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Application of Neural Network Techniques to Downscale Precipitation

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Application of Neural Network Techniques to Downscale Precipitation

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**Thesis submitted to the Benjamin M. Statler College of Engineering and Mineral
Resources at West Virginia University**

in partial fulfillment of the requirements for the degree of

**Master of Science in
Industrial Engineering**

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ABSTRACT

Application of Neural Network Techniques to Downscale Precipitation

Javad Rahimikollu

Global climate change is a major area of concern to public and climate researchers. It impacts flooding, crop yields, water based diseases, etc. Significant effort has gone into developing climate models. Global climate models use a coarse grid of 300x300 Km², while the resolution of interest for the hydrologist is 50x50 Km².

Downscaling is the tool to map the large-scale global climate properties to a finer grid size in order to accurately predict climate variables such as precipitation. This study utilized Artificial Neural Networks (ANN) and Hybrid Support Vector Regression (HSVR) methods to predict precipitation at a finer grid, based on the data from a coarse grid. Precipitation data for three stations (Dhaka, Comilla and Mymensingh in Bangladesh) was utilized. For each model, the raw data was partitioned into training and test datasets. Based on the R² values, the HSVR technique appears to be superior to other methods.

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List of Notations

Notation	Description
i	Indexes unit in input layer or the feature number
j	Indexes unit in hidden layer
net_j	Scalar net activation of neuron j in hidden layer
net_k	Scalar net activation of neuron k in output layer
N_h	Number of neurons in hidden layer
N_k	Number of neurons in output layer
O_i	Output of neuron i in input layer
O_j	Output of neuron j in hidden layer
δ_k	Sensitivity of neuron k at output layer
δ_j	Sensitivity of neuron j at hidden layer
η	Learning rate
J	Error function in ANNs
R	Risk function
N	Overall Number of data points
x	Space for input pattern
y	Output vector
t	Target vector
$\bar{t}$	The average of target values
XN	Normalized data
b	Regression line bias term
w_{SVR}	Regression line weight vector
ϵ	Acceptable deviation range
ζ_u^+	Positive deviation from acceptable region for u-th pattern
ζ_u^-	Negative deviation from acceptable region for u-th pattern
C	Cost of not falling inside the SVR tube
ϕ	The kernel function
c_1	Self-cognitive constant
w_{pso}	The inertia weight
$x_{l,d(pbestl)}$	Best experience of l^{th} particle

List of Notations (Cont'd)

$x_{l,d(gbestl)}$	Best experience of all particles
a_1, a_2	Randomly distributed uniform variables in [0,1]
σ	The sigma value of kernel function
F_i	Feature i
n	Number of features
u	Unit index of u^{th} pattern
v	Unit index of v^{th} pattern
ite	Iteration number for PSO
l	l^{th} particle of Particle Swarm Optimization
d	d^{th} element of particle
θ	Accepted Error for BP
r	r^{th} epoch in back propagation algorithm
W_{ji}	Weight for connection from neuron i to neuron j in hidden layer
W_{kj}	Weight for connection from neuron j in hidden layer to neuron k in output layer
FIT	Fitness function
$xp_{l,d}^{ite}$	d^{th} element of l^{th} particle at iteration ite
D	Dimension of feature space
Di	Dimension of the population in PSO
P	Number of selected features
$R^2_{(adj)tr}$	Adjusted R^2 for train data
$R^2_{(adj)val}$	Adjusted R^2 for validation data
v_{ld}^{ite}	Velocity of d^{th} dimension of l^{th} particle at iteration ite
x_{iu}	The amount of feature i on uth pattern.
y_u	The amount of output for uth pattern.
β_i	Regression coefficient for feature i

List of Acronyms

Acronym	Description
SVR	Support Vector Regression
HSVR	Hybrid Support Vector Regression
ANNs	Artificial Neural Networks
BP	Back Propagation
GCMs	Global Climate Models
RD	Regression Downscaling
CCCSN	Canadian Climate Change Scenarios Network

Chapter 1: Introduction

1.1 Literature Review

Global climate change is a major area of concern to public and climate researchers. It impacts flooding, crop yields, water based diseases, etc. Significant effort has gone into developing climate models [Ojha et. al., 2010]. The scope of these models ranges from the entire globe to a small region. There are numerous parameters, such as wind speed, wind direction, atmospheric pressure, associated with modeling climate change. The impact of climate change on precipitation is of special concern to the hydrology community [Ojha et. al., 2010].

Global Climate Models (GCMs) utilize the resolution of coarser than 2° (few hundred kilometers) in latitude and longitude while the scale of interest for hydrologists is finer than this resolution. Therefore hydrology community defines downscaling as the process of linking the free atmosphere properties of GCMs outputs to predict local meteorological conditions (i.e. precipitations and temperature) as shown in [Fowler et. al., 2007].

Figure 1 - Downscaling Process

Climate studies employ large scale free atmospheric variables to estimate climate changes. The Canadian Institute for Climate Studies (CICS) identified twenty six (26) free atmospheric variables for a land grid size of 300 km x 300 km [R.L. Wilby et. al, 2012]. These variables are capable of representing various earth segments; i.e. atmosphere land surface and humidity [Fowler et. al., 2007]. The main task of downscaling is to link the twenty six (26) free atmospheric

variables as identified by the CICS to local climate variables such as precipitation [Grotch et. al, 1991].

One of the main downscaling approaches is Statistical Downscaling (SD).SD employs nonlinear functions to translate large scale Global Climate Models (GCM) features to sub-grid local variables [Girvetz et. al., 2009]. Because SD pertains to less computationally expensive techniques, it is widely preferred over other downscaling techniques in hydrological studies and has gained increasing attention in recent studies [Girvetz et. al., 2009]. SD techniques can be grouped in three main categories [Fowler et. al, 2007]:

- Weather typing approaches;
- Stochastic weather generators;
- Regression-based downscaling (RD) methods.

RD methods utilize statistical and machine learning methods to map features from a large grid to a local one to estimate a climate variable such as precipitation. Each of these methods encompasses a wide variety of techniques relying on a function representing a relation of atmospheric and local climate variables. This relation can be expressed as $y=f(x)$ where the fine resolution y is a function of atmospheric large scale factors x [Busuioc et. al., 2001].

The main regression based downscaling methods are: [Wibly and Dawson, 2012]

- Linear Regression (LR)
- Artificial Neural Networks (ANNs)
- Support Vector Regression (SVR)

In many research studies, LR methods have been used for downscaling. Abaurrea and Asin (2005) utilized LR regression model to forecast a long term local daily precipitation in a climate change scenario in Spain using GCM as predictors. Huth (2002) applied LR to model daily precipitation.

Huth (2005) used multiple linear regression to downscale precipitation in the summer at four stations in Czech Republic and compared with the result of some other techniques. Kidson and Thompson (1998) applied LR to predict daily and monthly precipitation over New Zealand for 1980-94. Tatliet. al. (2001) applied LR to downscale monthly precipitation in Turkey. Schoof and Pryor (2004) used LR to downscale daily and monthly precipitation in Indianapolis, IN, USA for two different seasons.

ANNs has been utilized to precipitation downscaling problem. Cannon and Whitfield (2002) used ANNs to downscale streamflow during recent decades in British Columbia, Canada and it was concluded that ANN outperforms stepwise linear regression. Cawleyet. al. (2003) applied a neural network based technique to describe precipitation downscaling in England. Dibike and Coulibaly (2006) used neural networks approach to downscale GCM output to predict daily precipitation in Quebec, Canada and a comparison showed that ANNs outperformed LR. Hall et. al. (1999) used ANNs to downscale daily precipitation in Dallas-Fort Worth. Olsson et. al. (2004) applied artificial neural network based downscaling method to forecast rainfall in southern Japan. Tolikaet. al. (2007) used ANNs based downscaling technique to predict seasonal precipitation and rain days for specific sites in Greece.

SVR is also applied in recent studies. Lima et. al. (2012) applied SVR to precipitation downscaling problem and concluded that SVR outperform ANNs. Chen et. al. (2010) applied to downscale daily precipitation in Shih-Men Reservoir basin of Taiwan. Tripathi et. al. (2006) applied SVR to downscaling problem over India and it was shown that SVR outperforms ANNs in these stations.

Recently, linear regression is not used due to nonlinear relationship between the independent variables. ANN and SVR have been utilized by hydrologists to estimate precipitation. These methods are preferred due to their ability to handle nonlinear data. Recently, Hybrid Support Vector Regression (HSVR) was developed by [Lin et.al, 2008]. HSVR method shows a better performance compared to SVR, however HSVR has not been applied to the precipitation downscaling studies.

1.2 Problem Statement

The focus of this study is to perform the following task:

- Apply the following techniques to the downscaling problem:
 - Linear Regression (LR)
 - Artificial Neural Networks (ANN)
 - Hybrid Support Vector Regression (HSVR)
- Predict precipitation as a function of the twenty six (26) atmospheric variables as identified by Canadian Climate Change Scenarios Network (CCCSN).
- Compare HSVR prediction results obtained by LR, ANN methods and recommend the better technique.

1.3 Methodology

Methodology for conducting the above study is as follows:

- Conduct literature review in the area of precipitation on use of ANNs, LR and HSVR to estimate precipitation.
- Apply ANNs, LR and HSVR to precipitation estimation.
- Gather benchmark data from literature and industry archives.
- Compare the results estimated by the three methods.

Chapter 2: Downscaling Methods

Regression models are used to identify relationships between local variables and free atmospheric variables as predictors [Giorgi, et al, 2001]. Multi variable regression is the simplest form of regression technique. It is applied widely to predict local features such as temperature and precipitation [Hellstrom, et.al, 2001]. Techniques such as Artificial Neural Networks (ANNs) [Tatliet. al, 2004] and Support Vector Regression (SVR)[Chen, et.al, 2010] are also utilized to map large scale variables to fine scale climate variables using nonlinear functions. ANN and SVR offer natural flexibility and are capable to identify nonlinear patterns. These techniques are preferred over other methods by the climate study community [Chen, et.al, 2010], however they have some disadvantages. ANNs can fall inside a local minima and the model complexity can be a computationally expensive process [Suykens et.al, 2001]. Similarly, due to complexity of the parameter space, determining the appropriate SVR parameters is difficult [Boardman, et al, 2006]. Beside the aforementioned problems, all regression techniques require identification of predictor variable [Li, et.al, 2008]. A specific variable may not be significant for a location, but highly significant for another location. The HSVR method was developed by [Li, et.al, 2008] to automate the task of feature selection. Brief overview of LR, ANN, SVR and HSVR is provided below.

2.1 Linear Regression (LR)

LR is one of the oldest and simplest tools to analyze the data. A LR model is described in Equation (1):

$$y_1 = \beta_0 + \beta_1 x_{11} + \beta_2 x_{21} + \beta_3 x_{31} + \beta_4 x_{41} + \cdots + \beta_n x_{n1} + \varepsilon \quad (1)$$

Least square method determines the parameters in regression line using Equation (2). ε is the error term which is normally distributed.

$$\hat{\beta} = (X^T X)^{-1} X^T Y \quad (2)$$

The matrices X and vector Y can be defined as Equations (3) and (4) show:

$$X = \begin{bmatrix} 1 & x_{11} \cdots & x_{n1} \\ \vdots & \ddots & \vdots \\ 1 & x_1 \cdots & x_{nN} \end{bmatrix} \quad (3)$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} \quad (4)$$

In stepwise regression, several regression models are used to choose the variables that have the most impact on the model based on the R^2 value. This procedure can be automated using three different approaches: 1) Forward selection, 2) Backward elimination, and 3) Bi-directional elimination. [Ralston et. al, 1960]

2.2 Artificial Neural Networks

2.2.1 Neural Network structure

Artificial Neural Networks (ANNs) are mathematical models inspired by human brain nerve system. Neural Networks in human brain consists of nerve cells. Each nerve cell is connected to other nerve cells [Yegnanarayana, 2004]. Similar to the human neurons, ANNs consist of neurons which are connected to the neurons of previous or next layer. A weight is assigned to each connection. The output of each neuron is a function of these weights [Duda, et. al, 2012]. Each layer has a number of 'nodes'. Figure 2 shows a neural network structure. Each node contains an 'activation function'. Activation function specifies the output of a neuron. Different activation

functions can be utilized in a network. Some common activation functions are sigmoid, tangent and linear. Equations (5), (6) and (7) describe these functions [Duda, et. al, 2012]:

$$\text{Sigmoid: } f(x) = \frac{1}{1 + e^{-x}}; \quad 0 < f(x) < 1 \quad (5)$$

$$\text{Tangent: } f(x) = \frac{e^{-x} - e^x}{e^{-x} + e^x}, \quad 0 < f(x) < 1 \quad (6)$$

$$\text{Linear: } f(x) = x \quad (7)$$

Figure 2-Neural Network Structure

2.2.2 Artificial Neural Network Training

The main goal of neural network is to analyze the data and to adjust weights in a process called “network training”. Back Propagation Algorithm (BPA) is widely applied to train neural networks. Rumelhart (2002) proposed BPA to find the optimal values of connection weights. This method consists of two main steps:

Feed Forward

In this step, the output of each neuron of hidden layer will be obtained using Equation (9).

$$net_j = \sum_{i=0}^n x_i w_{ji} \quad (8)$$

$$O_j = f(net_j) \quad (9)$$

$$net_k = \sum_{j=0}^{n_h} O_j w_{kj} \quad (10)$$

Where w_{ji} is the weight of connection between neuron i to neuron j at hidden layer. net_j and net_{jk} represent the scalar net activation of neuron j and k in hidden layer and output layer, respectively. O_j also shows the output of neuron j in the hidden layer.

Feed Backward

Finally, the output from the output layer is calculated as shown in Equation (11):

$$y_k = f(net_k) \quad (11)$$

In this step, the error of the network is back propagated from output layer to input layer. According to Equation (12), sensitivity of neuron k at output layer (δ_k) is calculated for each output node:

$$\delta_k = y_k(1 - y_k)(y_k - t_k) \quad (12)$$

According to Duda (2012), sensitivity of neuron j at hidden layer (δ_j) is calculated for each node in hidden layer:

$$\delta_j = O_j(1 - O_j) \sum_{k=1}^{Nk} \delta_k w_{jk} \quad (13)$$

Cost function J is calculated by Equation (14). Based on Equation(11), y_k is a function of vector w , therefore J will be the function of w_{kj} and w_{ji} , $k=1, \dots, Nk$; $j=1, \dots, Nh$; and $i=1, \dots, n$.

$$J = \frac{1}{2} \sum (y_k - t_k)^2 \quad (14)$$

The back propagation algorithm can be summarized as follows [Duda, 2012]:

Back Propagation Algorithm

1. Initialize $w, \eta, r = 0$
2. Do $r \leftarrow r + 1$ (Update epoch)
3. $u \leftarrow 0; \Delta w_{ij} \leftarrow 0; \Delta w_{kj} \leftarrow 0$
4. Do $u \leftarrow u + 1$
5. $x^m \leftarrow \text{select pattern}$
6. $\Delta w_{ij} \leftarrow \Delta w_{ij} + \eta \delta_j x_i; \Delta w_{kj} \leftarrow \Delta w_{kj} + \eta \delta_k y_j$
7. Until $u = N$
8. $w_{ij} \leftarrow w_{ij} + \Delta w_{ij}; w_{kj} \leftarrow w_{kj} + \Delta w_{kj}$
9. Until $||\nabla J(w)|| < \theta, \nabla J(w) = J_{r+1}(w) - J_r(w)$
10. Return w

2.2.3 Over-fitting problem

Over-fitting problem occurs when the training data error is much less than validation data error. In this condition, neural networks lack the generalization power and predicting the results of new data is off target. To avoid such a problem, the following action can be taken:

Cross Validation: Data is divided in three sets: Training, validation and testing. The algorithm uses a certain portion of data as a training data. After each training process, the validation data is used to test the performance of the model. After some numbers of failures in validation data sets, the algorithm stops the training process. The model parameters can be estimated, i.e. number of nodes in the hidden layer [Duda, et. al, 2012].

2.3 Support Vector Regression (SVR)

Suppose $\{x_u, t_u\}$ is the given data where $x_u \in R^m$ and $t_u \in R$. The goal is to find a regression line that maps x_u to t_u , such that the points fall inside the predefined tube. For a LR problem where $y_u = w^T x + b$, w_{SVR} is the weight of x and b is the bias term, the accepted region of each point t_u is $\{y_u - \epsilon, y_u + \epsilon\}$. The value of ϵ can be selected by trial and error.

Vapnik (2000) proposed a function which is called ϵ intensive loss function to formulate the problem as shown in Equation (15):

$$L_{\epsilon}(t_u, y_u) = \begin{cases} 0 & |t_u - y_u| < \epsilon \\ |t_u - y_u| - \epsilon & \text{otherwise} \end{cases} \quad \forall_u \quad (15)$$

Equation (16) shows the deviation from tube for u^{th} point. Equations (17) and (18) represent the deviation from different side of the tube.

$$\zeta_u = |t_u - y_u| - \epsilon \quad \forall_u \quad (16)$$

$$\zeta_u^+ = t_u - y_u - \epsilon \quad \forall_u \quad (17)$$

$$\zeta_u^- = y_u - t_u - \epsilon \quad \forall_u \quad (18)$$

A data point can be in three possible areas:

- 1) If data point falls to the left side of the tube, ζ_u^+ will be positive, ζ_u^- will be zero and $L_\epsilon(t_u, y_u) = \zeta_u^+$.
- 2) If data point falls to the right side of the tube, ζ_u^- will be positive, ζ_u^+ will be zero and $L_\epsilon(t_u, y_u) = \zeta_u^-$.
- 3) If data point falls inside the tube, both ζ_u^+ and ζ_u^- will be zero and $L_\epsilon(t_u, y_u) = 0$.

We can conclude that ζ_u^+ and ζ_u^- cannot be simultaneously positive since the data can only be in one of the areas and $L_\epsilon(t_u, y_u) = \zeta_u^+ + \zeta_u^-$. The goal is to find regression line such that the data points fall inside the tube or do not fall in left or right area. To do so, a risk function is defined as follows based on equation (19):

$$R = \frac{1}{N} \sum_{u=1}^N L_\epsilon(x_u, y_u) = \frac{1}{N} \sum_{u=1}^N (\zeta_u^+ + \zeta_u^-) \quad (19)$$

The weight w_{SVR} indicates the complexity of the linear model. To find a simple and flat function without over-fitting, $|w|$ should be minimized. Minimizing $|w|$ is equivalent to minimizing $\frac{1}{2} w_{SVR}^T w_{SVR}$. Equations (21) and (22) represent the boundary of accepted region. By adding this term to our risk function an overall cost function can be obtained. Details of SVR are provided in [Vapnik, 2000].

$$\min z = \frac{1}{2} w_{SVR}^T w_{SVR} + c \sum_{i=1}^N (\zeta_u^+ + \zeta_u^-) \quad (20)$$

Subject To:

$$t_u - y_u < \epsilon + \zeta_u^+ \quad \forall_u \quad (21)$$

$$-\epsilon - \zeta_u^- < t_u - y_u \forall_u \quad (22)$$

$$\zeta_u^+ > 0 \quad \forall_u \quad (23)$$

$$\zeta_u^- > 0 \quad \forall_u \quad (24)$$

2.3.1 Non-linear models

SVR exploits a kernel function to map $x_u \in \mathbb{R}^n$ to higher dimensional feature space where theoretically exists a linear function f to formulate y as a function of input data x [Duda et.al, 2012].

The regression function can be written as Equation (25):

$$y = f(x) = w_{SVR} \varphi(x) + b \quad \forall_u \quad (25)$$

The kernel function can is defined by Equation (26):

$$\varphi(x_u, x_v) = \exp\left(-\frac{1}{2\sigma^2} ||x_u - x_v||^2\right) \quad \forall_u \quad (26)$$

x_u and x_v are all possible combinations of the existing data points. The value of σ is user identified. Using trial and error, one can obtain the value of σ in a time consuming process.

2.4 Hybrid Support Vector Regression

2.4.1 Particle Swarm Optimization

Particle swarm optimization (PSO) is an intelligent algorithm developed by [Eberhart, et al, 1995]. The PSO can be presented as follows:

$$V_{ld}^{ite} = w_{pso} \cdot V_{ld}^{ite-1} + c_1 \cdot a_1 (xp_{ld}^{ite} - x_{l,d(pbestl)}) + c_2 \cdot a_2 (xp_{l,d}^{ite} - xp_{l,d(gbest)}) \quad (27)$$

$$xp_{l,d}^{ite+1} = xp_{l,d}^{ite} + V_{l,d}^{ite+1} \quad (28)$$

PSO algorithm generates a random population in Di-dimensional search space. Particles are moving from their current position $xp_{l,d}^{ite}$ in the direction of the velocity vector $V_{l,d}^{ite}$ according to Equation (28). Particle's position and velocity are changing during the process until a termination criterion has been met.

2.4.2 Fitness criterion

FIT is the fitness criterion to optimize the HSVR. After obtaining adjusted R^2 , as in Equation (30), FIT is used to represent the model performance as in Equation (31). Using this fitness criterion, model one can increase the R^2 for train data and the validation data. R^2 can be represented as follows [Carpenter, 1960]:

$$R^2 = 1 - \frac{\sum (t_u - y_u)^2}{\sum (t_u - \bar{t})^2} \forall_u \quad (29)$$

$$R_{adjusted}^2 = 1 - \frac{(1 - R^2)(N - 1)}{N - p - 1} \forall_u \quad (30)$$

$$FIT = R_{(adj)val}^2 \forall_u \quad (31)$$

To tune SVR parameters using PSO, a solution representation is designed as follows. The model is an extension of work done by [Lin et al, 2008]. The main extension of the method is the fitness function shown in Equation (32). In the previous HSVR model, R^2 for validation data was used to optimize SVR. The optimization problem is described as:

$$Max FIT = g(C, \sigma, F_i) \forall_i = 1. \dots 26 \quad (32)$$

Subject to:

$$C < 1000 \quad (33)$$

$$\sigma < 10 \quad (34)$$

$$F_i = 0,1; C, \sigma > 0 \quad \forall_i = 1. \dots 26 \quad (35)$$

Function g is the fitness function. The first two parameters are SVR parameters. C is the cost and σ is the sigma value of kernel function. F_i is the feature selection variable. This variable represents the importance of a feature in the model. A value of one indicates the given feature is important and a value of zero indicates that the given feature is not important. The HSVR algorithm is as follows:

Step1: Initialize the solution randomly: Each variable will be randomized on a predefined range.

Step2: After acquiring the solution, algorithm will run the SVR. This will result in a fitness function value.

Step3: PSO will update velocity and position based on the fitness function.

Step4: Algorithm will stop after meeting the maximum number of iterations. The number of iterations for this problem is 100. A flowchart of the HSVR algorithm is shown in Figure 3.

Figure 3-HSVR algorithm

Chapter 3: Application of HSVR to Downscale Precipitation

3.1 Data Description

This study utilizes the following four datasets:

1. Data on 26 free atmospheric variables was obtained from <http://www.cccsn.ec.gc.ca> as shown in Table 1. This data uses large scale grid of size 300 km x 300 km
2. Monthly Precipitation data for Comilla, Bangladesh (The data ranges from 1961 to 2001)
3. Monthly Precipitation data for Dhaka, Bangladesh (The data ranges from 1961 to 2001)
4. Monthly Precipitation data for Mymensingh, Bangladesh (The data ranges from 1961 to 2001)
5. Each of the features has different scales. To prepare the data for analysis, they need to be normalized using Equation (36). After the analysis, they will be un-normalized using Equation (37) as follows:

$$XN = 2 * \frac{x - \min(x)}{\max(x) - \min(x)} - 1 \quad (36)$$

$$x = \frac{(XN + 1)(\max(x) - \min(x))}{2} + \min(x) \quad (37)$$

Table 1-List of large grid features

Feature Number	Feature	Description
1	Temp	Mean temperature at 2 m
2	MSLP	Mean sea level pressure
3	H850	850 hPageopotential heights
4	H500	500 hPageopotential heights
5	USUR	Near surface westerly wind
6	U850	Westerly wind at 850 pa
7	U500	Westerly wind at 500 pa
8	VSUR	Near surface southerly wind
9	V850	Southerly wind at 850 pa
10	V500	Southerly wind at 500 pa
11	FSUR	Near surface wind strength
12	F850	Wind strength at 850 pa
13	F500	Wind strength 500 pa
14	ZSUR	Near surface vorticity
15	Z850	Vorticity at 850 pa
16	Z500	Vorticity at 500 pa
17	DSUR	Near surface divergence
18	D850	Divergence at 850 pa
19	D500	Divergence at 500 pa
20	QSUR	Near surface specific humidity
21	Q850	Specific humidity at 850 pa
22	Q500	Specific humidity at 500 pa
23	RSUR	Near surface relative humidity
24	R850	Relative humidity at 850 pa
25	R500	Relative humidity at 500 pa
26	Temp	Mean Temperature at 2m

3.2 Results and Analysis

The SVR was applied to data for Comilla, Dhaka and Mymensingh cities in Bangladesh using MATLAB software and LibSVM toolbox. To implement Artificial Neural Network a demo version of commercial Dtrex software [Sherrod, Phil, 2014] was used. The data set was partitioned in three different sets: calibration, validation and testing. The percentage of each portion of data is 70, 20 and 10, respectively. The maximum iteration for PSO to train this data was determined as 100. Observed precipitation and NCEP data for Comilla, Dhaka and Mymensingh cities were used to model calibration and validation. Three measurements of Bias, RMSE and $R^2_{(adj)}$ is used to compare the results of each technique. Figure 4 displays the location of each city in the map.

Figure 4-Three Stations (Source: Nations Online Project)

3.2.1 Comilla Station

The Comilla station is located at $23^{\circ} 27' 20''$ N latitudes and $91^{\circ} 10' 58''$ E longitudes in Bengal delta. To construct the model, the NCEP data was trained for the period of 1961-2001 to find a link between large scale atmospheric variables and the local precipitation in Comilla station. Using stepwise regression, the following equation was obtained:

$$y = -0.53 + 0.19F_7 + 0.16F_{17} + 0.16F_{13} + 0.31F_{11} + 0.16F_5 - 0.16F_{10} \quad (38)$$

The optimum number of hidden neurons of 12 was found for ANNs based on validation data on the variables selected in stepwise regression method. $\epsilon = .001$, $C = 83.63$ and $\sigma = 0.63$ was found as optimal setting for SVR.

Figure 5- Monthly observed and downscaled precipitation for Comilla station

Table 2-Comparative results for test data in Comilla station

	LR	ANN	HSVR
R2	0.700	0.660	0.780
R2(adj)	0.650	0.600	0.740
Bias	1448	1592	1195
RMSE	151	163.170	132.910

Figure 5 demonstrates the acceptable performance of HSVR in downscaling GCM outputs to precipitation in Comilla station. Table 2 represents the results for test data which is a good indicator for generalization power of the HSVR which is better compared to other techniques.

3.2.2 Dhaka Station

The Dhaka station is located at 23.7000° N latitudes and 90.3750° E longitude. Similar to Comilla station, the NCEP data was trained for the period of 1961-2001 to seek out a link between large scale atmospheric variables and the local precipitation in Dhaka station. The optimum number of hidden neurons is 5 in this station. The optimum setting for SVR was obtained as $\epsilon = .001$, $C = 99.3745$ and $\sigma = 0.19$.

Equation (39) was obtained using stepwise regression for this station. Figure 6 represents the suitable performance of the HSVR method and compares the results of HSVR with previous methods in Dhaka station.

$$y = -0.54 + 0.36F_2 - 0.2F_{23} - 0.11F_{19} + 0.08F_{13} + \varepsilon \quad (39)$$

Table 3 represents the results for test data for Dhaka station showing better performance of HSVR compared to other techniques.

Table 3 - Comparative results for test data in Dhaka station

	LR	ANN	HSVR
R2	0.622	0.613	0.673
R2(adj)	0.582	0.573	0.638
Bias	453.770	451.820	406.950
RMSE	98.700	100.180	91.210

Figure 6-Monthly observed and downscaled precipitation for Dhaka City

3.2.3 Mymensingh Station

Mymensingh station is located at 24.7539° N latitude and 90.4031° E longitudes. Similar to other stations to build the model, the NCEP data was used to build the model for the period of 1961-2001 in Mymensingh station (Figure 7). In this station, table 4 shows that HSVR outperforms previous techniques. The optimum number of hidden neurons is 4 in this station and optimum setting for SVR is $\epsilon = .001$, $C = 89.2013$ and $\sigma = 0.3213$. After getting the model on the period of 1961-2001. Table 4 compares HSVR with previous methods in this station. Equation (40) shows the coefficients obtained using stepwise regression:

$$y = -0.58 + 0.06F_7 + 0.13F_{13} - 0.59F_{21} - 0.53F_{10} + 0.20F_{26} - 0.08F_8 + 0.71F_{11} + \varepsilon \quad (40)$$

Figure 7-Monthly observed and downscaled precipitation for Mymensingh City

Table 4-Comparative results for test data in Mymensingh station

	LR	ANN	HSVR
R2	0.640	0.720	0.770
R2(adj)	0.560	0.660	0.720
Bias	669.201	892.742	453.794
RMSE	25.870	29.880	21.300

3.2.4 Model for the features at surface level

In this section, large scale features of Table 4 are grouped based on the level of the feature: Surface level, 500pa and 850 pa.

Table 5 represents the surface level features. In this section, these features are used to downscale precipitation in three stations of Dhaka, Mymensingh and Comilla using LR, ANNs and HSVR. A comparative study is conducted to examine the performance of each technique results.

Table 5-Surface Level Features

Feature Number	Description
1	Mean sea level pressure
2	Surface sea level pressure
3	Surface airflow strength
4	Surface meridional velocity
5	Surface vorticity
6	Surface wind direction
7	Surface divergence
8	Near surface relative humidity
9	Surface specific humidity
10	Mean temperature at 2m

Comilla station

After the running of the stepwise linear regression the Equation (41) was obtained. The optimum number of neurons for hidden layer in neural network is 2. The optimum setting for SVR is as $\epsilon = .001$, $C = 23.32$ and $\sigma = 3.41$. From the values of RMSE and R2-adj on it can be concluded that the HSVR is more reliable technique for downscaling in this station. Results are shown in Table 5.

$$y = 0.28 * F_{surf_3} + 0.26 * F_{surf_5} + .81 * F_{surf_9} - .64 * F_{surf_{10}} - .455 \quad (41)$$

Table 6-Comparative results of surface features for Comilla station

	LR	ANN	HSVR
R2	0.487	0.478	0.595
R2-adj	0.340	0.329	0.479
MSE	20790.806	21536.484	17315.515
RMSE	144.190	146.753	131.588
Bias	1165.788	1501.866	1322.757

Dhaka station

After the running of the stepwise linear regression the Equation (42) was obtained. The optimum number of neurons for hidden layer in neural network is 2. The optimum setting for SVR is as $\epsilon = .001$, $C = 0.48$ and $\sigma = 0.16$. From the values of RMSE and R2-adj it can be concluded that the HSVR is more reliable technique for downscaling in this station. Results are shown in Table 7.

$$y = -0.34 * F_{surf_4} - 0.51 * F_{surf_8} + 1.23 * F_{surf_9} - .69 * F_{surf_{10}} - 0.50 \quad (42)$$

Table 7-Comparative results of surface features for Dhaka station

	LR	ANN	HSVR
R2	0.703	0.673	0.709
R2-adj	0.610	0.571	0.618
MSE	7913.045	9136.456	7719.496
RMSE	88.955	95.584	87.861
Bias	-796.233	-1180.258	317.035

Mymensingh station

After the running of the stepwise linear regression the Equation (43) was obtained.

$$y = 0.31 * F_{surf_1} + 0.20 * F_{surf_3} - 0.06 * F_{surf_6} - 0.66 * F_{surf_8} + 1.95 * F_{surf_9} - 1.24 * F_{surf_{10}} - .38 \quad (43)$$

The optimum number of neurons for hidden layer in neural network is 6. The optimum setting

for SVR is as $\epsilon = .001$, $C = 67.01$ and $\sigma = 1.16$. From the values of RMSE and R2-adj it can be concluded that the HSVR is more reliable technique for downscaling in this station. Results are shown in Table 8.

Table 8-Comparative results of surface features for Mymensingh station

	LR	ANN	HSVR
R2	0.663	0.660	0.713
R2-adj	0.558	0.554	0.623
MSE	11620.199	10541.092	7687.767
RMSE	107.797	102.670	87.67991
Bias	-73.780	-151.237	154.883

3.2.5 Model for the features at 500 Pa

Table 8 represents the features 500Pa far from the sea level. In this section, these features are used to downscale precipitation in three stations of Dhaka, Mymensingh and Comilla using LR, ANNs and HSVR. A comparative study is conducted to examine the performance of each technique. Results are shown in Table 9.

Table 9- 500Pa Large Scale Features

Feature Number	Description
1	500 hpa airflow strength
2	500 hpa zonal velocity
3	500 hpameridonia velocity
4	500 hpaveticity
5	500 hpadivergance
6	500 hpa wind direction
7	500 hpapeopotential height
8	Relative humidity at 500 hpa

Comilla station

After the running of the stepwise linear regression the Equation (44) was obtained. The optimum number of neurons for hidden layer in neural network is 2. The optimum setting for SVR is as $\epsilon = .001$, $C = 11.85$ and $\sigma = 26.12$. From the values of RMSE and R2-adj it can be concluded that the HSVR is more reliable technique for downscaling in this station.

$$y = -0.33 * F_{500paf_1} + 0.33 * F_{500paf_4} + 0.13 * F_{500paf_8} - .58 \quad (44)$$

Table 10-Comparative results of 500pa features for Comilla station

	LR	ANN	HSVR
R2	0.404	0.346	0.580
R2-adj	0.275	0.205	0.489
MSE	24036.218	25926.904	17106.678
RMSE	155.036	161.018	130.7925
Bias	405.251	782.164	856.2134

Dhaka station

After the running of the stepwise linear regression the Equation (45) was obtained. The optimum number of neurons for hidden layer in neural network is 2. The optimum setting for SVR is as $\epsilon = .001$, $C = 71.72$ and $\sigma = 2.44$. From the values of RMSE and R2-adj it can be concluded that the HSVR is more reliable technique for downscaling in this station. Results are shown in Table 11.

$$y = -0.42 * F_{500paf_1} + 0.18 * F_{500paf_4} + 0.14 * F_{500paf_5} - 0.62 \quad (45)$$

Table 11-Comparative results of 500pa features for Dhaka station

	LR	ANN	HSVR
R2	0.562	0.603	0.716
R2-adj	0.459	0.510	0.649
MSE	12457.214	10908.616	7354.510
RMSE	111.612	104.444	85.758
Bias	-1436.664	-1140.296	280.667

Mymensingh station

After the running of the stepwise linear regression the following Equation (46) was obtained. The optimum number of neurons for hidden layer in neural network is 5. The optimum setting for SVR is $\text{as } \epsilon = .001, C = 71.72 \text{ and } \sigma = 2.44$. From the values of MSE and R2-adj it can be concluded that the HSVR is more reliable technique for downscaling in this station. Results are shown in Table 12.

$$y = -0.71 * F_{500paf_1} + 0.50 * F_{500paf_2} + 0.33 * F_{500paf_4} + 0.19 * F_{500paf_5} + 0.12 * F_{500paf_8} - 0.62 \quad (46)$$

Table 12-Comparative results of 500pa features for Mymensingh station

	LR	ANN	HSVR
R2	0.478	0.491	0.634
R2-adj	0.343	0.360	0.540
MSE	16478.558	16227.306	10745.802
RMSE	128.369	127.386	103.662
Bias	-1240.054	-1386.111	-336.331

3.2.6 Model for the features at 850 Pa

Table 13-850 Large Scale Features

Feature Number	Description
1	850 hpa airflow strength
2	850 hpa zonal velocity
3	850 hpameridonia velocity
4	850 hpaveticity
5	850 hpadivergance
6	850 hpa wind direction
7	850 hpageopotential height
8	Relative humidity at 850 hpa

Comilla station

After the running of the stepwise linear regression the Equation (47) was obtained. The optimum number of neurons for hidden layer in neural network is 2. The optimum setting for SVR is as $\epsilon = .001$, $C = 15.04$ and $\sigma = 1.55$. From the values of RMSE and R2-adj it can be concluded that the HSVR is more reliable technique for downscaling in this station. Results are shown in Table 14.

$$y = -0.28 * F_{850paf_1} + 0.40 * F_{850paf_2} - 0.51 * F_{850paf_5} + 0.18 * F_{850paf_8} - 0.46 \quad (47)$$

Table 14-Comparative results of 850pa features for Comilla station

	LR	ANN	HSVR
R2	0.546	0.522	0.599
R2-adj	0.448	0.419	0.512
MSE	19607.983	20269.273	19233.151
RMSE	140.029	142.370	138.687
Bias	1954.459	1787.563	2451.614

Dhaka station

After the running of the stepwise linear regression the Equation (48) was obtained. The optimum number of neurons for hidden layer in neural network is 2. The optimum setting for SVR is as $\epsilon = .001$, $C = 85.8$ and $\sigma = 9.51$. From the values of RMSE and R2-adj it can be concluded that the HSVR is more reliable technique for downscaling in this station. Results are shown in Table 15.

$$y = -0.26 * F_{850paf_1} - 0.09 * F_{850paf_4} + 0.51 * F_{850paf_5} + .43 * F_{850paf_7} - 0.37(48)$$

Table 15-Comparative results of 850pa features for Dhaka station

	LR	ANN	HSVR
R2	0.611	0.592	0.683
R2-adj	0.519	0.496	0.608
MSE	10371.923	10479.498	8177.597
RMSE	101.843	102.369	90.430
Bias	-900.363	-184.099	-284.656

Mymensingh station

After the running of the stepwise linear regression the Equation (49) was obtained. The optimum number of neurons for hidden layer in neural network is 3. The optimum setting for SVR is as $\epsilon = .001$, $C = 31.46$ and $\sigma = 0.10$. Based on the values of RMSE and R2-adj, it can be concluded that the HSVR is a more reliable technique for downscaling in this station. Results are shown in

$$y = -0.18 * F_{850paf_1} + 0.25 * F_{850paf_4} - 0.14 * F_{850paf_5} - .42 * F_{850paf_7} + 0.16 * F_{850paf_8} - 0.48(49)$$

Table 16-Comparative results of 850 pa features for Mymensingh station

	LR	ANN	HSVR
R2	0.546	0.613	0.627
R2-adj	0.429	0.513	0.531
MSE	17224.264	13672.636	11503.708
RMSE	131.241	116.930	107.255
Bias	649.002	474.860	174.885

Chapter 4: Conclusion

Global climate change is a major area of concern to public and climate researchers. The focus of this study was to evaluate regression techniques appropriate for precipitation downscaling problem. This study compared Linear Regression (LR), Artificial Neural Networks (ANNs), Hybrid Support Vector Regression (HSVR) techniques to predict precipitation at three (Mymensingh, Comilla and Dhaka) stations in Bangladesh. The study utilized 26 atmospheric features identified by CCCSN. The metrics used for comparing these techniques were R^2 , R^2_{adj} , MSE, RMSE and bias.

The raw data was grouped at surface level, 500 Pa and 850 Pa for data for each group data was split into three portions: calibration, validation and test in the ratio of 70, 20 and 10, respectively. The performance of each model was compared to the performance of other models on the test data.

In Surface level model for city of Comilla, the R^2 in HSVR was higher than other techniques. The R^2 for ANNs was close to R^2 of LR. The bias term of HSVR was less than bias of other techniques. LR outperformed ANN in terms of bias. Surface level model for the city of Dhaka HSVR outperformed other techniques in terms of R^2 and RMSE and LR performed better compared to other techniques in terms of bias.

In Mymensingh city, the result of ANNs and LR represents an equal performance in terms of R^2 and RMSE, but the bias value indicates a better performance of ANNs compare to LR. HSVR outperformed both techniques in different terms.

For 500 Pa scenarios of Comilla, HSVR outperform other techniques in terms of R^2 and RMSE. LR outperforms other techniques in terms of bias. HSVR outperformed ANNs and LR in terms of R^2 , RMSE and bias in Dhaka station at 500 Pa. ANN showed a better result compared to LR. For 500 Pa scenarios of Mymensingh station, HSVR outperformed LR and ANNs in terms of R^2 , RMSE and bias. In this station, ANN showed a better performance compared to LR.

In 850 Pa scenarios of Comilla, ANN outperformed HSVR and LR in terms of RMSE and bias, but the R^2 for HSVR was higher than LR and ANNs. For 850 Pa scenarios of Dhaka station, HSVR outperformed ANNs and LR in terms of R^2 , MSE, although ANN showed a smaller bias value compared to other techniques. In Mymensingh city at 850 Pa, HSVR outperformed LR and ANNs in terms of R^2 RMSE and bias.

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Appendix

Result of Comilla Station

LR	ANN	HSVR	Actual
0	7.4919	0	0
38.40995	4.431394	26.68918	0
10.95342	0	21.75082	0
225.7063	202.2992	204.5633	116
395.2081	311.3735	320.8654	182
431.9948	379.3444	512.9184	639
406.0056	373.3253	449.9248	382
360.1953	365.6328	368.6153	577
344.3329	345.3186	320.9756	114
161.269	193.5954	164.4801	134
67.1821	54.39016	33.28611	0
61.11085	5.169233	16.2496	0
0	0	14.68669	0
54.82758	2.332845	13.44536	0
0	0	0	0
236.7803	223.4991	185.3211	189
333.0068	285.9164	228.2955	237
462.0446	370.3777	477.2742	446
337.4565	380.8558	345.7911	456
286.6099	348.4904	292.014	230
195.8975	313.1967	191.9974	98
124.0254	178.1816	112.144	231
0	27.04694	15.38518	0
0	11.3045	16.75667	0
23.95085	0.667379	13.51156	0
0	-4.01698	11.13109	0
33.46734	15.16414	6.634551	0
281.381	219.3297	228.2007	121
304.0275	269.2382	246.3138	343
445.9003	347.6506	521.0399	544
443.0747	346.9743	518.4423	816
359.3145	359.0128	387.8781	540
217.32	294.0573	210.6202	428
255.9528	223.0164	255.0374	533
32.38737	70.16078	35.13118	28
6.055585	3.154961	4.34259	0
0	1.706289	0	0

Result of Comilla Station (cont'd)

LR	ANN	HSVR	Actual
0	0	13.17921	0
0	0	7.814551	0
145.8676	162.8833	39.34804	0
224.7175	241.2593	161.6533	53
461.9884	373.2782	546.0972	717
381.5965	358.2795	402.2044	595
311.4961	313.2057	304.3034	810
204.845	258.4074	200.5517	329
118.4512	151.2181	105.0465	49
56.10876	55.01283	35.68073	99

Results for Dhaka Station

LR	ANN	HSVR	Actual
226.9091	218.6896	315.623	414
344.787	349.3528	340	323
341.684	339.89	328.1558	380
336.0109	348.1331	313.1402	269
260.5191	266.0759	251.7338	296
158.315	150.7382	149.2242	300
72.7099	64.97926	58.44648	0
21.66335	24.29814	26.59614	0
0	2.854333	38.54991	10
0	2.720116	39.74836	42
31.8439	28.59713	37.4208	109
180.0084	167.4901	200.6018	274
291.1107	289.7411	300.6371	272
347.6761	356.0117	312.503	168
348.9662	354.3548	380.0383	356
341.4519	343.1559	280.3275	188
276.1385	279.7307	294.4424	320
171.7293	165.5104	126.1096	82
59.60113	54.04271	50.04636	9
7.416278	13.04379	18.03288	35
16.49892	19.14078	14.6789	0
0	1.548599	0	15
23.4128	18.21724	37.04616	81
190.8914	186.2955	135.714	104
295.3557	297.9235	293.9117	154

Results for Dhaka Station (Cont'd)

LR	ANN	HSVR	Actual
360.4496	361.9288	307.2152	514
339.8575	347.0915	331.5223	136
325.9468	335.2291	316.0104	346
261.9377	264.7644	257.2453	258
125.936	119.9553	172.3593	146
53.76282	47.62211	19.58237	51
0	0	15.87201	0
0	0	37.84018	61
195.7	182.112	177.2174	318
228.8397	220.6717	310.1535	348
362.8828	366.4358	335.2329	300
346.7499	344.9664	325.1153	179
328.9846	349.9435	354.5373	437
276.2609	276.5543	262.8227	322
149.89	143.0574	154.7874	253
0	0	44.52219	13
0	4.758717	40.62641	1
17.93187	21.3467	47.72421	5
149.2771	137.9367	79.85005	124
327.1211	331.265	310.0757	637

Results for Mymensingh Station

ANNs	LR	HSVR	Actual
0.0	2.8	0.0	0
7.5	8.2	0.0	0
28.4	21.2	0.0	0
0.0	0.0	9.7	0
0.0	0.0	28.2	0
16.1	46.9	10.6	0
0.0	0.0	25.4	0
1.7	0.0	0.0	4
6.8	14.5	0.0	7
55.2	100.6	0.0	10
0.0	0.0	37.9	13
0.0	0.0	14.7	19
50.8	84.3	17.1	19
13.8	25.1	23.2	20
26.8	11.4	55.2	25

Results for Mymensingh Station
(Cont'd)

ANNs	LR	HSVR	Actual
46.3	42.3	9.8	31
81.7	114.7	50.6	46
0.0	0.0	31.1	49
83.7	53.3	44.5	59
144.3	145.9	54.8	74
157.9	156.6	113.3	76
225.2	216.5	129.4	101
285.0	281.3	252.1	179
135.9	187.4	111.4	189
236.4	184.5	232.3	204
352.7	410.4	405.6	211
458.1	464.4	487.6	218
334.8	280.0	287.8	237
338.4	329.0	408.1	283
250.1	235.1	268.7	297
219.5	210.8	289.1	368
369.6	352.0	297.7	386
201.9	184.2	270.6	396
288.1	227.0	377.6	409
234.9	174.2	245.1	448
377.9	354.6	422.7	451
369.6	382.5	359.1	472
416.7	411.1	516.4	488
422.4	357.7	495.4	558
416.3	387.1	446.7	598
290.4	263.5	330.6	669