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Impact of Stress-field Changes Induced by Hydraulic Fracturing on Subsurface Fault Stability

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**Impact of Stress-field Changes Induced by Hydraulic Fracturing on Subsurface
Fault Stability**

Qian Gao

**Thesis submitted to the
College of Engineering and Mineral Resources
at West Virginia University
In partial fulfillment of the requirements
For the degree of**

**Master of Science
In
Petroleum and Natural Gas Engineering**

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2012**

Keywords: Hydraulic Fracturing; Fault Stability; Finite Element Method

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Abstract

Impact of Stress-field Changes Induced by Hydraulic Fracturing on Subsurface Fault Stability

Qian Gao

Hydraulic fracturing is essential for economically producing hydrocarbons from shale reservoirs, and thus has been extensively used in completion of shale gas wells. However, fracturing formations can change in-situ stresses in the surrounding areas. There are concerns that hydraulic fracturing may potentially reactivate the subsurface faults nearby the treatment wells. The reactivated faults could cause many problems, such as early aborting/failure of fracturing treatment, fluid leakage along the fault and even seismicity events.

3D numerical models are developed based on finite element method. The comparison between analytical and numerical solutions indicates an excellent agreement has been achieved, which certifies the applicability of the numerical models to complex situations.

Stress redistribution around hydraulic fractures can occur due to the opening of hydraulic fractures and poroelastic effects. Only the opening of hydraulic fractures is considered in numerical models. The results indicate that the initial in-situ stress contrast in horizontal plane has a strong influence on the extent of stress-reversal region and reoriented-stress region. When the generated stress contrast is larger than the initial in-situ stress contrast, there will be stress-reversal region in the vicinity of fractures. As the distance from fractures increases, the generated stress contrasts become smaller. When the generated stress contrast is less than the initial in-situ stress contrast, the stress trajectories do not reverse. Usually the stress-reversal region is in the vicinity of fractures, the reoriented-stress region is beyond the stress-reversal region and around fracture tips.

In this study, fault stability during hydraulic fracturing was investigated using 3D numerical models. Three typical faulting environments were considered. They are normal, strike-slip, and strike-slip/reverse faults. The orientation and relative magnitudes of in-situ stress fields differ under different faulting environments, which in turn control the direction of fracture propagation. Three hydraulic fractures created simultaneously are considered in all case studies. It was found that the angle between fracture orientation and fault strike has a strong effect on the stability of a fault based on the change in the ratio of shear to effective normal stresses. Along the fault plane, the stability is strengthened in certain regions but weakened in other regions. The stress alteration patterns are different on the three types of fault. The normal faulting environment has the largest fluctuation in stresses and in the ratio of shear to effective normal stress. The reverse/strike-slip fault has the least perturbation on stresses and the ratio of shear to effective normal stress when hydraulic pressure is applied.

This study provides insight on the stress redistribution during hydraulic treatment and the impact of hydraulic fracturing on fault stability. The results indicate that it is feasible to manage the stability of faults by adjusting fracture design.

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1 Introduction

1.1 Problem Statement

Hydraulic stimulation is an important technology for extracting hydrocarbons from both conventional and unconventional reservoirs. For ultralow-permeability shale reservoir now being regularly exploited, hydraulic treatment is absolutely essential to obtain economic levels of production (Sutton, Cox, & Barree, 2010; N. Warpinski, Du, & Zimmer, 2012). An important concern regarding the deployment of hydraulic fracturing is that it could reactivate pre-existing faults and triggers microseismicity, low magnitude seismic events that sometimes can be felt at the surface (Majer et al., 2007; McClure & Horne, 2011). The potential impact on surface structures and shallow aquifers threatens public acceptance of it. Though induced seismicity associated with hydraulic fracturing is very small and generally not a problem (N. Warpinski et al., 2012), fault activations during hydraulic fracturing are reported (Downie, Kronenberger, & Maxwell, 2010), which caused the failure of the treatment. A better understanding of fault mechanics and in-situ stress changes during hydraulic fracturing will increase our ability to better predict the likelihood and characteristic of stress field underground allowing for better optimizing hydraulic treatment design.

1.2 Literature Review

Once a fault has been formed its further motion is controlled by friction. Friction is a contact property rather than a bulk property (Scholz, 2002). Friction experiments were first carried out by Leonardo da Vinci. Leonardo's discoveries remained hidden and were rediscovered 200 years later by Amontons. Da Vinci found that frictional sliding will occur on a plane when the ratio of shear to normal stress reaches a material property of the material, μ , the coefficient of friction (M. D. Zoback, 2010). A comprehensive summary of numerous laboratory experiments on friction on a wide variety of rock types indicates that at intermediate pressure and high pressure (≥ 100 MPa), surface roughness, rock type, normal stress, etc. have little or no effect on friction. The coefficient of friction is found to be within a relatively small range: $0.6 \leq \mu \leq 1.0$. However, if the sliding surfaces are separated by large thickness of gouge composed of minerals such as montmorillonite

and vermiculite the friction can be very low (Byerlee, 1978). For example, (Morrow, Shi, & Byerlee, 1982) found that samples of fault gouge with clays from the San Andreas fault have coefficients of friction ranging from 0.15 to 0.55. Fault gouges with a wide range of constituent minerals relevant to natural faults are found to have a strong influence on frictional stability, the friction of natural faults is strongly dependent on the composition of gouge (Ikari, Marone, & Saffer, 2011).

Recent development of unconventional tight gas reservoir makes researchers investigate the mechanical properties of gas shale reservoir rocks (Sone & Zoback, 2010; M. D. Zoback, Kohli, Das, & McClure, 2012). Samples tested differ in their mineralogical composition, the degree of diagenesis, the total organic content and the degree of maturity of the organic material. It is suggested by laboratory data that Young's modulus correlates well with the amount of viscoplastic creep and frictional strength, coefficient of friction and amount of viscoplastic creep vary strongly with clay content. Based on rate and state friction experiments in the laboratory using shale samples with a large range of clay content, it is indicated that clay content determines the deformation mechanism of pre-existing fractures and faults (M. D. Zoback et al., 2012). When shales comprised of less than about 30% clay, slip on faults is expected to propagate unstably, thus conventional microseismic events are generated. For shales containing more than about 30% clay, fault slip is expected to propagate slowly, which does not generate high frequency seismic waves.

Depending upon the in-situ stress regime, faults underground may or may not be tectonically active. Generally active faults are in critical stress state, even a slight stress perturbations may trigger such faults to slip (Townend & Zoback, 2000; M. D. Zoback, 2010). Fluid injection to subsurface reservoirs, such as hydraulic stimulation and geological sequestration of CO₂, raises pore pressure and causes in-situ stress field changes, which would tend to influence the stability of underground faults. Researchers have done a lot of work on fault stability for CO₂ sequestration (Bretan, Yielding, Mathiassen, & Thorsnes, 2011; Chiaramonte, Zoback, Friedmann, & Stamp, 2008; Rutqvist, Birkholzer, Cappa, & Tsang, 2007; Streit & Hillis, 2004). In order to prevent CO₂ escape due to fault reactivation, the determination of fault orientations, ambient pore pressure and in-situ stresses in a potential storage site is required. Stress field changes

induced by hydrocarbon production and associated pore pressure depletion are needed to be incorporated into fault stability analysis. Based on numerical and analytical methods, researchers estimated the maximum sustainable injection pressure during geological sequestration of CO₂ (Rutqvist et al., 2007).

In addition to fluid injection in reservoir, reservoir depletion can also cause fault reactivation. According to the theory of poroelasticity, depleting a hydrocarbon reservoir alters the state of in-situ stresses, which can sufficiently reactivate and induce slip of nearby faults (Segall, 1989; Segall, 1992; Talwani, 1997; Yerkes & Castle, 1976; M. D. Zoback & Zinke, 2002).

Horizontal wells with multiple fractures are commonly used in unconventional gas reservoirs, such as the ultralow-permeability shales. It is absolutely essential to perform hydraulic stimulation in order to achieve commercial gas production rates (Sutton et al., 2010; N. Warpinski et al., 2012). During this kind of stimulation, the local earth stresses are changed, which effects the stability of underground faults. For hydraulic treatment, there are two common effects causing stress change during hydraulic fracturing (Economides, Nolte, & Ahmed, 1989), thus effecting the stability of faults. The first one is the increase of minimum stress because of the poroelastic effect. During hydraulic treatment, fracturing fluid leaks into formations. Pore pressure increases around the hydraulic fractures due to the leakage, which results in dilation of the formation. The minimum stress thus increases in this stimulated region. When injection is stopped and the excess pore pressure spreads out into formation, this poroelastic effects disappear.

The second effect is the stress increase due to the opening of the fracture. If the induced fracture is held by proppant, this effect remains. These stresses can extend to significant distances into the formation due to the large area of fracture face (Palmer, 1993; N. Warpinski & Branagan, 1989).

Because of the high pressures needed for fracture dilation and the large fracture surface area created, a hydraulic fracture can effectively alter the stress field. This could mean that the original fracture alters the in-situ stress, and a subsequent vertical fracture will initially propagate orthogonal to the original vertical fracture.

When a fracture is reoriented, fracture width is expected to be reduced. The degree of reduction in the fracture width depends on the degree of orientation (Soliman, East, & Adams, 2004). A narrow effective fracture width tends to cause high fracture pressure, and consequently high fracture-propagation pressure.

1.3 Research Objectives

Previous studies in the literature on fault stability focus on the geological sequestration of CO₂, reservoir depletion and so on. A comprehensive analysis of how hydraulic stimulation influences the fault stability has not been fully investigated to date. Prediction of how pre-existing faults and fractures respond to hydraulic stimulation can help optimize field operations and improve recovery (M. D. Zoback et al., 2012).

The main purpose of the study is to investigate the in-situ stress change during hydraulic stimulation and evaluate its impact on the fault stability.

1.4 Outline of the Thesis

This thesis is presented in a series of manuscripts. This introductory chapter provides background information. Some basic knowledge involved in this study is provided in chapter 2. Chapter 3 presents the used methods, validation of the provided methods is performed through different examples. Chapter 4 investigates the stress distribution during hydraulic fracturing. Fault stability analysis is carried out in Chapter 5. The last chapter summarizes the study.

2 Fundamental Theories

2.1 Friction of Rocks

Friction is the resistance to relative motion by which a tangential shearing force is required in order to displace two contacting surfaces (Jaeger, Cook, & Zimmerman, 2007). At different stages of frictional sliding, there are three types of friction: initial friction, maximum friction and residual friction, respectively (Byerlee, 1978). The first two classical laws of friction, usually attributed to the Frenchman Guillaume Amontons, are:

1. The force of friction is directly proportional to the applied load.
2. The force of friction is independent of the apparent area of contact.

The coefficient of friction is defined as the ratio of shear to normal stress acting between the surfaces during sliding. Different physical mechanisms are involved in the sliding of rock at various pressures. At low pressure the surfaces can move with respect to one another by lifting over the interlocked irregularities, however, at very high pressure this effect is suppressed and the surfaces then slide by shearing through the irregularities.

Based on numerous laboratory experiments on a wide variety of rock types, Byerlee (1978) show that the frictional behavior of rocks could be fit by the following bilinear empirical expression:

$$\begin{aligned}\tau &= 0.85\sigma \quad \text{for } \sigma < 200 \text{ MPa} \\ \tau &= 50 \text{ MPa} + 0.6\sigma \quad \text{for } 200 < \sigma < 1700 \text{ MPa}\end{aligned}\tag{2.1}$$

Also, the friction can be very low if the sliding surfaces are separated by large thicknesses of gouge composed of minerals such as montmorillonite or vermiculite.

There are several different types of apparatus used to measure the friction of rocks. As shown in Figure 2.1, these are (a) triaxial compression, (b) direct shear, (c) biaxial loading, and (d) rotary. Each one has its own advantages and disadvantages. The triaxial test is best suitable for high-pressure and high-temperature studies. However, the amount of slip is limited and dynamic measurement during stick slip is doubtful due to the

possibility of sample misalignment occurring during rapid slip. The other three types of tests are applicable to lower loads limited by the uniaxial failure strength of the rock.

Figure 2.1 Common types of experimental configurations used in friction studies.

2.2 Mechanics of Faults

Geological discontinuities such as faults are inherent in most petroleum formations. Frictional sliding can be expected to occur on a fault plane when the ratio of shear to effective normal stress along the fault plane equals the experimentally determined “frictional coefficient”. This relationship is known as Amontons’ law

$$\frac{\tau}{\sigma_n} = \mu \quad (2.2)$$

where τ is the shear stress along the fault plane, σ_n the effective normal stress, defined as $(S_n - P_p)$, μ is a material property, called the coefficient of friction. S_n is the normal stress on the plane, P_p is the pore-pressure. Equation (2.2) is sometimes called the Coulomb criterion and is widely used in fault stability analysis (C. B. Raleigh, Healy, & Bredehoeft, 1976; Sibson, 1974).

When the ratio is less than μ , a fault is stable and no slip should occur, because the shear stress is insufficient to overcome the frictional resistance to sliding, $\mu\sigma_n$. However, as the ratio equals μ , frictional sliding will occur on a pre-existing fault plane since sufficient shear stress is available to overcome the frictional resistance to sliding. Confirmation of Equation (2.2) by in-situ measurements would mean that stress magnitudes at any given depth are limited by the frictional strength of underground discontinuities, such as faults, fractures.

E. M. Anderson developed the modern mechanical concepts of the origin of faults (Scholz, 2002). He applied the Coulomb criterion (Figure 2.2) to study faults, which is called Anderson's theory of faulting.

Figure 2.2 (a) Mohr diagram illustrating the Coulomb criterion; (b) Diagram showing the relationship between stress axes and shear angle. (Atsushi, 2007)

The basis of the theory is the assumption that the surface of the earth must be a principal plane of stress, the directions of other two principle stresses are parallel to the earth's surface. The third principal stress direction must be oriented normal to the earth's surface. Combining this with Coulomb criterion, an important relationship between the type of faulting and stress can be obtained.

The theory predicts that the direction of inter-mediate principal stress is parallel to fault planes. The fault plane has an angle $\frac{1}{2}\tan^{-1}(1/\mu)$ with the direction of maximum principal stress. Therefore, if the minimum principal stress is in vertical direction, the stress field gives rise to reverse faulting with dip angle of the faults less than 45° . If the

maximum principal stress or inter-mediate principal stress is in the vertical direction, normal or strike-slip faults are formed. The dip angle of thrust faults should be $\sim 30^\circ$, the dip angle of normal faults should be $\sim 60^\circ$, and the dip angle of strike-slip faults should be about vertical (Figure 2.3).

Figure 2.3 Types of faulting system predicted by Anderson's theory. (Van der Pluijm & Marshak, 2004)

It should be remembered that Figure 2.3 illustrates the idealized relationship between potentially active faults and the stress state that caused them. In reality, after the faults were formed, the principal stresses in-situ may be changed in both magnitude and directions. Thus, preexisting faults may be reactivated even if the fault planes are not inclined at well-oriented degree ($\sim 30^\circ$) to the direction of maximum principal stress. At the same time, the orientation of faults may change as the formation containing the faults undergoes progressive deformation.

As mentioned above, Equation (2.2) indicates that pore pressure takes a part of the absolute stress to lower the normal stress, thus raising pore pressure would tend to destabilize faults and make slip to occur by increasing the ratio of shear to effective normal stress. Figure 2.4 illustrates the effect of pore pressure increase on fault stability. Raising pore pressure shifts the Mohr circles leftward to traverse the failure envelope activating a pre-existing fault.

Figure 2.4 Effect of pore pressure increase on fault stability.

2.3 In-situ Stress Underground

In-situ stress acts as a load on the formation and is the dominant parameter that controls the propagation and geometry of hydraulic fractures (Economides et al., 1989). Fractures will always propagate along the path with the least resistance and create width in a direction that requires the least force. This means that a fracture will propagate parallel to the maximum principal stress and perpendicular to the minimum principal stress. Consequently, understanding of the direction of minimum principal stress allows prediction of the expected direction of hydraulic fracture propagating away from the wellbore. Also, it is generally accepted that the degree of fracture containment is determined primarily by the in-situ stress contrast between adjacent layers.

In practice, it is observed that the minimum principal stress is in the vertical direction at shallow depth, at which a hydraulic fracture is most likely to occur in a horizontal plane (Economides et al., 1989). In an extensional geologic environment, however, the minimum principal stress is generally in horizontal direction, even at shallow depths. In normally pressured sedimentary basins, the minimum principal stress is most probably in horizontal direction at depths greater than 3300 ft (Plumb, 1994).

Underground discontinuities, such as faults and fractures, are widely distributed at different scales and orientations, it is thought that stress magnitudes at depth are limited by the frictional strength of these discontinuities (Sibson, 1974; Townend & Zoback, 2000; M. D. Zoback, 2010). The in-situ stress measurements demonstrate that the stress

states predicted using Coulomb frictional-failure theory with laboratory-derived frictional coefficients of 0.6-1.0 are consistent with the measured stresses.

Coulomb failure criterion, the simplest, and still most widely used failure criterion, states that failure will occur along a plane if the following condition is satisfied:

$$\tau > \tau_f = \mu \sigma_n \quad (2.3)$$

where τ is the applied shear stress, τ_f is the frictional shear resistance, μ is the static frictional coefficient, and σ_n is the normal stress acting on the fault. It would be convenient to express the Coulomb failure criterion directly in terms of the principal stresses $\{ \sigma_1, \sigma_3 \}$, which is given by:

$$\frac{\sigma_1}{\sigma_3} = \frac{S_1 - P_p}{S_3 - P_p} \geq [(1 + \mu^2)^{1/2} + \mu]^2 \quad (2.4)$$

It has been shown that many geological faults fall into three classes termed normal, strike-slip and reverse faults. When a fault first forms, it does so at an angle θ to σ_1 (Figure 2.2), where θ is the optimum angle for frictional sliding,

$$\theta = \frac{1}{2} \tan^{-1} \left(\frac{1}{\mu} \right) \quad (2.5)$$

Using Anderson's faulting theory to determine which principal stress (S_{Hmax} , S_{hmin} , or S_v) corresponds to S_1 , S_2 and S_3 , respectively, Equation (2.6) can be used to estimate an upper bound of the ratio of the maximum to minimum effective stresses (M. D. Zoback, 2010), the following are the equations:

$$\begin{aligned} \text{Normal faulting } \frac{\sigma_1}{\sigma_3} &= \frac{S_v - P_p}{S_{hmin} - P_p} \leq \left[(\mu^2 + 1)^{\frac{1}{2}} + \mu \right]^2 \\ \text{Strik - slip faulting } \frac{\sigma_1}{\sigma_3} &= \frac{S_{Hmax} - P_p}{S_{hmin} - P_p} \leq \left[(\mu^2 + 1)^{1/2} + \mu \right]^2 \\ \text{Reverse faulting } \frac{\sigma_1}{\sigma_3} &= \frac{S_{Hmax} - P_p}{S_v - P_p} \leq \left[(\mu^2 + 1)^{\frac{1}{2}} + \mu \right]^2 \end{aligned} \quad (2.6)$$

The effective overburden pressure at a depth z is given by

$$\sigma_v = \rho g z (1 - \lambda) \quad (2.7)$$

where ρ is the formation density, g is the gravity acceleration, and the pore pressure factor $\lambda = P_p/\rho g z$, P_p being the pore pressure.

In a normal fault regime, where $\sigma_v = \sigma_1$, the limiting inequality for frictional sliding is

$$(\sigma_1 - \sigma_3) \geq \left[\frac{(R - 1)}{R} \right] \rho g z (1 - \lambda) \quad (2.8)$$

where $R = \left[(\mu^2 + 1)^{\frac{1}{2}} + \mu \right]^2$.

In a strike-slip fault regime, $\sigma_v = \sigma_2$, and σ_2 may be written in the form

$$\sigma_2 = \sigma_3 + k(\sigma_1 - \sigma_3) \quad (2.9)$$

where $0 < k < 1$. The limiting inequality for frictional sliding can be written as

$$(\sigma_1 - \sigma_3) \geq \left[\frac{(R - 1)}{k(R - 1) + 1} \right] \rho g z (1 - \lambda) \quad (2.10)$$

In a reverse fault regime, $\sigma_v = \sigma_3$, the limiting inequality for frictional sliding is

$$(\sigma_1 - \sigma_3) \geq [(R - 1)] \rho g z (1 - \lambda) \quad (2.11)$$

Based on the above relationships (Equations (2.8), (2.10) and (2.11)), limiting conditions for each of the three fault types are shown in Figure 2.5.

Figure 2.5 Differential stress required to initiate sliding on optimal oriented faults as a function of depth. ($\rho = 2.8 \times 10^3 \text{ kg/m}^3$; $g = 9.8 \text{ m/s}^2$; $\mu = 0.75$; $\theta = 27^\circ$.) (Sibson, 1974; Van der Pluijm & Marshak, 2004)

3 Methodology

The 3D model developed in this study is based on finite element method (FEM). The finite element method is a widely used technique which is suitable for modeling geomechanical cases (Dhondt, 2004; Gudehus, 1978; Hughes, 2000; Lewis & Schrefler, 1987; Smith & Griffiths, 1998).

3.1 Finite Element Principles

In finite element method the continuum is divided into a series of elements which are connected at a finite number of points known as nodal points.

The governing equilibrium equations can be obtained by minimizing the total potential energy of a system. The total potential energy, π , can be expressed as

$$\pi = \frac{1}{2} \int_V \{\boldsymbol{\sigma}\}^T \{\boldsymbol{\varepsilon}\} dV - \int_V \{\boldsymbol{\sigma}\}^T \{\mathbf{p}\} dV - \int_S \{\boldsymbol{\sigma}\}^T \{\mathbf{q}\} dS \quad (3.1)$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are the stress and strain vectors respectively, $\boldsymbol{\delta}$ the displacements at any point, $\mathbf{p}$ the body forces per unit volume and $\mathbf{q}$ the applied surface tractions. Integrations are taken over the volume V of the continuum and loaded surface area, S .

The first term on the right hand side of Equation (3.1) represents the internal strain energy and the second and third terms are respectively the work contributions of body forces and distributed surface loads (Hinton & Owen, 1977).

In the finite element displacement method, the displacement is assumed to have unknown values only at the nodal points, so that the variation within any element is described in terms of the nodal values by means of interpolation functions. Thus

$$\{\boldsymbol{\delta}\} = [\mathbf{N}] \{\boldsymbol{\delta}^e\} \quad (3.2)$$

where $\mathbf{N}$ is the set of interpolation functions termed the shape functions and $\boldsymbol{\delta}^e$ is the vector of nodal displacements of the element. For different element types, the shape functions and vector of nodal displacements are different. The explicit formats of Equation (3.2) for 4-node rectangular element and 4-node tetrahedron element are given as examples, listed as following:

For 4-node rectangular element:

$$\{\delta\} = \begin{Bmatrix} \mathbf{u} \\ \mathbf{v} \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix} = [\mathbf{N}] \{\delta^e\}$$

where

$\mathbf{u}, \mathbf{v}$ are x – and y – displacement at any position in the internal of a element.

u_i, v_i are x – and y – displacement at nodes of element.

$$N_1 = \frac{1}{4}(1 - \xi)(1 - \eta)$$

$$N_2 = \frac{1}{4}(1 - \xi)(1 + \eta)$$

$$N_3 = \frac{1}{4}(1 + \xi)(1 + \eta)$$

$$N_4 = \frac{1}{4}(1 + \xi)(1 - \eta)$$

For 4-node tetrahedron element:

$$\{\delta\} = \begin{Bmatrix} \mathbf{u} \\ \mathbf{v} \\ \mathbf{w} \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix} = [\mathbf{N}] \{\delta^e\}$$

where

$\mathbf{u}, \mathbf{v}, \mathbf{w}$ are x –, y – and z – displacements at any position in the internal of a element,

u_i, v_i, w_i are x –, y – and z – displacements at nodes of element.

$$N_1 = L_1$$

$$N_2 = L_2$$

$$N_3 = L_3$$

$$N_4 = (1 - L_1 - L_2 - L_3)$$

The strains within the element can be expressed in terms of the element nodal displacements as

$$\{\boldsymbol{\epsilon}\} = [\mathbf{B}]\{\boldsymbol{\delta}^e\} \quad (3.3)$$

where $\mathbf{B}$ is the strain matrix generally composed of derivatives of the shape functions. The explicit form of Equation (3.3) for 4-node rectangular element and 4-node tetrahedron element are listed respectively as follows:

For 4-node rectangular element:

$$\begin{aligned} \{\boldsymbol{\epsilon}\} = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} &= \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{v} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} [\mathbf{N}]\{\boldsymbol{\delta}^e\} \\ &= \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial x} & 0 & \frac{\partial N_3}{\partial x} & 0 & \frac{\partial N_4}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial y} & 0 & \frac{\partial N_3}{\partial y} & 0 & \frac{\partial N_4}{\partial y} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial y} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial y} & \frac{\partial N_3}{\partial x} & \frac{\partial N_4}{\partial y} & \frac{\partial N_4}{\partial x} \end{bmatrix} \{\boldsymbol{\delta}^e\} = [\mathbf{B}]\{\boldsymbol{\delta}^e\} \end{aligned}$$

where

$$\begin{aligned} \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial x} & \frac{\partial N_4}{\partial x} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \frac{\partial N_3}{\partial y} & \frac{\partial N_4}{\partial y} \end{bmatrix} &= \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial N_1}{\partial \xi} & \frac{\partial N_2}{\partial \xi} & \frac{\partial N_3}{\partial \xi} & \frac{\partial N_4}{\partial \xi} \\ \frac{\partial N_1}{\partial \eta} & \frac{\partial N_2}{\partial \eta} & \frac{\partial N_3}{\partial \eta} & \frac{\partial N_4}{\partial \eta} \end{bmatrix} \\ x &= N_1 x_1 + N_2 x_2 + N_3 x_3 + N_4 x_4 \\ y &= N_1 y_1 + N_2 y_2 + N_3 y_3 + N_4 y_4 \end{aligned}$$

For 4-node tetrahedron element:

$$\begin{aligned}
 \{\boldsymbol{\varepsilon}\} &= \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{v} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \end{bmatrix} [\mathbf{N}] \{\boldsymbol{\delta}^e\} \\
 &= \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & 0 & \frac{\partial N_2}{\partial x} & 0 & 0 & \frac{\partial N_3}{\partial x} & 0 & 0 & \frac{\partial N_4}{\partial x} & 0 & 0 \\ 0 & \frac{\partial N_1}{\partial y} & 0 & 0 & \frac{\partial N_2}{\partial y} & 0 & 0 & \frac{\partial N_3}{\partial y} & 0 & 0 & \frac{\partial N_4}{\partial y} & 0 \\ 0 & 0 & \frac{\partial N_1}{\partial z} & 0 & 0 & \frac{\partial N_2}{\partial z} & 0 & 0 & \frac{\partial N_3}{\partial z} & 0 & 0 & \frac{\partial N_4}{\partial z} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial y} & \frac{\partial N_2}{\partial x} & 0 & \frac{\partial N_3}{\partial y} & \frac{\partial N_3}{\partial x} & 0 & \frac{\partial N_4}{\partial y} & \frac{\partial N_4}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial z} & \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial z} & \frac{\partial N_2}{\partial y} & 0 & \frac{\partial N_3}{\partial z} & \frac{\partial N_3}{\partial y} & 0 & \frac{\partial N_4}{\partial z} & \frac{\partial N_4}{\partial y} \\ \frac{\partial N_1}{\partial z} & 0 & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial z} & 0 & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial z} & 0 & \frac{\partial N_3}{\partial x} & \frac{\partial N_4}{\partial z} & 0 & \frac{\partial N_4}{\partial x} \end{bmatrix} \{\boldsymbol{\delta}^e\} \\
 &= [\mathbf{B}] \{\boldsymbol{\delta}^e\}
 \end{aligned}$$

where

$$\begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial x} & \frac{\partial N_4}{\partial x} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \frac{\partial N_3}{\partial y} & \frac{\partial N_4}{\partial y} \\ \frac{\partial N_1}{\partial z} & \frac{\partial N_2}{\partial z} & \frac{\partial N_3}{\partial z} & \frac{\partial N_4}{\partial z} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial L_1} & \frac{\partial y}{\partial L_2} & \frac{\partial z}{\partial L_3} \\ \frac{\partial x}{\partial L_1} & \frac{\partial y}{\partial L_2} & \frac{\partial z}{\partial L_3} \\ \frac{\partial x}{\partial L_1} & \frac{\partial y}{\partial L_2} & \frac{\partial z}{\partial L_3} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial N_1}{\partial L_1} & \frac{\partial N_2}{\partial L_2} & \frac{\partial N_3}{\partial L_3} & \frac{\partial N_4}{\partial L_4} \\ \frac{\partial N_1}{\partial L_1} & \frac{\partial N_2}{\partial L_2} & \frac{\partial N_3}{\partial L_3} & \frac{\partial N_4}{\partial L_4} \\ \frac{\partial N_1}{\partial L_1} & \frac{\partial N_2}{\partial L_2} & \frac{\partial N_3}{\partial L_3} & \frac{\partial N_4}{\partial L_4} \end{bmatrix}$$

$$x = N_1 x_1 + N_2 x_2 + N_3 x_3 + N_4 x_4$$

$$y = N_1 y_1 + N_2 y_2 + N_3 y_3 + N_4 y_4$$

$$z = N_1 z_1 + N_2 z_2 + N_3 z_3 + N_4 z_4$$

Finally the stresses are related to the strains by use of an elasticity matrix $\mathbf{D}$, as follows

$$\{\boldsymbol{\sigma}\} = [\mathbf{D}]\{\boldsymbol{\varepsilon}\} \quad (3.4)$$

The explicit form of Equation (3.4) for 4-node rectangular element and 4-node tetrahedron element are listed respectively in the following:

For 4-node rectangular element:

$$\{\boldsymbol{\sigma}\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = [\mathbf{D}]\{\boldsymbol{\varepsilon}\} \quad (\text{plane stress})$$

For 4-node tetrahedron element:

$$\{\boldsymbol{\sigma}\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-\nu} & 1 & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = [\mathbf{D}]\{\boldsymbol{\varepsilon}\}$$

The total potential energy of the continuum is the sum of the energy contributions of the individual elements. Thus

$$\pi = \sum_e \pi_e \quad (3.5)$$

where π_e represents the total potential energy of element e which, on use of Equation (3.1), can be written

$$\pi_e = \frac{1}{2} \int_{V_e} \{\delta^e\}^T [\mathbf{B}]^T [\mathbf{D}] [\mathbf{B}] \{\delta^e\} dV - \int_{V_e} \{\delta^e\}^T [\mathbf{N}]^T \{\mathbf{p}\} dV - \int_{S_e} \{\delta^e\}^T [\mathbf{N}]^T \{\mathbf{q}\} dS \quad (3.6)$$

where V_e is the element volume and S_e the loaded element surface area. Performance of the minimization for element e with respect to the nodal displacement δ^e for the element results in

$$\frac{\pi_e}{\delta^e} = \int_{V_e} ([\mathbf{B}]^T \mathbf{D} \mathbf{B}) \delta^e dV - \int_{V_e} [\mathbf{N}]^T \mathbf{p} dV - \int_{S_e} [\mathbf{N}]^T \mathbf{q} dS = \mathbf{K}^e \delta^e - \mathbf{F}^e \quad (3.7)$$

where

$$\mathbf{F}^e = \int_{V_e} [\mathbf{N}]^T \mathbf{p} dV + \int_{S_e} [\mathbf{N}]^T \mathbf{q} dS \quad (3.8)$$

are the equivalent nodal forces for the element, and

$$\mathbf{K}^e = \int_{V_e} ([\mathbf{B}]^T \mathbf{D} \mathbf{B}) dV \quad (3.9)$$

is termed the element stiffness matrix. The summation of the terms in Equation (3.7) over all the elements, when equated to zero, results in a system of equilibrium equations for the complete continuum. These equations are then solved to yield the nodal displacements.

In a finite element model, the fracture surface applied hydraulic pressure could be treated as interior boundary condition. When hydraulic pressure is applied, the pressure is transferred into node forces, and the forces are applied along the surface of the hydraulic fractures. The remaining model should experience stress redistribution so that the surface of the region applied hydraulic pressure is indeed an ‘interior boundary’.

All element stiffness matrixes are assembled by a sparse matrix assembly procedure in this study. Iterative solution technique is adopted to solve the assembled global linear algebraic equations. For static equilibrium problems, the assembled system matrix is symmetric positive definite matrix (Smith & Griffiths, 1998; Zienkiewicz, Taylor, & Zhu, 2005). The Symmetric Successive Overrelaxation method, or SSOR, is then used, which is highly efficient to solve sparse symmetrical equations (Axelsson, 1972; Barrett et al., 1987; Chen & Yung, 2001).

The stresses within each element can then be calculated from the displacement using Equations (3.3) and (3.4).

3.2 Stress Recovery

In finite element method, the final results of stress and strain are presented in a Global Coordinate System. Each element (or cell) in a three dimension model has six components of stress, namely, the three normal stresses, σ_x , σ_y , σ_z , and the three shearing stresses $\tau_{xy} = \tau_{yx}$, $\tau_{xz} = \tau_{zx}$, $\tau_{yz} = \tau_{zy}$ (Figure 3.1).

Figure 3.1 Six Stress Components in Three-Dimensional Coordinate.

Due to numerical integration adopted in finite element calculation, the strains and stresses are more accurate at integration points than anywhere else (Dhondt, 2004). The field variables (stresses and strains) are usually evaluated at the integration points, and can be extrapolated to element nodes.

In fault stability analysis, it is required to calculate shear and normal stresses at nodes located on an inclined plane (fault plane), so the six stress components at element nodes should be determined first, based on which shear and normal stresses could be calculated.

3.2.1 Calculation of Element Strains and Stresses

Suppose that we have solved the master stiffness equations

$$[\mathbf{K}]\{\boldsymbol{\delta}\} = \{\mathbf{f}\} \quad (3.10)$$

for the node displacements $\boldsymbol{\delta}$, where $\mathbf{K}$ is the stiffness matrix.

Let $\boldsymbol{\delta}^e$ the vector of computed node displacements for one element, the strains at any point in the element may be related to these displacements as

$$\{\boldsymbol{\epsilon}\} = [\mathbf{B}]\{\boldsymbol{\delta}^e\} \quad (3.11)$$

A loop is performed over all defined elements, where $\mathbf{B}$ is the strain-displacement matrix.

The corresponding stresses for each element are given by

$$\{\boldsymbol{\sigma}\} = [\mathbf{D}]\{\boldsymbol{\epsilon}\} = [\mathbf{D}][\mathbf{B}]\{\boldsymbol{\delta}^e\} \quad (3.12)$$

where $\mathbf{D}$ is the stress-strain matrix.

3.2.2 Extrapolation of Integration Point Values to the Nodes

Usually two approaches are used in practice to compute element nodal stresses:

- (1). Calculate σ at element node locations by substituting natural coordinates of the nodal points as arguments to shape function modules.
- (2). Calculate σ at Gauss integration points used in the element stiffness integration rule and then extrapolate to the element node points.

For quadrilateral elements whose geometry departs substantially from the rectangular shape, empirical evidence indicates that the second approach generally delivers better stress values. In this report, the second approach is adopted.

The extrapolation is done on an element basis. Each node will obtain as many values as the number of elements it belongs to. The stresses computed at the same node from adjacent elements will not generally be the same, since stresses are not required to be continuous in displacement-assumed finite elements. Stress averaging should be used to improve the stress accuracy. This is done by calculating the mean value over all elements the node belongs to. The results from this averaging procedure are called nodal point stresses.

Assume that the shape functions that are used in Equation (3.2) for displacements are also used for stresses, strains and any other dependent fields and the field variables, such as stresses, are known at nodes. Then, the values at integration point could be obtained by

$$\{\sigma\}_{\text{integration points}} = [N] \{\sigma\}_{\text{nodes}} \quad (3.13)$$

where N is matrix for shape function.

Consequently, the values at nodes could be calculated by inverting the above equation

$$\{\sigma\}_{\text{nodes}} = [N]^{-1} \{\sigma\}_{\text{integration points}} \quad (3.14)$$

For tetrahedron elements with 4 integration points:

$$[N]^{-1} = \begin{bmatrix} +1.92705 & -0.30902 & -0.30902 & -0.30902 \\ -0.30902 & +1.92705 & -0.30902 & -0.30902 \\ -0.30902 & -0.30902 & +1.92705 & -0.30902 \\ -0.30902 & -0.30902 & -0.30902 & +1.92705 \end{bmatrix}$$

For hexahedron elements with 8 integration points:

$$[N]^{-1} = \begin{bmatrix} +2.549 & -0.683 & -0.683 & +0.183 & -0.683 & +0.183 & +0.183 & -0.049 \\ -0.683 & +2.549 & +0.183 & -0.683 & +0.183 & -0.683 & -0.049 & +0.183 \\ +0.183 & -0.683 & -0.683 & +2.549 & -0.049 & +0.183 & +0.183 & -0.683 \\ -0.683 & +0.183 & +2.549 & -0.683 & +0.183 & -0.049 & -0.683 & +0.183 \\ -0.683 & +0.183 & +0.183 & -0.049 & +2.549 & -0.683 & -0.683 & +0.183 \\ +0.183 & -0.683 & -0.049 & +0.183 & -0.683 & +2.549 & +0.183 & -0.683 \\ -0.049 & +0.183 & +0.183 & -0.683 & +0.183 & -0.683 & -0.683 & +2.549 \\ +0.183 & -0.049 & -0.683 & +0.183 & -0.683 & +0.183 & +2.549 & -0.683 \end{bmatrix}$$

The values in the middle nodes are found by taking the mean of the neighboring vertex nodal point values.

3.3 Shear and Normal Stresses Calculation on Fault Plane

If stress components at element nodes located on fault plane are known, the stress acting on the inclined plane through this point can be calculated from equations of statics.

Let O be a node on an inclined plane, and suppose the six stress components are known for the coordinate plane xy , xz , yz (Figure 3.1). To obtain shear and normal stresses for an

inclined plane through O , a plane BCD parallel to the inclined plane at a small distance from O is used. Plane BCD together with the coordinate planes cuts out a very small tetrahedron $BCDO$. The stresses acting on the plane BCD will approach the stresses on the parallel plane through O as the small tetrahedron $BCDO$ is made infinitesimal.

If N is the normal to the plane BCD in Figure 3.1, and it could be wrote that

$$\begin{aligned}\cos(N, x) &= l \\ \cos(N, y) &= m \\ \cos(N, z) &= n\end{aligned}\tag{3.15}$$

If the three components of stress are denoted by X, Y, Z , parallel to the Global Coordinate Axes, acting on the inclined face BCD , then the equations of equilibrium of the tetrahedron $BCDO$ can be written

$$\begin{aligned}X &= \sigma_x l + \tau_{xy} m + \tau_{xz} n \\ Y &= \tau_{xy} l + \sigma_y m + \tau_{zy} n \\ Z &= \tau_{xz} l + \tau_{yz} m + \sigma_z n\end{aligned}\tag{3.16}$$

Provided the six components of stress $\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yx}, \tau_{xz}$ at point O are known, the components of stress on any plane, defined by the direction cosines l, m, n , can be calculated according to Equation (3.16).

3.3.1 Normal Stress for Nodal Points Located on Inclined Plane

Any plane in 3-dimensional rectangular coordinate system is determined by the following geometric equation:

$$Ax + By + Cz + D = 0\tag{3.17}$$

By substituting four nodes on a plane, the constants A, B, C and D can be determined. The normal direction cosines for the plane (l, m, n) can be calculated through following equations:

$$\begin{aligned}
l &= \frac{A}{\sqrt{A^2 + B^2 + C^2}} \\
m &= \frac{B}{\sqrt{A^2 + B^2 + C^2}} \\
n &= \frac{C}{\sqrt{A^2 + B^2 + C^2}}
\end{aligned} \tag{3.18}$$

Projecting the three components of stress (X, Y, Z) determined by Equation (3.16) to the normal direction of the plane, the normal stress (σ_n) on this particular plane can be calculated as

$$\sigma_n = Xl + Ym + Zn \tag{3.19}$$

Substituting the values of X, Y, Z from Equation (3.16), then

$$\sigma_n = \sigma_x l^2 + \sigma_y m^2 + \sigma_z n^2 + 2\tau_{yz} mn + 2\tau_{xz} ln + 2\tau_{xy} lm \tag{3.20}$$

3.3.2 Shear Stress for Nodal Points Located on Inclined Plane

The magnitude of the total stress $\sigma_{(v)}$ on an inclined plane through nodal points is

$$\sigma_{(v)} = \sqrt{X^2 + Y^2 + Z^2} \tag{3.21}$$

In 3-dimensional coordinate system, the total stress could be represented by a vector, which has an ordered set of three components representing its projections on the coordinate axes. In this case, the total stress is written as

$$\underline{\sigma_{(v)}} = X\underline{i} + Y\underline{j} + Z\underline{k} \tag{3.22}$$

where $\underline{i}, \underline{j}, \underline{k}$ are unit vectors in the 3-dimensional coordinate system.

The normal stress on the inclined plane could also be written in the form of vector, listed as following:

$$\underline{\sigma_n} = (Xl^2)\underline{i} + (Ym^2)\underline{j} + (Zn^2)\underline{k} \tag{3.23}$$

According to vector operation, the shear stress τ on the inclined plane could be derived as

$$\underline{\tau} = \underline{\sigma_{(v)}} - \underline{\sigma_n} = (X - Xl^2)\underline{i} + (Y - Ym^2)\underline{j} + (Z - Zn^2)\underline{k} \tag{3.24}$$

For each nodes located on the inclined plane (fault plane), a loop is performed to carry out all the above procedures to calculate shear and normal stresses at these points.

3.4 Validation

Three different cases are used to validate the above mentioned methods. The case No. 1 and 2 are utilized to validate the shear and normal stresses calculation. In the case No. 3, the stress distribution of a 3D numerical model is compared to analytical solutions when a constant hydraulic pressure is acting on the surfaces of a fracture.

For case No. 1, the shear and normal stresses on an inclined plane could be calculated through analytical method. Numerical calculation using finite element method is also carried out. The results obtained from numerical calculation are compared to those computed through analytical method.

A field case is used in case No. 2, the results obtained from numerical simulation is compared with those calculated by analytical method.

3.4.1 Case No. 1

In this case, a square column is fixed at the bottom, the lateral boundaries are free (without restraint). A constant pressure is applied on the top of the column. There is an inclined plane in the column, dipped at an angle of 60° . The geometric model is illustrated in Figure 3.2.

Figure 3.2 Geometric model for Case No. One.

The boundary conditions are listed as following:

- Top surface: uniform distributed pressure in vertical downward direction, value of which is 6.94 psi.
- Four lateral surfaces: free (without restraint).
- Bottom: fixed in vertical direction.

Young's modulus and Poisson's ratio are 1.12×10^6 psi and 0.28, respectively.

3.4.1.1 Analytical Results

For the given geometric configuration and boundary conditions, the total force acting on the top surface should be equal to the one acting on the inclined plane. In other words, the following equilibrium equation should be satisfied:

Force Acting on Top Surface = Force Acting on the Inclined Inner Plane

or

Vertical Downward Pressure on Top Surface $\times$ Area of Top Surface =

Vertical Downward Pressure on Inclined Plane $\times$ Area of Inclined Plane

$\sigma_{(v)} = \text{Vertical Downward Pressure on Inclined Plane}$

$\sigma_{(v)} = \text{Vertical Downward Pressure on Top Surface} \times \frac{\text{Area of Top Surface}}{\text{Area of Inclined Plane}}$

$\sigma_{(v)} = 6.94 \times \cos 60^\circ = 3.47 \text{ psi}$

Derived from force equilibrium equations, the total stress acting on the lower part below the inclined plane is in the downward vertical direction, with a magnitude of 3.47 psi.

The shear and normal stresses on the inclined plane could be calculated as following:

$$\sigma_n = \sigma_{(v)} \times \cos 60^\circ = 3.47 \times 0.5 = 1.735 \text{ psi}$$

$$\tau = \sigma_{(v)} \times \cos 30^\circ = 3.47 \times 0.866 = 3.005 \text{ psi}$$

The ratio of shear to normal stress is $\tan 60^\circ = 1.732$. The directions of shear and normal stresses are illustrated in Figure 3.3.

Figure 3.3 Shear and normal stresses on the inclined plane.

3.4.1.2 Numerical Results

The discretized model is shown in Figure 3.4. There are 105 nodes located on the inclined inner plane. Using the method discussed before, the shear and normal stresses on the inclined plane are calculated at those nodes. The results are summarized in Table 3.1.

Figure 3.4 Discretized geometric model.

For Case No. 1, as can be seen from Table 3.1, the shear and normal stresses are almost the same as that calculated using analytical method. The maximum differences for shear and normal stresses between the analytical results and the numerical ones are 0.00273 psi and 0.00162 psi, respectively. For the ratio of shear to normal stress, the maximum difference is 0.000108.

It is thought that these differences between analytical and numerical results are acceptable for engineering application and the proposed methods for calculating shear and normal stresses are reliable.

Table 3.1 Summary of numerical computation results using finite element method.

Node No.	Shear Stress (psi)	Shear Stress Direction			Normal Stress (psi)	Normal Stress Direction			R *
		x-direction	y-direction	z-direction		x-direction	y-direction	z-direction	
5	3.00784	-0.50001	0.00005	-0.86602	-1.73662	-0.86602	0.00000	0.50001	-1.73201
6	3.00768	-0.50001	-0.00003	-0.86602	-1.73651	-0.86602	0.00000	0.50001	-1.73203
7	3.00709	-0.50001	0.00000	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73200
8	3.00709	-0.50001	0.00000	-0.86602	-1.73619	-0.86602	0.00000	0.50001	-1.73200
29	3.00749	-0.50001	0.00002	-0.86602	-1.73634	-0.86602	0.00000	0.50001	-1.73208
30	3.00708	-0.50001	0.00000	-0.86602	-1.73619	-0.86602	0.00000	0.50001	-1.73200
31	3.00727	-0.50001	0.00001	-0.86602	-1.73624	-0.86602	0.00000	0.50001	-1.73206
32	3.00735	-0.50001	-0.00002	-0.86602	-1.73638	-0.86602	0.00000	0.50001	-1.73197
33	3.00753	-0.50001	-0.00003	-0.86602	-1.73643	-0.86602	0.00000	0.50001	-1.73202
34	3.00753	-0.50001	0.00002	-0.86602	-1.73639	-0.86602	0.00000	0.50001	-1.73206
35	3.00723	-0.50001	0.00001	-0.86602	-1.73625	-0.86602	0.00000	0.50001	-1.73203
36	3.00713	-0.50001	0.00001	-0.86602	-1.73622	-0.86602	0.00000	0.50001	-1.73200
37	3.00707	-0.50001	0.00001	-0.86602	-1.73621	-0.86602	0.00000	0.50001	-1.73197
38	3.00704	-0.50001	0.00001	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73197
39	3.00703	-0.50001	0.00000	-0.86602	-1.73621	-0.86602	0.00000	0.50001	-1.73195
40	3.00705	-0.50001	0.00000	-0.86602	-1.73622	-0.86602	0.00000	0.50001	-1.73196
41	3.00708	-0.50001	0.00000	-0.86602	-1.73622	-0.86602	0.00000	0.50001	-1.73197
42	3.00709	-0.50001	0.00000	-0.86602	-1.73621	-0.86602	0.00000	0.50001	-1.73198
43	3.00710	-0.50001	0.00000	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73200
44	3.00710	-0.50001	0.00000	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73200
45	3.00709	-0.50001	0.00000	-0.86602	-1.73619	-0.86602	0.00000	0.50001	-1.73200
46	3.00709	-0.50001	0.00000	-0.86602	-1.73619	-0.86602	0.00000	0.50001	-1.73200
47	3.00709	-0.50001	0.00000	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73200
48	3.00709	-0.50001	0.00000	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73200
49	3.00709	-0.50001	0.00000	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73200
50	3.00709	-0.50001	0.00000	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73200
51	3.00709	-0.50001	0.00000	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73200
52	3.00745	-0.50001	-0.00001	-0.86602	-1.73631	-0.86602	0.00000	0.50001	-1.73209
53	3.00719	-0.50001	-0.00001	-0.86602	-1.73625	-0.86602	0.00000	0.50001	-1.73200
54	3.00712	-0.50001	-0.00001	-0.86602	-1.73623	-0.86602	0.00000	0.50001	-1.73199
55	3.00705	-0.50001	0.00000	-0.86602	-1.73621	-0.86602	0.00000	0.50001	-1.73197
56	3.00703	-0.50001	0.00000	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73196
57	3.00704	-0.50001	0.00000	-0.86602	-1.73622	-0.86602	0.00000	0.50001	-1.73195
58	3.00706	-0.50001	0.00000	-0.86602	-1.73622	-0.86602	0.00000	0.50001	-1.73196
59	3.00708	-0.50001	0.00000	-0.86602	-1.73622	-0.86602	0.00000	0.50001	-1.73197
60	3.00709	-0.50001	0.00000	-0.86602	-1.73621	-0.86602	0.00000	0.50001	-1.73199
61	3.00709	-0.50001	0.00000	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73200
62	3.00709	-0.50001	0.00000	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73200
63	3.00709	-0.50001	0.00000	-0.86602	-1.73619	-0.86602	0.00000	0.50001	-1.73200
64	3.00709	-0.50001	0.00000	-0.86602	-1.73619	-0.86602	0.00000	0.50001	-1.73200
125	3.00709	-0.50001	0.00000	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73200
126	3.00709	-0.50001	0.00000	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73200
127	3.00698	-0.50001	0.00000	-0.86602	-1.73616	-0.86602	0.00000	0.50001	-1.73197
128	3.00696	-0.50001	0.00002	-0.86602	-1.73607	-0.86602	0.00000	0.50001	-1.73205
129	3.00699	-0.50001	-0.00004	-0.86602	-1.73614	-0.86602	0.00000	0.50001	-1.73200
130	3.00694	-0.50001	-0.00003	-0.86602	-1.73608	-0.86602	0.00000	0.50001	-1.73203
131	3.00710	-0.50001	-0.00003	-0.86602	-1.73617	-0.86602	0.00000	0.50001	-1.73203
132	3.00725	-0.50001	-0.00003	-0.86602	-1.73616	-0.86602	0.00000	0.50001	-1.73213
133	3.00736	-0.50001	-0.00004	-0.86602	-1.73626	-0.86602	0.00000	0.50001	-1.73208
134	3.00702	-0.50001	0.00004	-0.86602	-1.73609	-0.86602	0.00000	0.50001	-1.73206
135	3.00710	-0.50001	0.00001	-0.86602	-1.73623	-0.86602	0.00000	0.50001	-1.73198
136	3.00717	-0.50001	0.00003	-0.86602	-1.73618	-0.86602	0.00000	0.50001	-1.73206
137	3.00742	-0.50001	0.00005	-0.86602	-1.73632	-0.86602	0.00000	0.50001	-1.73206
138	3.00692	-0.50001	0.00000	-0.86602	-1.73609	-0.86602	0.00000	0.50001	-1.73201
139	3.00695	-0.50001	-0.00001	-0.86602	-1.73613	-0.86602	0.00000	0.50001	-1.73199
140	3.00704	-0.50001	0.00000	-0.86602	-1.73621	-0.86602	0.00000	0.50001	-1.73196
141	3.00708	-0.50001	0.00000	-0.86602	-1.73621	-0.86602	0.00000	0.50001	-1.73197

Table 3.1 Summary of numerical computation results using finite element method.
(continued)

142	3.00709	-0.50001	0.00000	-0.86602	-1.73621	-0.86602	0.00000	0.50001	-1.73199
143	3.00709	-0.50001	0.00000	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73200
144	3.00695	-0.50001	0.00002	-0.86602	-1.73605	-0.86602	0.00000	0.50001	-1.73206
145	3.00696	-0.50001	0.00002	-0.86602	-1.73612	-0.86602	0.00000	0.50001	-1.73200
146	3.00702	-0.50001	0.00001	-0.86602	-1.73619	-0.86602	0.00000	0.50001	-1.73197
147	3.00708	-0.50001	0.00000	-0.86602	-1.73622	-0.86602	0.00000	0.50001	-1.73197
148	3.00709	-0.50001	0.00000	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73200
149	3.00709	-0.50001	0.00000	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73200
150	3.00709	-0.50001	0.00000	-0.86602	-1.73619	-0.86602	0.00000	0.50001	-1.73200
151	3.00693	-0.50001	-0.00002	-0.86602	-1.73608	-0.86602	0.00000	0.50001	-1.73202
152	3.00699	-0.50001	-0.00002	-0.86602	-1.73615	-0.86602	0.00000	0.50001	-1.73199
153	3.00707	-0.50001	0.00000	-0.86602	-1.73621	-0.86602	0.00000	0.50001	-1.73197
154	3.00709	-0.50001	0.00000	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73200
155	3.00706	-0.50001	0.00002	-0.86602	-1.73617	-0.86602	0.00000	0.50001	-1.73201
156	3.00712	-0.50001	-0.00001	-0.86602	-1.73612	-0.86602	0.00000	0.50001	-1.73210
157	3.00706	-0.50001	0.00002	-0.86602	-1.73617	-0.86602	0.00000	0.50001	-1.73201
158	3.00713	-0.50001	0.00003	-0.86602	-1.73616	-0.86602	0.00000	0.50001	-1.73206
159	3.00701	-0.50001	0.00001	-0.86602	-1.73618	-0.86602	0.00000	0.50001	-1.73196
160	3.00700	-0.50001	0.00002	-0.86602	-1.73614	-0.86602	0.00000	0.50001	-1.73200
161	3.00707	-0.50001	0.00000	-0.86602	-1.73622	-0.86602	0.00000	0.50001	-1.73196
162	3.00705	-0.50001	0.00000	-0.86602	-1.73622	-0.86602	0.00000	0.50001	-1.73196
163	3.00709	-0.50001	0.00000	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73199
164	3.00709	-0.50001	0.00000	-0.86602	-1.73621	-0.86602	0.00000	0.50001	-1.73198
165	3.00709	-0.50001	0.00000	-0.86602	-1.73619	-0.86602	0.00000	0.50001	-1.73200
166	3.00709	-0.50001	0.00000	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73200
167	3.00709	-0.50001	0.00000	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73200
168	3.00709	-0.50001	0.00000	-0.86602	-1.73619	-0.86602	0.00000	0.50001	-1.73200
169	3.00708	-0.50001	-0.00002	-0.86602	-1.73618	-0.86602	0.00000	0.50001	-1.73201
170	3.00701	-0.50001	-0.00002	-0.86602	-1.73616	-0.86602	0.00000	0.50001	-1.73200
171	3.00702	-0.50001	-0.00002	-0.86602	-1.73618	-0.86602	0.00000	0.50001	-1.73198
172	3.00704	-0.50001	-0.00001	-0.86602	-1.73622	-0.86602	0.00000	0.50001	-1.73194
173	3.00706	-0.50001	0.00000	-0.86602	-1.73622	-0.86602	0.00000	0.50001	-1.73196
174	3.00708	-0.50001	0.00000	-0.86602	-1.73621	-0.86602	0.00000	0.50001	-1.73198
175	3.00709	-0.50001	0.00000	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73199
176	3.00709	-0.50001	0.00000	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73200
177	3.00709	-0.50001	0.00000	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73200
178	3.00709	-0.50001	0.00000	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73200
179	3.00709	-0.50001	0.00000	-0.86602	-1.73621	-0.86602	0.00000	0.50001	-1.73199
180	3.00703	-0.50001	-0.00001	-0.86602	-1.73621	-0.86602	0.00000	0.50001	-1.73196
181	3.00697	-0.50001	-0.00003	-0.86602	-1.73612	-0.86602	0.00000	0.50001	-1.73200
182	3.00709	-0.50001	0.00000	-0.86602	-1.73620	-0.86602	0.00000	0.50001	-1.73200
183	3.00709	-0.50001	0.00000	-0.86602	-1.73621	-0.86602	0.00000	0.50001	-1.73199
184	3.00706	-0.50001	0.00000	-0.86602	-1.73622	-0.86602	0.00000	0.50001	-1.73196
185	3.00695	-0.50001	0.00001	-0.86602	-1.73613	-0.86602	0.00000	0.50001	-1.73198
186	3.00701	-0.50001	0.00004	-0.86602	-1.73610	-0.86602	0.00000	0.50001	-1.73205
187	3.00689	-0.50001	0.00000	-0.86602	-1.73603	-0.86602	0.00000	0.50001	-1.73206
188	3.00706	-0.50001	0.00005	-0.86602	-1.73614	-0.86602	0.00000	0.50001	-1.73204
189	3.00702	-0.50001	-0.00005	-0.86602	-1.73613	-0.86602	0.00000	0.50001	-1.73203
R [*] : Ratio of Shear Stress and Normal Stress.									

3.4.2 Case No. 2

The second case is a famous experiment of fluid injection induced earthquakes at Rangely oil field in Colorado (Haimson, 1972; C. Raleigh, Healy, & Bredehoeft, 1972; C. Raleigh, Healy, & Bredehoeft, 1976; M. D. Zoback & Healy, 1984). It thoroughly analyzed the mechanics of crustal earthquakes in terms of the magnitude of in-situ stresses and the frictional strength of faults. The researchers attempted to demonstrate

that raising pore pressure would tend to induce strike-slip faulting by increasing the ratio of shear to effective normal stress on a pre-existing fault.

Table 3.2 illustrates the orientation and magnitude of three principal stresses. The pre-existing fault plane strikes N50°E and dips 80°NW. Using tensor transformation (C. Raleigh et al., 1976; M. L. Zoback & Zoback, 1980), the shear and normal stresses resolved into the slip direction and normal to the fault plane are 1943.51 psi and 6874.79 psi, respectively.

Table 3.2 State of Stress at Rangely, Colorado. (C. Raleigh et al., 1976; M. D. Zoback & Healy, 1984)

Principal Stresses			Stress on Fault		Pore Pressure (psi)	Coefficient of Fiction
S_{Hmax} (psi)	S_{hmax} (psi)	S_v (psi)	Shear Stress (psi)	Normal Stress (psi)		
8557.23 (N80°W)	4554.18 (N10°E)	6193.11 (Vertical)	1943.51	6874.79	3988.54	0.67

Numerical analysis is also performed using the finite element method. Figure 3.5 shows the geometric model and the corresponding discretized grid system. Stress boundary conditions are used. S_{Hmax} is applied in y direction, S_{hmax} in x direction, S_v in z direction. The bottom has been restrained (zero displacement) in vertical direction.

Young's modulus and Poisson's ratio are 4.02×10^6 psi and 0.209, respectively.

Figure 3.5 (a) Geometric model for case 2; (b) Discretized grid system.

Numerical model was built to simulate the in-situ stress field, the normal and shear stresses distributed on the fault plane were then calculated according to the above mentioned method. As can be seen in Figure 3.6, the numerical results are almost the same as that calculated by tensor transformation.

Figure 3.6 Numerical results for Case No. Two. (a) Effective normal stress distribution on the fault plane, unit: psi. (b) Shear stress distribution on the fault plane, unit: psi. (c) Ratio of shear stress and effective normal stress.

3.4.3 Case No. 3

Sneddon and Elliott (1946) (Sneddon & Elliot, 1946) derived the analytical solutions for the stress distribution in the vicinity of a semi-infinite crack in an elastic medium. They assumed the fracture is rectangular, with limited height and infinite length, in order to simplify the problem. The fracture is open under uniform internal pressure. The final solution for semi-infinite crack given by Sneddon and Elliot (1946) is listed as following:

$$\begin{aligned}
\frac{1}{2}(\sigma_y + \sigma_x) &= p_0 \left\{ \frac{r}{\sqrt{r_1 r_2}} \cos(\theta - 0.5\theta_1 - 0.5\theta_2) - 1 \right\} \\
\frac{1}{2}(\sigma_y - \sigma_x) &= p_0 \frac{2r \sin\theta}{H} \left(\frac{H^2}{4r_1 r_2} \right)^{\frac{3}{2}} \sin\left(\frac{3}{2}(\theta_1 + \theta_2)\right) \\
\tau_{xy} &= -p_0 \frac{2r \sin\theta}{H} \left(\frac{H^2}{4r_1 r_2} \right)^{\frac{3}{2}} \sin\left(\frac{3}{2}(\theta_1 + \theta_2)\right) \\
\sigma_z &= \nu(\sigma_y + \sigma_x)
\end{aligned} \tag{3.25}$$

Figure 3.7 illustrates the mathematical interpretation of the crack stress function. The 3D geometric model is given in Figure 3.8. The results of the 3D numerical model were compared to analytical solutions by plotting normalized additional stresses as a function of the ratio of distance (L) to fracture height (H) along the line of symmetry (x-axis in Figure 3.7). The line of symmetry is horizontal and perpendicular to the fracture. As can be seen in Figure 3.9, there is an excellent unanimity between the analytical and the numerical solutions.

Figure 3.7 Illustration of the mathematical interpretation of the crack stress function.

Figure 3.8 3D geometric model.

Figure 3.9 Comparisons of analytical and numerical solutions along the symmetric line normal to the semi-infinite fracture.

4 Stress Reorientation

In-situ stress underground can be altered due to the opening of propped hydraulic fractures and poroelastic effects (Olson, 2008; Roussel & Sharma, 2010; Siebrits et al., 1998; Singh, Roussel, & Sharma, 2008; Soliman et al., 2004). When the production or injection of fluids is minimal, poroelastic effects can be neglected during hydraulic operations (Cheng, 2009; Roussel & Sharma, 2010; N. Warpinski & Wright, 2001). In this study, in-situ stress field alteration induced by the opening of propped fractures is considered. The extent of stress reversal and reorientation has been investigated for two different cases: (a) in-situ stress contrast equals to zero ($S_{Hmax} - S_{Hmin} = 0$), and (b) in-situ stress contrast does not equal to zero ($S_{Hmax} - S_{Hmin} \neq 0$).

The 3D geometric model is illustrated in Figure 3.8. One hydraulic fracture is created at the center of the model. A constant stress, equal to the minimum in-situ horizontal stress S_{Hmin} plus net pressure p_{net} is imposed on the faces of the fracture.

4.1 Zero Initial In-situ Stress Contrast

In-situ stresses are initialized before imposing hydraulic pressure, the values of which are $S_{Hmax} = S_{Hmin} = 4055$ psi, $S_v = 6373$ psi. After applying hydraulic pressure ($p_{net} = 1000$ psi), stress trajectory patterns on a horizontal plane through the center point of the model are found, as illustrated in Figure 4.1. The trajectories of minimum horizontal principal stress encircle the hydraulic fracture, converging near the fracture ends. The maximum horizontal principal stress is perpendicular to the fracture face along the fracture wall.

Figure 4.1 Stress trajectories on a horizontal plane through the center point of the model. (a) Minimum horizontal principal stress trajectories. (b) Maximum horizontal principal stress trajectories.

4.2 Nonzero Initial In-situ Stress Contrast

Three different stress contrasts are created when initiating the in-situ stresses before applying hydraulic pressure. They are (1) $S_{Hmax} = 4555$ psi, $S_{hmin} = 4055$ psi, $S_v = 6373$ psi; (2) $S_{Hmax} = 4655$ psi, $S_{hmin} = 4055$ psi, $S_v = 6373$ psi; and (3) $S_{Hmax} = 4755$ psi, $S_{hmin} = 4055$

psi, $S_v = 6373$ psi. After applying hydraulic pressure ($p_{net} = 1000$ psi), the stress trajectories for the three scenarios are presented on a horizontal plane through the center point of the model, they are presented in Figure 4.2, Figure 4.3, and Figure 4.4, respectively. The stress trajectory patterns of the three scenarios are different from that obtained with zero initial in-situ stress contrast.

For the scenario with 500 psi horizontal stress contrast (Figure 4.2), the minimum and maximum horizontal principal stresses are rotated 90° from their initial in-situ directions in the vicinity of the fracture. The extent of the stress-reversal region is 100 ft in this scenario. Outside the stress-reversal region, there are reoriented-stress regions which are around the fracture tips and confined to the vicinity of the fracture.

When the initial horizontal stress contrast is 600 psi (Figure 4.3), the minimum and maximum horizontal principal stresses are rotated 90° from their initial in-situ directions in the vicinity of the fracture. The extent of the stress-reversal region is 80 ft in this scenario. There is a very interested phenomenon that is the stress trajectories are oriented as initially on the face of the fracture.

For the scenario with 700 psi horizontal stress contrast (Figure 4.4), there is no stress-reversal region in the vicinity of the fracture. The reoriented-stress region is around the fracture tips.

(a)

(b)

Figure 4.2 Stress trajectories on a horizontal plane through the center point of the model with 500 psi horizontal stress contrast. (a) Minimum horizontal principal stress trajectories. (b) Maximum horizontal principal stress trajectories.

(a)

(b)

Figure 4.3 Stress trajectories on a horizontal plane through the center point of the model with 600 psi horizontal stress contrast. (a) Minimum horizontal principal stress trajectories. (b) Maximum horizontal principal stress trajectories.

(a)

(b)

Figure 4.4 Stress trajectories on a horizontal plane through the center point of the model with 700 psi horizontal stress contrast. (a) Minimum horizontal principal stress trajectories. (b) Maximum horizontal principal stress trajectories.

4.3 Discussion

Based on the 3D numerical model (Figure 3.8), the generated stress contrast ($\Delta S_{yy} - \Delta S_{xx}$) normalized by the net pressure ($p_{net} = 1000$ psi) is plotted along a symmetric line normal to the fracture face (Figure 4.5). As can be seen, the generated stress contrast ($\Delta S_{yy} - \Delta S_{xx}$) is less than 700 psi when the net pressure is 1000 psi. There is no stress-reversal region if the initial in-situ stress contrast ($S_{yy} - S_{xx}$) is 700 psi (Figure 4.4).

When the initial in-situ stress contrast is 600 psi, the extent of the stress-reversal region is 80 ft (Figure 4.3). The generated stress contrast on the surface of the fracture is 600 psi, which is equal to the initial in-situ stress contrast. In this situation, relatively small stress perturbations are sufficient to change the stress trajectories on the fracture surface (Figure 4.3).

When the initial in-situ stress contrast is 500 psi, the extent of the stress-reversal region is 100 ft (Figure 4.2). If the initial in-situ stress contrast is zero, the extent of the stress-reversal region could be infinite (Figure 4.1). The stress distribution in Figure 4.5 confirms the phenomena shown from Figure 4.1 to Figure 4.4.

Generally, the operation of hydraulic fracturing alters the in-situ stress field. The extent of the stress-reversal region and the reoriented-stress region largely depends on the initial in-situ stress contrast. As revealed in this study, when the horizontal stress contrast is zero, the stress trajectories are totally determined by the created hydraulic fracture. When the horizontal stress contrast is not zero, the generated stress contrast ($\Delta S_{yy} - \Delta S_{xx}$), difference between the stress change in the direction perpendicular to and in the direction parallel to the fracture (Figure 3.7), may cause the horizontal stress trajectories to rotate 90° in the vicinity of the fracture. As the distance from the fracture increases, the generated stress contrast ($\Delta S_{yy} - \Delta S_{xx}$) becomes smaller than the initial in-situ stress contrast ($S_{yy} - S_{xx}$), the stress trajectories are oriented in initial directions. When the initial in-situ stress contrast ($S_{yy} - S_{xx}$) is larger than the generated stress contrast ($\Delta S_{yy} - \Delta S_{xx}$), there will be no stress-reversal region in the vicinity of the fracture, however, reoriented-stress region still exists around the fracture tips.

The relationship between the generated stress contrast ($\Delta S_{yy} - \Delta S_{xx}$) and the initial in-situ stress contrast ($S_{yy} - S_{xx}$) determines the stress redistribution during hydraulic fracture. ($\Delta S_{yy} - \Delta S_{xx}$) is a function of net pressure and fracturing spacing, this indicates we can control the extent of stress-reversal region and reoriented-stress region by adjusting net pressure and fracturing spacing.

Further work is needed to investigate the relationships between net pressure, initial stress contrast and the extent of stress-reversal/reoriented-stress region.

Figure 4.5 Normalized stresses vs. distance normal to a fracture, fracture height used in the numerical model is 200 ft.

5 Fault Stability Simulation

In order to evaluate fault stability during hydraulic fracturing, 3D numerical models based on the finite element method were developed to simulate the stress distributions in reservoirs intersected by geological faults during hydraulic fracturing. The potential of reactivation of fault was evaluated based on the changes of stress state around the fault resulting from creation of multiple hydraulic fractures. Three basic types of faults (normal, strike-slip, and reverse/strike-slip) are considered and characterized with their typical corresponding configuration of in-situ stresses. The orientation and relative magnitudes of in-situ stress fields differ under different faulting environments, which in turn control the direction of fracture propagation.

Three hydraulic fractures created simultaneously are considered in all case studies. The closest distance between the hydraulic fracture and a fault is varied for each fault type so that effects of this factor on fault stability can be quantified. The closest distances used are 50 ft, 100 ft and 150 ft.

Poroelastic effects due to the leakoff of the fracturing fluid into reservoirs are neglected in this study because of the ultralow-permeability of shale gas reservoirs. Constant net pressure is assumed along the hydraulic fractures (uniform proppant distribution).

5.1 Strike-slip Fault

The case used to simulate strike-slip faulting environment is modified from the famous fluid injection experiment carried out at Rangely, Colorado (Haimson, 1972; C. Raleigh et al., 1976; M. D. Zoback & Healy, 1984). Hydrostatic pore pressure is used in the current case, which is different with the one used in validation analysis. The configuration of hydraulic fractures and a strike-slip fault plane is illustrated in Figure 5.1. The three fractures are perpendicular to the direction of horizontal minimum principal stress (S_{hmin}).

Figure 5.1 3D geometric model for strike-slip faulting environment.

The basic reservoir and fracture properties are listed in the following:

Reservoir Properties

Geometry	4,000×4,000×800 ft ³ (length/width/height)		
Depth of reservoir top	5794.15 ft	Fault dip angle	80°
Young's modulus	4.02×10 ⁶ psi	Poisson's Ratio	0.209
Bulk density of reservoir	156 lb/ft ³	Vertical Stress	6277 psi
Reservoir pore pressure	0.433 psi/ft		
Maximum Horizontal Stress	8673 psi (x-direction in the model)		
Minimum Horizontal Stress	4616 psi (y-direction in the model)		

Fracture Parameters

Geometry (length/height)	2000×200 ft ²	Number of fractures	3
Fracture spacing	100 ft	Net pressure	1000 psi

Figure 5.2 and Figure 5.3 illustrate the distribution of shear and normal stresses on the strike-slip fault plane. Before applying hydraulic pressure, the shear stress ranges from 1967 to 1974 psi. It changes when applying hydraulic pressure, it ranges from 1850 to 2000 psi for the scenario with the closest distance of 50 ft between hydraulic fractures and the fault plane. The change behavior on the fault plane is very similar to each other for the three different scenarios. The maximum change on the fault plane occurs at the

point that has the closest distance from hydraulic fractures. As shown in Figure 5.2, the shear stress increases on one side of the line AB. However, on the opposite side of the line AB, the shear stress decreases. The shear stress has a wider range of stress change than the normal stress.

Figure 5.2 Shear stress distribution on the strike-slip fault plane. (a) initial stress state; (b) after applying hydraulic pressure (50 ft); (c) after applying hydraulic pressure (100 ft); (d) after applying hydraulic pressure (150 ft). The number (50, 100, 150) indicates the closest distance between the hydraulic fractures and the fault plane.

Before applying hydraulic pressure, the normal stress ranges between 6850 and 7100 psi. This range changes when applying hydraulic pressure. The maximum value increases to 7340 psi, the minimum value decreases to 6820 psi. The normal stress decreases on the left side of the line CD in Figure 5.3, where the shear stress increases in Figure 5.2. On the right side of line CD, the normal stress increases and the shear stress decreases. This pattern is confirmed in Fig. 4.5 by the ratio of shear to normal stress.

Figure 5.3 Normal stress distribution on the strike-slip fault plane. (a) initial stress state; (b) after applying hydraulic pressure (50 ft); (c) after applying hydraulic pressure (100 ft); (d) after applying hydraulic pressure (100 ft). The number (50, 100, 150) indicates the closest distance between the hydraulic fractures and the fault plane.

Figure 5.4 gives the ratio of shear to effective normal stress distributed on the strike-slip fault plane. Before applying hydraulic pressure, the ratio ranges between 0.45 and 0.46. When applying hydraulic pressure, the ratio on the left side of the line EF increases (Figure 5.4), which means fault plane tends to become unstable; on the contrary, the ratio on the right side of the line EF decreases, which indicates the fault plane becomes more stable.

Figure 5.4 Ratio of shear to effective normal stress distributed on the strike-slip fault plane. (a) initial state; (b) after applying hydraulic pressure (50 ft); (c) after applying hydraulic pressure (100 ft); (d) after applying hydraulic pressure (100 ft). The number (50, 100, 150) indicates the closest distance between the hydraulic fractures and the fault plane.

5.2 Normal Fault

Stress regime for normal faulting environment is assumed based on the Anderson's theory. Figure 5.5 shows the geometric model. The basic reservoir and fracture properties are listed in the following:

Reservoir properties

Geometry	4,000×4,000×800 ft ³ (length/width/height)		
Depth of reservoir top	5794.15 ft	Fault dip angle	60°
Young's modulus	4.02×10 ⁶ psi	Poisson's Ratio	0.209
Bulk density of reservoir	156 lb/ft ³	Vertical Stress	1.1 psi/ft
Reservoir pore pressure	0.433 psi/ft		
Maximum Horizontal Stress	0.8 psi/ft		
Minimum Horizontal Stress	0.7 psi/ft		

Fracture Parameters

Geometry (length/height)	2000×200 ft ²	Number of fractures	3
Fracture spacing	100 ft	Net pressure	1000 psi

Figure 5.5 3D geometric model for normal faulting environment.

The distribution of shear and normal stresses on fault plane is shown in Figure 5.6 and Figure 5.7, respectively. The shear stress ranges from 860 to 1140 psi before applying hydraulic pressure. After applying hydraulic pressure, the shear stress decreases by a maximum value of 195 psi above the line AB as shown in Figure 5.6. Points on the line AB have the closest distance from hydraulic fractures to the fault plane. On the contrary, the region below the line AB has increased shear stress by a maximum value of 260 psi.

Before applying hydraulic pressure, the original normal stress on the normal fault plane ranges from 4430 to 4830 psi. The normal stress increases in the region around the line AB after applying hydraulic pressure. The maximum increased magnitude is 470 psi for the scenario with the closest distance between the hydraulic fractures and the fault plane being 50 ft. There is no region with decreased normal stress in the normal faulting environment.

Figure 5.6 Shear stress distribution on the normal fault plane. (a) initial stress state; (b) after applying hydraulic pressure (50 ft); (c) after applying hydraulic pressure (100 ft); (d) after applying hydraulic pressure (100 ft). The number (50, 100, 150) indicates the closest distance between the hydraulic fractures and the fault plane.

Figure 5.7 Normal stress distribution on the normal fault plane. (a) initial stress state; (b) after applying hydraulic pressure (50 ft); (c) after applying hydraulic pressure (100 ft); (d) after applying hydraulic pressure (100 ft). The number (50, 100, 150) indicates the closest distance between the hydraulic fractures and the fault plane.

The ratio of shear to effective normal stress distributed on the normal fault plane is illustrated in Figure 5.8. It ranges from 0.41 to 0.53 before applying hydraulic pressure. The ratio decreases with a maximum value of 0.14 above the line AB after applying hydraulic pressure. Below the line AB, the ratio increases with a maximum value of 0.08. Changes of the ratio conform to the alteration of shear and normal stresses.

Figure 5.8 Ratio of shear to effective normal stress distributed on the normal fault plane. (a) initial state; (b) after applying hydraulic pressure (50 ft); (c) after applying hydraulic pressure (100 ft); (d) after applying hydraulic pressure (150 ft). The number (50, 100, 150) indicates the closest distance between the hydraulic fractures and the fault plane.

5.3 Reverse/strike-slip Fault

A fault which has a component of dip-slip and a component of strike-slip is termed an oblique-slip fault. Reverse/strike-slip fault means the hanging wall moves in both upward and horizontal directions.

The geometric model for reverse/strike-slip fault is shown in Figure 5.9. Based on the Anderson's theory, stress regime for reverse/strike-slip fault is assumed. The basic reservoir and fracture properties are listed in the following:

Reservoir properties

Geometry	4,000×4,000×800 ft ³ (length/width/height)		
Depth of reservoir top	5794.15 ft	Fault dip angle	30°
Young's modulus	4.02×10 ⁶ psi	Poisson's Ratio	0.209
Bulk density of reservoir	156 lb/ft ³	Vertical Stress	1.05 psi/ft
Reservoir pore pressure	0.433 psi/ft		
Maximum Horizontal Stress	1.25 psi/ft		
Minimum Horizontal Stress	0.98 psi/ft		

Fracture Parameters

Geometry (length/height)	2000×200 ft ²	Number of fractures	3
Fracture spacing	100 ft	Net pressure	1000 psi

Figure 5.9 3D geometric model for reverse/strike-slip faulting environment.

Figure 5.10 illustrates the shear stress distribution on the reverse/strike-slip fault plane. The shear stress ranges from 363 to 640 psi before applying hydraulic pressure. After applying hydraulic pressure, the shear stress ranges from 342 to 640 psi, the lower bound of it decreases by a value of 21 psi.

Figure 5.10 Shear stress distribution on the reverse/strike-slip fault plane. (a) initial stress state; (b) after applying hydraulic pressure (50 ft); (c) after applying hydraulic pressure (100 ft); (d) after applying hydraulic pressure (100 ft). The number (50, 100, 150) indicates the closest distance between the hydraulic fractures and the fault plane.

Figure 5.11 Normal stress distribution on the reverse/strike-slip fault plane. (a) initial stress state; (b) after applying hydraulic pressure (50 ft); (c) after applying hydraulic pressure (100 ft); (d) after applying hydraulic pressure (100 ft). The number (50, 100, 150) indicates the closest distance between the hydraulic fractures and the fault plane.

The normal stress distribution on the reverse/strike-slip fault plane is shown in Figure 5.11. Before applying hydraulic pressure, the normal stress ranges from 6016 to 6723 psi. It ranges from 6015 to 6756 psi after applying hydraulic pressure. It should be noticed that though the range of stress does not alter a lot, the stress distribution on the fault plane has been changed. The normal stress on the fault plane decreases at the region where the closest distance between hydraulic fracture and the fault plane exists.

The ratio of shear to effective normal stress distributed on the reverse/strike-slip fault plane is presented in Figure 5.12. It ranges from 0.09 to 0.17 before applying hydraulic pressure. After applying hydraulic pressure, the range does not change, however, the ratio at the area A (Figure 5.12) on the fault plane increases, which means it tends to become unstable.

Figure 5.12 Ratio of shear to effective normal stress distributed on the reverse/strike-slip fault plane. (a) initial state; (b) after applying hydraulic pressure (50 ft); (c) after applying hydraulic pressure (100 ft); (d) after applying hydraulic pressure (100 ft). The number (50, 100, 150) indicates the closest distance between the hydraulic fractures and the fault plane.

5.4 Discussion

Due to the alteration of stress field caused by hydraulic fracturing, faults nearby the treatment wells have potential to be reactivated. Numerical results indicate that the distance between the fracture and the fault plane has influence on the change of stress magnitude distributed on the fault plane, but it does not change the patterns of stress distribution. Comparing the three types of fault environment, it is indicated that the patterns of stress distribution on the fault planes are different, which depend on the angle between fracture orientation and fault strike.

Table 5.1 summarized the shear and normal stress changes, also the alterations of the ratio of shear to effective normal stress are included. As can be seen, the normal faulting environment has the largest fluctuation in stresses and the ratio of shear to effective normal stress. The reverse/strike-slip fault has the least perturbation when hydraulic pressure is applied.

In-situ stresses control the propagative direction of the created fractures and thus determine the angle between fracture orientation and fault strike. Numerical simulations demonstrate that the angle has a strong influence on the stability of a fault based on the change in the ratio of shear to effective normal stresses. When the created fractures parallel to the normal fault plane (Figure 5.5), the fault plane has the largest extent of stress change range compared to other types of faulting environment. In the reversal/strike-slip faulting environment (Figure 5.9), the hydraulic fracture is perpendicular to the fault plane, the creation of the fracture has the least influence on the fault stability. However, there is a great possibility that the fracture propagating perpendicularly to the fault plane may extent to the fault, which may cause early aborting/failure of fracturing treatment, fluid leakage along the fault.

Table 5.1 Summary of stress and ratio changes for different fault types. (Positive represents increased value, negative represents decreased value.)

Fault Type	Shear Stress (psi)		Normal Stress (psi)		Ratio	
	Upper Bound	Lower Bound	Upper Bound	Lower Bound	Upper Bound	Lower Bound
Strike-slip	+ 26	- 117	+ 240	- 30	+ 0.02	- 0.07
Normal	+ 260	- 195	+ 470	0	+ 0.08	- 0.14
Reverse/strike-slip	0	- 21	+ 33	- 1	0	0

6 Conclusions

This study investigates the stress distribution underground during hydraulic stimulation and the effect of stress change on fault stability is also analyzed. 3D numerical models based on the finite element method are developed and validated. An excellent agreement between analytical and numerical solutions has been obtained, which demonstrates the suitability of using the numerical models to simulate complex situations.

The opening of propped hydraulic fractures generally alters in-situ stresses. Larger additional stresses perpendicular to the fracture wall can be generated compared to the additional stresses parallel to the fracture wall. It is found that the initial in-situ stress contrast has a strong influence on the extent of stress-reversal region and reoriented-stress region. If the generated stress contrast is larger than the initial in-situ stress contrast in horizontal plane, the directions of horizontal principal stresses will be reversed. When the generated stress contrast is less than the initial in-situ stress contrast in horizontal plane, the directions of horizontal principal stresses are oriented as initially except the region around fracture tips, which generally have stress-reoriented region due to stress concentration. Usually the stress-reversal region is in the vicinity of fractures, the reoriented-stress region is beyond the stress reversal region and around fracture tips.

Three typical faulting environments were considered in this study. They are normal, strike-slip, and strike-slip/reverse faults. The orientation and relative magnitudes of in-situ stress fields differ under different faulting environments, which in turn control the direction of fracture propagation. Three hydraulic fractures created simultaneously are considered in all case studies.

It was found that the angle between fracture orientation and fault strike has a strong effect on the stability of a fault based on the change in the ratio of shear to effective normal stresses. Along the fault plane, the stability is strengthened in certain regions but weakened in other regions. The stress alteration patterns are different on the three types of fault. The normal faulting environment has the largest fluctuation in stresses and in the ratio of shear to effective normal stress. The reverse/strike-slip fault has the least perturbation when hydraulic pressure is applied.

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