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Consistent Regional Commodity-by-Industry Input-Output Accounts

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Foundations for Impacts and Structural Change Analyses

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Abstract

A long-standing regional science problem domain focuses on the identification of structural economic change. One of several approaches reliles on the use of historical final demand series and a comparison of observed inudstry output to an estimate of what output would have been were economic structure static. However, these methods were first developed before the introduction of today's commonly used commodity-by-industry (CxI)input-output (IO) accounting frameworks, and before the application of these methods to regional economies. Correctly formulating the supporting accounting structures for these analyses is essential, but can be challenging even for experienced analysts. Related textbook and journal articles often imply a simplicity that belies two important barriers to understanding. First, although modern IO accounts are now almost universally compiled and distributed as CxI accounts, IO methods presentations are very commonly founded on inter*industry* accounts. Second, introductions to many IO-based methods tend to focus on national IO accounting, for which the implications of degree of openness of the economy are seldom – if ever – discussed. The implication is that the path from published national data to a coherent set of regional CxI accounts is simple and straightforward when, in fact, there are several key considerations to be taken and assumptions to be made along the way. In this paper, we lay out the mathematical foundations of a CxI version of traditional interindustry regionalization and structural change analyses, and in so doing, clarify appropriate regional commodity-by-industry impacts assessment formulations.

1 Introduction

Identifying and understanding the nature of structural economic change is a longstanding research objective. Leontief (1965), Tilanus and Harkema (1966), and Carter (1967) laid early foundations for such analyses in their studies of changes in the structure of the American and Dutch economies. Given the inter-industrial input-output (IO)structure for an economy for time t along with final demands for time t and $t-n$, the industrial output and factor input requirements by industry for each year can be estimated. Differences between expected (estimated) and observed values for year $t-n$ and are a measure of structural change from $t-n$ to t.

In developing his regional hybrid econometric IO model, Conway (1990) overcame the objections to static IO structure in projection and simulation models by making use of this approach to modeling structural change. Given a time series of final demands and IO accounts for a model calibration year, the relationships among observed and expected values for each industry can be quantified econometrically and the resulting specifications can then be embedded within the model to account for anticipated system-wide structural economic change.

Building the database for these econometric estimations for open economies requires a thorough understanding of the correct formulation for generating expected output values. In the following sections, we develop comprehensively the systems of equations that give rise to this correct formulation, recognizing explicitly the commodity-by-industry (CxI) source data that modern analysts will need to use.

2 Regional Interindustry IO Accounting

As a point of reference, we begin with the general foundations for regionalizing interindustry IO accounts. If we assume zero re-exports (exported imports), regional output for exports can be expressed as

$$(I - \hat{P}A)^{-1}E \tag{1}$$

and regional output for regional demand will be

$$(I - \hat{P}A)^{-1}\hat{P}(C + I + G), \tag{2}$$

where variables C, I, and G are regional consumption, investment, and government expenditures by industry, and the regional supply percentages (RSP) are defined by

$$P = (X - E)(X - E + M)^{-1}\mathbf{i}. \tag{3}$$

Variables E, M, and X are regional industry exports, imports, and output, respectively. The complete regional industry output balance equation can now be expressed as

$$X = (I - \hat{P}A)^{-1}(\hat{P}(C + I + G) + E), \tag{4}$$

which establishes the fundamental accounting relationships that form the basis of the standard interindustry impacts formulation,

$$\Delta X = (I - \hat{P}A)^{-1}[\hat{P}\Delta(C + I + G) + \Delta E] \tag{5}$$

Equation 5 clarifies an area of common confusion in application, namely when and how to modify demands by RSP; they should modify all but export final demand. The confusion arises in part due to a tendency in many presentations to focus on components of equation systems, e.g., coefficients matrices, multiplier matrices, and so on, without placing them in the context of the complete accounting systems and identities. In the following section, we shift the focus to open regional economies and corresponding CxI accounts and regionalization methods.

3 Commodity-by-Industry Frameworks

3.1 Closed Economy

To derive a parallel regionalization formulation for Stone-type CxI accounts (Stone, 1961), we begin with the conventional CxI IO accounting framework for a closed economy as shown in Figure 1.

Figure 1: The Commodity-by-Industry Framework

	Commodity	Industries	Final Demand	Totals
Commodity		U	e	q
Industries	V			g
Primary Inputs		va		
Totals	q'	g'		

Source: Adapted from United Nations (1968)

In matrix notation, we have the following identities:

$$U\mathbf{i} + e \equiv q \tag{6}$$

$$V\mathbf{i} \equiv g \tag{7}$$

$$V'\mathbf{i} \equiv q \tag{8}$$

where U, V, g, q, and e are the Use and Make matrices, industry and commodity output, and a commodity final demand vector, respectively, and va is value added by industry. $\mathbf{i}$ is a summing vector, and $'$ signifies the transpose operation. Next, behavioral relationships are indicated as follows:

$$B = U\hat{g}^{-1} \tag{9}$$

$$U = B\hat{g} \tag{10}$$

$$D = V\hat{q}^{-1} \tag{11}$$

$$V = D\hat{q} \tag{12}$$

where $\hat{}$ indicates vector diagonalization. Equation 9 defines the production requirements of commodities per industry output dollar, and equation 11 is a statement of the industry-based technology assumption that commodities are produced by industries in fixed proportions.[1] Note that the effect of pre-multiplication of a commodity vector or matrix by D results in a transformation from commodity-space to industry-space, so $V\mathbf{i} = g = Dq$. This system allows us to formulate the following:

$$q = Bg + e \tag{13}$$

$$q = BDq + e \tag{14}$$

$$q = (I - BD)^{-1}e \tag{15}$$

Similarly, premultiplying Equation (13) by D yields

$$Dq = DBg + De \tag{16}$$

$$g = DBg + De \tag{17}$$

$$g = (I - DB)^{-1}De \tag{18}$$

3.2 Open Economy

Equations 6 through 18 describe fully an economic system closed to trade. Analysts who use IO for impact assessment will nearly always need to reformulate the system representation to accommodate trade with the rest of the world. In the process, technical coefficients, a_{ij} are effectively bifurcated such that they become equal to the sum of the domestic input per dollar output coefficient, r_{ij}, and the import coefficient, m_{ij}, or $a_{ij} = r_{ij} + m_{ij}$. This bifurcating procedure is commonly called *regionalization* when its goal is the parameterization of a subnational region, but the approach we use can be implemented in similar fashion with national "regions."

Jackson's (1998) method was the first to address explicitly the regionalization of CxI as opposed to interindustry national accounts. His approach offers a number of advantages in terms of transparency in exposition, formal representation, and algebraic manipulation. It involves the modification of *make* and *use* tables to correspond to estimated regional output by industry, estimation of one set of regional final demand activities based on regional production levels and a second set of regional final demand activities related

[1] The alternative is the commodity-based technology assumption, which while not used here, could be developed in parallel fashion.

more directly to the economic size of the region (e.g., gross regional product relative to the national economy), and lastly, a supply-demand pooling method modified by cross-hauling estimates. In his presentation, Jackson (1998) introduced a convenient mechanism in matrix notation for bifurcating the technical coefficients as described above. The core of this representation lies in the standardization of the *make* table not by domestic commodity output but instead by total regional supply, which is consistent with the RSP denominator in equation 3. The effect, elaborated further below, is analagous to other "rows-only" adjustment methods, where each row is multiplied by a value between zero and 1.0 that reflects the regional supply percentage – the proportion of local commodity demand that is supplied locally, i.e., produced within the region.

Jackson's (1998) paper originally defined $\tilde{D}$ as the *make* matrix standardized by $s = q + m$, or $\tilde{D} = V\hat{s}^{-1}$. However, further analysis reveals that the original formulation implied that imports would partially satisfy not only domestic demand but also export export demand. A formulation where export demand will be satisfied instead by domestic production within the CxI framework is obtained by recasting the RPS in commodity space.[2] To do so, begin by specifying a a commodity-space counterpart to equation 3, as shown in equation 19. Redefine the units of vectors M and E as import and export values now by *commodity*, and let

$$Q = \mathbf{i}(V - D\hat{E})(\widehat{\mathbf{i}V - E + M})^{-1} = \mathbf{i}(\widehat{q - E})(\widehat{q - E + M})^{-1} \tag{19}$$

where V is defined as in equation 12, q is a vector of commodity output, and

$$D = V\hat{q}^{-1}. \tag{20}$$

Now define $\tilde{D}$ as

$$\tilde{D} \equiv D\hat{Q}. \tag{21}$$

Substituting equation 19 into equation 21, we obtain

$$\tilde{D} = D(\widehat{q - E})(\widehat{q - E + M})^{-1} = (V - D\hat{E})(\widehat{\mathbf{i}V - E + M})^{-1} \tag{22}$$

Next, we draw from the standard CxI accounting relationships, beginning with the commodity balance equation.

[2]This again effectively disregards re-exports.

$$U\mathbf{i} + C + G + I \equiv q - E + M \tag{23}$$

Now multiply equation (23) by $\tilde{D}$ and substitute $D\hat{Q}$ from equation 21 on the RHS to obtain

$$\tilde{D}U\mathbf{i} + \tilde{D}(C + G + I) = D\hat{Q}(q - E + M) \tag{24}$$

Substituting Bg for $U\mathbf{i}$, where g denotes industry output, and using equation 19, we obtain

$$\tilde{D}Bg + \tilde{D}(C + G + I) = D(\widehat{q - E})(\widehat{q - E + M})^{-1}(q - E + M) \tag{25}$$

$$\tilde{D}Bg + \tilde{D}(C + G + I) = D(q - E) = g - DE \tag{26}$$

And rearranging, we obtain

$$g - \tilde{D}Bg = \tilde{D}(C + G + I) + DE \tag{27}$$

or,

$$g = (I - \tilde{D}B)^{-1}[\tilde{D}(C + G + I) + DE] \tag{28}$$

The regional impact of new export demand is

$$\Delta g^E = (I - \tilde{D}B)^{-1}D\Delta E \tag{29}$$

the regional impact of new intra-regional final demand is

$$\Delta g^R = (I - \tilde{D}B)^{-1}\tilde{D}\Delta(C + G + I), \tag{30}$$

and the comprehensive impact assessment equation is

$$\Delta g = (I - \tilde{D}B)^{-1}[\tilde{D}\Delta(C + G + I) + D\Delta E] \tag{31}$$

Finally, equation (32) shows the form of the equation used in forecasting a time series of *expected* output as used in the construction of hybrid econometric IO models (e.g., Conway, 1990). Again, the objective is the generation of a time series of what would have been the outputs required to satisfy a time series of final demand vectors for $t = \{[t-n, ...t]\}$ given a fixed interindustry and trade structure. Holding D, B, and $\tilde{D}$ constant at the reference year, the result is

$$g_t = (I - \tilde{D}B)^{-1}[\tilde{D}(C + G + I)_t + DE_t]. \quad (32)$$

4 Summary

In this paper, we have provided the foundations for a CxI version of traditional interindustry IO regionalization methods. The path from national CxI accounts to regional analytical formulations is now explicit, as are supporting frameworks for impacts and traditional structural change analyses.

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