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Abstract. There has been a recent resurgence of interest in biproportional adjustment methods for updating and interpreting change in matrix representations of regional structures, most commonly input-output accounts. Although the biproportional method, commonly called the RAS technique in the input-output literature, has been shown to have a number of theoretically appealing properties, various alternatives do exist. In this paper, we develop and assess empirically a number of alternatives, comparing performance and examining attributes of these adjustment methods. Two of these are sign-preserving updating methods for use when tables contain both positive and negative entries. One of these is shown to generate less information gain than Junius and Oosterhaven's generalized RAS method which was formulated to deal with matrices with both positive and negative values. Overall, while the RAS method continues to be commonly used and its choice is often rational, alternative methods can perform as well or better along certain dimensions and in certain contexts.

1. Introduction*

Biproportional adjustment techniques are commonly used in a variety of modeling frameworks and in areas as diverse as demography, transportation research, and economic analysis. In input-output analysis, a particular form of biproportional analysis was developed and introduced to the literature by Stone (1961) and Stone and Brown (1962). Their objective was to devise a procedure that could be used to update a given input-output (IO) table without having to generate a completely new set of interindustry data. The method they devised, which has come to be known as the RAS method, generates new IO coefficients for a target year using a prior year table in conjunction with target year total intermediate industry inputs and outputs, and total industry outputs. Hence, this method is able to produce estimates of n^2 pieces of information using just $3n$ pieces of new information.

Bacharach (1970) later showed that the RAS procedure generated a solution equivalent to minimizing information gain (see Theil 1967) expressed in terms of the prior and target year IO coefficients. Hewings and Janson (1980) recast the updating procedure more generally in terms of finding matrix solutions that minimize particular measures of fit. They also show, *inter alia,* that minimizing information gain given by

$$\sum\sum q_{ij} \ln {}^{q_{ij}}\!/_{a_{ij}}$$

results in the solution $\ln q_{ij} = \ln a_{ij} + \lambda_i + \mu_j$, which can be converted directly into $q_{ij} = r_i a_{ij} s_j$, with $\ln r = \lambda$ and $\ln s = \mu$. This, of course, is in the form of the traditional RAS solution.

The RAS procedure has thus been shown to have some desirable qualities. First, the result can be described in information gain language. As Bacharach (1970) notes, "one estimates the unknown matrix as that value which, if realized, would occasion the least 'surprise' in view of the prior". The target matrix is, in this sense, as "close" as possible to the prior. Second, the pre- and post-multiplicands have clear-cut effects that have motivated some to offer economic interpretations. Third, the RAS procedure preserves the signs of the original matrix elements. And fourth, the iterative solution for the RAS method is simple to understand and relatively straightforward to compute. These last two qualities are somewhat interrelated. Even when the original matrix is non-negative, negative elements can appear in the updated matrix. When non-negativity constraints "are imposed, the simple iterative adjustment is no longer available …. the resultant problem makes heavy demands on computer time and programming ingenuity" (Lecomber, 1975).

These qualities notwithstanding, the information gain measure of closeness remains but one of many possible specifications (Bacharach 1970) and, as Lecomber (1975) notes, "'closeness' is not a very clearly defined concept." Indeed, one can offer a counterpoint to each of the motivations for using RAS listed above. First, just as different location analysis problems demand differing objective functions (such as min-max, max-min, and max cover), minimizing information gain may well not be the preferred objective function for IO matrix updating. Second, economic interpretations followed the introduction of the method, and might well follow the introduction of other methods. And third, although an iterative and computationally accessible solution is convenient, present-day computer hardware and software development have been rapid over the past two decades, such that a reassessment of computational constraints might well be productive.

In this paper, we review previous matrix updating formulations, and develop and empirically assess the traditional approaches along with a new set of alternatives for this problem as it arises in the context of input-output modeling. We compare the performance and attributes of these alternatives to one another and to the commonly applied RAS method.

2. The Traditional RAS Method

For notational simplicity, let $a_{ij} \in A$ be the known IO coefficients matrix, and let $q_{ij} \in Q$ be the new matrix that is the target of the procedure. Let u, v, and x be a column vector of intermediate outputs, a row vector of intermediate inputs, and a column vector of total outputs corresponding to the target year, respectively. Then the RAS procedure can be expressed generally as $Q = f(A,u,v,x)$. The iterative procedure is defined by the following steps, where superscripts on variables denote the iterative step corresponding to the temporary values for those variables.

The first step is to compute the intermediate output values that would have been observed had there been no change in the intermediate IO structure, or $u^1 = Ax$. The changes that have occurred in the intervening period are responsible for differences between u and u^1. Let $r_1 = \hat{u}\left(\hat{u}^1\right)^{-1}$, where ^ denotes diagonalization. Then our first estimate of the new IO structure will be $Q^1 = r_1 A$. The row sums of $Q^1 x$ now equal the known values in u. The column sums of $Q^1 x$, however, will typically not equal the known values of intermediate industry inputs, v. The next step in the iterative procedure is thus to calculate $v^1 = eQ^1\hat{x}$, where e is a summing vector. Let $s_1 = \hat{v}\left(\hat{v}^1\right)^{-1}$ and compute the next estimate of Q as $Q^2 = Q^1 s_1$. Column sums of $Q^2 x$ are now equal to v, but the row sums are no longer necessarily equal to u. Follow the format of the steps described above to compute successive estimates of r_m and s_m. The procedure will normally converge to a stable estimate of Q after a relatively small number of steps.

One might reasonably ask why, given the available data, one does not update the matrix of intermediate transactions directly, and then derive the associated new coefficients matrix. Okuyama et al (2002) suggest that because the RAS adjustment process "operates on the **A** matrices, the adjustment process is conservative, making only the minimally necessary adjustments to ensure agreement with the vectors u and v."

3. Alternative Formulations

Lecomber (1975) presented the RAS approach along with three alternative minimands (see Almon 1968, Friedlander 1961, Matuszewski et al 1964) to be used in the matrix adjustment procedure. In this paper, we present Lecomber's alternatives along with six additional minimands. To simplify presentation, variables A, Q, x, u, and v denote the input-output coefficients table for the prior period, the estimate of the coefficients table for the target period, and the gross output, intermediate output, and intermediate input vectors for the target year, respectively.

Model 1: Absolute Differences

The objective in Model 1 is to minimize the sum of absolute differences between prior and target period element pairs.

$$\min z = \sum\sum\left|a_{ij} - q_{ij}\right|$$
$$s.t. \qquad \sum_i q_{ij}x_j = v_j \quad \text{for all j}$$
$$\sum_j q_{ij}x_j = u_i \quad \text{for all i}$$
$$q_{ij} \ge 0 \quad \text{for all i,j}$$

A linearized version of Model 1 is more efficient computationally:

$$\min z = \sum\sum\left(t_{ij}^{+} + t_{ij}^{-}\right)$$
$$s.t. \qquad \sum_i q_{ij}x_j = v_j \text{ for all j}$$
$$\sum_j q_{ij}x_j = u_i \text{ for all i}$$
$$t_{ij}^{+} \ge a_{ij} - q_{ij} \quad \text{for all i,j}$$
$$t_{ij}^{-} \ge q_{ij} - a_{ij} \quad \text{for all i,j}$$
$$q_{ij}, t_{ij}^{+}, t_{ij}^{-} \ge 0 \quad \text{for all i,j}$$

This model can therefore be solved optimally using commercial linear programming software.

Model 2: Weighted Absolute Differences

All differences in Model 1 are given equal weight, irrespective of coefficient size. Since accuracy in large coefficients is generally a higher priority than accuracy in small coefficients, an alternative approach is to weight some deviations more heavily than others in the objective function. Model 2 weights the absolute differences by the prior year coefficient value, which has the effect of weighting more heavily differences in larger coefficients. This approach is consistent with the spirit of weighted absolute difference (WAD) measure of inter-matrix distance included in Lahr (2001). However, there is an important distinction. In estimating the new matrix, we generate a new matrix as close (by some definition) as possible to the prior. When the method is evaluated, however, we seek a matrix that is as close as possible (adjudged by alternative matrix distance measures) to the observed target year table. When differences in a minimand are weighted by coefficient size, an implicit expectation is that large coefficients should not experience large changes. Although there is some empirical evidence of a tendency towards temporal coefficient stability (see Tilanus 1966), we are aware of no empirical evidence concerning the stability of coefficients relative to coefficient size.[1] The weighted absolute differences measure is expressed as:

$$
\begin{aligned}
&\min z = \sum\sum a_{ij}\left|a_{ij} - q_{ij}\right| \\
&s.t. \qquad \sum_i q_{ij}x_j = v_j \quad \text{for all j} \\
&\qquad\qquad \sum_j q_{ij}x_j = u_i \quad \text{for all i} \\
&\qquad\qquad q_{ij} \geq 0 \quad \text{for all i,j}
\end{aligned}
$$

The following linearized version of this problem is as follows:

$$
\begin{aligned}
&\min z = \sum\sum a_{ij}\left(t_{ij}^{+} + t_{ij}^{-}\right) \\
&s.t. \qquad \sum_i q_{ij}x_j = v_j \text{ for all j} \\
&\qquad\qquad \sum_j q_{ij}x_j = u_i \text{ for all i} \\
&\qquad\qquad t_{ij}^{+} \geq a_{ij} - q_{ij} \quad \text{for all i,j} \\
&\qquad\qquad t_{ij}^{-} \geq q_{ij} - a_{ij} \quad \text{for all i,j} \\
&\qquad\qquad q_{ij}, t_{ij}^{+}, t_{ij}^{-} \geq 0 \quad \text{for all i,j}
\end{aligned}
$$

Again, this linear formulation can be solved optimally using commercial software.

Model 3: Normalized Absolute Differences (Matuszewski 1964)

A third alternative is to modify the weighting in Model 2. Model 3 is attributed to Matuszewski et al (1964), and is a normalized version of Model 1. In this formulation, differences in large coefficients will contribute *less* to the value of the objective function than will equally sized differences in small coefficients. This imposes a greater penalty on changes in *small* coefficients, resulting in updated matrices whose changes are more concentrated in the larger coefficients.

$$
\begin{aligned}
&\min z = \sum\sum \frac{|a_{ij} - q_{ij}|}{a_{ij}} \\
&s.t. \qquad \sum_i q_{ij} x_j = v_j \quad \text{for all j} \\
&\qquad\qquad \sum_j q_{ij} x_j = u_i \quad \text{for all i} \\
&\qquad\qquad q_{\mathrm{ij}} \geq 0 \quad \text{for all i,j}
\end{aligned}
$$

An element a_{ij} cannot not be zero, or this fraction is undefined. In instances where this assumption is violated, some modification is necessary. As with the previous models, a linearized version of Model 3 may be structured:

$$
\begin{aligned}
&\min z = \sum\sum \frac{\left(t_{ij}^{+} + t_{ij}^{-}\right)}{a_{ij}} \\
&s.t. \qquad \sum_i q_{ij} x_j = v_j \ \text{for all j} \\
&\qquad\qquad \sum_j q_{ij} x_j = u_i \ \text{for all i} \\
&\qquad\qquad t_{ij}^{+} \geq a_{ij} - q_{ij} \ \text{for all i,j} \\
&\qquad\qquad t_{ij}^{-} \geq q_{ij} - a_{ij} \ \text{for all i,j} \\
&\qquad\qquad q_{ij,}\ t_{ij}^{+}, t_{ij}^{-} \geq 0 \ \text{for all i,j}
\end{aligned}
$$

Model 4: Squared Differences (Almon 1968)

A fourth alternative is Model 4, Almon's (1968) formulation, which is a squared differences version of Model 1. There is no explicit differential weighting according to coefficient size. Since differences will always be less than unity, squaring the differences results in a nonlinear weighting of coefficient change by size. That is, larger changes are weighted in less than proportion to their size.

$$
\begin{aligned}
&\min z = \sum\sum\left(a_{ij} - q_{ij}\right)^2 \\
&s.t. \qquad \sum_i q_{ij} x_j = v_j \quad \text{for all j} \\
&\qquad\qquad \sum_j q_{ij} x_j = u_i \quad \text{for all i} \\
&\qquad\qquad q_{ij} \geq 0 \ \text{for all i,j}
\end{aligned}
$$

Model 4 poses a solution challenge since there is no way to linearize the problem. There is commercial software to solve non-linear optimization problems, but current capabilities are not equivalent to those for linear programs. Although it is possible to solve some instances of non-linear problems, solutions can be local rather than global optima. Indeed, despite computational advances, capabilities to solve non-linear optimization models still may not be adequate to the task. The experiments in this paper will provide a good indication of the viability of non-linear formulations in this context.

Model 5: Weighted Squared Differences

Model 5 weights the squared differences by the size of the corresponding coefficient, weighting changes in larger coefficients more heavily than those in smaller coefficients. Otherwise, its properties are identical to that of Model 4.

$$\min z = \sum \sum a_{ij} \left(a_{ij} - q_{ij} \right)^2$$
$$s.t. \quad \sum_i q_{ij} x_j = v_j \quad \text{for all j}$$
$$\sum_j q_{ij} x_j = u_i \quad \text{for all i}$$
$$q_{ij} \geq 0 \text{ for all i,j}$$

Model 6: Normalized Squared Differences (Friedlander 1961)

Model 6, Friedlander's minimand, amounts to a normalized version of Almon's formulation, and thus it shares most of Model 4's properties.

$$\min z = \sum \sum \frac{(a_{ij} - q_{ij})^2}{a_{ij}}$$
$$s.t. \quad \sum_i q_{ij} x_j = v_j \quad \text{for all j}$$
$$\sum_j q_{ij} x_j = u_i \quad \text{for all i}$$
$$q_{ij} \geq 0 \text{ for all i,j}$$

Although Lecomber (1975) suggests that "the Friedlander minimand is perhaps the most appealing", there is again an inverse weighting imposed on differences in large coefficients, and solutions are not likely to be global optima.

Model 7: Global Change Constant

Although we generally expect input-output coefficient change to be marked by some degree of substitution, a growing regional economy might be expected to exhibit uniformly increasing coefficients. Model 7 accounts for this possibility by assigning a global proportional change constant that minimizes the sum of errors (ε) in $\gamma A = Q + \varepsilon$.

$$
\begin{aligned}
&\min z = \sum_i (\alpha_i^+ + \alpha_i^-) + \sum_i (\beta_i^+ + \beta_i^-) \\
&s.t. \quad \gamma \sum_j a_{ij} x_j = u_i - \alpha_i^+ + \alpha_i^- \quad \text{for all i} \\
&\qquad \gamma \sum_i a_{ij} x_j = v_j - \beta_j^+ + \beta_j^- \quad \text{for all j} \\
&\qquad \gamma \geq 0 \\
&\qquad \alpha_i^+, \alpha_i^- \geq 0 \quad \text{for all i} \\
&\qquad \beta_j^+, \beta_j^- \geq 0 \quad \text{for all j}
\end{aligned}
$$

Model 8 : RAS

Model 8 is the equivalent optimization representation corresponding to the traditional RAS approach. In the context of the properties of the other minimands, changes in large estimated coefficients are weighted more heavily than in small estimated coefficients.

$$
\begin{aligned}
&\min \quad \sum\sum q_{ij} \ln {q_{ij}}/{a_{ij}} \\
&s.t. \qquad \sum_i q_{ij} x_j = v_j \quad \text{for all j} \\
&\qquad\qquad \sum_j q_{ij} x_j = u_i \quad \text{for all i} \\
&\qquad\qquad q_{ij} \geq 0 \text{ for all i,j}
\end{aligned}
$$

Again, this model can be solved approximately using the iterative technique described previously.

Model 9: Sign Preserving Absolute Difference Formulation

Junius and Oosterhaven (2002) recently studied the problem of updating a matrix with both positive and negative entries. Model 9 is a reformulation of Model 1 in which the signs of matrix elements are preserved. Additionally, equal valued changes contribute more strongly to the objective function when associated with large coefficients. In this formulation, $a_{ij} y_{ij} = q_{ij}$. This problem is given by:

$$
\begin{aligned}
&\min z = \sum\sum \left| a_{ij} - y_{ij} a_{ij} \right| = \sum_i \sum_j \left| a_{ij} \right| \left| 1 - y_{ij} \right| \\
&s.t. \qquad \sum_i y_{ij} a_{ij} x_j = v_j \quad \text{for all } j \\
&\qquad\qquad \sum_j y_{ij} a_{ij} x_j = u_i \quad \text{for all } i \\
&\qquad\qquad y_{ij} \geq 0 \quad \text{for all } i,j
\end{aligned}
$$

The linear equivalent to this model is:

$$
\begin{aligned}
&\min z = \sum\sum \left| a_{ij} \right| \left(t_{ij}^{+} + t_{ij}^{-} \right) \\
&s.t. \qquad \sum_i y_{ij} a_{ij} x_j = v_j \quad \text{for all } j \\
&\qquad\qquad \sum_j y_{ij} a_{ij} x_j = u_i \quad \text{for all } i \\
&\qquad\qquad t_{ij}^{+} \geq 1 - y_{ij} \quad \text{for all } i,j \\
&\qquad\qquad t_{ij}^{-} \geq y_{ij} - 1 \quad \text{for all } i,j \\
&\qquad\qquad y_{ij}, t_{ij}^{+}, t_{ij}^{-} \geq 0 \quad \text{for all } i,j
\end{aligned}
$$

Although Models 1 and 9 will generate identical solutions with respect to objective value performance, the resulting matrix coefficients may differ because of alternate optima.

In developing Models 1-6, we presented unweighted, weighted, and normalized versions of the minimands. In the sign-preserving context, however, neither a weighted ($\min z = \sum\sum a_{ij} \left| a_{ij} \right| \left(t_{ij}^{+} + t_{ij}^{-} \right)$) nor a normalized absolute difference minimand ($\sum\sum \frac{\left| a_{ij} \right| \left(t_{ij}^{+} + t_{ij}^{-} \right)}{a_{ij}}$) makes any intuitive sense. With the former, the differences would be weighted by a negative value for negative original coefficients, and with the latter, the differences would be weighted by values of 1.0 or –1.0. Both possibilities would lead to a maximization of differences associated with negative coefficients.

Model 10: Sign Preserving Squared Differences

Model 10 is the squared difference version of Model 9, so it also preserves the signs of the original matrix elements. Equal changes in large coefficients are weighted more heavily, but in less than proportion to their size. Again as in Model 6, $a_{ij} y_{ij} = q_{ij}$.

$$
\begin{aligned}
\min z &= \sum\sum\left(a_{ij} - y_{ij}a_{ij}\right)^2 = \sum_i\sum_j a_{ij}^{\,2}\left(1 - y_{ij}\right)^2 \\
s.t. \quad & \sum_i y_{ij}a_{ij}x_j = v_j \quad \text{for all j} \\
& \sum_j y_{ij}a_{ij}x_j = u_i \quad \text{for all i} \\
& y_{ij} \geq 0 \quad \text{for all i,j}
\end{aligned}
$$

Eight of the ten updating methods are clearly interrelated. The methods involve weighting and normalization of change terms, which are absolute squared, and sign preserving or not (sign floating). Table 1 summarizes the relationships among these eight approaches.[2]

Table 1. Interrelationships among input-output updating methods

	Sign Floating		Sign Preserving	
	Absolute	Squared	Absolute	Squared
Unweighted	Model 1	Model 4	Model 9	Model 10
Weighted	Model 2	Model 5		
Normalized	Model 3	Model 6		

4. **Empirical Assessment**

To assess the relative performance of the alternative matrix updating methods presented in the previous section, we used four primary matrix comparison methods. For comparing entire matrices, we calculated Theil's U (Theil 1971), the weighted absolute difference (WAD, Lahr 2001), and a somewhat less commonly used index of fit, C, introduced by Roy, et al (1982) in the context of evaluating input-output aggregation error. Finally, we used standardized total percentage error (STPE) for multiplier assessment (see Miller and Blair, 1985). In all instances, we compared the estimated target year table, **Q**, to the known target year table. Since the prior year table is no longer used, and for notational simplicity, **A** refers hereafter to the known target year table. Theil's U, and the WAD are defined as follow:

$$U = \sqrt{\frac{\sum\sum\left(a_{ij} - q_{ij}\right)^2}{\sum\sum a_{ij}{}^2}}$$

$$WAD = \frac{\sum\sum a_{ij}\left|a_{ij} - q_{ij}\right|}{\sum\sum\left(a_{ij} + q_{ij}\right)}$$

The C index is the difference in entropy between the target year and its estimate as a proportion of the target entropy, as follows:

$$C = \frac{H_{\hat{Q}} - H_A}{H_A}$$

where $H_Q = -\sum\sum q_{ij}\log q_{ij}$ and $H_A = -\sum\sum a_{ij}\log a_{ij}$. The formula for standard total percentage error is:

$$STPE = 100\frac{\sum\sum\left|a_{ij} - \hat{q}_{ij}\right|}{\sum\sum a_{ij}}$$

The 23-sector US data drawn from Miller and Blair (1985) for 1967 → 1972 and for 1972 → 1977 were used in this analysis.[3] The results are presented in Tables 2, 3, and 4. Error levels are ranked, both individually and cumulatively. For solutions, we used LINGO, a commercial optimization software package operating on a Pentium 4 at 1.8 GHz with 512 MB RAM PC, solution times and statistics are shown in Tables 5. Solution times are omitted for the linear formulations, as all of these solved in seconds or less. Solution times for nonlinear models are substantial, even though the analysis was based only on 23 sectors. While software and hardware have indeed advanced significantly, solutions for large matrices still appear to consume inordinate amounts of time for non-linear models.

Table 2. Matrix Updating Results: US 1967 → 1972

	STPE	Rank	WAD	Rank	Theil's U	Rank	C	Rank	Average Rank	Combined Rank
Model 1	8.969	6	0.013	7	0.335	7	0.042	7	6.8	8
Model 2	9.597	8	0.012	5	0.335	8	0.032	4	6.3	7
Model 3	4.873	1	0.014	8	0.198	1	0.004	1	2.8	2
Model 4	8.277	4	0.012	3	0.310	4	0.053	9	5.0	5
Model 5	9.301	7	0.012	6	0.331	6	0.035	5	6.0	6
Model 6	43.125	10	0.116	10	1.235	10	0.191	10	10.0	10
Model 7	17.352	9	0.015	9	0.339	9	0.042	6	8.3	9
Model 8	7.013	2	0.011	1	0.261	2	0.024	3	2.0	1
Model 9	8.305	5	0.012	4	0.310	5	0.023	2	4.0	3
Model 10	8.275	3	0.012	2	0.309	3	0.048	8	4.0	4

Table 3. Matrix Updating Results: US 1972 → 1977

	STPE	Rank	WAD	Rank	Theil's U	Rank	C	Rank	Average Rank	Combined Rank
Model 1	1.054	3	0.014	4	0.174	5	0.002	1	3.3	2
Model 2	1.854	8	0.020	9	0.235	9	0.010	2	7.0	7
Model 3	1.538	6	0.014	3	0.166	4	0.030	8	5.3	5
Model 4	32.889	10	0.119	10	1.265	10	0.084	10	10.0	10
Model 5	1.653	7	0.020	7	0.207	8	0.028	7	7.3	8
Model 6	1.157	5	0.011	2	0.125	2	0.016	5	3.5	4
Model 7	6.718	9	0.020	8	0.190	7	0.033	9	8.3	9
Model 8	0.725	1	0.009	1	0.100	1	0.010	3	1.5	1
Model 9	1.037	2	0.015	5	0.158	3	0.012	4	3.5	3
Model 10	1.147	4	0.019	6	0.187	6	0.022	6	5.5	6

Table 4. Summary Updating Performance, Both Periods

	67-72	72-77	Overall Average	Combined Rank
Model 1	8	2	5	4
Model 2	7	7	7	6
Model 3	2	5	4	3
Model 4	5	10	8	9
Model 5	6	8	7	7
Model 6	10	4	7	8
Model 7	9	9	9	10
Model 8	1	1	1	1
Model 9	3	3	3	2
Model 10	4	6	5	5

Table 5. Solution Statistics

US 66-72

Model	Objective	Status	Iterations	Solution Time (min)
4	0.023	local	943	55
5	0.002	local	1674	172
6	0.465	local	529	122
10	0.024	local	2054	100

US 72-77

Model	Objective	Status	Iterations	Solution Time (min)
4	0.003	local	616	35
5	0.000	local	1522	142
6	0.257	local	1082	78
10	0.003	local	2815	271

Ideally, a single model would outperform the rest uniformly. Indeed, the combined rankings of the models in Table 4 indicate that the RAS approach performed best not only overall, but for both updated matrices. There are several cases, however, in which a model ranked highly for one period but performed poorly for the other. There also are cases in which RAS did not rank highest for all measures of fit. This lack of consistency suggests that the suitability of a particular model formulation for a given transition period will be a function of the distinct nature of that inter-period structural change. In evaluating the remaining models, then, we might well place a higher value on those that performed relatively consistently well for the two periods.

Models 1 and 3, absolute differences and normalized absolute differences, each ranked second for one transition period, but ranked much lower for the alternate period. In the case of Model 1, its performance for 1972 → 1977 was quite strong, but for 1967 →1972, it ranked a very poor eighth, while Model 3 ranked as low as fifth for 1972 →1977. Model 9, the sign-preserving absolute differences model, was thus second best overall, ranking third in both updating trials. The strong performance of Models 9 bodes well for its use in cases where sign preservation is needed. Of course, this conclusion can only be tentative until the models are tested on matrices with both positive and negative entries.

A final result of note is the poor performance of Models 4, 5, and 6, variants of the squared differences models. Models 4 and 6 performed even worse for one period each than did Model 7, the global change constant model. Given their nonlinearity, the explanation may lie in the possibility of local rather than global solutions for these models.[4]

5. Matrices with Negative Values

To gain preliminary insight into the problem where negative entries are present in the matrices, we analyzed the example used by Junius and Oosterhaven (2002). Table 6 presents the relevant data and results for our analysis. The data include the original table that was to be updated, the updated tables using a) traditional RAS, b) Junius and Oosterhaven's GRAS, c) sign preserving absolute differences, and d) sign preserving squared differences. Since Junius and Oosterhaven chose to maintain consistency, to the greatest degree possible, with the minimum information gain objective, Table 6 also presents the information gain measures associated with each updating method. The RAS method is known to behave erratically in the presence of negative values, and as expected, it performed most poorly (generated the largest information gain value), followed by Model 9, the GRAS method, and Model 10. Since there is no known true matrix for the hypothetical example, we can only assess performance on the basis of information gain.

Although what can be said definitively concerning these results is limited, we can conclude at the very least that: 1) there are available matrix updating methods that are superior to RAS in some instances; and 2) the GRAS procedure does *not* generate a minimum information gain solution, since Model 10 generated a feasible solution with a smaller information gain value. Nevertheless, given the nonlinearity of Model 10, it is once again subject to non-global solutions for a given problem, and it may be computationally intractable for problems of any substantial size. The GRAS solution is much simpler computationally, and generates an information gain value close to that of Model 10 and far better than that of the RAS.

Table 6. Negative Entries Assessment

Original Table

0.778	0.200	0.294	1.500
0.222	0.600	0.471	-0.500
-0.222	0.000	0.118	-0.500

RAS (J&O) Table

0.937	0.238	0.353	1.500
0.286	0.762	0.602	-0.385
-0.222	0.000	0.045	-0.115

Model 9 Absolute Differences Table

0.633	0.150	0.456	3.000
0.167	0.450	0.544	-1.000
-0.200	0.000	0.000	-1.000

Model 10 d Differences Table

0.648	0.179	0.400	2.992
0.196	0.421	0.560	-1.003
-0.244	0.000	0.040	-0.989

GRAS (J&O) Table

0.877	0.228	0.348	1.120
0.282	0.772	0.628	-0.100
-0.158	0.000	0.024	-0.015

Information Gain

Model 9	0.642
Model 10	0.536
GRAS	0.540
RAS	0.908

6. Summary

This paper set out to re-examine a range of alternatives to the RAS, biproportional matrix updating procedure. The study was motivated by a consideration of the reasons why the RAS method ultimately became the approach of choice for updating input-output models. Given a wide range of possible minimands in an optimization framework, we sought to determine whether advances in computer hardware and software might have offset the convenience of the iterative biproportional solution. Further motivation for the study came from an interest in alternative procedures that could accommodate matrices containing both positive and negative entries.

The performance of the sign-preserving alternatives was encouraging, suggesting that they might be well useful for matrices with both positive and negative elements. This is especially true for Model 9, since it is linear and can be solved easily. Model 10 was shown to outperform the GRAS method in terms of information gain, but only in the small table example. Solutions for Model 10 may not be feasible for matrices of much larger size. The relative utility of Models 9 and 10 and of the GRAS method given negative values awaits further analysis.

In combined ranks, none of the alternative minimands outperformed the RAS approach when all values in the matrices were positive. Further, despite substantial advances over the last three decades, nonlinear formulations continue to stretch computational and methodological limits, and global optima are not assured. The results of our analysis, albeit limited in variety of matrices studied, suggest that while the RAS method continues to be commonly used and its choice is often rational, alternative methods can perform as well or better along certain dimensions and in certain contexts.

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Endnotes

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1 Thanks to an anonymous reviewer for raising this issue. Further work oriented toward identifying theoretically desirable characteristics of matrix updating models is clearly warranted.

2 A reviewer notes that absolute and squared differences can be grouped into the same category, the Hölder norm at the power p: $\sum_i \sum_j \left| a_{ij} - q_{ij} \right|^p$ with p varying from 1 to infinity. Likewise, the difference weighting term could be represented generally by adding the term a_{ij}^m, i.e., $\sum_i \sum_j a_{ij}^m \left| a_{ij} - q_{ij} \right|^p$ with m varying from -1 to 1.

3 Preliminary analyses using the 49-sector Washington state data for 1963 → 1967 and 1967 → 1972 generated similar results for the linear models, but exceedingly long solution times for the nonlinear formulations. We therefore present results only for the US table analyses.

4 The additive components of Theil's U were also computed. The RAS approach results in a near-zero bias estimate for the 1972→77 update, but an observable bias for the US 1967→72 update, although all models generated observable bias estimates for that period. The most striking variance proportion was associated with Model 3 for the early period matrix pair, but there were almost universally larger variances for the later matrix pair. This indicates an error structure that is correlated with the actual values for this period, suggesting that structural change processes in the later period were fundamentally different than in the earlier period.