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Three Essays on Forecasting in Nonlinear Models

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Three Essays on Forecasting in Nonlinear Models

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Dissertation submitted to the
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West Virginia University
in fulfillment of the requirements
for the degree of

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in
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Abstract

Three Essays on Forecasting in Nonlinear Models

Scott T. Murdoch

Nonlinear models have many applications in the economic and financial fields. The following works focus on their use for forecasting. Neural networks, in conjunction with an Affine Term Structure Model, are used to discover possible yield curve arbitrage opportunities. A G/ARCH model is employed to test and forecast the conditional variance of state-level employment growth. A Space-Time Autoregressive (STAR) model is applied to state employment growth to ensure that any measure of volatility in the series is not misdirected as employment movements between neighboring states.

Contents

Introduction	iv
1 NN Approach Yield Curve Arbitrage	1
1.1 Review of the Literature	2
1.2 Empirical Analysis	4
1.2.1 ATSM Architecture	4
1.2.2 Basic Neural Network	6
1.2.3 Neural Network Architecture	7
1.3 Empirical Model and Results	11
1.3.1 Data	11
1.3.2 Empirical Results	12
1.4 Conclusion	22
1.5 Appendix A: Additional Tables	23
2 State Employment Volatility	25
2.1 Review of the Literature	25
2.2 Empirical Analysis	27
2.2.1 Empirical Data	27
2.2.2 Empirical Model	28
2.2.3 ARCH Length Specification Algorithm	30
2.3 Results of Empirics	33
2.4 Conclusion	45
2.5 Appendix A: Additional Tables	46
3 State Spatial Employment Volatility	50
3.1 Review of the Literature	51
3.2 Empirical Analysis	52
3.2.1 Data	52
3.2.2 Empirical Model	52
3.3 Results of Empirics	54
3.4 Conclusion	58

Introduction

In today's economy, forecasting is both an art and a science. The science side deals with the analytical modeling of economic and financial variables such as employment, personal income, interest rates, and countless others. The art side addresses the deficiencies in the current knowledge in scientific modeling. This is rhythmically seen in the relationship between variables; the structural relationship between variables is often nonlinear. The variables and structure of regression equation(s) used in forecasting are ultimately up to the forecaster.

The essays that follow focus on the scientific side of forecasting; in particular, model methodology. The initial argument for an increased understanding in forecast modeling is to lower the mean squared error (MSE) of the forecast residuals. The argument, while valid, is misleading. The reason for improving a forecast is not for the forecast's sake; it is to have a better and more certain understanding of the future. For example, if one discovers a revolutionary employment forecasting model, the average MSE may be reduced by one half; however, the value added by the revolution is so the individual can make more informed decisions about future employment. Improved decision-making is the ultimate goal. Throughout the three subsequent essays the main focus at times may be perceived as MSE reduction, but in fact is just a tool used for the bigger picture.

The first chapter centers around the prediction of yield curve arbitrage opportunities. The point of convergence of these opportunities is the difference between the estimated Affine Term Structure Model and realized yield curve. Affine Term Structure Model (ATSM) generates a yield curve constructed under a no-arbitrage assumption, it assumes markets are efficient. Discovering yield curve arbitrage can be achieved by comparing the yield curve constructed by the ATSM and the realized yield curve. If the curves differ then there are arbitrage opportunities available for exploitation. However, given the speed of global markets, arbitrage opportunities disappear almost instantaneously. Therefore, the most profitable form of arbitrage is to have a landscape of future opportunities.

Unfortunately, one of the major downfalls of ATSM's is poor out-of-sample forecast performance. The unknown precise relationship between yields to maturity of different terms is undoubtedly one of the largest factors in forecast performance. To address this problem, neural networks will be tested for prediction; one of the key strengths of neural networks is finding relationships among variables when an exact mathematical relationship is unknown. Neural networks are used to forecast both the Affine Term Structure Model curve and the actual yield curve.

The second chapter centers on the structure of state-level employment volatility; assessing whether the conditional variance is anything other than a white noise process. If the assumption of independently and identically distributed errors (iid) is violated, what is the correct structure of the error? The degree of erroneousess in the conditional variance will be tested using methods described in Engle (1982). If the assumption is violated the biased portion of the MSE will be nonzero. The correct specification of the conditional variance would minimize the bias term to zero or at a minimum decrease the magnitude of the term.

The "optimal" volatility structure, if ARCH effects are shown to be present, will be determined by an algorithm described by Figures 2.1 & 2.2. The chapter ends by comparing the forecast error, both the Mean Squared Error and Percent Forecast Error, of the volatility structure using the ARCH specification algorithm, a GARCH(1,1) process, and white noise estimation for each state.

The final chapter serves as an extension of the second. The purpose of this chapter is to examine if the findings of Chapter 2 hold when accounting for spatial spillover effects. If ARCH effects are shown to be present, these effects could be misclassified as changes in employment growth from neighboring states.

Furthermore, the effects of the 'Great Moderation' on the conditional variance for each state are examined. The 'Great Moderation' refers to the reduction in the variance, or volatility, of real U.S. GDP growth due to the Federal Reserve policy changes. The chapter will conclude by comparing the number of the states that display ARCH effects accounting for spatial spillovers and the 'Great Moderation.'

Chapter 1

Neural Network Approach to Yield Curve Arbitrage

Hedge funds and other investment firms seek out arbitrage opportunities to increase their own and their clients' wealth with as little risk as possible. However, it can be seen from the demise of Long Term Capital Management, and more recently, large losses written off by investment banks due to the default subprime lending market. That there is room for improvement in modeling arbitrage opportunities.

Intelligence algorithms are being used more frequently outside of the artificial intelligence field mainly because of their adaptability. Intelligence algorithms attempt to model the human thought process and human logic, and are more flexible than traditional algorithms. This paper focuses on the performance of integrating intelligence algorithms with traditional methods: in particular Affine Term Structure Models (ATSM) of discovering yield curve arbitrage opportunities.

The set of intelligence algorithms discussed in this paper are neural networks. Neural networks (NN) are useful when the relationships between variables are unknown. The criticism of neural networks is that they are a so-called 'Black Box,' meaning it is unknown to the user what exactly the procedure is doing. However, while there is some validity to this criticism, this criticism is also an over-simplification. Neural networks are a nonparametric technique, which differs from regression analysis used in traditional economic and finance studies.

The neural networks illustrated here can be explained in relation to a classical regression setting, refer to Section 1.2.2. These networks will be used for out-of-sample forecasting for both the actual yield curve and the curve estimated by ATSM. Additionally, neural networks with only autoregressive terms and a network with macroeconomic variables in addition to the au-

toregressive terms will be tested. The design being that the macroeconomic variables should increase the forecast accuracy.

The remainder of this paper is structured as follows. The next section reviews the literature; followed by a characterization of the intelligence algorithms. After the description of the model structure the empirical results are presented.

1.1 Review of the Literature

The type of integration between yield curve arbitrage and intelligence algorithms discussed in this paper is not discussed in any great detail in the past literature. There are several articles that aided in the integration between these methods, which can be divided into two subsets; articles that deal with yield curve arbitrage and that deal with intelligence algorithms. There are several articles in the finance literature that use intelligence algorithms with basic finance concepts. In these types of articles neural networks are often used for predicting a variable that has unknown relationship(s) with other variable(s).

One of the most comprehensive articles on arbitrage opportunities is Duarte, Longstaff, and Yu (2007). It discusses five different forms of fixed income arbitrage and whether these arbitrage methods provide significant excess returns. It finds that yield curve, mortgage, and capital structure arbitrage provide significant excess returns because they require the most 'intellectual capital.' In other words, finding arbitrage opportunities in these three markets requires significant knowledge, which creates a barrier to entry. These three forms of arbitrage produce excess returns even after accounting for transaction costs (including a 2/20 hedge fund fee), and market risk.

Of the three forms of arbitrage that demonstrate excess returns, yield curve arbitrage is the most accessible for this study. The yield curve arbitrage involves comparing the yields from an Affine Term Structure Model with the actual yields and is the form of arbitrage that is focused on in the remainder of this paper. Mortgage arbitrage is relevant given the current state of the market and introduces an interesting dynamic due to the fact that the borrowers (households) can pay loans back before they are due. This has the possibility of disturbing the lenders cash flow, adding uncertainty on the lender's end. Capital structure arbitrage is relatively new form of arbitrage that involves looking at the differences between a company's debt and its assets, such as equity. Yield curve arbitrage was chosen for this study because of the relevant ease of accessibility compared to the other two forms; however, the integration of intelligence algorithm(s) can be applied to any

form of the arbitrage given the proper methodology and data.

Extending the discussion of yield curve arbitrage, Dai and Singleton (2000) discuss the technique of Affine Term Structure Models (ATSMs) and some goodness-of-fit measures for evaluating ATSMs performance. Affine Term Structure Models are a way of modeling the price of a zero-coupon bond. ATSM's carry the assumption of no arbitrage, or in other words, markets are assumed to be efficient. The ATSM model, in particular Vasicek, is formulated for efficient markets. If the ATSM yield curve does not perfectly fit the actual yield curve then there are arbitrage opportunities available, because the actual yield curve is in a inefficient market structure. This formulation is presented in 1.3.2. The instantaneous interest rates are given by a linear or 'affine' function that is made up of N unobserved state variables. The state variables are assumed to follow a Brownian motion. A factor in an Affine Term Structure Model is a particular term-to-maturity, for example a 10-year bond.

The number of factors in an ATSM is equal to the number of unobserved state variables. Solving an ATSM requires solving the differential equations of the corresponding unobserved state variable such that the past prices of the state variable are perfectly fitted by the model. The coefficients determined in each of the state variables are used to calculate the prices of other terms-to-maturity (TTM) bonds that are not included in the factors. For example, if you have a two factor ATSM that fits the prices of a 6-month and 5-year TTM perfectly, you would use the coefficients found in the differential equations of these yields to find prices of a 1-year bond, 2-year bond, etc.

On the intelligence side, Amilon (2003) discuss a neural network approach to pricing options and compares this approach to the traditional Black-Scholes method. The results of this paper are that the neural network approach, on the whole, outperform the Black-Scholes method. However, not all of the results were statistically significant at the 5% level. Anders, Korn, and Schmitt (1998) also show statistically significant results that a neural network approach to pricing options outperforms the traditional Black-Scholes. The interesting point of their study is the variables used in the neural networks. The neural networks, in addition to using the past prices such variables like the stock price over the option strike price, the time to maturity, 30 day standard deviation, Black-Scholes, and DAX volatility index are used as input variables. The data used in this study was call options for 30 of the largest German companies from January 1992 to December 1994. The authors used statistical inference to test performance of the neural networks specification versus the Black-Scholes. All four of the neural network specified in this article outperform the Black-Scholes in terms of RMSE for both in-sample and out-of-sample.

1.2 Empirical Analysis

The empirical analysis is intertwined between estimating a no-arbitrage yield curve and forecasting with neural networks. To illustrate the global model it is best, at the beginning, to separate the analysis into two parts. A detailed description of the ATSM estimation is presented in Section 1.2.1. The architecture of the neural network system is discussed in Section 1.2.2 & 1.2.3.

1.2.1 ATSM Architecture

The term structure model chosen for this study is a two factor Vasicek Model, refer to Vasicek (1977). The instantaneous interest rate is given by the Ornstein-Uhlenbeck process:

$$dr = \alpha_i(\gamma_i - r)dt + \rho_i dz \quad (1.1)$$

$$a_{i,t} = \gamma_i + (r(t) - \gamma_i)e^{-\alpha_i \Delta} \quad (1.2)$$

$$\sigma_{i,t}^2 = \frac{\rho_i^2}{2\alpha_i}(1 - e^{-2\alpha_i \Delta}) \quad (1.3)$$

Where **Equation 1.2 & 1.3** are the conditional expectation and conditional variance of the Ornstein-Uhlenbeck process given by **equation 1.1**. There is a set of the above equations for each factor or yield solved for; and the unknown variables are α_i , ρ_i , and γ_i . In **equation 1.1**, $\alpha_i(\gamma_i - r)$ is the drift term, α_i is coined the elastic random walk or speed of reversion (term). γ_i is often called mean reversion or long-term mean. The instantaneous variance is represented by ρ_i^2 . The multi-factor version of equation (1.1) can be demonstrated as follows:

$$\begin{pmatrix} r_{1,t} \\ r_{10,t} \end{pmatrix} = \begin{pmatrix} \alpha_1(\gamma_1 - r_{1,t}) & 0 \\ 0 & \alpha_{10}(\gamma_{10} - r_{10,t}) \end{pmatrix} dt + \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_{10} \end{pmatrix} \begin{pmatrix} dz_1 \\ dz_{10} \end{pmatrix} \quad (1.4)$$

In the above example, the ordinary differential equations (ODEs) are for a 1 and 10 year bond. This is an example of a two-factor model because there is two yields being solved. The unknown variables in the ordinary differential equations (ODEs) are solved through a maximum likelihood estimation (MLE). When the estimation is complete and the parameters of the ODEs are known, the ATSM curve should fit the realized yields perfectly for each factor, which in this case is a 1 and 10 year bond. The known parameters are then held constant and used to unravel the ODEs for yields of bonds with different terms to maturity. For example, the six parameters estimated by

the two-factor model above could be used to construct a no-arbitrage yield curve with terms to maturity such as 3-month, 6-month, 1 year, 2 year, 5 year, and 10 year.

As stated before, the yields for the 1 and 10 year bonds will be the same as the realized yield curve. However, if there are discrepancies between the affine term structure model yield curve and the realized yield curve for some term to maturity (TTM), then there may be arbitrage opportunities that can be exploited. If the estimated ATSM curve is accurate, once the market clears, the yield curve will reach the same point as the ATSM curve. A point on the ATSM curve where the yield is higher than the realized yield curve, one could formulate an arbitrage profit by buying the asset(s) and holding it until the time where the realized yield curve moves up to the ATSM curve. Conversely, if the realized yield curve is higher than the ASTM curve it is possible to make an arbitrage profit by short selling the asset.

While this method for ascertaining yield curve arbitrage is useful, the arbitrage possibilities are given in the current period. Given the computational time for ATSMs and the speed of global markets it is possible that the arbitrage opportunities realized will disappear by the time that they would have been discovered. Therefore, the most constructive use of the Affine Term Structure Model is to forecast it forward. If a forecast of the ATSM yield curve and a forecast of the realized yield curve were available, one would be able to exploit arbitrage opportunities of the future. However while ATSM forecasts traditionally perform well in-sample, they do not perform well out-of-sample.¹ Since practitioners don't have much use for arbitrage opportunities that have passed, the out-of-sample forecast performance is the most important.

The next step of this empirical model is to forecast both yield curves, actual and ASTM, and then judge the accuracy of the forecast. If the out-of-sample performance is established to be satisfactory, then the forecast can be used to predict changes in both the ATSM curve and the expected yield curve. If one knows the direction and magnitude of change in these two curves, one can exploit future arbitrage opportunities before they are realized and/or vanish.

One of the reasons for this poor out-of-sample performance of the ATSMs may be the term structure of interest rates. There are four main theories on the term structure of interest rates, which make different arguments for the curvature of the yield curve. The four main theories of term structure usually

¹Special thanks to Dr. Egorov, while at West Virginia University, for helping with yield curve arbitrage modeling and data. After discussing yield curve arbitrage with him, I found out that Affine Term Structure Models perform well in-sample, but not out-of-sample.

considered are: expectations, liquidity premium, market segmentation, and preferred habitat theory.

Given the fact that none or all of these theories could be true during a given time period; it is rather difficult to model the term structure of interest rates. For example, in the short run the preferred habitat or market segmentation theory may be the best explanation for changes in the yield curve, while the expectations and/or liquidity premium theory may be a better explanation for long-term movements in the yield curve. Incidentally, the distinction between these theories makes it very difficult to choose a single correct model for forecasting the yield curve.

1.2.2 Basic Neural Network

There are many different types of neural networks. The optimal network type and structure are dependent on the relationship between the input variables. For example, some neural networks are used primarily for pattern recognition; others are used primarily for model estimation, and some are used for both. Neural networks are used for the purpose of modeling when the mathematical relationship between inputs and the output(s) is unknown or undefined. This is advantageous when forecasting of Affine Term Structure Models since the forward-looking relationship within the yield curve has inconsistent definitions.

One of the most famous neural networks is the backpropagation or feed-forward network. The backpropagation neural network is a multiple layer network. **Figure 1.1** graphically illustrates a basic network with one hidden layer.² The first layer is the input layer, represented by $x_{1...n}$. The number of nodes in the input layer is equal to the number of inputs the user believes is necessary to reach the desired output(s). The subsequent layer(s) are hidden layer(s), $z_{1...p}$, where the connections between different inputs are found through a recursive algorithm. There is a weight associated with each node, or unit, when connecting to a layer. The final layer of the network is the output layer, $y_{1...m}$. The nodes on the side, represented by w_{01} & v_{01} , are known as the bias. The bias is commonly identified with the intercept in regression analysis; it can be seen as a garbage collector.

The inputs $x_{1...n}$ are the exogenous variables, similar to the right hand side (RHS) variables of a traditional regression. Each input node represents a single input value. For example, it could be the first autoregressive lag for a 3-month yield during January 1995 (i.e. December 1994). If the second

² Figure 1.1, and Algorithms 1 & 2 are cited from Fausett (1993). Please refer to Chapter 6 for greater detail on other variations on backpropagation networks.

and third AR lags are included, there would be two more nodes. The target in this case would be the actual 3-month yield for January 1995. The node in the output layer represents the estimated 3-month yield, similar to the $\hat{y}$ of an OLS regression.

The variables and terminology used for the backpropagation training algorithm are defined in **Table 1.1**. The input vector x will include all independent variables at a specific point in time for each iteration, call it time r . The target vector t represents the dependent variable, at time r per iteration. The output vector represents the final estimates, $\hat{y}$. The weight vectors v & w are randomly initialized.

Algorithm 1 illustrates the training methodology for backpropagation; **Algorithm 2** illustrates the method of inputting new data once the network has been trained. The next section illustrates a time-series variation of the neural network described here.

Figure 1.1: Standard Backpropagation Network with 1 Hidden Layer

1.2.3 Neural Network Architecture

The two main types of neural networks will enter the discussion here; the nonlinear autoregressive (NAR) and nonlinear autoregressive with exogenous

Table 1.1: Variables & Definition for Backpropagation Training Algorithm, with 1 hidden layer ²

$x = [x_1, \dots, x_n]$	Inputs
$t = [t_1, \dots, t_m]$	Targets
$z = [z_1, \dots, z_p]$	Hidden Nodes
$v = [v_{01}, \dots, w_{jk}, \dots, w_{np}]$	Weights for hidden layer from input layer. Note: v_{01} is the bias for the hidden layer.
$w = [w_{01}, \dots, w_{ij}, \dots, w_{pm}]$	Weights for output layer from hidden layer. Note: w_{01} is the bias for the output layer.
δ	Error: δ_k & δ_j denotes the errors associated with the weights on output node k and hidden node j , respectively. δ represents the total error.
$y = [y_1, \dots, y_q]$	Output(s)
η	Learning rate, which is the rate at which weights can be adapted.
$f(x)$	Activation function for hidden layer
$g(x)$	Activation function for output layer. The symbol primary ' denotes the first derivative
ξ	Error tolerance, as defined by user

Algorithm 1 Training Algorithm for Backpropagation Network²

- 1: Randomly initialize weights
 - 2: **while** $\delta^2 > \xi$ **do**
 - 3: **for** $\forall x, y$ pairs **do**
 - 4: Pass x into z . Each x_i in feed into each z_j , if connected, where $i = 1 \dots n$ & $j = 1 \dots p$. Calculate each z_j as the sum of the input connected to the hidden node, then feed into the activation function.

$$z_j = f(v_{0j} + \sum_{i=1}^n x_i v_{ij})$$
For example, if Hyperbolic tangent activation function is use:

$$z_j = \frac{e^{2(v_{0j} + \sum_{i=1}^n x_i v_{ij})} - 1}{e^{2(v_{0j} + \sum_{i=1}^n x_i v_{ij})} + 1}$$
 - 5: Pass z into y . Each z_j is feed into each y_k , where $j = 1 \dots p$ and $k = 1 \dots q$. Similarly to the last step, here the hidden nodes are summed and transferred to the outputs' activation function.

$$y_k = g(w_{0k} + \sum_{j=1}^p z_j w_{jk})$$
A linear activation function is often used as the output function:

$$g(w_{0k} + \sum_{j=1}^p z_j w_{jk}) \rightarrow w_{0k} + \sum_{j=1}^p z_j w_{jk}$$

$$y_k = w_{0k} + \sum_{j=1}^p z_j w_{jk}$$
 - 6: **Error Backpropagation**
 δ_k is evaluated by the difference between y_k and t_k ; this can be written as:

$$\delta_k = (t_k - y_k)g'$$
Therefore the change in w is:

$$\Delta w_{jk} = \eta \delta_k z_j$$
For the bias term the z is not connected:

$$\Delta w_{0k} = \eta \delta_k$$
 - 7: Next, compute the Δ , change, for each v_{ij} ; this requires to rearward work. Sum the weights of the output layer for each hidden node j

$$\delta_j = \sum_{k=1}^m \delta_k w_{jk} f'$$
Change in v is:

$$\Delta v_{ij} = \eta \delta_j x_i$$
For the bias term the x is not connected:

$$\Delta v_{0j} = \eta \delta_j$$
 - 8: Apply the change to get the new weights (aka update):
Let r denote the iteration.

$$v_{ij,r} = v_{ij,r-1} + \Delta v_{ij}$$

$$w_{jk,r} = w_{jk,r-1} + \Delta w_{jk}$$
Similarly, for the bias terms:

$$v_{0j,r} = v_{0j,r-1} + \Delta v_{0j}$$

$$w_{0k,r} = w_{0k,r-1} + \Delta w_{0k}$$
 - 9: **end for**
 - 10: **end while**
-

Algorithm 2 Estimation of a Trained Backpropagation Network²

- 1: Fix the weights from the last iteration of training
 - 2: **for** $\forall x$ **do**
 - 3: $z_j = f(v_{0j} + \sum_{i=1}^n x_i v_{ij})$
 - 4: $y_k = g(w_{0k} + \sum_{j=1}^p z_j w_{jk})$
 - 5: **end for**
-

variables (NARX). The NARX network will be used for forecasting that includes macroeconomic variables, while the NAR does not.³

Figure 1.2: Example of a Open NARX Network

Figure 1.3: Example of a Close (Out-of-Sample) NARX Network

³It should be noted that all of the neural networks solved in this paper are done using the neural network toolbox in MATLAB. Additionally, the nonlinear autoregressive with exogenous variables network function is NARXNET, while the nonlinear autoregressive network is NARNET.

Figure 1.2 gives a visual representation of a NARX network. This is an 'Open Loop' network, which means the network is being trained. This is analogous to **Algorithm 1**. The $x(t)$ refers to the exogenous variables, while $y(t)$ refers to the autoregressive terms of output. That is why $y(t)$ is displayed as an input and output. The input layer uses the 1st & 2nd lags of the yield being estimated and the 1st lag of the exogenous data. Each yield for both the ASTM and realized curve will be modeled using their own network, univariately.

Figure 1.3 gives a visual of "Close Loop" form. This is an out-of-sample forecast configuration and analogous to **Algorithm 2**. Notice the feedback loop at the bottom of the diagram. This is to illustrate that the output at time t is used as an autoregressive input during at time $t + 1$.

1.3 Empirical Model and Results

The data used for modeling and the results of that data using the above methodology are presented.

1.3.1 Data

The data is split into two categories, the yield curve and macroeconomic variables. The yield curve data used for the remainder of this chapter is the same data used in Duffee (2002).⁴ This data has a long-standing history and wide acceptance, which was the main driver for using it in this study. The data from January 1952 to 1991 is from McCulloch and Kwon (1993). A subset of this dataset, before 1983, was taken from McCulloch (1990) in the Handbook of Monetary Economics. All these datas were gleaned using the methodology of McCulloch (1975). The methodology uses a cubic spline discount function that is adjusted for tax. The price for a Treasury Bill is given in **equation 1.5**; where p is the price, t is the tax rate, and $\delta(m)$ is the discount function. The discount function, displayed in **equation 1.6**, follows a cubic spline. The data used after February 1991 is from Bliss (1997), whom also follows McCulloch (1975).

$$p = [100(1 - t) + tp]\delta(m) \quad (1.5)$$

⁴This data was provided by Dr. Egorov and was downloaded from Gregory R. Duffee's, Ph.D website. Click here for link.

$$\delta(m) = 1 + \sum_k^{j=1} a_j f_j(m) \quad (1.6)$$

Each $f_j(m)$ need to satisfy $f_j(0) = 0$

The zero-coupon yield curve estimated cited by Duffee (2002) includes 6 different terms to maturity 3-month, 6-month, 1 year, 2 year, 5 year, and 10 year. Each term includes monthly yields to maturity from January 1952 to December 1998. The zero-coupon curve provides out-of-sample estimates from January 1995 to December 1998. Therefore, this period will also be set aside for analyzing out-of-sample forecast. Additionally, it should be noted that all yields are presented in nominal value, divided by 100.

Five macroeconomic variables have been chosen to assist forecasting the yield curves. These variables are Debt of Domestic (Non-financial Sector, in % change, SA), Effective Federal Fund Rate, Housing Starts (New Privately Owned, in % change, SAAR), Real Personal Consumption Expenditures (in % change, SAAR), and Civilian Unemployment Rate (in % change, SA). This data has been gathered from the FRED database at the Federal Reserve Bank of St. Louis. Both Debt of Domestic and Real Personal Consumption Expenditures have the shortest time series of the data series beginning in March 1959. For uniformity, all datas range from March 1959 to December 1994 for the in-sample estimation, or 429 observations.

1.3.2 Empirical Results

As stated above, the first step of the empirical model is to calculate a no-arbitrage yield curve defined by a two-factor model, where the 1 year and 10 year yields are used as the two factors. The structure of the two-factor and its closed form solution is the $A_0(2)Model$ defined by At-Sahalia and Kimmel (2010). **Equation 1.7** lays out the $A_0(2)Model$. The state variables, $X_{1,t}$ & $X_{2,t}$, here are represented by the 1 and 10-year terms-to-maturity.⁵

$$\begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix} = \begin{bmatrix} b_{1,1} & 0 \\ b_{2,1} & b_{2,2} \end{bmatrix} \begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix} dt + d \begin{bmatrix} W_{1,t}^p \\ W_{2,t}^p \end{bmatrix} \quad (1.7)$$

⁵At-Sahalia and Kimmel (2010) define the notation of A_M^N where N is the number of state variables, 'factors,' and M is the number of the linear combinations in the diffusion process for directed at the state variables. So in the case here, there are 2 factors with no linear combinations of the state variables in the diffusion process. Described as $dX_t = (\tilde{A} + \tilde{B}X_t)dt + \Sigma\sqrt{S(X_t; \alpha, \beta)}dW_t^Q$. Furthermore, M can also be described as rank of β . Please see At-Sahalia and Kimmel (2010) for more depth.

The Taylor series expansion presented by At-Sahalia and Kimmel (2010) for **equation 1.7** is given in **equation 1.8**. The coefficients of the Taylor series C_x^K , for $K=-1 \dots 2$ are presented beginning at **equation 1.9**. The estimates are then listed in **Table 1.2**, as well in matrix form in **equation 1.13**. Note here, the notation of 1 and 10 year TTMs have been put in place.

$$l_x^{(K)}(\Delta, x|x_0; \theta) = -\frac{m}{2} \ln(2\pi\Delta) - D_v(x; \theta) + \frac{C_x^{(-1)}(x|x_0; \theta)}{\Delta} \quad (1.8)$$

$$+ \sum_{k=0}^K C_x^{(k)}(x|x_0; \theta) \frac{\Delta^k}{k!}$$

$$\text{Where } D_v(x; \theta) \equiv \frac{1}{2} \ln(\text{Det}[v(x; \theta)]),$$

$$v(x; \theta) \equiv \sigma(x; \theta) \sigma^T(x; \theta),$$

$$l_x \equiv \ln(p_x)$$

$$c_X^{(-1)}(x|x_0; \theta) = -\frac{1}{2}(x_1 - x_{10})^2 - \frac{1}{2}(x_2 - x_{20})^2 \quad (1.9)$$

$$c_X^{(0)}(x|x_0; \theta) = -\frac{1}{2}(x_1 - x_{10})^2 b_{11} - (x_1 - x_{10})x_{10}b_{11} - \frac{1}{2}(x_1 - x_{10})(x_2 - x_{20})b_{21}$$

$$- \frac{1}{2}(x_2 - x_{20})^2 b_{22} + (x_2 - x_{20})(-x_{10}b_{21} - x_{20}b_{22}) \quad (1.10)$$

$$c_X^{(1)}(x|x_0; \theta) = \frac{1}{24}(x_1 - x_{10})^2(-4b_{11}^2 - 3b_{21}^2) - \frac{1}{3}(x_1 - x_{10})(x_2 - x_{20})b_{21}b_{22}$$

$$- \frac{1}{2}(x_2 - x_{20})b_{22}(x_{10}b_{21} + x_{20}b_{22}) + \frac{1}{2}(x_1 - x_{10})(-x_{10}(b_{11}^2 + b_{21}^2)) - x_{20}b_{21}b_{22}$$

$$+ \frac{1}{24}(x_2 - x_{20})^2(b_{21}^2 - 4b_{22}^2) + \frac{1}{2}(b_{11} - x_{10}^2b_{11}^2 + b_{22} - (x_{10}b_{21} + x_{20}b_{22})^2) \quad (1.11)$$

$$c_X^{(2)}(x|x_0; \theta) = -\frac{1}{12}((x_1 - x_{10})^2 b_{21}^2 b_{22}) + \frac{1}{12}(x_2 - x_{20})^2 b_{21}^2 b_{22}$$

$$- \frac{1}{6}(x_1 - x_{10})b_{21}b_{22}(x_{10}b_{21} + x_{20}b_{22}) + \frac{1}{6}(x_2 - x_{20})b_{21}(x_{10}(b_{11}^2 + b_{21}^2) + x_{20}b_{21}b_{22})$$

$$+ \frac{1}{12}(-2b_{11}^2 - b_{21}^2 - 2b_{22}^2) + \frac{1}{12}(x_1 - x_{10})(x_2 - x_{20})b_{21}(b_{11}^2 + b_{21}^2 - b_{22}^2) \quad (1.12)$$

Table 1.2

Sample Estimates of Coefficients	
Parameter	Value
b_{11}	2.955e-06
b_{21}	0
b_{22}	0.881

Table 1.3

Ljung-Box test for all terms
Results are at the first lag level.

Term	1-lag Q-stat	2-lags Q-stat
3-month	1.63	0.02
6-month	4.79*	0.05
1-year	6.19*	0.07
2-year	6.50*	0.09
5-year	2.78*	0.02
10-year	1.59	0.01

*Statistically Significant
at the 10% level.

$$\begin{bmatrix} X_{1,t} \\ X_{10,t} \end{bmatrix} = \begin{bmatrix} 2.955e-06 & 0 \\ 0 & 0.881 \end{bmatrix} \begin{bmatrix} X_{1,t} \\ X_{10,t} \end{bmatrix} dt + d \begin{bmatrix} W_{1,t}^p \\ W_{10,t}^p \end{bmatrix} \quad (1.13)$$

The estimated ATSM curve and the actual (zero-coupon) curve will be trained and forecasted through a system of neural networks. First, the proper lag-length of the autoregressive portion of the neural nets needs to be determined. This will be done with the traditional time-series analysis of the autocorrelation function (ACF), and the Ljung-Box Q-statistic. The ACF does not show any major persistence in any of the yields, while the ACF of the shortest term, 3-months, shows minor persistence at the 1st lag. The results of the Ljung-Box test are shown in **Table 1.3**.

For concerns of serial correlation, there is no need to go beyond the second lag. This AR(2) is displayed in regression form in Eq. 1.14. For uniformity, all yields will use two autoregressive terms.

$$y_{it} = \beta_0 + \beta_1 y_{it-1} + \beta_2 y_{it-2} \quad (1.14)$$

Table 1.4 list the variables to be used in an array of neural networks. For the NAR networks only the own lags will be included. The NARX networks will use all of the variables. The networks are configured as in **Figures 1.2 & 1.3**. The number of nodes in the hidden layer is an additional variant for a neural network. A standard practice is to use between 1.5 to 2 times the number of input nodes, variables, for the number of hidden nodes. For each yield, there will be networks with a different number of nodes in the hidden layers. An algorithm will test a set of networks with the number of hidden nodes in the range between $2 \pm \frac{1}{2}$ of the inputs. The network with the best out-of-sample performance, based on MSE, is chosen for that yield. Furthermore, the training function, how the NN is updated through each epoch, for all networks is the Levenberg-Marquardt (LM).⁶ The LM training algorithm is described in Equation 1.15.

$$x_{k+1} = x_k - [J^T J + \mu I]^{-1} J^T e \quad (1.15)$$

Note: J represents the Jacobian, the Hessian is estimated as $J^T J$, and the gradient can be defined as $J^T e$

Table 1.5 & 1.6 display the in-sample MSE for the realized (zero-coupon) and ATSM estimated curve, respectively. In **Table 1.5**, it is shown that when macroeconomic variables are included to the neural network the performance increases, except for the 1-year bond. **Table 1.6** demonstrates there are significant gains from incorporating the macroeconomic variables for the 3-month and 6-month terms. The 1-year and 10-year are not displayed in **Table 1.6**, because these were the two yields, factors, that were used to solve the ATSM curve. Therefore, they will only be forecasted in the realized sample.

In-sample mean squared error is valuable information in some respects, however, the out-of-sample performance is key for model validation. **Table**

⁶This is a traditional training algorithm for backpropagation NN's. The definition provided here is from mat (2011). The "Gradient descent with momentum and adaptive learning rate" training algorithm was also tested. However, initial testing suggested the Levenberg-Marquardt provided better results.

Table 1.4

List of Variables to be Used in NNs

Variables	Format	Seasonal Adj.
1 st lag	/100	NA
2 nd lag	/100	NA
Debt of Domestic (Non-financial Sector)	% change	SA
Effective Federal Fund Rate	/100	NA
Housing Starts: New Privately Owned	% change	SAAR
Real Personal Consumption Expenditures	% change	SA
Civilian Unemployment Rate	% change	SA

Table 1.5

In-sample MSE for Yield Curve

Term	# of Nodes	NAR	# of Nodes	NARX
	NAR	w/o Macro vars.	NARX	w Macro vars.
3-month	12	0.325	11	0.214
6-month	10	0.258	12	0.219
1-year	16	0.280	14	0.332
2-year	13	0.200	18	0.200
5-year	11	0.149	17	0.121
10-year	14	0.104	13	0.097

*MSE is at 1E-04, was multiple for readability

Note: The number of hidden nodes were chosen on the out-of-sample performance.

Table 1.6

In-sample MSE for ATSM Curve				
Term	# of Nodes	NAR	# of Nodes	NARX
	NAR	w/o Macro vars.	NAR	w Macro vars.
3-month	11	0.474	11	0.372
6-month	11	0.544	11	0.313
2-year	14	0.183	14	0.150
5-year	12	0.115	12	0.175

*MSE is at 1E-04, was multiple for readability
Note: The number of hidden nodes were chosen on the out-of-sample performance.

1.7 displays the out-of-sample MSE of each curve. The forecast here is 3-months ahead; a 3-month rolling window for 15 different periods is calculated. The MSE displayed in the tables is the mean of those 15 period MSEs.

Each curve displays better performance using only the autoregressive terms in the network for the short terms, while the use of the macroeconomic variables out-performed in the longer terms. The NAR network displays the lowest MSE for 3-month and 6-month terms for the realized yields and again for the 3-month in the ATSM curve. These results are inline with the intuition that shorter terms are more closely based on speculation while longer terms are more heavily based on the macro economy.

As a comparison, a simple random walk forecast is also displayed in **Table 1.7**. For both curves the short terms, 1-year or less, are significantly out-performed by the use of either neural network structure. The longest term, 10-year, is underperformed with the use of neural networks. The MSE, remember, is based on a rolling 3-month, during this time the YTM on a 10-year zero-coupon bond is not likely to be strongly volatile. Therefore, a random walk will perform well.

Lastly, the difference between the two forecasted curves will be examined. **Figure 1.4** exhibits both the realized and ATSM yield curves during the first 1-month forecast for the neural network using macroeconomic variables. As shown, there is a considerable gap, around 100 bases points, in the 3-month and 6-month periods of the curves. The 2-year term demonstrates a similar YTM between the two curves, while the difference reverses and grows for the 5-year bond. The greatest room for financial arbitrage here is seen in the three and six month TTMs. **Tables 1.8 & 1.9** in Appendix A list the difference between the curves for each yield and forecast time-period.

Table 1.7

Out-of-sample MSE Comparison for Yield & ATSM Curve
Based on 15 samples of a 3-month ahead forecast

Yield Curve						
Term	# of Nodes NAR	NAR w/o Macro vars.	# of Nodes NAR	NARX w Macro vars.	Term	Random Walk Actual Yields
3-month	11	0.030	11	0.031	3-month	0.265
6-month	15	0.033	12	0.034	6-month	0.281
1-year	12	0.051	14	0.049	1-year	0.308
2-year	18	0.131	18	0.102	2-year	0.139
5-year	13	0.157	17	0.116	5-year	0.105
10-year	13	0.140	13	0.121	10-year	0.085
ATSM Curve						
Term	# of Nodes NAR	NAR textbfw/o Macro vars.	# of Nodes NAR	NARX w Macro vars.	Term	Random Walk Actual Yields
3-month	11	0.043	11	0.075	3-month	0.478
6-month	11	0.050	11	0.047	6-month	0.309
2-year	14	0.085	14	0.070	2-year	0.302
5-year	12	0.130	12	0.115	5-year	0.114

*MSE is at 1E-04, was multiple for readability

Note: The number of hidden nodes were chosen on the out-of-sample performance.

Figure 1.4: First 1-Month Out-of-sample Yield Curve for Actual and ATSM

Figure 1.5: Timeseries of 1-Month Out-of-sample Yield Curve for Actual and ATSM

Figure 1.6: Out-of-sample Yield Curve for Actual and ATSM

Figure 1.5 & 1.6 cultivate the visualization of several arbitrage opportunities referenced in Appendix A. The out-of-sample 1-month and 3-month over an expanding forecast is illustrated in figures. Only five of the 15 periods shown in Appendix A have been displayed to keep the graphs legible. **Figure 1.5** conveys wider gap between the realized and ATSM curve at the 5 year TTM versus the 3-month or 6-month consistency. Furthermore, **Figure 1.6** illustrates a similar pattern.

1.4 Conclusion

To conclude here, the estimation and prediction of yield curve arbitrage opportunities are examined. The availability of arbitrage is seen by estimating an Affine Term Structure Model and comparing it to the actual yield curve. Here a two factor Vasicek Model is used to generate the ATSM curve, see Section 1.2.1.

These two curves are then forecasted through a system of neural networks, with and without the use of macroeconomic variables. A nonlinear autoregressive (NAR) and nonlinear autoregressive with exogenous variables (NARX) networks are employed for both the realized and ATSM yield curve. The out-of-sample MSE suggests that only the autoregressive variables are needed for forecasting short-term bonds, 1-year or less. For longer maturities, macroeconomic variables are advantageous for yield prediction.

As a comparison, the 3-month forecast MSE for the neural networks was compared to a random walk forecast. The neural networks outperformed the random walk significantly for TTM 1-year or less. The 10-year term random walk out-performed the NNs due to the stability of the YTM in a short-range forecast.

Finally, the predicted ATSM and actual yield curve were compared to find possible arbitrage opportunities. The three and six month terms-to-maturity displayed the greatest opportunity. The five-year also illustrated chances for arbitrage.

For future works, it would be compelling to see if other macroeconomic variables increase the predictability for TTMs beyond 1-year. Additionally, analyzing the MSE of longer forecast periods, 6-month or 1-year, would be interesting. Greater foresight will increase the space of arbitrage opportunities.

1.5 Appendix A: Additional Tables

Table 1.8

Results for NARX network, w/ Macro Variables
Difference between Forecasted Actual & ASTM Yields
for each 3-month Ahead Forecast for All Periods.
(Actual Yield - ASTM Yields) in %

Forecast Period	Month Ahead	3-month	6-month	2-year	5-year
1	1	-0.944	-0.673	0.007	0.329
1	2	-0.841	-0.612	-0.213	0.363
1	3	-0.699	-0.668	-0.288	0.269
2	1	-0.234	-0.084	0.235	-0.010
2	2	-0.120	-0.132	0.242	-0.074
2	3	0.110	-0.378	0.207	-0.499
3	1	0.298	0.182	0.034	-0.141
3	2	0.418	0.211	0.002	-0.286
3	3	0.648	0.152	-0.008	-0.502
4	1	0.120	-0.088	0.049	-0.039
4	2	0.223	-0.141	0.100	-0.016
4	3	0.313	-0.195	0.130	-0.037
5	1	0.338	0.168	-0.025	-0.316
5	2	0.537	0.126	-0.030	-0.305
5	3	0.649	0.003	-0.088	-0.279
6	1	0.017	-0.077	-0.102	-0.120
6	2	0.139	-0.105	-0.093	-0.368
6	3	0.260	-0.197	-0.175	-0.509
7	1	-0.083	-0.180	0.105	0.008
7	2	0.032	-0.182	0.098	0.110
7	3	0.200	-0.232	0.014	-0.225
8	1	-0.329	-0.178	0.047	0.336
8	2	-0.211	-0.326	-0.006	0.221
8	3	-0.121	-0.422	0.018	0.110
9	1	0.084	-0.111	-0.005	-0.275
9	2	0.133	-0.139	0.036	-0.314
9	3	0.205	-0.190	0.009	-0.200
10	1	-0.299	-0.288	0.020	0.333
10	2	-0.187	-0.297	-0.018	0.141
10	3	-0.044	-0.287	-0.055	0.151
11	1	-0.150	-0.139	0.166	0.309
11	2	0.038	-0.183	0.037	0.152
11	3	0.178	-0.172	0.057	0.135
12	1	-0.093	-0.139	0.190	0.118
12	2	0.060	-0.240	0.076	0.065
12	3	0.158	-0.276	0.097	0.222
13	1	-0.270	-0.297	0.191	0.231
13	2	-0.123	-0.234	0.156	-0.139
13	3	0.072	-0.210	0.173	-0.021
14	1	-0.124	-0.139	0.131	-0.080
14	2	0.001	-0.069	0.114	-0.348
14	3	0.025	-0.065	0.349	-0.016
15	1	-0.212	-0.204	0.083	0.013
15	2	0.062	-0.087	0.128	-0.354
15	3	0.281	-0.111	0.056	-0.297

Table 1.9

Results for NAR network, w/o Macro Variables
Difference between Forecasted Actual & ASTM Yields
for each 3-month Ahead Forecast for All Periods.
(Actual Yield - ASTM Yields) in %

Forecast Period	Month Ahead	3-month	6-month	2-year	5-year
1	1	-0.830	-0.714	0.006	0.297
1	2	-0.770	-0.648	-0.105	0.360
1	3	-0.695	-0.499	-0.200	0.410
2	1	-0.281	0.090	0.087	0.200
2	2	-0.178	0.283	0.056	0.254
2	3	-0.090	0.487	0.022	0.310
3	1	0.317	0.353	-0.067	-0.013
3	2	0.408	0.497	-0.042	0.050
3	3	0.471	0.641	-0.019	0.125
4	1	-0.002	0.123	0.066	0.071
4	2	0.005	0.230	0.100	0.142
4	3	0.016	0.338	0.133	0.220
5	1	0.182	0.158	-0.096	0.038
5	2	0.186	0.242	-0.137	0.117
5	3	0.180	0.322	-0.175	0.195
6	1	-0.020	0.010	-0.138	-0.083
6	2	-0.011	0.082	-0.180	-0.019
6	3	0.010	0.154	-0.203	0.056
7	1	-0.135	-0.036	0.209	0.181
7	2	-0.117	0.070	0.266	0.223
7	3	-0.102	0.177	0.326	0.267
8	1	-0.198	-0.034	0.215	0.163
8	2	-0.178	0.085	0.267	0.201
8	3	-0.159	0.202	0.318	0.242
9	1	-0.086	0.010	0.063	0.110
9	2	-0.085	0.095	0.040	0.194
9	3	-0.075	0.175	0.009	0.281
10	1	-0.272	-0.091	0.226	0.197
10	2	-0.243	0.015	0.282	0.260
10	3	-0.213	0.118	0.344	0.326
11	1	-0.362	0.006	0.278	0.194
11	2	-0.332	0.122	0.346	0.248
11	3	-0.295	0.232	0.412	0.306
12	1	-0.184	-0.016	0.180	0.193
12	2	-0.180	0.086	0.217	0.266
12	3	-0.166	0.187	0.270	0.345
13	1	-0.339	-0.143	0.218	0.251
13	2	-0.319	-0.033	0.226	0.341
13	3	-0.288	0.080	0.245	0.432
14	1	-0.239	-0.052	0.103	0.179
14	2	-0.216	0.046	0.064	0.257
14	3	-0.192	0.144	0.033	0.336
15	1	-0.336	0.002	0.133	0.219
15	2	-0.296	0.103	0.095	0.296
15	3	-0.259	0.200	0.056	0.373

Chapter 2

Estimating and Predicting State Employment Volatility

When forecasting on the regional level there are several dynamic relationships to consider. The prime example is the relationship between the regions in question and the parent economy. The main focus of this study is to consider the structure of the error term in state level employment growth.

The importance of volatility employment is stated in the accuracy of the estimation and prediction of regional employment. Erroneous assumptions about the volatility structure will bias estimation results and ultimately lead to unsatisfactory forecasts.

Unsatisfactory forecast errors are the main motivation for this study. One could have an excellent forecasting model that is backed by sound economic theory. However, if the assumption of white noise in the error term is violated, the forecast is doomed to produce inefficient results. It is important to have an accurate forecast, not for the forecast itself, but for the implications and policies that are based on the findings of the forecast. The user of economic forecasts must be confident in the results; otherwise employers and policy maker will to rely on the current set of information. It will be illustrated that measuring the ARCH effects have improved the forecast accuracy.

The remainder of this paper is structured as follows: Section 2.1 discusses the current state of the literature, Section illustrates the empirical analysis, Section 2.3 reports the empirical results, and Section 2.4 concludes the study.

2.1 Review of the Literature

The literature focused on here deals with forecasting state-level employment and the umbrella idea of volatility time variance and clustering.

The seminal work on measuring a variance conditional on the past is by Engle (1982). The model is coined Autoregressive Conditional Heteroscedasticity (ARCH). The author points to several applications of the model, the first being economic forecasters have notice a clustering of uncertainty over different forecast intervals. Secondly, the ARCH model could be a better approximation for real world complexity is a subject of structural breaks or omitted variables bias. The variance clustering is of particular interest to forecasters since incorrectly modeling the error distribution will lend too less accurate forecasting. This inefficiency can be corrected by ARCH since the volatility clustering of high or low variance periods will be captured by the model, including the forecast. Higher accuracy allows forecasters and users of the forecast to be more confident in the results.

On the state-level, Carlino, DeFina, and Sill (2007) examined the reduction in employment growth volatility. The data used ES202 state employment ranging from 1960 to 2002. State level volatility was measured by regressing employment growth on state and time dummies, then using the absolute value of the error term as volatility. A panel regression with a two fixed effects model that accounts for structural breaks (i.e. the 'Great Moderation'), the manufacturing to total employment share, and the deregulation of each state were used to determine the source of the volatility. The authors concluded that both state and national factors influence state volatility; however, national factors have a greater effect on volatility than state factors. The statistically significant national factors were monetary policy, oil prices, fluctuations in trade as a share of GDP, and fluctuations in inventory as a share of sales. The key state factors were the loosening of banking regulations and the level of manufacturing employment to total employment.

Blanchard and Simon (2001) studied the reduction in GDP volatility for the U.S. The study begins by examining the decline in U.S. output volatility as measured by a rolling standard deviation of twenty quarters beginning in the first quarter of 1952. The authors also looked at the rolling standard deviation of residuals from an AR(1) process, which also concluded a decline in GDP volatility. In the multivariate case, the authors measured output volatility by using a panel regression, keeping country and time fixed. The regressors in the panel regression were the rolling mean and rolling standard deviations of the inflation rate. The volatility of output was also divided into the components of GDP. The authors concluded that more efficient monetary policy played a role in the reduction of GDP volatility. Blanchard and Simon (2001) also found that developments in financial markets caused investment and consumption volatility to decline.

Morgan, Rime, and Strahan (2004) studied the effects of interstate banking integration on state growth volatility. The authors' main question was

whether allowing interstate banking softened supply or demand shocks within a state because of diversification. The dataset ranged from 1976 to 1994, except for GDP, which began in 1978. For growth variables the authors looked at employment, real personal income, and real GDP. The volatility of state growth was measured as the magnitude of divergence from the average state growth. Morgan, Rime, and Strahan (2004) used two different measures of banking integration, interstate asset ratio and other state ratio. The first ratio measured the financial assets in a state relative to a holding company that holds assets in at least two states. The second ratio measured the financial assets held by a holding company outside of the state in question compared to all the financial assets within the state. The authors found that allowing for interstate banking has decreased state volatility growth and decreased the divergence between states.

The literature surrounding the decline in employment and GDP volatility here focuses on structural breaks. There is no doubt that changes in banking regulation and monetary policy have played a role in the decline of employment and GDP volatility. However, if one was to forecast these variances without including the presence of ARCH effects, the accuracy is at-risk as stated in Engle (1982).

2.2 Empirical Analysis

2.2.1 Empirical Data

As stated previously, the analysis of this study centers around state-level employment volatility. The data series is from the Bureau of Labor Statistics (BLS) Current Employment Statistics (CES). The data is monthly non-seasonally adjusted total non-farm payroll employment for the U.S. 51 states. The time series ranges from January 1939 to December 2009, which 46 among the 51 states demonstrate. The shortest series length is Alaska, which begins in January 1960 to December 2009.

For the analysis to follow, the growth rate of employment is used in place of the levels to ensure issues of stationarity do not arise. The growth rates are calculated by taking the first difference of natural logarithm in the levels. After studying the graphical representation of the states' employment growth, the fluctuation in the growth was significantly wide for several states became apparent. Given this dynamic, the best course of action for a reliable analysis is to use the growth rate from January 1960 to December 2009 for all states, which is 599 observations per state.¹

¹The analysis below indicates the full time series for those 46 state. Wide fluctuation

2.2.2 Empirical Model

At the heart of the empirical model in this study is the structure of the error term because the variable of interest is employment volatility. Focusing solely on state-level employment growth, other factors must be accounted for as to not skew the interpretation of state employment volatility. In the regression the US employment growth rate is added to account for national factors that affect the state level economy. Eleven seasonal dummies are also included to account for seasonal fluctuations in employment. Lastly, state lags are added to ensure that serial correlation is not an issue. The statistic used to test for serial correlation is the Ljung-Box Q-statistic lack-of-fit test defined as follows:

$$\tilde{Q}(\hat{r}) = n(n+2) \sum_{k=1}^m \frac{r_k^2}{n-k} \quad (2.1)$$

where

$$r_k = \frac{\sum_{t=k+1}^n \epsilon_t \epsilon_{t-k}}{\sum_{t=1}^n \epsilon_t^2}$$

The H_o is no serial correlation in the residuals; the H_a is serial correlation in the residuals. The statistical testing is comparing the Q -statistic with the chi-square distribution; the null is rejected in the case of $Q > \chi_{m-q}^2$. Since our interest is in determining the optimal lag structure for the error term, every lag 1 through 12 inclusive, are tested. The dynamics of the autocorrelation and partial autocorrelation functions are also used to judge the possibility of serial correlation. The number of the state lag length will depend on the results of these tests.

The structure of the volatility equation is an ARCH model specification with restricted coefficients to satisfy non-negativity. These constraints are put in place to ensure the forecasts of the conditional variance will remain positive. This set of equations is estimated through a maximum likelihood estimation. The number of ARCH coefficients can range from 2 to 12. The specific ARCH structure will vary from state to state, and the criteria on which the specification is determine by an algorithm are discussed in the

pre-1960 counted as outliers rather than a separate process; therefore, it was excluded. Because the main purpose of the study is a forecast of the current period, it is reasonable to begin the series where the data follows the process found in today's economy.

next section. The general form of the model specification is as follows:

$$y_{i,t} = \beta_{i,0} + \beta_{i,1}y_{us,t-1} + \sum_{m=1}^{\leq 12} \beta_{i,m+1}y_{i,t-m} + \sum_{s=1}^{11} \beta_{i,c+s+1}dum_{s,t} + \epsilon_{i,t} \quad (2.2)$$

$$\sigma_{i,t}^2 = \omega_i + \sum_{n=1}^q \alpha_{i,n}\epsilon_{i,t-n}^2 \quad (2.3)$$

$$\omega > 0$$

$$\alpha_n \geq 0 \forall n, \sum_{n=1}^q \alpha_{i,n} < 1$$

Where $i = i^{th}$ State, $q = 1 \dots 12$, $\{\epsilon_{i,t} | \mathbb{I}_{i,t-1}\} \sim \mathcal{N}(0, \sigma_{i,t}^2)$

Lagrange multiplier test as defined by Engle (1982) is used to judge the statistical significance of the ARCH effect.

It should be addressed at this point the choice for this particular ARCH model. It is reasonable to place the restrictions on coefficients in the variance to provide a reasonable and stable forecast. However, there are other, more advanced, configurations of the G/ARCH such as the threshold-GARCH or exponential GARCH. Enders (2010) correctly coins the issue confronted by the use of the TGARCH as the leverage effect. The effect being that moments in variance are more affected when the residual is negative than when it is positive. This is a case often seen with the pricing of securities. However, it is not believe to be the case for the state level employment growth discussed here. Furthermore, exponential-GARCH (EGARCH) has the agreeable feature that the volatility can never go negative. The EGARCH, similarly to TGARCH, has the ability to incorporate leverage effects. Finally, the variance equations for EGARCH are in the log-linear form. Given that the forecast and interpretation of the variance equation is key to this study and leverage effects are not needed, equation 2.3 is a reasonable choice for modeling.

Once the β 's and α 's have been estimated and issues of statistical significance have been addressed, the inquiry continues into the usefulness of the predictability in the error structure to decrease the mean squared error (MSE) of the forecast. In other words, does having a better understanding of the error structure lower forecast errors? In theory, the answer is unequivocally yes; however, the analysis will tell us if the empirics match the theory.

2.2.3 ARCH Length Specification Algorithm

As stated above, the degree of the ARCH structure is allowed to change across states. The addition of this feature allows for more flexibility in each state. There is no reason to assume that the employment growth for each individual state will follow the same ARCH process. However, the criteria and process for which the ARCH structure is chosen for each state should be the same.

A flow chart of the algorithm charged with finding the ARCH process for each state is displayed below. The primary criterion of this search algorithm is the Akaike Information Criterion (AIC). There are many decisions and processes employed throughout the algorithm; however, the basic concept is relatively simple.

The starting ARCH length is 4 lags, then $-1/+1$ lags are estimated (3 and 5 lags). While 4 is a somewhat arbitrary choice it allows the algorithm to move in either direction, increasing the flexibility of the algorithm. There are two restrictions placed on the number of ARCH coefficients, a minimum of 1 lags and maximum of 12. The minimum of 1 is to ensure that ARCH effects are still captured and the maximum of 12 is to ensure overlapping effects from year to year do not occur.

Once the $4 -1/+1$ lags have been estimated and their respective AIC have been calculated, estimates will continue along the path of lowest AIC. For example, assume the ARCH structure of a particular state follows the characteristic $AIC_{ARCH(3)} > AIC_{ARCH(4)} > AIC_{ARCH(5)}$. Since the autoregressive conditional heteroskedasticity model with 5 lags produces the lowest AIC value, a model with 6 lags will be estimated and so on will be calculated as long as the AIC decreases. Once the AIC of a model increases from the last lag, this portion of the algorithm is terminated and the previous lag length is used. The next portion of the algorithm calculates the significance level of the last lag in the length divulged in the previous portion and test to see if it is significant at the 10% level. If the lag is significant then this lag length is used as the optimal ARCH structure. If the lag is deemed insignificant then the algorithm cuts that lag off and will continue to do so until the last lag in the sequence is significant at the 10% level. When a lag is insignificant and the minus one process has not already been estimated from the previous step in the algorithm then it will be estimated. There are several different decisions that must be managed when working through this procedure, but these are the basic concepts the algorithm faces.

One could compose an argument for simply calculating all of the possible different ARCH structures and then comparing them. However, the computational time for this endeavor is massive and it is not backed by sound

NOTE: In the decision tree the number refers to the AIC value of the associated ARCH lag length. For Example, 3<4 means of AIC for an ARCH(3) model is strictly less than the AIC for an ARCH(4) model. The statistical significance level is denoted as α . The last decision states if the last lag of the Optimal ARCH is still significance at the 10% level, if it is not then the last lag is removed until it is significance. v is the logic operator that denotes or. & is the logic operator that denotes and.

Figure 2.1: Flow Chart of the ARCH Length Specification Algorithm

NOTE: In the decision tree the number refers to the AIC value of the associated ARCH lag length. For Example, $3 < 4$ mean of AIC for an ARCH(3) model is strictly less than the AIC for an ARCH(4) model. $\vee$ is the logic operator that denotes or. $\wedge$ is the logic operator that denotes and.

Figure 2.2: Sub-routine for equals decision in ARCH algorithm flow chart

theory. These are the main issues this algorithm aims to address. Now that the theory behind the empirical model has been discussed, the results will be presented.

2.3 Results of Empirics

Equation 2.2 was estimated for each state and tested for serial correlation. The evidence of correlation between the errors is mixed when studying the progression of the autocorrelation and partial autocorrelation functions. There is an emphasis placed on the first few lags and primarily the mid-year lags when studying the correlation. The Ljung-Box test does find evidence of serial correlation in the first 12 lags. This evidence is sufficiently reduced when an autoregressive model of order 6 is used for Equation 2.2. Therefore, c in equation 2.2 is equal to 6 for the regression model, yielding:

$$y_{i,t} = \beta_{i,0} + \beta_{i,1}y_{us,t-1} + \sum_{m=1}^6 \beta_{i,m+1}y_{i,t-m} + \sum_{s=1}^{11} \beta_{i,c+s+1}dum_{s,t} + \epsilon_{i,t} \quad (2.4)$$

In addition to serial correlation, the stationary of employment growth has also been tested for. **Table 2.1** displays the results of Phillips-Perron test for unit root for each state and the US. The results are using the growth rate to be inline with rest of the analysis. Each state and the United States rejected the null hypothesis of unit root. The conclusions are unchanged using Newey-West at 0 or 1 lag. Therefore, concerns of the unit root in the mean regression are no longer an issue.

With corrections made to account for serial correlation, forward progression leads us to the feasibility of ARCH effects in state employment growth. To begin, Engle (1982) Lagrange multiplier is tested for each state at a 12 lag-length and the results are displayed in **Table 2.2** below. If the ARCH test displays the presence of ARCH effects at the 12 lag-length, the specification algorithm will refine the model for each state and Engle's ARCH test will be re-implemented at each state specific length.

The results from **Table 2.2** are promising, 47 out of 51 states are statistically significant at the 10% level for ARCH effects. Additionally, the significant 47 states were also statistically significant at the 1% level. Given the strong evidence for ARCH effects at the 12 lag-length, the ARCH Length Specification Algorithm is implemented. **Table 2.3** displays the optimal ARCH length chosen by the algorithm and the associated Engle (1982) Lagrange multiplier test for that lag.

Table 2.1

Phillips-Perron test for Unit Root					
State	NW 0-lag	NW 1-lag	State	NW 0-lag	NW 1-lag
	PP-statistic	PP-statistic		PP-statistic	PP-statistic
Alabama	-22.68*	-22.68*	Montana	-15.89*	-15.81*
Alaska	-10.16*	-10.66*	Nebraska	-19.84*	-19.91*
Arizona	-20.12*	-20.15*	Nevada	-15.05*	-14.97*
Arkansas	-20.51*	-20.53*	New Hampshire	-18.22*	-18.16*
California	-25.31*	-25.31*	New Jersey	-20.75*	-20.78*
Colorado	-24.44*	-24.44*	New Mexico	-23.28*	-23.28*
Connecticut	-24.27*	-24.27*	New York	-24.80*	-24.80*
Delaware	-24.00*	-24.00*	North Carolina	-23.55*	-23.55*
Washington D.C.	-25.54*	-25.55*	North Dakota	-16.32*	-16.53*
Florida	-18.75*	-18.74*	Ohio	-21.63*	-21.65*
Georgia	-23.68*	-23.68*	Oklahoma	-22.93*	-22.94*
Hawaii	-23.24*	-23.24*	Oregon	-21.29*	-21.28*
Idaho	-17.24*	-17.23*	Pennsylvania	-21.71*	-21.73*
Illinois	-22.01*	-22.02*	Rhode Island	-23.92*	-23.92*
Indiana	-20.88*	-20.92*	South Carolina	-23.75*	-23.75*
Iowa	-20.82*	-20.87*	South Dakota	-17.37*	-17.35*
Kansas	-23.81*	-23.81*	Tennessee	-22.28*	-22.29*
Kentucky	-21.80*	-21.82*	Texas	-23.36*	-23.36*
Louisiana	-23.25*	-23.25*	Utah	-22.46*	-22.46*
Maine	-18.62*	-18.50*	Vermont	-23.02*	-23.02*
Maryland	-23.19*	-23.20*	Virginia	-21.70*	-21.72*
Massachusetts	-24.03*	-24.03*	Washington	-21.63*	-21.63*
Michigan	-23.43*	-23.44*	West Virginia	-21.44*	-21.46*
Minnesota	-18.03*	-18.08*	Wisconsin	-19.71*	-19.72*
Mississippi	-21.68*	-21.69*	Wyoming	-13.87*	-13.89*
Missouri	-21.86*	-21.89*	US	-22.35*	-22.36*

*Statistically Significant at the 10% level.

As **Table 2.3** exhibits the length of the ARCH process ranges widely from state to state, from Maine with 1 lag to Alaska with 9 lags. One of the most notable observations from **Table 2.3** is that New Jersey is not included; this is because the model of New Jersey did not converge. Given no-convergence for New Jersey, the state will be excluded from the remainder of the analysis. There are 2 states that chose the minimum lag length of 1, and Alaska is the only state with 9 lags. Hence, none of the states reached the maximum constraint of 12 lags. The second lengthiest ARCH process is Utah with 8 lags. Additionally, 16 and 7 states have lag lengths of 3 and 2, respectively. Meaning 25 states have a volatility structure of lag length less than or equal to 3, or 50%.

The District of Columbia, Idaho, Nebraska, and New Mexico became insignificant at the 10% level when the ARCH process was allowed to deviate from a lag length of 12. The higher ARCH-statistic at the 12-lag length for these states versus their respective chosen lags from the algorithm is likely do to residual seasonal overlaps from year to year. Louisiana, Maryland, and Virginia remain statistically insignificant at the 10% level after the ARCH process was allowed to fluctuate. However, it is interesting to note these states chose lag lengths relatively small compared to 12. Louisiana and Maryland chose 3 lags and Virginia chose 5 lags. Tennessee is significant

Table 2.2

ARCH LM test ARCH-statistics for all 51 States			
State	12-Lags ARCH-statistic	State	12-Lags ARCH-statistic
Alabama	42.76*	Montana	67.91*
Alaska	105.75*	Nebraska	35.87*
Arizona	34.59*	Nevada	111.94*
Arkansas	101.36*	New Hampshire	323.47*
California	63.01*	New Jersey	50.75*
Colorado	49.88*	New Mexico	97.11*
Connecticut	53.13*	New York	71.04*
Delaware	66.73*	North Carolina	67.87*
Washington D.C.	122.50*	North Dakota	142.47*
Florida	100.46*	Ohio	54.61*
Georgia	51.95*	Oklahoma	97.28*
Hawaii	204.56*	Oregon	160.96*
Idaho	63.64*	Pennsylvania	37.06*
Illinois	41.42*	Rhode Island	43.45*
Indiana	43.64*	South Carolina	43.57*
Iowa	35.49*	South Dakota	52.34*
Kansas	118.86*	Tennessee	15.21
Kentucky	37.67*	Texas	74.03*
Louisiana	4.42	Utah	83.67*
Maine	80.82*	Vermont	256.86*
Maryland	4.74	Virginia	9.35
Massachusetts	72.99*	Washington	103.27*
Michigan	83.76*	West Virginia	74.28*
Minnesota	37.77*	Wisconsin	55.05*
Mississippi	34.73*	Wyoming	80.26*
Missouri	55.61*		

*Statistically Significant at the 10% level.

Table 2.3

Optimal ARCH length as defined by the algorithm and its associated ARCH test statistic					
State	Optimal-Lags	ARCH-Statistic	State	Optimal-Lags	ARCH-Statistic
Alabama	4	22.35*	Missouri	3	5.82
Alaska	9	74.79*	Montana	3	22.60*
Arizona	3	6.81*	Nebraska	4	7.21
Arkansas	6	44.63*	Nevada	5	51.30*
California	6	20.44*	New Hampshire	4	57.14*
Colorado	7	25.66*	New Mexico	5	7.57
Connecticut	3	8.85*	New York	2	13.28*
Delaware	5	39.56*	North Carolina	3	24.02*
District of Columbia	4	3.62	North Dakota	6	24.41*
Florida	5	38.91*	Ohio	6	48.14*
Georgia	3	38.59*	Oklahoma	3	23.39*
Hawaii	2	10.92*	Oregon	5	37.05*
Idaho	2	3.04	Pennsylvania	3	14.03*
Illinois	4	23.04*	Rhode Island	1	14.43*
Indiana	4	24.46*	South Carolina	3	15.15*
Iowa	6	27.69*	South Dakota	3	10.75*
Kansas	2	111.29*	Tennessee	2	8.04*
Kentucky	5	22.89*	Texas	6	30.79*
Louisiana	3	4.36	Utah	8	72.83*
Maine	1	25.90*	Vermont	2	29.90*
Maryland	3	3.32	Virginia	5	7.81
Massachusetts	3	19.28*	Washington	6	51.06*
Michigan	2	68.09*	West Virginia	3	25.36*
Minnesota	3	17.05*	Wisconsin	7	56.75*
Mississippi	3	10.53*	Wyoming	4	25.87*

*Statistically Significant at the 10% level.

with 2 lags compare to insignificant at 12.

Table 2.4 displays the results of the volatility regression, Equation 2.3, for each state. As illustrated by **Table 2.4** the constraints set on the volatility equation have been satisfied, $\omega > 0$ and $\alpha_n \geq 0$. The ω for each state has been scaled by 1E04, which mean's ω is very small for each state. Nevertheless, they are greater than zero for each state and statistically significant at the 10% level. The last lag in the ARCH process for each state must be significant as part of the conditions set by the algorithm. The only exception to this rule is Michigan; the 1st ARCH term has a coefficient very close to one. This suggest this second term is in place to satisfy the constraint place on equation 2.3, that sum of α must be less then 1. While it is impossible for states with other lag lengths to have the final lag insignificant, it is possible for intermediate lag(s) to be insignificant. The 2nd is the lag that is most often insignificant, 21 out of the 50 states found this lag to be insignificant. However, this results are somewhat skewed given that every state must include this lag. Case in point, Alaska has insignificant lags at 2,5,6, and 7. In this case, Alaska's volatility model may be improved by ending at the 4th lag, this could be discussed in future research endeavors.

Table 2.11 in the Appendix displays the results from the state employment growth regression, Equation 2.2. The constant term on this regression is insignificant for 6 states. The first lag of the United States employment growth is significant at the 10% level for 47 out of the 50 states. Every regression coefficient of United States employment growth displayed a positive and correct signs, meaning US employment growth aids in a individual state's employment growth. The insignificant states include the District of Columbia, Washington, and Wyoming. The significance level of the state employment lags are mixed. The 5th lag is the most often to be insignificant of any lag. The first and second quarter lags (the 3th and 6th) are most often to be significant; 32 out of the 50 states have these lags significant at the 10% level. While the contribution of each individual lag on the current period varies, the sum of the lagged coefficients does not exceed 1 or -1. Additionally, no single lagged coefficients exceed 1 or -1, ensuring the issue of stationarity does not arise. The remaining β 's of these regressions are the seasonal dummies. The most noteworthy trend of this set of β 's is that the final dummy was statistically significant at the 10% for all 50 states, suggesting that annual effects play a strong role in the current employment growth rate.

At this juncture the next question to be answered is whether the knowledge of ARCH effects being present allows for greater predictability in volatility. In judging the relevant predictability of the ARCH effects, several forecasts are implemented for comparability. First we examine performance of a

Table 2.4

Equation 2.3 estimates, the volatility equation, displayed for AL-MS										
State	$\omega^\dagger$	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9
Alabama	0.074 (0.012)*	0.044 (0.039)	0.164 (0.047)*	0.330 (0.083)*	0.150 (0.052)*					
Alaska	0.088 (0.032)*	0.255 (0.062)*	0.003 (0.029)	0.174 (0.048)*	0.203 (0.059)*	0.000 (0.000)	0.000 (0.042)	0.055 (0.048)	0.166 (0.060)*	0.144 (0.046)*
Arizona	0.135 (0.017)*	0.138 (0.045)*	0.203 (0.057)*	0.261 (0.078)*						
Arkansas	0.056 (0.015)*	0.111 (0.035)*	0.039 (0.032)	0.503 (0.088)*	0.054 (0.038)	0.209 (0.045)*	0.084 (0.044)*			
California	0.042 (0.009)*	0.216 (0.077)*	0.051 (0.052)	0.074 (0.060)	0.149 (0.055)*	0.043 (0.054)	0.200 (0.078)*			
Colorado	0.033 (0.009)*	0.300 (0.073)*	0.000 (0.038)	0.068 (0.044)	0.050 (0.049)	0.156 (0.062)*	0.259 (0.071)*	0.111 (0.059)*		
Connecticut	0.105 (0.009)*	0.089 (0.052)*	0.000 (0.046)	0.267 (0.076)*						
Delaware	0.209 (0.031)*	0.233 (0.069)*	0.108 (0.045)*	0.030 (0.036)	0.191 (0.059)*	0.145 (0.057)*				
District of Columbia	0.409 (0.034)*	0.083 (0.054)	0.000 (0.043)	0.000 (0.031)	0.058 (0.025)*					
Florida	0.070 (0.018)*	0.415 (0.093)*	0.111 (0.061)*	0.062 (0.055)	0.000 (0.046)	0.280 (0.068)*				
Georgia	0.107 (0.008)*	0.142 (0.037)*	0.000 (0.038)	0.178 (0.057)*						
Hawaii	0.301 (0.033)*	0.200 (0.054)*	0.616 (0.093)*							
Idaho	0.334 (0.027)*	0.151 (0.058)*	0.159 (0.060)*							
Illinois	0.072 (0.009)*	0.219 (0.057)*	0.122 (0.048)*	0.100 (0.045)*	0.163 (0.047)*					
Indiana	0.189 (0.018)*	0.015 (0.042)	0.160 (0.056)*	0.000 (0.039)	0.102 (0.047)*					
Iowa	0.049 (0.010)*	0.112 (0.059)*	0.091 (0.060)	0.286 (0.058)*	0.230 (0.062)*	0.022 (0.050)	0.169 (0.071)*			
Kansas	0.171 (0.014)*	0.132 (0.053)*	0.369 (0.062)*							
Kentucky	0.063 (0.016)*	0.130 (0.049)*	0.110 (0.024)*	0.190 (0.051)*	0.098 (0.045)*	0.471 (0.090)*				
Louisiana	0.121 (0.016)*	0.081 (0.045)*	0.021 (0.036)	0.898 (0.125)*						
Maine	0.343 (0.020)*	0.132 (0.040)*								
Maryland	0.101 (0.011)*	0.030 (0.024)	0.068 (0.039)*	0.583 (0.105)*						
Massachusetts	0.089 (0.010)*	0.209 (0.060)*	0.063 (0.039)	0.177 (0.046)*						
Michigan	0.208 (0.016)*	0.999 (0.123)*	0.000 (0.027)							
Minnesota	0.074 (0.008)*	0.084 (0.026)*	0.071 (0.042)*	0.587 (0.104)*						
Mississippi	0.170 (0.018)*	0.142 (0.054)*	0.044 (0.037)	0.169 (0.051)*						

*Statistically Significant at the 10% level.

[†] The estimates of ω are scaled by 1E04

Table 2.4 Continued

Equation 2.3 estimates, the volatility equation, displayed for MO-WY										
State	$\omega^\dagger$	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9
Missouri	0.136 (0.016)*	0.062 (0.052)	0.104 (0.038)*	0.204 (0.065)*						
Montana	0.452 (0.045)*	0.006 (0.039)	0.115 (0.040)*	0.170 (0.073)*						
Nebraska	0.136 (0.014)*	0.000 (0.039)	0.000 (0.033)	0.074 (0.043)*	0.185 (0.058)*					
Nevada	0.140 (0.022)*	0.104 (0.045)*	0.255 (0.060)*	0.102 (0.054)*	0.119 (0.053)*	0.243 (0.062)*				
New Hampshire	0.099 (0.020)*	0.119 (0.044)*	0.015 (0.022)	0.814 (0.109)*	0.052 (0.028)*					
New Mexico	0.128 (0.020)*	0.094 (0.055)*	0.227 (0.062)*	0.090 (0.056)	0.000 (0.042)	0.143 (0.051)*				
New York	0.088 (0.007)*	0.302 (0.077)*	0.066 (0.031)*							
North Carolina	0.110 (0.012)*	0.141 (0.057)*	0.077 (0.059)	0.182 (0.069)*						
North Dakota	0.201 (0.023)*	0.301 (0.080)*	0.000 (0.051)	0.054 (0.036)	0.000 (0.039)	0.081 (0.035)*	0.063 (0.031)*			
Ohio	0.051 (0.010)*	0.135 (0.051)*	0.112 (0.041)*	0.000 (0.046)	0.040 (0.043)	0.040 (0.041)	0.403 (0.081)*			
Oklahoma	0.154 (0.014)*	0.082 (0.047)*	0.113 (0.039)*	0.256 (0.055)*						
Oregon	0.078 (0.018)*	0.121 (0.057)*	0.150 (0.052)*	0.000 (0.034)	0.634 (0.101)*	0.095 (0.047)*				
Pennsylvania	0.085 (0.009)*	0.131 (0.043)*	0.000 (0.034)	0.254 (0.070)*						
Rhode Island	0.253 (0.018)*	0.259 (0.056)*								
South Carolina	0.162 (0.018)*	0.157 (0.057)*	0.089 (0.042)*	0.103 (0.057)*						
South Dakota	0.134 (0.023)*	0.148 (0.052)*	0.187 (0.058)*	0.304 (0.078)*						
Tennessee	0.128 (0.012)*	0.078 (0.042)*	0.242 (0.051)*							
Texas	0.033 (0.006)*	0.136 (0.045)*	0.003 (0.048)	0.268 (0.076)*	0.007 (0.040)	0.160 (0.061)*	0.236 (0.075)*			
Utah	0.031 (0.010)*	0.018 (0.033)	0.152 (0.045)*	0.105 (0.042)*	0.063 (0.045)	0.054 (0.052)	0.262 (0.073)*	0.074 (0.045)*	0.192 (0.064)*	
Vermont	0.209 (0.032)*	0.671 (0.096)*	0.329 (0.074)*							
Virginia	0.085 (0.011)*	0.044 (0.037)	0.248 (0.061)*	0.040 (0.037)	0.000 (0.021)	0.236 (0.072)*				
Washington	0.082 (0.018)*	0.351 (0.086)*	0.151 (0.060)*	0.041 (0.043)	0.005 (0.039)	0.010 (0.027)	0.234 (0.062)*			
West Virginia	0.248 (0.032)*	0.107 (0.052)*	0.052 (0.020)*	0.841 (0.088)*						
Wisconsin	0.036 (0.009)*	0.145 (0.059)*	0.051 (0.051)	0.272 (0.075)*	0.038 (0.049)	0.161 (0.060)*	0.096 (0.054)*	0.072 (0.043)*		
Wyoming	0.442 (0.056)*	0.250 (0.064)*	0.331 (0.083)*	0.062 (0.036)*	0.079 (0.031)*					

*Statistically Significant at the 10% level.

$\dagger$ The estimates of ω are scaled by 1E04

1-month, 3-month, 6-month and 12-month (or 1-year) ahead forecasts.

These are out-of-sample; forecast and the initial regressions are calculated using the data through November 2008. Given that 6 own lags are needed for the employment growth rate regression, the data begins in June of 1960. This leaves 581 obs. per state for the first estimates of the state employment growth regression and the volatility equation. The estimated coefficients of these equations are held constant and forecast estimates are produced until the end of the data series is reached. As an example, in the estimation of this first volatility equation, a 1-month, 2-month, 3-month, and so on through 12-month ahead forecast will be produced. Through this forecast structure in total 12 1-month, 10 3-month, 7 6-month, and 1 12-month ahead forecast will be produced for each state. This means the 1-Months MSE will have higher robustness than the other forecast because of the higher number of the forecast observations. **Table 2.5** through **Table 2.10** display the resulting mean square errors (MSE) of these forecasts. The first displayed is the MSE adjusted for the number of forecast, Equation 2.5. The mean square errors have been scaled by 1E10 to make them user friendly. Also displayed, is the forecast error in terms of percentage, Equation 2.6.

$$MSE = \frac{\sum_{k=1}^n (\sigma_{i,t+k}^2 - \hat{\sigma}_{i,t+k}^2)^2}{n} \quad (2.5)$$

$$Error\% = \left(\frac{\sigma_{i,t+k}^2 - \hat{\sigma}_{i,t+k}^2}{\sigma_{i,t+k}^2} \right) \frac{100}{n} \quad (2.6)$$

Table 2.5 & 2.6 displays the results from forecast models based on the ARCH length specification algorithm.

As a comparison model, **Tables 2.7 & 2.8** display the results from forecast using a GARCH(1,1). There are many choices in the selection of a comparison model; the GARCH(1,1) is chosen for a few strategic reason. Enders (2010) refers to Bollerslev (1986) seminal paper as a juxtaposition between an ARCH model, a traditional AR model, and this new GARCH. Bollerslev (1986) position the GARCH model as a natural extend to the ARCH, it allows for the past conditional variance(s) to effect the current. The author further states the GARCH(1,1) is a simple but effective model.

This is demonstrated by an empirical example comparing the works of Engle and Karft (1983), an analysis of an ARCH framework on the growth rate of U.S. implicit GNP deflator from 1948Q2 to 1980Q3. Bollerslev (1986) changes the ARCH(8) specification suggested by Engle and Karft (1983) to GARCH formulation. The GARCH(1,2)/GARCH(2,1) forms are insignificant according to the LM test. The GARCH(1,1) is used as the comparison

model. The author demonstrates the GARCH(1,1) provides a better model fit and lag structure than the ARCH(8).

For the specification of GARCH(1,1), the focus here is on the presence of autoregressive conditional heteroskedastic models and their use in predicting state level employment growth. The addition of 1 generalized autoregressive and restricting the autoregressive term to 1, demonstrates the persistence of autoregressive error. Similar to an ARMA process for a mean equation fitted while with a long series of AR terms, the addition of a MA term can help reduce the persistence autoregressive terms. Therefore, the use of GARCH(1,1) comparison can demonstrate value of the generalized volatility for those states with high ARCH lag length. For instance, if a comparison of a GARCH(1,0) was used it would not be readily apparent that the benefit of the addition of the generalized moment to the autoregressive in the difference of the forecast error.

In this model, the state employment growth regression is the same, however, the volatility equation is specified as follows:

$$\sigma_{i,t}^2 = \omega_i + \vartheta_{i,1}\sigma_{i,t-1}^2 + \alpha_{i,1}\epsilon_{i,t-1}^2 \quad (2.7)$$

Lastly, **Tables 2.9 & 2.10** displays results from a forecast comparison using a white noise process. The advocacy of this theory stems from the phenomenon that a perfectly theoretically sound regression produces unbiased and efficient estimators, which means the error term should follow a white noise process. To test this hypothesis, an error term is generated using a random number generator with mean 0 and a standard deviation equal to the state employment regression residual standard deviation.

$$\hat{\epsilon}_{581+q} \sim \mathcal{N}(0, \sigma_{\epsilon_{1...580+q}}^2) \quad (2.8)$$

Tables 2.5 & 2.6 illustrates the forecast based on ARCH length specification algorithm performed well overall. The mean of the 6-month ahead forecast is slightly higher than the mean of the 12-month. However, this is due solely to the outlying error of Nevada. The state average of a 6-month ahead forecast drops to -8.35%, out-performing the 12-month forecast, when Nevada is excluded from the mean. It is noteworthy to point out that all of the state means in **Table 2.5** are negative, indicating this model has a trend to under-predict the volatility of state employment growth. Also displayed in **Table 2.6** is the mean of the absolute percentage change, ABS Mean. At each step-ahead forecast the absolute percentage change is greater than the percentage change, suggesting there is some cancellation between positive and negative changes. The 12-step ahead showed the largest difference between these two values, jumping to 19.56% for the absolute percentage

Table 2.5: [†]

1-Month, 3-Month, 6-Month, & 12-Month Ahead Forecast MSE for the Algorithm Specified ARCH Model									
State	1-Month	3-Month	6-Month	12-Month	State	1-Month	3-Month	6-Month	12-Month
Alabama	0.0122	0.0087	0.0202	0.0145	Missouri	0.0010	0.0014	0.0012	0.0015
Alaska	0.1256	1.1162	3.1887	0.0267	Montana	0.0768	0.0528	0.2991	0.3712
Arizona	0.0507	0.0292	0.1348	0.1854	Nebraska	0.0007	0.0012	0.0163	0.0000
Arkansas	0.1010	0.3856	0.6218	0.0005	Nevada	0.1417	0.8998	36.1431	1.2489
California	0.0045	0.0088	0.0696	0.1672	New Hampshire	0.0171	0.0293	0.4384	0.1206
Colorado	0.0141	0.0291	0.1164	0.5145	New Mexico	0.0126	0.0373	0.2935	0.0032
Connecticut	0.0002	0.0021	0.0143	0.0154	New York	0.0049	0.0325	0.1666	0.0022
Delaware	0.2036	0.3058	0.3808	0.3178	North Carolina	0.0008	0.0087	0.1078	0.0560
District of Columbia	0.0014	0.0228	0.1528	0.0337	North Dakota	0.0193	0.2149	0.5984	0.0754
Florida	0.0273	2.5413	1.8000	1.3656	Ohio	0.0055	0.0179	0.0614	0.1125
Georgia	0.0021	0.0071	0.0226	0.0586	Oklahoma	0.0009	0.0043	0.0165	0.0057
Hawaii	0.0394	0.2896	1.7139	6.2214	Oregon	1.0214	1.4549	1.8963	10.4299
Idaho	0.0359	0.2454	0.5589	0.0176	Pennsylvania	0.0002	0.0005	0.0024	0.0048
Illinois	0.0012	0.0040	0.0180	0.1073	Rhode Island	0.0020	0.1146	0.2752	0.2584
Indiana	0.0008	0.0012	0.0653	0.0604	South Carolina	0.0005	0.0027	0.0089	0.0247
Iowa	0.0363	0.0490	0.0404	0.2409	South Dakota	0.0046	0.0100	0.0663	0.0014
Kansas	0.0072	0.0495	0.2573	0.4724	Tennessee	0.0010	0.0073	0.0472	0.0004
Kentucky	0.1128	0.0998	0.2499	0.9221	Texas	0.0056	0.0056	0.0230	0.0331
Louisiana	9.0004	0.1662	2.8956	17.0499	Utah	0.0045	0.0123	0.2674	1.0959
Maine	0.0011	0.0035	0.0098	0.0002	Vermont	0.0698	0.0962	1.0592	0.1506
Maryland	0.0043	0.0027	0.0987	0.0071	Virginia	0.4279	0.5304	0.3470	0.0098
Massachusetts	0.0046	0.0040	0.0094	0.0023	Washington	0.0172	0.0716	0.4802	0.8588
Michigan	0.2825	2.1449	0.9658	9.2416	West Virginia	0.0240	0.0809	5.6357	0.0054
Minnesota	0.0010	0.0013	0.0137	0.6648	Wisconsin	0.0048	0.0135	0.0186	0.2461
Mississippi	0.0063	0.0213	0.0214	0.0504	Wyoming	0.0648	3.5373	28.0114	8.9208
Mean MSE	0.2401	0.2955	1.7944	1.2359					

[†]All MSEs in this table have been scaled by 1E10
Hence, there is 11 one-step ahead forecast instead of 12 and 9 three-step ahead instead of 10.

change. However, the 12-step ahead MSE is only based on one forecast obs., which makes it less robust than the other steps. The smallest forecast MSE is demonstrated by the 12-month forecast for Nebraska, followed by the 1-month forecast for Connecticut. The largest forecast MSE was displayed by the 6-month forecast for Nevada.

The results of the GARCH(1,1) also performed very well. In fact, the average state forecast MSE and percent error out-performed the forecast based on the ARCH length specification algorithm for each step. This evidence suggests that using the first lag of the volatility itself for estimating the current volatility is a very powerful tool. The mean of the forecast MSE once again follows the correct progression of increasing over the forecast horizon, except for the 12-month ahead has a lower MSE than the 3-month and 6-month.

As **Tables 2.7 & 2.8** shows, the mean of the forecast percent error does increase over the forecast horizon. The absolute percentage change in the forecast error was once again higher on all counts, suggesting some cancellation between the errors. The forecast error absolute percentage change is still lower for the GARCH (1,1) then for the algorithm based model. The lowest forecast MSE is held by the 12-month ahead forecast for Louisiana, followed closely by the 1-step ahead forecast for Minnesota. The highest MSE for the

Table 2.6

1-Month, 3-Month, 6-Month, & 12-Month Ahead Percent Forecast Error for the Algorithm Specified ARCH Model									
State	1-Month	3-Month	6-Month	12-Month	State	1-Month	3-Month	6-Month	12-Month
Alabama	-0.65	-2.19	-4.30	10.15	Missouri	-1.54	-2.07	-0.22	-2.56
Alaska	-2.53	-10.34	-23.28	2.31	Montana	2.75	1.18	-3.67	9.23
Arizona	-1.42	-3.06	-7.58	-14.65	Nebraska	0.11	0.34	-1.90	-0.27
Arkansas	-8.36	-15.79	-30.16	-1.59	Nevada	-6.94	-16.63	-110.27	-21.32
California	-2.62	-5.51	-19.89	-37.70	New Hampshire	-2.69	-4.79	0.11	24.56
Colorado	-6.66	-8.56	-18.54	-73.66	New Mexico	-1.82	-4.37	-13.07	3.22
Connecticut	-0.55	-2.45	-6.52	-7.39	New York	-0.37	-3.22	-5.58	-5.06
Delaware	-5.55	-5.30	-6.83	-22.85	North Carolina	-0.30	-2.88	-12.33	-14.12
District of Columbia	-0.21	-1.22	-3.75	-4.34	North Dakota	0.09	-1.33	-0.87	-7.04
Florida	-0.08	-3.14	1.85	14.22	Ohio	-0.36	-3.43	-12.76	-28.71
Georgia	-0.37	-0.86	-3.87	-17.21	Oklahoma	-0.71	-1.95	-3.58	-3.96
Hawaii	-1.55	-4.90	0.32	30.31	Oregon	-5.31	-1.68	-15.42	30.35
Idaho	-2.08	-5.59	-7.93	-3.84	Pennsylvania	-0.35	-0.79	-2.30	6.15
Illinois	-0.82	-4.30	-9.81	-30.88	Rhode Island	-0.15	-2.34	-3.72	-14.25
Indiana	0.07	-1.05	-4.77	-7.39	South Carolina	-0.80	-2.50	-4.35	-8.74
Iowa	-1.95	-2.83	-5.77	44.46	South Dakota	1.45	-1.03	-3.48	1.90
Kansas	0.64	-2.16	-4.63	-16.98	Tennessee	-0.34	-1.70	-6.82	-1.50
Kentucky	-5.67	-7.30	-10.63	-33.26	Texas	0.67	-0.22	-3.62	25.86
Louisiana	6.96	-2.62	-6.99	-71.12	Utah	-3.13	-4.34	-19.06	-64.31
Maine	-0.83	-1.37	-0.59	-0.34	Vermont	-2.69	-5.47	-19.39	-15.15
Maryland	-1.53	-1.96	-13.34	-8.12	Virginia	-4.57	-6.58	-24.48	-6.41
Massachusetts	-0.46	-2.21	-3.77	3.97	Washington	-1.07	-3.51	-18.30	-48.87
Michigan	-0.60	-25.19	-18.40	-111.56	West Virginia	-2.21	-0.13	10.73	-2.78
Minnesota	-0.14	0.02	2.48	12.81	Wisconsin	-0.14	-0.36	1.83	-15.26
Mississippi	0.23	-1.15	0.60	-5.70	Wyoming	-0.49	-13.64	-40.63	29.83
Mean	-1.35	-4.09	-10.39	-9.59	ABS Mean	3.53	6.50	14.98	19.56

Hence, there is 11 one-step ahead forecast instead of 12 and 9 three-step ahead instead of 10.

Table 2.7: [†]

1-Month, 3-Month, 6-Month, & 12-Month Ahead Forecast MSE for the GARCH(1,1) Specified Model									
State	1-Month	3-Month	6-Month	12-Month	State	1-Month	3-Month	6-Month	12-Month
Alabama	0.0041	0.0053	0.0062	0.0006	Missouri	0.0012	0.0016	0.0020	0.0042
Alaska	0.0295	0.1222	0.3544	0.0088	Montana	0.0143	0.0068	0.0037	0.1229
Arizona	0.0089	0.0129	0.0184	0.0209	Nebraska	0.0008	0.0004	0.0034	0.0084
Arkansas	0.0002	0.0011	0.0042	0.0005	Nevada	0.0133	0.0975	2.1822	4.0063
California	0.0007	0.0011	0.0069	0.0380	New Hampshire	0.0039	0.0154	0.0207	0.0146
Colorado	0.0017	0.0044	0.0218	0.0626	New Mexico	0.0004	0.0024	0.0206	0.0194
Connecticut	0.0002	0.0011	0.0052	0.0204	New York	0.0005	0.0008	0.0032	0.0198
Delaware	0.1415	0.1494	0.2395	0.5334	North Carolina	0.0007	0.0066	0.0571	0.0829
District of Columbia	0.0012	0.0028	0.0405	0.1718	North Dakota	0.0116	0.0163	0.0374	0.0211
Florida	0.0188	0.0810	0.1894	0.0325	Ohio	0.0004	0.0022	0.0253	0.0658
Georgia	0.0011	0.0034	0.0143	0.0160	Oklahoma	0.0004	0.0014	0.0070	0.0077
Hawaii	0.0391	0.0583	0.0857	0.0097	Oregon	0.0056	0.0137	0.0909	0.0430
Idaho	0.0085	0.0164	0.0340	0.0129	Pennsylvania	0.0003	0.0006	0.0011	0.0005
Illinois	0.0005	0.0009	0.0030	0.0151	Rhode Island	0.0012	0.0039	0.0097	0.0421
Indiana	0.0031	0.0041	0.0175	0.0475	South Carolina	0.0005	0.0026	0.0079	0.0206
Iowa	0.0005	0.0004	0.0001	0.0017	South Dakota	0.0011	0.0004	0.0016	0.0046
Kansas	0.0012	0.0019	0.0087	0.0013	Tennessee	0.0276	0.0789	0.0371	0.0029
Kentucky	0.0053	0.0105	0.0279	0.0998	Texas	0.0004	0.0002	0.0041	0.0070
Louisiana	0.0002	0.0035	0.0119	0.0000	Utah	0.0012	0.0071	0.1430	0.1717
Maine	0.0126	0.0172	0.0300	0.0375	Vermont	0.1168	0.1384	0.2840	0.5499
Maryland	0.0003	0.0007	0.0031	0.0121	Virginia	0.0003	0.0007	0.0016	0.0005
Massachusetts	0.0072	0.0059	0.0076	0.0051	Washington	0.0040	0.0074	0.0416	0.2083
Michigan	0.0057	0.0038	0.0034	0.0360	West Virginia	0.0285	14.4433	31.5146	0.0045
Minnesota	0.0001	0.0004	0.0043	0.0151	Wisconsin	0.0002	0.0030	0.0080	0.0089
Mississippi	0.0019	0.0133	0.0213	0.0418	Wyoming	0.0390	0.0381	0.8568	0.2743
Mean MSE	0.0114	0.3082	0.7305	0.1391					

[†]All MSEs in this table have been scaled by 1E10

Table 2.8

1-Month, 3-Month, 6-Month, & 12-Month Ahead Percent Forecast Error for the GARCH(1,1) Specified Model									
State	1-Month	3-Month	6-Month	12-Month	State	1-Month	3-Month	6-Month	12-Month
Alabama	-2.85	-3.70	-4.01	-1.69	Missouri	-2.63	-3.34	-4.01	-6.03
Alaska	-2.95	-5.38	-11.36	1.86	Montana	3.67	2.39	1.02	10.02
Arizona	-0.98	-2.30	-3.94	5.06	Nebraska	2.04	0.19	-1.75	-6.68
Arkansas	-1.00	-2.04	-4.05	-1.93	Nevada	-2.97	-8.71	-45.79	-57.93
California	-2.13	-3.50	-9.68	-22.01	New Hampshire	-2.16	-4.81	-6.56	6.11
Colorado	-3.91	-5.94	-11.69	-27.59	New Mexico	-1.08	-2.90	-8.39	-8.79
Connecticut	-1.44	-3.25	-7.05	-15.26	New York	0.81	-0.44	-2.61	-10.88
Delaware	-11.35	-13.44	-18.97	-33.08	North Carolina	-0.85	-2.82	-9.39	-14.28
District of Columbia	-0.20	-0.85	-3.35	-9.24	North Dakota	-2.03	-2.63	-2.87	-3.29
Florida	-1.74	-2.56	-0.16	-2.53	Ohio	0.90	-1.92	-11.24	-20.88
Georgia	-0.14	-1.35	-4.34	8.14	Oklahoma	-0.91	-2.24	-4.11	-5.51
Hawaii	-2.81	-3.65	-5.08	-1.67	Oregon	-3.31	-5.04	-12.43	-8.85
Idaho	-2.85	-4.23	-6.73	-4.09	Pennsylvania	-1.79	-2.60	-3.59	-2.33
Illinois	-1.89	-3.12	-5.92	-14.43	Rhode Island	-1.03	-1.73	-3.35	-8.77
Indiana	-1.86	-2.53	-5.36	-9.65	South Carolina	-0.66	-2.40	-4.11	-8.16
Iowa	1.25	0.39	-0.64	3.58	South Dakota	1.70	0.48	-1.10	-4.40
Kansas	1.55	0.92	0.46	-1.56	Tennessee	1.93	2.03	-2.60	-3.80
Kentucky	-2.99	-4.39	-6.49	-14.31	Texas	1.30	0.54	-3.60	5.95
Louisiana	-0.36	-1.39	-3.01	0.01	Utah	-2.10	-4.75	-18.92	-28.40
Maine	-8.38	-11.18	-15.49	-17.19	Vermont	-11.38	-13.94	-21.82	-32.06
Maryland	-1.17	-2.06	-4.52	-9.55	Virginia	-1.46	-2.16	-3.37	-2.16
Massachusetts	-3.82	-4.39	-5.06	-5.89	Washington	-1.74	-2.58	-5.50	-14.15
Michigan	2.84	1.60	-0.96	-6.91	West Virginia	-0.97	6.07	-6.45	1.71
Minnesota	0.32	-0.13	-0.25	6.20	Wisconsin	0.13	-0.20	1.76	-4.41
Mississippi	0.19	-1.72	0.20	-8.88	Wyoming	6.36	-1.45	-24.64	-12.14
Mean MSE	-1.34	-2.70	-6.86	-8.45	ABS Mean	2.58	4.26	8.54	10.40

Table 2.9: [†]

1-Month, 3-Month, 6-Month, & 12-Month White Noise Process Forecast MSE									
State	1-Month	3-Month	6-Month	12-Month	State	1-Month	3-Month	6-Month	12-Month
Alabama	8.2219	11.7834	12.3633	1.9799	Missouri	4.8739	9.6795	5.7987	0.0028
Alaska	268.4360	42.2698	943.3265	0.0786	Montana	182.2295	17.9328	63.0195	43.0603
Arizona	73.9078	9.5004	6.8604	8.3784	Nebraska	3.1217	4.8689	8.7968	0.7098
Arkansas	23.1969	2.1730	50.6233	1.7126	Nevada	14.2164	25.5968	62.3315	7.5469
California	16.4316	4.0893	10.5261	0.1707	New Hampshire	43.5609	239.5309	88.5697	3.3501
Colorado	5.2305	6.2011	30.4681	0.6301	New Mexico	16.2836	2.5454	18.3348	3.1220
Connecticut	3.1863	9.8658	3.8473	1.9729	New York	6.8060	3.0857	2.3301	0.7612
Delaware	19.4312	44.7284	66.0319	947.5934	North Carolina	2.7646	7.1811	3.9912	1.1559
District of Columbia	14.8130	37.7972	108.8174	17.6396	North Dakota	32.1408	23.6519	11.5053	10.5565
Florida	33.4324	27.3232	42.0500	20.6800	Ohio	5.4891	14.5968	1.4617	0.5076
Georgia	3.8916	3.8590	2.3906	0.5966	Oklahoma	9.2177	3.5918	8.0098	0.9436
Hawaii	73.6731	66.2649	135.0967	50.7524	Oregon	37.4819	314.9361	130.7312	110.4641
Idaho	16.8459	35.5788	55.2799	255.2506	Pennsylvania	2.9463	1.5035	1.6705	1.0057
Illinois	1.6905	6.2790	3.2411	0.8796	Rhode Island	13.8014	5.4828	27.2803	12.7185
Indiana	21.6205	120.1084	5.5448	11.0537	South Carolina	6.4346	23.8473	3.2133	1.8402
Iowa	2.3645	52.5359	4.3578	0.8825	South Dakota	7.4803	10.7488	9.0898	3.2748
Kansas	18.0717	14.7262	6.4631	11.5081	Tennessee	9.8356	9.5833	19.0329	1.4889
Kentucky	64.2997	45.3653	12.9329	1.8602	Texas	1.7718	15.1218	1.0768	0.4934
Louisiana	32.1043	69.3320	71.5334	33.4044	Utah	3.9940	20.9371	11.4376	0.8297
Maine	7.6018	7.9137	9.0293	6.2197	Vermont	413.5752	55.0391	150.5772	832.6872
Maryland	6.0321	16.1595	2.0660	0.3970	Virginia	7.9841	1.4229	0.9122	0.2011
Massachusetts	6.2731	15.8457	0.6681	1.4209	Washington	9.5452	35.1210	51.8624	0.9063
Michigan	90.5795	51.3749	111.4256	3.8160	West Virginia	558.4355	286.0652	621.9184	27.2076
Minnesota	6.5433	15.3731	5.9778	39.6160	Wisconsin	4.4943	5.7581	5.6213	4.1582
Mississippi	4.1236	8.4343	7.9908	0.5947	Wyoming	167.8514	200.2519	62.3286	71.1594
Mean MSE	47.7668	41.2593	61.5963	51.1848					

[†]All MSEs in this table have been scaled by 1E10

Hence, there is 11 one-step ahead forecast instead of 12 and 9 three-step ahead instead of 10.

Table 2.10

1-Month, 3-Month, 6-Month, & 12-Month White Noise Process Percent Forecast Error									
State	1-Month	3-Month	6-Month	12-Month	State	1-Month	3-Month	6-Month	12-Month
Alabama	-30.22	-83.63	-64.34	-118.56	Missouri	-19.45	-66.41	-50.16	3.44
Alaska	-59.82	16.68	-242.01	3.97	Montana	-118.06	27.67	-8.70	99.46
Arizona	-150.19	8.12	52.72	98.50	Nebraska	35.95	2.24	-50.50	-43.55
Arkansas	-166.23	27.27	-228.51	89.01	Nevada	14.21	-10.62	-26.72	52.40
California	-91.10	-32.35	-151.82	-38.08	New Hampshire	-101.76	-254.56	-113.48	-129.43
Colorado	-33.84	-16.19	-162.71	81.51	New Mexico	-1.86	3.96	-9.00	100.00
Connecticut	-7.08	-61.71	-7.78	83.62	New York	-41.34	3.93	33.15	93.45
Delaware	-11.19	-65.88	-120.80	-1247.56	North Carolina	55.53	-10.99	4.84	64.15
District of Columbia	65.39	12.16	-70.24	99.45	North Dakota	-14.56	-10.79	-26.89	83.31
Florida	23.04	59.55	74.56	55.33	Ohio	25.11	-65.00	60.12	-60.98
Georgia	-8.49	27.16	-20.13	54.92	Oklahoma	-62.90	51.10	38.77	-50.80
Hawaii	16.84	-1.75	14.73	86.58	Oregon	-17.85	-191.94	-76.63	98.79
Idaho	18.46	6.07	-89.23	-463.32	Pennsylvania	6.03	12.56	15.11	-88.58
Illinois	5.99	-24.34	-2.98	88.39	Rhode Island	10.05	10.74	16.10	-99.99
Indiana	-43.78	-128.03	81.94	100.00	South Carolina	-33.35	-152.08	-15.41	75.43
Iowa	37.79	-255.54	18.58	85.10	South Dakota	13.25	-14.53	86.33	90.57
Kansas	18.81	17.53	39.66	-83.82	Tennessee	-91.49	-43.30	-99.97	92.04
Kentucky	-136.94	-68.62	-8.37	47.24	Texas	33.34	-107.27	27.75	99.78
Louisiana	10.82	-35.44	-109.87	99.55	Utah	22.19	-87.05	-69.83	55.96
Maine	41.59	72.26	40.95	69.29	Vermont	-362.92	-90.38	-45.99	-1126.80
Maryland	-40.79	-74.27	5.95	60.72	Virginia	-6.03	34.89	37.33	29.09
Massachusetts	4.04	-131.02	37.45	98.88	Washington	38.87	-27.05	-32.42	-50.20
Michigan	2.55	-46.36	-210.25	71.69	West Virginia	-139.62	-262.88	-209.61	-197.58
Minnesota	24.22	-63.60	31.31	98.89	Wisconsin	42.97	80.70	80.82	62.74
Mississippi	57.16	15.04	94.02	-19.57	Wyoming	-13.25	-26.55	56.12	84.26
Mean MSE	-23.60	-40.41	-27.52	-23.23	ABS Mean	115.85	130.33	123.63	129.53

Hence, there is 11 one-step ahead forecast instead of 12 and 9 three-step ahead instead of 10.

GARCH(1,1) model is held by the 3 month and 6 month forecast for West Virginia.

The mean percent error in **Tables 2.6 & 2.8** is particularly interesting since it speaks to the bias of the forecast. As stated before, the trend of the forecast error is correct across the horizon. However, it should be noted that each of the mean percent error under both the algorithm specified ARCH and GARCH(1,1) are negative at each forecast horizon. This implies that the forecasts of state-level employment growth that use these methods are biased downward. Therefore, forecasts will on average understate the magnitude of employment fluctuation. Seeing that the magnitude of the mean percent error is smaller under the GARCH(1,1), the bias is reduced under this model specification.

Finally, both of the volatility models are compared to the results of a randomly generated white noise process displayed in **Tables 2.9 & 2.10**. The results illustrate the use of a ARCH or GARCH model in defining volatility increases the predictability of state employment growth greatly. The algorithm generated ARCH model and the GARCH(1,1) outperform the white noise. The state average forecast MSEs are significantly higher than the volatility models, as well as the forecast percentage error and absolute forecast percentage error.

2.4 Conclusion

The question addressed in this essay is whether state-level employment growth volatility follows an autoregressive conditional heteroskedastic (ARCH) process. An ARCH length specification algorithm was developed to determine the optimal lag length based on AIC and significance level. Forecasts were produced of this model and for two comparison models. The first comparison model was a standard GARCH(1,1) model to compare the feasibility of the algorithm's criteria and the addition of a generalized autoregressive moment. The second comparison is a randomly generated white noise process to analyze the performance of the volatility models.

The results demonstrated the GARCH(1,1) model outperformed the ARCH algorithm model, suggesting the generalized autoregressive term increases the predictability of state employment growth volatility. Second, the results state that both volatility models significantly over-performed the white noise. Given the information found in this study, when studying the effect of the state-level employment growth rate it is advantageous to also study its variance for greater forecast accuracy.

In concluding this article, it is interesting to consider some additional research. In the future, it would be beneficial to study the effects of including spatial spillover to the conditional mean regression. Second, it would be interesting to implement this study on other variables, such as state-level per capita personal income growth.

2.5 Appendix A: Additional Tables

Table 2.11

Equation 2.2 estimates, displayed for AL-MS β_0 - β_7								
State	β_0	β_1	AR Lags					
			β_2	β_3	β_4	β_5	β_6	β_7
Alabama	0.011 (0.002)*	0.437 (0.077)*	-0.081 (0.052)	0.092 (0.044)*	0.118 (0.042)*	-0.066 (0.043)	-0.020 (0.043)	0.177 (0.034)*
Alaska	0.014 (0.004)*	-0.242 (0.134)*	0.200 (0.043)*	0.002 (0.037)	-0.019 (0.042)	-0.131 (0.044)*	-0.154 (0.039)*	-0.213 (0.036)*
Arizona	0.018 (0.002)*	0.458 (0.077)*	0.023 (0.046)	-0.027 (0.043)	0.154 (0.039)*	-0.080 (0.037)*	0.137 (0.032)*	0.308 (0.035)*
Arkansas	0.019 (0.002)*	0.333 (0.061)*	0.169 (0.044)*	0.052 (0.042)	0.009 (0.048)	-0.033 (0.038)	-0.046 (0.041)	0.022 (0.030)
California	0.013 (0.002)*	0.419 (0.079)*	-0.013 (0.055)	0.088 (0.044)*	0.318 (0.047)*	0.013 (0.046)	-0.069 (0.042)	-0.003 (0.047)
Colorado	0.011 (0.002)*	0.376 (0.081)*	0.000 (0.051)	0.131 (0.037)*	0.264 (0.041)*	-0.105 (0.043)*	-0.049 (0.042)	0.172 (0.043)*
Connecticut	-0.001 (0.002)	0.389 (0.064)*	-0.227 (0.050)*	0.070 (0.044)	0.142 (0.049)*	0.166 (0.047)*	0.130 (0.041)*	0.063 (0.040)
Delaware	0.000 (0.003)	0.600 (0.120)*	-0.348 (0.050)*	-0.063 (0.050)	0.109 (0.044)*	-0.037 (0.043)	0.016 (0.046)	0.049 (0.033)
District of Columbia	0.003 (0.003)	0.003 (0.104)	-0.162 (0.049)*	-0.180 (0.050)*	0.003 (0.046)	-0.087 (0.053)	0.038 (0.052)	0.055 (0.047)
Florida	0.013 (0.002)*	0.200 (0.075)*	0.237 (0.049)*	0.122 (0.041)*	0.194 (0.042)*	-0.133 (0.041)*	-0.063 (0.042)	0.071 (0.040)*
Georgia	0.006 (0.002)*	0.321 (0.071)*	-0.098 (0.057)*	0.045 (0.039)	0.191 (0.047)*	0.065 (0.040)	0.079 (0.039)*	0.189 (0.041)*
Hawaii	0.022 (0.003)*	0.354 (0.105)*	0.080 (0.040)*	-0.083 (0.048)*	-0.072 (0.035)*	-0.187 (0.042)*	-0.018 (0.034)	0.093 (0.035)*
Idaho	0.027 (0.003)*	0.759 (0.106)*	0.125 (0.052)*	-0.099 (0.050)*	0.094 (0.042)*	-0.074 (0.040)*	-0.090 (0.040)*	-0.200 (0.037)*
Illinois	0.005 (0.002)*	0.524 (0.060)*	-0.264 (0.059)*	0.046 (0.045)	0.056 (0.041)	0.002 (0.047)	0.103 (0.038)*	0.269 (0.034)*
Indiana	0.013 (0.002)*	0.461 (0.096)*	0.065 (0.052)	0.174 (0.047)*	-0.040 (0.044)	-0.128 (0.047)*	0.039 (0.039)	0.141 (0.037)*
Iowa	0.012 (0.002)*	0.531 (0.062)*	-0.030 (0.046)	0.103 (0.039)*	-0.100 (0.046)*	-0.018 (0.038)	0.141 (0.035)*	0.080 (0.044)*
Kansas	0.013 (0.002)*	0.572 (0.071)*	-0.131 (0.048)*	-0.064 (0.051)	-0.015 (0.041)	-0.046 (0.037)	0.033 (0.035)	0.079 (0.029)*
Kentucky	0.005 (0.002)*	0.368 (0.072)*	-0.131 (0.041)*	-0.049 (0.041)	0.034 (0.041)	0.133 (0.023)*	0.092 (0.041)*	0.090 (0.034)*
Louisiana	0.005 (0.002)*	0.107 (0.063)*	-0.013 (0.045)	0.136 (0.038)*	0.150 (0.052)*	-0.004 (0.041)	0.066 (0.039)*	0.081 (0.036)*
Maine	0.017 (0.003)*	0.864 (0.096)*	-0.018 (0.050)	-0.115 (0.040)*	-0.217 (0.046)*	-0.192 (0.043)*	0.066 (0.040)*	0.036 (0.047)
Maryland	0.002 (0.002)	0.318 (0.066)*	-0.204 (0.046)*	0.003 (0.040)	0.137 (0.047)*	0.137 (0.036)*	0.030 (0.039)	0.137 (0.035)*
Massachusetts	0.005 (0.002)*	0.284 (0.067)*	-0.091 (0.052)*	0.046 (0.045)	0.190 (0.051)*	0.121 (0.040)*	-0.013 (0.039)	0.100 (0.035)*
Michigan	0.003 (0.002)	0.548 (0.095)*	-0.230 (0.039)*	0.002 (0.033)	0.136 (0.036)*	0.071 (0.033)*	0.194 (0.024)*	0.140 (0.029)*
Minnesota	0.016 (0.001)*	0.596 (0.056)*	0.054 (0.045)	0.105 (0.039)*	-0.100 (0.047)*	-0.060 (0.037)	-0.057 (0.032)*	0.015 (0.031)
Mississippi	0.009 (0.002)*	0.383 (0.083)*	-0.081 (0.054)	0.141 (0.044)*	0.076 (0.049)	0.003 (0.047)	0.043 (0.041)	0.053 (0.045)

*Statistically Significant at the 10% level.

Table 2.11 Continued

Equation 2.2 estimates, displayed for AL-MS β_8 - β_{18}											
State	Seasonal Dummies										
	β_8	β_9	β_{10}	β_{11}	β_{12}	β_{13}	β_{14}	β_{15}	β_{16}	β_{17}	β_{18}
Alabama	-0.005 (0.002)*	-0.005 (0.002)*	-0.013 (0.002)*	-0.013 (0.002)*	-0.016 (0.002)*	-0.007 (0.001)*	-0.008 (0.002)*	-0.013 (0.002)*	-0.012 (0.002)*	-0.011 (0.002)*	-0.029 (0.002)*
Alaska	-0.021 (0.005)*	-0.014 (0.005)*	0.012 (0.005)*	0.014 (0.005)*	0.003 (0.005)	-0.012 (0.004)*	-0.025 (0.005)*	-0.038 (0.005)*	-0.017 (0.005)*	-0.008 (0.004)*	-0.038 (0.004)*
Arizona	-0.020 (0.003)*	-0.015 (0.002)*	-0.028 (0.003)*	-0.031 (0.002)*	-0.027 (0.003)*	-0.011 (0.002)*	-0.002 (0.003)	-0.011 (0.003)*	-0.012 (0.003)*	-0.009 (0.003)*	-0.037 (0.002)*
Arkansas	-0.012 (0.002)*	-0.015 (0.002)*	-0.018 (0.002)*	-0.022 (0.002)*	-0.032 (0.002)*	-0.007 (0.001)*	-0.008 (0.002)*	-0.023 (0.002)*	-0.024 (0.002)*	-0.021 (0.002)*	-0.041 (0.002)*
California	-0.007 (0.002)*	-0.007 (0.002)*	-0.012 (0.003)*	-0.014 (0.002)*	-0.026 (0.002)*	-0.009 (0.001)*	-0.007 (0.002)*	-0.010 (0.002)*	-0.012 (0.002)*	-0.013 (0.002)*	-0.035 (0.002)*
Colorado	-0.004 (0.003)	-0.002 (0.003)	-0.015 (0.003)*	-0.004 (0.003)*	-0.015 (0.003)*	-0.007 (0.001)*	-0.014 (0.002)*	-0.011 (0.003)*	-0.012 (0.003)*	-0.007 (0.002)*	-0.035 (0.002)*
Connecticut	0.006 (0.002)*	0.012 (0.003)*	0.011 (0.003)*	0.011 (0.003)*	-0.016 (0.003)*	-0.005 (0.001)*	0.010 (0.002)*	0.004 (0.003)*	0.006 (0.003)*	0.006 (0.002)*	-0.027 (0.002)*
Delaware	0.008 (0.004)*	0.016 (0.004)*	0.011 (0.004)*	0.013 (0.004)*	-0.005 (0.004)	0.003 (0.002)	-0.002 (0.003)	-0.006 (0.004)	-0.001 (0.003)	0.003 (0.003)	-0.031 (0.003)*
District of Columbia	0.002 (0.004)	0.003 (0.004)	-0.001 (0.004)	0.007 (0.004)*	0.015 (0.004)*	-0.007 (0.003)*	-0.013 (0.003)*	-0.007 (0.004)*	-0.002 (0.004)	0.000 (0.004)	-0.023 (0.003)*
Florida	-0.007 (0.002)*	-0.016 (0.003)*	-0.019 (0.003)*	-0.018 (0.003)*	-0.028 (0.003)*	-0.005 (0.002)*	0.000 (0.002)	-0.007 (0.003)*	-0.009 (0.003)*	-0.008 (0.003)*	-0.026 (0.002)*
Georgia	0.000 (0.002)	0.003 (0.003)	-0.004 (0.003)	-0.004 (0.002)	-0.012 (0.002)*	0.000 (0.001)	-0.004 (0.002)*	-0.008 (0.002)*	-0.006 (0.002)*	-0.003 (0.002)*	-0.027 (0.002)*
Hawaii	-0.015 (0.004)*	-0.029 (0.003)*	-0.024 (0.003)*	-0.018 (0.003)*	-0.024 (0.004)*	-0.020 (0.003)*	-0.039 (0.003)*	-0.013 (0.004)*	-0.017 (0.004)*	-0.016 (0.003)*	-0.045 (0.003)*
Idaho	-0.020 (0.004)*	-0.017 (0.004)*	-0.023 (0.004)*	-0.018 (0.004)*	-0.042 (0.004)*	-0.008 (0.002)*	-0.014 (0.003)*	-0.032 (0.003)*	-0.032 (0.003)*	-0.031 (0.003)*	-0.058 (0.003)*
Illinois	0.002 (0.003)	0.004 (0.003)	0.001 (0.003)	0.003 (0.003)	-0.005 (0.003)*	-0.002 (0.002)	-0.006 (0.002)*	-0.010 (0.002)*	-0.008 (0.002)*	-0.007 (0.002)*	-0.032 (0.002)*
Indiana	-0.003 (0.003)	-0.005 (0.003)	-0.012 (0.003)*	-0.015 (0.003)*	-0.023 (0.003)*	-0.001 (0.002)	0.001 (0.002)	-0.023 (0.003)*	-0.020 (0.002)*	-0.012 (0.002)*	-0.036 (0.003)*
Iowa	-0.004 (0.002)	-0.003 (0.003)	-0.006 (0.003)*	-0.007 (0.003)*	-0.029 (0.003)*	-0.006 (0.001)*	0.001 (0.002)	-0.016 (0.002)*	-0.015 (0.002)*	-0.011 (0.002)*	-0.040 (0.002)*
Kansas	-0.008 (0.003)*	-0.005 (0.003)*	-0.012 (0.003)*	-0.012 (0.003)*	-0.033 (0.003)*	-0.010 (0.001)*	0.002 (0.002)	-0.014 (0.003)*	-0.013 (0.003)*	-0.012 (0.003)*	-0.036 (0.002)*
Kentucky	0.000 (0.002)	0.007 (0.002)*	0.005 (0.002)*	-0.002 (0.003)	-0.016 (0.003)*	0.000 (0.001)	-0.001 (0.002)	-0.007 (0.002)*	-0.004 (0.002)*	-0.003 (0.002)	-0.033 (0.002)*
Louisiana	0.004 (0.002)*	0.003 (0.002)	-0.002 (0.002)	0.001 (0.002)	-0.012 (0.002)*	-0.004 (0.001)*	0.002 (0.002)	-0.002 (0.002)	-0.005 (0.002)*	-0.003 (0.002)*	-0.028 (0.002)*
Maine	-0.021 (0.004)*	-0.014 (0.004)*	-0.009 (0.004)*	0.010 (0.004)*	-0.019 (0.004)*	0.010 (0.003)*	-0.018 (0.004)*	-0.024 (0.004)*	-0.030 (0.003)*	-0.022 (0.003)*	-0.057 (0.003)*
Maryland	0.007 (0.002)*	0.015 (0.003)*	0.009 (0.003)*	0.005 (0.003)*	-0.007 (0.002)*	-0.006 (0.001)*	-0.005 (0.002)*	-0.002 (0.002)	0.000 (0.002)	0.001 (0.002)	-0.033 (0.002)*
Massachusetts	0.001 (0.003)	0.012 (0.003)*	0.005 (0.003)	0.000 (0.003)	-0.018 (0.003)*	-0.006 (0.001)*	-0.001 (0.002)	-0.004 (0.003)	-0.003 (0.002)	-0.005 (0.002)*	-0.036 (0.002)*
Michigan	-0.002 (0.003)	0.007 (0.003)*	0.008 (0.004)*	0.006 (0.003)*	-0.021 (0.003)*	-0.002 (0.002)	0.006 (0.003)*	0.000 (0.003)	-0.005 (0.003)*	-0.002 (0.002)	-0.033 (0.003)*
Minnesota	-0.010 (0.002)*	-0.010 (0.002)*	-0.008 (0.003)*	-0.014 (0.002)*	-0.032 (0.002)*	-0.005 (0.001)*	-0.007 (0.002)*	-0.017 (0.002)*	-0.021 (0.002)*	-0.020 (0.002)*	-0.042 (0.002)*
Mississippi	-0.001 (0.003)	0.000 (0.003)	-0.006 (0.003)*	-0.010 (0.003)*	-0.020 (0.003)*	-0.001 (0.001)	0.003 (0.002)	-0.012 (0.003)*	-0.012 (0.002)*	-0.009 (0.003)*	-0.028 (0.002)*

*Statistically Significant at the 10% level.

Table 2.11 Continued

Equation 2.2 estimates, displayed for MO-WY β_0 - β_7								
State	AR Lags							
	β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7
Missouri	0.009	0.532	-0.190	0.016	0.048	-0.016	-0.003	0.039
	(0.002)*	(0.080)*	(0.052)*	(0.046)	(0.049)	(0.040)	(0.039)	(0.039)
Montana	0.015	0.383	0.109	-0.005	-0.014	-0.104	-0.096	-0.239
	(0.003)*	(0.119)*	(0.048)*	(0.046)	(0.055)	(0.045)*	(0.049)*	(0.042)*
Nebraska	0.012	0.557	-0.135	0.006	0.051	-0.111	-0.067	0.041
	(0.002)*	(0.064)*	(0.043)*	(0.039)	(0.043)	(0.043)*	(0.037)*	(0.036)
Nevada	0.025	0.278	0.389	0.214	-0.055	-0.105	-0.222	0.047
	(0.002)*	(0.110)*	(0.048)*	(0.052)*	(0.051)	(0.047)*	(0.046)*	(0.038)
New Hampshire	0.015	0.569	0.171	0.064	-0.152	-0.047	0.028	0.292
	(0.002)*	(0.102)*	(0.042)*	(0.036)*	(0.044)*	(0.041)	(0.034)	(0.027)*
New Mexico	0.014	0.222	0.072	0.120	-0.042	-0.135	-0.024	0.067
	(0.002)*	(0.081)*	(0.048)	(0.049)*	(0.048)	(0.042)*	(0.044)	(0.038)*
New York	0.007	0.290	-0.097	0.099	0.102	-0.139	0.075	0.084
	(0.002)*	(0.055)*	(0.055)*	(0.046)*	(0.043)*	(0.041)*	(0.039)*	(0.038)*
North Carolina	0.018	0.790	-0.118	-0.066	0.137	0.003	0.091	-0.047
	(0.002)*	(0.065)*	(0.051)*	(0.042)	(0.045)*	(0.037)	(0.038)*	(0.032)
North Dakota	0.017	0.248	0.198	0.071	-0.025	-0.094	-0.151	-0.268
	(0.002)*	(0.083)*	(0.047)*	(0.042)*	(0.045)	(0.041)*	(0.041)*	(0.043)*
Ohio	0.007	0.576	-0.226	0.118	0.198	0.001	-0.006	0.139
	(0.001)*	(0.074)*	(0.054)*	(0.041)*	(0.037)*	(0.039)	(0.033)	(0.045)*
Oklahoma	0.009	0.296	-0.009	-0.072	0.108	-0.062	0.045	0.148
	(0.002)*	(0.079)*	(0.048)	(0.045)	(0.052)*	(0.044)	(0.036)	(0.036)*
Oregon	0.021	0.665	-0.025	0.102	0.282	-0.025	-0.131	-0.224
	(0.002)*	(0.096)*	(0.046)	(0.040)*	(0.036)*	(0.035)	(0.040)*	(0.032)*
Pennsylvania	0.008	0.293	-0.031	0.042	0.020	-0.087	0.134	0.066
	(0.001)*	(0.065)*	(0.052)	(0.042)	(0.048)	(0.041)*	(0.038)*	(0.034)*
Rhode Island	0.005	0.375	-0.148	0.064	0.083	0.042	-0.013	-0.042
	(0.003)*	(0.094)*	(0.058)*	(0.042)	(0.039)*	(0.040)	(0.040)	(0.032)
South Carolina	0.020	0.938	-0.261	-0.048	0.087	0.003	0.064	0.067
	(0.002)*	(0.071)*	(0.052)*	(0.040)	(0.047)*	(0.039)	(0.036)*	(0.035)*
South Dakota	0.010	0.401	-0.002	0.046	0.082	-0.136	-0.058	0.023
	(0.002)*	(0.070)*	(0.047)	(0.043)	(0.046)*	(0.042)*	(0.036)	(0.037)
Tennessee	0.012	0.554	-0.124	0.104	0.107	0.005	-0.096	0.111
	(0.002)*	(0.082)*	(0.052)*	(0.049)*	(0.042)*	(0.040)	(0.037)*	(0.034)*
Texas	0.010	0.278	-0.040	0.029	0.139	0.067	-0.047	0.368
	(0.001)*	(0.052)*	(0.043)	(0.037)	(0.039)*	(0.037)*	(0.036)	(0.039)*
Utah	0.009	0.204	0.066	0.138	0.141	-0.035	-0.069	0.224
	(0.002)*	(0.068)*	(0.044)	(0.046)*	(0.047)*	(0.043)	(0.045)	(0.043)*
Vermont	0.031	0.995	-0.017	-0.215	-0.024	-0.234	-0.033	-0.132
	(0.002)*	(0.093)*	(0.045)	(0.038)*	(0.036)	(0.028)*	(0.029)	(0.027)*
Virginia	0.009	0.266	0.066	0.096	0.062	0.112	-0.047	0.038
	(0.002)*	(0.067)*	(0.061)	(0.057)*	(0.047)	(0.046)*	(0.049)	(0.041)
Washington	0.008	0.066	0.016	0.107	0.200	0.036	-0.059	0.068
	(0.002)*	(0.085)	(0.052)	(0.046)*	(0.041)*	(0.044)	(0.037)	(0.044)
West Virginia	0.003	0.721	-0.348	-0.099	-0.168	-0.139	-0.099	-0.041
	(0.002)	(0.091)*	(0.030)*	(0.041)*	(0.046)*	(0.032)*	(0.032)*	(0.031)
Wisconsin	0.007	0.416	-0.121	0.078	0.073	0.022	0.087	0.069
	(0.002)*	(0.068)*	(0.048)*	(0.038)*	(0.044)*	(0.041)	(0.044)*	(0.044)
Wyoming	0.010	0.161	0.315	0.018	-0.056	-0.080	-0.100	-0.153
	(0.004)*	(0.152)	(0.052)*	(0.051)	(0.050)	(0.044)*	(0.044)*	(0.039)*

*Statistically Significant at the 10% level.

Table 2.11 Continued

Equation 2.2 estimates, displayed for MO-WY β_8 - β_{18}											
State	Seasonal Dummies										
	β_8	β_9	β_{10}	β_{11}	β_{12}	β_{13}	β_{14}	β_{15}	β_{16}	β_{17}	β_{18}
Missouri	0.001 (0.003)	0.004 (0.003)	-0.004 (0.003)	-0.008 (0.003)*	-0.022 (0.002)*	-0.005 (0.001)*	0.001 (0.002)	-0.009 (0.002)*	-0.012 (0.002)*	-0.010 (0.002)*	-0.037 (0.002)*
Montana	-0.007 (0.004)*	-0.005 (0.004)	-0.006 (0.004)	-0.002 (0.004)	-0.031 (0.004)*	0.000 (0.003)	-0.009 (0.004)*	-0.015 (0.004)*	-0.019 (0.004)*	-0.016 (0.004)*	-0.049 (0.003)*
Nebraska	-0.005 (0.002)*	0.000 (0.003)	-0.006 (0.003)*	-0.009 (0.003)*	-0.024 (0.003)*	-0.007 (0.001)*	-0.004 (0.002)*	-0.011 (0.002)*	-0.014 (0.002)*	-0.015 (0.002)*	-0.036 (0.002)*
Nevada	-0.013 (0.003)*	-0.025 (0.003)*	-0.022 (0.003)*	-0.029 (0.003)*	-0.028 (0.003)*	-0.014 (0.002)*	-0.017 (0.003)*	-0.028 (0.003)*	-0.025 (0.003)*	-0.028 (0.003)*	-0.043 (0.003)*
New Hampshire	-0.012 (0.003)*	-0.014 (0.003)*	-0.006 (0.003)*	-0.007 (0.003)*	-0.018 (0.003)*	-0.001 (0.002)	-0.011 (0.003)*	-0.024 (0.003)*	-0.024 (0.003)*	-0.011 (0.002)*	-0.042 (0.002)*
New Mexico	-0.004 (0.003)	-0.010 (0.003)*	-0.011 (0.003)*	-0.011 (0.003)*	-0.020 (0.003)*	-0.006 (0.002)*	-0.005 (0.002)*	-0.017 (0.003)*	-0.016 (0.002)*	-0.012 (0.002)*	-0.037 (0.002)*
New York	0.003 (0.003)	0.000 (0.003)*	-0.006 (0.003)*	0.001 (0.003)	-0.015 (0.003)*	-0.007 (0.002)*	-0.007 (0.002)*	-0.002 (0.003)*	-0.007 (0.002)*	-0.006 (0.002)*	-0.040 (0.002)*
North Carolina	-0.013 (0.002)*	-0.012 (0.003)*	-0.018 (0.003)*	-0.016 (0.002)*	-0.039 (0.002)*	-0.002 (0.001)*	-0.008 (0.002)*	-0.016 (0.002)*	-0.020 (0.002)*	-0.018 (0.002)*	-0.041 (0.002)*
North Dakota	-0.004 (0.003)	-0.004 (0.003)*	-0.006 (0.003)*	-0.022 (0.004)*	-0.039 (0.003)*	-0.009 (0.002)*	0.004 (0.003)	-0.008 (0.003)*	-0.020 (0.003)*	-0.022 (0.003)*	-0.048 (0.003)*
Ohio	0.001 (0.002)	0.007 (0.002)*	0.000 (0.002)	-0.006 (0.002)*	-0.019 (0.002)*	-0.006 (0.001)*	-0.001 (0.002)	-0.006 (0.002)*	-0.010 (0.002)*	-0.010 (0.002)*	-0.036 (0.002)*
Oklahoma	-0.005 (0.002)*	-0.002 (0.003)	-0.005 (0.003)*	-0.010 (0.003)*	-0.018 (0.003)*	-0.003 (0.002)*	-0.002 (0.002)	-0.008 (0.003)*	-0.010 (0.003)*	-0.007 (0.002)*	-0.027 (0.002)*
Oregon	-0.010 (0.003)*	-0.009 (0.003)*	-0.020 (0.004)*	-0.019 (0.003)*	-0.046 (0.003)*	-0.009 (0.002)*	-0.012 (0.003)*	-0.015 (0.003)*	-0.026 (0.003)*	-0.027 (0.003)*	-0.053 (0.002)*
Pennsylvania	-0.001 (0.002)	0.001 (0.002)	-0.006 (0.002)*	-0.001 (0.002)	-0.017 (0.002)*	-0.006 (0.001)*	-0.004 (0.002)*	-0.007 (0.002)*	-0.009 (0.002)*	-0.008 (0.002)*	-0.035 (0.002)*
Rhode Island	0.004 (0.003)	0.011 (0.004)*	0.006 (0.004)	0.000 (0.004)	-0.028 (0.004)*	0.005 (0.002)*	0.005 (0.003)	-0.004 (0.003)	-0.004 (0.003)	-0.006 (0.003)*	-0.045 (0.003)*
South Carolina	-0.014 (0.003)*	-0.012 (0.003)*	-0.019 (0.003)*	-0.021 (0.003)*	-0.037 (0.003)*	-0.011 (0.002)*	-0.014 (0.002)*	-0.022 (0.003)*	-0.022 (0.002)*	-0.018 (0.002)*	-0.040 (0.003)*
South Dakota	-0.003 (0.003)	0.006 (0.003)*	0.005 (0.003)	0.003 (0.003)	-0.027 (0.003)*	-0.002 (0.002)	-0.011 (0.003)*	-0.010 (0.003)*	-0.019 (0.003)*	-0.014 (0.002)*	-0.037 (0.002)*
Tennessee	-0.002 (0.003)	-0.002 (0.003)	-0.011 (0.003)*	-0.016 (0.003)*	-0.020 (0.003)*	-0.001 (0.002)	-0.005 (0.002)*	-0.016 (0.002)*	-0.015 (0.002)*	-0.013 (0.002)*	-0.038 (0.002)*
Texas	-0.006 (0.002)*	-0.005 (0.002)*	-0.008 (0.002)*	-0.012 (0.002)*	-0.011 (0.002)*	-0.007 (0.001)*	-0.007 (0.001)*	-0.010 (0.001)*	-0.009 (0.002)*	-0.008 (0.001)*	-0.026 (0.001)*
Utah	-0.001 (0.002)	0.000 (0.003)	-0.011 (0.002)*	-0.007 (0.002)*	-0.017 (0.003)*	-0.001 (0.001)	0.003 (0.002)	-0.011 (0.002)*	-0.007 (0.002)*	-0.012 (0.002)*	-0.033 (0.002)*
Vermont	-0.040 (0.003)*	-0.039 (0.003)*	-0.039 (0.003)*	-0.027 (0.003)*	-0.049 (0.003)*	-0.018 (0.002)*	-0.015 (0.003)*	-0.030 (0.004)*	-0.042 (0.003)*	-0.019 (0.003)*	-0.055 (0.003)*
Virginia	0.001 (0.002)	0.000 (0.003)	-0.002 (0.003)	-0.003 (0.003)	-0.022 (0.002)*	-0.005 (0.001)*	-0.003 (0.002)	-0.010 (0.002)*	-0.004 (0.002)*	-0.008 (0.002)*	-0.035 (0.002)*
Washington	0.004 (0.003)	0.006 (0.003)*	0.001 (0.003)	-0.001 (0.003)	-0.021 (0.003)*	-0.006 (0.002)*	0.003 (0.002)	-0.008 (0.003)*	-0.010 (0.002)*	-0.013 (0.003)*	-0.032 (0.002)*
West Virginia	0.001 (0.003)	0.002 (0.003)	0.006 (0.003)*	-0.001 (0.003)	-0.006 (0.003)*	0.005 (0.002)*	0.004 (0.003)	-0.004 (0.003)	-0.005 (0.003)*	-0.005 (0.003)*	-0.029 (0.003)*
Wisconsin	0.000 (0.002)	0.006 (0.003)*	0.004 (0.003)	0.006 (0.003)*	-0.017 (0.003)*	-0.004 (0.001)*	-0.004 (0.002)*	-0.010 (0.002)*	-0.012 (0.002)*	-0.011 (0.002)*	-0.038 (0.002)*
Wyoming	-0.003 (0.005)	-0.006 (0.005)	0.012 (0.005)*	0.017 (0.005)*	-0.037 (0.005)*	0.001 (0.004)	-0.005 (0.005)	-0.021 (0.005)*	-0.013 (0.005)*	-0.002 (0.005)	-0.040 (0.005)*

*Statistically Significant at the 10% level.

Chapter 3

Spatial Approach to Estimating State Employment Volatility

In Chapter 2, the conditional variance was shown to have an effect on the estimation and prediction of state level employment growth. Here we examine the role of the conditional variance, if any, when accounting for spatial spillover effects. It is well known that interaction and dependences between state economic growth indicators such as employment, personal per capita income, unemployment insurance claims, etc. are highly correlated with neighboring states.

Given the likelihood of spatial presence in state employment growth, this chapter will focus on whether G/ARCH effects are still present when accounting for these effects. If the G/ARCH effects disappear in the presence of spatial effects, then they are misclassified spatial spillovers. Conceptually, the volatility cluster as stated by Engle (1982) could be misidentified as dependences between states. The impact of the "Great Moderation" will be examined in conjunction.

Work has been completed on the volatility structure of employment on the national and state-level. The present state of the literature focuses on the change in employment volatility around the 'Great Moderation.' The 'Great Moderation' refers to the reduction in volatility in 1984 due to the U.S. Federal Reserve policy change to concentrate on inflation stability and reduction. The majority of the literature centers on national employment. There are a few key articles that focus on the effects of the 'Great Moderation' on state-level employment: Carlino, DeFina, and Sill (2007), Owyang, Piger, and Wall (2008), and Morgan, Rime, and Strahan (2004). While it is important to keep in mind that current research of reduction in volatility is due to the 'Great Moderation', the prime focus here is to test for autoregressive conditional heteroskedasticity (ARCH) processes at the state-level.

The literature base for this work is closely tied to Chapter 2. Next, a description of the data and the empirical model is given; followed by results of the model. The chapter concludes with a synopsis of the findings.

3.1 Review of the Literature

The relevant literature gives evidence that after 1984 both national and state-level employment volatility made a statistically significant drop, hence the 'Great Moderation.'

On the national level, Stock and Watson (2002) examined the reasoning behind the 'Great Moderation.' They use a substantial array of macro variables. In testing the hypothesis that a change in policy caused a structural break in the GDP growth, the authors used a structural VAR with a monetary rule similar to Taylor's Rule, a New Keynesian Phillips curve, and an IS equation. The U.S. conditional mean and variance were measured using an autoregressive model in the univariate case and a VAR in the multivariate case. Volatility was measured as the absolute value of the error. The statistically significant changes in volatility were measured using the Quandt likelihood ratio statistic. The results of the study led Stock and Watson (2002) to conclude that better inflation targeting by the Fed accounts for 20 to 30 percent of the reduction in GDP volatility, which began in 1984. Furthermore, the reductions in the magnitude of shocks in productivity and commodities played a role in the decline of GDP volatility. The conclusions also stated that better inventory schedules and the move from manufacturing to services are insignificant in aiding the decline of volatility.

Campbell (2004) took a different approach to measuring the 'Great Moderation,' by using data from the Survey of Professional Forecasters. The data ranges from the first quarter of 1969 to the second quarter of 2003 for the quarterly survey. Data for the annual survey ranged from the first quarter of 1971 to the fourth quarter of 2002. The author defined uncertainty as the expected error variance of output growth regressed on the expected future output growth, which is conditional on the information in the current period, to determine the predictability components of the 'Great Moderation.' The analysis stated that an autoregressive model of order 1 in testing the magnitude of the decline in output volatility inflates the magnitude of the decline; in other words, part of the 'Great Moderation' was due to an increase in predictability.

Owyang, Piger, and Wall (2008) also studied the 'Great Moderation' on the regional level. The data used was total non-farm employment from February 1956 to December 2004. The study used a panel regression with a

Markov-switching process and structural breaks for state employment growth. The residuals from this panel regression were measured as volatility. They discovered that 38 states saw a reduction in volatility at the same time as the U.S. This article also looks at five possible causes of the decrease in volatility: for example, the deregulation of Regulation Q, improved inventory practices, and improved monetary policy. The authors discovered that of these five theories only the improvement in monetary policy was statistically significant in explaining the decreases in volatility.

When studying the 'Great Moderation' it is important to notice that structural breaks have been included for most of the above articles. However, the assumption of a homoskedastic error has not changed. Given that the structure of volatility has changed over time, so should the structure of the volatility equation. An autoregressive conditional heteroskedastic (ARCH) model would be appropriate for analyzing changes in the conditional variance.

3.2 Empirical Analysis

3.2.1 Data

This is the same data used in Chapter 2, described in section 2.2.1 . For the purpose of uniformity in the spatial weight matrix, all states time-series will range from January 1960 to December 2009. Alaska and Hawaii have been removed from the sample since they have no neighbors in the United States. If the number of neighboring states is zero, then the resulting regression is the same as in Chapter 2. This leaves 49 states with 599 observations per state.

The spatial weight matrix is constructed using queen contiguity, meaning if two states have a common border or vertex they are counted as neighbors. Any column vector, neighbors for a particular state, must sum to one. For simplification, all n neighboring states for state i are given equal weighting, $\frac{1}{n}$.

3.2.2 Empirical Model

The structure of the empirical model is analogous the model described in section 2.2.2 with the addition of the spatial weight matrix. The Ljung-Box Q-statistic is used to test for serial correlation, refer to equation 2.1. The regression specification is as follows:

$$y_{i,t} = \beta_{i,0} + \beta_{i,1}y_{us,t-1} + \sum_{a=1}^6 \beta_{i,a+1}y_{i,t-a} + \sum_{d=1}^{11} \beta_{i,c+d+1}dum_{d,t} + \sum_{f=1}^{\leq 6} \lambda_{i,f}W y_{i-f} + \epsilon_{i,t} \quad (3.1)$$

$$\sigma_{i,t}^2 = \omega_i + \sum_{n=1}^{q=12} \alpha_{i,n} \epsilon_{i,t-n}^2 \quad (3.2)$$

$$\begin{aligned} \omega &> 0 \\ \alpha_n &\geq 0 \forall n, \sum_{n=1}^q \alpha_{i,n} < 1 \end{aligned}$$

Where $i = i^{th}$ State, $\{\epsilon_{i,t} | \mathcal{I}_{i,t-1}\} \sim \mathcal{N}(0, \sigma_{i,t}^2)$

The additional term of $\sum_{f=1}^{g=6} \lambda_{i,f}W y_{i-f}$ is the weighted average of employment growth for the neighboring state(s) in the previous period. This makes equation 3.1 a variation of a space-time autoregressive model (STAR).¹ It should be noted, that own state employment growth has been set from 1 to 6, inclusive. This is to keep consistency and continuity equation 2.4 from Chapter 2. Remember, this lag length is when the indication of serial correlation is greatly reduced. Each lag coefficient in the volatility structure is restricted to be positive and the sum of the coefficients on the lags is restricted to be less than one. This is to ensure that the forecast(s) of the employment growth volatility is positive and stationary.

Moreover, the inclusion of the neighboring states' growth will help to identify whether or not the ARCH effects are misclassified intertemporal space-time dependences. Failure to model these space-time dependences, if present, could cause a state's own variance to display time-dependence. That is, if a spatial time-dependencies are not properly modeled, they could appear as own time-dependence in the error distribution.

The standard spatial autoregressive process assumes a iid error, as described in LeSage and Pace (2009). Recall, Engle (1982) states that ARCH ef-

¹The basis of this Chapter, using a STAR implementation of Chapter 2, was suggested by James LeSage PhD. Chapter 2 was rejected from being submitted at a journal. The reject letter included the comment that it would be more powerful if spatial spillover effects were accounted for.

Table 3.1

Ljung Box Test Q-statistics for all 49 States
Using 1-Lag of neighboring states

State	1-Lag Q-statistic	6-Lag Q-statistic	State	1-Lag Q-statistic	6-Lag Q-statistic
Alabama	0.54	1.77	Nebraska	0.22	2.60
Arizona	0.81	18.45*	Nevada	0.00	6.83
Arkansas	0.36	1.53	New Hampshire	0.21	5.19
California	0.65	4.73	New Jersey	1.05	3.33
Colorado	0.00	2.86	New Mexico	0.00	1.13
Connecticut	0.03	4.06	New York	2.49	3.67
Delaware	0.17	0.33	North Carolina	0.42	3.92
District of Columbia	0.00	0.92	North Dakota	2.92*	34.76*
Florida	0.16	4.44	Ohio	0.11	6.15
Georgia	0.08	3.89	Oklahoma	0.02	5.01
Idaho	0.68	7.96	Oregon	5.93*	13.90*
Illinois	3.85*	6.43	Pennsylvania	0.52	4.50
Indiana	0.02	3.50	Rhode Island	0.44	1.10
Iowa	3.93*	7.76	South Carolina	0.03	4.32
Kansas	0.57	3.32	South Dakota	1.06	3.56
Kentucky	0.48	1.50	Tennessee	0.11	1.37
Louisiana	0.15	0.71	Texas	1.71	12.19*
Maine	0.55	16.60*	Utah	0.20	12.62*
Maryland	0.05	1.79	Vermont	0.83	16.36*
Massachusetts	0.03	1.78	Virginia	0.04	1.64
Michigan	0.15	1.90	Washington	0.12	3.11
Minnesota	0.43	4.95	West Virginia	0.01	2.70
Mississippi	0.12	0.60	Wisconsin	0.41	2.23
Missouri	0.43	3.48	Wyoming	0.14	20.53*
Montana	3.30*	15.28*			

*Statistically Significant at the 10% level.

fects can approximate real world complexity when structural breaks or omitted variables bias exist. Therefore with the spatial dependences accounted for, if ARCH effects are still present and the neighbor states employment growth effects are significant; then it is clear that the ARCH effects in Chapter 2 are not from omitted variable bias.

3.3 Results of Empirics

The results of the Ljung-Box Q-statistic using 1 lag of the neighboring states are displayed in 3.1.

For simplicity, all states will be evaluated using the same lag length of the spatial weight matrix. As shown in **Table 3.2**, 44 out of the 49 states show no indication of autocorrelation. The 5 other states will keep the same structure for uniformity. With evidence of the serial correlation reduce, any time-dependences in the variance is not misread as autoregressive lags in the states own growth.² Therefore, there is no need to go beyond the first lag of

²Each shows statistically significant autocorrelation at the 12th, suggesting strong sea-

Table 3.2

ARCH LM test ARCH-statistics for all 49 State					
State	1-Lag	6-Lag	State	1-Lag	6-Lag
	ARCH-statistic	ARCH-statistic		ARCH-statistic	ARCH-statistic
Alabama	2.93*	18.37*	Nebraska	1.08	8.41
Arizona	3.29*	9.23	Nevada	37.50*	53.27*
Arkansas	5.07*	44.63*	New Hampshire	2.03	81.65*
California	6.01*	12.02*	New Jersey	4.40*	21.75*
Colorado	2.50	19.77*	New Mexico	0.22	7.65
Connecticut	6.12*	11.28*	New York	3.22*	7.07
Delaware	23.05*	42.37*	North Carolina	20.93*	31.43*
District of Columbia	2.69	6.45	North Dakota	15.68*	23.71*
Florida	9.86*	41.92*	Ohio	11.66*	47.33*
Georgia	28.21*	41.58*	Oklahoma	8.93*	25.49*
Idaho	0.53	4.96	Oregon	7.93*	39.60*
Illinois	1.15	27.61*	Pennsylvania	5.03*	16.30*
Indiana	0.29	26.44*	Rhode Island	14.48*	16.06*
Iowa	17.12*	20.85*	South Carolina	5.02*	16.32*
Kansas	75.09*	89.43*	South Dakota	3.04*	14.64*
Kentucky	3.56*	22.67*	Tennessee	0.30	9.10
Louisiana	3.06*	4.58	Texas	6.54*	27.03*
Maine	22.04*	30.68*	Utah	0.09	45.57*
Maryland	0.63	3.80	Vermont	4.76*	44.61*
Massachusetts	2.69	62.11*	Virginia	0.00	7.39
Michigan	34.11*	79.02*	Washington	7.05*	53.38*
Minnesota	18.02*	23.39*	West Virginia	0.15	71.57*
Mississippi	3.93*	14.95*	Wisconsin	2.86*	48.35*
Missouri	1.25	6.52	Wyoming	5.58*	24.22*
Montana	1.05	23.88*			

*Statistically Significant at the 10% level.

neighboring state(s) employment growth. The mean equation of 3.1 for each state will be structured as follows.

$$y_{i,t} = \beta_{i,0} + \beta_{i,1}y_{us,t-1} + \sum_{a=1}^6 \beta_{i,a+1}y_{i,t-a} + \sum_{d=1}^{11} \beta_{i,c+d+1}dum_{d,t} + \lambda_{i,1}W_{y_{i,t-1}} + \epsilon_{i,t}$$

The next step is to test the persistence in the conditional volatility for employment growth, with the ARCH LM test as specified by Engle (1982). The results are displayed in **Table 3.2**.

As illustrated in **Table 3.2** at the first lag, the null hypothesis of homoscedasticity is rejected at the 10% level for 33 of the 49 states. For 6-lags, results indicate heteroskedasticity 38 of the 49 states at a 10% level of significance. Additionally, for one lag 16 states are significant at the 1% level, while 31 states are statistically significant at the 1% level when using 6 lags for the volatility structure. These test results indicate that the employment growth

rate demonstrates a time-dependent volatility even when serial correlation, unit root, and spatial spillovers effect are accounted for.

Evidence suggests that ARCH effects are still present when accounting for spatial spillovers; this is an important discovery for several reasons. First, it supports the hypothesis that volatility cluster occurs in state-level employment growth. The existence of clustering with associated state neighbors, means cross border employment is not the main driver for it. Additionally, this provides additional evidence that volatility clustering should be accounted for measuring the Great Moderation. As discussed earlier Carlinio, DeFina, and Sill (2007), and Morgan, Rime, and Strahan (2004), there has been a reduction in state-level employment growth volatility at the end of 1984. However, the effects of time-varying volatility were not accounted for, which means effects of the decline could be overstated. At a minimum, the regression is inefficient and since they are measuring the conditional variance, there is the potential bias.

The Great Moderation's effect on state-level employment. This should also be accounted for when studying the volatility in state-level employment. This effect will be examined by separating the dataset into two and excluding the permanent shock that occurred at the end of 1984, at the outset of the Great Moderation. Omitting this permanent shock is the final step in analyzing the ARCH effects of state employment growth. If ARCH effects are still present when excluding the Great Moderation, then it is not the only driver for the time-dependent volatility.

The first dataset ranges from January 1960 through December 1983, the second ranges from January 1985 to December 2009. This leaves 287 and 294 observations for the first and second datasets, respectively. The same mean regression will be used to compute the ARCH LM test for these two datasets to keep the results comparable. **Tables 3.3 & 3.4** displays the ARCH LM test results for the pre and post 1984 periods.

Referring to **Table 3.2** when the full sample is included, January 1960 to December 2009, 33 out of the 49 states illustrate ARCH effects. As shown in **Table 3.3** at 1-lag length, 16 out of the 49 states exhibit ARCH effects statistically significant at the 10% level when using the pre-1984 data. This increases to only 18 states when using 6-lags. For post-1984, see **Table 3.4**, 18 and 22 states are significant at the 1 and 6 lag lengths, respectively.

The separation of the data into pre and post 1984, excluding the year 1984, causes the magnitude of states with ARCH effects to decrease from about two-thirds to about one-third. This steep decline would suggest that the "Great Moderation" caused a single permanent shock, resulting in a need to forecast only the conditional mean equation to predict the state level employment growth for the majority of the states when this period is not

Table 3.3

ARCH LM test ARCH-statistics for all 49 States
Time Period: January 1960 through December 1983

State	1-Lag	6-Lag	State	1-Lag	6-Lag
	ARCH-statistic	ARCH-statistic		ARCH-statistic	ARCH-statistic
Alabama	0.29	6.98	Nebraska	0.11	2.40
Arizona	0.09	2.09	Nevada	12.05*	15.98*
Arkansas	0.02	10.29	New Hampshire	0.60	15.58*
California	0.28	1.39	New Jersey	0.79	9.16
Colorado	0.00	6.02	New Mexico	3.93*	10.21
Connecticut	8.65*	8.47	New York	1.62	4.59
Delaware	9.21*	10.91*	North Carolina	4.35*	6.62
District of Columbia	0.67	2.87	North Dakota	0.47	7.47
Florida	1.15	3.69	Ohio	0.01	40.43*
Georgia	5.66*	9.83	Oklahoma	6.29*	15.80*
Idaho	0.89	3.88	Oregon	0.52	8.37
Illinois	0.88	12.39*	Pennsylvania	0.19	12.68*
Indiana	0.36	17.45*	Rhode Island	10.08*	10.44
Iowa	2.73*	6.31	South Carolina	3.99*	6.74
Kansas	30.13*	34.88*	South Dakota	0.44	3.50
Kentucky	0.84	7.49	Tennessee	0.10	3.53
Louisiana	0.41	2.14	Texas	6.45*	7.64
Maine	38.96*	50.21*	Utah	1.46	14.24*
Maryland	0.02	0.99	Vermont	0.11	11.73*
Massachusetts	0.10	10.51	Virginia	0.04	0.49
Michigan	7.39*	27.77*	Washington	0.01	5.28
Minnesota	17.81*	21.13*	West Virginia	0.13	31.27*
Mississippi	3.63*	20.29*	Wisconsin	0.00	15.19*
Missouri	1.59	3.92	Wyoming	0.20	7.06
Montana	0.38	14.27*			

*Statistically Significant at the 10% level.

Table 3.4

ARCH LM test ARCH-statistics for all 49 States					
Time Period: January 1985 to December 2009					
State	1-Lag	6-Lag	State	1-Lag	6-Lag
	ARCH-statistic	ARCH-statistic		ARCH-statistic	ARCH-statistic
Alabama	5.39*	16.02*	Nebraska	0.44	16.40*
Arizona	0.24	10.64	Nevada	0.01	3.52
Arkansas	0.21	3.20	New Hampshire	2.38	12.42*
California	1.18	8.74	New Jersey	0.40	13.32*
Colorado	3.06*	29.83*	New Mexico	0.11	10.29
Connecticut	0.01	5.53	New York	0.02	20.00*
Delaware	21.54*	35.40*	North Carolina	0.25	4.47
District of Columbia	7.86*	9.74	North Dakota	1.87	10.58
Florida	0.39	18.79*	Ohio	0.33	13.12*
Georgia	45.48*	50.93*	Oklahoma	14.75*	40.30*
Idaho	7.29*	6.99	Oregon	3.83*	7.47
Illinois	0.01	6.52	Pennsylvania	12.31*	22.15*
Indiana	0.50	3.74	Rhode Island	9.62*	12.78*
Iowa	0.02	2.82	South Carolina	0.01	2.66
Kansas	0.62	30.26*	South Dakota	5.39*	8.84
Kentucky	0.02	2.84	Tennessee	0.85	3.39
Louisiana	0.09	0.35	Texas	6.26*	29.66*
Maine	4.44*	8.22	Utah	1.99	13.61*
Maryland	9.86*	16.69*	Vermont	3.93*	15.78*
Massachusetts	10.07*	19.70*	Virginia	3.71*	7.20
Michigan	1.64	6.62	Washington	0.11	12.34*
Minnesota	0.11	7.38	West Virginia	0.45	4.89
Mississippi	3.66*	4.52	Wisconsin	1.96	19.47*
Missouri	1.64	21.81*	Wyoming	0.47	8.33
Montana	0.35	7.92			

*Statistically Significant at the 10% level.

include. However for the insignificant states and a sample that includes the moderation period, a simultaneous estimation of the conditional mean and variance is needed.

3.4 Conclusion

This chapter is aimed at answering whether ARCH effects are present when accounting for spatial spillovers. In Chapter 2, we saw that 92% of the 51 states demonstrated ARCH effects at the 10% level according to the Engle (1982) Lagrange Multiplier test. However, one could argue the presence of time-dependent volatility is misclassified by employment movements between the host state and its neighbors.

To test this hypothesis, a STAR model was employed. A single lag of the neighboring state(s) employment was used, with the remainder of the regression being a duplicate of that used in Chapter 2. This setup ensured any reduction in time-dependent volatility is attributed to movements in their neighboring state(s) employment growth. **Table 3.2** illustrates that

the presence of ARCH effects declines to 67% of the 49 states, excluding Alaska & Hawaii.

Furthermore, the effect of the "Great Moderation" is examined by dividing the sample into two separate periods, pre and post 1984. The results, shown in **Tables 3.3 & 3.4**, are a further decline in the number of states with statistically significant ARCH effects. The pre and post 1984 samples, tested at 1-lag length have 33% and 37% of the 49 states with significant ARCH effects.

It would be interesting to study the volatility structure of the remaining 18 states that still display statistically significant ARCH effects in the post 1984 sample. The question would be, does this knowledge improve forecast performance, i.e. decrease MSE, of employment growth? Additionally, does the volatility structure itself tell a story about a particular state's employment growth?

Final Remarks

The collaboration of works are amalgamated and summarized. The principle of using non-linear models here was to increase our knowledge of economic forecasting, and reduce the mean squared error (MSE).

The first chapter illustrates the use of neural networks for discovering possible yield curve arbitrage opportunities. There are many theories about the dynamics of the yield curve. Given this heterogeneity, neural networks are a good choice for their adaptability. For each of the 6 different TTMs the ATSM and realized yield curves were through NAR and NARX networks. The difference between these two networks is NAR only includes autoregressive terms, while NARX also includes the use of external inputs. For NARX networks estimated here five macroeconomic variables are used, see Section 1.3.1.

The forecast results of each curve share advantages and disadvantages. For short-term bonds, 1-year or less, only the autoregressive terms are needed for forecasting a 3-month window. Macroeconomic variables aid in forecasting longer terms. Accompanied by comparing the forecast actual and ATSM yield curves, future arbitrage prospects can be surveyed. As shown in Figure 1.4, the three and six months TTM illustrate the greatest profitability. This concludes the first chapter.

The results of Chapter 2 & 3 are interlinked. In Chapter 2, the conditional variance for 47 out of the 51 states follow an autoregressive conditional heteroskedasticity (ARCH) process when studying the univariate state-level employment growth. This is according to the Engle (1982) LM test and is displayed in Table 2.2. Given this knowledge the next step was to create forecasts of the volatility structure. The algorithm illustrated in Figures 2.1 & 2.2 was used to determine the optimal ARCH structure for each state. Using a GARCH(1,1) as the volatility structure proved to provide the best out-of-sample forecast MSE, outperforming the ARCH structure and white noise process.

Chapter 3 arose from a criticism, by James LeSage Ph.D., that the volatility structure seen in Chapter 2 could be misclassified as spatial spillover ef-

fects. The basis of this theory was tested using a STAR model. All this required was to add a spatial lag to the mean regression, which is shown in equation 3.1. The results, shown in Table 3.2, displayed a sharp decline in the number of the states with ARCH effects. At 1-lag length, 33 of the 49 states were statistically significant.

As an extension, the effects of the 'Great Moderation' were examined. This was done by splitting the dataset into before and after 1984. The pre-1984 dataset only had 16 states with statistically significant ARCH effects, while post-1984 slightly increased to 18 states. This decline demonstrates that the 'Great Moderation' was a permanent shock on state-level employment growth. This shock should be properly accounted for in state employment growth forecast to ensure an unbiased MSE.

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