the head side of a layer is adjacent to the tail side of its neighbor (head-to-tail rule [10]). By summing layers' energies, we obtain the *full* nonlinear, rotationally invariant smectic elastic Hamiltonian

$$E_{\text{sm}}=\int dz\,d^{d-1}\mathbf{x}\{1+(\nabla_{\mathbf{x}}h)^2\}^{1/2}\left[\frac{B_{\text{sm}}}{2}e^2+\frac{K_{\text{sm}}}{2}H^2+\gamma_{\text{sm}}H\right]$$
$$=\int dz\,d^{d-1}\mathbf{x}\{1+(\nabla_{\mathbf{x}}h)^2\}^{1/2}\left[\frac{B_{\text{sm}}}{2}e^2+\frac{K_{\text{sm}}}{2}H^2-\gamma_{\text{sm}}eH\right]. \quad (1)$$

Here $B_{\text{sm}}=B/l$, $K_{\text{sm}}=K/l$, and $\gamma_{\text{sm}}=KH_0/l$. The two forms of H_{sm} in Eq. (1) differ by a surface term not affecting bulk fluctuations. The eH terms in (1), with $\gamma_{\text{sm}}\sim H_0$, is *odd* under spatial inversion $h(\mathbf{x},z)\to -h(-\mathbf{x},-z)$ under which $e\to e$, $H\to -H$. $\gamma_{\text{sm}}=0$ in ordinary, symmetric smectics A because either the heads (tails) are equally distributed between layer sides (so $H_0=0$), or, if this is not the case, asymmetric layers are arranged head to head, tail to tail, so that spontaneous curvature contributions of neighboring layers cancel.

In 2D (and *only* in 2D) the γ_{sm} term in (1) becomes a derivative contributing only to the boundary energy: Note that the spontaneous curvature of each layer contributes the energy $\sim\int dx[1+(\partial_x h)^2]^{1/2}H=\int ds\,\partial\theta/\partial s$, with s the arclength, and θ the local layer tilt angle $[=\tan^{-1}(\partial_x h)]$. Thus, the inversion-symmetry-breaking term does not affect bulk fluctuations in 2D. From the point of view of bulk phonons, the ordinary ($\gamma_{\text{sm}}=0$) and asymmetric smectics A ($\gamma_{\text{sm}}\neq 0$) are *identical* in 2D.

We now relate the nonlinear smectic Hamiltonian (1) to the KPZ model for the evolution of the profile $h(\mathbf{x},t)$ of a $(d-1)$-dimensional growing interface [7]. The full nonlinear, rotationally invariant dynamical model for the interface of an isotropic (amorphous) cluster growing in an isotropic depositing flux has the form

$$\frac{\partial h}{\partial t}=[1+(\nabla_{\mathbf{x}}h)^2]^{1/2}(\lambda+\nu H)+[1+(\nabla_{\mathbf{x}}h)^2]^{1/4}\eta(\mathbf{x},t). \quad (2)$$

λ is the (bare) mean velocity of the interface. In the following, we chose a time unit such that $\lambda=1$. ν is a surface tension, and η is a noise, with the distribution

$$P(\eta)\sim\exp\left[-\frac{1}{4D}\int dt\,d^{d-1}\mathbf{x}\,\eta^2(\mathbf{x},t)\right]. \quad (3)$$

To relate (2) and (3) to the smectic model (1), we first identify the *time* coordinate t of the KPZ model with the smectic *spatial* coordinate z, $t=z$. Next, we map the *dynamical* problem (2) and (3) in $d-1$ spatial dimensions $(\mathbf{x})$ into an *equilibrium* statistical mechanics problem for the field $h(\mathbf{x},z)$ in d spatial dimensions $(\mathbf{x},z)$. This can be accomplished by applying the classical dynamics path integral formalism [15], yielding the probability weight $P(h)$ of a field configuration h. $h(\mathbf{x},z)$ can be interpreted now as the smectic layer height function. (So, we identify the sequence of snapshots of the KPZ interfaces taken at equal time intervals with a stack of smectic layers.) $P(h)$ is simply obtained from $P(\eta)$ in (3), by changing variable $\eta\to h$. By (2), with $\lambda=1$, $\eta(h)=[1+(\nabla_{\mathbf{x}}h)^2]^{1/4}(e-\nu H)$, where e and H are smectic local strain and layer curvature. Thus, $P(h)=P(\eta)\times J(h)$, with $J(h)=|D\eta/Dh|$. The Jacobian J can be ignored without qualitative consequences for the following [16]. Thus, eventually, $P(h)$ assumes the form of a Boltzmann factor, $P(h)\sim\exp[-H_{\text{eff}}(h)]$, with the effective Hamiltonian

$$H_{\text{eff}}(h)=\frac{1}{4D}\int dz\,d^{d-1}\mathbf{x}\{1+(\nabla_{\mathbf{x}}h)^2\}^{1/2}[e-\nu H]^2. \quad (4)$$

Note that $H_{\text{eff}}(h)$ is equivalent to the smectic Hamiltonian (1) for a special choice of B_{sm}, K_{sm}, and γ_{sm} ensuring that the expression in square brackets in the second line of Eq. (1) is, as in Eq. (4), a *full square.* It follows that for the special value of γ_{sm},

$$\gamma_{\text{sm}}=\gamma_c=\pm(K_{\text{sm}}B_{\text{sm}})^{1/2}, \quad (5)$$

the *equilibrium* behavior of the smectic elastic model (1) is directly related to the dynamical *behavior* of the KPZ model (2) and (3) in *any d.*

Recall now that, for $d=2$, the γ_{sm} term in (1) contributes only to the boundary energy. Thus, in 2D, Eq. (4) reduces to the ordinary smectic Hamiltonian, Eq. (1) with $\gamma_{\text{sm}}=0$, $B_{\text{sm}}=1/2D$, and $K_{\text{sm}}=\nu^2/2D$. So, the standard 2D smectic elastic model is *one to one* related to the KPZ model. Using this and the *exact* results for the KPZ model in 1+1 dimensions [7], we arrive at the following conclusions about 2D smectics: (I) Correlations of smectic displacements, $u(x,z)=h(x,z)-z$, are given by $\langle[u(x,z)-u(0,0)]^2\rangle^{1/2}=K(x,z)$, with

$$K(x,z)=|x|^{\alpha}\phi(|z|/|x|^{\alpha/\beta}), \quad (6)$$

where $\phi(s)\to s^{\beta}$ for $s\to\infty$, while $\phi(s)\to\text{const}$ for

$s \to 0$. [Thus, $K(x,0) \sim |x|^{\alpha}$, $K(0,z) \sim |z|^{\beta}$.] Here $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{3}$ exactly [7]. (II) Smectic elastic constants undergo a nontrivial renormalization at long length scales (small wave vectors q) of the form $K_{sm}(q) \sim |q_x|^{-1/2}$, for $q_z = 0$, and $K_{sm} \sim |q_z|^{-1/3}$, for $q_x = 0$, while $B_{sm}(q) \sim |q_x|^{1/2}$, for $q_z = 0$, and $B_{sm} \sim |q_z|^{1/3}$, for $q_x = 0$ [17]. This renormalization causes a breakdown of linear elasticity theory: The strain responds *nonlinearly* to *weak* external stresses S along the z direction [18],

$$\langle e \rangle \sim S^{2/3}.$$

(III) The above results hold for 2D smectics with purely phonon-type excitations. Behavior at long length scales is, however, affected by dislocations with are free in 2D smectics at any finite temperature T [4]. They convert a 2D smectic A, at large length scales, into a nematic, so that the smectic order still persists inside anisotropic domains (cybotactic groups) with sizes ξ_z and ξ_x along z and x directions: $\xi_x \xi_z = \xi_D^2 = 1/n_D$, where $n_D \sim \exp(-\text{const}/T)$ is the density of free dislocations. Harmonic elasticity theory, with $\xi_z \sim \xi_x^2$, then yields $\xi_x \sim \xi_D^{2/3}$ and $\xi_z \sim \xi_D^{4/3}$, as found by Toner and Nelson [4]. The above anharmonic effects produce a different scaling for large enough cybotactic groups. According to Eq. (6), $\xi_z \sim \xi_x^{\alpha/\beta} = \xi_x^{3/2}$, and thus $\xi_x \sim \xi_D^{4/5}$ and $\xi_z \sim \xi_D^{6/5}$.

We now turn to 3D smectics A with broken inversion symmetry [10-12], by first discussing their behavior for the *special* value of the symmetry-breaking coupling $\gamma_{sm} = \gamma_c = \pm (K_{sm} B_{sm})^{1/2}$, Eq. (5), for which they are related to the KPZ model in 2+1 dimensions. By using this relationship, we conclude the following for $\gamma_{sm} = \gamma_c$: (I) The displacement correlations have the form of Eq. (6), with $\beta = \alpha/(2-\alpha)$ (in any d [7]). For $d = 3$, $\alpha \approx 0.40$ and $\beta \approx 0.25$ [19]. (II) For $\gamma_{sm} = \gamma_c$, elastic constants of 3D smectics A undergo a nontrivial renormalization at small wave vectors of the form ($d = 3$) $K_{sm} \sim q_x^{\alpha}$, for $q_z = 0$, and $K_{sm} \sim q_z^{\alpha/(2-\alpha)}$, for $q_x = 0$, whereas $B_{sm}(q) \sim [K_{sm}(q)]^3$ [17]. So, both K_{sm} and B_{sm} vanish at long length scales ($q \to 0$). (III) This softening of elastic constants causes a breakdown of Hooke's law: A weak stress S normal to layers produces a strain $\langle e \rangle \sim S^{\eta_S}$, with $\eta_S = 2(1-\alpha)/(d-1+\alpha)$ [18]. This, with $\alpha \approx 0.4$ in $d = 3$ [19], gives $\eta_S \approx 0.5$, i.e., $\langle e \rangle \sim S^{1/2}$. (IV) The softening of elastic constants produces violent displacement fluctuations diverging for $\gamma_{sm} = \gamma_c$ as *power laws* of the system sizes: $\langle u^2 \rangle^{1/2} \sim L_x^{\alpha}$ for $L_z \gg L_x$, and $\langle u^2 \rangle^{1/2} \sim L_z^{\beta}$ for $L_x \gg L_z$. This divergence is much stronger than the well-known Landau-Peierls logarithmic divergence in ordinary smectics A having $\gamma_{sm} = 0$ [13]. For $\gamma_{sm} = \gamma_c$, strong displacement fluctuations destroy long-range translational order and produce *exponentially* decaying translational correlations. This is in marked contrast to the situation for $\gamma_{sm} = 0$ with a power-law decay of translational correlations [13]. The state at $\gamma_{sm} = \gamma_c$ would appear like a nematic. Nonetheless, thermal undulations dephasing translational correlations do not destroy the integrity of smectic layers which only assume a rough appearance similar to that of successive snapshots of the KPZ model interfaces [6]. Correlations of director fluctuations $\langle \nabla_{\mathbf{x}} h(\mathbf{x},z) \nabla_{\mathbf{x}} h(\mathbf{0},0) \rangle$ decay, for $\gamma_{sm} = \gamma_c$, as $|\mathbf{x}|^{-2(1-\alpha)}$, for $z = 0$, and as $|z|^{-2(1-\alpha)/(2-\alpha)}$, for $\mathbf{x} = \mathbf{0}$, in contrast to 3D nematics where these correlations decay as $|\mathbf{x}|^{-1}$ and $|z|^{-1}$.

Thus, the relationship to the KPZ model provides an understanding of 3D smectics for the *particular* value of $\gamma_{sm} = \gamma_c = \pm (K_{sm} B_{sm})^{1/2}$, Eq. (5). For a *general* γ_{sm}, we analyzed Eq. (1) by a one-loop renormalization-group (RG) transformation. For $d = 3$ (and for any $d \neq 2$), the RG flow pattern has a *separatrix* occurring for $|\gamma_{sm}| = |\gamma_c| = (K_{sm} B_{sm})^{1/2}$: For $|\gamma_{sm}| < |\gamma_c|$, flows iterate to the symmetric, ordinary smectic line with $\gamma_{sm} = 0$. So, for $|\gamma_{sm}| < |\gamma_c|$, at the longest length scales, one has ordinary Landau-Peierls behavior [13] and logarithmic corrections of Grinstein and Pelcovits [1]. Along the separatrix $|\gamma_{sm}| = |\gamma_c|$, our RG actually reduces to that of KPZ [7], and one has the behavior discussed above [20]. The region $|\gamma_{sm}| > |\gamma_c|$ is a runaway region: the one-loop RG flows drive $|\gamma_{sm}|$ to ∞. So, the KPZ separatrix $|\gamma_{sm}| = |\gamma_c| = (K_{sm} B_{sm})^{1/2}$ is, in fact, a *critical line* between a Landau-Peierls phase ($|\gamma_{sm}| < |\gamma_c|$) and an "unstable" region ($|\gamma_{sm}| > |\gamma_c|$). A preliminary mean-field investigation of Eq. (1) in the region $|\gamma_{sm}| > |\gamma_c|$ indicates the onset of an undulated phase, with $\langle u(\mathbf{x},z) \rangle \neq 0$, in the form of a superposition of modulations with wave vectors normal to the z axis.

We thank Tom Lubensky and Robijn Bruinsma for useful discussions. This work was supported in part by the Camille and Henry Dreyfus Foundation.

Note added.—After this work was done, a related work, Ref. [21], was brought to our attention.

[1] G. Grinstein and R. A. Pelcovits, Phys. Rev. Lett. **47**, 856 (1981); Phys. Rev. A **26**, 915 (1982).

[2] Y. Park, T. C. Lubensky, P. Barois, and J. Prost, Phys. Rev. A **37**, 2197 (1988); L. B. Ioffe and M. V. Feigel'man, Zh. Eksp. Teor. Fiz. **85**, 1801 (1983) [Sov. Phys. JETP **58**, 1047 (1983)]; L. Golubović and M. Kulić, Phys. Rev. B **37**, 7582 (1988); D. R. Nelson and L. Peliti, J. Phys. (Paris) **48**, 1085 (1987); J. Aronovitz and T. C. Lubensky, Phys. Rev. Lett. **60**, 2634 (1988); J. Aronovitz, L. Golubović, and T. C. Lubensky, J. Phys. (Paris) **50**, 609 (1989); E. Guitter, *ibid.* **51**, 2407 (1990); J. Toner, Phys. Rev. Lett. **68**, 1331 (1992).

[3] J. Swift and P. C. Hohenberg, Phys. Rev. A **15**, 319 (1977).

[4] J. Toner and D. R. Nelson, Phys. Rev. B **23**, 316 (1981).

[5] G. Baym, B. Friman, and G. Grinstein, Nucl. Phys. **B210/FS6**, 193 (1982).

[6] For a review, see *Dynamics of Fractal Surfaces,* edited by F. Family and T. Vicsek (World Scientific, Singapore, 1991).

[7] M. Kardar, G. Parisi, and Y.-C. Zhang, Phys. Rev. Lett. **56**, 889 (1986).

[8] M. Kardar and Y.-C. Zhang, Phys. Rev. Lett. **58**, 2087 (1987); D. Foster, D. R. Nelson, and M. J. Stephen, Phys. Rev. A **16**, 732 (1977); E. Medina, M. Kardar, Y. Shapir, and X.-R. Wang, Phys. Rev. Lett. **62**, 941 (1989).

[9] M. Seul, L. R. Monar, L. O'Gorman, and R. Wolfe, Science **254**, 1616 (1991); M. Seul and R. Wolfe, Phys. Rev. Lett. **68**, 2460 (1992).

[10] S. A. Pikin and V. L. Indenbom, Usp. Fiz. Nauk. **126**, 251 (1978) [Sov. Phys. Usp. **21**, 487 (1978)]; R. Blinc and B. Žekš, *Soft Modes in Ferroelectrics and Antiferroelectrics* (North-Holland, Amsterdam, 1974).

[11] R. G. Petschek and K. M. Wiefling, Phys. Rev. Lett. **59**, 343 (1987); P. Pallfy-Muhoray, M. A. Lee, and R. G. Petschek, *ibid.* **60**, 2303 (1988); Lin Lei, Mol. Cryst. Liq. Cryst. **146**, 41 (1987); L. M. Leung and Lin Lei, *ibid.* **146**, 71 (1987).

[12] F. Tournilhac, L. M. Blinov, J. Simon, and S. V. Yablonsky (unpublished).

[13] A. Caille, C. R. Acad. Ser. B **274**, 891 (1972).

[14] W. Helfrich, Z. Naturforsch. **28C**, 693 (1973).

[15] P. C. Martin, E. D. Siggia, and H. A. Rose, Phys. Rev. A **8**, 423 (1973); C. De Dominicis, Phys. Rev. B **18**, 4913 (1978).

[16] The rotationally invariant KPZ model (2) is usually represented in a truncated form; only the terms up to the second order in $u(\mathbf{x},z)=h(\mathbf{x},z)-z$ are kept since higher-order terms are irrelevant [7]. The rotationally invariant smectic model (1) can be (with the same justification) truncated up to terms of the fourth order in u. The truncated models (the KPZ and the smectic one) preserve infinitesimal rotational invariance (known as "Galilean" invariance in the KPZ case). By applying the formalism of Refs. [15] to the truncated KPZ model, one gets the truncated smectic model as $H_{\rm eff}$, while the Jacobian $J(h)=|D\eta/Dh|$ turns out to be a constant. In the case of the full nonlinear KPZ model (2), J is a functional of h, which, however, contributes only irrelevant terms not affecting the universal scaling.

[17] The $\gamma_{\rm sm}$ term in (1) contributes, to the *lowest* order in u, the *cubic* term $(\nabla_{\mathbf{x}}u)^2\nabla_{\mathbf{x}}^2u$. Thus, the quadratic, harmonic approximation to (1) is *insensitive* to the value of $\gamma_{\rm sm}$, and the harmonic propagator is of the standard form $G_0=(B_{\rm sm}q_z^2+K_{\rm sm}q_x^4)^{-1}$. The renormalized propagator G_R is obtained by replacing $B_{\rm sm}\to B_{\rm sm}(q)$, $K_{\rm sm}\to K_{\rm sm}(q)$, and the form of the renormalized elastic constants is extracted by requiring G_R to yield the scaling of corrections as given by (6). This yields

$$K_{\rm sm}\sim q_x^{d-3+\alpha}\sim q_z^{(d-3+\alpha)/(2-\alpha)},$$

$$B_{\rm sm}\sim q_x^{d-3+3\alpha}\sim q_z^{(d-3+3\alpha)/(2-\alpha)}.$$

[18] This result is obtained along the lines of the discussion of stress-strain relationship in systems with nonclassical elasticity in Aronovitz, Golubović, and Lubensky, Ref. [2].

[19] J. M. Kim and J. M. Kosterlitz, Phys. Rev. Lett. **62**, 2289 (1989); B. M. Forest and L.-H. Tang, *ibid.* **64**, 1405 (1990); J. G. Amar and F. Family, Phys. Rev. A **41**, 3399 (1990).

[20] For $d<3$, our one-loop RG gives a *perturbative* anharmonic fixed point (A) with $\gamma_{\rm sm}^*=0$, yielding, in the region $|\gamma_{\rm sm}|<|\gamma_c|$, the scaling as in Eq. (6), with $\alpha=3(3-d)/(8-d)$, $\beta=3(3-d)/(7+d)$. Scaling along the critical, KPZ line ($|\gamma_{\rm sm}|=|\gamma_c|$) is regulated by a different, *nonperturbative* fixed point (KPZ) with $\gamma_{\rm sm}^*\neq0$. For $d>2$, KPZ is unstable with respect to A. As $d\to2$, this instability vanishes, and a *line* of fixed points containing both A and KPZ is formed in $d=2$. The appearance of this line is related to the fact that, in 2D, the $\gamma_{\rm sm}$ term in (1) becomes a boundary term not affecting bulk fluctuations. So, all the points of the fixed line yield the *same* scaling behavior with $\alpha=1/2$, $\beta=1/3$ *exactly*. Note also that, in 2D, the above one-loop results for α and β for A agree with the exact values for KPZ. In 2D, the one-loop theory becomes exact in the continuum (see also Ref. [7]).

[21] F. Tournilhac, J. Prost, and R. Bruinsma (unpublished) considered the effects of unscreened dipolar interactions. Weak unscreened dipolar interactions have a similar effect as the external stress considered here.