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Analysis of Production Data from Horizontal Shale Wells Using Decline Curves

Atoosa Mashayekhi
West Virginia University

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Analysis of Production Data from Horizontal Shale Wells Using Decline Curves

Atoosa Mashayekhi

Thesis submitted to the
Benjamin M. Statler College of Engineering and Mineral Resources
at West Virginia University

In Partial fulfillment of the requirements for the degree of

Master of Science
In
Petroleum and Natural Gas Engineering

Khashayar Aminian, Ph.D., Chair
Samuel Ameri, M.S.
Alan Brannon, Ph.D.

Department of Petroleum and Natural Gas Engineering

Morgantown, West Virginia
2014

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ABSTRACT

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Atoosa Mashayekhi

Recent interest in the exploitation of Marcellus shale play, using horizontal drilling and multistage hydraulic fracturing, has increased the demand for reliable estimation of recoverable reserves from ultra-low permeability shale gas formations. Due to the limited field experience, the production performances of Marcellus shale gas reservoirs as well as the key parameters that affect the long-term production of the horizontal wells have not been well-established.

Among all the prediction methods, only the Decline Curve Analysis (DCA) technique has proved successful in forecasting production data rapidly and to a high degree of accuracy. Several DCA models including conventional Arps, PLE, and Duong have been utilized in this study to determine the most appropriate method for production data from horizontal Marcellus shale wells. Fekete (Fast Evolution) simulator has been used to generate the thirty year production data from 3000 feet of horizontal lateral. The two base scenarios include seven and thirteen hydraulic fracture stages. The gas adsorbed to shale is also considered. The applicability of several DCA models to shale gas production history was examined using the simulated production profiles. The impact of the permeability, fracture half length, and matrix porosity on DCA models constants were also investigated. Finally, the proposed ratio methodology was applied to the limited production profile (3.5 years of production history) from a well in Upshur County, WV to estimate the DCA constants, and predict the long-term production performance. The prediction results from DCA models then compared to the history-matched simulation model predictions for confirmation.

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Nomenclature

d_i = Loss Ratio, 1/ time

q_i = Initial flow rate, volume/time

n = Derivative of Loss Ratio, dimensionless

G_p = Cumulative gas production, volume

D_∞ = Loss ratio at ($t=\infty$), 1/time

D_1 = Loss ratio at ($t=1$), 1/time

q = Flow rate, volume/time

a = Vertical axis intercept of q/G_p vs. time plot, dimensionless

m = Slope of log-log plot of q/G_p vs. time plot, dimensionless

$t(a,m)$ = Duong time function, dimensionless

q_1 = Slope of q vs. $t(a,m)$ plot, volume

q_∞ = Vertical axis intercept of q vs. $t(a,m)$ plot, dimensionless

t = time, years

K = permeability, m-Darcy

X_f = Fracture Half-length, feet

Φ = Porosity, Fraction

S_{SE} = Sum of error squared

S_{ST} = Total of error squared

R^2 = Regression Coefficient, dimensionless

1. Introduction and Problem Statement

Shale gas reservoirs have become a significant source of gas supply in North America after development of the Barnett Shale in Texas, USA (Belvalkar 2010). The exploitation of these ultra-low permeability formations has been possible due to advancement of drilling and stimulation techniques. The key to maximizing gas production in these reservoirs is the development of an interconnected fracture system through effective stimulation treatment. The main objective of stimulation treatments is to obtain a large, highly fractured network that can produce from the ultra-low permeability rock. The primary method of stimulating these formations is by means of horizontal wells with multi-stage hydraulic fractures. Large water-fracs with light proppant are currently being used with good success in many areas (Belyadi et al, 2012). Although both vertical and horizontal wells can be used, horizontal wells are considered to be more cost effective in achieving commercial production from the shale (Belyadi et al, 2010).

Analytical and empirical approaches for forecasting production for horizontal wells with multi-stage hydraulic fractures are not well established due to the limited field experience. Yet, rigorous techniques are highly in demand from the industry to assist with forecasting of horizontal wells with multiple fractures, which is a critical step for reserve estimation and development planning (Nobakht, 2011).

Various techniques have been discussed in the literature for production analysis and estimating the shale-gas reserves. One of the most commonly used procedures is decline-curve analysis (DCA). Prior to the development of DCA models, estimation of oil reserves was accomplished by calculating the contents of a reservoir based on saturation and percentage of recoverable oil over a certain known area (Valko, 2010). This resulted in a very rough estimate of recoverable hydrocarbons (Valko, 2010). Traditional Arps' Decline Analysis equation (1945) estimates a reliable ultimate recovery for conventional oil and gas wells; since they exhibit boundary dominated flow (BDF) to abandonment. Unlike conventional reservoirs, analyzing shale production data using traditional decline curve methods is problematic because of the nature of the reservoir properties and flow behavior. Shale wells have a long transient flow due to the very low matrix permeability (Kanfar 2012). Therefore, application of Arps' DCA to production data from the unconventional reservoirs results in significant overestimation of reserves (Okouma, 2012).

The issues related to misapplications of the Arps' equation have led to the development of new empirical equations more suitable for shale production decline analysis. The recent methods are the Power Law Exponential Decline (PLE) and Duong's method which account for production during transient flow followed by the boundary dominated flow (Joshi, 2013). While it has been demonstrated

that these new decline curves can be used to successfully match the complete production history of the shale wells, their ability to predict future production rates from the limited production history remain uncertain. In addition, the impact of the formation and hydraulic fracture characteristics cannot be incorporated in the production forecasts due to empirical nature of the decline curves.

2. Literature Review

Production decline analysis is probably the most prominent method for forecasting future production from tight gas and shale formations (Lee and Sidle, 2010; Mishra, 2012). Arps' hyperbolic model is used for estimating ultimate recovery of conventional oil and gas wells. However, this model usually results in overestimation of the reserves for shale gas due to the long transient flow. This has brought about several alternative decline curve analysis methods which are empirically formulated for shale and tight gas wells. The two recent methods considered here besides the traditional Arps' DCA are: the Power Law Exponential Decline and Duong's method. Each of these methods might be applicable for specific cases (Kanfar, 2012).

2.1. Arps' Model

Production rates exhibit three different types of declines during BDF: Exponential, Hyperbolic and Harmonic. Arps models these declines using the concept of loss ratio and its derivative (Kanfar, 2012).

Johnson and Bollens (1928) and later Arps (1945) presented the well's "loss ratio (rate of change of the reciprocal of instantaneous decline rate)" and the "derivative of the loss-ratio" functions respectively as:

$$\frac{1}{D} = -\frac{q}{dq/dt} \dots \dots \dots (1)$$

$$n = \frac{d}{dt} \left[-\frac{q}{dq/dt} \right] \dots \dots \dots (2)$$

Equations 1 and 2 are empirical results based on observations (Ilk, 2008).

When D is constant, equation 1 leads to an exponential decline which can be derived for the case of pseudo-steady state (or boundary dominated) flow in a closed reservoir containing a constant compressibility liquid and being produced at a constant wellbore flowing pressure (Ilk, 2008). The other two cases, on the other hand, were not totally based on any physical basis, but can be utilized in

multilayered reservoirs during BDF. Decline exponent (n - values) ranging from 0 to 1 for these three cases. Where $n=0$ indicates the exponential decline, $0<n<1$ the hyperbolic decline, and $n=1$ the harmonic decline (Kanfar, 2012).

Assuming constant loss ratio, n , Arps derived his hyperbolic decline model in equation 3.

$$q = q_i(1 + nd_it)^{-1/n} \dots \dots \dots (3)$$

For the boundary dominated flow regime the n value lies between 0 and 1, depending on the reservoir fluid, reservoir characteristics, and operating conditions (Fetkovich et al., 1996). However, “high” n values ($n>1$) seem to match the field data from the transient-flow period. For instance, transient linear flow, multi-fracture flow, and bilinear flow which often observed in shale gas reservoirs can be matched with Arps hyperbolic model with n values of 2,3, and 4 respectively (Okouma, 2012).

Integration of Eq. (3) leads to an expression for cumulative production, G_p (Mishra, 2012):

$$G_p = \frac{q_i^n}{d_i(n-1)} \left(q(t)^{(1-n)} - q_i^{(1-n)} \right) \dots \dots \dots (4)$$

2.2. Power Law Exponential Decline Model (PLE)

Power Law Exponential has been proposed by Ilk et al. (2008) to address the problem with over-estimating the reserves from the wells with transient flow (Seshadri, 2010). Traditional exponential decline often does not fit production rates from unconventional reservoirs; since they have a large decline rate in early production period, but over time the decline rate becomes smaller. Power Law Exponential decline model is empirically developed for tight/shale gas production data analysis by matching early transient data without over-estimating reserves as compared to hyperbolic decline prediction with a high n -exponent.

Loss ratio during transient linear and bilinear flow has a power law relation with time. Therefore, the PLE loss ratio (D) model developed by the following equation:

$$D = D_\infty + D_1 t^{-(1-n)} \dots \dots \dots (5)$$

In case $D_\infty = 0$ and $n=1$, the Power Law loss ratio turns into traditional exponential.

This forecast is sensitive to D_∞ values; since it sets a limit on how low the loss ratio can become. Therefore, it prevents the reserves over-prediction (Seshadri, 2010). In early production time, D_∞ is

insignificant; so the model becomes a power law function to match linear or bilinear flow. On the other hand, at late times, D_∞ is more effective and Eq. 5 becomes constant and fit the loss ratio of exponential decline (Kanfar, 2012).

The Power Law rate-time relation is given in equation 6 below:

$$q = q_i \exp\left(-D_\infty t - \frac{D_1}{n} t^n\right) \dots \dots \dots (6)$$

The n value typically lies between 0 and 1. As n gets closer to zero, the power law's decline rates start off large, and then gets smaller over time. This is very similar to how tight gas reservoirs behave. Although zero value for n makes the Eq. 5 undefined, it can be shown analytically that transient linear flow corresponds to $n=0$ (Mattar and Moghadam, 2009).

2.3. Duong Model

The dominant flow in shale gas reservoirs with hydraulic fractures is the fracture flow, and it rarely reaches pseudo-radial and boundary dominated flow even over several years of production. This suggests that matrix contribution is not considerable in compare to fracture contribution. Therefore, there should be an expansion in connected fracture area or stimulated rock volume (SRV) over time to support these fracture flows. This increase is possible by reactivating existing faults and fractures through changing depletion pressure and local stress. Duong introduced an empirically derived decline model based on the SRV expansion which leads to long-term linear flow (Dung, 2012; Kanfar, 2012).

In case of finite conductivity fractures, the flow will be bilinear which can be shown as a quarter-slope line on a log-log plot of rate vs. time; while for infinite conductivity fractures, flow will be linear and indicates by a half-slope line on the same plot.

If a fracture flow regime (either linear or bilinear) extends through the life of the well, production-rate and time would have a power law relation and form a straight line on a log-log scale with a slope of -1 (Dung, 2012; Kanfar, 2012). However, by analyzing of field data from different shale gas wells, Duong substitute the time exponent (slope) with a negative slope of $-m$, and an intercept of a . The modified equation is:

$$\frac{q}{G_p} = at^{-m} \dots \dots \dots (7)$$

The log-log plot of q/G_p vs. time is generated to determine the values of a and m through a linear regression analysis. The R^2 value is used to determine the best fit of data, and it is recommended to accept

R^2 more than 0.95. The slope, m , is always greater than unity for shale reservoirs; while m less than unity indicates a conventional tight well (Duong, 2012).

Once a and m are determined, by plotting flow rate versus t (a, m) and using following equations q_1 and q_∞ will be obtained. During production, due to many factors such as water production or liquid dropout/hold up in the fracture system and wellbore there might be a deviation from fracture flow which results in high decline rate. Thus, the values of q_1 and q_∞ (the rate at infinite time) are designed to represent this increase in production (Duong, 2012).

$$t(a, m) = t^{-m} e^{\frac{a}{1-m}(t^{1-m}-1)} \dots \dots \dots (8)$$

$$q = q_1 t(a, m) + q_\infty \dots \dots \dots (9)$$

The q -vs.- t (a, m) plot will give a straight line with slope of q_1 and intercept of q_∞ . Since q_∞ is the rate at infinite time, it can be zero, positive, or negative. Nonetheless, the original model did not include the term q_∞ . It has been added to the equation to consider the cases that showed an intercept instead of a straight line to the origin on the plot of q vs. t (a, m) (Duong, 2012).

Duong (2011) derived expressions for production rate and cumulative production vs. time that satisfy Eq. 10 as given below:

$$G_p = \frac{q_1 t(a, m)}{a t^{-m}} \dots \dots \dots (10)$$

As shown in Eq. 10, cumulative production is not related to q_∞ and can only be used when q_∞ is zero (Kanfar, 2012).

3. Objective and Methodology

3.1. Objectives

The three main goals of this study are:

- 1) To determine which decline curve model is most appropriate for Marcellus shale production data analysis.
- 2) To evaluate the impact of the formation and hydraulic fracture properties (X_f , K , ϕ) on decline curve model parameters.
- 3) To predict Marcellus shale production from limited production history using different decline curve model.

3.2. Methodology

In order to study the decline behavior of a shale gas reservoir three steps have been undertaken:

- 1) A simulator developed by Fekete (Fast Evolution), was utilized to generate simulated production profile for a horizontal well with multiple hydraulic fractures in an ultra-low permeability formation which contains adsorbed gas (Marcellus shale). Thirty years of production data for a horizontal well with seven and thirteen fracture stages have been generated. For each scenario (7 and 13 stages), production profiles were generated for three different values of fracture half length (X_f), permeability (K), and matrix porosity (ϕ).
- 2) Three decline curve models which were discussed earlier are used to match the thirty-year production profiles. Comparing these DCA models will determine the best model that provides the minimum error in predicting the future production.
- 3) Correlations were established to adjust DCA model parameters obtained by fitting limited-duration production profile (3-5 years) to estimate DCA model parameters for long term production prediction.

- 4) In the final step, the effect of different formation and hydraulic fracture properties (X_f , K , Φ) on production decline model parameters is examined. Model parameters are estimated via non-linear regression using Excel-SOLVER.

3.2.1. Development of a Marcellus Shale Reservoir Model

In the first step a base model for a horizontal well with multiple hydraulic fracture stages in a shale formation was developed. The Fekete simulation model, Fast Evolution, was selected to generate 30-year production profiles. The model consisted of a 4000 feet by 2000 feet rectangular area with 3000 feet of horizontal lateral (Figure 3-1 and Figure 3-2). Two base scenarios include seven- and thirteen-stage hydraulic fracture, in a single porosity system with adsorbed gas.

Values of permeability, fracture half length, and matrix porosity were changed as presented in the Table 3-1 along with the base model parameters (fracture half-length of 500-ft, permeability of 0.002-mD, and porosity of 5%).

It should be noted that previous studies (Belyadi, 2011) shows that in the ultra-low permeability, natural fractures disguise the effect of dual porosity system.

Figure 3-1: Illustration of the Base Model with 13 Stages

Figure 3-2: Illustration of the Base Model with 7 Stages

Table 3-1: Base Model Parameters

Base Model Parameters		
Reservoir Parameters		Ranges of Properties
Layer Name	Marcellus	
Depth, ft.	7000	
Thickness, ft.	75	
Length, ft.	4000	
Width, ft.	2000	
Initial pressure, psia	3000	
Water saturation, fraction	0.15	
Temperature, °F	120	
Rock Properties		
Type	Single Porosity	
Fracture Porosity, frac.	0.005	
Matrix Porosity, frac.	0.05	0.04, 0.05, 0.06, 0.08
Fracture Bulk Permeability, i, j, k, md	0.002, 0.002, 0.0002	0.0008, 0.001, 0.002, 0.004, 0.006
Matrix Permeability, i, j, k, md	0.0004, 0.0004, 0.00004	
Matrix fracture sigma, σ , 1/ft ²	0.0073	
Coal Compressibility, 1/psia	1x10 ⁻⁶	
Rock Density, lb/ft ³	150	
Hydraulic Fracture Properties		
Half length, ft.	500	300, 400, 500, 600, 700
Width, in	0.01	
Top of Fracture, ft.	7000	
Bottom of Fracture, ft.	7075	
X Center, ft.	500	
Y Center, ft.	1000	
Permeability, md	20,000	
Porosity, fraction	0.2	
Number of fracture stages	7 & 13	
Stage Spacing, ft.	500 & 250	
Well Production Controls		
P_{wf} , psia	500	
Adsorption		
Diffusion Coefficient, ft ² /day	1	
Sorption Time, day	62	
Langmuir Pressure, psia	635	
Langmuir Concentration, MSCF/ton	0.08899	

3.2.2. Application of Decline Curve Analysis to All Simulated Data

Three different DCA models: Arps, PLE, and Duong have been applied to the simulated production profiles. Towler (2002) introduced a non-linear regression technique using Excel-Solver for decline curve parameters determination which was discussed more in the Appendix. These three DCA models were compared in terms of a “goodness of fit” to the history data as well as production forecast.

3.2.3. Application of Selected DCA Models to Limited Production Profiles

The 30-year simulated production profile for each case (7 and 13 stages) with various fracture and formation properties combination were truncated at different production times including two, three, four, five, seven, ten, fifteen, twenty, and twenty-five. All three DCA models were then analyzed for each limited production profile. This analysis allows for determination of the appropriate constants for each DCA model as well as studying the trend of each constant as a function of production time. By performing regression curve fit on the constants from each of the DCA models, a technique can be developed to predict future production performance with limited production history.

3.2.4. Adjusting Decline Curve Constants using Correlations

Since horizontal Shale wells often exhibit long transient flow regimes, and do not reach BDF for most of their production life; decline curve constants which were evaluated based on the early production periods such as 2, 3, 4 years most probably will overestimate the later production. In order to overcome this problem the decline curve constants should be adjusted for prediction purposes. One possible approach is to generate a ratio of each decline curve constant to the value of the same constant for the 30-year production profile.

3.2.5. Effect of Formation and Hydraulic Fracture Properties on Production Decline Behavior

After obtaining the various decline curves constant ratios for each time interval, these ratios were compared at each production time interval by plotting them over varied fracture half-lengths from 300-ft to 700-ft, permeability from 0.0008-mD to 0.006-mD, and porosity from 0.04 to 0.08 in order to show the effect of these properties on decline curve constants.

3.2.6. Error Compared to the Base Case Model

Since detailed information on fracture half-length, permeability, and porosity is not readily available for Marcellus wells, there would be a potential error in each DCA constant if these properties are not chosen precisely. In order to evaluate the significance of these errors, the higher and lower values of the base model's assumed fracture half-length, permeability, and porosity were compared to the base case model's corresponding properties value. These errors then plotted versus production time.

3.2.7. Confirmation with Field Data

In order to understand the behavior of the aforementioned decline curve methodology, it is necessary to validate the results with an actual field data. Since shale plays are among the most recently developed type of resources, there is a lack of long term field production data, formation properties, and fracture spacing. Therefore, an actual Marcellus horizontal well containing seven or thirteen stages data was not available. In order to investigate the reliability of this method, data from a well in Upshur County, West Virginia (Well 1) was used containing 3.5 years of production history and five fracture stages. Well 1 has 1812 feet of horizontal lateral, thus having a fracture spacing of 453 feet.

Well 1 production history data matched with single porosity model using Fekete (Fast Evolution) in order to predict the production behavior in time beyond 3.5 years. Arps, PLE, and Duong decline curves were also fitted to the same 3.5 years of well 1 monthly production data and the constants for each decline curve were estimated. Finally, the decline curve constants were adjusted using the established correlations from previous steps. The production rates for 30 years were predicted based on the adjusted constants. The predicted production rates by the simulator were then compared with predictions by different decline curves to evaluate the accuracy of these DCA models.

4. Results and Discussion

4.1. Development of a Marcellus Shale Reservoir Model

Various models were generated for the two scenarios of seven and thirteen stages based on the ranges which were given in Table 3-1.

Figure 4-1 and Figure 4-2 show the thirty year production profile generated from Fekete (Fast Evolution) simulator for the seven and thirteen stages scenarios (for various fracture half-length).

Figure 4-1: 7 Stages Production Profile- Various Fracture Half Length

Figure 4-2: 13 Stages Production Profile-Various Fracture Half Length

Thirty year production profile generated from Fekete (Fast Evolution) simulator for the seven and thirteen stages scenarios (for various permeability and porosity) shown in Figure B-1 through Figure B-4, located in the Appendix.

4.2. Application of Selected DCA Models to All Simulated Data

Three different DCA models: Arps, PLE, and Duong have been applied to the simulated production profiles using Excel-Solver method. Matching these DCA models to all simulated production history yielded the results presented below for seven stages (Base Model) in Figure 4-3. Results for thirteen stages shown in Figure C-1, located in the Appendix. The correspondent DCA models' constants at thirty years of production has been shown in Table 4-1 through Table 4-3 for seven stages case; and Table 4-13 through Table 4-15 for thirteen stages case.

Using this technique for all simulated production data, it was found that these three decline models can provide equally good fits to the history data for both seven- and thirteen- stage cases.

Figure 4-3 : Arps, PLE, and Duong DCAs Simulated Data History Match - Seven Stages

4.3. Application of Selected DCA Models to Limited Production Profiles

As it is mentioned in the previous sections, the Arps DCA model depends on three parameters: “ n ”, “ d_i ”, and “ q_i ”. The PLE model depends on four variables: “ D_∞ ”, “ n ”, “ q_i ”, and “ D_I ”. The Duong model also depends on four variables: “ a ”, “ m ”, “ q_I ”, and “ q_∞ ”.

Since these three decline models can provide equally good fits to the simulated history data, using Excel-Solver technique, another method has been acquired to select the best DCA model to predict the future production behavior. So, the production data has been truncated into two, three, five, seven, ten, fifteen, twenty, and twenty-five of production time in order to get a correlation between the decline curves’ constants versus the production time. The DCA models which present the highest R^2 on the regression curve of their constants vs. time will be chosen as the most appropriate model to predict the future production performance.

4.3.1. Seven Stages Model

For seven fracture model with various fracture half-length, permeability and porosity the Arps, PLE, and Duong models were compared. The results for the basic model over two, three, five, seven, ten, fifteen, twenty, twenty-five, and thirty years of production time are shown in Table 4-1 through Table 4-3, respectively as a sample. Results for the rest of the cases are shown in Table D-1 through Table D-42, located in the Appendix.

Table 4-1: 7 Stages (Base Model) - Arps

7 Fractures (Base Model)- Arps				
Years	n	d_i	q_i	R^2
3	2.1055	0.0259	6.0292	0.9950
5	1.7534	0.0149	5.3604	0.9902
7	1.5965	0.0118	5.0889	0.9885
10	1.4858	0.0099	4.8918	0.9885
15	1.4195	0.0089	4.7677	0.9895
20	1.3931	0.0085	4.7153	0.9903
25	1.3769	0.0083	4.6821	0.9909
30	1.3647	0.0081	4.6569	0.9912

Table 4-2: 7 Stages (Base Model) - PLE

7 Fractures (Base Model)- PLE					
Years	D_{∞}	n	q_i	D_1	R^2
2	-0.000994	0.397486	7.754779	0.077834	0.998041
3	-0.000460	0.362622	7.766450	0.078970	0.997015
4	-0.000242	0.343609	7.778605	0.079894	0.996721
5	-0.000141	0.333277	7.788613	0.080465	0.996772
7	-0.000077	0.325283	7.801243	0.081104	0.997153
10	-0.000081	0.326343	7.806431	0.080959	0.997574
15	-0.000100	0.329156	7.806475	0.080775	0.997838
20	-0.000116	0.333219	7.788616	0.080075	0.997987
25	-0.000118	0.333970	7.788459	0.079975	0.998105
30	-0.000114	0.332869	7.789941	0.080144	0.998170

Table 4-3: 7 Stages (Base Model) - Duong

7 Fractures (Base Model) - Duong					
Years	a	m	q_1	q_{∞}	R^2
2	0.808	0.989	12.151	-3.519	0.970
3	0.812	0.992	11.050	-2.853	0.973
4	0.850	1.011	9.802	-1.851	0.994
5	0.891	1.029	8.870	-1.109	0.999
7	0.940	1.044	8.021	-0.776	0.999
10	1.022	1.066	7.025	-0.461	0.998
15	1.117	1.089	6.175	-0.247	0.997
20	1.202	1.106	5.501	-0.151	0.996
25	1.261	1.122	5.074	-0.061	0.988
30	1.320	1.130	4.650	-0.035	0.991

The obtained correlations from plotting DCA models' constants versus time (for various X_f) as shown in Figure D-1 through Figure D-15, located in the Appendix, are summarized in Table 4-4 through Table 4-6 for Arps, PLE, and Duong models, respectively.

Table 4-4: 7 Stages (Fracture Half Length) - Arps- Correlations

7 Fractures (Fracture Half Length) - Arps			
X_f (ft.)	n	d_i	q_i
300	$y = 3.0275x^{-0.311}$	$y = 0.1333x^{-1.081}$	$y = 6.8693x^{-0.289}$
	$R^2 = 0.9942$	$R^2 = 0.9917$	$R^2 = 0.9978$
400	$y = 2.6555x^{-0.24}$	$y = 0.069x^{-0.76}$	$y = 6.8735x^{-0.19}$
	$R^2 = 0.9699$	$R^2 = 0.9704$	$R^2 = 0.9836$
500	$y = 2.3629x^{-0.176}$	$y = 0.0335x^{-0.458}$	$y = 6.415x^{-0.103}$
	$R^2 = 0.8977$	$R^2 = 0.8706$	$R^2 = 0.8847$
600	$y = 2.3646x^{-0.13}$	$y = 0.0224x^{-0.207}$	$y = 6.204x^{-0.031}$
	$R^2 = 0.9638$	$R^2 = 0.8926$	$R^2 = 0.8182$
700	$y = 2.237x^{-0.12}$	$y = 0.014x^{-0.146}$	$y = 5.8513x^{-0.015}$
	$R^2 = 0.9887$	$R^2 = 0.9914$	$R^2 = 0.9794$

Table 4-5: 7 Stages (Fracture Half Length) - PLE- Correlations

7 Fractures (Fracture Half Length) - PLE				
X_f (ft.)	D_∞	n	D_1	q_i
300	$y = 6E-05\ln(x) - 0.0004$	$y = -0.008\ln(x) + 0.3216$	$y = 0.0007\ln(x) + 0.1017$	$y = 0.0024\ln(x) + 6.9017$
	$R^2 = 0.8368$	$R^2 = 0.8383$	$R^2 = 0.6886$	$R^2 = 0.0965$
400	$y = 5E-05\ln(x) - 0.0003$	$y = -0.004\ln(x) + 0.3266$	$y = 4E-05\ln(x) + 0.092$	$y = -0.007\ln(x) + 7.6086$
	$R^2 = 0.3245$	$R^2 = 0.2679$	$R^2 = 0.0125$	$R^2 = 0.494$
500	$y = 0.0002\ln(x) - 0.0007$	$y = -0.008\ln(x) + 0.3534$	$y = 0.0002\ln(x) + 0.0798$	$y = 0.0072\ln(x) + 7.7741$
	$R^2 = 0.5143$	$R^2 = 0.3204$	$R^2 = 0.0673$	$R^2 = 0.2114$
600	$y = 0.0002\ln(x) - 0.0005$	$y = -0.017\ln(x) + 0.3908$	$y = 0.0006\ln(x) + 0.0673$	$y = -0.009\ln(x) + 7.6669$
	$R^2 = 0.6006$	$R^2 = 0.5802$	$R^2 = 0.3669$	$R^2 = 0.1273$
700	$y = 0.0002\ln(x) - 0.0007$	$y = -0.034\ln(x) + 0.4448$	$y = 0.0027\ln(x) + 0.0528$	$y = 0.0641\ln(x) + 7.2263$
	$R^2 = 0.7116$	$R^2 = 0.7539$	$R^2 = 0.7361$	$R^2 = 0.638$

Table 4-6: 7 Stages (Fracture Half Length) - Duong- Correlations

7 Fractures (Fracture Half Length) - Duong				
X_f (ft.)	a	m	q_1	q^∞
300	$y = 0.1034\ln(x) + 0.6565$	$y = 0.0499\ln(x) + 0.9267$	$y = -2.343\ln(x) + 12.999$	$y = 1.6809\ln(x) - 4.9752$
	$R^2 = 0.6253$	$R^2 = 0.9075$	$R^2 = 0.999$	$R^2 = 0.8761$
400	$y = 0.1778\ln(x) + 0.5838$	$y = 0.0547\ln(x) + 0.9336$	$y = -2.525\ln(x) + 13.258$	$y = 1.1572\ln(x) - 3.4918$
	$R^2 = 0.8814$	$R^2 = 0.9674$	$R^2 = 1$	$R^2 = 0.8935$
500	$y = 0.2005\ln(x) + 0.594$	$y = 0.056\ln(x) + 0.9383$	$y = -2.75\ln(x) + 13.706$	$y = 1.2274\ln(x) - 3.7281$
	$R^2 = 0.9614$	$R^2 = 0.99$	$R^2 = 0.9866$	$R^2 = 0.8598$
600	$y = 0.2205\ln(x) + 0.5613$	$y = 0.0564\ln(x) + 0.9323$	$y = -3.396\ln(x) + 16.043$	$y = 1.5355\ln(x) - 4.8051$
	$R^2 = 0.8981$	$R^2 = 0.9709$	$R^2 = 0.9821$	$R^2 = 0.8259$
700	$y = 0.2073\ln(x) + 0.542$	$y = 0.0587\ln(x) + 0.9148$	$y = -3.94\ln(x) + 18.393$	$y = 2.3453\ln(x) - 7.277$
	$R^2 = 0.8973$	$R^2 = 0.9258$	$R^2 = 0.9781$	$R^2 = 0.8653$

The obtained correlations from plotting DCA models' constants vs. time (for various **K**) as shown in Figure D-16 through Figure D-30, located in the Appendix, are summarized in Table 4-7 through Table 4-9 for Arps, PLE, and Duong models, respectively.

Table 4-7: 7 Stages (Permeability) - Arps- Correlations

7 Fractures (Permeability) - Arps			
K (md)	n	di	qi
0.0008	$y = 2.9269x^{-0.16}$ $R^2 = 0.9811$	$y = 0.1972x^{-0.782}$ $R^2 = 0.9908$	$y = 6.6519x^{-0.197}$ $R^2 = 0.9917$
0.001	$y = 2.7097x^{-0.157}$ $R^2 = 0.9959$	$y = 0.1345x^{-0.746}$ $R^2 = 0.9913$	$y = 6.6857x^{-0.186}$ $R^2 = 0.9927$
0.002	$y = 2.3117x^{-0.169}$ $R^2 = 0.9124$	$y = 0.0306x^{-0.427}$ $R^2 = 0.8965$	$y = 6.2848x^{-0.096}$ $R^2 = 0.9101$
0.004	$y = 1.4461x^{-0.055}$ $R^2 = 0.7784$	$y = 0.01x^{-0.091}$ $R^2 = 0.8169$	$y = 6.754x^{-0.019}$ $R^2 = 0.8424$
0.006	$y = 1.2484x^{-0.024}$ $R^2 = 0.6793$	$y = 0.0087x^{-0.041}$ $R^2 = 0.6724$	$y = 7.8428x^{-0.01}$ $R^2 = 0.6684$

Table 4-8: 7 Stages (Permeability) - PLE - Correlations

7 Fractures (Permeability) - PLE				
K (md)	D_{∞}	n	D_1	q_i
0.0008	$y = 0.0004\ln(x) - 0.0013$ $R^2 = 0.8495$	$y = -0.047\ln(x) + 0.4048$ $R^2 = 0.9299$	$y = 0.003\ln(x) + 0.0943$ $R^2 = 0.9508$	$y = 0.032\ln(x) + 5.8492$ $R^2 = 0.9519$
0.001	$y = 0.0004\ln(x) - 0.0011$ $R^2 = 0.8069$	$y = -0.047\ln(x) + 0.4125$ $R^2 = 0.8911$	$y = 0.0027\ln(x) + 0.0903$ $R^2 = 0.937$	$y = 0.0331\ln(x) + 6.2825$ $R^2 = 0.964$
0.002	$y = 1E-04\ln(x) - 0.0004$ $R^2 = 0.4324$	$y = -0.007\ln(x) + 0.3529$ $R^2 = 0.2894$	$y = 2E-05\ln(x) + 0.08$ $R^2 = 0.0004$	$y = -0.007\ln(x) + 7.788$ $R^2 = 0.1074$
0.004	$y = -8E-05\ln(x) - 3E-05$ $R^2 = 0.4152$	$y = 0.0074\ln(x) + 0.3876$ $R^2 = 0.2353$	$y = -6E-04\ln(x) + 0.0623$ $R^2 = 0.1442$	$y = -0.049\ln(x) + 9.4046$ $R^2 = 0.8203$
0.006	$y = 4E-05\ln(x) - 0.0005$ $R^2 = 0.0355$	$y = -0.014\ln(x) + 0.4935$ $R^2 = 0.2429$	$y = 0.0023\ln(x) + 0.0457$ $R^2 = 0.4151$	$y = 0.0657\ln(x) + 10.163$ $R^2 = 0.2844$

Table 4-9: 7 Stages (Permeability) - Duong- Correlations

7 Fractures (Permeability) - Duong				
K (md)	α	m	q_1	q^{∞}
0.0008	$y = 0.1557\ln(x) + 0.718$ $R^2 = 0.9771$	$y = 0.0167\ln(x) + 1.0526$ $R^2 = 0.8126$	$y = -1.295\ln(x) + 7.2718$ $R^2 = 0.9906$	$y = -0.087\ln(x) + 0.2203$ $R^2 = 0.8302$
0.001	$y = 0.1835\ln(x) + 0.6816$ $R^2 = 0.9947$	$y = 0.0246\ln(x) + 1.0344$ $R^2 = 0.9812$	$y = -1.325\ln(x) + 7.6966$ $R^2 = 0.9595$	$y = -0.147\ln(x) + 0.3145$ $R^2 = 0.9608$
0.002	$y = 0.2282\ln(x) + 0.7322$ $R^2 = 0.985$	$y = 0.0408\ln(x) + 1.013$ $R^2 = 0.9827$	$y = -2.03\ln(x) + 9.879$ $R^2 = 0.927$	$y = 0.3542\ln(x) - 1.0735$ $R^2 = 0.9435$
0.004	$y = 0.2633\ln(x) + 0.798$ $R^2 = 0.9738$	$y = 0.0478\ln(x) + 1.0166$ $R^2 = 0.991$	$y = -2.3\ln(x) + 11.306$ $R^2 = 0.988$	$y = 0.7135\ln(x) - 2.1718$ $R^2 = 0.898$
0.006	$y = 0.2721\ln(x) + 0.7984$ $R^2 = 0.9396$	$y = 0.051\ln(x) + 1.0133$ $R^2 = 0.9764$	$y = -2.889\ln(x) + 13.859$ $R^2 = 0.9753$	$y = 1.033\ln(x) - 3.1298$ $R^2 = 0.8331$

The obtained correlations from plotting DCA models' constants vs. time (for various Φ) as shown in Figure D-31 through Figure D-42, located in the Appendix, are summarized in Table 4-10, Table 4-11, and Table 4-12 for Arps, PLE, and Duong models, respectively.

Table 4-10: 7 Stages (Porosity) - Arps- Correlations

7 Fractures (Porosity) - Arps			
ϕ (%)	n	di	qi
4	$y = 2.0438x^{-0.137}$	$y = 0.0234x^{-0.321}$	$y = 5.5356x^{-0.072}$
	$R^2 = 0.8655$	$R^2 = 0.8552$	$R^2 = 0.871$
5	$y = 2.3123x^{-0.169}$	$y = 0.0307x^{-0.427}$	$y = 6.2876x^{-0.096}$
	$R^2 = 0.9132$	$R^2 = 0.8976$	$R^2 = 0.9117$
6	$y = 2.5379x^{-0.19}$	$y = 0.0386x^{-0.513}$	$y = 6.9906x^{-0.114}$
	$R^2 = 0.9456$	$R^2 = 0.9297$	$R^2 = 0.9414$
8	$y = 2.8603x^{-0.211}$	$y = 0.0537x^{-0.627}$	$y = 8.1968x^{-0.138}$
	$R^2 = 0.9803$	$R^2 = 0.9702$	$R^2 = 0.9773$

Table 4-11: 7 Stages (Porosity) - PLE- Correlations

7Fractures (Porosity) - PLE				
ϕ (%)	D_{∞}	n	D_1	q_i
4	$y = 0.0001\ln(x) - 0.0005$	$y = -0.008\ln(x) + 0.3596$	$y = -2E-04\ln(x) + 0.0819$	$y = -0.008\ln(x) + 7.4105$
	$R^2 = 0.3596$	$R^2 = 0.2322$	$R^2 = 0.0522$	$R^2 = 0.2443$
5	$y = 0.0002\ln(x) - 0.0007$	$y = -0.017\ln(x) + 0.3784$	$y = 0.0006\ln(x) + 0.0788$	$y = 0.0098\ln(x) + 7.7634$
	$R^2 = 0.5061$	$R^2 = 0.4914$	$R^2 = 0.3216$	$R^2 = 0.4788$
6	$y = 0.0003\ln(x) - 0.0009$	$y = -0.026\ln(x) + 0.3974$	$y = 0.0013\ln(x) + 0.0757$	$y = 0.0214\ln(x) + 8.0566$
	$R^2 = 0.6064$	$R^2 = 0.6514$	$R^2 = 0.6412$	$R^2 = 0.7462$
8	$y = 0.0002\ln(x) - 0.0008$	$y = -0.028\ln(x) + 0.4009$	$y = 0.0017\ln(x) + 0.072$	$y = 0.0187\ln(x) + 8.5758$
	$R^2 = 0.699$	$R^2 = 0.7379$	$R^2 = 0.7061$	$R^2 = 0.6224$

Table 4-12: 7 Stages (Porosity) - Duong- Correlations

7Fractures (Porosity) - Duong				
ϕ (%)	α	m	q_1	q^{∞}
4	$y = 0.2582\ln(x) + 0.697$	$y = 0.0483\ln(x) + 1.0013$	$y = -1.922\ln(x) + 9.1644$	$y = 0.3232\ln(x) - 1.0152$
	$R^2 = 0.9877$	$R^2 = 0.9801$	$R^2 = 0.9509$	$R^2 = 0.9413$
5	$y = 0.239\ln(x) + 0.6433$	$y = 0.047\ln(x) + 0.9864$	$y = -2.168\ln(x) + 10.725$	$y = 0.3599\ln(x) - 1.1833$
	$R^2 = 0.9949$	$R^2 = 0.9982$	$R^2 = 0.987$	$R^2 = 0.981$
6	$y = 0.2114\ln(x) + 0.689$	$y = 0.0443\ln(x) + 0.9779$	$y = -2.258\ln(x) + 11.876$	$y = 0.4165\ln(x) - 1.3863$
	$R^2 = 0.9871$	$R^2 = 0.9944$	$R^2 = 0.9943$	$R^2 = 0.9801$
8	$y = 0.1592\ln(x) + 0.6128$	$y = 0.0394\ln(x) + 0.9596$	$y = -2.526\ln(x) + 14.791$	$y = 0.7121\ln(x) - 2.3848$
	$R^2 = 0.8887$	$R^2 = 0.9404$	$R^2 = 0.9673$	$R^2 = 0.9768$

According to Table 4-4 through Table 4-12, trends of the constants plotted versus time show a high R^2 only for Arps and Duong models. Arps model constants have a more consistent trend with time at a higher porosity cases as well as a lower permeability ranges which is probably due to the accuracy of Arps model at BDF period.

4.3.2. Thirteen Stages Model

For thirteen stages model with various fracture half-length, permeability and porosity the Arps, PLE, and Duong models were compared. The results for the basic model over two, three, five, seven, ten, fifteen, twenty, twenty-five, and thirty years of production time are shown in Table 4-13 through Table 4-15, respectively as a sample. Results for the rest of the cases are shown in Table D-43 through Table D-48, located in the Appendix.

Table 4-13: 13 Stages (Base Model) - Arps

13 Fractures (Base Model)- Arps				
Years	n	d_i	q_i	R^2
3	1.5027	0.0281	11.76	0.971
5	1.4045	0.0265	11.71	0.971
7	1.3575	0.0252	11.61	0.974
10	1.3221	0.0240	11.48	0.978
15	1.3046	0.0233	11.41	0.980
20	1.2946	0.0229	11.35	0.982
25	1.2800	0.0221	11.25	0.984
30	1.2765	0.0220	11.23	0.984

Table 4-14: 13 Stages (Base Model) - PLE

13 Fractures (Base Model)- PLE					
Years	D_{∞}	n	q_i	D_1	R^2
2	0.0004	0.3897	13.9609	0.0730	0.9997
3	0.0003	0.3970	13.9637	0.0729	0.9997
4	0.0001	0.4138	13.9337	0.0719	0.9995
5	-0.0001	0.4286	13.9121	0.0710	0.9991
7	-0.0005	0.4574	13.8354	0.0685	0.9984
10	-0.0008	0.4830	13.7405	0.0656	0.9980
15	-0.0008	0.4834	13.7383	0.0656	0.9980
20	-0.0007	0.4728	13.7375	0.0668	0.9976
25	-0.0006	0.4615	13.7385	0.0682	0.9968
30	-0.0006	0.4526	13.7400	0.0694	0.9958

Table 4-15: 13 Stages (Base Model) - Duong

13 Fractures (Base Model)- Duong					
Years	a	m	q ₁	q _∞	R ²
2	0.946	1.049	16.1	-2.577	0.954
3	0.986	1.051	15.0	-2.556	0.982
4	1.063	1.081	14.0	-1.591	0.996
5	1.118	1.099	13.2	-1.081	0.994
7	1.199	1.124	12.3	-0.609	0.990
10	1.262	1.143	11.8	-0.324	0.986
15	1.340	1.162	11.2	-0.144	0.983
20	1.399	1.181	10.9	-0.038	0.977
25	1.465	1.185	10.2	-0.024	0.980
30	1.650	1.220	9.8	0.050	0.963

The obtained correlations from plotting DCA models constants versus time (for various X_f) as shown in Figure D-43 through Figure D-57, located in the Appendix, are summarized in Table 4-16 through Table 4-18 for Arps, PLE, and Duong models, respectively.

Table 4-16: 13 Stages (Fracture Half-Length) - Arps - Correlations

13 Fractures (Fracture Half Length) - Arps			
X_f (ft.)	n	d _i	q _i
300	$y = 1.5095x^{-0.079}$	$y = 0.0413x^{-0.26}$	$y = 9.4778x^{-0.075}$
	$R^2 = 0.9299$	$R^2 = 0.9853$	$R^2 = 0.9924$
400	$y = 1.5816x^{-0.075}$	$y = 0.041x^{-0.192}$	$y = 11.481x^{-0.049}$
	$R^2 = 0.9543$	$R^2 = 0.9896$	$R^2 = 0.9807$
500	$y = 1.571x^{-0.066}$	$y = 0.0313x^{-0.107}$	$y = 12.073x^{-0.021}$
	$R^2 = 0.9112$	$R^2 = 0.9859$	$R^2 = 0.9815$
600	$y = 1.5452x^{-0.063}$	$y = 0.0248x^{-0.089}$	$y = 12.314x^{-0.014}$
	$R^2 = 0.9911$	$R^2 = 0.988$	$R^2 = 0.9218$
700	$y = 1.5782x^{-0.057}$	$y = 0.021x^{-0.083}$	$y = 12.351x^{-0.011}$
	$R^2 = 0.9816$	$R^2 = 0.9292$	$R^2 = 0.9227$

Table 4-17: 13 Stages (Fracture Half-Length) - PLE - Correlations

13 Fractures (Fracture Half Length) - PLE				
X_f (ft.)	D _∞	n	D ₁	q _i
300	$y = -3E-05x + 4E-05$	$y = 0.0005x + 0.3751$	$y = 0.0003x + 0.0958$	$y = 0.0562\ln(x) + 12.266$
	$R^2 = 0.2008$	$R^2 = 0.0198$	$R^2 = 0.5073$	$R^2 = 0.22$
400	$y = -3E-05x + 0.0001$	$y = 0.0017x + 0.3906$	$y = -8E-06x + 0.0842$	$y = -0.041\ln(x) + 13.666$
	$R^2 = 0.3383$	$R^2 = 0.2034$	$R^2 = 0.0008$	$R^2 = 0.2736$
500	$y = -3E-05x + 4E-05$	$y = 0.0021x + 0.4181$	$y = -0.0002x + 0.0711$	$y = -0.102\ln(x) + 14.049$
	$R^2 = 0.4498$	$R^2 = 0.3774$	$R^2 = 0.2787$	$R^2 = 0.8824$
600	$y = -2E-05x - 1E-04$	$y = 0.0017x + 0.4543$	$y = -8E-05x + 0.0578$	$y = -0.005\ln(x) + 13.639$
	$R^2 = 0.4616$	$R^2 = 0.3555$	$R^2 = 0.1385$	$R^2 = 0.0033$
700	$y = -2E-05x - 0.0002$	$y = 0.0017x + 0.4849$	$y = -0.0001x + 0.0471$	$y = -0.092\ln(x) + 13.196$
	$R^2 = 0.4733$	$R^2 = 0.4751$	$R^2 = 0.5158$	$R^2 = 0.9722$

Table 4-18: 13 Stages (Fracture Half-Length) - Duong - Correlations

13 Fractures (Fracture Half Length) - Duong				
X_f (ft.)	a	m	q_1	q_∞
300	$y = 0.1484\ln(x) + 0.7232$	$y = 0.058\ln(x) + 0.983$	$y = -1.614\ln(x) + 15.764$	$y = 1.1358\ln(x) - 3.1835$
	$R^2 = 0.9384$	$R^2 = 0.970$	$R^2 = 0.9838$	$R^2 = 0.6807$
400	$y = 0.2009\ln(x) + 0.7566$	$y = 0.0612\ln(x) + 0.9962$	$y = -1.81\ln(x) + 16.176$	$y = 1.1225\ln(x) - 3.235$
	$R^2 = 0.9115$	$R^2 = 0.9748$	$R^2 = 0.9763$	$R^2 = 0.7797$
500	$y = 0.2347\ln(x) + 0.7415$	$y = 0.0621\ln(x) + 0.9968$	$y = -2.212\ln(x) + 17.156$	$y = 1.0272\ln(x) - 3.0835$
	$R^2 = 0.958$	$R^2 = 0.9793$	$R^2 = 0.974$	$R^2 = 0.8807$
600	$y = 0.2543\ln(x) + 0.732$	$y = 0.0627\ln(x) + 0.9916$	$y = -2.775\ln(x) + 18.281$	$y = 1.7826\ln(x) - 5.2705$
	$R^2 = 0.9945$	$R^2 = 0.9924$	$R^2 = 0.9931$	$R^2 = 0.7891$
700	$y = 0.2756\ln(x) + 0.6725$	$y = 0.0649\ln(x) + 0.978$	$y = -3.393\ln(x) + 20.304$	$y = 2.2697\ln(x) - 6.7527$
	$R^2 = 0.9669$	$R^2 = 0.9781$	$R^2 = 0.9701$	$R^2 = 0.8047$

The obtained correlations from plotting DCA models constants versus time (for various K) as shown in Figure D-58 through Figure D-72, located in the Appendix, are summarized in Table 4-19 through Table 4-21 for Arps, PLE, and Duong models, respectively.

Table 4-19: 13 Stages (Permeability) - Arps - Correlations

13 Fractures (Permeability) - Arps			
K (md)	n	d_1	q_1
0.0008	$y = 1.8353x^{-0.084}$	$y = 0.0309x^{-0.102}$	$y = 7.7348x^{-0.014}$
	$R^2 = 0.954$	$R^2 = 0.9778$	$R^2 = 0.9773$
0.001	$y = 1.8246x^{-0.08}$	$y = 0.0338x^{-0.094}$	$y = 8.8291x^{-0.012}$
	$R^2 = 0.9471$	$R^2 = 0.9939$	$R^2 = 0.9223$
0.002	$y = 1.8166x^{-0.07}$	$y = 0.0461x^{-0.059}$	$y = 13.434x^{-0.011}$
	$R^2 = 0.9366$	$R^2 = 0.9553$	$R^2 = 0.9934$
0.004	$y = 1.5519x^{-0.061}$	$y = 0.0451x^{-0.049}$	$y = 17.203x^{-0.008}$
	$R^2 = 0.9927$	$R^2 = 0.9735$	$R^2 = 0.9492$
0.006	$y = 1.7655x^{-0.052}$	$y = 0.0712x^{-0.032}$	$y = 23.294x^{-0.007}$
	$R^2 = 0.9923$	$R^2 = 0.8745$	$R^2 = 0.8741$

Table 4-20: 13 Stages (Permeability) - PLE - Correlations

13 Fractures (Permeability) - PLE				
K (md)	D_∞	n	D_1	q_1
0.0008	$y = -5E-06x - 5E-05$	$y = 0.0009x + 0.3271$	$y = -0.0001x + 0.0967$	$y = -0.0035x + 10.759$
	$R^2 = 0.2013$	$R^2 = 0.5959$	$R^2 = 0.9101$	$R^2 = 0.8741$
0.001	$y = -1E-05x + 7E-05$	$y = 0.0018x + 0.3372$	$y = -0.0002x + 0.0915$	$y = -0.0057x + 11.523$
	$R^2 = 0.8285$	$R^2 = 0.8901$	$R^2 = 0.9364$	$R^2 = 0.9513$
0.002	$y = -3E-05x + 4E-05$	$y = 0.0015x + 0.429$	$y = -5E-05x + 0.0701$	$y = 0.0003x + 13.87$
	$R^2 = 0.444$	$R^2 = 0.2358$	$R^2 = 0.0407$	$R^2 = 0.001$
0.004	$y = 6E-05x - 0.0024$	$y = -0.0052x + 0.6253$	$y = 0.0006x + 0.0468$	$y = 0.0265x + 16.363$
	$R^2 = 0.6197$	$R^2 = 0.8148$	$R^2 = 0.9003$	$R^2 = 0.8146$
0.006	$y = 0.0002x - 0.0063$	$y = -0.0105x + 0.7614$	$y = 0.0009x + 0.0391$	$y = 0.0561x + 17.585$
	$R^2 = 0.816$	$R^2 = 0.9099$	$R^2 = 0.9169$	$R^2 = 0.896$

Table 4-21: 13 Stages (Permeability) - Duong – Correlations

13 Fractures (Permeability) - Duong				
K (md)	a	m	q ₁	q _∞
0.0008	y = 0.2551ln(x) + 0.7188	y = 0.0477ln(x) + 1.0193	y = -2.264ln(x) + 11.873	y = 0.3464ln(x) - 1.1335
	R ² = 0.9788	R ² = 0.9815	R ² = 0.9925	R ² = 0.9898
0.001	y = 0.3114ln(x) + 0.7383	y = 0.055ln(x) + 1.0242	y = -2.493ln(x) + 11.97	y = 0.3998ln(x) - 1.2821
	R ² = 0.9832	R ² = 0.9942	R ² = 0.9749	R ² = 0.9724
0.002	y = 0.3667ln(x) + 0.8288	y = 0.0601ln(x) + 1.0508	y = -2.841ln(x) + 13.502	y = 0.4413ln(x) - 1.3594
	R ² = 0.9969	R ² = 0.9884	R ² = 0.9841	R ² = 0.9379
0.004	y = 0.4439ln(x) + 0.9146	y = 0.0634ln(x) + 1.082	y = -3.861ln(x) + 16.425	y = 0.8428ln(x) - 2.3521
	R ² = 0.9926	R ² = 0.9813	R ² = 0.9938	R ² = 0.7202
0.006	y = 0.4523ln(x) + 0.8899	y = 0.0641ln(x) + 1.0859	y = -4.886ln(x) + 20.43	y = 0.5404ln(x) - 1.5052
	R ² = 0.9915	R ² = 0.9678	R ² = 0.991	R ² = 0.6603

The obtained correlations from plotting DCA models constants versus time (for various ϕ) as shown in Figure D-73 through Figure D-84, located in the Appendix, are summarized in Table 4-22 through Table 4-24 for Arps, PLE, and Duong models, respectively.

Table 4-22: 13 Stages (Porosity) - Arps - Correlations

13 Fractures (Porosity) - Arps			
ϕ (%)	n	d _i	q _i
4	y = 1.8713x ^{-0.098}	y = 0.0775x ^{-0.265}	y = 14.476x ^{-0.065}
	R ² = 0.9757	R ² = 0.9869	R ² = 0.9764
5	y = 1.9621x ^{-0.117}	y = 0.0699x ^{-0.295}	y = 14.874x ^{-0.069}
	R ² = 0.9845	R ² = 0.9711	R ² = 0.9396
6	y = 2.0256x ^{-0.132}	y = 0.0616x ^{-0.315}	y = 15.171x ^{-0.073}
	R ² = 0.9653	R ² = 0.9898	R ² = 0.9772
8	y = 2.2511x ^{-0.169}	y = 0.0592x ^{-0.384}	y = 16.014x ^{-0.082}
	R ² = 0.9925	R ² = 0.9852	R ² = 0.967

Table 4-23: 13 Stages (Porosity) - PLE - Correlations

13 Fractures (Porosity) - PLE				
ϕ (%)	D _∞	n	D ₁	q _i
4	y = -3E-04ln(x) + 6E-05	y = 0.0103ln(x) + 0.4369	y = 0.0008ln(x) + 0.0702	y = 0.0548ln(x) + 13.11
	R ² = 0.3228	R ² = 0.0934	R ² = 0.0421	R ² = 0.1632
5	y = -4E-04ln(x) + 0.0005	y = 0.0257ln(x) + 0.3904	y = -0.002ln(x) + 0.0729	y = -0.045ln(x) + 13.96
	R ² = 0.6264	R ² = 0.4964	R ² = 0.2419	R ² = 0.1639
6	y = -4E-04ln(x) + 0.0006	y = 0.0327ln(x) + 0.3639	y = -0.003ln(x) + 0.0735	y = -0.08ln(x) + 14.588
	R ² = 0.7889	R ² = 0.7316	R ² = 0.6062	R ² = 0.4764
8	y = -3E-04ln(x) + 0.0006	y = 0.0378ln(x) + 0.3378	y = -0.004ln(x) + 0.0737	y = -0.156ln(x) + 15.634
	R ² = 0.9176	R ² = 0.9156	R ² = 0.9088	R ² = 0.8961

Table 4-24: 13 Stages (Porosity) - Duong - Correlations

13 Fractures (Porosity) - Duong				
ϕ (%)	a	m	q_1	q^∞
4	$y = 0.1862\ln(x) + 0.9319$	$y = 0.0496\ln(x) + 1.044$	$y = -1.686\ln(x) + 14.013$	$y = 0.8214\ln(x) - 2.4067$
	$R^2 = 0.9825$	$R^2 = 0.9721$	$R^2 = 0.9757$	$R^2 = 0.8092$
5	$y = 0.175\ln(x) + 0.9575$	$y = 0.0469\ln(x) + 1.0423$	$y = -1.665\ln(x) + 14.217$	$y = 0.9216\ln(x) - 2.754$
	$R^2 = 0.9908$	$R^2 = 0.9808$	$R^2 = 0.9382$	$R^2 = 0.8447$
6	$y = 0.1643\ln(x) + 0.9051$	$y = 0.0472\ln(x) + 1.0246$	$y = -1.822\ln(x) + 16.036$	$y = 1.1095\ln(x) - 3.3831$
	$R^2 = 0.9859$	$R^2 = 0.9889$	$R^2 = 0.9809$	$R^2 = 0.8914$
8	$y = 0.1558\ln(x) + 0.9207$	$y = 0.0432\ln(x) + 1.0252$	$y = -1.91\ln(x) + 16.678$	$y = 1.1611\ln(x) - 3.6625$
	$R^2 = 0.9838$	$R^2 = 0.9867$	$R^2 = 0.972$	$R^2 = 0.9424$

According to Table 4-16 through Table 4-24, trends of the constants plotted vs. time show a high R^2 only for Arps and Duong models.

Additionally, values of “ n ” in the Arps model begins at approximately two, indicating transient linear flow. The “ n ” values approach a harmonic decline ($n=1$) as time progresses, especially for a higher ranges of fracture half-length and permeability; and lower porosity.

The Duong and Arps models were finally selected due to the consistent trend of constants plotted versus time at various fracture half-length, permeability, and porosity.

4.4. Effect of Formation and Hydraulic Fracture Properties on Production

Decline Behavior

In order to accurately predict the long term (30-year) decline model parameters based on the limited production histories (2, 3, and 4 years), the ratio of decline curve constants obtained from fitting the limited production history (Table E-1 through Table E-28 for seven stages case; and Table E-57 through Table E-84 for thirteen stages case, located in the Appendix), to the same constant obtained from fitting the 30-year production profile were determined for Arps and Duong decline curve models. However, the constant values were re-calculated based on the obtained correlations in Table 4-4 through Table 4-12. This re-calculation results in a higher R^2 when the regression curve applied to the plot of ratios vs. various ranges of fracture half-length, permeability, and porosity at each truncated production time (2, 3, 4, 5, 7, 10, 15, 20, 25, and 30 years). Comparing these plots will assist us to investigate the effect of formation and hydraulic fracture properties on the decline curve constants as well as the changing behavior of the constants over 30-year production time.

4.4.1. Seven Stages Model

The results for re-calculated Arps and Duong constants and their ratios are shown in Table 4-25 through Table 4-28, for the base case only. The ratio calculation for other ranges of X_f , K , and ϕ can be seen in Table E-29 through Table E-56, located in the Appendix.

Table 4-25: 7 Stages (Base Model) - Arps- Recalculated Constants

7 Fractures (Base Model)- Arps						
Years	n	$n(Eq.)$	d_i	$d_i(Eq.)$	q_i	$q_i(Eq.)$
3	2.1055	1.9475	0.0259	0.0203	6.0292	5.7287
5	1.7534	1.7800	0.0149	0.0160	5.3604	5.4350
7	1.5965	1.6777	0.0118	0.0137	5.0889	5.2499
10	1.4858	1.5756	0.0099	0.0117	4.8918	5.0605
15	1.4195	1.4671	0.0089	0.0097	4.7677	4.8535
20	1.3931	1.3946	0.0085	0.0085	4.7153	4.7118
25	1.3769	1.3409	0.0083	0.0077	4.6821	4.6048
30	1.3647	1.2986	0.0081	0.0071	4.6569	4.5191

Table 4-26: 7 Stages (Base Model) - Arps - Table of Ratios

7 Fractures (Base Model)- Arps- Table of Ratios			
Years	n - Ratio	d_i - Ratio	q_i - Ratio
3	1.499685	2.870781	1.267652
5	1.370739	2.271921	1.202679
7	1.291921	1.947451	1.161712
10	1.213314	1.653947	1.119808
15	1.129747	1.373636	1.074004
20	1.07397	1.204065	1.042647
25	1.032609	1.087089	1.018957
30	1	1	1

Table 4-27: 7 Stages (Base Model) - Duong- Recalculated Constants

7 Fractures (Base Model)- Duong								
Years	a	$a(Eq.)$	m	$m(Eq.)$	q_1	$q_1(Eq.)$	q_∞	$q_\infty(Eq.)$
2	0.808	0.733	0.989	0.979	12.151	11.800	-3.519	-2.877
3	0.812	0.814	0.992	1.001	11.050	10.685	-2.853	-2.380
4	0.850	0.872	1.011	1.017	9.802	9.894	-1.851	-2.027
5	0.891	0.917	1.029	1.030	8.870	9.280	-1.109	-1.753
7	0.940	0.984	1.044	1.049	8.021	8.355	-0.776	-1.340
10	1.022	1.056	1.066	1.069	7.025	7.374	-0.461	-0.902
15	1.117	1.137	1.089	1.091	6.175	6.259	-0.247	-0.404
20	1.202	1.195	1.106	1.108	5.501	5.468	-0.151	-0.051
25	1.261	1.239	1.122	1.120	5.074	4.854	-0.061	0.223
30	1.320	1.276	1.130	1.130	4.650	4.353	-0.035	0.447

Table 4-28: 7 Stages (Base Model) - Duong - Table of Ratios

7 Fractures (Base Model)- Duong- Table of Ratios				
Years	α-Ratio	m-Ratio	q_1- Ratio	q_∞ - Ratio
2	0.57446	0.865827	2.710921	-6.4437626
3	0.638174	0.885917	2.454752	-5.3292389
4	0.68338	0.90017	2.272997	-4.5384718
5	0.718445	0.911226	2.132017	-3.9251052
7	0.771318	0.927897	1.919437	-3.0002259
10	0.827365	0.945568	1.694093	-2.0198144
15	0.89108	0.965657	1.437924	-0.9052908
20	0.936286	0.979911	1.256169	-0.1145237
25	0.97135	0.990967	1.115189	0.49884298
30	1	1	1	1

Section 4.4.1 and section 4.4.2 show the changes of Arps and Duong models' constants with fracture half-length, permeability, and porosity. However, the plot of the initial flow rate (q_i) has been only presented in this chapter. The rest of the plots for other Arps and Duong models' constants were shown in the Figure F-1 through Figure F-21, located in the Appendix.

4.4.1.1. Arps Decline Curve Analysis

Figure 4-4 shows that initial flow rate (q_i) ratio does not change significantly at higher ranges of fracture half length; while lower ranges of fracture half-length has more impact on the constant. This is probably due to the Arps model assumption that the boundary-dominated flow regime exists. At the higher ranges of fracture half-length, well reaches the boundary flow sooner; and the constant ratios at each production time interval are closer to 1. Whereas, for lower ranges of fracture half-length reservoir exhibits a long linear transient flow because of the domination of the fracture flow. This results in an overestimation of the Arps constants, and the constants ratios are further from 1 (Figure F-1 through Figure F-3). Thus, adjusting Arps constants need to be done at lower ranges of fracture half-length.

Figure 4-4: q_i -Ratio vs. Fracture Half-length – 7 Stages

Figure 4-5 shows that higher ranges of permeability slightly impact the initial flow rate (q_i) ratio; and the constant ratio at each production time interval is closer to 1. So, the Arps model is more precise for higher ranges of permeability; since the reservoir reaches the boundaries sooner. On the other hand, lower ranges of permeability impact the Arps model constants more significantly; and the constant ratios are further from 1 (Figure F-4 through Figure F-6). This is probably because of longer linear transient flow in horizontal shale wells with lower ranges of permeability which makes Arps model not as accurate. Therefore, adjusting Arps constants need to be done at lower range of permeability.

Figure 4-5: q_i -Ratio vs. Permeability - 7 Stages

Figure 4-6 shows that changing porosity doesn't seem to have a significant impact on initial flow rate (q_i) ratio. However, the higher ranges of porosity means higher gas volume which causes long linear transient flow in a low-permeability reservoir. This will result in an overestimation of the Arps model constants; and the constant ratios are further from 1 at each production time interval. The lower ranges of porosity, on the other hand, causes lower gas volume and consequently a low-permeability reservoir reaches the boundary dominated flow sooner. Therefore, Arps' constants are closer to 1 and not much adjustment is needed compare to higher ranges of porosity (Figure F-7 through Figure F-9).

Figure 4-6: q_i -Ratio vs. Porosity - 7 Stages

As shown in Figure F-1 through Figure F-9, the changes of “ d_i -Ratio” and “ q_i -Ratio” with fracture half-length and permeability present a higher slope compare to changes of “ n -Ratio”. So, “ d_i ” and “ q_i ” constants seem to get more effected with increasing the corresponding properties, over thirty years of production. However, according to Figure F-7 through Figure F-9, increasing porosity will have more or less same impact on the Arps model constants (n , d_i , q_i).

It is also clear that constants change with fracture half-length, permeability, and porosity at a higher slope during the early years and then the changes approach to a zero slope by thirty years of production.

4.4.1.2. Duong Decline Curve Analysis

Figure 4-7 shows that initial flow rate (q_i) ratio does not change significantly at lower ranges of fracture half length; while higher ranges of fracture half-length has more impact on the constant. This is probably due to the accuracy of Duong model at transient linear flow. At the lower ranges of fracture half-length owing to fracture dominated flow, Duong model constant ratios are closer to 1. On the contrary, at the higher ranges of fracture half-length well reaches the boundary flow sooner; and the constants ratios at each production time interval are further from 1 (Figure F-10 through Figure F-13). Therefore, adjusting Duong constants need to be done at higher ranges of fracture half-length.

Figure 4-7: q_1 -Ratio vs. Fracture Half Length- 7 Stages

Figure 4-8 shows that lower ranges of permeability slightly impact initial flow rate (q_i) ratio compared to higher ranges of permeability. This is probably because of Duong model accuracy for transient linear flow. At the lower ranges of permeability owing to fracture dominated flow, Duong model constant ratios are closer to 1. On the other hand, at the higher ranges of permeability well reaches the boundary dominated flow sooner; and the constant ratios at each production time interval are further from 1 (Figure F-14 through Figure F-17). Therefore, adjusting Duong constants need to be done at lower ranges of permeability.

Figure 4-8: q_1 -Ratio vs. Permeability - 7 Stages

Figure 4-9 shows initial flow rate (q_i) ratio get less affected at higher ranges of porosity which is probably due to accuracy of Duong model at longer linear transient flow. On the other hand, lower ranges of porosity have more impact on the constant. The lower ranges of porosity causes lower gas volume and consequently a low-permeability reservoir reaches the boundary dominated flow sooner. This will result in an overestimation of the Duong model constants, and the constants ratios are further from 1 at each production time interval. On the contrary, the higher ranges of porosity means higher gas volume which causes long linear transient flow in a low-permeability reservoir. So, the Duong constant ratios are closer to 1; and not much adjustment is needed (Figure F-18 through Figure F-21).

Figure 4-9: q_1 -Ratio vs. Porosity - 7 Stages

As shown in Figure F-10 through Figure F-21, the changes of “ a -Ratio” and “ m -Ratio” with fracture half length, permeability, and porosity present a lower slope compare to changes of “ q_1 -Ratio” and “ q_∞ -Ratio”. So, “ a ” and “ m ” constants seem to get less effected with increasing the corresponding properties, over thirty years of production.

It is also clear that constants change with fracture half-length, permeability, and porosity at a higher slope during the early years and then the changes approach to a zero slope by thirty years of production.

It should be noted that the reason q_∞ -Ratio values don’t seem reasonable is due to the fact that q_∞ doesn’t have any physical meaning. It is just added to the Duong Equation to provide a better fit to some field data that showed an intercept instead of a straight line to the origin when plotting (q vs. t (a , m)).

4.4.2. Thirteen Stages Model

The results for re-calculated Arps and Duong constants and their ratios are shown in Table 4-29 through Table 4-32, for the base case only. The ratio calculation for other ranges of X_f , K , and Φ can be seen in Table E-57 through Table E-112, located in the Appendix.

Table 4-29: 13 Stages (Base Model) - Arps- Recalculated Constants

13 Fractures (Base Model)- Arps						
Years	n	n(Eq.)	d _i	d _i (Eq.)	q _i	q _i (Eq.)
3	1.5027	1.4611	0.0281	0.0278	11.7560	11.7977
5	1.4045	1.4127	0.0265	0.0263	11.7097	11.6718
7	1.3575	1.3817	0.0252	0.0254	11.6079	11.5896
10	1.3221	1.3495	0.0240	0.0245	11.4800	11.5031
15	1.3046	1.3139	0.0233	0.0234	11.4094	11.4056
20	1.2946	1.2892	0.0229	0.0227	11.3525	11.3369
25	1.2800	1.2703	0.0221	0.0222	11.2519	11.2839
30	1.2765	1.2551	0.0220	0.0218	11.2320	11.2408

Table 4-30: 13 Stages (Base Model) - Arps- Table of Ratios

13Fractures (Base Model)- Arps- Table of Ratios			
Years	n- Ratio	d _i - Ratio	q _r - Ratio
3	1.164126	1.279381	1.049542
5	1.125532	1.211329	1.038344
7	1.100813	1.168494	1.031033
10	1.075202	1.12474	1.023339
15	1.04681	1.076986	1.014663
20	1.027122	1.04434	1.008551
25	1.012106	1.0197	1.003836
30	1	1	1

Table 4-31: 13 Stages (Base Model) - Duong- Recalculated Constants

13 Fractures (Base Model)- Duong								
Years	a	a(Eq.)	m	m(Eq.)	q ₁	q ₁ (Eq.)	q _∞	q _∞ (Eq.)
2	0.9457	0.9042	1.0494	1.0398	16.0602	15.6228	-2.5770	-2.3715
3	0.9859	0.9993	1.0506	1.0650	15.0210	14.7259	-2.5563	-1.9550
4	1.0626	1.0669	1.0809	1.0829	14.0400	14.0895	-1.5909	-1.6595
5	1.1180	1.1192	1.0992	1.0967	13.1801	13.5959	-1.0808	-1.4303
7	1.1985	1.1982	1.1236	1.1176	12.2501	12.8516	-0.6089	-1.0847
10	1.2621	1.2819	1.1430	1.1398	11.8010	12.0627	-0.3241	-0.7183
15	1.3396	1.3771	1.1621	1.1650	11.1505	11.1658	-0.1441	-0.3018
20	1.3988	1.4446	1.1808	1.1828	10.8500	10.5294	-0.0376	-0.0063
25	1.4654	1.4970	1.1851	1.1967	10.1590	10.0358	-0.0240	0.2229
30	1.6505	1.5398	1.2199	1.2080	9.8000	9.6326	0.0500	0.4102

Table 4-32: 13 Stages (Base Model) - Duong- Table of Ratios

13 Fractures (Base Model)- Duong- Table of Ratios				
Years	α-Ratio	m-Ratio	q_1-Ratio	q_∞- Ratio
2	0.587222	0.860788	1.621871	-5.781184
3	0.649026	0.881632	1.528761	-4.7658655
4	0.692876	0.896421	1.462698	-4.0454856
5	0.726889	0.907892	1.411456	-3.4867155
7	0.778176	0.925189	1.334189	-2.6441609
10	0.832543	0.943524	1.252283	-1.751017
15	0.894346	0.964368	1.159173	-0.7356985
20	0.938196	0.979156	1.09311	-0.0153185
25	0.972209	0.990627	1.041868	0.54345158
30	1	1	1	1

Section 4.4.2.1 and section 4.4.2.2 show the changes of Arps and Duong models' constants with fracture half-length, permeability and porosity. However, the plot of the initial flow rate (q_i) has been only presented in this chapter. The rest of the plots for other Arps and Duong models' constants were shown in the Figure F-22 through Figure F-42.

4.4.2.1. Arps Decline Curve Analysis

Figure 4-10 shows that initial flow rate " q_i " ratio does not change significantly at higher ranges of fracture half-length; while lower ranges of fracture half-length has more impact on the constant. This is probably due to Arps model assumption that boundary dominated flow exists. At the lower ranges of fracture half-length owing to fracture dominated flow, Arps model constant ratios are further from 1. On the other hand, at the higher ranges of fracture half-length well reaches the boundary dominated flow sooner; and the constants ratios at each production time interval are further from 1 (Figure F-22 through Figure F-24). Therefore, adjusting Arps constants need to be done at lower ranges of fracture half-length.

Figure 4-10: q_i -Ratio vs. Fracture Half-length - 13 Stages

Figure 4-11 shows that at higher ranges of permeability slightly impact the initial flow rate “ q_i – ratio” compare to lower ranges of permeability. This is probably due to Arps model assumption that boundary dominated flow exist. At the lower ranges of permeability owing to fracture dominated flow, Arps model constants ratios are further from 1. Oppositely, at the higher ranges of permeability, well reaches the boundary dominated flow sooner; and the constant ratios at each production time interval are closer to 1 (Figure F-25 through Figure F-27). Therefore, adjusting Arps constants need to be done at lower ranges of permeability.

Figure 4-11: q_i -Ratio vs. Permeability - 13 Stages

Figure 4-12 shows that initial flow rate “ q_i ” ratio does not change significantly at lower ranges of porosity; while higher ranges of porosity has more impact on the constants. Since at the lower ranges of porosity the reservoir will contain less gas volume, it reaches the boundaries sooner and therefore the Arps model predict the future production more accurately (constants ratios closer to 1). However, as higher gas volume in the reservoir causes the longer linear flow from stimulated parts, the Arps model is not as accurate; and the constants ratios further from 1 (Figure F-28 through Figure F-30). So, it can be concluded that Arps’ model constants need more adjustment when the matrix porosity is in higher ranges.

Figure 4-12: q_i -Ratio vs. Porosity - 13 Stages

As shown in Figure F-22 through Figure F-30, the changes of “ d_i -Ratio” and “ q_i -Ratio” with fracture half- length and permeability present a higher slope compare to changes of “ n -Ratio”. So, “ d_i ” and “ q_i ” constants seem to get more effected with increasing the corresponding properties, over thirty years of production. However, according to Figure F-28 through Figure F-30, increasing porosity will have more or less same impact on the Arps model constants (n , d_i , q_i).

It is also clear that constants change with fracture half-length, permeability, and porosity at a higher slope during the early years and then the changes approach to a zero slope by thirty years of production.

4.4.2.2. Duong Decline Curve Analysis

Figure 4-13 shows that initial flow rate “ q_i ” ratio does not change significantly at lower ranges of fracture half-length (ratios are closer to 1); while higher ranges of fracture half-length has more impact on the constant (ratios are further from 1). Since the higher the fracture half length, the sooner the reservoir reaches the boundaries; Duong model does not estimate the future reserves as accurate as when the

fracture half-length is shorter. This is probably because of the accuracy of Duong model at transient linear flow which is longer at lower ranges of fracture half length. Therefore, adjusting Duong constants need to be done at higher ranges of fracture half-length (Figure F-31 through Figure F-34).

Figure 4-13: q_1 -Ratio vs. Fracture Half Length - 13 Stages

Figure 4-14 shows that initial flow rate “ q_i ” constant ratio does not change significantly at lower ranges of permeability (ratios are closer to 1); while higher ranges of permeability has more impact on the constants (ratios are further from 1). As at the lower ranges of permeability, production would be mainly from the stimulated section; Duong model predict the future production more exact. This is probably due to the accuracy of the Duong model at transient linear flow which is longer at lower ranges of permeability. Therefore, adjusting Duong constants need to be done at higher ranges of permeability (Figure F-35 through Figure F-38).

Figure 4-14: q_1 -Ratio vs. Permeability - 13 Stages

Figure 4-15 shows that initial flow rate “ q_i ” ratio get less effected at higher ranges of porosity which is probably due to accuracy of Duong model at longer linear transient flow. On the other hand, lower ranges of porosity have more impact on the constant. The lower ranges of porosity causes lower gas volume and consequently a low-permeability reservoir reaches the boundary dominated flow sooner. This will result in an overestimation of the Duong model constants, and the constants ratios are further from 1 at each production time interval. On the contrary, the higher ranges of porosity means higher gas volume which causes long linear transient flow in a low-permeability reservoir. So, the Duong constant ratios are closer to 1; and not much adjustment is needed (Figure F-39 through Figure F-42).

Figure 4-15: q_1 -Ratio vs. Porosity - 13 Stages

As shown in Figure F-31 through Figure F-42, the changes of “ a -Ratio” and “ m -Ratio” with fracture half length, permeability, and porosity present a lower slope compare to changes of “ q_1 -Ratio” and “ q_∞ -Ratio”. So, “ a ” and “ m ” constants seem to get less effected with increasing the corresponding properties, over thirty years of production.

It is also clear that constants change with fracture half-length, permeability, and porosity at a higher slope during the early years and then the changes approach to a zero slope by thirty years of production.

It should be noted that the reason q_∞ -Ratio values don’t seem reasonable is due to the fact that q_∞ doesn’t have any physical meaning. It is just added to the Duong Equation to provide a better fit to some field data that showed an intercept instead of a straight line to the origin when plotting (q vs. t (a , m)).

4.5. Error Compared to the Base Case Model

The effect of underestimating or overestimating the fracture and formation properties on each Arps and Duong model constants studied by comparing the cases were shown in the following table, for both seven- and thirteen- stage models.

Table 4-33 : Error Comparison Cases

Lower Value Case	Base Case Model Properties Value	Higher Value Case
<i>Fracture Half-Length (ft.)</i>		
400	500	600
<i>Natural Fracture Permeability in x-direction (m-D)</i>		
0.001	0.002	0.004
<i>Matrix Porosity (%)</i>		
4	5	6

4.5.1. Seven Stages Model

4.5.1.1. Arps Decline Curve Analysis

According to Figure 4-16 through Figure 4-18, underestimating fracture half-length results in a larger error in calculating the Arps model constants, particularly for initial decline rate (d_i) values. However, after 15 or 20 years the effect of fracture half-length reduced and there is not much difference in errors compared to the base case model (less than 10% for d_i , and 5% for q_i and n).

Figure 4-16: n-Ratio Error with the Base Case Model- 7 Stages

Figure 4-17: d_i -Ratio Error with the Base Case Model- 7 Stages

Figure 4-18: q_i -Ratio Error with the Base Case Model- 7 Stages

According to Figure 4-19 through Figure 4-21, underestimating permeability results in a larger error in calculating the (q_i) and (d_i) constants, and adversely for (n). Although, after 15 or 20 years the effect of permeability will be diminished and there is not much difference in errors compared to the base case model (less than 1% for n , and 5% for q_i), d_i values deviate from the base case model with 20% error.

Figure 4-19: n-ratio Error with the Base Case Model – Permeability- 7 Stages

Figure 4-20: di-ratio Error with the Base Case Model – Permeability- 7 Stages

Figure 4-21: qi-ratio Error with the Base Case Model - Permeability

As it is shown in Figure 4-22 through Figure 4-24, estimating a lower porosity causes a larger error in calculating the Arps model constants; yet same effect on d_i . However, after 15 years the effect of porosity will be diminished and there is not much difference in errors compared to the base case model (less than 1% for n and q_i ; and 10% for d_i).

Figure 4-22: n- Ratio Error with the Base Case Model – 7 Stages

Figure 4-23: di- Ratio Error with the Base Case Model – 7 Stages

Figure 4-24: qi- Ratio Error with the Base Case Model – 7 Stages

4.5.1.2. Duong Decline Curve Analysis

As it is shown in Figure 4-25 through Figure 4-28, estimating a higher fracture half-length generates a larger error in calculating the Arps model constants; particularly for initial and infinite flow rate. However, after 15 or 20 years the effect of fracture half-length will be diminished and there is not much difference in errors compared to the base case model (less than 0.5% for a , 0.05% for m , 10% for q_i and q_∞).

Figure 4-25: a- Ratio Error with the Base Case Model - Fracture Half length – 7 Stages

Figure 4-26: m- Ratio Error with the Base Case Model - Fracture Half length- 7 Stages

Figure 4-27: q_1 - Ratio Error with the Base Case Model - Fracture Half length- 7 Stages

Figure 4-28: q_{∞} - Ratio Error with the Base Case Model - Fracture Half length – 7 Stages

According to Figure 4-29 through Figure 4-32, underestimating permeability causes a larger error in calculating the Duong model constants; particularly for initial and infinite flow rates. However, after 15 years the effect of permeability will be diminished and there is not much difference in errors compared to the base case model (less than 0.5% for a and m ; and 10% for q_1 and q_{∞}).

Figure 4-29: a- Ratio Error with the Base Case Model- Permeability – 7 Stages

Figure 4-30: m- Ratio Error with the Base Case Model- Permeability – 7 Stages

Figure 4-31: q_1 - Ratio Error with the Base Case Model- Permeability – 7 Stages

Figure 4-32: q_{∞} - Ratio Error with the Base Case Model- Permeability – 7 Stages

4.5.2. Thirteen Stages Model

4.5.2.1. Arps decline Curve Analysis

According to Figure 4-33 through Figure 4-35, underestimating the fracture half-length results in a larger error in calculating the Arps model constants, particularly for initial decline rate (d_i) values. However, as hydraulic fractures are spaced closely and the BDF period happens earlier with 13 stages, the errors are very minimal in compare to the seven stages scenario (Nelson, 2014).

Figure 4-33: n- Ratio Error with the Base Case Model - Fracture Half length – 13 Stages

Figure 4-34: d_i - Ratio Error with the Base Case Model - Fracture Half length- 13 Stages

Figure 4-35: q_i - Ratio Error with the Base Case Model- Fracture Half Length – 13 Stages

According to Figure 4-36 through Figure 4-38, underestimating the natural fracture permeability results in a larger error in calculating the Arps model constants, except for initial flow rate (q_i) values. However, after 15 years the effect of permeability will be diminished and there is not much difference in errors compared to the base case model (less than 1% for n and d_i ; and 0.2% for q_i).

Figure 4-36: n- Ratio Error with the Base Case Model – Permeability – 13 Stages

Figure 4-37: d_i- Ratio Error with the Base Case Model – Permeability – 13 Stages

Figure 4-38: q_i- Ratio Error with the Base Case Model – Permeability – 13 Stages

According to Figure 4-39 through Figure 4-41, underestimating the porosity results in a larger error in calculating the Arps model constants, except for initial flow rate (q_i) values. However, after 15 years the effect of porosity will be diminished and there is not much difference in errors compared to the base case model (less than 1% for n and d_i ; and 0.2% for q_i). Yet, the errors are very minimal compared to seven stages scenario, particularly for initial flow rate (q_i).

Figure 4-39: n- Ratio Error with the Base Case Model – Porosity – 13 Stages

Figure 4-40: di- Ratio Error with the Base Case Model – Porosity – 13 Stages

Figure 4-41: qi- Ratio Error with the Base Case Model – Porosity – 13 Stages

4.5.2.2. Duong Decline Curve Analysis

As it is shown in Figure 4-42 through Figure 4-45, overestimating the fracture half- length generates a significant error in calculating the Duong model constants; particularly for initial and infinite flow rates. However, underestimating the fracture half-length value does not have much effect on Duong model constants (less than 2% error in all the constants). Unlike Arps model, even after 15 or 20 years, a higher fracture half-length value will deflect from the base model with a large error.

Figure 4-42: a- Ratio Error with the Base Case Model - Fracture Half length – 13 Stages

Figure 4-43: m- Ratio Error with the Base Case Model - Fracture Half length – 13 Stages

Figure 4-44: q_1 - Ratio Error with the Base Case Model - Fracture Half length – 13 Stages

Figure 4-45: q_{∞} - Ratio Error with the Base Case Model - Fracture Half length – 13 Stages

As it is shown in Figure 4-46 through Figure 4-49, overestimating permeability causes a larger error in calculating the (a) and (q_1) constants and adversely affected the (m) and (q_{∞}) constants. However, after 20 years of production the effect of permeability reduces, and there is not much difference in errors compared to the base case model (less than 0.3% for a and m ; and 10% for q_i), except for infinite flow rate values.

Figure 4-46: a- Ratio Error with the Base Case Model - Permeability– 13 Stages

Figure 4-47: m- Ratio Error with the Base Case Model - Permeability– 13 Stages

Figure 4-48: q₁- Ratio Error with the Base Case Model - Permeability– 13 Stages

Figure 4-49: q_{∞} - Ratio Error with the Base Case Model - Permeability– 13 Stages

As it is shown in Figure 4-50 through Figure 4-53, overestimating porosity causes a larger error in calculating the (q_1) and (q_{∞}) constants and adversely affected the (a) and (m) constants. However, after 15 years of production the effect of porosity reduces, and there is not much difference in errors compared to the base case model (less than 0.5% for a , m , and q ; and 30% for q_{∞}). Unlike other properties, incorrect estimation of porosity does not have a significant error on Duong models.

Figure 4-50: a - Ratio Error with the Base Case Model – Porosity– 13 Stages

Figure 4-51: m- Ratio Error with the Base Case Model – Porosity– 13 Stages

Figure 4-52: q₁- Ratio Error with the Base Case Model – Porosity– 13 Stages

Figure 4-53: q_∞- Ratio Error with the Base Case Model – Porosity– 13 Stages

4.6. Confirmation with Field Data

Well 1 production data and the corresponding Fekete (Fast Evolution) history match for 3.5 years as well as the simulator prediction for 30 years is shown in Figure 4-54 and Figure 4-55, respectively.

Figure 4-54: Well 1 Production History Match at 3.5 years (By the Fekete Simulator)

Figure 4-55: Well 1 Production Prediction for 360 Months (By the Fekete Simulator)

Figure 4-56 and Figure 4-57 illustrate the Arps and Duong DCA models fitting to the same 3.5 years of monthly production rate for well 1.

Figure 4-56: Arps DCA Production History Match at 3.5 Years

Figure 4-57: Duong DCA Production History Match at 3.5 Years

Arps and Duong models fit the production history data with high R^2 values ($R^2 = 0.966$).

The final reservoir parameters after history matching are 350-ft fracture half-length, 0.002 m-D natural fracture permeability, and 4.5% matrix porosity. Therefore, the comparison has been done with 400-ft fracture half-length, 0.002 m-D natural fracture permeability, and 5% matrix porosity, for both seven and thirteen fracture stages.

The constants ratios obtained in section 4.4 for each DCA model plotted versus time; and the correlation was then derived for each DCA constants. By inserting the production history of Well-1 (3.5 years production time) into these correlations, a value of the ratio at this point in time was calculated. Actual Well 1 value of the constant obtained via the nonlinear-regression method (Microsoft Excel-Solver) at 3.5 years was then divided by the value of the ratio at 3.5 years. So, future reservoir performance (at 30 years) can be determined by applying predicted constant values to DCA models (Nelson, 2013). The results were shown in Table G-1 through Table G-8.

In order to confirm the obtained results via ratio method for well 1 with five fracture stages (453-ft spacing), the well 1 history match data generated from Fekete (Fast Evolution). Arps and Duong models were then compared with the simulated data to determine which DCA model matches most closely. Figure 4-58 compares these two DCA models prediction for well 1, with respect to stage spacing, against the simulator data for both seven (500-ft spacing) and thirteen (250-ft spacing) stage cases from 3.5 years until 30 years of production history.

Figure 4-58: Well 1 Production Prediction Comparison

As it can be observed, Arps DCA model prediction is close to the predicted data obtained from Fekete simulator, especially in the case of 13 stages. On the contrary, Duong appears to overestimate the rates, particularly at later production time. The decrease in spacing between stages results in earlier BDF period which makes Arps prediction more accurate in case of 13 stages (250-ft spacing).

As stated previously, the predicted DCA model constants' values were derived from 400-ft fracture half-length, 0.002 m-D natural fracture permeability, and 5% matrix porosity horizontal well in order to forecast the Well 1 production performance.

In order to confirm the reliability of the results regarding to the effect of underestimating or overestimating the fracture half-length, permeability, and porosity on the accuracy of the DCA models, the higher and lower values of the Well 1 history matched corresponding properties were considered (Table 4-34).

Table 4-34: Comparison Cases

<i>Lower Value Case</i>	<i>Well #1 History Matched Properties</i>	<i>Higher Value Case</i>
<i>Fracture Half-Length (ft.)</i>		
300	350	500
<i>Natural Fracture Permeability in x-direction (m-D)</i>		
0.001	0.002	0.004
<i>Matrix Porosity (%)</i>		
4	4.5	6

Firstly, the generated production data (at 3.5 years) by Fekete simulator, for each case, were matched to Well 1 production history at 3.5 years. As it is shown in Figure G-1 through Figure G-3, the simulated production data matches the Well 1 production history to some extent of accuracy. However, the case of 0.001-mD permeability is not well-matched. Therefore, 0.001-m-D has not been applied for future prediction in the next step.

Figure 4-59 through Figure 4-61 illustrate that Arps model predicts the future production more accurately in the case of overestimating the fracture half-length value (400-ft compared to 300-ft) after 10 years of production; and permeability value (0.004 m-D compared to 0.002 m-D) during the early years; as well as underestimating the porosity value (4% compared to 6%).

On the Other hand, Duong model seems to be more precise when the fracture half-length value is underestimated (300-ft compared to 400-ft) only during the very short time; and the porosity value is

overestimated (6% compared to 4%). It should be noted that as the lower value permeability (0.001-mD) history match was not achieved, it would be difficult to confirm the results in this case.

Figure 4-59: Well 1 Production Prediction Comparison at 30 years (Various Fracture Half length)

Figure 4-60: Well 1 Production Prediction Comparison at 30 years (Various Permeability)

Figure 4-61: Well 1 Production Prediction Comparison at 30 years (Various Porosity)

5. Conclusions

- The three DCA models (Arps, PLE, and Duong) can provide equally good fits to the single porosity simulated production history of a horizontal well with multiple hydraulic fractures in a Marcellus shale formation.
- Developing a DCA model constants correlation based on a single porosity simulated production history shows that Arps and Duong models appear to have a better trend for their constants over 30 years of production time than PLE model.
- Comparing Arps and Duong models at various fracture half length, permeability, and porosity indicates that for both cases of seven and thirteen stage:
 - ✓ Duong model is more accurate at lower ranges of fracture half length, permeability and higher ranges of porosity.
 - ✓ Arps model predicts with higher precession at higher ranges of fracture half length, permeability and lower ranges of porosity
- The impact of formation and fracture properties was found to be small on (*a*) and (*m*) constants in Duong model; while (*q₁*) and (*q_∞*) seem to get more effected particularly by fracture half-length and natural fracture permeability changes.
- The loss ratio (*d_i*) in Arps model get impacted significantly by the changes in formation and fracture properties compare to initial flow rate (*q_i*) and derivative of loss ratio (*n*).
- Generally, the effect of changes in fracture half-length and permeability are more substantial on DCA model constants compared to matrix porosity.
- Underestimating the fracture half-length, natural fracture permeability, and matrix porosity values results in a larger error in calculating the Arps models constants.
- Overestimating the fracture half-length, natural fracture permeability, and matrix porosity values results in a larger error in calculating the Duong models constants.
- The effect of formation and fracture properties on DCA models constants will be diminished after fifteen or twenty years of production time.
- Incorrect estimation of the fracture half length, permeability, and porosity will result in smaller error in DCA models constants in case of 13 stages rather than 7 stages.
- The applicability of the method of ratios to the limited production data in order to predict the future production has been confirmed in this study.
- The Arps hyperbolic decline curve provides a more reliable production prediction for a limited Well 1 Marcellus data with five fracture stages compare to Duong model.

6. Recommendations

- It is recommended to investigate the effect of other reservoir and formation properties such as drainage area, desorption, lateral length, and reservoir thickness on decline curve models constants as well as hydraulic fractures characteristics like fracture conductivity and width of the fracture.
- Future research should be conducted with other simulators to test the results, particularly for dual porosity reservoirs.
- Additional research is needed to be done to obtain a more accurate numerical optimization method.
- Further analysis can be performed if a longer production history of a Marcellus horizontal well would be available.

7. References

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Appendix

A. Application of Decline Curve Analysis to Simulated Data Using Excel-Solve Method

Towler (2002) introduced a non-linear regression technique using Excel- Solver for decline curve parameters determination. The technique is presented as follows.

A spreadsheet was set up containing cells for each parameter of every DCA model. For every model, a column with the appropriate production rate equation, q' , the errors squared, $(q-q')^2$, and total errors squared, $(q-q_{average})^2$, was then generated, where q corresponds to the flow rate from simulated production data. A cell for the sum of errors squared, S_{SE} , and total errors squared, S_{ST} , was also created. The regression coefficient was also generated in a cell by using the following formula:

$$R^2 = 1 - \frac{S_{SE}}{S_{ST}} \dots \dots \dots (1)$$

The Solver works by optimizing R^2 with non-linear equations to indicate the best curve fit to production history of a well; while varying only each DCA model parameters (Nelson, 2013).

B. Development of a Marcellus Shale Reservoir Model

Various models were generated for the two scenarios of seven and thirteen stages (for various permeability and porosity values) based on the ranges which were given in Table 3-1.

Figure B-1: 7 Stages Production Profile- Various Permeability

Figure B-2: 7 Stages Production Profile- Various Porosity

Figure B-3: 13 Stages Production Profile- Various Permeability

Figure B-4: 13 Stages Production Profile- Various Porosity

C. Application of Selected DCA Models to All Simulated Data

Matching Arps, PLE, and Duong DCA models to all simulated production history yielded the results presented below for thirteen-stage case (Base Model).

Figure C-1: Arps, PLE, and Duong DCA Models Simulated Data History Match - Thirteen Stages

D. DCA Models Constants Correlations

For both seven and thirteen stage cases with various fracture half-lengths, permeability, and porosity, the DCA models constants over two, five, seven, ten, eleven, twelve, fifteen, twenty, twenty-five, and thirty years of production time are shown in the following tables.

D.1. Seven Stages Model

Table D-1: 7 Stages (300 ft.) - Arps

7 Fractures (300 ft.)- Arps				
Years	n	d_i	q_i	R^2
3	2.1669	0.0436	5.0052	0.9928
5	1.8742	0.0247	4.3838	0.9951
7	1.6453	0.0154	3.9078	0.9958
10	1.4477	0.0100	3.4948	0.9904
15	1.2704	0.0065	3.0975	0.9744
20	1.1807	0.0051	2.8748	0.9594
25	1.1270	0.0043	2.7301	0.9474
30	1.0795	0.0038	2.6105	0.9363

Table D-2: 7 Stages (300 ft.) - PLE

7 Fractures (300 ft.)- PLE					
Years	D_∞	n	q_i	D_1	R^2
2	-0.00056	0.32722	6.88861	0.10258	0.99943
3	-0.00034	0.31392	6.90018	0.10266	0.99926
4	-0.00025	0.30712	6.90898	0.10296	0.99926
5	-0.00023	0.30590	6.90989	0.10284	0.99930
7	-0.00023	0.30557	6.89710	0.10272	0.99938
10	-0.00023	0.30510	6.90718	0.10292	0.99945
15	-0.00021	0.30401	6.92042	0.10303	0.99946
20	-0.00019	0.29906	6.90695	0.10343	0.99925
25	-0.00016	0.29361	6.90687	0.10408	0.99881
30	-0.00014	0.28892	6.90722	0.10465	0.99754

Table D-3: 7 Stages (300 ft.) - Duong

7 Fractures (300 ft.)- Duong					
Years	a	m	q_1	q_∞	R^2
2	0.8370	0.9883	11.4065	-4.5662	0.9997
3	0.8116	0.9874	10.4424	-3.4514	0.9997
4	0.7944	0.9878	9.7563	-2.7111	0.9994
5	0.7671	0.9880	9.2240	-1.9608	0.9988
7	0.7377	1.0007	8.4220	-0.7879	0.9974
10	0.7992	1.0331	7.5720	-0.2725	0.9964
15	0.9062	1.0656	6.6051	-0.0834	0.9962
20	0.9884	1.0846	5.9190	-0.0243	0.9968
25	1.0540	1.0972	5.3870	0.0011	0.9974
30	1.0774	1.1014	5.2052	0.0079	0.9977

Figure D-1: 7 Stages - PLE Constants Correlations (300ft.)

Figure D-2: 7 Stages - Duong Constants Correlations (300ft.)

Figure D-3: 7 Stages - Arps Constants Correlations (300ft.)

Table D-4: 7 Stages (400 ft.) - Arps

7 Fractures (400 ft.)- Arps				
Years	n	di	qi	R²
3	2.1317	0.0349	5.7257	0.9930
5	1.8148	0.0203	5.0730	0.9910
7	1.6213	0.0143	4.6758	0.9913
10	1.4605	0.0105	4.3258	0.9913
15	1.3394	0.0080	4.0284	0.9891
20	1.2832	0.0070	3.8694	0.9865
25	1.2501	0.0063	3.7674	0.9842
30	1.2278	0.0060	3.6982	0.9825

Table D-5: 7 Stages (400 ft.) - PLE

7 Fractures (400 ft.)- PLE					
Years	D_∞	n	q_i	D₁	R²
2	-0.00076	0.35784	7.58525	0.09139	0.99843
3	-0.00035	0.33202	7.59484	0.09181	0.99790
4	-0.00019	0.31973	7.59416	0.09210	0.99785
5	-0.00014	0.31482	7.61079	0.09241	0.99796
7	-0.00012	0.31342	7.59811	0.09232	0.99823
10	-0.00015	0.31760	7.59367	0.09189	0.99841
15	-0.00017	0.31946	7.58652	0.09175	0.99855
20	-0.00017	0.31946	7.58652	0.09175	0.99865
25	-0.00015	0.31653	7.58643	0.09213	0.99862
30	-0.00014	0.31314	7.58639	0.09259	0.99848

Table D-6: 7 Stages (400 ft.) - Duong

7 Fractures (400 ft.)- Duong					
Years	a	m	q₁	q_∞	R²
2	0.8075	0.9904	11.5016	-3.2483	0.9996
3	0.8082	0.9959	10.5101	-2.4452	0.9997
4	0.8104	1.0023	9.7481	-1.8054	0.9997
5	0.8165	1.0099	9.1861	-1.3209	0.9996
7	0.8498	1.0281	8.3380	-0.7414	0.9990
10	0.9226	1.0540	7.4454	-0.3634	0.9979
15	1.0356	1.0835	6.4180	-0.1491	0.9967
20	1.1255	1.1022	5.6940	-0.0714	0.9964
25	1.1985	1.1149	5.1320	-0.0364	0.9966
30	1.2611	1.1245	4.6720	-0.0185	0.9968

Figure D-4: 7 Stages - PLE Constants Correlations (400ft.)

Figure D-5: 7 Stages - Duong Constants Correlations (400ft.)

Figure D-6: 7 Stages - Arps Constants Correlations (400ft.)

Table D-77: 7 Stages (500ft.) - Arps

7 Fractures (500 ft.)- Arps				
Years	n	d_i	q_i	R^2
3	2.1055	0.0259	6.0292	0.9950
5	1.7534	0.0149	5.3604	0.9902
7	1.5965	0.0118	5.0889	0.9885
10	1.4858	0.0099	4.8918	0.9885
15	1.4195	0.0089	4.7677	0.9895
20	1.3931	0.0085	4.7153	0.9903
25	1.3769	0.0083	4.6821	0.9909
30	1.3647	0.0081	4.6569	0.9912

Table D-8 : 7 Stages (500ft.) - PLE

7 Fractures (500 ft.)- PLE					
Years	D_∞	n	q_i	D_1	R^2
2	-0.000994	0.397486	7.754779	0.077834	0.998041
3	-0.000460	0.362622	7.766450	0.078970	0.997015
4	-0.000242	0.343609	7.778605	0.079894	0.996721
5	-0.000141	0.333277	7.788613	0.080465	0.996772
7	-0.000077	0.325283	7.801243	0.081104	0.997153
10	-0.000081	0.326343	7.806431	0.080959	0.997574
15	-0.000100	0.329156	7.806475	0.080775	0.997838
20	-0.000116	0.333219	7.788616	0.080075	0.997987
25	-0.000118	0.333970	7.788459	0.079975	0.998105
30	-0.000114	0.332869	7.789941	0.080144	0.998170

Table D-9: 7 Stages (500ft.) - Duong

7 Fractures (500 ft.)- Duong					
Years	a	m	q_1	q_∞	R^2
2	0.808	0.989	12.151	-3.519	0.970
3	0.812	0.992	11.050	-2.853	0.973
4	0.850	1.011	9.802	-1.851	0.994
5	0.891	1.029	8.870	-1.109	0.999
7	0.940	1.044	8.021	-0.776	0.999
10	1.022	1.066	7.025	-0.461	0.998
15	1.117	1.089	6.175	-0.247	0.997
20	1.202	1.106	5.501	-0.151	0.996
25	1.261	1.122	5.074	-0.061	0.988
30	1.320	1.130	4.650	-0.035	0.991

Figure D-7: 7 Stages - PLE Constants Correlations (500ft.)

Figure D-8: 7 Stages - Duong Constants Correlations (500ft.)

Figure D-9: 7 Stages - Arps Constants Correlations (500ft.)

Table D-10: 7 Stages (600 ft.) - Arps

7 Fractures (600 ft.)- Arps				
Years	n	d_i	q_i	R^2
3	2.0700	0.0181	6.0055	0.9963
5	1.9707	0.0175	6.0129	0.9897
7	1.8060	0.0142	5.7744	0.9860
10	1.7032	0.0128	5.6735	0.9826
15	1.6397	0.0124	5.6647	0.9792
20	1.5905	0.0118	5.6180	0.9783
25	1.5702	0.0117	5.6271	0.9770
30	1.5601	0.0118	5.6488	0.9756

Table D-11: 7 Stages (600 ft.) - PLE

7 Fractures (600 ft.)- PLE					
Years	D_∞	n	q_i	D_1	R^2
2	-0.001210	0.441118	7.605094	0.064286	0.998296
3	-0.000560	0.393394	7.653150	0.066939	0.997020
4	-0.000309	0.370971	7.652832	0.067932	0.996492
5	-0.000175	0.356099	7.657943	0.068790	0.996384
7	-0.000074	0.342357	7.675244	0.069826	0.996650
10	-0.000044	0.340433	7.617607	0.069180	0.997148
15	-0.000048	0.341435	7.616927	0.069063	0.997627
20	-0.000055	0.341618	7.658626	0.069477	0.997854
25	-0.000067	0.344964	7.659044	0.069041	0.997970
30	-0.000066	0.344877	7.622074	0.068830	0.998069

Table D-12: 7 Stages (600 ft.) - Duong

7 Fractures (600 ft.)- Duong					
Years	a	m	q_1	q_∞	R^2
2	0.812	0.983	14.655	-5.219	0.973
3	0.831	1.000	12.084	-2.788	0.998
4	0.855	1.008	11.062	-2.294	0.999
5	0.881	1.018	10.065	-1.720	0.999
7	0.922	1.032	9.015	-1.219	0.999
10	0.980	1.049	8.022	-0.798	0.999
15	1.081	1.074	7.015	-0.575	0.978
20	1.221	1.111	6.060	-0.213	0.973
25	1.330	1.121	5.205	-0.211	0.991
30	1.410	1.130	4.709	-0.217	0.981

Figure D-10: 7 Stages - PLE Constants Correlations (600ft.)

Figure D-11: 7 Stages - Duong Constants Correlations (600ft.)

Figure D-12: 7 Stages - Arps Constants Correlations (600ft.)

Table D-13: 7 Stages (700 ft.)- Arps

7 Fractures (700 ft.)- Arps				
Years	n	d_i	q_i	R^2
3	1.9283	0.0118	5.7419	0.9930
5	1.8589	0.0112	5.7172	0.9919
7	1.8021	0.0106	5.6887	0.9877
10	1.7065	0.0100	5.6524	0.9821
15	1.6045	0.0093	5.5957	0.9790
20	1.5561	0.0091	5.5946	0.9730
25	1.5124	0.0087	5.5612	0.9712
30	1.4896	0.0086	5.5577	0.9696

Table D-14: 7 Stages (700 ft.)- PLE

7 Fractures (700 ft.)- PLE					
Years	D_∞	n	q_i	D_1	R^2
2	-0.001320	0.479142	7.308632	0.052714	0.998766
3	-0.000661	0.433825	7.247723	0.053863	0.997449
4	-0.000376	0.404813	7.266815	0.055553	0.996813
5	-0.000215	0.379885	7.380440	0.058351	0.996635
7	-0.000078	0.360111	7.398304	0.059706	0.996692
10	-0.000014	0.348555	7.411853	0.060621	0.997109
15	0.000009	0.343733	7.421391	0.061089	0.997659
20	0.000010	0.342378	7.411506	0.061315	0.997954
25	0.000003	0.345321	7.411598	0.060867	0.998108
30	-0.000006	0.348579	7.411705	0.060474	0.998183

Table D-15: 7 Stages (700 ft.)- Duong

7 Fractures (700 ft.)- Duong					
Years	a	m	q_1	q_∞	R^2
2	0.889	0.996	16.505	-7.556	0.997
3	0.839	0.995	14.451	-4.487	0.974
4	0.846	1.000	12.541	-3.305	0.977
5	0.876	1.010	11.207	-2.570	0.981
7	0.905	1.019	10.000	-1.939	0.999
10	0.944	1.032	9.054	-1.371	0.999
15	1.033	1.050	8.051	-1.099	0.974
20	1.151	1.100	7.051	-0.124	0.981
25	1.222	1.110	6.014	-0.123	0.975
30	1.346	1.129	4.890	-0.102	0.953

Figure D-13: 7 Stages - PLE Constants Correlations (700ft.)

Figure D-14: 7 Stages - Duong Constants Correlations (700ft.)

Figure D-15: 7 Stages - Arps Constants Correlations (700ft.)

Table D-16: 7 Stages (0.0008 md) - Arps

7 Fractures (0.0008 md)- Arps				
Years	n	d_i	q_i	R^2
3	2.377	0.076	5.240	0.999
5	2.299	0.061	4.945	0.999
7	2.195	0.046	4.605	0.996
10	2.056	0.033	4.228	0.991
15	1.895	0.023	3.854	0.986
20	1.796	0.018	3.643	0.983
25	1.733	0.016	3.511	0.982
30	1.693	0.014	3.430	0.981

Table D-17: 7 Stages (0.0008 md) - PLE

7 Fractures (0.0008 md)- PLE					
Years	D_{∞}	n	q_i	D_1	R^2
2	-0.0017599	0.4041707	5.8630480	0.0951762	0.9987945
3	-0.0010798	0.3673193	5.8842922	0.0970671	0.9972487
4	-0.0007545	0.3450723	5.8879911	0.0981314	0.9957509
5	-0.0005413	0.3269072	5.8970867	0.0992108	0.9944219
7	-0.0003101	0.3024638	5.9140685	0.1007829	0.9925840
10	-0.0001595	0.2823156	5.9306075	0.1020690	0.9913864
15	-0.0000733	0.2677478	5.9434058	0.1030337	0.9913360
20	-0.0000473	0.2622412	5.9431991	0.1034574	0.9918924
25	-0.0000386	0.2594209	5.9552019	0.1039534	0.9924197
30	-0.0000389	0.2603138	5.9476042	0.1036524	0.9927936

Table D-18: 7 Stages (0.0008 md) - Duong

7 Fractures (0.0008 md)- Duong					
Years	a	m	q_1	q_{∞}	R^2
2	0.814	1.052	6.500	0.110	0.9941
3	0.876	1.070	6.000	0.138	0.9990
4	0.965	1.088	5.300	0.154	0.9989
5	0.983	1.087	5.100	0.115	0.9982
7	1.034	1.090	4.605	0.057	0.9962
10	1.067	1.089	4.228	-0.013	0.9940
15	1.105	1.090	3.854	-0.064	0.9928
20	1.157	1.096	3.500	-0.064	0.9923
25	1.237	1.108	3.060	-0.048	0.9912
30	1.270	1.113	2.900	-0.042	0.9911

Figure D-16: 7 Stages - PLE Constants Correlations (0.0008 m-D)

Figure D-17: 7 Stages - Duong Constants Correlations (0.0008 m-D)

Figure D-18: 7 Stages - Arps Constants Correlations (0.0008 m-D)

Table D-19: 7 Stages (0.001 md) - Arps

7 Fractures (0.001 md)- Arps				
Years	n	di	qi	R²
3	2.2579	0.0607	5.4598	0.9907
5	2.1343	0.0431	5.0242	0.9915
7	1.9900	0.0302	4.6638	0.9904
10	1.9027	0.0231	4.3044	0.9877
15	1.7674	0.0166	3.9711	0.9842
20	1.6800	0.0138	3.7939	0.9829
25	1.6292	0.0124	3.6905	0.9828
30	1.5980	0.0116	3.6266	0.9829

Table D-20: 7 Stages (0.001 md) - PLE

7 Fractures (0.001 md)- PLE					
Years	D_∞	n	qi	D₁	R²
2	-0.0016583	0.4049781	6.3078249	0.0913779	0.9985639
3	-0.0009749	0.3674505	6.3117454	0.0929166	0.9968762
4	-0.0006447	0.3433544	6.3281281	0.0942091	0.9954338
5	-0.0004511	0.3262473	6.3384936	0.0951939	0.9943143
7	-0.0002410	0.3035609	6.3505370	0.0964695	0.9930946
10	-0.0001180	0.2870777	6.3502003	0.0973312	0.9927224
15	-0.0000583	0.2760628	6.3829838	0.0985132	0.9932919
20	-0.0000461	0.2737594	6.3855497	0.0986443	0.9939106
25	-0.0000461	0.2739570	6.3851936	0.0985882	0.9943316
30	-0.0000471	0.2739408	6.3900587	0.0987206	0.9946006

Table D-21: 7 Stages (0.001 md) - Duong

7 Fractures (0.001 md)- Duong					
Years	a	m	q1	q[∞]	R²
2	0.812	1.046	6.800	0.200	0.951
3	0.876	1.062	6.500	0.185	0.959
4	0.929	1.071	6.000	0.131	0.978
5	0.982	1.079	5.500	0.098	0.979
7	1.025	1.080	5.050	-0.021	0.994
10	1.136	1.096	4.160	-0.061	0.994
15	1.180	1.099	3.800	-0.089	0.993
20	1.230	1.107	3.700	-0.132	0.984
25	1.268	1.113	3.600	-0.154	0.957
30	1.299	1.117	3.550	-0.154	0.943

Figure D-19: 7 Stages - PLE Constants Correlations (0.001 m-D)

Figure D-20: 7 Stages - Duong Constants Correlations (0.001 m-D)

Figure D-21: 7 Stages - Arps Constants Correlations (0.001 m-D)

Table D-22: 7 Stages (0.002 md) - Arps

7 Fractures (0.002 md) - Arps				
Years	n	di	qi	R²
3	2.0494	0.0231	5.8789	0.9951
5	1.7531	0.0149	5.3601	0.9902
7	1.5962	0.0118	5.0870	0.9885
10	1.4851	0.0099	4.8906	0.9885
15	1.4190	0.0089	4.7662	0.9895
20	1.3929	0.0085	4.7148	0.9903
25	1.3769	0.0083	4.6826	0.9909
30	1.3646	0.0081	4.6567	0.9912

Table D-23: 7 Stages (0.002 md) - PLE

7 Fractures (0.002 md)- PLE					
Years	D_∞	n	qi	D₁	R²
2	-0.000995	0.397612	7.753027	0.077808	0.998041
3	-0.000459	0.362615	7.765285	0.078960	0.997015
4	-0.000242	0.343634	7.777479	0.079884	0.996721
5	-0.000141	0.333309	7.787229	0.080450	0.996773
7	-0.000077	0.325932	7.791395	0.080849	0.997153
10	-0.000078	0.326346	7.774932	0.080706	0.997577
15	-0.000109	0.332615	7.770000	0.079955	0.997830
20	-0.000113	0.335953	7.756657	0.079415	0.997657
25	-0.000120	0.335184	7.737065	0.079441	0.998099
30	-0.000111	0.331992	7.789662	0.080253	0.998176

Table D-24: 7 Stages (0.002 md) - Duong

7 Fractures (0.002 md)- Duong					
Years	a	m	q1	q_∞	R²
2	0.894	1.040	9.300	-0.877	0.960
3	0.952	1.051	8.000	-0.791	0.941
4	1.043	1.070	6.750	-0.584	0.960
5	1.090	1.078	6.250	-0.421	0.998
7	1.213	1.100	5.150	-0.294	0.997
10	1.261	1.109	4.800	-0.233	0.997
15	1.390	1.130	4.000	-0.012	0.957
20	1.432	1.137	3.800	0.026	0.951
25	1.456	1.142	3.700	0.010	0.973
30	1.465	1.143	3.670	0.008	0.978

Figure D-22: 7 Stages - PLE Constants Correlations (0.002 m-D)

Figure D-23: 7 Stages - Duong Constants Correlations (0.002 m-D)

Figure D-24: 7 Stages - Arps Constants Correlations (0.002 m-D)

Table D-25: 7 Stages (0.004 md) - Arps

7 Fractures (0.004 md)- Arps				
Years	n	di	qi	R²
3	1.4226	0.0097	6.6927	0.9939
5	1.2860	0.0083	6.4906	0.9947
7	1.2603	0.0080	6.4478	0.9955
10	1.2553	0.0080	6.4386	0.9962
15	1.2482	0.0079	6.4238	0.9968
20	1.2355	0.0077	6.3942	0.9970
25	1.2203	0.0076	6.3577	0.9970
30	1.2071	0.0074	6.3261	0.9969

Table D-26: 7 Stages (0.004 md) - PLE

7 Fractures (0.004 md)- PLE					
Years	D_∞	n	qi	D₁	R²
2	-0.00015	0.39455	9.34784	0.06228	0.99911
3	-0.00004	0.38593	9.34969	0.06255	0.99929
4	-0.00008	0.39036	9.34302	0.06216	0.99938
5	-0.00014	0.39683	9.35595	0.06189	0.99936
7	-0.00027	0.41293	9.30001	0.06029	0.99923
10	-0.00033	0.42337	9.25717	0.05888	0.99922
15	-0.00030	0.41868	9.25710	0.05948	0.99929
20	-0.00026	0.41149	9.25690	0.06032	0.99913
25	-0.00023	0.40444	9.25688	0.06115	0.99883
30	-0.00020	0.39852	9.25688	0.06190	0.99851

Table D-27: 7 Stages (0.004 md) - Duong

7 Fractures (0.004)- Duong					
Years	a	m	q₁	q_∞	R²
2	1.039	1.053	9.900	-2.019	0.989
3	1.090	1.066	8.900	-1.543	0.991
4	1.146	1.081	8.000	-1.100	0.988
5	1.212	1.095	7.300	-0.794	0.999
7	1.252	1.106	6.950	-0.527	0.998
10	1.381	1.131	5.850	-0.268	0.997
15	1.462	1.145	5.300	-0.148	0.997
20	1.647	1.168	4.030	-0.013	0.989
25	1.674	1.171	3.880	-0.012	0.991
30	1.700	1.173	3.820	-0.053	0.991

Figure D-25: 7 Stages - PLE Constants Correlations (0.004 m-D)

Figure D-26: 7 Stages - Duong Constants Correlations (0.004 m-D)

Figure D-27: 7 Stages - Arps Constants Correlations (0.004 m-D)

Table D-28: 7 Stages (0.006 md) - Arps

7 Fractures (0.006 md)- Arps				
Years	n	d_i	q_i	R²
3	1.2020	0.0081	7.7187	0.9972
5	1.1908	0.0081	7.7039	0.9980
7	1.2009	0.0082	7.7231	0.9983
10	1.2038	0.0082	7.7289	0.9985
15	1.1885	0.0080	7.6917	0.9987
20	1.1671	0.0078	7.6365	0.9985
25	1.1471	0.0075	7.5823	0.9982
30	1.1307	0.0073	7.5361	0.9978

Table D-29: 7 Stages (0.006 md) - PLE

7 Fractures (0.006 md)- PLE					
Years	D_∞	n	q_i	D₁	R²
2	0.00009	0.42243	10.35337	0.05241	0.99981
3	-0.00017	0.44563	10.31848	0.05117	0.99968
4	-0.00038	0.46432	10.29341	0.05007	0.99948
5	-0.00056	0.48445	10.23376	0.04823	0.99935
7	-0.00063	0.49091	10.25994	0.04792	0.99937
10	-0.00058	0.48603	10.24710	0.04821	0.99940
15	-0.00045	0.46876	10.24625	0.04987	0.99901
20	-0.00036	0.45438	10.24649	0.05126	0.99842
25	-0.00027	0.43311	10.48946	0.05502	0.99800
30	-0.00023	0.42474	10.48919	0.05611	0.99760

Table D-30: 7 Stages (0.006 md) - Duong

7 Fractures (0.006 md)- Duong					
Years	a	m	q₁	q[∞]	R²
2	1.0943	1.0565	11.5000	-3.2125	0.9997
3	1.1183	1.0684	10.5000	-2.1135	0.9987
4	1.1323	1.0772	10.1000	-1.4970	0.9973
5	1.1644	1.0885	9.6000	-1.0468	0.9962
7	1.2213	1.1051	8.9500	-0.6195	0.9950
10	1.4099	1.1382	6.9000	-0.2861	0.9960
15	1.5558	1.1592	5.6400	-0.1560	0.9970
20	1.6885	1.1748	4.6400	-0.1050	0.9977
25	1.7021	1.1764	4.5500	-0.0993	0.9979
30	1.7098	1.1773	4.5000	-0.0964	0.9980

Figure D-28: 7 Stages - PLE Constants Correlations (0.006 m-D)

Figure D-29: 7 Stages - Duong Constants Correlations (0.006 m-D)

Figure D-30: 7 Stages - Arps Constants Correlations (0.006 m-D)

Table D-31: 7 Stages (4%) - Arps

7 Fractures (4%)- Arps				
Years	n	d_i	q_i	R^2
3	1.8915	0.0198	5.3124	0.9934
5	1.6118	0.0133	4.8797	0.9899
7	1.4882	0.0111	4.6859	0.9895
10	1.4145	0.0099	4.5653	0.9903
15	1.3751	0.0093	4.4970	0.9914
20	1.3585	0.0090	4.4656	0.9921
25	1.3452	0.0088	4.4399	0.9926
30	1.3341	0.0086	4.4160	0.9928

Table D-32: 7 Stages (4%) - PLE

7 Fractures (4%)- PLE					
Years	D_{∞}	n	q_i	D_1	R^2
2	-0.000785	0.382315	7.387753	0.080651	0.998044
3	-0.000300	0.349959	7.393483	0.081541	0.997443
4	-0.000155	0.337680	7.399518	0.082001	0.997442
5	-0.000097	0.332164	7.403981	0.082281	0.997630
7	-0.000076	0.329848	7.417353	0.082481	0.997985
10	-0.000108	0.334174	7.407473	0.082157	0.998199
15	-0.000145	0.340823	7.399531	0.081305	0.998320
20	-0.000154	0.343527	7.372863	0.080675	0.998438
25	-0.000147	0.342295	7.374503	0.080814	0.998502
30	-0.000132	0.338189	7.372705	0.081450	0.998507

Table D-33: 7 Stages (4%) - Duong

7 Fractures (4%)- Duong					
Years	a	m	q_1	q^{∞}	R^2
2	0.885	1.032	8.300	-0.863	0.999
3	0.955	1.048	7.400	-0.735	0.999
4	1.033	1.066	6.500	-0.571	0.999
5	1.116	1.083	5.700	-0.431	0.998
7	1.201	1.099	5.000	-0.312	0.998
10	1.324	1.121	4.200	-0.179	0.997
15	1.439	1.141	3.600	-0.056	0.996
20	1.489	1.149	3.380	-0.056	0.996
25	1.510	1.152	3.300	-0.014	0.992
30	1.532	1.155	3.200	-0.034	0.996

Figure D-31: 7 Stages - PLE Constants Correlations (4%)

Figure D-32: 7 Stages - Duong Constants Correlations (4%)

Figure D-33: 7 Stages - Arps Constants Correlations (4%)

Table D-34: 7 Stages (5%) - Arps

7 Fractures (5%)- Arps				
Years	n	d_i	q_i	R^2
3	2.0494	0.0231	5.8798	0.9951
5	1.7531	0.0149	5.3586	0.9902
7	1.5968	0.0118	5.0887	0.9885
10	1.4859	0.0099	4.8925	0.9885
15	1.4192	0.0089	4.7670	0.9895
20	1.3931	0.0085	4.7149	0.9903
25	1.3767	0.0083	4.6806	0.9909
30	1.3644	0.0081	4.6547	0.9912

Table D-35: 7 Stages (5%) - PLE

7 Fractures (5%)- PLE					
Years	D_{∞}	n	q_i	D_1	R^2
2	-0.0010263	0.3996857	7.7542692	0.0777371	0.9980356
3	-0.0004567	0.3622116	7.7651826	0.0790257	0.9970165
4	-0.0002323	0.3418896	7.7922867	0.0802047	0.9967238
5	-0.0001454	0.3335663	7.7906324	0.0805192	0.9967706
7	-0.0000751	0.3253991	7.7886584	0.0809130	0.9971546
10	-0.0000803	0.3266889	7.7902631	0.0807726	0.9975767
15	-0.0001043	0.3306330	7.7904639	0.0804448	0.9978375
20	-0.0001133	0.3321755	7.7904542	0.0802676	0.9979881
25	-0.0001145	0.3329079	7.7911305	0.0800973	0.9981090
30	-0.0001145	0.3330758	7.7898979	0.0801135	0.9981683

Table D-36: 7 Stages (5%) - Duong

7 Fractures (5%)- Duong					
Years	a	m	q_1	q^{∞}	R^2
2	0.8409	1.0205	9.5000	-0.9938	0.9992
3	0.9044	1.0369	8.5000	-0.7985	0.9990
4	0.9563	1.0485	7.7923	-0.7067	0.9987
5	1.0199	1.0627	7.0000	-0.5678	0.9984
7	1.0946	1.0782	6.2000	-0.4359	0.9981
10	1.1860	1.0966	5.4000	-0.2880	0.9976
15	1.2742	1.1139	4.8000	-0.1671	0.9968
20	1.3695	1.1295	4.2000	-0.0986	0.9961
25	1.4261	1.1380	3.8800	-0.0681	0.9958
30	1.4648	1.1434	3.6700	-0.0225	0.9913

Figure D-34: 7 Stages - PLE Constants Correlations (5%)

Figure D-35: 7 Stages - Duong Constants Correlations (5%)

Figure D-36: 7 Stages - Arps Constants Correlations (5%)

Table D-37: 7 Stages (6%) - Arps

7 Fractures (6%)- Arps				
Years	n	di	qi	R²
3	2.16593	0.02604	6.37471	0.9967
5	1.87686	0.01673	5.81062	0.9913
7	1.70174	0.01278	5.48117	0.9885
10	1.56138	0.01024	5.21353	0.9874
15	1.46689	0.00876	5.02701	0.9880
20	1.42918	0.00820	4.94638	0.9888
25	1.40816	0.00791	4.90262	0.9894
30	1.39377	0.00770	4.87033	0.9898

Table D-38: 7 Stages (6%) - PLE

7 Fractures (6%)- PLE					
Years	D_∞	n	q_i	D₁	R²
2	-0.00118	0.41305	8.05189	0.07497	0.99816
3	-0.00059	0.37403	8.07836	0.07673	0.99685
4	-0.00033	0.35199	8.09137	0.07780	0.99625
5	-0.00020	0.33805	8.10059	0.07860	0.99610
7	-0.00009	0.32480	8.12288	0.07951	0.99635
10	-0.00007	0.32136	8.10653	0.07958	0.99688
15	-0.00008	0.32361	8.10605	0.07934	0.99733
20	-0.00008	0.32361	8.12000	0.07934	0.99748
25	-0.00010	0.32668	8.12288	0.07920	0.99768
30	-0.00009	0.32670	8.12286	0.07918	0.99778

Table D-39: 7 Stages (6%) - Duong

7 Fractures (6%)- Duong					
Years	a	m	q1	q[∞]	R²
2	0.815	1.013	10.500	-1.195	0.996
3	0.866	1.028	9.500	-0.945	0.987
4	0.924	1.042	8.600	-0.711	0.999
5	0.952	1.047	8.200	-0.702	0.999
7	1.021	1.062	7.300	-0.554	0.998
10	1.093	1.078	6.500	-0.400	0.998
15	1.173	1.095	5.800	-0.247	0.997
20	1.241	1.108	5.300	-0.106	0.986
25	1.356	1.126	4.450	-0.060	0.990
30	1.371	1.129	4.380	-0.045	0.990

Figure D-37: 7 Stages - PLE Constants Correlations (6%)

Figure D-38: 7 Stages - Duong Constants Correlations (6%)

Figure D-39: 7 Stages - Arps Constants Correlations (6%)

Table D-40: 7 Stages (8%) - Arps

7 Fractures (8%) - Arps				
Years	n	d_i	q_i	R^2
3	2.3116	0.0298	7.1613	0.9986
5	2.0681	0.0201	6.6052	0.9941
7	1.8845	0.0150	6.2081	0.9901
10	1.7091	0.0113	5.8304	0.9870
15	1.5665	0.0089	5.5192	0.9860
20	1.5029	0.0079	5.3734	0.9864
25	1.4696	0.0075	5.2974	0.9870
30	1.4494	0.0072	5.2475	0.9874

Table D-41: 7 Stages (8%) - PLE

7 Fractures (8%)- PLE					
Years	D_{∞}	n	q_i	D_1	R^2
2	-0.0014276	0.4373642	8.5418185	0.0700473	0.9985068
3	-0.0007734	0.3934334	8.5773798	0.0724611	0.9969567
4	-0.0004615	0.3660959	8.5921826	0.0739963	0.9959223
5	-0.0002932	0.3469751	8.6233565	0.0755427	0.9953371
7	-0.0001445	0.3287410	8.6250558	0.0765437	0.9950712
10	-0.0000757	0.3184932	8.6287014	0.0772133	0.9954898
15	-0.0000565	0.3150625	8.6311078	0.0774690	0.9962281
20	-0.0000689	0.3183689	8.6320722	0.0770969	0.9965933
25	-0.0000694	0.3184869	8.6287030	0.0770455	0.9968308
30	-0.0000700	0.3182458	8.6299384	0.0771257	0.9969869

Table D-42: 7 Stages (8%) - Duong

7 Fractures (8%)- Duong					
Years	a	m	q_1	q^{∞}	R^2
2	0.7961	0.9998	12.5002	-1.9834	0.9979
3	0.7984	1.0034	12.1001	-1.6560	0.9985
4	0.8141	1.0102	11.5440	-1.4846	0.9769
5	0.8487	1.0212	10.6869	-1.0469	0.9971
7	0.9037	1.0352	9.7614	-0.8557	0.9969
10	0.9037	1.0352	9.6140	-0.7681	0.9839
15	0.9666	1.0535	8.6569	-0.4529	0.9847
20	1.1037	1.0819	6.9500	-0.2087	0.9807
25	1.1899	1.0976	6.1000	-0.0912	0.9657
30	1.2034	1.1010	6.0300	-0.0911	0.9797

Figure D-40: 7 Stages - PLE Constants Correlations (8%)

Figure D-41: 7 Stages - Duong Constants Correlations (8%)

Figure D-42: 7 Stages - Arps Constants Correlations (8%)

D.2. Thirteen Stages Model

Table D-43: 13 Stages (300 ft.)- Arps

13 Fractures (300 ft.)- Arps				
Years	n	d_i	q_i	R^2
3	1.422	0.032	8.748	0.974
5	1.325	0.027	8.443	0.982
7	1.278	0.025	8.206	0.987
10	1.226	0.022	7.899	0.988
15	1.209	0.020	7.717	0.986
20	1.194	0.019	7.571	0.983
25	1.180	0.018	7.452	0.980
30	1.178	0.018	7.406	0.978

Table D-44: 13 Stages (300 ft.)- PLE

13 Fractures (300 ft.)- PLE					
Years	D_∞	n	q_i	D_1	R^2
2	0.00108	0.31242	12.40479	0.09891	0.99956
3	0.00039	0.34859	12.38347	0.09934	0.99849
4	-0.00018	0.38027	12.34272	0.09822	0.99724
5	-0.00018	0.38027	12.34272	0.09822	0.99559
7	-0.00094	0.42481	12.26754	0.09468	0.99595
10	-0.00096	0.42618	12.27122	0.09445	0.99610
15	-0.00077	0.40981	12.31815	0.09686	0.99533
20	-0.00059	0.38989	12.41823	0.10050	0.99344
25	-0.00049	0.37774	12.46526	0.10256	0.99110
30	-0.00040	0.36136	12.65075	0.10718	0.98874

Table D-45: 13 Stages (300 ft.)- Duong

13 Fractures (300 ft.)- Duong					
Years	a	m	q_1	q_∞	R^2
2	0.908	1.024	14.505	-3.934	0.997
3	0.855	1.032	14.205	-1.972	0.991
4	0.881	1.057	13.650	-0.977	0.985
5	0.935	1.082	13.010	-0.558	0.982
7	1.008	1.109	12.400	-0.245	0.978
10	1.065	1.130	12.040	-0.082	0.974
15	1.122	1.146	11.608	0.040	0.973
20	1.162	1.155	11.208	0.040	0.974
25	1.215	1.164	10.500	0.055	0.976
30	1.249	1.170	10.040	0.060	0.977

Figure D-43: 13 Stages - PLE Constants Correlation (300ft.)

Figure D-44: 13 Stages - Duong Constants Correlation (300ft.)

Figure D-45: 13 Stages - Arps Constants Correlation (300ft.)

Table D-46: 13 Stages (400 ft.)- Arps

13 Fractures (400 ft.)- Arps				
Years	n	d_i	q_i	R^2
3	1.485	0.033	10.781	0.968
5	1.405	0.031	10.714	0.970
7	1.345	0.028	10.444	0.977
10	1.305	0.026	10.217	0.981
15	1.287	0.024	10.070	0.984
20	1.271	0.023	9.928	0.985
25	1.250	0.022	9.764	0.986
30	1.240	0.021	9.680	0.986

Table D-47: 13 Stages (400 ft.)- PLE

13 Fractures (400 ft.)- PLE					
Years	D_{∞}	n	q_i	D_1	R^2
2	0.00080	0.34496	13.67353	0.08701	0.99958
3	0.00049	0.36265	13.66283	0.08684	0.99937
4	0.00013	0.38490	13.63423	0.08602	0.99873
5	-0.00021	0.40703	13.60437	0.08463	0.99795
7	-0.00070	0.43970	13.52472	0.08163	0.99692
10	-0.00093	0.45725	13.45377	0.07947	0.99672
15	-0.00084	0.44949	13.48119	0.08054	0.99667
20	-0.00069	0.43334	13.58492	0.08325	0.99571
25	-0.00057	0.42064	13.58613	0.08509	0.99420
30	-0.00049	0.41009	13.58668	0.08682	0.99261

Table D-48: 13 Stages (400 ft.)- Duong

13 Fractures (400 ft.)- Duong					
Years	a	m	q_1	q_{∞}	R^2
2	0.973	1.043	15.050	-3.557	0.999
3	0.965	1.055	14.501	-2.142	0.996
4	0.998	1.075	13.850	-1.292	0.993
5	1.058	1.097	13.020	-0.815	0.990
7	1.137	1.123	12.254	-0.413	0.985
10	1.216	1.146	11.600	-0.183	0.981
15	1.276	1.163	11.202	-0.054	0.978
20	1.316	1.172	10.860	-0.006	0.978
25	1.347	1.178	10.530	0.015	0.978
30	1.572	1.218	10.232	0.071	0.976

Figure D-46: 13 Stages - PLE Constants Correlation (400ft.)

Figure D-47: 13 Stages - Duong Constants Correlation (400ft.)

Figure D-48: 13 Stages - Arps Constants Correlation (400ft.)

Table D-49: 13 Stages (500 ft.)- Arps

13 Fractures (500 ft.)- Arps				
Years	n	d_i	q_i	R^2
3	1.5027	0.0281	11.76	0.971
5	1.4045	0.0265	11.71	0.971
7	1.3575	0.0252	11.61	0.974
10	1.3221	0.0240	11.48	0.978
15	1.3046	0.0233	11.41	0.980
20	1.2946	0.0229	11.35	0.982
25	1.2800	0.0221	11.25	0.984
30	1.2765	0.0220	11.23	0.984

Table D-50: 13 Stages (500 ft.)- PLE

13 Fractures (500 ft.)- PLE					
Years	D_∞	n	q_i	D_1	R^2
2	0.00043	0.38974	13.96094	0.07301	0.99967
3	0.00031	0.39698	13.96372	0.07287	0.99968
4	0.00007	0.41378	13.93370	0.07191	0.99946
5	-0.00014	0.42864	13.91210	0.07096	0.99909
7	-0.00052	0.45740	13.83536	0.06846	0.99837
10	-0.00083	0.48304	13.74054	0.06558	0.99797
15	-0.00084	0.48344	13.73831	0.06556	0.99803
20	-0.00073	0.47279	13.73751	0.06678	0.99765
25	-0.00063	0.46149	13.73848	0.06819	0.99680
30	-0.00055	0.45262	13.74003	0.06942	0.99581

Table D-51: 13 Stages (500 ft.)- Duong

13 Fractures (500 ft.)- Duong					
Years	a	m	q_1	q_∞	R^2
2	0.946	1.049	16.1	-2.577	0.954
3	0.986	1.051	15.0	-2.556	0.982
4	1.063	1.081	14.0	-1.591	0.996
5	1.118	1.099	13.2	-1.081	0.994
7	1.199	1.124	12.3	-0.609	0.990
10	1.262	1.143	11.8	-0.324	0.986
15	1.340	1.162	11.2	-0.144	0.983
20	1.399	1.181	10.9	-0.038	0.977
25	1.465	1.185	10.2	-0.024	0.980
30	1.650	1.220	9.8	0.050	0.963

Figure D-49: 13 Stages - PLE Constants Correlation (500ft.)

Figure D-50: 13 Stages - Duong Constants Correlation (500ft.)

Figure D-51: 13 Stages - Arps Constants Correlation (500ft.)

Table D-52: 13 Stages (600 ft.)- Arps

13 Fractures (600 ft.)- Arps				
Years	n	d_i	q_i	R^2
3	1.4450	0.0224	12.0700	0.9779
5	1.4031	0.0214	12.0418	0.9739
7	1.3678	0.0210	12.0225	0.9737
10	1.3229	0.0200	11.9181	0.9767
15	1.3027	0.0198	11.9095	0.9785
20	1.2795	0.0190	11.8087	0.9810
25	1.2648	0.0186	11.7405	0.9825
30	1.2538	0.0182	11.6853	0.9836

Table D-53: 13 Stages (600 ft.)- PLE

13 Fractures (600 ft.)- PLE					
Years	D_{∞}	n	q_i	D_1	R^2
2	0.00022	0.42758	13.69827	0.05990	0.99983
3	0.00007	0.43851	13.67661	0.05928	0.99983
4	-0.00009	0.45032	13.65668	0.05860	0.99973
5	-0.00026	0.46464	13.61662	0.05747	0.99957
7	-0.00052	0.48787	13.54504	0.05542	0.99924
10	-0.00073	0.50604	13.46277	0.05359	0.99902
15	-0.00075	0.50428	13.60774	0.05453	0.99903
20	-0.00068	0.49576	13.67713	0.05575	0.99893
25	-0.00061	0.48764	13.67656	0.05662	0.99862
30	-0.00055	0.48126	13.67509	0.05725	0.99817

Table D-54: 13 Stages (600 ft.)- Duong

13 Fractures (600 ft.)- Duong					
Years	a	m	q_1	q_{∞}	R^2
2	1.021	1.041	16.507	-5.493	0.963
3	1.034	1.051	15.500	-4.020	0.985
4	1.085	1.078	14.540	-1.983	0.996
5	1.138	1.094	13.540	-1.390	0.995
7	1.213	1.116	12.541	-0.837	0.992
10	1.303	1.139	11.640	-0.479	0.988
15	1.395	1.159	10.750	-0.253	0.986
20	1.505	1.181	9.970	-0.121	0.984
25	1.565	1.200	9.520	-0.020	0.976
30	1.606	1.199	9.020	-0.032	0.984

Figure D-52: 13 Stages - PLE Constants Correlation (600ft.)

Figure D-53: 13 Stages - Duong Constants Correlation (600ft.)

Figure D-54: 13 Stages - Arps Constants Correlation (600ft.)

Table D-55: 13 Stages (700 ft.)- Arps

13 Fractures (700 ft.)- Arps				
Years	n	d_i	q_i	R^2
3	1.46852	0.01906	12.22260	0.97786
5	1.44685	0.01902	12.18208	0.97294
7	1.42000	0.01770	12.06787	0.97043
10	1.38458	0.01694	12.00566	0.96882
15	1.36408	0.01649	11.97663	0.96604
20	1.32989	0.01631	11.97121	0.96935
25	1.30288	0.01621	11.95697	0.97233
30	1.29766	0.01606	11.93092	0.97197

Table D-56: 13 Stages (700 ft.)- PLE

13 Fractures (700 ft.)- PLE					
Years	D_{∞}	n	q_i	D_1	R^2
2	0.00009	0.46229	13.11657	0.04833	0.99991
3	-0.00005	0.47230	13.10195	0.04792	0.99988
4	-0.00021	0.48640	13.06952	0.04703	0.99980
5	-0.00035	0.49873	13.06428	0.04634	0.99972
7	-0.00049	0.51175	13.02549	0.04534	0.99962
10	-0.00063	0.52632	12.98356	0.04403	0.99958
15	-0.00067	0.53208	12.92281	0.04323	0.99961
20	-0.00063	0.52686	12.92406	0.04374	0.99960
25	-0.00058	0.52271	12.88195	0.04396	0.99949
30	-0.00053	0.51545	12.90238	0.04482	0.99935

Table D-57: 13 Stages (700 ft.)- Duong

13 Fractures (700 ft.)- Duong					
Years	a	m	q_1	q_{∞}	R^2
2	1.023	1.036	18.500	-7.214	0.947
3	1.020	1.045	17.420	-4.633	0.997
4	1.071	1.065	15.142	-2.765	0.996
5	1.118	1.081	14.014	-1.890	0.995
7	1.180	1.101	13.015	-1.169	0.993
10	1.252	1.121	12.125	-0.725	0.991
15	1.353	1.139	11.405	-0.474	0.972
20	1.495	1.185	10.458	-0.063	0.966
25	1.606	1.196	9.700	-0.091	0.979
30	1.653	1.197	8.790	-0.022	0.977

Figure D-55: 13 Stages - PLE Constants Correlation (700ft.)

Figure D-56: 13 Stages - Duong Constants Correlation (700ft.)

Figure D-57: 13 Stages - Arps Constants Correlation (700ft.)

Table D-58: 13 Stages (0.0008 md) - Arps

13 Fractures (0.0008 md)- Arps				
Years	n	d_i	q_i	R^2
3	1.7081	0.0279	7.6043	0.9914
5	1.6072	0.0266	7.5911	0.9822
7	1.5350	0.0249	7.5138	0.9776
10	1.4861	0.0242	7.4964	0.9744
15	1.4426	0.0232	7.4440	0.9742
20	1.4236	0.0227	7.4178	0.9751
25	1.4126	0.0224	7.3962	0.9762
30	1.4055	0.0222	7.3783	0.9770

Table D-59: 13 Stages (0.0008 md) - PLE

13 Fractures (0.0008 md)- PLE					
Years	D_∞	n	q_i	D_1	R^2
2	-0.00079	0.37450	10.71868	0.09489	0.99830
3	-0.00027	0.34598	10.72667	0.09543	0.99772
4	-0.00009	0.33348	10.73402	0.09582	0.99763
5	-0.00001	0.32773	10.74286	0.09596	0.99772
7	0.00003	0.32376	10.75309	0.09623	0.99797
10	0.00000	0.32810	10.74030	0.09585	0.99814
15	-0.00009	0.33908	10.71571	0.09473	0.99788
20	-0.00016	0.34824	10.68225	0.09349	0.99754
25	-0.00020	0.35350	10.66887	0.09286	0.99738
30	-0.00021	0.35676	10.64840	0.09224	0.99737

Table D-60: 13 Stages (0.0008 md) - Duong

13 Fractures (0.0008 md)- Duong					
Years	a	m	q_1	q_∞	R^2
2	0.96	1.07	10.50	-0.90	0.97
3	1.00	1.07	9.50	-0.73	1.00
4	1.05	1.08	8.80	-0.69	1.00
5	1.11	1.09	8.00	-0.59	1.00
7	1.19	1.11	7.20	-0.45	1.00
10	1.27	1.13	6.50	-0.31	1.00
15	1.37	1.15	5.87	-0.18	0.99
20	1.50	1.17	5.00	-0.04	0.98
25	1.58	1.18	4.55	-0.02	0.99
30	1.61	1.18	4.45	-0.01	0.99

Figure D-58: 13 Stages - PLE Constants Correlation (0.0008 m-D)

Figure D-59: 13 Stages - Duong Constants Correlation (0.0008 m-D)

Figure D-60: 13 Stages - Arps Constants Correlation (0.0008 m-D)

Table D-61: 13 Stages (0.001 md) - Arps

13 Fractures (0.001 md)- Arps				
Years	n	d_i	q_i	R^2
3	1.71228	0.03078	8.68725	0.98508
5	1.59603	0.02896	8.66456	0.97525
7	1.53583	0.02796	8.64567	0.97119
10	1.48984	0.02717	8.62811	0.96961
15	1.45077	0.02612	8.56997	0.97098
20	1.43475	0.02569	8.54586	0.97242
25	1.41867	0.02491	8.48038	0.97430
30	1.41237	0.02467	8.46194	0.97520

Table D-62: 13 Stages (0.001 md) - PLE

13 Fractures (0.001 md)- PLE					
Years	D_∞	n	q_i	D_1	R^2
2	-0.000474	0.372956	11.478445	0.089993	0.998487
3	-0.000075	0.350434	11.488406	0.090392	0.998288
4	0.000036	0.342631	11.497691	0.090685	0.998382
5	0.000069	0.339972	11.498864	0.090782	0.998524
7	0.000037	0.343018	11.497754	0.090579	0.998670
10	-0.000056	0.352353	11.482352	0.089846	0.998505
15	-0.000204	0.369531	11.437436	0.087887	0.997936
20	-0.000278	0.379788	11.391280	0.086325	0.997675
25	-0.000301	0.383195	11.375149	0.085816	0.997677
30	-0.000298	0.382883	11.362997	0.085791	0.997719

Table D-63: 13 Stages (0.001 md) - Duong

13 Fractures (0.001 md)- Duong					
Years	a	m	q_1	q_∞	R^2
2	0.99	1.06	10.80	-1.06	1.00
3	1.07	1.08	9.50	-0.92	1.00
4	1.18	1.10	8.10	-0.71	1.00
5	1.25	1.12	7.40	-0.59	1.00
7	1.30	1.13	7.02	-0.46	1.00
10	1.42	1.15	6.02	-0.28	1.00
15	1.52	1.17	5.50	-0.15	0.99
20	1.73	1.20	4.20	-0.07	0.99
25	1.77	1.20	4.02	-0.04	0.99
30	1.80	1.21	3.90	-0.02	0.99

Figure D-61: 13 Stages - PLE Constants Correlation (0.001 m-D)

Figure D-62: 13 Stages - Duong Constants Correlation (0.001 m-D)

Figure D-63: 13 Stages - Arps Constants Correlation (0.001 m-D)

Table D-64: 13 Stages (0.002 md) - Arps

13 Fractures (0.002 md)- Arps				
Years	n	d_i	q_i	R^2
3	1.7204	0.0432	13.2629	0.9511
5	1.6219	0.0424	13.2042	0.9472
7	1.5506	0.0412	13.1656	0.9518
10	1.5202	0.0394	13.1046	0.9548
15	1.4899	0.0390	13.0411	0.9590
20	1.4711	0.0384	12.9876	0.9618
25	1.4602	0.0382	12.9649	0.9634
30	1.4502	0.0381	12.9520	0.9646

Table D-65: 13 Stages (0.002 md) - PLE

13Fractures (0.002 md)- PLE					
Years	D_∞	n	q_i	D_1	R^2
2	0.0004	0.3897	13.9609	0.0730	0.9997
3	0.0003	0.3970	13.9637	0.0729	0.9997
4	0.0001	0.4138	13.9337	0.0719	0.9995
5	-0.0001	0.4286	13.9121	0.0710	0.9991
7	-0.0005	0.4574	13.8354	0.0685	0.9984
10	-0.0008	0.4816	13.7358	0.0657	0.9980
15	-0.0008	0.4838	13.7626	0.0656	0.9980
20	-0.0007	0.4677	13.8786	0.0682	0.9977
25	-0.0006	0.4585	13.8818	0.0693	0.9968
30	-0.0005	0.4476	13.9559	0.0711	0.9959

Table D-66: 13 Stages (0.002 md) - Duong

13Fractures (0.002 md)- Duong					
Years	a	m	q_1	q_∞	R^2
2	1.112	1.091	11.850	-1.001	0.980
3	1.192	1.108	10.850	-1.009	0.990
4	1.324	1.132	9.500	-0.824	0.995
5	1.431	1.151	8.500	-0.696	0.996
7	1.543	1.174	7.600	-0.395	0.994
10	1.692	1.199	6.500	-0.183	0.991
15	1.821	1.218	5.704	-0.056	0.989
20	1.919	1.231	5.100	-0.006	0.989
25	2.003	1.240	4.600	0.002	0.989
30	2.083	1.247	4.140	0.001	0.989

Figure D-64: 13 Stages - PLE Constants Correlation (0.002 m-D)

Figure D-65: 13 Stages - Duong Constants Correlation (0.002 m-D)

Figure D-66: 13 Stages - Arps Constants Correlation (0.002 m-D)

Table D-67: 13 Stages (0.004 md) - Arps

13 Fractures (0.004 md)- Arps				
Years	n	d_i	q_i	R^2
3	1.4505	0.0428	17.0135	0.9453
5	1.4059	0.0418	17.0062	0.9521
7	1.3851	0.0411	16.9626	0.9563
10	1.3451	0.0401	16.9083	0.9588
15	1.3042	0.0391	16.8576	0.9596
20	1.2874	0.0388	16.7685	0.9588
25	1.2764	0.0387	16.7524	0.9584
30	1.2654	0.0386	16.7465	0.9577

Table D-68: 13 Stages (0.004 md) - PLE

13 Fractures (0.004 md)-PLE					
Years	D_∞	n	q_i	D_1	R^2
2	0.00021	0.49681	16.52229	0.05390	0.99945
3	-0.00119	0.56492	16.40861	0.05096	0.99857
4	-0.00239	0.61786	16.25132	0.04785	0.99808
5	-0.00255	0.61550	16.56712	0.04965	0.99787
7	-0.00251	0.61148	16.69012	0.05039	0.99788
10	-0.00199	0.58632	16.69210	0.05221	0.99734
15	-0.00138	0.54895	16.66632	0.05539	0.99491
20	-0.00069	0.48187	17.06158	0.06432	0.98844
25	-0.00081	0.49566	17.02566	0.06251	0.99592
30	-0.00069	0.48187	17.06158	0.06432	0.98789

Table D-69: 13 Stages (0.004 md) - Duong

13 Fractures (0.004 md)- Duong					
Years	a	m	q_1	q_∞	R^2
2	1.285	1.114	14.100	-2.803	0.997
3	1.387	1.145	12.500	-1.426	0.994
4	1.509	1.172	11.000	-0.809	0.991
5	1.618	1.192	9.840	-0.486	0.989
7	1.765	1.217	8.500	-0.199	0.988
10	1.920	1.239	7.200	-0.038	0.988
15	2.077	1.256	6.000	0.040	0.989
20	2.220	1.269	5.000	0.062	0.991
25	2.360	1.281	4.150	0.069	0.992
30	2.487	1.290	3.500	0.071	0.993

Figure D-67: 13 Stages - PLE Constants Correlation (0.004 m-D)

Figure D-68: 13 Stages - Duong Constants Correlation (0.004 m-D)

Figure D-69: 13 Stages - Arps Constants Correlation (0.004 m-D)

Table D-70: 13 Stages (0.006 md) - Arps

13 Fractures (0.006 md)- Arps				
Years	n	d_i	q_i	R^2
3	1.670760	0.067481	23.046770	0.932476
5	1.630571	0.068697	23.005184	0.937313
7	1.584310	0.067430	23.001984	0.943918
10	1.560853	0.066324	22.980259	0.947709
15	1.534700	0.065168	22.894356	0.950518
20	1.514508	0.064501	22.785205	0.951413
25	1.490434	0.064057	22.694780	0.951903
30	1.482164	0.063557	22.684934	0.951872

Table D-71: 13 Stages (0.006 md) - PLE

13 Fractures (0.006 md)- PLE					
Years	D_∞	n	q_i	D_1	R^2
2	-0.00244	0.64996	17.97412	0.04281	0.99868
3	-0.00602	0.73628	17.80197	0.04149	0.99826
4	-0.00703	0.75759	17.64590	0.04081	0.99835
5	-0.00575	0.72998	17.82385	0.04168	0.99832
7	-0.00418	0.68948	17.97193	0.04310	0.99730
10	-0.00279	0.61914	18.00000	0.05290	0.98016
15	-0.00146	0.55660	18.70211	0.05457	0.99080
20	-0.00103	0.51952	18.99970	0.05942	0.98755
25	-0.00083	0.50188	18.99947	0.06136	0.98509
30	-0.00071	0.48995	19.00348	0.06274	0.98330

Table D-72: 13 Stages (0.006 md) - Duong

13 Fractures (0.006 md)- Duong					
Years	a	m	q_1	q_∞	R^2
2	1.252	1.115	17.531	-2.965	0.993
3	1.369	1.145	15.500	-1.584	0.981
4	1.495	1.180	13.700	-0.659	0.986
5	1.621	1.202	12.000	-0.349	0.986
7	1.776	1.226	10.200	-0.101	0.986
10	1.919	1.245	8.700	0.017	0.988
15	2.067	1.260	7.250	0.061	0.990
20	2.211	1.272	6.000	0.069	0.992
25	2.338	1.282	5.050	0.068	0.993
30	2.511	1.299	4.013	0.100	0.989

Figure D-70: 13 Stages - PLE Constants Correlation (0.006 m-D)

Figure D-71: 13 Stages - Duong Constants Correlation (0.006 m-D)

Figure D-72: 13 Stages - Arps Constants Correlation (0.006 m-D)

Table D-73: 13 Stages (4%) - Arps

13 Fractures (4%) - Arps				
Years	n	d_i	q_i	R^2
3	1.7081	0.0570	13.3167	0.9439
5	1.6020	0.0528	13.2247	0.9451
7	1.5292	0.0466	12.8230	0.9525
10	1.4691	0.0412	12.4245	0.9605
15	1.4210	0.0371	12.0791	0.9678
20	1.3910	0.0345	11.8450	0.9722
25	1.3710	0.0329	11.6904	0.9749
30	1.3651	0.0325	11.6599	0.9759

Table D-74: 13 Stages (4%) - PLE

13Fractures ($\phi=4\%$)- PLE					
years	D_∞	n	q_i	D_1	R^2
2	0.0006	0.3932	13.3007	0.0750	0.9998
3	0.0002	0.4131	13.2775	0.0744	0.9996
4	-0.0002	0.4369	13.2433	0.0730	0.9992
5	-0.0005	0.4584	13.2024	0.0714	0.9987
7	-0.0009	0.4876	13.1329	0.0687	0.9981
10	-0.0011	0.5006	13.0790	0.0672	0.9980
15	-0.0010	0.4879	13.1193	0.0688	0.9978
20	-0.0008	0.4646	13.2900	0.0731	0.9969
25	-0.0007	0.4529	13.3916	0.0746	0.9956
30	-0.0006	0.4422	13.3910	0.0762	0.9943

Table D-75: 13 Stages (4%) - Duong

13 Fractures ($\phi=4\%$)- Duong					
Years	a	m	q_1	q_∞	R^2
2	1.087	1.074	13.000	-2.560	0.999
3	1.113	1.091	12.400	-1.592	0.997
4	1.161	1.109	11.800	-1.039	0.994
5	1.223	1.127	11.100	-0.701	0.992
7	1.298	1.149	10.500	-0.386	0.988
10	1.385	1.170	9.800	-0.186	0.985
15	1.458	1.186	9.278	-0.025	0.983
20	1.512	1.195	8.800	-0.025	0.983
25	1.525	1.198	8.750	-0.008	0.984
30	1.533	1.200	8.700	0.000	0.984

Figure D-73: 13 Stages - PLE Constants Correlation (4%)

Figure D-74: 13 Stages – Duong Constants Correlation (4%)

Figure D-75: 13 Stages - Arps Constants Correlation (4%)

Table D-76: 13 Stages (5%) - Arps

13 Fractures (5%) - Arps				
Years	n	di	qi	R²
3	1.7421	0.0475	13.4537	0.9498
5	1.6410	0.0467	13.6124	0.9449
7	1.5517	0.0405	13.1748	0.9518
10	1.4807	0.0355	12.7633	0.9600
15	1.4107	0.0305	12.2690	0.9694
20	1.3765	0.0282	12.0222	0.9741
25	1.3523	0.0266	11.8401	0.9772
30	1.3452	0.0263	11.7978	0.9783

Table D-77: 13 Stages (5%) - PLE

13 Fractures ($\phi=5\%$)- PLE					
Years	D_∞	n	qi	D₁	R²
2	0.0004	0.3896	13.9740	0.0731	0.9997
3	0.0003	0.3981	13.9621	0.0728	0.9997
4	0.0001	0.4135	13.9369	0.0720	0.9995
5	-0.0001	0.4286	13.9203	0.0710	0.9991
7	-0.0005	0.4578	13.8467	0.0685	0.9984
10	-0.0008	0.4829	13.7437	0.0656	0.9980
15	-0.0009	0.4855	13.7134	0.0651	0.9980
20	-0.0007	0.4713	13.8168	0.0674	0.9977
25	-0.0006	0.4602	13.8203	0.0688	0.9968
30	-0.0005	0.4475	13.9572	0.0711	0.9959

Table D-78: 13 Stages (5%) - Duong

13 Fractures ($\phi=5\%$)- Duong					
Years	a	m	q₁	q_∞	R²
2	1.086	1.071	13.700	-2.801	1.000
3	1.126	1.087	12.700	-1.875	0.999
4	1.189	1.106	11.700	-1.257	0.997
5	1.243	1.122	11.050	-0.893	0.995
7	1.309	1.141	10.500	-0.526	0.992
10	1.380	1.160	10.000	-0.278	0.988
15	1.449	1.172	9.561	-0.120	0.975
20	1.488	1.185	9.300	-0.059	0.984
25	1.515	1.190	9.100	-0.030	0.984
30	1.529	1.193	9.000	-0.016	0.984

Figure D-76: 13 Stages - PLE Constants Correlation (5%)

Figure D-77: 13 Stages - Duong Constants Correlation (5%)

Figure D-78: 13 Stages - Arps Constants Correlation (5%)

Table D-79: 13 Stages (6%) - Arps

13 Fractures (6%) - Arps				
Years	n	d_i	q_i	R^2
3	1.8065	0.0444	13.8793	0.9528
5	1.6359	0.0379	13.7552	0.9504
7	1.5368	0.0329	13.1509	0.9564
10	1.4502	0.0283	12.6930	0.9648
15	1.4039	0.0261	12.4598	0.9706
20	1.3625	0.0238	12.1694	0.9759
25	1.3404	0.0226	12.0141	0.9787
30	1.3209	0.0216	11.8694	0.9809

Table D-80: 13 Stages (6%) - PLE

13 Fractures ($\phi=6\%$)-PLE					
Years	D_∞	n	q_i	D_1	R^2
2	0.00028	0.38915	14.53514	0.07150	0.99946
3	0.00029	0.38869	14.53413	0.07149	0.99957
4	0.00016	0.39894	14.51876	0.07090	0.99953
5	0.00001	0.41162	14.47904	0.06995	0.99932
7	-0.00028	0.43504	14.43713	0.06808	0.99870
10	-0.00059	0.46575	14.25055	0.06440	0.99804
15	-0.00070	0.47362	14.30886	0.06398	0.99797
20	-0.00064	0.46715	14.34853	0.06495	0.99793
25	-0.00058	0.46031	14.34864	0.06571	0.99749
30	-0.00051	0.45007	14.40841	0.06729	0.99686

Table D-81: 13 Stages (6%) - Duong

13 Fractures ($\phi=6\%$)- Duong					
Years	a	m	q_1	q_∞	R^2
2	1.057	1.062	15.000	-3.184	1.000
3	1.075	1.071	14.300	-2.394	0.999
4	1.114	1.086	13.450	-1.704	0.998
5	1.145	1.098	13.000	-1.268	0.996
7	1.218	1.120	12.100	-0.752	0.993
10	1.283	1.139	11.600	-0.418	0.989
15	1.359	1.158	11.000	-0.197	0.985
20	1.414	1.169	10.500	-0.107	0.983
25	1.441	1.175	10.300	-0.065	0.983
30	1.455	1.178	10.200	-0.043	0.983

Figure D-79: 13 Stages - PLE Constants Correlation (6%)

Figure D-80: 13 Stages - Duong Constants Correlation (6%)

Figure D-81: 13 Stages - Arps Constants Correlation (6%)

Table D-82: 13 Stages (8%) - Arps

13 Fractures (8%) - Arps				
Years	n	d_i	q_i	R^2
3	1.9041	0.0387	14.4732	0.9610
5	1.7050	0.0326	14.1389	0.9542
7	1.5960	0.0264	13.5263	0.9549
10	1.5045	0.0252	13.4493	0.9612
15	1.4325	0.0223	13.0902	0.9684
20	1.3515	0.0187	12.5148	0.9773
25	1.3024	0.0167	12.1619	0.9823
30	1.2834	0.0160	12.0275	0.9841

Table D-83: 13 Stages (8%) - PLE

13Fractures ($\phi=8\%$)- PLE					
Years	D_∞	n	q_i	D_1	R^2
2	-0.00005	0.39867	15.42605	0.06837	0.99907
3	0.00015	0.38618	15.43693	0.06860	0.99917
4	0.00016	0.38510	15.43725	0.06869	0.99929
5	0.00013	0.38955	15.37688	0.06806	0.99932
7	-0.00004	0.40579	15.37621	0.06701	0.99908
10	-0.00026	0.42954	15.28329	0.06462	0.99841
15	-0.00047	0.45512	15.18778	0.06162	0.99785
20	-0.00051	0.46111	15.08417	0.06053	0.99788
25	-0.00048	0.45725	15.11713	0.06108	0.99788
30	-0.00045	0.45225	15.17413	0.06192	0.99768

Table D-84: 13 Stages (8%) - Duong

13Fractures ($\phi=8\%$)- Duong					
Years	a	m	q_1	q_∞	R^2
2	1.07	1.06	15.42	-2.99	0.99
3	1.08	1.07	15.00	-2.79	0.99
4	1.11	1.08	14.20	-2.08	1.00
5	1.15	1.09	13.40	-1.61	1.00
7	1.21	1.11	12.60	-1.06	1.00
10	1.28	1.13	11.90	-0.63	0.99
15	1.36	1.15	11.22	-0.32	0.99
20	1.40	1.16	10.92	-0.19	0.99
25	1.43	1.16	10.70	-0.09	0.98
30	1.45	1.17	10.62	-0.07	0.98

Figure D-82: 13 Stages - PLE Constants Correlation (8%)

Figure D-83: 13 Stages - Duong Constants Correlation (8%)

Figure D-84: 13 Stages - Arps Constants Correlation (8%)

E. DCA Models Constants Ratios

The results for re-calculated Arps and Duong constants and their ratios were summarized in the following tables, for both seven and thirteen stages.

E.1. Seven Stages Model

Table E-1: 7 Stages (300 ft.) - Arps- Recalculated Constants

7 Fractures (300 ft.)- Arps						
Years	n	$n(Eq.)$	di	$di(Eq.)$	qi	$qi(Eq.)$
3	2.1669	2.1513	0.0436	0.0407	5.0052	5.0006
5	1.8742	1.8353	0.0247	0.0234	4.3838	4.3143
7	1.6453	1.6529	0.0154	0.0163	3.9078	3.9145
10	1.4477	1.4794	0.0100	0.0111	3.4948	3.5311
15	1.2704	1.3041	0.0065	0.0071	3.0975	3.1407
20	1.1807	1.1925	0.0051	0.0052	2.8748	2.8901
25	1.1270	1.1126	0.0043	0.0041	2.7301	2.7096
30	1.0795	1.0512	0.0038	0.0034	2.6105	2.5705

Table E-2: 7 Stages (300 ft.) - Duong- Recalculated Constants

7 Fractures (300 ft.)- Duong								
Years	a	$a(Eq.)$	m	$m(Eq.)$	q_1	$q_1(Eq.)$	q_∞	$q_\infty(Eq.)$
2	0.8370	0.7282	0.9883	0.9613	11.4065	11.3750	-4.5662	-3.8101
3	0.8116	0.7701	0.9874	0.9815	10.4424	10.4250	-3.4514	-3.1285
4	0.7944	0.7998	0.9878	0.9959	9.7563	9.7509	-2.7111	-2.6450
5	0.7671	0.8229	0.9880	1.0070	9.2240	9.2281	-1.9608	-2.2699
7	0.7377	0.8577	1.0007	1.0238	8.4220	8.4397	-0.7879	-1.7043
10	0.7992	0.8946	1.0331	1.0416	7.5720	7.6040	-0.2725	-1.1048
15	0.9062	0.9365	1.0656	1.0618	6.6051	6.6540	-0.0834	-0.4232
20	0.9884	0.9663	1.0846	1.0762	5.9190	5.9800	-0.0243	0.0603
25	1.0540	0.9893	1.0972	1.0873	5.3870	5.4572	0.0011	0.4354
30	1.0774	1.0082	1.1014	1.0964	5.2052	5.0300	0.0079	0.7419

Table E-3: 7 Stages (400 ft.) - Arps- Recalculated Constants

7 Fractures (400 ft.)- Arps						
Years	n	$n(Eq.)$	di	$di(Eq.)$	qi	$qi(Eq.)$
3	2.1317	2.0400	0.0349	0.0299	5.7257	5.5786
5	1.8148	1.8047	0.0203	0.0203	5.0730	5.0626
7	1.6213	1.6646	0.0143	0.0157	4.6758	4.7491
10	1.4605	1.5281	0.0105	0.0120	4.3258	4.4379
15	1.3394	1.3864	0.0080	0.0088	4.0284	4.1089
20	1.2832	1.2939	0.0070	0.0071	3.8694	3.8903
25	1.2501	1.2264	0.0063	0.0060	3.7674	3.7288
30	1.2278	1.1739	0.0060	0.0052	3.6982	3.6018

Table E-4: 7 Stages (400 ft.) – Duong - Recalculated Constants

7 Fractures (400 ft.)- Duong								
Years	a	$a(Eq.)$	m	$m(Eq.)$	q_1	$q_1(Eq.)$	q_∞	$q_\infty(Eq.)$
2	0.808	0.707	0.990	0.972	11.502	11.508	-3.248	-2.690
3	0.808	0.779	0.996	0.994	10.510	10.484	-2.445	-2.220
4	0.810	0.830	1.002	1.009	9.748	9.758	-1.805	-1.888
5	0.816	0.870	1.010	1.022	9.186	9.194	-1.321	-1.629
7	0.850	0.930	1.028	1.040	8.338	8.345	-0.741	-1.240
10	0.923	0.993	1.054	1.060	7.445	7.444	-0.363	-0.827
15	1.036	1.065	1.083	1.082	6.418	6.420	-0.149	-0.358
20	1.125	1.116	1.102	1.097	5.694	5.694	-0.071	-0.025
25	1.198	1.156	1.115	1.110	5.132	5.130	-0.036	0.233
30	1.261	1.189	1.124	1.120	4.672	4.670	-0.018	0.444

Table E-5: 7 Stages (500 ft.) - Arps- Recalculated Constants

7 Fractures (500 ft.)- Arps						
Years	n	$n(Eq.)$	di	$di(Eq.)$	qi	$qi(Eq.)$
3	2.1055	1.9475	0.0259	0.0203	6.0292	5.7287
5	1.7534	1.7800	0.0149	0.0160	5.3604	5.4350
7	1.5965	1.6777	0.0118	0.0137	5.0889	5.2499
10	1.4858	1.5756	0.0099	0.0117	4.8918	5.0605
15	1.4195	1.4671	0.0089	0.0097	4.7677	4.8535
20	1.3931	1.3946	0.0085	0.0085	4.7153	4.7118
25	1.3769	1.3409	0.0083	0.0077	4.6821	4.6048
30	1.3647	1.2986	0.0081	0.0071	4.6569	4.5191

Table E-6: 7 Stages (500 ft.) - Duong- Recalculated Constants

7 Fractures (500 ft.)- Duong								
Years	a	$a(Eq.)$	m	$m(Eq.)$	q_1	$q_1(Eq.)$	q_∞	$q_\infty(Eq.)$
2	0.808	0.733	0.989	0.979	12.151	11.800	-3.519	-2.877
3	0.812	0.814	0.992	1.001	11.050	10.685	-2.853	-2.380
4	0.850	0.872	1.011	1.017	9.802	9.894	-1.851	-2.027
5	0.891	0.917	1.029	1.030	8.870	9.280	-1.109	-1.753
7	0.940	0.984	1.044	1.049	8.021	8.355	-0.776	-1.340
10	1.022	1.056	1.066	1.069	7.025	7.374	-0.461	-0.902
15	1.117	1.137	1.089	1.091	6.175	6.259	-0.247	-0.404
20	1.202	1.195	1.106	1.108	5.501	5.468	-0.151	-0.051
25	1.261	1.239	1.122	1.120	5.074	4.854	-0.061	0.223
30	1.320	1.276	1.130	1.130	4.650	4.353	-0.035	0.447

Table E-7: 7 Stages (600 ft.) - Arps- Recalculated Constants

7 Fractures (600 ft.)- Arps						
Years	n	n(Eq.)	di	di(Eq.)	qi	qi(Eq.)
3	2.0700	2.0499	0.0181	0.0178	6.0055	5.9963
5	1.9707	1.9182	0.0175	0.0161	6.0129	5.9021
7	1.8060	1.8361	0.0142	0.0150	5.7744	5.8408
10	1.7032	1.7529	0.0128	0.0139	5.6735	5.7766
15	1.6397	1.6629	0.0124	0.0128	5.6647	5.7044
20	1.5905	1.6019	0.0118	0.0120	5.6180	5.6538
25	1.5702	1.5561	0.0117	0.0115	5.6271	5.6148
30	1.5601	1.5196	0.0118	0.0111	5.6488	5.5832

Table E-8: 7 Stages (600 ft.) - Duong- Recalculated Constants

7 Fractures (600 ft.)- Duong								
Years	a	a(Eq.)	m	m(Eq.)	q1	q1 (Eq.)	q[∞]	q[∞] (Eq.)
2	0.812	0.714	0.983	0.971	14.655	13.689	-5.219	-3.741
3	0.831	0.804	1.000	0.994	12.084	12.312	-2.788	-3.118
4	0.855	0.867	1.008	1.010	11.062	11.335	-2.294	-2.676
5	0.881	0.916	1.018	1.023	10.065	10.577	-1.720	-2.334
7	0.922	0.990	1.032	1.042	9.015	9.435	-1.219	-1.817
10	0.980	1.069	1.049	1.062	8.022	8.223	-0.798	-1.269
15	1.081	1.158	1.074	1.085	7.015	6.846	-0.575	-0.647
20	1.221	1.222	1.111	1.101	6.060	5.869	-0.213	-0.205
25	1.330	1.271	1.121	1.114	5.205	5.112	-0.211	0.137
30	1.410	1.311	1.130	1.124	4.709	4.493	-0.217	0.417

Table E-9: 7 Stages (700 ft.) - Arps- Recalculated Constants

7 Fractures (700 ft.)- Arps						
Years	n	n(Eq.)	di	di(Eq.)	qi	qi(Eq.)
3	1.9283	1.9607	0.0118	0.0119	5.7419	5.7557
5	1.8589	1.8441	0.0112	0.0111	5.7172	5.7117
7	1.8021	1.7711	0.0106	0.0105	5.6887	5.6830
10	1.7065	1.6969	0.0100	0.0100	5.6524	5.6527
15	1.6045	1.6163	0.0093	0.0094	5.5957	5.6184
20	1.5561	1.5615	0.0091	0.0090	5.5946	5.5942
25	1.5124	1.5202	0.0087	0.0088	5.5612	5.5755
30	1.4896	1.4873	0.0086	0.0085	5.5577	5.5603

Table E-10: 7 Stages (700 ft.) - Duong- Recalculated Constants

7 Fractures (700 ft.)- Duong								
Years	α	$\alpha(Eq.)$	m	$m(Eq.)$	q_1	$q_1 (Eq.)$	q^∞	$q^\infty (Eq.)$
2	0.889	0.686	0.996	0.955	16.505	15.662	-7.556	-5.651
3	0.839	0.770	0.995	0.979	14.451	14.064	-4.487	-4.700
4	0.846	0.829	1.000	0.996	12.541	12.931	-3.305	-4.026
5	0.876	0.876	1.010	1.009	11.207	12.052	-2.570	-3.502
7	0.905	0.945	1.019	1.029	10.000	10.726	-1.939	-2.713
10	0.944	1.019	1.032	1.050	9.054	9.321	-1.371	-1.877
15	1.033	1.103	1.050	1.074	8.051	7.723	-1.099	-0.926
20	1.151	1.163	1.100	1.091	7.051	6.590	-0.124	-0.251
25	1.222	1.209	1.110	1.104	6.014	5.711	-0.123	0.272
30	1.346	1.247	1.129	1.114	4.890	4.992	-0.102	0.700

Table E-11: 7 Stages (0.0008 m-D) - Arps- Recalculated Constants

7 Fractures (0.0008 m-D)- Arps						
Years	n	$n(Eq.)$	d_i	$d_i(Eq.)$	q_i	$q_i(Eq.)$
3	2.3770	2.4549	0.0762	0.0835	5.2399	5.3574
5	2.2994	2.2623	0.0610	0.0560	4.9447	4.8445
7	2.1953	2.1437	0.0462	0.0431	4.6050	4.5338
10	2.0564	2.0248	0.0328	0.0326	4.2277	4.2262
15	1.8951	1.8976	0.0226	0.0237	3.8544	3.9017
20	1.7957	1.8122	0.0181	0.0189	3.6427	3.6867
25	1.7325	1.7487	0.0157	0.0159	3.5110	3.5282
30	1.6930	1.6984	0.0143	0.0138	3.4299	3.4037

Table E-12: 7 Stages (0.0008 m-D) - Duong- Recalculated Constants

7 Fractures (0.0008 m-D)- Duong								
Years	α	$\alpha(Eq.)$	m	$m(Eq.)$	q_1	$q_1 (Eq.)$	q^∞	$q^\infty (Eq.)$
2	0.8135	0.8259	1.0520	1.0643	6.50	6.3742	0.1098	0.1600
3	0.8763	0.8891	1.0698	1.0710	6.00	5.8491	0.1385	0.1247
4	0.9649	0.9338	1.0878	1.0759	5.30	5.4765	0.1538	0.0997
5	0.9825	0.9686	1.0873	1.0796	5.10	5.1876	0.1153	0.0803
7	1.0339	1.0210	1.0902	1.0852	4.60	4.7518	0.0571	0.0510
10	1.0668	1.0765	1.0887	1.0912	4.23	4.2900	-0.0133	0.0200
15	1.1050	1.1396	1.0896	1.0979	3.85	3.7649	-0.0643	-0.0153
20	1.1571	1.1844	1.0964	1.1027	3.50	3.3923	-0.0643	-0.0403
25	1.2370	1.2192	1.1080	1.1065	3.06	3.1034	-0.0484	-0.0597
30	1.2696	1.2476	1.1126	1.1095	2.90	2.8672	-0.0416	-0.0756

Table E-13: 7 Stages (0.001 m-D) - Arps- Recalculated Constants

7 Fractures (0.001 m-D)- Arps						
Years	n	n(Eq.)	di	di(Eq.)	qi	qi(Eq.)
3	2.2579	2.2804	0.0607	0.0593	5.4598	5.4501
5	2.1343	2.1047	0.0431	0.0405	5.0242	4.9561
7	1.9900	1.9964	0.0302	0.0315	4.6638	4.6554
10	1.9027	1.8876	0.0231	0.0241	4.3044	4.3566
15	1.7674	1.7712	0.0166	0.0178	3.9711	4.0401
20	1.6800	1.6930	0.0138	0.0144	3.7939	3.8296
25	1.6292	1.6347	0.0124	0.0122	3.6905	3.6739
30	1.5980	1.5886	0.0116	0.0106	3.6266	3.5514

Table E-14: 7 Stages (0.001 m-D) - Duong- Recalculated Constants

7 Fractures (0.001 m-D)- Duong								
Years	a	a(Eq.)	m	m(Eq.)	q1	q1 (Eq.)	q∞	q∞ (Eq.)
2	0.8117	0.8091	1.0458	1.0515	6.8000	6.9782	0.2000	0.2126
3	0.8763	0.8835	1.0624	1.0614	6.5000	6.3676	0.1851	0.1530
4	0.9285	0.9363	1.0710	1.0685	6.0000	5.9343	0.1310	0.1107
5	0.9823	0.9772	1.0787	1.0740	5.5000	5.5983	0.0983	0.0779
7	1.0245	1.0390	1.0804	1.0823	5.0500	5.0915	-0.0212	0.0285
10	1.1360	1.1044	1.0955	1.0910	4.1600	4.5544	-0.0613	-0.0240
15	1.1804	1.1788	1.0994	1.1010	3.8000	3.9438	-0.0885	-0.0836
20	1.2299	1.2316	1.1066	1.1081	3.7000	3.5105	-0.1321	-0.1259
25	1.2684	1.2726	1.1127	1.1136	3.6000	3.1745	-0.1542	-0.1587
30	1.2986	1.3060	1.1174	1.1181	3.5500	2.8999	-0.1535	-0.1855

Table E-15: 7 Stages (0.002 m-D) - Arps- Recalculated Constants

7 Fractures (0.002 m-D)- Arps						
Years	n	n(Eq.)	di	di(Eq.)	qi	qi(Eq.)
3	2.0494	1.8041	0.0231	0.0191	5.8789	5.6557
5	1.7531	1.6864	0.0149	0.0154	5.3601	5.3851
7	1.5962	1.6132	0.0118	0.0133	5.0870	5.2139
10	1.4851	1.5390	0.0099	0.0114	4.8906	5.0384
15	1.4190	1.4588	0.0089	0.0096	4.7662	4.8460
20	1.3929	1.4044	0.0085	0.0085	4.7148	4.7140
25	1.3769	1.3637	0.0083	0.0077	4.6826	4.6141
30	1.3646	1.3312	0.0081	0.0072	4.6567	4.5341

Table E-16: 7 Stages (0.002 m-D) - Duong- Recalculated Constants

7 Fractures (0.002 m-D)- Duong								
Years	a	a(Eq.)	m	m(Eq.)	q1	q1 (Eq.)	q ∞	q ∞ (Eq.)
2	0.8937	0.8904	1.0404	1.0413	9.3000	8.1230	-0.8769	-0.8280
3	0.9522	0.9829	1.0511	1.0578	8.0000	7.3697	-0.7907	-0.6844
4	1.0426	1.0486	1.0698	1.0696	6.7500	6.8352	-0.5840	-0.5825
5	1.0902	1.0995	1.0779	1.0787	6.2500	6.4206	-0.4212	-0.5034
7	1.2132	1.1763	1.0999	1.0924	5.1500	5.7954	-0.2945	-0.3843
10	1.2615	1.2576	1.1088	1.1069	4.8000	5.1327	-0.2333	-0.2579
15	1.3897	1.3502	1.1301	1.1235	4.0000	4.3793	-0.0123	-0.1143
20	1.4321	1.4158	1.1375	1.1352	3.8000	3.8448	0.0259	-0.0124
25	1.4556	1.4667	1.1416	1.1443	3.7000	3.4302	0.0100	0.0666
30	1.4652	1.5084	1.1435	1.1518	3.6700	3.0915	0.0081	0.1312

Table E-17: 7 Stages (0.004 m-D) - Arps- Recalculated Constants

7 Fractures (0.004 m-D)- Arps						
Years	n	n(Eq.)	di	di(Eq.)	qi	qi(Eq.)
3	1.4226	1.3613	0.0097	0.0090	6.6927	6.6145
5	1.2860	1.3236	0.0083	0.0086	6.4906	6.5506
7	1.2603	1.2993	0.0080	0.0084	6.4478	6.5088
10	1.2553	1.2741	0.0080	0.0081	6.4386	6.4649
15	1.2482	1.2460	0.0079	0.0078	6.4238	6.4153
20	1.2355	1.2264	0.0077	0.0076	6.3942	6.3803
25	1.2203	1.2115	0.0076	0.0075	6.3577	6.3533
30	1.2071	1.1994	0.0074	0.0073	6.3261	6.3313

Table E-18: 7 Stages (0.004 m-D) - Duong- Recalculated Constants

7 Fractures (0.004 m-D)- Duong								
Years	a	a(Eq.)	m	m(Eq.)	q1	q1 (Eq.)	q ∞	q ∞ (Eq.)
2	1.0394	0.9805	1.0525	1.0497	9.9000	9.7118	-2.0193	-1.6772
3	1.0903	1.0873	1.0660	1.0691	8.9000	8.7792	-1.5433	-1.3879
4	1.1460	1.1630	1.0810	1.0829	8.0000	8.1175	-1.1003	-1.1827
5	1.2122	1.2218	1.0947	1.0935	7.3000	7.6043	-0.7940	-1.0235
7	1.2521	1.3104	1.1064	1.1096	6.9500	6.8304	-0.5266	-0.7834
10	1.3810	1.4043	1.1307	1.1267	5.8500	6.0101	-0.2680	-0.5289
15	1.4622	1.5110	1.1451	1.1460	5.3000	5.0775	-0.1481	-0.2396
20	1.6475	1.5868	1.1681	1.1598	4.0300	4.4158	-0.0126	-0.0343
25	1.6741	1.6455	1.1713	1.1705	3.8800	3.9026	-0.0123	0.1249
30	1.7000	1.6935	1.1725	1.1792	3.8200	3.4832	-0.0526	0.2550

Table E-19: 7 Stages (0.006 m-D) - Arps- Recalculated Constants

7 Fractures (0.006 m-D)- Arps						
Years	<i>n</i>	<i>n</i>(Eq.)	<i>d_i</i>	<i>d_i</i>(Eq.)	<i>q_i</i>	<i>q_i</i>(Eq.)
3	1.2020	1.2159	0.0081	0.0083	7.7187	7.7571
5	1.1908	1.2011	0.0081	0.0081	7.7039	7.7176
7	1.2009	1.1914	0.0082	0.0080	7.7231	7.6917
10	1.2038	1.1813	0.0082	0.0079	7.7289	7.6643
15	1.1885	1.1698	0.0080	0.0078	7.6917	7.6333
20	1.1671	1.1618	0.0078	0.0077	7.6365	7.6113
25	1.1471	1.1556	0.0075	0.0076	7.5823	7.5944
30	1.1307	1.1505	0.0073	0.0076	7.5361	7.5805

Table E-20: 7 Stages (0.006 m-D) - Duong- Recalculated Constants

7 Fractures (0.006 m-D)- Duong								
Years	<i>a</i>	<i>a</i>(Eq.)	<i>m</i>	<i>m</i>(Eq.)	<i>q₁</i>	<i>q₁</i> (Eq.)	<i>q_∞</i>	<i>q_∞</i> (Eq.)
2	1.0943	0.9870	1.0565	1.0487	11.5000	11.8565	-3.2125	-2.4138
3	1.1183	1.0973	1.0684	1.0693	10.5000	10.6851	-2.1135	-1.9949
4	1.1323	1.1756	1.0772	1.0840	10.1000	9.8540	-1.4970	-1.6978
5	1.1644	1.2363	1.0885	1.0954	9.6000	9.2093	-1.0468	-1.4673
7	1.2213	1.3279	1.1051	1.1125	8.9500	8.2373	-0.6195	-1.1197
10	1.4099	1.4249	1.1382	1.1307	6.9000	7.2068	-0.2861	-0.7512
15	1.5558	1.5353	1.1592	1.1514	5.6400	6.0354	-0.1560	-0.3324
20	1.6885	1.6135	1.1748	1.1661	4.6400	5.2043	-0.1050	-0.0352
25	1.7021	1.6743	1.1764	1.1775	4.5500	4.5597	-0.0993	0.1953
30	1.7098	1.7239	1.1773	1.1868	4.5000	4.0329	-0.0964	0.3836

Table E-21: 7 Stages (4%) - Arps- Recalculated Constants

7 Fractures (4%)- Arps						
Years	<i>n</i>	<i>n</i>(Eq.)	<i>d_i</i>	<i>d_i</i>(Eq.)	<i>q_i</i>	<i>q_i</i>(Eq.)
3	1.8915	1.7582	0.0198	0.0164	5.3124	5.1146
5	1.6118	1.6394	0.0133	0.0140	4.8797	4.9299
7	1.4882	1.5655	0.0111	0.0125	4.6859	4.8119
10	1.4145	1.4909	0.0099	0.0112	4.5653	4.6899
15	1.3751	1.4103	0.0093	0.0098	4.4970	4.5550
20	1.3585	1.3558	0.0090	0.0089	4.4656	4.4616
25	1.3452	1.3150	0.0088	0.0083	4.4399	4.3905
30	1.3341	1.2825	0.0086	0.0079	4.4160	4.3332

Table E-22: 7 Stages (4%) – Duong - Recalculated Constants

7 Fractures (4%)- Duong								
Years	a	a(Eq.)	m	m(Eq.)	q1	q1 (Eq.)	q [∞]	q [∞] (Eq.)
2	0.8853	0.8760	1.0319	1.0348	8.3000	7.8322	-0.8628	-0.7912
3	0.9550	0.9807	1.0481	1.0544	7.4000	7.0529	-0.7345	-0.6601
4	1.0331	1.0549	1.0656	1.0683	6.5000	6.4999	-0.5706	-0.5671
5	1.1156	1.1126	1.0828	1.0790	5.7000	6.0711	-0.4313	-0.4950
7	1.2013	1.1994	1.0994	1.0953	5.0000	5.4244	-0.3116	-0.3863
10	1.3235	1.2915	1.1215	1.1125	4.2000	4.7388	-0.1788	-0.2710
15	1.4386	1.3962	1.1406	1.1321	3.6000	3.9595	-0.0556	-0.1400
20	1.4891	1.4705	1.1487	1.1460	3.3800	3.4066	-0.0556	-0.0470
25	1.5102	1.5281	1.1521	1.1568	3.3000	2.9777	-0.0142	0.0251
30	1.5325	1.5752	1.1552	1.1656	3.2000	2.6273	-0.0340	0.0841

Table E-23: 7 Stages (5%) - Arps- Recalculated Constants

7 Fractures (5%)- Arps						
Years	n	n(Eq.)	di	di(Eq.)	qi	qi(Eq.)
3	2.0494	1.9205	0.0231	0.0192	5.8798	5.6582
5	1.7531	1.7616	0.0149	0.0154	5.3586	5.3875
7	1.5968	1.6643	0.0118	0.0134	5.0887	5.2162
10	1.4859	1.5669	0.0099	0.0115	4.8925	5.0406
15	1.4192	1.4631	0.0089	0.0097	4.7670	4.8482
20	1.3931	1.3937	0.0085	0.0085	4.7149	4.7161
25	1.3767	1.3421	0.0083	0.0078	4.6806	4.6162
30	1.3644	1.3014	0.0081	0.0072	4.6547	4.5361

Table E-24: 7 Stages (5%) - Duong- Recalculated Constants

7 Fractures (5%)- Duong								
Years	a	a(Eq.)	m	m(Eq.)	q1	q1 (Eq.)	q [∞]	q [∞] (Eq.)
2	0.8409	0.8090	1.0205	1.0190	9.5000	9.2223	-0.9938	-0.9338
3	0.9044	0.9059	1.0369	1.0380	8.5000	8.3432	-0.7985	-0.7879
4	0.9563	0.9746	1.0485	1.0516	7.7923	7.7195	-0.7067	-0.6844
5	1.0199	1.0280	1.0627	1.0620	7.0000	7.2357	-0.5678	-0.6041
7	1.0946	1.1084	1.0782	1.0779	6.2000	6.5063	-0.4359	-0.4830
10	1.1860	1.1936	1.0966	1.0946	5.4000	5.7330	-0.2880	-0.3546
15	1.2742	1.2905	1.1139	1.1137	4.8000	4.8539	-0.1671	-0.2087
20	1.3695	1.3593	1.1295	1.1272	4.2000	4.2303	-0.0986	-0.1051
25	1.4261	1.4126	1.1380	1.1377	3.8800	3.7465	-0.0681	-0.0248
30	1.4648	1.4562	1.1434	1.1463	3.6700	3.3512	-0.0225	0.0408

Table E-25: 7 Stages (6%) - Arps- Recalculated Constants

7 Fractures (6%)- Arps						
Years	n	n(Eq.)	di	di(Eq.)	qi	qi(Eq.)
3	2.1659	2.0598	0.0260	0.0220	6.3747	6.1677
5	1.8769	1.8693	0.0167	0.0169	5.8106	5.8188
7	1.7017	1.7535	0.0128	0.0142	5.4812	5.5998
10	1.5614	1.6386	0.0102	0.0118	5.2135	5.3767
15	1.4669	1.5171	0.0088	0.0096	5.0270	5.1338
20	1.4292	1.4364	0.0082	0.0083	4.9464	4.9682
25	1.4082	1.3768	0.0079	0.0074	4.9026	4.8434
30	1.3938	1.3299	0.0077	0.0067	4.8703	4.7438

Table E-26: 7 Stages (6%) - Duong- Recalculated Constants

7 Fractures (6%)- Duong								
Years	a	a(Eq.)	m	m(Eq.)	q1	q1 (Eq.)	q ∞	q ∞ (Eq.)
2	0.8152	0.8355	1.0128	1.0086	10.5000	10.3109	-1.1949	-1.0976
3	0.8663	0.9212	1.0282	1.0266	9.5000	9.3953	-0.9451	-0.9287
4	0.9240	0.9821	1.0422	1.0393	8.6000	8.7457	-0.7110	-0.8089
5	0.9520	1.0292	1.0474	1.0492	8.2000	8.2419	-0.7019	-0.7160
7	1.0208	1.1004	1.0624	1.0641	7.3000	7.4821	-0.5542	-0.5758
10	1.0930	1.1758	1.0780	1.0799	6.5000	6.6768	-0.4004	-0.4273
15	1.1735	1.2615	1.0953	1.0979	5.8000	5.7612	-0.2471	-0.2584
20	1.2407	1.3223	1.1083	1.1106	5.3000	5.1116	-0.1062	-0.1386
25	1.3562	1.3695	1.1259	1.1205	4.4500	4.6078	-0.0598	-0.0456
30	1.3714	1.4080	1.1288	1.1286	4.3800	4.1961	-0.0453	0.0303

Table E-27: 7 Stages (8%) - Arps- Recalculated Constants

7 Fractures (8%)- Arps						
Years	n	n(Eq.)	di	di(Eq.)	qi	qi(Eq.)
3	2.3116	2.2685	0.0298	0.0270	7.1613	7.0437
5	2.0681	2.0367	0.0201	0.0196	6.6052	6.5643
7	1.8845	1.8971	0.0150	0.0159	6.2081	6.2664
10	1.7091	1.7596	0.0113	0.0127	5.8304	5.9655
15	1.5665	1.6153	0.0089	0.0098	5.5192	5.6408
20	1.5029	1.5202	0.0079	0.0082	5.3734	5.4213
25	1.4696	1.4503	0.0075	0.0071	5.2974	5.2569
30	1.4494	1.3955	0.0072	0.0064	5.2475	5.1263

Table E-28: 7 Stages (8%) - Duong- Recalculated Constants

7 Fractures (8%)- Duong								
Years	<i>a</i>	<i>a</i>(Eq.)	<i>m</i>	<i>m</i>(Eq.)	<i>q</i>₁	<i>q</i>₁ (Eq.)	<i>q</i>_∞	<i>q</i>_∞ (Eq.)
2	0.7961	0.7231	0.9998	0.9869	12.5002	13.0401	-1.9834	-1.8912
3	0.7984	0.7877	1.0034	1.0029	12.1001	12.0159	-1.6560	-1.6025
4	0.8141	0.8335	1.0102	1.0142	11.5440	11.2892	-1.4846	-1.3976
5	0.8487	0.8690	1.0212	1.0230	10.6869	10.7256	-1.0469	-1.2387
7	0.9037	0.9226	1.0352	1.0363	9.7614	9.8756	-0.8557	-0.9991
10	0.9037	0.9794	1.0352	1.0503	9.6140	8.9747	-0.7681	-0.7451
15	0.9666	1.0439	1.0535	1.0663	8.6569	7.9505	-0.4529	-0.4564
20	1.1037	1.0897	1.0819	1.0776	6.9500	7.2238	-0.2087	-0.2515
25	1.1899	1.1252	1.0976	1.0864	6.1000	6.6601	-0.0912	-0.0926
30	1.2034	1.1543	1.1010	1.0936	6.0300	6.1996	-0.0911	0.0372

Table E-29: 7 Stages (300 ft.)- Arps- Table of Ratios

7 Fractures (300 ft.)- Arps- Table of Ratios			
Years	<i>n</i>-Ratio	<i>d</i>_i- Ratio	<i>q</i>_i-Ratio
3	2.046445	12.05036	1.94536
5	1.745842	6.937157	1.678362
7	1.572386	4.821888	1.522842
10	1.407294	3.279201	1.373688
15	1.240567	2.115502	1.221793
20	1.134395	1.550082	1.124321
25	1.05834	1.217853	1.054104
30	1	1	1

Table E-30: 7 Stages (300 ft.)- Duong- Table of Ratios

7 Fractures (300 ft.)- Duong- Table of Ratios				
Years	<i>a</i>-Ratio	<i>m</i>-Ratio	<i>q</i>₁- Ratio	<i>q</i>_∞ - Ratio
2	0.722260576	0.876752	2.261425	-5.13577
3	0.763845346	0.895205	2.072557	-4.21709
4	0.79335021	0.908298	1.938553	-3.56527
5	0.816235961	0.918454	1.834612	-3.05968
7	0.850744776	0.933767	1.677881	-2.29732
10	0.887325595	0.95	1.51174	-1.48918
15	0.928910366	0.968454	1.322872	-0.5705
20	0.95841523	0.981547	1.188868	0.081316
25	0.98130098	0.991702	1.084926	0.586904
30	1	1	1	1

Table E-31: 7 Stages (400 ft.)- Arps- Table of Ratios

7 Fractures (400 ft.)- Arps- Table of Ratios			
Years	n-Ratio	di- Ratio	qi-Ratio
3	1.737801	5.754399	1.548817
5	1.537292	3.902968	1.405558
7	1.41803	3.022301	1.318513
10	1.301695	2.304688	1.23212
15	1.180993	1.693491	1.140764
20	1.102204	1.36091	1.080084
25	1.044729	1.148624	1.035248
30	1	1	1

Table E-32: 7 Stages (400 ft.)- Duong- Table of Ratios

7 Fractures (400 ft.)- Duong- Table of Ratios				
Years	a-Ratio	m-Ratio	q ₁ - Ratio	q _∞ - Ratio
2	0.594886	0.867699	2.46421	-6.05697
3	0.655542	0.887508	2.24498	-5.00036
4	0.698578	0.901562	2.089434	-4.25068
5	0.73196	0.912464	1.968783	-3.66918
7	0.782295	0.928902	1.786856	-2.79236
10	0.835652	0.946328	1.594006	-1.8629
15	0.896308	0.966136	1.374776	-0.80629
20	0.939344	0.980191	1.21923	-0.05661
25	0.972725	0.991093	1.098579	0.524884
30	1	1	1	1

Table E-33: 7 Stages (500 ft.)- Arps- Table of Ratios

7 Fractures (500 ft.)- Arps- Table of Ratios			
Years	n-Ratio	di- Ratio	qi-Ratio
3	1.499685	2.870781	1.267652
5	1.370739	2.271921	1.202679
7	1.291921	1.947451	1.161712
10	1.213314	1.653947	1.119808
15	1.129747	1.373636	1.074004
20	1.07397	1.204065	1.042647
25	1.032609	1.087089	1.018957
30	1	1	1

Table E-34: 7 Stages (500 ft.)- Duong- Table of Ratios

7 Fractures (500 ft.)- Duong- Table of Ratios				
Years	a-Ratio	m-Ratio	q1- Ratio	q∞ - Ratio
2	0.57446	0.865827	2.710921	-6.4437626
3	0.638174	0.885917	2.454752	-5.3292389
4	0.68338	0.90017	2.272997	-4.5384718
5	0.718445	0.911226	2.132017	-3.9251052
7	0.771318	0.927897	1.919437	-3.0002259
10	0.827365	0.945568	1.694093	-2.0198144
15	0.89108	0.965657	1.437924	-0.9052908
20	0.936286	0.979911	1.256169	-0.1145237
25	0.97135	0.990967	1.115189	0.49884298
30	1	1	1	1

Table E-35: 7 Stages (600 ft.)- Arps- Table of Ratios

7 Fractures (600 ft.)- Arps- Table of Ratios			
Years	n-Ratio	di- Ratio	qi-Ratio
3	1.348963	1.610646	1.073989
5	1.262292	1.44903	1.057116
7	1.208267	1.35154	1.046147
10	1.153522	1.255348	1.034644
15	1.094294	1.154285	1.02172
20	1.054124	1.087554	1.012649
25	1.023985	1.038462	1.005668
30	1	1	1

Table E-36: 7 Stages (600 ft.)- Duong- Table of Ratios

7 Fractures (600 ft.)- Duong- Table of Ratios				
Years	a-Ratio	m-Ratio	q1- Ratio	q∞ - Ratio
2	0.544619	0.864131	3.047072	-8.9612525
3	0.612801	0.884474	2.740572	-7.4697955
4	0.661177	0.898908	2.523107	-6.41159
5	0.698701	0.910103	2.354428	-5.590782
7	0.755281	0.926985	2.100082	-4.3531074
10	0.815259	0.94488	1.830464	-3.0411195
15	0.883442	0.965223	1.523964	-1.5496625
20	0.931818	0.979657	1.3065	-0.491457
25	0.969341	0.990853	1.137821	0.32935104
30	1	1	1	1

Table E-37: 7 Stages (700 ft.)- Arps- Table of Ratios

7 Fractures (700 ft.)- Arps- Table of Ratios			
Years	n-Ratio	di- Ratio	qi-Ratio
3	1.318257	1.399587	1.035142
5	1.239876	1.299003	1.027241
7	1.190811	1.236731	1.022069
10	1.140918	1.173977	1.016616
15	1.086735	1.106497	1.010451
20	1.049859	1.060985	1.006101
25	1.02212	1.026976	1.002739
30	1	1	1

Table E-38: 7 Stages (700 ft.)- Duong- Table of Ratios

7 Fractures (700 ft.)- Duong- Table of Ratios				
Years	a-Ratio	m-Ratio	q1- Ratio	q∞ - Ratio
2	0.549841	0.857362	3.137242	-8.0753559
3	0.617242	0.878719	2.817242	-6.7165405
4	0.665063	0.893872	2.590198	-5.7524457
5	0.702156	0.905625	2.414089	-5.0046357
7	0.758088	0.923348	2.148539	-3.8770328
10	0.817378	0.942134	1.867045	-2.6817255
15	0.884778	0.963491	1.547044	-1.3229102
20	0.9326	0.978643	1.32	-0.3588153
25	0.969693	0.990397	1.143891	0.38899471
30	1	1	1	1

Table E-39: 7 Stages (0.0008 m-D) - Arps- Table of Ratios

7 Fractures (0.0008 m-D)- Arps- Table of Ratios			
Years	n-Ratio	di- Ratio	qi-Ratio
3	1.44544	6.053409	1.573983
5	1.332	4.05989	1.423298
7	1.262187	3.120629	1.332013
10	1.192173	2.36107	1.241632
15	1.117287	1.719513	1.146312
20	1.067025	1.373104	1.083153
25	1.029601	1.15324	1.03657
30	1	1	1

Table E-40: 7 Stages (0.0008 m-D) - Duong- Table of Ratios

7 Fractures (0.0008 m-D)- Duong- Table of Ratios				
Years	a-Ratio	m-Ratio	q1- Ratio	q[∞] - Ratio
2	0.662027	0.959239	2.223097	-2.116235
3	0.712631	0.965342	2.039968	-1.6496541
4	0.748534	0.969672	1.910036	-1.3186096
5	0.776383	0.973031	1.809252	-1.0618316
7	0.818376	0.978095	1.657284	-0.6746429
10	0.86289	0.983464	1.496191	-0.2642063
15	0.913493	0.989567	1.313062	0.20237464
20	0.949397	0.993897	1.183129	0.53341908
25	0.977246	0.997256	1.082346	0.79019709
30	1	1	1	1

Table E-41: 7 Stages (0.001 m-D) - Arps- Table of Ratios

7 Fractures (0.001 m-D)- Arps- Table of Ratios			
Years	n-Ratio	di- Ratio	qi-Ratio
3	1.435489	5.571857	1.534616983
5	1.324859	3.806281	1.395520217
7	1.256689	2.961347	1.31086002
10	1.188251	2.269512	1.226717479
15	1.114966	1.677136	1.137605228
20	1.065728	1.353207	1.078333194
25	1.029038	1.145696	1.03449337
30	1	1	1

Table E-42: 7 Stages (0.001 m-D) - Duong- Table of Ratios

7 Fractures (0.001 m-D)- Duong- Table of Ratios				
Years	a-Ratio	m-Ratio	q1- Ratio	q[∞] - Ratio
2	0.61951	0.940417	2.406373	-1.1462796
3	0.676479	0.949338	2.195803	-0.824926
4	0.7169	0.955668	2.046401	-0.5969221
5	0.748252	0.960577	1.930516	-0.4200685
7	0.795527	0.96798	1.755775	-0.1533956
10	0.845641	0.975828	1.570543	0.129289
15	0.902611	0.984749	1.359973	0.45064252
20	0.943031	0.991079	1.21057	0.67864648
25	0.974383	0.995989	1.094685	0.85550008
30	1	1	1	1

Table E-43: 7 Stages (0.002 m-D) - Arps- Table of Ratios

7 Fractures (0.002 m-D)- Arps- Table of Ratios			
Years	n-Ratio	di- Ratio	qi-Ratio
3	1.355189	2.673006	1.247384
5	1.266823	2.149169	1.187688
7	1.211789	1.861547	1.149937
10	1.156059	1.598567	1.111229
15	1.095812	1.344435	1.068806
20	1.05498	1.189025	1.039692
25	1.024358	1.080962	1.017657
30	1	1	1

Table E-44: 7 Stages (0.002 m-D) - Duong- Table of Ratios

7 Fractures (0.002 m-D)- Duong- Table of Ratios				
Years	a-Ratio	m-Ratio	q1- Ratio	q∞ - Ratio
2	0.590297	0.904071	2.627559	-6.3106808
3	0.65164	0.918434	2.383871	-5.2160829
4	0.695164	0.928625	2.210972	-4.4394534
5	0.728923	0.936529	2.076861	-3.8370527
7	0.779828	0.948448	1.874639	-2.9287087
10	0.83379	0.961083	1.660274	-1.9658253
15	0.895133	0.975446	1.416587	-0.8712274
20	0.938657	0.985637	1.243688	-0.0945979
25	0.972416	0.993541	1.109577	0.50780281
30	1	1	1	1

Table E-45: 7 Stages (0.004 m-D) - Arps- Table of Ratios

7 Fractures (0.004 m-D)- Arps- Table of Ratios			
Years	n-Ratio	di- Ratio	qi-Ratio
3	1.135011	1.233105	1.04472
5	1.103566	1.177096	1.03463
7	1.083331	1.1416	1.028036
10	1.062287	1.105142	1.021093
15	1.038859	1.065108	1.013257
20	1.022551	1.037586	1.007734
25	1.010078	1.01673	1.00347
30	1	1	1

Table E-46: 7 Stages (0.004 m-D) - Duong- Table of Ratios

7 Fractures (0.004 m-D)- Duong- Table of Ratios				
Years	a-Ratio	m-Ratio	q1- Ratio	q∞ - Ratio
2	0.57897	0.890224	2.788135	-6.5785879
3	0.642009	0.906661	2.520405	-5.4438774
4	0.686736	0.918322	2.330448	-4.6387875
5	0.721429	0.927368	2.183105	-4.0143113
7	0.773741	0.941007	1.960931	-3.0726801
10	0.829195	0.955466	1.725418	-2.0745109
15	0.892234	0.971902	1.457688	-0.9398004
20	0.936961	0.983564	1.26773	-0.1347105
25	0.971654	0.992609	1.120388	0.48976576
30	1	1	1	1

Table E-47: 7 Stages (0.006 m-D) - Arps- Table of Ratios

7 Fractures (0.006 m-D)- Arps- Table of Ratios			
Years	n-Ratio	di- Ratio	qi-Ratio
3	1.056818	1.099006	1.023293
5	1.04394	1.076228	1.018079
7	1.035544	1.061483	1.014659
10	1.026717	1.046073	1.011047
15	1.016775	1.028827	1.006956
20	1.009779	1.016763	1.004063
25	1.004385	1.007503	1.001825
30	1	1	1

Table E-48: 7 Stages (0.006 m-D) - Duong- Table of Ratios

7 Fractures (0.006 m-D)- Duong- Table of Ratios				
Years	a-Ratio	m-Ratio	q1- Ratio	q∞ - Ratio
2	0.572553	0.883624	2.939914	-6.2918322
3	0.636553	0.901048	2.649458	-5.2000564
4	0.681962	0.913411	2.443377	-4.4254292
5	0.717183	0.923001	2.283528	-3.8245817
7	0.770293	0.93746	2.042496	-2.9185796
10	0.826592	0.952788	1.786992	-1.9581787
15	0.890592	0.970213	1.496536	-0.866403
20	0.936	0.982575	1.290455	-0.0917757
25	0.971222	0.992165	1.130606	0.50907181
30	1	1	1	1

Table E-49: 7 Stages (4%) - Arps- Table of Ratios

7 Fractures (4%)- Arps- Table of Ratios			
Years	n-Ratio	di- Ratio	qi-Ratio
3	1.370882	2.094112	1.180321
5	1.278223	1.777406	1.137698
7	1.220639	1.595436	1.110467
10	1.162427	1.42284	1.082313
15	1.099616	1.249196	1.051173
20	1.057121	1.139004	1.029624
25	1.025293	1.060272	1.013214
30	1	1	1

Table E-50: 7 Stages (4%) - Duong- Table of Ratios

7 Fractures (4%)- Duong- Table of Ratios				
Years	a-Ratio	m-Ratio	q1- Ratio	q[∞] - Ratio
2	0.556105	0.887782	2.981074	-9.4112421
3	0.622568	0.9045839	2.684456	-7.8524101
4	0.669724	0.9165051	2.474002	-6.7464011
5	0.7063	0.9257519	2.310761	-5.888514
7	0.761454	0.9396948	2.064615	-4.5949287
10	0.819919	0.954475	1.80369	-3.223673
15	0.886382	0.9712769	1.507072	-1.664841
20	0.933537	0.9831981	1.296618	-0.558832
25	0.970114	0.9924448	1.133377	0.29905514
30	1	1	1	1

Table E-51: 7 Stages (5%) - Arps- Table of Ratios

7 Fractures (5%)- Arps- Table of Ratios			
Years	n- Ratio	di- Ratio	qi- Ratio
3	1.475707	2.673006	1.247384
5	1.353654	2.149169	1.187688
7	1.278827	1.861547	1.149937
10	1.204019	1.598567	1.111229
15	1.124279	1.344435	1.068806
20	1.070926	1.189025	1.039692
25	1.031292	1.080962	1.017657
30	1	1	1

Table E-52: 7 Stages (5%) - Duong- Table of Ratios

7 Fractures (5%)- Duong- Table of Ratios				
Years	a-Ratio	m-Ratio	q1- Ratio	q ∞ - Ratio
2	0.555535	0.8889617	2.751923	-22.89323
3	0.622083	0.905587	2.489615	-19.315796
4	0.669299	0.9173828	2.303505	-16.777566
5	0.705923	0.9265324	2.159146	-14.808762
7	0.761148	0.9403288	1.941471	-11.840055
10	0.819688	0.9549535	1.710727	-8.6930982
15	0.886236	0.9715789	1.448419	-5.115664
20	0.933452	0.9833747	1.262308	-2.5774341
25	0.970076	0.9925243	1.11795	-0.6086301
30	1	1	1	1

Table E-53: 7 Stages (6%) - Arps- Table of Ratios

7 Fractures (6%)- Arps- Table of Ratios			
Years	n- Ratio	di- Ratio	qi- Ratio
3	1.548817	3.258367	1.30017
5	1.405558	2.507215	1.226618
7	1.318513	2.109735	1.180458
10	1.23212	1.756965	1.133422
15	1.140764	1.427015	1.082225
20	1.080084	1.231218	1.047308
25	1.035248	1.098045	1.021002
30	1	1	1

Table E-54: 7 Stages (6%) - Duong- Table of Ratios

7 Fractures (6%)- Duong- Table of Ratios				
Years	a-Ratio	m-Ratio	q1- Ratio	q ∞ - Ratio
2	0.593412	0.8937006	2.457254	-36.226104
3	0.654288	0.9096164	2.239065	-30.652394
4	0.697481	0.9209088	2.084258	-26.697784
5	0.730984	0.9296679	1.96418	-23.63035
7	0.781502	0.9428755	1.783118	-19.005048
10	0.835054	0.9568761	1.591184	-14.10203
15	0.89593	0.9727918	1.372996	-8.52832
20	0.939123	0.9840842	1.218189	-4.5737099
25	0.972626	0.9928433	1.098111	-1.506276
30	1	1	1	1

Table E-55: 7 Stages (8%) - Arps- Table of Ratios

7 Fractures (8%)- Arps- Table of Ratios			
Years	n- Ratio	di- Ratio	qi- Ratio
3	1.625549	4.23643	1.374042
5	1.459452	3.075395	1.280516
7	1.35943	2.490454	1.222417
10	1.260877	1.991384	1.163705
15	1.15749	1.54435	1.100379
20	1.089319	1.289464	1.057549
25	1.039219	1.121106	1.02548
30	1	1	1

Table E-56: 7 Stages (8%) - Duong- Table of Ratios

7 Fractures (8%)- Duong- Table of Ratios				
Years	α-Ratio	m-Ratio	q1- Ratio	q_{∞} - Ratio
2	0.626499	0.9024356	2.103388	-50.849015
3	0.682421	0.9170435	1.938182	-43.085877
4	0.722099	0.927408	1.820967	-37.577843
5	0.752876	0.9354473	1.730047	-33.305481
7	0.799283	0.9475695	1.592953	-26.863298
10	0.848477	0.9604197	1.447627	-20.034309
15	0.904399	0.9750276	1.282421	-12.271171
20	0.944077	0.9853921	1.165206	-6.7631376
25	0.974854	0.9934314	1.074286	-2.4907747
30	1	1	1	1

E.2. Thirteen Stages Model

Table E-57: 13 Stages (300 ft.) - Arps- Recalculated Constants

13 Fractures (300 ft.)- Arps						
Years	<i>n</i>	<i>n</i> (Eq.)	<i>d_i</i>	<i>d_i</i> (Eq.)	<i>q_i</i>	<i>q_i</i> (Eq.)
3	1.4221	1.3840	0.0319	0.0310	8.7480	8.7282
5	1.3251	1.3293	0.0274	0.0272	8.4425	8.4001
7	1.2777	1.2944	0.0247	0.0249	8.2065	8.1908
10	1.2260	1.2584	0.0216	0.0227	7.8986	7.9746
15	1.2087	1.2188	0.0201	0.0204	7.7170	7.7357
20	1.1936	1.1914	0.0189	0.0190	7.5712	7.5706
25	1.1800	1.1706	0.0180	0.0179	7.4521	7.4450
30	1.1784	1.1538	0.0177	0.0171	7.4063	7.3438

Table E-58: 13 Stages (300 ft.) - Duong- Recalculated Constants

13 Fractures (300 ft.)- Duong								
Years	<i>a</i>	<i>a</i> (Eq.)	<i>m</i>	<i>m</i> (Eq.)	<i>q₁</i>	<i>q₁</i> (Eq.)	<i>q_∞</i>	<i>q_∞</i> (Eq.)
2	0.908	0.826	1.024	1.023	14.505	14.645	-3.934	-2.396
3	0.855	0.886	1.032	1.047	14.205	13.991	-1.972	-1.936
4	0.881	0.929	1.057	1.063	13.650	13.527	-0.977	-1.609
5	0.935	0.962	1.082	1.076	13.010	13.166	-0.558	-1.356
7	1.008	1.012	1.109	1.096	12.400	12.623	-0.245	-0.973
10	1.065	1.065	1.130	1.117	12.040	12.048	-0.082	-0.568
15	1.122	1.125	1.146	1.140	11.608	11.393	0.040	-0.108
20	1.162	1.168	1.155	1.157	11.208	10.929	0.040	0.219
25	1.215	1.201	1.164	1.170	10.500	10.569	0.055	0.472
30	1.249	1.228	1.170	1.180	10.040	10.274	0.060	0.680

Table E-59: 13 Stages (400 ft.) - Arps- Recalculated Constants

13 Fractures (400 ft.)- Arps						
Years	<i>n</i>	<i>n</i> (Eq.)	<i>d_i</i>	<i>d_i</i> (Eq.)	<i>q_i</i>	<i>q_i</i> (Eq.)
3	1.4853	1.4565	0.0329	0.0332	10.7806	10.8793
5	1.4045	1.4018	0.0311	0.0301	10.7141	10.6104
7	1.3446	1.3668	0.0280	0.0282	10.4438	10.4369
10	1.3052	1.3308	0.0258	0.0264	10.2169	10.2560
15	1.2869	1.2909	0.0245	0.0244	10.0698	10.0543
20	1.2705	1.2633	0.0233	0.0231	9.9284	9.9136
25	1.2500	1.2424	0.0220	0.0221	9.7641	9.8057
30	1.2400	1.2255	0.0214	0.0213	9.6799	9.7185

Table E-60: 13 Stages (400 ft.) - Duong- Recalculated Constants

13 Fractures (400 ft.)- Duong								
Years	a	$a(Eq.)$	m	$m(Eq.)$	$q1$	$q1 (Eq.)$	q^∞	$q^\infty (Eq.)$
2	0.9725	0.8959	1.0430	1.0386	15.0501	14.9214	-3.5566	-2.4569
3	0.9647	0.9773	1.0552	1.0634	14.5010	14.1875	-2.1417	-2.0018
4	0.9976	1.0351	1.0750	1.0810	13.8504	13.6668	-1.2916	-1.6789
5	1.0576	1.0799	1.0967	1.0947	13.0200	13.2629	-0.8145	-1.4284
7	1.1370	1.1475	1.1231	1.1153	12.2540	12.6539	-0.4128	-1.0507
10	1.2155	1.2192	1.1458	1.1371	11.6000	12.0083	-0.1834	-0.6503
15	1.2763	1.3006	1.1626	1.1619	11.2020	11.2744	-0.0539	-0.1952
20	1.3162	1.3584	1.1717	1.1795	10.8601	10.7537	-0.0061	0.1277
25	1.3474	1.4033	1.1775	1.1932	10.5303	10.3498	0.0153	0.3782
30	1.5718	1.4399	1.2182	1.2044	10.2320	10.0198	0.0708	0.5828

Table E-61: 13 Stages (500 ft.) - Arps- Recalculated Constants

13 Fractures (500 ft.)- Arps						
Years	n	$n(Eq.)$	d_i	$d_i(Eq.)$	q_i	$q_i(Eq.)$
3	1.5027	1.4611	0.0281	0.0278	11.7560	11.7977
5	1.4045	1.4127	0.0265	0.0263	11.7097	11.6718
7	1.3575	1.3817	0.0252	0.0254	11.6079	11.5896
10	1.3221	1.3495	0.0240	0.0245	11.4800	11.5031
15	1.3046	1.3139	0.0233	0.0234	11.4094	11.4056
20	1.2946	1.2892	0.0229	0.0227	11.3525	11.3369
25	1.2800	1.2703	0.0221	0.0222	11.2519	11.2839
30	1.2765	1.2551	0.0220	0.0218	11.2320	11.2408

Table E-62: 13 Stages (500 ft.) - Duong- Recalculated Constants

13 Fractures (500 ft.)- Duong								
Years	a	$a(Eq.)$	m	$m(Eq.)$	$q1$	$q1 (Eq.)$	q^∞	$q^\infty (Eq.)$
2	0.9457	0.9042	1.0494	1.0398	16.0602	15.6228	-2.5770	-2.3715
3	0.9859	0.9993	1.0506	1.0650	15.0210	14.7259	-2.5563	-1.9550
4	1.0626	1.0669	1.0809	1.0829	14.0400	14.0895	-1.5909	-1.6595
5	1.1180	1.1192	1.0992	1.0967	13.1801	13.5959	-1.0808	-1.4303
7	1.1985	1.1982	1.1236	1.1176	12.2501	12.8516	-0.6089	-1.0847
10	1.2621	1.2819	1.1430	1.1398	11.8010	12.0627	-0.3241	-0.7183
15	1.3396	1.3771	1.1621	1.1650	11.1505	11.1658	-0.1441	-0.3018
20	1.3988	1.4446	1.1808	1.1828	10.8500	10.5294	-0.0376	-0.0063
25	1.4654	1.4970	1.1851	1.1967	10.1590	10.0358	-0.0240	0.2229
30	1.6505	1.5398	1.2199	1.2080	9.8000	9.6326	0.0500	0.4102

Table E-63: 13 Stages (600 ft.) - Arps- Recalculated Constants

13 Fractures (600 ft.)- Arps						
Years	n	n(Eq.)	d_i	d_i(Eq.)	q_i	q_i(Eq.)
3	1.4450	1.4419	0.0224	0.0225	12.0700	12.1261
5	1.4031	1.3962	0.0214	0.0215	12.0418	12.0396
7	1.3678	1.3669	0.0210	0.0209	12.0225	11.9831
10	1.3229	1.3365	0.0200	0.0202	11.9181	11.9234
15	1.3027	1.3028	0.0198	0.0195	11.9095	11.8559
20	1.2795	1.2794	0.0190	0.0190	11.8087	11.8082
25	1.2648	1.2616	0.0186	0.0186	11.7405	11.7714
30	1.2538	1.2472	0.0182	0.0183	11.6853	11.7414

Table E-64: 13 Stages (600 ft.) - Duong- Recalculated Constants

13 Fractures (600 ft.)- Duong								
Years	a	a(Eq.)	m	m(Eq.)	q₁	q₁ (Eq.)	q_∞	q_∞ (Eq.)
2	1.0207	0.9083	1.0409	1.0351	16.5065	16.3575	-5.4934	-4.0349
3	1.0341	1.0114	1.0506	1.0605	15.5000	15.2324	-4.0200	-3.3121
4	1.0848	1.0845	1.0775	1.0785	14.5404	14.4340	-1.9835	-2.7993
5	1.1384	1.1413	1.0943	1.0925	13.5400	13.8148	-1.3903	-2.4015
7	1.2131	1.2268	1.1163	1.1136	12.5405	12.8811	-0.8371	-1.8017
10	1.3027	1.3175	1.1386	1.1360	11.6400	11.8913	-0.4788	-1.1659
15	1.3954	1.4207	1.1590	1.1614	10.7500	10.7662	-0.2525	-0.4431
20	1.5050	1.4938	1.1808	1.1794	9.9700	9.9678	-0.1206	0.0697
25	1.5652	1.5506	1.1998	1.1934	9.5200	9.3486	-0.0200	0.4675
30	1.6060	1.5969	1.1986	1.2049	9.0200	8.8427	-0.0320	0.7925

Table E-65: 13 Stages (700 ft.) - Arps- Recalculated Constants

13 Fractures (700 ft.)- Arps						
Years	n	n(Eq.)	d_i	d_i(Eq.)	q_i	q_i(Eq.)
3	1.4685	1.4824	0.0191	0.01917	12.2226	12.1671
5	1.4469	1.4399	0.0190	0.01837	12.1821	12.1480
7	1.4200	1.4125	0.0177	0.01787	12.0679	12.1031
10	1.3846	1.3841	0.0169	0.01735	12.0057	12.0557
15	1.3641	1.3525	0.0165	0.01677	11.9766	12.0021
20	1.3299	1.3305	0.0163	0.01638	11.9712	11.9642
25	1.3029	1.3136	0.0162	0.01608	11.9570	11.9348
30	1.2977	1.3001	0.0161	0.01584	11.9309	11.9109

Table E-66: 13 Stages (700 ft.) - Duong- Recalculated Constants

13 Fractures (700 ft.)- Duong								
Years	a	a(Eq.)	m	m(Eq.)	q1	q1 (Eq.)	q ∞	q ∞ (Eq.)
2	1.0230	0.8635	1.0359	1.0230	18.5000	17.9522	-7.2145	-5.1795
3	1.0204	0.9753	1.0446	1.0493	17.4200	16.5764	-4.6330	-4.2592
4	1.0712	1.0546	1.0649	1.0680	15.1420	15.6003	-2.7645	-3.6062
5	1.1180	1.1161	1.0812	1.0825	14.0142	14.8432	-1.8896	-3.0998
7	1.1803	1.2088	1.1011	1.1043	13.0151	13.7015	-1.1692	-2.3361
10	1.2522	1.3071	1.1206	1.1274	12.1250	12.4913	-0.7251	-1.5265
15	1.3525	1.4188	1.1386	1.1538	11.4051	11.1156	-0.4736	-0.6062
20	1.4950	1.4981	1.1853	1.1724	10.4580	10.1395	-0.0630	0.0467
25	1.6055	1.5596	1.1964	1.1869	9.7000	9.3824	-0.0910	0.5532
30	1.6532	1.6099	1.1974	1.1987	8.7900	8.7637	-0.0218	0.9670

Table E-67: 13 Stages (0.0008 m-D) - Arps- Recalculated Constants

13 Fractures (0.0008 m-D)- Arps						
Years	n	n(Eq.)	di	di(Eq.)	qi	qi (Eq.)
3	1.7081	1.6735	0.0279	0.0276	7.6043	7.6167
5	1.6072	1.6032	0.0266	0.0262	7.5911	7.5625
7	1.5350	1.5585	0.0249	0.0253	7.5138	7.5269
10	1.4861	1.5125	0.0242	0.0244	7.4964	7.4894
15	1.4426	1.4619	0.0232	0.0234	7.4440	7.4470
20	1.4236	1.4270	0.0227	0.0228	7.4178	7.4171
25	1.4126	1.4005	0.0224	0.0223	7.3962	7.3940
30	1.4055	1.3792	0.0222	0.0218	7.3783	7.3751

Table E-68: 13 Stages (0.0008 m-D) - Duong- Recalculated Constants

13 Fractures (0.0008 m-D)- Duong								
Years	a	a(Eq.)	m	m(Eq.)	q1	q1 (Eq.)	q ∞	q ∞ (Eq.)
2	0.9613	0.8956	1.0659	1.0524	10.5000	10.3037	-0.8960	-0.8934
3	0.9966	0.9991	1.0690	1.0717	9.5000	9.3857	-0.7347	-0.7529
4	1.0467	1.0724	1.0786	1.0854	8.8000	8.7344	-0.6917	-0.6533
5	1.1117	1.1294	1.0919	1.0961	8.0000	8.2292	-0.5881	-0.5760
7	1.1886	1.2152	1.1079	1.1121	7.2000	7.4675	-0.4537	-0.4594
10	1.2727	1.3062	1.1254	1.1291	6.5000	6.6599	-0.3139	-0.3359
15	1.3701	1.4096	1.1453	1.1485	5.8650	5.7420	-0.1826	-0.1954
20	1.5005	1.4830	1.1660	1.1622	5.0000	5.0907	-0.0357	-0.0958
25	1.5811	1.5399	1.1782	1.1728	4.5500	4.5855	-0.0246	-0.0185
30	1.6079	1.5864	1.1830	1.1815	4.4500	4.1727	-0.0143	0.0447

Table E-69: 13 Stages (0.001 m-D) - Arps- Recalculated Constants

13 Fractures (0.001 m-D)- Arps						
Years	n	n(Eq.)	di	di(Eq.)	qi	qi(Eq.)
3	1.7123	1.6711	0.0308	0.0305	8.6873	8.7135
5	1.5960	1.6042	0.0290	0.0291	8.6646	8.6602
7	1.5358	1.5616	0.0280	0.0281	8.6457	8.6253
10	1.4898	1.5176	0.0272	0.0272	8.6281	8.5885
15	1.4508	1.4692	0.0261	0.0262	8.5700	8.5468
20	1.4348	1.4358	0.0257	0.0255	8.5459	8.5173
25	1.4187	1.4104	0.0249	0.0250	8.4804	8.4946
30	1.4124	1.3899	0.0247	0.0246	8.4619	8.4760

Table E-70: 13 Stages (0.001 m-D) - Duong- Recalculated Constants

13 Fractures (0.001 m-D)- Duong								
Years	a	a(Eq.)	m	m(Eq.)	q1	q1 (Eq.)	q∞	q∞ (Eq.)
2	0.9885	0.9541	1.0637	1.0695	10.8000	9.8069	-1.0585	-0.8994
3	1.0744	1.0804	1.0813	1.0904	9.5000	8.8821	-0.9246	-0.7583
4	1.1836	1.1700	1.1034	1.1052	8.1000	8.2259	-0.7052	-0.6581
5	1.2484	1.2395	1.1159	1.1167	7.4000	7.7169	-0.5872	-0.5805
7	1.2953	1.3443	1.1267	1.1341	7.0200	6.9494	-0.4570	-0.4633
10	1.4233	1.4553	1.1504	1.1525	6.0200	6.1358	-0.2781	-0.3392
15	1.5183	1.5816	1.1689	1.1734	5.5000	5.2109	-0.1489	-0.1980
20	1.7270	1.6712	1.1963	1.1883	4.2000	4.5547	-0.0666	-0.0979
25	1.7728	1.7407	1.2033	1.1998	4.0200	4.0457	-0.0364	-0.0202
30	1.8038	1.7974	1.2079	1.2092	3.9000	3.6299	-0.0194	0.0433

Table E-71: 13 Stages (0.002 m-D) - Arps- Recalculated Constants

13 Fractures (0.002 m-D)- Arps						
Years	n	n(Eq.)	di	di(Eq.)	qi	qi(Eq.)
3	1.7204	1.6748	0.0432	0.0432	13.2629	13.2726
5	1.6219	1.6176	0.0424	0.0419	13.2042	13.1983
7	1.5506	1.5810	0.0412	0.0411	13.1656	13.1495
10	1.5202	1.5431	0.0394	0.0402	13.1046	13.0980
15	1.4899	1.5012	0.0390	0.0393	13.0411	13.0397
20	1.4711	1.4721	0.0384	0.0386	12.9876	12.9985
25	1.4602	1.4499	0.0382	0.0381	12.9649	12.9667
30	1.4502	1.4321	0.0381	0.0377	12.9520	12.9407

Table E-72: 13 Stages (0.002 m-D) - Duong- Recalculated Constants

13 Fractures (0.002 m-D)- Duong								
Years	<i>a</i>	<i>a</i> (Eq.)	<i>m</i>	<i>m</i> (Eq.)	<i>q</i> 1	<i>q</i> 1 (Eq.)	<i>q</i> ∞	<i>q</i> ∞ (Eq.)
2	1.1123	1.0930	1.0908	1.0925	11.8500	11.5328	-1.0014	-1.0535
3	1.1920	1.2417	1.1081	1.1168	10.8500	10.3808	-1.0091	-0.8746
4	1.3236	1.3472	1.1317	1.1341	9.5000	9.5635	-0.8244	-0.7476
5	1.4314	1.4290	1.1512	1.1475	8.5000	8.9296	-0.6958	-0.6492
7	1.5432	1.5524	1.1739	1.1677	7.6000	7.9737	-0.3947	-0.5007
10	1.6922	1.6832	1.1988	1.1892	6.5000	6.9604	-0.1829	-0.3433
15	1.8213	1.8318	1.2185	1.2136	5.7040	5.8084	-0.0557	-0.1643
20	1.9194	1.9373	1.2308	1.2308	5.1000	4.9911	-0.0063	-0.0374
25	2.0028	2.0192	1.2397	1.2443	4.6000	4.3572	0.0017	0.0611
30	2.0830	2.0860	1.2474	1.2552	4.1400	3.8392	0.0013	0.1415

Table E-73: 13 Stages (0.004 m-D) - Arps- Recalculated Constants

13 Fractures (0.004 m-D)- Arps						
Years	<i>n</i>	<i>n</i> (Eq.)	<i>d</i> i	<i>d</i> i(Eq.)	<i>q</i> i	<i>q</i> i (Eq.)
3	1.4505	1.4513	0.0428	0.0427	17.0135	17.0525
5	1.4059	1.4068	0.0418	0.0417	17.0062	16.9829
7	1.3851	1.3782	0.0411	0.0410	16.9626	16.9373
10	1.3451	1.3485	0.0401	0.0403	16.9083	16.8890
15	1.3042	1.3156	0.0391	0.0395	16.8576	16.8343
20	1.2874	1.2927	0.0388	0.0389	16.7685	16.7956
25	1.2764	1.2752	0.0387	0.0385	16.7524	16.7657
30	1.2654	1.2611	0.0386	0.0382	16.7465	16.7412

Table E-74: 13 Stages (0.004 m-D) - Duong- Recalculated Constants

13 Fractures (0.004 m-D)- Duong								
Years	<i>a</i>	<i>a</i> (Eq.)	<i>m</i>	<i>m</i> (Eq.)	<i>q</i> 1	<i>q</i> 1 (Eq.)	<i>q</i> ∞	<i>q</i> ∞ (Eq.)
2	1.2850	1.2223	1.1144	1.1259	14.1000	13.7488	-2.8031	-1.7679
3	1.3870	1.4023	1.1450	1.1517	12.5000	12.1833	-1.4261	-1.4262
4	1.5092	1.5300	1.1719	1.1699	11.0000	11.0725	-0.8094	-1.1837
5	1.6182	1.6290	1.1924	1.1840	9.8400	10.2110	-0.4860	-0.9957
7	1.7648	1.7784	1.2171	1.2054	8.5000	8.9118	-0.1985	-0.7121
10	1.9200	1.9367	1.2385	1.2280	7.2000	7.5347	-0.0380	-0.4115
15	2.0771	2.1167	1.2563	1.2537	6.0000	5.9692	0.0398	-0.0698
20	2.2197	2.2444	1.2693	1.2719	5.0000	4.8585	0.0622	0.1727
25	2.3604	2.3435	1.2805	1.2861	4.1500	3.9969	0.0695	0.3608
30	2.4872	2.4244	1.2898	1.2976	3.5000	3.2930	0.0710	0.5144

Table E-75: 13 Stages (0.006 m-D) - Arps- Recalculated Constants

13 Fractures (0.006 m-D)- Arps						
Years	n	n(Eq.)	di	di(Eq.)	qi	qi(Eq.)
3	1.6708	1.6675	0.0675	0.0687	23.0468	23.1155
5	1.6306	1.6238	0.0687	0.0676	23.0052	23.0330
7	1.5843	1.5956	0.0674	0.0669	23.0020	22.9789
10	1.5609	1.5663	0.0663	0.0661	22.9803	22.9216
15	1.5347	1.5336	0.0652	0.0653	22.8944	22.8566
20	1.5145	1.5108	0.0645	0.0647	22.7852	22.8106
25	1.4904	1.4934	0.0641	0.0642	22.6948	22.7750
30	1.4822	1.4793	0.0636	0.0639	22.6849	22.7460

Table E-76: 13 Stages (0.006 m-D) - Duong- Recalculated Constants

13Fractures (0.006 m-D)- Duong								
Years	a	a(Eq.)	m	m(Eq.)	q1	q1 (Eq.)	q[∞]	q[∞] (Eq.)
2	1.2522	1.2034	1.1154	1.1303	17.5310	17.0433	-2.9652	-1.1306
3	1.3691	1.3868	1.1452	1.1563	15.5000	15.0622	-1.5843	-0.9115
4	1.4946	1.5169	1.1799	1.1748	13.7000	13.6566	-0.6592	-0.7560
5	1.6210	1.6178	1.2023	1.1891	12.0000	12.5663	-0.3494	-0.6355
7	1.7764	1.7700	1.2264	1.2106	10.2000	10.9223	-0.1013	-0.4536
10	1.9187	1.9314	1.2447	1.2335	8.7000	9.1796	0.0167	-0.2609
15	2.0668	2.1148	1.2600	1.2595	7.2500	7.1985	0.0615	-0.0418
20	2.2106	2.2449	1.2723	1.2779	6.0000	5.7929	0.0694	0.1137
25	2.3385	2.3458	1.2821	1.2922	5.0500	4.7026	0.0680	0.2343
30	2.5112	2.4283	1.2995	1.3039	4.0125	3.8117	0.0997	0.3328

Table E-77: 13 Stages (4%) - Arps- Recalculated Constants

13 Fractures (4%)- Arps						
Years	n	n(Eq.)	di	di(Eq.)	qi	qi(Eq.)
3	1.7081	1.6803	0.0570	0.0579	13.3167	13.4783
5	1.6020	1.5982	0.0528	0.0506	13.2247	13.0381
7	1.5292	1.5464	0.0466	0.0463	12.8230	12.7561
10	1.4691	1.4933	0.0412	0.0421	12.4245	12.4637
15	1.4210	1.4351	0.0371	0.0378	12.0791	12.1396
20	1.3910	1.3952	0.0345	0.0350	11.8450	11.9147
25	1.3710	1.3650	0.0329	0.0330	11.6904	11.7431
30	1.3651	1.3409	0.0325	0.0315	11.6599	11.6047

Table E-78: 13 Stages (4%) - Duong- Recalculated Constants

13 Fractures (4%)- Duong								
Years	a	a(Eq.)	m	m(Eq.)	q1	q1 (Eq.)	q ∞	q ∞ (Eq.)
2	1.0875	1.0610	1.0742	1.0784	13.0000	12.8444	-2.5596	-1.8373
3	1.1131	1.1365	1.0908	1.0985	12.4000	12.1607	-1.5919	-1.5043
4	1.1605	1.1900	1.1090	1.1128	11.8000	11.6757	-1.0387	-1.2680
5	1.2235	1.2316	1.1273	1.1238	11.1000	11.2995	-0.7009	-1.0847
7	1.2984	1.2942	1.1488	1.1405	10.5000	10.7322	-0.3861	-0.8083
10	1.3853	1.3606	1.1696	1.1582	9.8000	10.1308	-0.1860	-0.5154
15	1.4579	1.4361	1.1857	1.1783	9.2783	9.4472	-0.0251	-0.1823
20	1.5124	1.4897	1.1953	1.1926	8.8000	8.9622	-0.0251	0.0540
25	1.5248	1.5313	1.1982	1.2037	8.7500	8.5860	-0.0079	0.2373
30	1.5331	1.5652	1.1999	1.2127	8.7000	8.2786	0.0000	0.3870

Table E-79: 13 Stages (5%) - Arps- Recalculated Constants

13 Fractures (5%)- Arps						
Years	n	n(Eq.)	di	di(Eq.)	qi	qi(Eq.)
3	1.7421	1.7254	0.0475	0.0506	13.4537	13.7882
5	1.6410	1.6253	0.0467	0.0435	13.6124	13.3106
7	1.5517	1.5626	0.0405	0.0394	13.1748	13.0052
10	1.4807	1.4987	0.0355	0.0354	12.7633	12.6890
15	1.4107	1.4293	0.0305	0.0314	12.2690	12.3389
20	1.3765	1.3820	0.0282	0.0289	12.0222	12.0964
25	1.3523	1.3464	0.0266	0.0270	11.8401	11.9116
30	1.3452	1.3179	0.0263	0.0256	11.7978	11.7627

Table E-80: 13 Stages (5%) - Duong- Recalculated Constants

13Fractures (5%)- Duong								
Years	a	a(Eq.)	m	m(Eq.)	q1	q1 (Eq.)	q ∞	q ∞ (Eq.)
2	1.0862	1.0788	1.0705	1.0748	13.7000	13.0629	-2.8015	-2.1152
3	1.1260	1.1498	1.0869	1.0938	12.7000	12.3878	-1.8750	-1.7415
4	1.1887	1.2001	1.1062	1.1073	11.7000	11.9088	-1.2571	-1.4764
5	1.2427	1.2392	1.1217	1.1178	11.0500	11.5373	-0.8933	-1.2707
7	1.3090	1.2980	1.1411	1.1336	10.5000	10.9771	-0.5259	-0.9606
10	1.3804	1.3605	1.1600	1.1503	10.0000	10.3832	-0.2776	-0.6319
15	1.4489	1.4314	1.1716	1.1693	9.5611	9.7081	-0.1199	-0.2583
20	1.4882	1.4818	1.1848	1.1828	9.3000	9.2291	-0.0589	0.0069
25	1.5146	1.5208	1.1899	1.1933	9.1000	8.8576	-0.0302	0.2125
30	1.5286	1.5527	1.1926	1.2018	9.0000	8.5540	-0.0162	0.3805

Table E-81: 13 Stages (6%) - Arps- Recalculated Constants

13 Fractures (6%)- Arps						
Years	n	n(Eq.)	di	di(Eq.)	qi	qi(Eq.)
3	1.8065	1.7522	0.0444	0.0436	13.8793	14.0018
5	1.6359	1.6379	0.0379	0.0371	13.7552	13.4893
7	1.5368	1.5668	0.0329	0.0334	13.1509	13.1620
10	1.4502	1.4947	0.0283	0.0298	12.6930	12.8237
15	1.4039	1.4168	0.0261	0.0262	12.4598	12.4497
20	1.3625	1.3640	0.0238	0.0240	12.1694	12.1910
25	1.3404	1.3244	0.0226	0.0223	12.0141	11.9940
30	1.3209	1.2929	0.0216	0.0211	11.8694	11.8354

Table E-82: 13 Stages (6%) – Duong - Recalculated Constants

13 Fractures (6%)- Duong								
Years	a	a(Eq.)	m	m(Eq.)	q1	q1 (Eq.)	q∞	q∞ (Eq.)
2	1.0571	1.0190	1.0616	1.0573	15.0000	14.7731	-3.1843	-2.6143
3	1.0752	1.0856	1.0714	1.0765	14.3000	14.0343	-2.3936	-2.1643
4	1.1137	1.1329	1.0858	1.0900	13.4500	13.5102	-1.7041	-1.8450
5	1.1446	1.1695	1.0975	1.1006	13.0000	13.1036	-1.2677	-1.5973
7	1.2183	1.2248	1.1196	1.1164	12.1000	12.4906	-0.7515	-1.2238
10	1.2827	1.2834	1.1387	1.1333	11.6000	11.8407	-0.4184	-0.8280
15	1.3587	1.3500	1.1579	1.1524	11.0000	11.1019	-0.1971	-0.3779
20	1.4137	1.3973	1.1694	1.1660	10.5000	10.5778	-0.1069	-0.0586
25	1.4406	1.4340	1.1752	1.1765	10.3000	10.1712	-0.0647	0.1890
30	1.4555	1.4639	1.1784	1.1851	10.2000	9.8390	-0.0432	0.3914

Table E-83: 13 Stages (8%) - Arps- Recalculated Constants

13 Fractures (8%)- Arps						
Years	n	n(Eq.)	di	di(Eq.)	qi	qi(Eq.)
3	1.9041	1.8697	0.0387	0.0388	14.4732	14.6344
5	1.7050	1.7150	0.0326	0.0319	14.1389	14.0341
7	1.5960	1.6202	0.0264	0.0280	13.5263	13.6522
10	1.5045	1.5254	0.0252	0.0245	13.4493	13.2587
15	1.4325	1.4244	0.0223	0.0209	13.0902	12.8251
20	1.3515	1.3568	0.0187	0.0187	12.5148	12.5261
25	1.3024	1.3066	0.0167	0.0172	12.1619	12.2990
30	1.2834	1.2670	0.0160	0.0160	12.0275	12.1165

Table E-84: 13 Stages (8%) – Duong - Recalculated Constants

13 Fractures (8%)- Duong								
Years	<i>a</i>	<i>a</i> (Eq.)	<i>m</i>	<i>m</i> (Eq.)	<i>q</i> ₁	<i>q</i> ₁ (Eq.)	<i>q</i> _∞	<i>q</i> _∞ (Eq.)
2	1.0689	1.0287	1.0645	1.0553	15.4200	15.3541	-2.9859	-2.8577
3	1.0830	1.0919	1.0688	1.0727	15.0000	14.5797	-2.7861	-2.3869
4	1.1138	1.1367	1.0788	1.0851	14.2000	14.0302	-2.0824	-2.0529
5	1.1534	1.1715	1.0905	1.0946	13.4000	13.6040	-1.6133	-1.7938
7	1.2091	1.2239	1.1075	1.1091	12.6000	12.9613	-1.0579	-1.4031
10	1.2765	1.2794	1.1266	1.1244	11.9000	12.2801	-0.6278	-0.9890
15	1.3553	1.3426	1.1465	1.1418	11.2200	11.5056	-0.3203	-0.5182
20	1.3994	1.3874	1.1573	1.1541	10.9200	10.9562	-0.1906	-0.1842
25	1.4300	1.4222	1.1643	1.1637	10.7000	10.5299	-0.0910	0.0749
30	1.4459	1.4506	1.1698	1.1715	10.6200	10.1817	-0.0679	0.2866

Table E-85: 13 Stages (300 ft.)- Arps- Table of Ratios

13Fractures (300 ft.)- Arps- Table of Ratios			
Years	<i>n</i> - Ratio	<i>di</i> - Ratio	<i>qi</i> - Ratio
3	1.199499	1.819701	1.188502
5	1.152057	1.59338	1.14383
7	1.121837	1.45991	1.115326
10	1.090668	1.330612	1.085886
15	1.056286	1.197479	1.053361
20	1.03255	1.111178	1.030877
25	1.014508	1.048545	1.013768
30	1	1	1

Table E-86: 13 Stages (300 ft.)- Duong- Table of Ratios

13Fractures (300 ft.)- Duong- Table of Ratios				
Years	<i>a</i> -Ratio	<i>m</i> -Ratio	<i>q</i> ₁ -Ratio	<i>q</i> _∞ - Ratio
2	0.672724	0.866923	1.425403	-3.52604
3	0.721726	0.886848	1.361709	-2.84837
4	0.756493	0.900985	1.316518	-2.36756
5	0.78346	0.911951	1.281465	-1.99461
7	0.824124	0.928485	1.228609	-1.43226
10	0.867229	0.946013	1.172579	-0.83614
15	0.916231	0.965938	1.108885	-0.15848
20	0.950998	0.980075	1.063694	0.322335
25	0.977966	0.99104	1.028641	0.695281
30	1	1	1	1

Table E-87: 13 Stages (400 ft.)- Arps- Table of Ratios

13Fractures (400 ft.)- Arps- Table of Ratios			
Years	n- Ratio	di- Ratio	qi- Ratio
3	1.188502	1.555966	1.119438
5	1.14383	1.410604	1.091766
7	1.115326	1.322356	1.073913
10	1.085886	1.23483	1.055307
15	1.053361	1.142346	1.034548
20	1.030877	1.08096	1.020066
25	1.013768	1.035626	1.008974
30	1	1	1

Table E-88: 13 Stages (400 ft.)- Duong- Table of Ratios

13 Fractures (400 ft.)- Duong-Table of Ratios				
Years	a-Ratio	m-Ratio	q₁-Ratio	q_∞- Ratio
2	0.622163	0.862389	1.489187	-4.21544
3	0.678735	0.882993	1.415943	-3.43455
4	0.718874	0.897611	1.363976	-2.8805
5	0.750007	0.908951	1.323667	-2.45075
7	0.796953	0.926049	1.262886	-1.80274
10	0.846718	0.944173	1.198455	-1.11582
15	0.90329	0.964777	1.125211	-0.33493
20	0.943428	0.979396	1.073244	0.219114
25	0.974562	0.990735	1.032935	0.648867
30	1	1	1	1

Table E-89: 13 Stages (500 ft.)- Arps- Table of Ratios

13Fractures (500 ft.)- Arps- Table of Ratios			
Years	n- Ratio	di- Ratio	qi- Ratio
3	1.164126	1.279381	1.049542
5	1.125532	1.211329	1.038344
7	1.100813	1.168494	1.031033
10	1.075202	1.12474	1.023339
15	1.04681	1.076986	1.014663
20	1.027122	1.04434	1.008551
25	1.012106	1.0197	1.003836
30	1	1	1

Table E-90: 13 Stages (500 ft.)- Duong- Table of Ratios

13 Fractures (500 ft.)- Duong- Table of Ratios				
Years	α -Ratio	m -Ratio	q_1 -Ratio	q_{∞} - Ratio
2	0.587222	0.860788	1.621871	-5.781184
3	0.649026	0.881632	1.528761	-4.7658655
4	0.692876	0.896421	1.462698	-4.0454856
5	0.726889	0.907892	1.411456	-3.4867155
7	0.778176	0.925189	1.334189	-2.6441609
10	0.832543	0.943524	1.252283	-1.751017
15	0.894346	0.964368	1.159173	-0.7356985
20	0.938196	0.979156	1.09311	-0.0153185
25	0.972209	0.990627	1.041868	0.54345158
30	1	1	1	1

Table E-91: 13 Stages (600 ft.)- Arps- Table of Ratios

13Fractures (600 ft.)- Arps- Table of Ratios			
Years	n - Ratio	di - Ratio	qi - Ratio
3	1.156112	1.227439	1.032761
5	1.119499	1.172885	1.025402
7	1.096017	1.138283	1.020583
10	1.071664	1.102716	1.015499
15	1.044636	1.063633	1.009751
20	1.025873	1.036745	1.005693
25	1.011552	1.016359	1.002556
30	1	1	1

Table E-92: 13 Stages (600 ft.)- Duong- Table of Ratios

13 Fractures (600 ft.)- Duong- Table of Ratios				
Years	α -Ratio	m -Ratio	q_1 -Ratio	q_{∞} - Ratio
2	0.56876	0.859075	1.849838	-5.0915153
3	0.633328	0.880175	1.722595	-4.1794581
4	0.67914	0.895146	1.632315	-3.5323431
5	0.714674	0.906758	1.562288	-3.0304018
7	0.768255	0.924268	1.456697	-2.2735377
10	0.825053	0.942829	1.344765	-1.4712295
15	0.889621	0.963929	1.217523	-0.5591722
20	0.935432	0.9789	1.127243	0.08794271
25	0.970966	0.990512	1.057216	0.58988406
30	1	1	1	1

Table E-93: 13 Stages (700 ft.)- Arps- Table of Ratios

13Fractures (700 ft.)- Arps- Table of Ratios			
Years	n- Ratio	di- Ratio	qi- Ratio
3	1.14025	1.210598	1.021505
5	1.107528	1.160343	1.019905
7	1.086489	1.128387	1.016137
10	1.064623	1.095471	1.012158
15	1.0403	1.059218	1.007654
20	1.023381	1.034226	1.00447
25	1.010447	1.015248	1.002008
30	1	1	1

Table E-94: 13 Stages (700 ft.)- Duong- Table of Ratios

13 Fractures (700 ft.)- Duong- Table of Ratios				
Years	a-Ratio	m-Ratio	q₁-Ratio	q_∞- Ratio
2	0.536398	0.853385	2.048458	-5.3562318
3	0.605811	0.875337	1.891477	-4.4045396
4	0.655061	0.890913	1.780097	-3.7293033
5	0.693262	0.902994	1.693704	-3.2055493
7	0.750864	0.92121	1.563434	-2.4157945
10	0.811924	0.940521	1.425343	-1.5786207
15	0.881337	0.962473	1.268361	-0.6269285
20	0.930587	0.978048	1.156981	0.04830781
25	0.968788	0.990129	1.070588	0.57206182
30	1	1	1	1

Table E-95: 13 Stages (0.0008 m-D) - Arps-Table of Ratios

13Fractures (0.0008 m-D)- Arps-Table of Ratios			
Years	n- Ratio	di- Ratio	qi- Ratio
3	1.213389	1.264736	1.032761
5	1.162424	1.200526	1.025402
7	1.13003	1.160022	1.020583
10	1.096676	1.118578	1.015499
15	1.059953	1.07326	1.009751
20	1.034646	1.042225	1.005693
25	1.015433	1.018771	1.002556
30	1	1	1

Table E-96: 13 Stages (0.0008 m-D) – Duong -Table of Ratios

13 Fractures (0.0008 m-D)- Duong- Table of Ratios				
Years	α-Ratio	m-Ratio	q_1-Ratio	q_∞- Ratio
2	0.564546	0.890673	2.469322	-19.997725
3	0.629745	0.907042	2.249327	-16.853823
4	0.676004	0.918656	2.093238	-14.623189
5	0.711886	0.927665	1.972165	-12.892974
7	0.76599	0.941248	1.789604	-10.28403
10	0.823343	0.955648	1.59608	-7.5184383
15	0.888542	0.972017	1.376085	-4.3745362
20	0.934801	0.983631	1.219996	-2.1439021
25	0.970683	0.992639	1.098923	-0.4136879
30	1	1	1	1

Table E-97: 13 Stages (0.001 m-D) – Arps -Table of Ratios

13Fractures (0.001 m-D)- Arps-Table of Ratios			
Years	n- Ratio	di- Ratio	qi- Ratio
3	1.202264	1.241652	1.028016
5	1.154123	1.18344	1.021734
7	1.123471	1.146595	1.017617
10	1.091867	1.10879	1.013271
15	1.057018	1.067325	1.008352
20	1.032969	1.038849	1.004877
25	1.014693	1.017286	1.00219
30	1	1	1

Table E-98: 13 Stages (0.001 m-D) – Duong -Table of Ratios

13 Fractures (0.001 m-D)- Duong- Table of Ratios				
Years	α-Ratio	m-Ratio	q_1-Ratio	q_∞- Ratio
2	0.530838	0.88444	2.701732	-20.79246
3	0.601084	0.901742	2.446938	-17.529566
4	0.650924	0.914018	2.26616	-15.214505
5	0.689583	0.923541	2.125937	-13.418805
7	0.747876	0.937899	1.914499	-10.711116
10	0.809669	0.953119	1.690365	-7.8408496
15	0.879914	0.970421	1.435572	-4.577955
20	0.929754	0.982698	1.254793	-2.2628945
25	0.968413	0.99222	1.11457	-0.4671941
30	1	1	1	1

Table E-99: 13 Stages (0.002 m-D) – Arps -Table of Ratios

13Fractures (0.002 m-D)- Arps-Table of Ratios			
Years	n- Ratio	di- Ratio	qi- Ratio
3	1.169499	1.145513	1.025652
5	1.129573	1.111504	1.019905
7	1.104022	1.089656	1.016137
10	1.077567	1.066965	1.012158
15	1.048262	1.041743	1.007654
20	1.027955	1.024211	1.00447
25	1.012475	1.010815	1.002008
30	1	1	1

Table E-100: 13 Stages (0.002 m-D) – Duong -Table of Ratios

13 Fractures (0.002 m-D)- Duong- Table of Ratios				
Years	a-Ratio	m-Ratio	q₁-Ratio	q_∞- Ratio
2	0.523954	0.870338	3.003952	-7.4427836
3	0.59523	0.889751	2.703909	-6.1786807
4	0.645801	0.903526	2.491025	-5.2817854
5	0.685028	0.91421	2.325899	-4.5860994
7	0.744176	0.93032	2.07691	-3.5370929
10	0.806876	0.947398	1.812971	-2.4251011
15	0.878152	0.966812	1.512928	-1.1609982
20	0.928724	0.980586	1.300043	-0.2641029
25	0.96795	0.99127	1.134918	0.43158312
30	1	1	1	1

Table E-101: 13 Stages (0.004 m-D) – Arps -Table of Ratios

13Fractures (0.004 m-D)- Arps-Table of Ratios			
Years	n- Ratio	di- Ratio	qi- Ratio
3	1.1508	1.119438	1.018591
5	1.115494	1.091766	1.014437
7	1.092832	1.073913	1.01171
10	1.069312	1.055307	1.008828
15	1.043189	1.034548	1.005561
20	1.025042	1.020066	1.003249
25	1.011184	1.008974	1.00146
30	1	1	1

Table E-102: 13 Stages (0.004 m-D) – Duong -Table of Ratios

13 Fractures (0.004 m-D)- Duong- Table of Ratios				
Years	α-Ratio	m-Ratio	q_1-Ratio	q_{∞}- Ratio
2	0.504163	0.86769	4.175176	-3.4366551
3	0.578402	0.8875	3.69977	-2.7723731
4	0.631076	0.901556	3.362464	-2.3010576
5	0.671933	0.912458	3.10083	-1.9354769
7	0.733541	0.928897	2.706317	-1.3842274
10	0.798847	0.946324	2.288118	-0.7998794
15	0.873086	0.966134	1.812712	-0.1355975
20	0.92576	0.98019	1.475406	0.33571807
25	0.966617	0.991092	1.213771	0.70129879
30	1	1	1	1

Table E-103: 13 Stages (0.006 m-D) – Arps -Table of Ratios

13Fractures (0.006 m-D)- Arps-Table of Ratios			
Years	n- Ratio	di- Ratio	qi- Ratio
3	1.127197	1.076465	1.016249
5	1.09765	1.059012	1.012621
7	1.078612	1.047671	1.010239
10	1.058791	1.035781	1.00772
15	1.036701	1.022429	1.004864
20	1.021308	1.013059	1.002842
25	1.009526	1.005851	1.001277
30	1	1	1

Table E-104: 13 Stages (0.006 m-D) – Duong -Table of Ratios

13 Fractures (0.006 m-D)- Duong- Table of Ratios				
Years	α-Ratio	m-Ratio	q_1-Ratio	q_{∞}- Ratio
2	0.495585	0.866873	4.471249	-3.3972334
3	0.571109	0.886806	3.951514	-2.7388539
4	0.624694	0.900948	3.582755	-2.2717262
5	0.666258	0.911918	3.296724	-1.9093938
7	0.728931	0.928459	2.865425	-1.3630424
10	0.795367	0.945993	2.40823	-0.7838867
15	0.870891	0.965925	1.888494	-0.1255072
20	0.924476	0.980068	1.519736	0.34162051
25	0.96604	0.991037	1.233705	0.70395289
30	1	1	1	1

Table E-105: 13 Stages (4%) - Arps-Table of Ratios

13Fractures (4%)- Arps-Table of Ratios			
Years	n- Ratio	di- Ratio	qi- Ratio
3	1.253141	1.840772	1.161449
5	1.191952	1.607719	1.123517
7	1.153289	1.470572	1.099212
10	1.113673	1.337942	1.074021
15	1.070289	1.201636	1.046085
20	1.040536	1.113433	1.026706
25	1.018028	1.049501	1.011921
30	1	1	1

Table E-106: 13 Stages (4%) – Duong -Table of Ratios

13 Fractures (4%)- Duong- Table of Ratios				
Years	a-Ratio	m-Ratio	q₁-Ratio	q_∞- Ratio
2	0.677844	0.8892394	1.551516	-4.7471376
3	0.726079	0.9058231	1.46894	-3.8866426
4	0.760303	0.9175895	1.410351	-3.2761116
5	0.786848	0.9267162	1.364906	-2.802547
7	0.826876	0.940478	1.296381	-2.0884716
10	0.869307	0.9550662	1.223741	-1.331521
15	0.917542	0.9716499	1.141165	-0.471026
20	0.951765	0.9834163	1.082576	0.13950495
25	0.978311	0.992543	1.037131	0.61306955
30	1	1	1	1

Table E-107: 13 Stages (5%) - Arps-Table of Ratios

13Fractures (5%)- Arps-Table of Ratios			
Years	n- Ratio	di- Ratio	qi- Ratio
3	1.309182	1.972423	1.172195
5	1.233229	1.696503	1.131599
7	1.185623	1.536197	1.105629
10	1.137164	1.382773	1.078751
15	1.084477	1.226885	1.048989
20	1.048583	1.12706	1.028372
25	1.021561	1.055258	1.01266
30	1	1	1

Table E-108: 13 Stages (5%) - Duong-Table of Ratios

13 Fractures (5%)- Duong- Table of Ratios				
Years	α-Ratio	m-Ratio	q_1-Ratio	q_∞- Ratio
2	0.694786	0.8943203	1.52711	-5.5583541
3	0.740484	0.9101433	1.448188	-4.576399
4	0.772908	0.9213699	1.392192	-3.8796907
5	0.798058	0.9300779	1.348758	-3.3392818
7	0.83598	0.9432085	1.283265	-2.5244136
10	0.87618	0.9571275	1.21384	-1.6606185
15	0.921878	0.9729504	1.134918	-0.6786634
20	0.954302	0.984177	1.078922	0.01804488
25	0.979451	0.992885	1.035488	0.55845378
30	1	1	1	1

Table E-109: 13 Stages (6%) - Arps-Table of Ratios

13Fractures (6%)- Arps-Table of Ratios			
Years	n- Ratio	d_i- Ratio	q_i- Ratio
3	1.355189	2.06538	1.183042
5	1.266823	1.7584	1.139738
7	1.211789	1.581566	1.112084
10	1.156059	1.413491	1.083502
15	1.095812	1.244012	1.051902
20	1.05498	1.136237	1.030041
25	1.024358	1.059112	1.013398
30	1	1	1

Table E-110: 13 Stages (6%) – Duong -Table of Ratios

13 Fractures (6%)- Duong- Table of Ratios				
Years	α-Ratio	m-Ratio	q_1-Ratio	q_∞- Ratio
2	0.696067	0.8921475	1.50148	-6.6794829
3	0.741574	0.9082958	1.426395	-5.5296658
4	0.773861	0.9197532	1.373122	-4.7138576
5	0.798905	0.9286402	1.3318	-4.0810676
7	0.836669	0.9420408	1.269492	-3.1269004
10	0.876699	0.956246	1.203442	-2.1154423
15	0.922206	0.9723943	1.128358	-0.9656253
20	0.954493	0.9838517	1.075084	-0.1498171
25	0.979537	0.9927387	1.033763	0.48297293
30	1	1	1	1

Table E-111: 13 Stages (8%) - Arps-Table of Ratios

13Fractures (8%)- Arps-Table of Ratios			
Years	n- Ratio	di- Ratio	qi- Ratio
3	1.475707	2.421029	1.207814
5	1.353654	1.989803	1.158266
7	1.278827	1.748626	1.126746
10	1.204019	1.524806	1.094269
15	1.124279	1.304955	1.058484
20	1.070926	1.168474	1.033807
25	1.031292	1.07252	1.015063
30	1	1	1

Table E-112: 13 Stages (8%) – Duong -Table of Ratios

13 Fractures (8%)- Duong- Table of Ratios				
Years	α-Ratio	m-Ratio	q_1-Ratio	q_{∞}- Ratio
2	0.709146	0.9008329	1.508006	-9.9699404
3	0.752695	0.9156808	1.431945	-8.3274568
4	0.783593	0.9262155	1.377978	-7.1620961
5	0.807559	0.9343869	1.336118	-6.2581722
7	0.843697	0.9467083	1.272999	-4.8951692
10	0.882005	0.9597695	1.20609	-3.4503279
15	0.925554	0.9746174	1.130028	-1.8078443
20	0.956452	0.9851521	1.076062	-0.6424836
25	0.980418	0.9933235	1.034202	0.26144035
30	1	1	1	1

F. Effect of Formation and Hydraulic Fracture Properties on Production Decline Behavior

The changes of Arps and Duong models' constant ratios with fracture half-length, permeability and porosity at each production time interval were presented in the following figures, for both seven and thirteen stage cases.

F.1. Seven Stages Model

F.1.1. Arps Decline Curve Analysis

Figure F-1: *n*-Ratio vs. Fracture Half-length - 7 Stages

Figure F-2: d_i -Ratio vs. Fracture Half-length - 7 Stages

Figure F-3: q_i -Ratio vs. Fracture Half-length - 7 Stages

Figure F-4: *n*-Ratio vs. Permeability- 7 Stages

Figure F-5: *d_f*-Ratio vs. Permeability - 7 Stages

Figure F-6: q_i -Ratio vs. Permeability - 7 Stages

Figure F-7: n-Ratio vs. Porosity- 7 Fractures

Figure F-8: d_f-Ratio vs. Porosity-- 7 Fractures

Figure F-9: q_i -Ratio vs. Porosity - 7 Fractures

F.1.2. Duong Decline Curve Analysis

Figure F-10: a-Ratio vs. Fracture Half Length - 7 Stages

Figure F-11: m-Ratio vs. Fracture half-length- 7 Stages

Figure F-12: q_1 -Ratio vs. Fracture Half Length- 7 Stages

Figure F-13: q_∞ vs. Fracture Half Length- 7 Stages

Figure F-14: *a*-Ratio vs. Permeability - 7 Stages

Figure F-15: *m*-Ratio vs. Permeability - 7 Stages

Figure F-16: q_1 -Ratio vs. Permeability - 7 Stages

Figure F-17: q_{∞} -Ratio vs. Permeability - 7 Stages

Figure F-18: a-Ratio vs. Porosity - 7 Stages

Figure F-19: m-Ratio vs. Porosity - 7 Stages

Figure F-20: q_1 -Ratio vs. Porosity - 7 Stages

Figure F-21: q_{∞} - Ratio vs. Porosity - 7 Stages

F.2. Thirteen Stages Model

F.2.1. Arps Decline Curve Analysis

Figure F-22: n-Ratio vs. Fracture Half-length- 13 Stages

Figure F-23: d_f-Ratio vs. Fracture Half-length - 13 Stages

Figure F-24: q_i -Ratio vs. Fracture Half-length - 13 Stages

Figure F-25: n-Ratio vs. Permeability - 13 Stages

Figure F-26: d_i-Ratio vs. Permeability - 13 Fractures

Figure F-27: q_i -Ratio vs. Permeability - 13 Stages

Figure F-28: n -Ratio vs. Porosity - 13 Stages

Figure F-29: d_i -Ratio vs. Porosity - 13 Stages

Figure F-30: q_i -Ratio vs. Porosity - 13 Stages

F.2.2. Duong Decline Curve Analysis

Figure F-31: a-Ratio vs. Fracture half-Length - 13 Stages

Figure F-32: m-Ratio vs. Fracture Half Length - 13 Stages

Figure F-33: q_1 -Ratio vs. Fracture Half Length - 13 Stages

Figure F-34: q_{∞} -Ratio vs. Fracture Half Length - 13 Stages

Figure F-35: a-Ratio vs. Permeability - 13 Stages

Figure F-36 : m-Ratio vs. Permeability - 13 Stages

Figure F-37: q_1 -Ratio vs. Permeability - 13 Stages

Figure F-38: q_∞ -Ratio vs. Permeability - 13 Stages

Figure F-39: *a*-Ratio vs. Porosity - 13 Stages

Figure F-40: *m*-Ratio vs. Porosity - 13 Stages

Figure F-41: q_1 -Ratio vs. Porosity - 13 Stages

Figure F-42: q_{∞} -Ratio vs. Porosity - 13 Stages

G. Confirmation with Field Data

Values of the ratios when computed at 3.5 years of production history using Arps and Duong models' derived correlations both for seven- and thirteen- stage cases were shown in Table G-1 through Table G-4.

Table G-1: 7 Stages- Arps Well #1 Ratio Values

<i>7 Stages- Well# 1 - Ratio Values</i>			
<i>Years</i>	<i>n</i>	<i>d_i</i>	<i>q_i</i>
3.45205	1.441131	2.5175246	1.230663

Table G-2: 7 Stages- Duong Well #1 Ratio Values

<i>7 Stages - Well #1- Ratio Values</i>				
<i>Years</i>	<i>a</i>	<i>m</i>	<i>q₁</i>	<i>q[∞]</i>
3.45205	0.645115	0.911297756	2.398687	-18.0775708

Table G-3: 13 Stages- Arps Well #1 Ratio Values

<i>13 Stages - Well# 1 - Ratio Values</i>			
<i>Years</i>	<i>n</i>	<i>d_i</i>	<i>q_i</i>
3.45205	1.158378	1.1360454	1.024048

Table G-4: 13 Stages- Duong Well #1 Ratio Values

<i>13 Stages - Well #1- Ratio Values</i>				
<i>Years</i>	<i>a</i>	<i>m</i>	<i>q₁</i>	<i>q[∞]</i>
3.45205	0.619911	0.896446646	2.600062	-5.74106433

In the next step, the predicted Arps and Duong models constants were shown along with the actual constants values at 3.5 years. As stated previously, predicted values were obtained by dividing the actual value by the ratio for both seven and thirteen fractures cases.

Table G-5: 7 Stages - Arps Well 1 Actual and Predicted Values

7 Stages - Well #1		
Arps Constants	Actual - 3.5 Years	Predicted - 30 Years
n	1.540921	1.0692443
d_i	0.006155	0.0024447
q_i	1.965478	1.597089

Table G-6: 7 Stages - Duong Well 1 Actual and Predicted Values

13 Stages - Well #1		
Duong Constants	Actual - 3.5 years	predicted - 30 years
a	0.717052	1.11151008
m	0.995263	1.092137896
q_1	6.53	2.72277692
q_∞	-0.61	0.033515431

Table G-7: 13 Stages - Arps Well 1 Actual and Predicted Values

13 Stages - Well #1		
Arps Constants	Actual - 3.5 Years	Predicted - 30 Years
n	1.54092089	1.33024045
d_i	0.006154637	0.005417598
q_i	1.965478132	1.919322169

Table G-8: 13 Stages - Duong Well 1 Actual and Predicted Values

13 Stages - Well #1		
Duong Constants	Actual - 3.5 years	predicted - 30 years
a	0.717052	1.156701354
m	0.995263	1.110230952
q_1	6.53	2.511896816
q_∞	-0.61	0.105534015

The generated production data (at 3.5 years) by Fekete simulator, for each case (Table 4-34), were matched to Well 1 production history at 3.5 years (Figure G-1 through Figure G-3).

Figure G-1: Well 1 History Match Comparison (at 3.5 years) - Various Fracture half Length

Figure G-2: Well 1 History Match Comparison (at 3.5 years) - Various Permeability

Figure G-3: Well 1 History Match Comparison (at 3.5 years) - Various Porosity