An Empirical Analysis of County-Level Determinants of Small Business Growth and Poverty in Appalachia: A Spatial Simultaneous-Equations Approach

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Abstract

A spatial simultaneous-equations growth equilibrium model estimated by GS2SLS and GS3SLS estimators is used to determine the interdependence between small business growth and poverty. The parameter estimates are mostly consistent with the theoretical expectations. The coefficients for the endogenous variables of the model are positive and significant indicating strong interdependence (feedback simultaneity) between small business and median household income growth rates. The results also show the presence of spatial autoregressive lag simultaneity and spatial cross-regressive lag simultaneity, with respect to both small business and median household income growth rates, and the existence of spatial correlation in the error terms. In addition, the estimates of the structural parameters show that there were strong agglomerative effects and significant conditional convergence with respect to both small business growth and median household income growth in Appalachia during the study period.

Key Words: Small business growth, poverty, spatial analysis, Appalachia

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Introduction

Persistent poverty is one of the most critical social problems facing policy makers in the United States. Despite decades of government intervention, and billions of dollars spent, many communities remain in poverty. The economic boom of the 1990s failed to reduce poverty in many counties in Appalachia maintained above average poverty rates (Rupasingha and Goetz, 2003). In spite of an expanding economy, many counties in Appalachia continue to suffer from high unemployment rates, a shrinking economic base, deeply rooted poverty, low human capital formation, and out migration (Deavers and Hoppe, 1992; Hayness, 1997; Dilger and Witt, 1994; Maggard, 1990). The slow growth of income and employment, out-migration, and the disappearance of rural households are both causes and effects of persistently high rates of poverty (Cushing and Rogers, 1996).

The changing structure of traditional industries and the impact of those changes on local communities have been sources of concern to many groups interested in the welfare of small communities. State policy makers and local leaders have long placed a high priority on local economic development (Pulver, 1989; Ekstrom and Leistritz, 1988). Consequently, a better understanding of factors that influence local employment, earning capacity, and quality of life issues has become important from county, state, and regional policy perspectives with respect to designing human capital development programs for rural community development. Since many of the forces responsible for past economic and social changes in small communities will
continue to affect families in small community, it is necessary to study the rural economy and evaluate alternative policy measures to promote diverse and resilient local communities.

Improving the economic base of a region requires an environment where business can prosper. Appalachia, however, despite efforts of national, state, and local policy programs to induce economic prosperity and ameliorate poverty, still has many economically depressed communities. To strengthen and diversify the economy, policy makers and local leaders need to know the characteristics and impact of small businesses on the local economy, to determine how small businesses can contribute to the growth of local economies.

**Literature Review**

**County-Level Determinants of Small Business Growth**

Confronted with concerns about unemployment, job creation, economic growth and international competitiveness, policy makers at all levels have responded with a mandate to promote the creation of new businesses (see Reynolds, 2000). Empirical studies show that most new business start small and that they generate the majority of new jobs created (Acs and Audretsch, 2001; Audretsch et al., 2000; Carree and Thurik, 1998, 1999; Wennekers and Thurik, 1999; Fritsch and Falck, 2003). These studies also indicate that there has been a structural shift in the industrial sector towards a more flexibility and knowledge-intensive production. This is considered to favor the small business sector. The recognition of the potential importance of new business formation for regional development motivates our interest to investigate the reasons why some economic spaces show high rates of new business formation while others do not.

A long tradition of studies of the determinants of new plant entry has focused on tax rates, transportation costs and scale economies at the plant level (Bartik, 1989; Kieschnick, 1981; Harrison and Kanter, 1978). More recently, a growing literature has sought the determinants of variation in new business formation on a regional basis (Reynolds, 1994; Acs and Armington, 2002; Fritsch, 1992; Audretsch and Fritsch, 1994; Hart and Gudgin, 1994; Keeble and Walker,
1994; Johnson and Parker, 1996; Davidson et al., 1994; Guesnier, 1994; Garofoli, 1994; Kangasharju, 2000; Fotopoulos and Spence, 1999; and Callejon and Segarra, 2001). Each of these studies attempted to identify the most important influences underpinning spatial variations in new business formation.

On the demand-side, most of these studies suggest that new and small businesses tend to serve restricted geographical markets, and are therefore influenced by local variations in the level and growth of market demand as measured by variables such as family median income, GDP, and resident population statistics. Increases in the demand for goods and services that result from increases in per capita income or GDP per capita are associated with higher business formation (Armington and Acs, 2002). As wealth increases consumer demand for a variety of products and services increases and small businesses are well equipped to supply these new and specialized goods and services (Curie, 2000). The employment share of the service sector, which is characterized by presence of small business, also increases with per capita income (Wennekers, Uhlaner and Thurik, 2002). In general, a growing population increases the demand for consumer goods and services and it is positively related to business formation (Acs and Armington, 2004a).

In addition to demand-side influences, both population growth and net migration measures incorporate supply-side influences. This is because population growth, which often includes net migration, also increases the local pool of potential entrepreneurs. Entrepreneurship and small business formation are strongly associated with previous population in-migration, which is stimulated by residential amenities and preference considerations (Keeble, Broom and Lewis, 1992).

Supply-side variables included in the studies reflect the supply of resources required to set up a new business. These include measures of aggregation/externalities, of unemployment, of the structure of production, of availability of capital and entrepreneurial culture.
The concentration of people and firms in an area decreases both the cost of access to customers and to suppliers (Reynolds, 1994). Both consumers and producers benefit from the easy availability of pooled services in such areas. This encourages new firm formation as a result of agglomeration effects that come from either demand effects, such as increase in population, or from regional spillovers, such as labor market characteristics. Krugman (1991a and 1991b) identified three types of spillovers within a region that may lead to the localization of economic activities. First, a pooled labor market most commonly associated with agglomerations yields increasing returns at a spatial level (Marshall, 1920). Second, agglomerations enable the production and provision of non-traded specialized inputs at a greater variety and lower cost. And third, spillovers emanate from economics in information flows, or what Jaffe (1989) and Acs, Audretsch and Feldman, (1992, 1994) term technological spillovers. Technological spillovers are more beneficial to new small firms than to incumbent large enterprises (Acs et al., 1994). Thus, regions where such spillovers are greatest are more conducive for new business locations.

Regional spillovers are likely to be most prevalent in areas with high population density because the infrastructure of services and inputs is more developed in such areas. The concentration of firms in a single location, for example, offers a pooled market for workers with industry-specific skills, ensuring both a lower probability of unemployment and a lower probability of labor shortage (Krugman, 1991a). Localized industries can also support the production of non-tradable specialized inputs. Besides, the informational spillovers that can give clustered firms a better production function than isolated firms. That is, economies of localization and urbanization yield reduced transaction cost. This suggests that both population density and population growth are positively related to new firm start-ups (Reynolds, 1991). Such agglomerations would also tend to exist where output per capita is relatively high.
The agglomeration effects that contribute to new firm formation can also come from supply factors related to the quality of the local labor market and business climate. Regions with similar demand and business climate patterns still differ in the rates of new firm formation, survival, and growth as a result of differences in their human capital endowment, and the propensity of locally available knowledge to spill over and stimulate new firm formation and growth. An educated population provides more human capital for implementing new ideas for creating and growing new businesses (Acs and Armington 2004b). A number of empirical studies have found a strong association between human capital and new firm formation and growth. Cross (1981), for example, argues that the availability of specialized labor influences the birth of new firms because there is a larger supply of potential entrepreneurs. Specialized workers are better prepared than non-specialized workers to create their own businesses, and workers with management skills are more likely to create a new firm (Lloyd and Mason, 1984).

Human capital studies have found that entrepreneurship is related to educational attainment and work experience. People with more educational attainment tend to found businesses more often than those with less educational attainment (Evans and Leighton, 1990). In the 1990s, there were increases in the incidence of highly educated people starting new businesses, especially in the highly advanced sectors of the economy, like computers, biotechnology, and internet-dependent businesses. Guesnier (1994) finds that the propensity to create a new firm is positively associated with adults with a bachelor degree. Highly educated people in most cases have easier access to research and development facilities, and perhaps insight into the business world and thus a clear idea about the present and the future needs of the market. Entrepreneurs with good education are also likely to know how to transform innovative ideas into marketable products (Christensen, 2000). People in regions that have a high percentage of college graduates are much more likely to start a business than those living in regions with high concentration of less skilled workers (Armington and Acs, 2002). Although the actual knowledge acquired with a
college degree seldom suffices as the basis for a successful new business, the analytical methods learned in college facilitates both future acquisition of knowledge and openness to new ideas received as spillover from other activities in the region (Acs and Armington 2004b). However, studies by Hart and Gudgin (1994) have shown that the percentage of the population with college degree is inversely associated with the rate of new firm formation. A comparative study by Uhlaner, Thurik and Hutjes, 2002) in fourteen OECD countries has also shown that countries with higher level of education tend to have a smaller proportion of self-employed. While the educational level of the entrepreneurs may not, however, play a specific role in the survival of individual firms, the general consensus is that education more broadly influences the overall probability of survival of new firms in a region (Storey, 1994).

Past research has found conflicting evidence about whether higher unemployment leads to more or to less new firm formation. Traditionally, regional unemployment rate has been used as a measure of regional economic distress, with high unemployment rates indicating slack growth, thereby dampening the incentives for new firms to locate within the region. Higher levels of unemployment might also indicate a reduction in aggregate demand throughout a regional economy, thereby putting downward pressure on the rate of new firm formation (Storey and Johnson, 1987). Moreover, unemployed individuals may not have the capital necessary to start their own business (Storey and Jones, 1987; Audretsch and Fritsch, 1994; Garofoli, 1994). Nevertheless, there is a substantial literature that indicates that higher levels of unemployment may lead to higher levels of firm formation. Actually, in many studies of new firm formation in the 1980s, there was a heavy emphasis on the possible positive explanatory power of unemployment (Evans and Leighton, 1990; Storey, 1991). High unemployment may lower labor costs, which favor the creation of new firms (Highfield and Smiley, 1987). A higher rate of unemployment also indicates that more people have reason to search for alternative ways to make a living. In the absence of alternative job opportunities, some workers may take steps to
start their own businesses (Davidsson, Lindmark and Olofsson, 1994; Beesley and Hamilton, 1994; Storey, 1994). This activity, in turn, reduces the unemployment rate as the resulting new firm employs not only the owners, but also others.

The empirical evidence provided at best depends on the methods it is followed to calculate the rate of new firm formation and on the data type used. If the rate of new firm formation is calculated with respect to the number of existing firms/establishments in the region, then higher rates of unemployment are positively associated with new firm formation. However, it is negatively associated with the rate of new firm formation if the latter is calculated with respect to number of employees in the region. Time series analyses point to unemployment being, ceteris paribus, positively associated with new firm formation, whereas cross sectional, or pooled cross sectional studies appear to indicate the reverse (Storey, 1991). Cross sectional studies by Armington and Acs (2002), however, indicate that the unemployment rate is positively related to new firm formation in US in the 1990s. Acs and Armington (2004b) also found that the unemployment rate is positively associated with the rate of new firm formation during recession and negatively associated during growth periods. The impact of the unemployment rate on the rate of new firm formation also depends on the type of the sector of activity, with industries that require small capital being more suitable for new firm formation during periods of higher unemployment (Armington and Acs, 2002). Thus, the direction of the effect of a region’s unemployment rate on new firm formation is indeterminate.

Higher personal household wealth can provide either the financial resources, as equity or loans to finance new business, that is required to start new firm or it reflects wealth and income that can create demand for goods and services that encourages entrepreneurship. In order to capture the availability of finance, several variables have been used in the empirical studies. These include variables such as the distribution of wealth at regional level (Fotopoulos and Spence, 1999); percentage of homes owned by their occupants (Storey, 1982; Ashcroft, Love and
The percentage of homes owned by their occupants is the variable that is frequently used in empirical analyses and it captures two different effects. A higher percentage of homes owned by their occupants may be an indication that there is a capacity to finance new businesses by potential entrepreneurs. Besides, a higher proportion of home ownership influences positively the formation of new firms because homes may be used as collateral for loans to start new business. In his study of the United States, Reynolds (1994) found that personal household wealth is associated with higher new firm formation in traditional rural regions. The local availability of personal finance, epitomized and embodied in the value of local owner-occupied housing, appears to play an important role in enabling or inhibiting new business creation (Keeble and Walker, 1994).

Guesnier (1994) and Garofoli (1994) have, however, found a negative relationship between home ownership and new firm formation. If houses already serve as collateral of bank loans and the burden imposed by those loans is too heavy for families, the ability to finance a new business is scant. Besides, the consumption of other goods is lower, influencing therefore the rate of new firm formation through the demand side. The other possibility where we can obtain a negative relationship between homeownership and the rate of new firm formation is that the young with the higher probability of becoming entrepreneurs tend to live in rented homes more than older individuals. This effect may be captured in the variable related with property ownership if we do not control for the percentage of the young individuals in our regression (Guesnier, 1994).
The size structure of existing enterprises can be a factor influencing the rate of new business formation. The shift from manufacturing to services that has resulted from industrial restructuring in the 1980s increased the rate of new firm formation. And many researchers suggest that areas having many small firms are likely to have high rates of new firm formation (Cross, 1981; Storey, 1982; Lloyd and Mason, 1984; O'Farrel and Crouchley, 1984; Garofoli, 1994; Keeble and Walker, 1994; Audretsch and Fritsch, 1994; Hart and Gudgin, 1994; Reynolds, 1994; Armington and Acs, 2002; Acs and Armington 2004b). A local business structure with no dominant large firms may offer fewer barriers to entry of new firms. In a region dominated by small firms there is a much broader population of business owners and more individuals may visualize their own careers as leading to the founding of independent new firms (Acs and Armington 2004b). Whereas regions that are dominated by large branch plants or firms will have less new firm formation (Gudgin, 1978; Mason, 1994, Garofoli, 1994; Keeble and Walker, 1994; Audretsch and Fritsch, 1994; Hart and Gudgin, 1994; Reynolds, 1994; Armington and Acs, 2002; Acs and Armington 2004b). This is because large firms both provide employment for highly skilled workers in the economy but they fail to provide a suitable training ground for new entrepreneurs. Cross (1981) argues that the small firm is the best incubator of entrepreneurial capacity. A large proportion of entrepreneurs usually spring from having had prior experience in small firms.

The importance of public services for regional growth stems from their effect on production and location decisions of private firms. Public services such as education, highways, public safety, sewer and, water treatment services can be viewed as unpaid inputs in the process of production of private businesses that contribute independently to output.

Many studies have shown that public services have positive and statistically significant effects on business location and growth (Fox, 1979; Charney, 1983; Bartik, 1985, 1989; Helms, 1985; Newman, 1983; Papke, 1991; Deich, 1989; Fisher, 1997; Gaygısız and Koksal, 2003;
Gabe and Bell, 2004). Fox (1979), for example, finds a positive location effect for local public services consumed by firms as measured by the expenditures for police and fire protection. A study by Charney (1983) also shows significant positive effects of the availability of water and sanitation infrastructure on location decision by firms. Similarly, Bartik (1991) found that fire protection services and local school spending have the strongest positive effects on small business start-ups. Among nineteen studies reviewed by Fisher (1997), education spending has a positive effect on business activities in twelve of them, and a positive and significant effect in six. More recently, a study by Gabe and Bell (2004) shows a positive and significant effect of local public spending on business location. Besides, Gabe and Bell (2004) find that the benefits of tax-financed public services are more important than the costs (taxes) as determinants of business location. Helms (1985) also found that local tax revenues used to fund transfer payments tend to reduce economic growth, whereas local tax revenues used to finance improvement in public services such as highways, education and public health tend to have a positive growth impact and concluded that a high public service level attracts businesses and economic activity, whereas transfer payments do not have the same positive effect on economic growth. Besides, Helms study shows that the net impact of tax-financed increases in government services is positive.

Studies by Reynolds (1994) Keeble and Walker (1994) and Audretsch and Fritsch, (1994), however, show that there is little evidence that variations in local government spending (on education, highways, public safety) have statistically significant effect on business growth.

**County-Level Determinants of Median Household Income**

The literature on economic growth at the regional level has focused attention on the so-called convergence hypothesis predicted by neoclassical growth theory: poorer regions grow faster than richer regions, that is, poorer regions tend to catch up with the richer regions in per capita income as time passes. Barro and Sala-i-Martin (1992, 2004), for example, find such
convergence for U.S. states, Japanese prefectures and between European countries. The results from the studies by Persson (1997) and by Aronsson et al. (2001) also show income convergence across Swedish counties. Study by Glaeser et al. (1995), however, does not show significant evidence of income convergence between U.S. cities. There are also other studies on regional/local income growth which have focused attention on a broader set of possible average income growth determinants, which include among others, geographic characteristics, initial conditions describing the regions (such as the average income, regional/local public expenditure, regional/local income tax rates, educational status of the population, resource endowment, etc.) and national policies directed towards the regional level (for example, Helms, 1985; Glaeser et al., 1995; Fagerberg, et al., 1997; Aronsson et al., 2001; Lundberg, 2003).

The initial average level of income is negatively related to the subsequent average income growth rate (Aronsson et al., 2001; Lundberg, 2003). Capital mobility tends to make regions more homogeneous over time, which leads to convergences in the sense that regions with low initial income levels tend to grow faster than regions with high initial income (Barro and Sala-i-Martin, 1992, 2004; Persson, 1997; Aronsson et al., 2001). Subsequent income growth is also positively related with initial unemployment rate. Unemployment causes out-migration, which decreases labor supply and increases wages during a subsequent period. The out-migration of unemployed persons changes the population composition such that average income increase for a given structure of wage among the employed (Aronsson et al., 2001). A rise in the regional or area unemployment rate, indicating a slack labor market, however, leads to low average per capita income, primarily through the depressing effect on wages (Duffy-Deno and Eberts, 1991; Bilger, Genosko and Hirte, 1991; Chalmers and Greenwood, 1980, 1984).

The level of human capital and physical capital and the underlying level of productivity in the long run determine per capita incomes. Particularly, the role of education and human capital externalities has been emphasized as a key variable in recent theories of economic
growth. Romer (1986), Lucas (1993), and Krugman (1991), for example, developed models that link such externalities within geographically bounded region to higher rates of growth in per capita income. Rauch (1993) also finds that cities with higher average levels of human capital also have higher wages. Similarly, Glaeser et al. (1995) find that for a cross-section of cities a key economic determinant of growth is the initial level of schooling of the population. Simon and Nardinelli (2002, 1996) also find historical evidence for both the United States and the United Kingdom that cities with more knowledgeable people grow faster in the long run. Duffy-Deno and Eberts (1991) find that the average year of education is positively related to the average per capita personal income. A study by Aronsson et al. (2001), however, shows that human capital, measured as the initial percentage of the population with a degree from university or college, has no effect on subsequent growth in per capita income. But, Aronsson et al. find that counties adjacent to regions where the major city areas are located tend to have higher growth rates of average income than other counties.

The size of the population of a region is positively correlated with real per capita personal income due to the beneficial effects of agglomeration economies of firm location (Duffy-Deno and Eberts, 1991). Population growth captures the extent to which regions are relatively attractive to migrants and a growing population increases the demand for consumer services which in turn leads to growth in business and employment. Incremental employment opportunities in turn provide a strong attraction for migrants that lead to net in-migration. While net in-migration increases local labor supply, it also increases local labor demand. If migrants possess differential endowments of human capital in form of education, accumulated skills, or entrepreneurial talent compared to the receiving population, then their skills, inventiveness and innovativeness will contribute to local productivity. Migrants may also own physical and financial capital that they bring with them and invest them in the receiving area. Moreover, migrants may contribute to the growth of markets and to the achievement of scale and
agglomeration economies. Such demand effects of net in-migration are sources of growth in per capita personal income. Greenwood et al. (1986), for example, find results that are consistent with a migrant-induced labor demand shift that offsets the migrant-induced labor supply shift. Bilger et al. (1991), and Chalmers and Greenwood, (1984) also find that regional net migration rates positively and significantly influence the change in regional median earnings. Similarly, Duffy-Deno and Eberts (1991) find that the proportion of manufacturing employment in a metropolitan area is positively correlated with per capita personal income.

The literature also provides evidence that local public expenditures on public health and hospitals, highways, local schools, higher education, police/fire protection, transfer payments/welfare, and other public services affect economic development as measured by different indicators such as net business establishments created (lost), net employment gains (losses), change in personal income, or/and change in per capita personal income (Duffy-Deno and Eberts, 1991; Jones, 1990). Government expenditures for certain aspects of the physical and human infrastructure such as expenditures on highways, education, and health can promote growth (Dye, 1980; Helms, 1985; Blair and Premus, 1987; Erickson, 1987; Schneider, 1987; Jones and Vedlitz, 1988, Jones, 1990; Glaeser et al., 1995). Dye’s (1980) early study, for example, found that increases in highway spending in the 1960s yielded relatively greater economic growth in the 1970s. Highway expenditures add to the capital investment in the local transportation system, whether these expenditures are for construction or maintenance. Public investment increases personal income by increasing employment and wages in the construction industry (Duffy-Deno and Eberts, 1991). The studies by Helms (1985) and Jones (1990), for example, report a significant and positive impact of highway expenditure on per capita personal income. Helms and Jones also find similar results for the impacts of public expenditures on education and health services on per capital personal income growth. Education expenditures add to the quality of the labor force. Health service expenditure is growth-enhancing due to the
externalities associated with preventive and primary care, which increases labor productivity and reduce the lose of working hours due to illness. Using a panel of 260 U.S. cities, Glaeser et al. (1995) find that economic growth is positively affected by public expenditures on sanitation, infrastructure, housing and urbanization, and transport services. Public expenditures on police and fire protection are positively related to per capita personal income growth (Jones, 1990). The impact of welfare spending on per capita personal income growth, however, is negative in most studies (Dye, 1980; Helms, 1985; Jones, 1990).

The Model: Small Business and Median Household Income Growth

The relationship between economic growth and its determinants has been studied extensively in the economic literature. The issue whether regional development can be associated with population driving employment changes or employment driving population changes (do ‘jobs follow people’ or ‘people follow jobs’?) has, for example, recently attracted considerable interest. Empirical works on identification of the direction of causality in this ‘jobs follow people’ or ‘people follow jobs’ literature (Steinnes and Fischer, 1974) have resulted in the view that empirical models of regional development often reflect the interdependence between household residential choices and firm location choices. To account for this causation and interdependency, Carlino and Mills (1987) suggested and constructed a two-equation simultaneous system with the two partial location equations as its components. This model has subsequently been used by a number of regional science researchers in order to examine regional economic growth (Boarnet, 1994; Duffy, 1994; Henry, Barkley, and Bao, 1997; Duffy-Dino, 1998; Barkley, Henry and Bao 1998; Henry et al. 1999; Edmiston, 2004). More recently, Deller, Tsai, Marcouiller, and English (2001) have expanded upon the original Carlino-Mills model to capture explicitly the role of income. According to the proposition of utility maximization in the traditional migration literature, households migrate to capture higher wages or income. The model expanded by Deller et al, (2001) is three-dimensional (jobs-people-income) and explicitly
traces the role of income in regional growth process. It also explicitly captures the increasing concerns about job quality as measured by income levels those jobs can support. There have also been efforts to model the interactions between employment growth and human migration (MacDonald, 1992; Clark and Murphy, 1996), per capita personal income and public expenditures (Duffy-Deno and Eberts, 1991), net migration, employment growth, and average income (earnings) (Greenwood and Hunt 1984; Greenwood et al., 1986; and Lewis, Hunt and Plantigna, 2002) in simultaneous-equations methods.

The theoretical base for the interdependencies between employment and income is the idea that households and firms are both mobile and that household location decisions maximize utility while firm location decisions maximize profits. That is, households migrate to capture higher wages or income and firms migrate to be near growing consumer markets. These actions in turn generate income to the regional (local) economy. The location decisions of firms, however, are expected to be influenced not only by population and income (i.e., growing consumer markets) but also by other factors such as local business climate, wage rates, tax rates, local public services, and regional location. Firm location decisions are also influenced by the substantial financial incentive that local governments offer in an effort to create jobs, spur income growth, and enhance the economic opportunities of the local population. Based upon these influences, the following hypotheses are constructed:

1. Business growth and median household income growth are interdependent and are jointly determined by regional covariates;

2. Business and median household income growth in a county are conditional upon initial conditions of county; and

3. Business and median household income growth in county are conditional upon business and median household income growth in neighboring counties
These hypotheses form the core research agenda for this study. Specifically, emphasis is put not only on examining the linkages among business growth and household median income, but also on investigating the elasticity of these variables with respect to each of the regional covariates. The elasticity analyses help to draw some policy recommendations for regional and rural development.

To test these hypotheses, a spatial simultaneous equations model of business growth and household median income is used. Following in the Carlino and Mills tradition and building upon Deller et al. (2001) and Lewis et al. (2002), the basic model is specified as

\[
\begin{align*}
EMP_{it}^* &= f_1(MHY_{it}^* | X_{it}^{em}) \\
MHY_{it}^* &= f_2(EMP_{it}^* | X_{it}^{mh})
\end{align*}
\]...........(3.1)

where \( EMP_{it}^* \) and \( MHY_{it}^* \) are equilibrium levels of private business employment and median household income, respectively, and \( i \) and \( t \) index county and time respectively. The vectors of additional exogenous variables that are included in the respective equations of the system of simultaneous equations are \( X_{it}^{em} \) and \( X_{it}^{mh} \), respectively.

The system of equations in (3.1) captures the simultaneous nature of the interactions between employment growth and median household income at equilibrium. The nature of interaction among the endogenous variables is dependent upon the initial conditions of a county.

In order to reduce the effects of the large diversity found in the data used in empirical analysis, a log-linear form of the model is used. Such specification also implies a constant-elasticity form for the equilibrium conditions given in (3.1). A log-linear (i.e., log-log) representation of these equilibrium conditions can thus be expressed as:
\[ EMP_{it}^* = \left( MHY_{it}^* \right)^{d_i} \times \prod_{k_i=3}^{K_i} \left( X_{kiit}^e \right)^{x_{ki}} \rightarrow \ln \left( EMP_{it}^* \right) = d_i \ln \left( MHY_{it}^* \right) + \sum_{k_i=3}^{K_i} x_{ki} \ln \left( X_{kiit}^e \right) \] (3.2a)

\[ MHY_{it}^* = \left( EMP_{it}^* \right)^{c_2} \times \prod_{k_i=3}^{K_i} \left( X_{kiit}^m \right)^{x_{ki}} \rightarrow \ln \left( MHY_{it}^* \right) = c_2 \ln \left( EMP_{it}^* \right) + \sum_{k_i=3}^{K_i} x_{ki} \ln \left( X_{kiit}^m \right) \] (3.2b)

where \( c_2 \) and \( d_i \) are the exponents on the endogenous variables, \( x_{ki} \) for \( i, j = 1, 2 \) are vectors of exponents on the exogenous variables, \( \prod \) is the product operator, and \( K_i \) for \( i = 1, 2 \) are the number of exogenous variables in the employment growth and median household income equations, respectively. The log-linear specification has an advantage of yielding a log-linear reduced form for estimation, where the estimated coefficients represent elasticities. Duffy-Deno (1998) and MacKinnon, White, and Davidson (1983) also show that a log-linear specification is more appropriate for models involving population and employment densities than a linear specification.

The literature (Edmiston, 2004; Hamalainen and Bockerman, 2004; Aronsson, Lundberg, and Wikstrom, 2001; Deller et al., 2001; Henry et al., 1999; Duffy-Deno, 1998; Barkley et al., 1998; Henry et al., 1997; Boarnet, 1994; Duffy, 1994, Carlino and Mills, 1987; Mills and Price, 1984) suggests that employment and median household income likely adjust to their equilibrium levels with a substantial lag (i.e., initial conditions). Following the literature a distributed lag adjustment is introduced and the corresponding partial-adjustment process for each of the equations given in (3.1) is of the form:

\[ \frac{EMP_{it}}{EMP_{it-1}} = \left( \frac{EMP_{it}^*}{EMP_{it-1}} \right)^{\eta_{em}} \rightarrow \ln \left( \frac{EMP_{it}}{EMP_{it-1}} \right) = \eta_{em} \ln \left( \frac{EMP_{it}^*}{EMP_{it-1}} \right) - \eta_{em} \left( EMP_{it-1} \right) \] (3.3a)

\[ \frac{MHY_{it}}{MHY_{it-1}} = \left( \frac{MHY_{it}^*}{MHY_{it-1}} \right)^{\eta_{mh}} \rightarrow \ln \left( \frac{MHY_{it}}{MHY_{it-1}} \right) = \eta_{mh} \ln \left( \frac{MHY_{it}^*}{MHY_{it-1}} \right) - \eta_{mh} \ln \left( MHY_{it-1} \right) \] (3.3b)
where the subscript \( t-1 \) refers to the indicated variable lagged one period, one decade in this study, and \( \eta_{em} \) and \( \eta_{mh} \) are the speed of adjustment parameters that represent, respectively, employment and median household income adjust to their respective desired equilibrium levels. They are interpreted as the shares or proportions of the respective equilibrium rate of growth that were realized each period.

Solving equations (3.3a) and (3.3b) for the equilibrium values gives:

\[
\ln \left( EMP^*_t \right) = \frac{1}{\eta_{em}} \left( \ln \left( EMP^*_{t-1} \right) - \ln \left( EMP_{t-1} \right) + \eta_{em} \ln \left( EMP_{t-1} \right) \right)
\]

\[
= \frac{1}{\eta_{em}} EMPR_{it} + \ln \left( EMP_{t-1} \right) \tag{3.4a}
\]

\[
\ln \left( MHY^*_t \right) = \frac{1}{\eta_{mh}} \left( \ln \left( MHY^*_{t-1} \right) - \ln \left( MHY_{t-1} \right) + \eta_{mh} \ln \left( MHY_{t-1} \right) \right)
\]

\[
= \frac{1}{\eta_{mh}} MHYR_{it} + \ln \left( MHY_{t-1} \right) \tag{3.4b}
\]

where EMPR and MHYR denote the employment growth rate and median household income growth rate, respectively.

Substituting from equations (3.4a) and (3.4b) into equations (3.2a) and (3.2b) gives:

**Business (Employment) Growth Equation:**

\[
\frac{1}{\eta_{em}} EMPR_{it} + \ln \left( EMP_{t-1} \right) = d_i \left( \frac{1}{\eta_{mh}} MHYR_{it} + \ln \left( MHY_{t-1} \right) \right) + \sum_{k_i=3}^{K_i} \gamma_i \ln \left( X_{k_i}^{em} \right) - \ln \left( EMP_{t-1} \right)
\]

\[
EMPR_{it} = \eta_{em} \left\{ d_i \left( \frac{1}{\eta_{mh}} MHYR_{it} + \ln \left( MHY_{t-1} \right) \right) + \sum_{k_i=3}^{K_i} \gamma_i x_{i,k_i} \ln \left( X_{k_i}^{em} \right) - \ln \left( EMP_{t-1} \right) \right\}
\]

\[
EMPR_{it} = \beta_{11} MHYR_{it} + \gamma_{11} \ln \left( EMP_{t-1} \right) + \gamma_{12} \ln \left( MHY_{t-1} \right) + \sum_{k_i=3}^{K_i} \gamma_{1,k_i} \ln \left( X_{k_i}^{em} \right) \tag{3.5a}
\]

**Median Household Income Growth Equation:**
\[
\frac{1}{\eta_{mh}} \text{MHYR}_{it} + \ln \left( \text{MHY}_{it-1} \right) = c_2 \left( \frac{1}{\eta_{em}} \text{EMPR}_{it} + \ln \left( \text{EMP}_{it-1} \right) \right) + \sum_{k_2=3}^{K_2} x_{2k_2} \ln \left( \text{X}^{mh}_{k2it} \right) \\
\text{MHYR}_{it} = \eta_{mh} \left\{ c_2 \left( \frac{1}{\eta_{em}} \text{EMPR}_{it} + \ln \left( \text{EMP}_{it-1} \right) \right) + \sum_{k_2=3}^{K_2} x_{2k_2} \ln \left( \text{X}^{mh}_{k2it} \right) - \ln \left( \text{MHY}_{it-1} \right) \right\} \\
\text{MHYR}_{it} = \beta_{21} \text{EMPR}_{it} + \gamma_{21} \ln \left( \text{EMP}_{it-1} \right) + \gamma_{22} \ln \left( \text{MHY}_{it-1} \right) + \sum_{k_2=3}^{K_2} \gamma_{2k_2} \ln \left( \text{X}^{mh}_{k2it} \right) 
\]

Equations (3.5a) and (3.5b) are the structural equations which constitute the basic simultaneous-equations model in this study. Thus, the general form of the model to be estimated and extended (to accommodate spatial effect) in subsequent sections can be given by:

\[
\begin{bmatrix}
\text{EMPR}_{it} = \beta_{11} \text{MHYR}_{it} + \gamma_{11} \ln \left( \text{EMP}_{it-1} \right) + \gamma_{12} \ln \left( \text{MHY}_{it-1} \right) + \sum_{k_1=3}^{K_1} \gamma_{1k_1} \ln \left( \text{X}^{em}_{k1it} \right) \\
\text{MHYR}_{it} = \beta_{21} \text{EMPR}_{it} + \gamma_{21} \ln \left( \text{EMP}_{it-1} \right) + \gamma_{22} \ln \left( \text{MHY}_{it-1} \right) + \sum_{k_2=3}^{K_2} \gamma_{2k_2} \ln \left( \text{X}^{mh}_{k2it} \right)
\end{bmatrix} \tag{3.6}
\]

Note that the speed of adjustment parameters \{\eta\} become embedded in the coefficient parameters, \beta and \gamma.

Equations in (3.6) are estimated using data collected from cross sectional observations on aggregate spatial units such as counties. Such data sets, however, are likely to exhibit a lack of independence in the form of spatial autocorrelation. To capture such spatial autocorrelation effects (using a contiguity weight matrix \(W\)), (3.6) is extended as follows:

\[
\begin{bmatrix}
\text{EMPR}_{it} = \beta_{11} \text{MHYR}_{it} + \lambda_{11} \text{W} \left( \text{EMPR}_{it} \right) + \lambda_{12} \text{W} \left( \text{MHYR}_{it} \right) + \gamma_{11} \ln \left( \text{EMP}_{it-1} \right) + \gamma_{12} \ln \left( \text{MHY}_{it-1} \right) + \sum_{k_1=3}^{K_1} \gamma_{1k_1} \ln \left( \text{X}^{em}_{k1it} \right) + u_{it}^{em}, \text{ where } u_{it}^{em} = \rho_{1} \text{W} u_{it}^{em} + \epsilon_{it}^{em} \\
\text{MHYR}_{it} = \beta_{21} \text{EMPR}_{it} + \lambda_{21} \text{W} \left( \text{EMPR}_{it} \right) + \lambda_{22} \text{W} \left( \text{MHYR}_{it} \right) + \gamma_{21} \ln \left( \text{EMP}_{it-1} \right) + \gamma_{22} \ln \left( \text{MHY}_{it-1} \right) + \sum_{k_2=3}^{K_2} \gamma_{2k_2} \ln \left( \text{X}^{mh}_{k2it} \right) + u_{it}^{mh}, \text{ where } u_{it}^{mh} = \rho_{2} \text{W} u_{it}^{mh} + \epsilon_{it}^{mh}
\end{bmatrix} \tag{3.7}
\]

where \(\beta, \gamma, \lambda, \text{ and } \rho\) are unobserved parameters \(u_{it}^{em}\) and \(u_{it}^{mh}\) are vectors of disturbances, and
\( \varepsilon^m_k \) and \( \varepsilon^m_{kh} \) are vectors of innovations. \( K_j, \ j = 1, 2 \) represents the number of exogenous variables included in the jth equation. The system in (3.7) is a spatial autoregressive model in which both the spatial lags in the dependent variables and spatial autoregressive error terms are incorporated. It can also be written more compactly as:

\[
\begin{align*}
\mathbf{y}_j &= \mathbf{Z}_j \delta_j + \mathbf{u}_j, \\
\mathbf{u}_j &= \rho_j \mathbf{Wu}_j + \varepsilon_j, \quad j = 1, 2
\end{align*}
\]  

(3.8)

where

\[
\mathbf{Z}_j = (\mathbf{Y}_j, \mathbf{X}_j, \mathbf{WY}_j) \quad \text{and} \quad \delta_j = (\mathbf{b}_j', \mathbf{c}_j', \mathbf{d}_j')'
\]

with \( \mathbf{Y}_j, \mathbf{X}_j \) and \( \mathbf{WY}_j \) representing the matrices of observations for the endogenous variables, exogenous variables and the spatially lagged endogenous variables that appear in the jth equation, respectively.

**Data Type and Sources**

The data for the empirical analysis are for the 417 Appalachian counties, which are collected and compiled from County Business Patterns, Bureau of Economic Analysis, Bureau of Labor Statistics, Current Population Survey Reports, County and City Data Book, U.S. Census of Population and Housing, U.S. Small Business Administration, and Department of Employment Security. Data for county employment and county median household income are collected for 1990 and 2000. The dependent variables of the model, employment growth rate (EMPR) and median household growth rate (MHYR), are computed by taking the log difference of the respective 2000 and the 1990 levels. In addition, data for the control variables are collected for 1990 from different sources (see table 1 for description of variables).

**Table 1 about here**

**Empirical Estimation and Result Analysis**
The model given in (3.8) is estimated using generalized spatial two stage least squares (GS2SLS) and generalized spatial three stage least squares (GS3SLS) procedures for data from Appalachian counties for 1990-2000. To determine whether a linear or log-linear specification is appropriate for this model, a PE test is undertaken following Kmenta (1986). The test indicates that the log-linear specification is preferred to the linear form for both equations. Thus, the model is specified in log-linear form with two modifications involving the measurement of the explanatory variables. First, the natural log formulation is dropped for the explanatory variables that can take negative or zero values. Second, lagged 1990 values are used for all of the explanatory variables to avoid simultaneity bias. Hausman’s (1978) specification test is used to test for the endogeneity of the several of the explanatory variables and found that the two equations are appropriately chosen. Tests for over-identifying restrictions also suggest a proper specification of the model. The presence of spatial autocorrelation in the disturbances is tested using Moran’s I test for models with endogenous regressors as suggested in Anselin and Kelejian (1997). Both equations of the model show spatial error dependence.

The GS2SLS and GS3SLS procedures are done in a three and a four step routines, respectively. The first three steps are common for both routines. In the first step, the parameter vector consisting of betas, lambdas and gammas \([\beta', \lambda', \gamma']\) are estimated by two stage least squares (2SLS) using an instrument matrix \(N\) that consists of a subset of \(X, WX, W^2X\), where \(X\) is the matrix that includes all control variables in the model, and \(W\) is a weight matrix. The disturbances for each equation in the model are computed by using the estimates for betas, lambdas and gammas from the first step. In the second step, these estimates of the disturbances are used to estimate the autoregressive parameter rho \((\rho)\) for each equation using Kelejian and Prucha’s (2004) generalized moments procedure. In the third step, a Cochran-Orcutt-type

---

2 The details of the steps are given in Appendix 1.
transformation is done by using the estimates for rhos from the second step to account for the spatial autocorrelation in the disturbances. The GS2SLS estimators for betas, lambdas and gammas are then obtained by estimating the transformed model using $[X, WX, W^2X]$ as the instrument matrix.

Although the GS2SLS takes the potential spatial correlation into account, it does not utilize the information available across equation because it does not take into account the potential cross equation correlation in the innovation vectors $\left(\epsilon_{it}^{cm}, \epsilon_{it}^{mb}\right)$. The full system information is utilized by stacking the Cochran-Orcutt-type transformed equations (from the second step) in order to estimate them jointly. Thus, in the fourth step, the GS3SLS estimator of betas, lambdas, and gammas is obtained by estimating this stacked model. The GS3SLS estimator is more efficient relative to GS2SLS estimator.

Table 2 about here

The GS2SLS and GS3SLS parameter estimates of the system given in (3.8) are reported in Table 2. The estimated equations explained 42 and 34 percent of the variations in employment growth (small business growth) and median household income growth respectively. These values are similar to results from many studies on cross-sectional analyses of this sort (Deller, et al., 2001; Henry et al., 1997, Boarnet, 1994). The parameter estimates are mostly consistent with the theoretical expectations. The coefficients on the endogenous variables (EMPR and MHYR) are positive indicating the interdependence of the small business growth and median household income growth. However, the attractive effect of median household income growth on small business growth is stronger than that of small business growth on median household income growth.

In the EMPR equation, ten of the coefficient estimates are significantly different from zero at the ten percent level or higher. The results suggest a positive and significant parameter
estimate for lambda11 that indicate that employment growth rate tends to spillover to neighboring counties and have a positive effect on their employment growth rates. The results also show a positive parameter estimate for lambda12 that indicate that median household income growth rates (MHYR) in neighboring counties tends to affect favorably EMPR in a given county. These are important from a policy perspective as they indicate that employment growth and growth in median household incomes in one county have positive spillover effects to EMPRs in neighboring counties. The results are also important from an economic perspective because these significant spatial lag effects indicate that EMPR does not only depend on characteristics within the county, but also on that of its neighbors. Hence, spatial effects should be tested for in empirical works involving employment growth rates and household income growth rates. The model specification in this study also incorporates spatially autoregressive spatial process (effect) besides the spatial lag in the dependent variables. The results in Table 2 suggest a negative parameter estimate for rho1 indicating that random shocks into the system with respect to EMPR do not only affect the county where the shocks originated and its neighbors, but create negative shock waves across Appalachia.

The elasticity of EMPR with respect to the initial employment level (EMP) is negative and statistically significant indicating convergence in the sense that counties with initial low level of employment at the beginning of the period (1990) tend to show higher rate of growth of business than counties with high initial level of employment conditional on the other explanatory variables in the model. This result supports prior results of rural renaissance in the literature (Deller et al., 2001; Lunderberg, 2003).

To control for agglomeration effects, the model includes measure of population statistics such as initial county population size (POPs). The results show that POPs have positive and significant effects on EMPR. This result is in line with the literature (Acs and Armington, 2004a) which indicates that a growing population increases the demand for consumer goods and
services, as well as the pool of potential entrepreneurs which encourage business formation. This result is important from a policy perspective. It indicates that counties with high population concentration are benefiting from the resulting agglomeration and spillover effects that lead to localization of economic activities, in line with Krugman’s (1991a, 1991b) argument on regional spillover effects. In contradiction to the theoretical expectations, the results show initial human capital endowment as measured by the percentage of adults (over 23 years old) with only high school education (POPHD) has the unexpected sign. The coefficient on POPCD (the percentage of adults (over 23 years old) with high school diploma or higher), however, is positive as expected. One interpretation of these results is that the jobs created in Appalachia during the study period might require high education levels. These results support the findings in the literature. In the 1990s, there were increases in the incidence of highly educated people starting new businesses, especially in sectors such as computers, biotechnology and internet dependent businesses (Christensen, 2000).

We have also included the county unemployment rate (UNE) in our vector of exogenous variables as a measure of local economic distress. Our results suggest that a high unemployment rate is associated with low business growth. This indicates that the poor economic environment in Appalachia did not provide incentive for individuals to form new business that can employ not only the owner, but others. Unemployed individuals may not have the capital to start a business. A high level of unemployment is also an indication of a reduction in aggregate demand in the region which puts downward pressure on new firm formation. This result is also in line with the study by Acs and Armington (2004b) which found that unemployment is associated negatively with new firm formation during growth periods and positively during recession periods.

Establishment density (ESBd), which is the total number of private sector establishments in the county divided by the total county’s population, is included in our model to capture the degree of competition among firms and crowding of businesses relative to the population. The
average size of establishment (ESBs), defined as the total private sector employment divided by the total number of private establishments in the county, is also included to capture the effects of barriers to entry of new small firms on employment growth. The coefficient on ESBD is positive and significant indicating that Appalachia region is far below the threshold where competition among firms for consumer demands crowds businesses. According to our results, high ESBd is associated with growth in Employment (business growth), indicating that firms tend to locate near each other possibly due to localization and agglomeration economies of scale. The coefficient on ESBs is also positive and significant indicating existence of low barrier to new firm formation and employment generation in Appalachia during the study period.

The coefficient on the variable representing the percentage of home owned by their occupants (OWHU) is negative and significant at ten percent level. This result indicates that high home ownership is negatively associated with business formation in Appalachia. This is contrary to theoretical expectation that high home ownership is an indication that there is a capacity to finance new business by potential entrepreneurs, either by using the house as collateral for loan or as indication of availability of personal financial resources to start new business. The result, however, shows the reality in Appalachia. During the study period, in Appalachia, home ownership was positively correlated with level of economic distress - home ownership was higher in distressed counties (76 percent) and lowest in attainment counties (69%); higher in central Appalachia (76 percent) than in northern or southern sub regions (more developed); and Appalachia non-metro areas had higher ownership rates (76 percent) than its metro areas (72 percent) (Pollard, 2003). Thus, the research result indicates that home ownership is not a good indicator of the availability of resources to start new business in Appalachia.

The coefficients for MANU and WHRT are positive and significant at the 5 percent and 1 percent levels respectively. These results indicate that counties with initial higher percentage of
their labor force employed in manufacturing and in wholesale and retail trade showed higher growth rate in business than other counties.

An interesting observation from the results pertains to the role of local government on business growth. The model predicts that local governments, through their spending and taxation functions, have critical roles in creating and enabling economic environments for businesses to prosper. The empirical results, however, indicate that local governments have not played significant roles in employment growth in Appalachia. Given the economic hardship and high level of underdevelopment in Appalachia, these results are indications that local governments may need to step up their efforts and create incentives in order to encourage business growth in the region.

The results for the MHYR equation suggest a negative parameter estimate for lambda22, an indication that MHYR tends to spillover to neighboring counties with negative effect on employment growth rates, although the coefficient is insignificant. The results also show a negative parameter estimate for lambda21, an indication that EMPR in neighboring counties tends to affect unfavorably, MHYR in a given county. These are important from a policy perspective as they indicate that employment growth and median household income growth in one county are unfavorable to MHYRs in neighboring counties.

The coefficient for the MHYR equation also indicates a positive parameter estimate for rho2 indicating that random shocks into the system, with respect to MHYR, do not only affect the county where the shocks originated and its neighbors, but create positive spillover effect across Appalachia. The elasticity of EMPR with respect to the initial median household income (MHY) is negative and statistically significant indicating convergence in the sense that counties with initial low level of median household income at the beginning of the period (1990) tend to show higher rate of growth of median household income than counties with high initial level of median household income conditional on the other explanatory variables in the model.
The coefficient for the index of social capital (SCIX) is positive and significant indicating that counties with high level of social capital increase the wellbeing of their communities. The coefficients on the proportion of population of school age (POP5-17), the proportion of population above 65 years old (POP>65), and the proportion of female headed households (FHHF) indicate the expected signs, negative, positive and negative, respectively. Counties with high proportions for POP5-17 and FHHF tend to have low level of median household income. Whereas, counties with high proportion of POP>65 tend to have high levels of MHY. These results are consistent with the results in the literature.

**Conclusions**

The main objective in this paper was to test the hypotheses that (1) business growth and median household income growth are interdependent and are jointly determined by regional covariates; (2) business and median household income growth in a county are conditional upon initial conditions of the county; and (3) business and median household income growth in a county are conditional upon business and median household income growth in neighboring counties. To test these hypotheses, a spatial simultaneous equations model was developed. GS2SLS and GS3SLS estimates of the parameters are obtained by estimating the model using data covering the 417 Appalachian counties for the 1990-2000 period. The empirical results of the study verify the three hypotheses. In particular, it was verified that EMPR in one county is positively affected by EMPR and MHYR in neighboring counties, whereas, MHYR in one county is negatively affected by EMPR and MHYR in neighboring counties. The policy implication of the finding is that neighboring counties may need to pool their resources in creating and enabling environments (business climate) to make their counties attractive to firms. The results also indicate the presence of spatial correlation in the error terms. This implies that a random shock into the system spreads across the region. The results also indicate convergence across counties in Appalachia with respect to EMPR and MHYR conditional upon the initial
conditions of the explanatory variables in the model. This information might be important to Appalachia Regional Commission as it indicates that the divergence in the level of economic status of the Appalachian counties is narrowing.

The results also indicate the presence of significant agglomerative effects. Counties with higher population concentration showed significant business growth. This information may encourage policy makers at the county level to design policies that can attract people to their respective counties.

The results of the study also produced important information for individuals who want to start or expand business in Appalachia. Establishment density which captures the degree of competition among firms and crowding of businesses relative to the population indicates that Appalachia region is far below the threshold where competition among firms for consumer demands crowds businesses. Besides, the results indicate the existence of low barrier to new firm formation and employment generation in Appalachia during the study period. Hence, the availability of this information to potential entrepreneurs, policy makers, development agents as well as to local authorities may be important to help them make informed decisions with respect to their efforts in creating enabling environments for business and income growth.
References


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Whittington, R.C., 1984, “Regional Bias in New Firm Formation in the UK,” *Regional Studies*,
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Table 1: Descriptive statistics

<table>
<thead>
<tr>
<th>Variable Code</th>
<th>Variable Description</th>
<th>Mean</th>
<th>Std Dev</th>
<th>Minimum</th>
<th>Maximum</th>
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<tbody>
<tr>
<td>Constant</td>
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<td>1.00</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
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<td>EMPR</td>
<td>Employment growth rate 1990-2000</td>
<td>0.17</td>
<td>0.25</td>
<td>-0.69</td>
<td>1.79</td>
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<td>MHYR</td>
<td>Median Household income growth rate 1990-2000</td>
<td>0.48</td>
<td>0.31</td>
<td>-0.49</td>
<td>1.40</td>
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<tr>
<td>WEMPR</td>
<td>Spatial Lag of EMPR</td>
<td>0.18</td>
<td>0.14</td>
<td>-0.18</td>
<td>0.81</td>
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<td>WMHYR</td>
<td>Spatial lag of MHYR</td>
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<td>0.19</td>
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<td>POPs</td>
<td>Population 1990</td>
<td>10.30</td>
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<td>POPd</td>
<td>Population density 1990</td>
<td>4.28</td>
<td>0.90</td>
<td>1.85</td>
<td>7.75</td>
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<td>POP5-17</td>
<td>Percent of population between 5-17 years 1990</td>
<td>2.92</td>
<td>0.12</td>
<td>2.17</td>
<td>3.22</td>
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<tr>
<td>POP25-44</td>
<td>Percent of population between 25-44 years old 1990</td>
<td>3.38</td>
<td>0.08</td>
<td>2.79</td>
<td>3.74</td>
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<tr>
<td>POP&gt;65</td>
<td>Percent of population above 65 years old 1990</td>
<td>2.60</td>
<td>0.20</td>
<td>1.55</td>
<td>3.20</td>
</tr>
<tr>
<td>FHHF</td>
<td>Percent of female householder, family householder, 1990</td>
<td>2.32</td>
<td>0.20</td>
<td>1.81</td>
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</tr>
<tr>
<td>POPHD</td>
<td>Persons 25 years and over, % high school or higher, 1990</td>
<td>4.10</td>
<td>0.17</td>
<td>3.57</td>
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<tr>
<td>POPCDD</td>
<td>Persons 25 years and over, % Bachelor's degree or above, 1990</td>
<td>2.27</td>
<td>0.41</td>
<td>1.31</td>
<td>3.73</td>
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<td>OWHU</td>
<td>Owner-Occupied Housing Unit in percent, 1990</td>
<td>4.33</td>
<td>0.08</td>
<td>3.87</td>
<td>4.47</td>
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<td>MHU</td>
<td>Median Value of owner occupied housing 1990</td>
<td>10.74</td>
<td>0.26</td>
<td>9.67</td>
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<td>UNEMP</td>
<td>Unemployment rate 1990</td>
<td>2.15</td>
<td>0.35</td>
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<tr>
<td>AGFF</td>
<td>% employed in Agr., forestry and fisheries 1990</td>
<td>3.62</td>
<td>2.66</td>
<td>0.00</td>
<td>17.10</td>
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<tr>
<td>MANU</td>
<td>% employed in manufacturing 1990</td>
<td>3.14</td>
<td>0.57</td>
<td>0.79</td>
<td>3.98</td>
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<tr>
<td>WHRT</td>
<td>% employed in wholesale and retail trade 1990</td>
<td>2.92</td>
<td>0.19</td>
<td>2.16</td>
<td>3.32</td>
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<td>FIRE</td>
<td>% employed Finance, Insurance and Real Estate 1990</td>
<td>1.23</td>
<td>0.33</td>
<td>0.00</td>
<td>2.23</td>
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<td>HLTH</td>
<td>% employed Health service 1990</td>
<td>1.95</td>
<td>0.34</td>
<td>0.74</td>
<td>3.44</td>
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<td>NAIX</td>
<td>Natural Amenities Index 1990</td>
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<td>1.16</td>
<td>-3.72</td>
<td>3.55</td>
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<td>ESBD</td>
<td>Establishment density 1990</td>
<td>2.93</td>
<td>0.34</td>
<td>1.87</td>
<td>4.09</td>
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<td>EFIR</td>
<td>Earnings in Finance Insurance and real Estate 1990</td>
<td>21075.08</td>
<td>96011.09</td>
<td>0.00</td>
<td>1638807.0</td>
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<td>CSBD</td>
<td>Commercial and Saving Banks deposits 1990</td>
<td>12.21</td>
<td>1.07</td>
<td>8.83</td>
<td>16.95</td>
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<td>DFEG</td>
<td>Direct federal expenditure and grants per capita 1990</td>
<td>7.99</td>
<td>0.38</td>
<td>6.98</td>
<td>10.18</td>
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<td>FGCE</td>
<td>Federal govern't civilian employment per 10,000 pop. 1990</td>
<td>60.48</td>
<td>101.03</td>
<td>0.00</td>
<td>1295.00</td>
</tr>
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<td>PCTAX</td>
<td>Per capital local tax 1990</td>
<td>5.91</td>
<td>0.53</td>
<td>4.51</td>
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<td>ESBs</td>
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<td>AWSR</td>
<td>Average annual wage and salary rate 1990</td>
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<td>0.19</td>
<td>9.31</td>
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<td>EMP</td>
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<td>8.83</td>
<td>1.25</td>
<td>5.42</td>
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<tr>
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<td>7.09</td>
<td>1.00</td>
<td>4.54</td>
<td>10.52</td>
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<td>OTMG</td>
<td>out-migration 1990</td>
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<td>0.97</td>
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<td>MHY</td>
<td>Median Household income 1989</td>
<td>9.94</td>
<td>0.23</td>
<td>9.06</td>
<td>10.68</td>
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<td>DGEX</td>
<td>Direct general exp. Per capita 1992</td>
<td>7.23</td>
<td>0.28</td>
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Note: All the variables are expressed in log terms except AGFF, EFIR, FGCE, SCIX, and NAIX
Table 2: Generalized Spatial 2SLS (GS2SLS) and Full Information Generalized Spatial 3SLS (GS3SLS) Estimation Results

<table>
<thead>
<tr>
<th>Variables</th>
<th>GS2SLS EMPR Equation</th>
<th>Coefficient</th>
<th>t-statistic</th>
<th>GS2SLS MHYR Equation</th>
<th>Coefficient</th>
<th>t-statistic</th>
<th>GS3SLS EMPR Equation</th>
<th>Coefficient</th>
<th>GS3SLS MHYR Equation</th>
<th>Coefficient</th>
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$\sigma^2$  

| $\rho$ | 0.0319 | 0.0527 |

N 417  417

F-Statistic 10.88  10.46

Adj. R2 0.42  0.34

Note: *, **, and *** denote statistical significance level at 10 percent, 5 percent and 1 percent respectively.
Appendix 1: Steps in GS2SLS and GS3SLS estimation

This appendix defines limited information and full information instrumental variables estimators for the parameters of the system given in (3.8). In the case of limited information (single equation) estimation, a three step generalized spatial two-stage least squares (GS2SLS) procedure is used to estimate the unknown parameters in the jth equation of the model in equation (3.8). The first step consists of the estimation of the model parameter vector $\delta_j$ in equation (3.8) by two-stage least squares (2SLS) using the instruments $N$, where $N$ is chosen as a subset of the linearly independent columns of $X, WX, W^2 X$, with $X$ representing the matrix that includes all control variables in the model, and $W$ a weight matrix. The resulting 2SLS estimator is given by

$$\hat{\delta}_j = (\bar{Z}_{j}'\bar{Z}_{j})^{-1}\bar{Z}_{j}'y_j \quad (3.9)$$

where $\bar{Z}_{j} = P_N Z_j = (\bar{Y}_{j}, \bar{X}_{j}, \bar{WY})$ with $\bar{Y}_{j} = P_N Y_j, \bar{WY} = P_N WY_j$, and $P_N = N(N'N)^{-1}N'$ is a projection matrix. Although $\hat{\delta}_j$ is consistent estimator of $\delta_j$, it does not utilize information relating to the spatial correlation of the disturbance terms. These 2SLS estimates are used to compute estimates for the disturbances $u_j$ which in turn are used to estimate the autoregressive parameter $\rho_j$ in the second step of the procedure. The resulting 2SLS residuals are hence given by

$$\bar{u}_j = y_j - Z_j \hat{\delta}_j \quad (3.10)$$

In the second step, the generalized moments procedure is used to estimate the spatial autoregressive parameter of the disturbances of the jth equation, for $j = 1, 2$, of the model in equation (3.8). Note that from the relation in equation (3.8) we have

$$u_j - \rho_j W u_j = \varepsilon_j \quad (3.11)$$

and pre-multiplication by the weights matrix $W$ gives
The following three-equation system is obtained from the relationships between equations (3.11) and (3.12):

\[
\frac{\varepsilon_j'}{n} = \frac{u_j'u_j}{n} + \rho_j^2 \left( \left(Wu_j\right)' \left(Wu_j\right) \right) - 2\rho_j \left( Wu_j \right)
\]

\[
\frac{\left(Wu_j\right)' \left(Wu_j\right)}{n} = \frac{\left(Wu_j\right)' \left(Wu_j\right)}{n} + \rho_j^2 \left( \left(W^2u_j\right)' \left(W^2u_j\right) \right) - 2\rho_j \left( W^2u_j \right) \left(Wu_j\right)
\]

\[
\frac{\varepsilon_j' \cdot \left(Wu_j\right)}{n} = \frac{u_j' \left(Wu_j\right)}{n} + \rho_j^2 \left( Wu_j \right)' \left(W^2u_j\right) - \rho_j \left( u_j' \left(W^2u_j\right) + \left(Wu_j\right)' \left(Wu_j\right) \right)
\]

Taking expectations across equation (3.13)

\[
E \left[ \frac{\varepsilon_j'}{n} \right] = \frac{u_j'u_j}{n} + \rho_j^2 \left( \left(Wu_j\right)' \left(Wu_j\right) \right) - 2\rho_j \left( Wu_j \right)
\]

\[
E \left[ \frac{\left(Wu_j\right)' \left(Wu_j\right)}{n} \right] = \frac{\left(Wu_j\right)' \left(Wu_j\right)}{n} + \rho_j^2 \left( \left(W^2u_j\right)' \left(W^2u_j\right) \right) - 2\rho_j \left( W^2u_j \right) \left(Wu_j\right)
\]

\[
E \left[ \frac{\varepsilon_j' \cdot \left(Wu_j\right)}{n} \right] = \frac{u_j' \left(Wu_j\right)}{n} + \rho_j^2 \left( Wu_j \right)' \left(W^2u_j\right) - \rho_j \left( u_j' \left(W^2u_j\right) + \left(Wu_j\right)' \left(Wu_j\right) \right)
\]

yields

\[
\begin{bmatrix}
\sigma_j^2 \\
\sigma_j^{tr} (W'W) \\
\sigma_j^{tr} (W) = 0
\end{bmatrix}
= E \begin{bmatrix}
\frac{u_j'u_j}{n} + \rho_j^2 \left( \left(Wu_j\right)' \left(Wu_j\right) \right) - 2\rho_j \left( Wu_j \right) \\
\frac{\left(Wu_j\right)' \left(Wu_j\right)}{n} + \rho_j^2 \left( \left(W^2u_j\right)' \left(W^2u_j\right) \right) - 2\rho_j \left( W^2u_j \right) \left(Wu_j\right) \\
\frac{u_j' \left(Wu_j\right)}{n} + \rho_j^2 \left( Wu_j \right)' \left(W^2u_j\right) - \rho_j \left( u_j' \left(W^2u_j\right) + \left(Wu_j\right)' \left(Wu_j\right) \right)
\end{bmatrix}
\]

(3.15)
and after rearranging

\[
\begin{bmatrix}
\frac{u_j'u_j}{n} \\
\frac{(W_j)'(W_j)}{n} \\
\frac{u_j'(W_j)}{n}
\end{bmatrix}
= 
\begin{bmatrix}
\frac{E(u_j'(W_j))}{n} \\
\frac{E((W^2u_j)'(W_j))}{n} \\
\frac{E(u_j'(W^2u_j)+(W_j)'(W_j))}{n}
\end{bmatrix}
- \frac{2\rho_j}{n}
\begin{bmatrix}
E(W_j)'
E(W^2u_j)'(W_j)
\rho_j
\end{bmatrix}
= 
\begin{bmatrix}
\frac{E(W_j)'}{n} \\
\frac{E(W^2u_j)'(W_j)}{n} \\
\frac{E(W_j)'(W^2u_j)}{n}
\end{bmatrix}
- \frac{\rho_j^2}{n}
\begin{bmatrix}
\sigma_j \\
\sigma_j \\
\sigma_j \\
\end{bmatrix}
\begin{bmatrix}
\frac{tr(WW)}{n}
0
\end{bmatrix}
\]

(3.16)

\[
\begin{bmatrix}
\frac{u_j'u_j}{n} \\
\frac{(W_j)'(W_j)}{n} \\
\frac{u_j'(W_j)}{n}
\end{bmatrix}
= 
\begin{bmatrix}
\frac{E(u_j'(W_j))}{n} \\
\frac{E((W^2u_j)'(W_j))}{n} \\
\frac{E(u_j'(W^2u_j)+(W_j)'(W_j))}{n}
\end{bmatrix}
- \frac{2\rho_j}{n}
\begin{bmatrix}
E(W_j)'
E(W^2u_j)'(W_j)
\rho_j
\end{bmatrix}
= 
\begin{bmatrix}
\frac{E(W_j)'}{n} \\
\frac{E(W^2u_j)'(W_j)}{n} \\
\frac{E(W_j)'(W^2u_j)}{n}
\end{bmatrix}
- \frac{\rho_j^2}{n}
\begin{bmatrix}
\sigma_j \\
\sigma_j \\
\sigma_j \\
\end{bmatrix}
\begin{bmatrix}
\frac{tr(WW)}{n}
0
\end{bmatrix}
\]

(3.17)

Thus the system in equation (3.17) can be rewritten as (j = 1,2)

\[
\tau_j = Y_j a_j = a_j = Y_j^{-1} \tau_j
\]

(3.18)

The parameter vector \( a_j = (\rho_j, \rho_j^2, \sigma_j^2)' \) would be completely determined in terms of the relation in equation (3.18) if \( \tau_j \) and \( Y_j \) were known. Note that \( \tau_j \) and \( Y_j \) are not observable. Following the
suggestions in Kelejian and Prucha (2004), however, the following estimators for \( \tau_j \) and \( \Upsilon_j \) in terms of sample moments can be defined as:

\[
o_j = \frac{1}{n} \left[ \begin{array}{c}
\bar{u}_j \bar{u}_j \prime (W \bar{u}_j) \prime (W \bar{u}_j) , \bar{u}_j \prime (W \bar{u}_j) \\
2 (W^2 \bar{u}_j) \prime (W \bar{u}_j) , - (W^2 \bar{u}_j) \prime (W \bar{u}_j) , n \\
\end{array} \right] (3.19)
\]

\[
O_j = \frac{1}{n} \left[ \begin{array}{c}
2 \bar{u}_j \prime (W \bar{u}_j) , - (W \bar{u}_j) \prime (W \bar{u}_j) \\
2 (W^2 \bar{u}_j) \prime (W \bar{u}_j) , - (W^2 \bar{u}_j) \prime (W \bar{u}_j) , \text{tr} (W'W) \\
\left[ \bar{u}_j \prime (W^2 \bar{u}_j) + (W \bar{u}_j) \prime (W \bar{u}_j) \right] , - (W \bar{u}_j) \prime (W^2 \bar{u}_j) , 0 \\
\end{array} \right]
\]

Thus, given the estimates in equation (3.19), the empirical form of the relationship in equation (3.18) can be given by

\[
o_j = O_j \alpha_j + \xi_j \\
\]

(3.20)

Since \( o_j \) and \( O_j \) are observable and \( \alpha_j \) is vector of parameters to be estimated, \( \xi_j \) can be viewed as a vector of regression residuals. Thus, the second step estimators of \( \rho_j \) and \( \sigma_j^2 \), say, \( \hat{\rho}_j \) and \( \hat{\sigma}_j^2 \), are nonlinear least squares estimators defined as the minimizers of

\[
\begin{bmatrix}
o_j - O_j \\
\rho_j^2 \\
\sigma_j^2
\end{bmatrix} \prime 
\begin{bmatrix}
o_j - O_j \\
\rho_j^2 \\
\sigma_j^2
\end{bmatrix} \]

(3.21)

or

\[
(\hat{\rho}_j, \hat{\sigma}_j^2) = \text{argmin} \left[ o_j - O_j \\
\rho_j^2 \\
\sigma_j^2 \right] \prime 
\left[ o_j - O_j \\
\rho_j^2 \\
\sigma_j^2 \right]
\]

In the third step of the procedure, a Cochrane-Orcutt type transformation is applied to the model in equation (3.8). More specifically, let
\[ y_j^*(\rho_j) = y_j - \rho_j W y_j \] and \[ Z_j^*(\rho_j) = Z_j - \rho_j W Z_j \]

Then, equation (3.8) becomes

\[ y_j^*(\rho_j) = Z_j^*(\rho_j) \delta_j + \epsilon_j \quad (3.22) \]

If \( \rho_j \) were known we could perform 2SLS on equation (3.22) to obtain the generalized spatial two-stage squares (GS2SLS) estimator for \( \delta_j \). That is

\[ \hat{\delta}_j = (Z_j^*(\rho_j)'Z_j^*(\rho_j))^{-1} Z_j^*(\rho_j)'y_j^*(\rho_j) \quad (3.23) \]

where \( Z_j^*(\rho_j) = P_N Z_j^*(\rho_j) \) and \( P_N = N (N'N)^{-1} N' \). But, since in practical applications \( \rho_j \) is not known, we replace it with its estimate as defined in equation (3.21) and estimate the model in equation (3.22) using 2SLS. The resulting estimator is termed as the feasible GS2SLS and is given by

\[ \hat{\delta}^F_j = (Z_j^*(\hat{\rho}_j)'Z_j^*(\hat{\rho}_j))^{-1} Z_j^*(\hat{\rho}_j)'y_j^*(\hat{\rho}_j) \quad (3.24) \]

where \( Z_j^*(\hat{\rho}_j) = P_N [Z_j - \hat{\rho}_j W Z_j] \) and \( y_j^*(\hat{\rho}_j) = y_j - \hat{\rho}_j W y_j \).

One of the limitations of the limited information (single equation) estimation technique is that it does not take into account the information provided by the potential cross equation correlation in the innovation vectors \( \epsilon_j \). In order to use the information from such cross equation correlations, it is important to stack the equations given in equation (3.22) as follows:

\[ y^*(\rho) = Z^*(\rho) \delta + \epsilon \quad (3.25) \]

where

\[ y^*(\rho) = (y_1^*(\rho_1)', y_2^*(\rho_2)')', Z^*(\rho) = \text{diag}_{j=1}^2 (Z_j^*(\rho_j)), \rho = (\rho_1, \rho_2)' \text{ and } \delta = (\delta_1, \delta_2)' \]

Note that \( E(\epsilon) = 0 \) and \( E(\epsilon \epsilon') = \Sigma \otimes I_n \). Assuming that \( \rho \) and \( \Sigma \) were known, equation (3.25) could
be estimated using the instrumental variable technique. In that case, the resulting systems
instrumental variable estimator of $\delta$ would be the generalized spatial three-stage least squares
(GS3SLS) estimator which can be given by (all notations as defined before)

$$
\hat{\delta} = \left( \bar{Z}^* (\hat{\rho}^* (\Sigma^{-1} \otimes I_n) \bar{Z}^* (\hat{\rho}))^{-1} \bar{Z}^* (\hat{\rho}) \right) y^* (\hat{\rho})
$$

(3.26)

Since in practical applications $\rho$ and $\Sigma$ are not known, their estimators are required to obtain the feasible estimator for $\delta$. The generalized moments estimators for $\rho_j$ and $\sigma^2_j$ are defined in equation (3.21). Note that $\sigma^2_j$ is the jth diagonal element of $\Sigma$. Besides, a consistent estimator for $\Sigma$ can be derived by combining equations (3.22) and (3.24) as

$$
\hat{\sigma}^2_{jl} = \frac{1}{n} \hat{\epsilon}_j^l \hat{\epsilon}_i^l, j, l = 1, 2
$$

(3.27)

where $\hat{\epsilon}_j = y_j (\hat{\rho}, \Sigma) - Z_j (\hat{\rho}, \Sigma) \hat{\delta}^F_j$. Then, the 2 by 2 matrix whose $(j,l)$th element is $\hat{\sigma}^2_{jl}$ defines a consistent estimator for $\Sigma$ denoted by $\hat{\Sigma}$. Substituting $\Sigma$ with $\hat{\Sigma}$ in equation (3.26) gives the feasible generalized spatial three-stage least squares (FGS3SLS) estimator for $\delta$. That is

$$
\hat{\delta}^F = \left( \bar{Z}^* (\hat{\rho}^* (\hat{\Sigma}^{-1} \otimes I_n) \bar{Z}^* (\hat{\rho}))^{-1} \bar{Z}^* (\hat{\rho}) \right) y^* (\hat{\rho})
$$

(3.28)