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**FORECASTING OF WORK IN PROCESS QUALITY USING
HOLT-WINTERS METHOD FOR MISSING OBSERVATIONS**

Sarang R. Kayande

**Thesis Submitted to the
College of Engineering and Mineral Resources
At West Virginia University
In Partial fulfillment of the Requirements for
the Degree of**

**Master of Science
in
Industrial Engineering**

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**Morgantown, West Virginia
1999**

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ABSTRACT

Forecasting of Work in Process Quality Using Holt-Winters Method for Missing Observations

Sarang R. Kayande

Since the pace of technological change is so great, and since new products and processes may be key to a company's future plans, an increasing number of companies are emphasizing regular and complete technological forecasts affecting their industry. Those companies which have gone far in developing planning premises from their technological forecasts have tended to be high-technology enterprises. What has been done in these instances is to encourage members of their technical staffs to be alert to future developments; to think in terms of the impact of current scientific developments on the future state of technology; and to develop orderly forecasts of how these developments affect the company's products, processes or markets. Many attempts have been made to accurately forecast future and some of the accurate and meaningful methods used to forecast the state of technology are the Delphi technique, the opportunity and goal oriented techniques.

The objective of this research was to develop a forecasting model using extension of Holt-Winters method for missing data. The variable of interest considered was the fraction non-conforming of a process. Initial values were generated using Beta distribution. Values of fraction non-conforming for future periods were generated using different processes such as Autoregressive and Autoregressive Moving Average process. Some of the values in each data set were assumed to be missing. The factors that were considered for forecasting using this method were the level, trend and the seasonal factor. Forecasting was done for at least one period ahead and at the most twelve periods ahead. The developed models were found to give acceptable results with as many as 40% of the total observations missing and this was validated by performing tracking signal analysis.

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1. INTRODUCTION AND RESEARCH OBJECTIVES

1.1 BACKGROUND

One of the principal activities for improving productivity in manufacturing systems is to forecast the product defects during processes and to recover the processes. The product defects come from typically even small changes during manufacturing processes that cause big variations in the product that in turn result in out of acceptable limits. It is therefore necessary to forecast the product quality during processes and finding desirable process conditions that result in high yield. Forecasting the in-process product quality requires a predictive model that identifies the relationship between a set of productivity measures and a set of process variables. Casual models are generally based on multiple regression. However, since the forecasting difficulty grows exponentially with the number of predictors, it is necessary to have various techniques so as to reduce the number of variables. Further the accuracy of forecasting results is truly dependant on the determination of the true nature of the relationships between the productivity measures (or “predictee”) and the process variables (or “predictor”). It is also important to know as to how well the model accounts for real manufacturing processes. So a casual forecasting model based on multiple regression needs to be developed to predict work-in-process quality under the current process conditions.

Much of the statistical methodology is concerned with models in which the observations are assumed to vary independently. In many applications dependence between the observations is regarded as a nuisance, and in planned experiments,

randomization of the experimental design is introduced to validate analysis conducted as if the observations were independent. However a great deal of data occur in the form of time series where observations are dependent and where the nature of this dependence is of interest in itself. The body of techniques available for the analysis of such series of dependent observations is time series analysis.

Time series are best represented by nonstationery models in which trends and other pseudo-systematic characteristic, which can change with time, are treated as statistical rather than as deterministic phenomena. Furthermore, some time series often posses marked seasonal or periodic components themselves capable of changing and needing seasonal statistical models for their description.

The process of model building is concerned with relating a class of statistical models to the data at hand and involves much more than model fitting. Thus, identification techniques, designed to suggest what particular kind of model might be worth considering, need to be developed first. The fitting of the identified model to a time series using likelihood function can then supply maximum likelihood estimates of the parameters. When accuracy of forecasts is the objective, the fitted statistical model is used directly to generate optimal forecasts by simple recursive calculation. In particular, this model completely determines whether the forecast projections should follow a straight line, an exponential curve, and so on. In addition, the fitted model allows one to see exactly how the forecasts utilize past data, to determine the variance of the forecast errors, and to calculate limits within which a future value of the series will lie with a given probability.

1.2 DEFINING THE FORECASTING PROBLEM

The use of available observations from a time series to forecast its value at some future time can provide a basis for planning, control, and optimization of industrial processes. Defining a forecasting problem starts with the decision problem. Information from the forecasting process is to be used to improve the decision process. In determining what is to be forecast, it is necessary to define the variables that are to be analyzed and predicted. The level of detail required is also an important consideration. Various factors influence the level of detail used: availability of data, accuracy attainable, cost of analysis, etc. In cases where the appropriate choice of variables is not clear, it is necessary to try several alternatives and select the one giving the best performance. This is generally done during the development of the forecasting system through simulation using historical data.

Another important class of decision involves elements such as the forecasting period, the forecasting horizon, and the forecasting interval. The forecasting period and horizon are usually dictated by the decision process requiring the forecast. For a forecast to be of value, the horizon must be no less than the lead-time for implementing the decision. The forecasting interval is determined by the operating mode of the data processing system that provides information on the variable being forecasted.

One more aspect of the forecasting problem relates to the required form of the forecast. It is generally convenient to conceive the variable of interest as being a random variable having a particular probability distribution. The decision problem requires an estimate of some characteristic of the distribution, such as the mean, median, or most

probable value for use as a forecast of the variable. Usually the forecast demands an estimate of the expected value of the variable, plus an estimate of the standard deviation of the forecast error.

1.3 TIME SERIES MODELS

A time series is a sequence of observations on a variable of interest. The variable is observed at discrete time points, usually equally spaced. Time series analysis involves describing the process or phenomena that generate the sequence. To forecast time series, it is necessary to represent the behavior of the process by a mathematical model that can be extended into the future.

It is required that the model be a good representation of the observations in any local segment of time close to the present. Once a valid model for the time series process has been established, an appropriate forecasting technique can be developed. Many of the models used to represent time series are algebraic or transcendental functions of time, or some composite model that combines both algebraic and transcendental components.

The idea of using a mathematical model to describe the behavior of a physical phenomenon is well established. In particular, it is sometimes possible to derive a model based on physical laws, which are capable of calculating the value of some time-dependant variable, such as quality, nearly exactly at any instant of time. Probably no phenomenon is deterministic because of the presence of unknown factors. In many problems it is required to consider a time-dependant phenomenon in which there are many unknown factors and for which it is not possible to write a deterministic model that

allows exact calculation of the future behavior of the phenomenon. Nevertheless, it may be possible to derive a model that can be used to calculate the probability of a future value. The models for time series that are needed are stochastic models.

In a given lot of any commodity the number of parts that lie outside the mentioned specification limits is a very vital information as far as the work in process quality is concerned. This information which is usually specified in terms of percent defective i.e. the percentage of parts lying outside the specification limits is calculated by figuring out the dispersion of the specification limits from the process average. One can mention numerous factors like the workers capability, the machine condition etc, which are very obvious and are bound to affect the percent defective. But we can state some of the very fundamental factors, which will definitely control the value of percent defective in a lot.

Primarily the discipline of acceptance sampling relies on different distributions for its calculations. The value of percent defective for a lot size certainly depends upon the probability distributions, which are being followed. It is very well known that a process can be said to be composed of different stages. These stages can be primarily stratified into three stages viz. the infant or the early stage, the matured or the stable stage and finally the wear out or the decline stage. The percent defective during these stages of the process life differ a lot like for example one will agree that the percent defectives during the decline stage will be more than during the stable stage.

Process capability can be said to be the measure of the spread of the process. By this study, it becomes possible to know the percentage of the products, which will be produced within specified limits on either side of the process mean. Thus it also

influences the value of percent defective to a certain extent. For example higher the process capability lower will be the value of percent defective for that lot.

1.4 MODEL SELECTION

In regression analysis the dependent variables Y_i are assumed to be independently distributed, but in reality they are not. They are correlated and follow a certain distribution. In the proposed model the dependent variable (Y_i) is the percent defective which depends on the factors as explained above. The basic aim is to develop a regression model, which will explain the relationship between the dependent variable and the various independent variables. Since there exists a correlation between Y_i 's a separate model needs to be developed to cater this aspect of the problem. This is being accomplished by developing a stochastic model. Some of the models considered are autoregressive model, moving average model, and ARIMA. Autoregressive model is a stochastic model, which can be extremely useful in representation of certain practically occurring series. To achieve greater flexibility in fitting of actual time series, it is sometimes advantageous to include both autoregressive and moving average terms in the model. ARIMA provides a powerful model for describing stationary and nonstationary time series. These models will be viewed as univariate time series models or for more than one time series. It is possible to build a model that uses the information content in other models to help forecast the dependent variable. These models are called transfer function models. The ARIMA model postulates that time series can be successfully represented as an output from a dynamic system to which the input is white noise and for which the transfer function can be parsimoniously expressed as the ratio of two

polynomials. The data analysis tool employed for the identification of transfer function model is the cross correlation function between the input and the output. Multivariate transfer function models are used to relate several mutually interacting output time series to one or more time series.

With suitable inputs and outputs, a dynamic system can represent any process. The value $Y_{\infty}(X)$ at which the discrete output from a stable system eventually comes to equilibrium when the input is held at a fixed level X is the steady state level of the output. Very often, over the range of interest, the relationship between $Y_{\infty}(X)$ and X will be approximately linear. Hence if we use Y and X to denote deviations from convenient origins situated on the line, we can write the steady state relationship as

$$Y_{\infty} = gX$$

where g is the steady state gain, and it is understood that Y_{∞} is a function of X .

Corresponding to the continuous representation, discrete dynamic systems are often parsimoniously represented by the general linear difference equation also referred to as a transfer function model.

$$\begin{aligned} (1 + \xi_1 \nabla + \dots + \xi_r \nabla^r) Y_t &= g (1 + \eta_1 \nabla + \dots + \eta_s \nabla^s) X_{t-b} \\ (1 - \partial_1 B - \dots - \partial_r B^r) Y^t &= (\omega_0 - \omega_1 B - \dots - \omega_s B^s) X_{t-b} \end{aligned} \quad (1.1)$$

In terms of backward shift operator $B=1-\nabla$, the above equation may also be written as
or as

$$\partial(B)Y_t = \omega(B)X_{t-b} \quad (1.2)$$

In these models X_t and Y_t were the deviations from equilibrium of the system input and output. In practice the system will be infected by disturbances, or noise, whose net effect it to corrupt the output predicted by the transfer function model by an amount N_t . The combined transfer function-noise model may then be written as

$$Y_t = \partial^{-1}(B)\omega(B)X_{t-b} \quad (1.3)$$

The basic aim of this research is to develop a regression model in which the dependant variable is the percent defective and independent variables are as mentioned before. To incorporate the correlation between the independent variables, i.e., to predict percent defective from a set of previously recorded percent defective values, a time series model is being developed. From the above models a third model, which will cover the relationship of these two models, will be developed. This is being done using transfer function model.

In developing the final model, noise factor will be considered which will also take into account the disturbances in the correlation as well as the regression model.

1.5 NEED FOR RESEARCH

Most of the traditional forecasting approaches deal with inventory management, production planning, financial planning, staff scheduling, facilities planning or process control. In today's world where process and product quality is of utmost importance, it is necessary to forecast process quality as well so as to take corrective measures and also

improve upon the existing process conditions. An approximate model can be developed using historical data to forecast work in process quality to be used in various applications.

In this research a similar model is being developed to forecast work in process quality. The variable of interest considered here is the percent non-conforming. This is assumed to follow beta distribution, which is a very general distribution. Not much research has been done in the past considering a variable percentage defective. This assumption is far more realistic as opposed to assuming a constant value of percent defective.

Forecasting using time series analysis and multiple regression is one of the most widely used approaches. Unfortunately very less research has been done in the field of forecasting work in process quality that lays out conditions under which effectiveness of a process can be studied. The objective of this research is to develop a model to forecast work in process quality and take corrective measures so as to improve over the existing conditions under the assumption of percent defective, which is the variable of interest being a random variable.

1.6 RESEARCH OBJECTIVES

The objective of this thesis is to develop a forecasting model using Holt-Winters method. This research will be aimed to present a procedure for interpolating, smoothing, and predicting work in process quality in seasonal time series with missing observations. The approach suggested by Wright (1986) for the Holt's method with nonseasonal data

published at irregular time intervals will be extended to the Holt-Winters method in the seasonal case. The variable considered will be percent defective and the data will be generated using various processes like Autoregressive process and Autoregressive Moving Average process. The exponential smoothing approaches, including the Holt's and Holt-Winters methods, constitute the basic equipment of practical time series analysis. Wright has suggested such extensions for simple exponential smoothing and for Holt's method. This research extends this approach to the Holt-Winters method for seasonal method for seasonal data with missing observations.

1.7 ORGANIZATION

The organization of this thesis consists of five chapters. Chapter 2 comprises of the literature regarding the basic forecasting concepts and various approaches to forecasting. Time series analysis and exponential smoothing approaches are presented in this chapter. Chapter 3 presents the methodology followed to develop the model and lists the various steps involved. Chapter 4 consists of the results obtained using the methodology. Chapter 5 summarizes the conclusions drawn from this research and provides recommendations for future research.

2. LITERATURE SURVEY

One of the operators which are employed extensively for stochastic models for forecasting and control is the backward shift operator B which is defined by $Bz_t = z_{t-1}$; hence $B^m z_t = z_{t-m}$. The inverse operation is the reverse of the forward shift operator $F = B^{-1}$ given by $Fz_t = z_{t+1}$. Another important operator is the backward difference operator ∇ , which can be written in terms of B , since

$$\nabla z_t = z_t - z_{t-1} = (1 - B)z_t \quad (2.1)$$

In turn, ∇ has for its inverse the summation operator S given by

$$\nabla^{-1} z_t = Sz_t = \sum_{j=0}^{\infty} z_{t-j} = (1 - B)^{-1} z_t \quad (2.2)$$

2.1 LINEAR FILTER MODEL

The stochastic models that are employed are based on the idea that a time series in which successive values are highly dependent can be usefully regarded as having been generated from a series of independent “shocks” a_t . These shocks are random drawings from a fixed distribution usually assumed normal and having mean zero and variance σ_a^2 . Such a sequence of random variables is called a white noise process. The white noise process is supposed to be transformed to the process z_t by what is called a linear filter as shown in figure 2.1. The linear filtering operation takes a weighted sum of previous observations so that

$$z_t = \mu + a_t + \psi_1 a_{t-1} + \psi_2 a_{t-2} + \dots = \mu + \psi(B)a_t \quad (2.3)$$

In general, μ is a parameter that determines the “level” of the process and

$$\psi(B) = 1 + \psi_1 B + \psi_2 B^2 + \dots \quad (2.4)$$

is the linear operator that transforms a_t into z_t and is called the transfer function of the filter.

Figure 2.1

The sequence Ψ_i formed by the weights may, theoretically, be finite or infinite. If this sequence is finite, or infinite and convergent, the filter is said to be stable and the process to be stationary. The parameter μ is then the mean about which the process varies. Otherwise, z_t is non-stationary and μ has no specific meaning except as a reference point for the level of the process.

2.1.1 AUTOREGRESSIVE PROCESS

A stochastic model, which can be extremely useful in the representation of certain practically occurring series, is the so-called autoregressive model. In this model, the current value of the process is expressed as a finite, linear aggregate of previous values of the process and a shock. If the values of a process at equally spaced times $t, t-1, \dots$ are z_t ,

$$\tilde{z}_t = \phi_1 \tilde{z}_{t-1} + \phi_2 \tilde{z}_{t-2} + \dots + \phi_p \tilde{z}_{t-p} + a_t \quad (2.5)$$

$z_{t-1} \dots$ then,

Where

$\tilde{z}_t = z_t - \mu$ is the deviation from the mean

is called an autoregressive process of order p . The reason for this name is that a linear model relating a dependent variable z to a set of independent variables plus an error term is a regression model and z is said to be regressed on the independent variables. In equation (2.5) the variable z is regressed on the previous values of itself; hence the model is autoregressive. An autoregressive operator of order p can be defined by

$$\Phi(B) = 1 - \Phi_1 B - \Phi_2 B^2 - \dots - \Phi_p B^p \quad (2.6)$$

and the autoregressive model may be written economically as

$$\Phi(B)\tilde{z}_t = a_t \quad (2.7)$$

The model contains $p+2$ unknown parameters $\mu, \phi_1 \dots \phi_p, \sigma_a^2$ which in practice have to be estimated from the data. The autoregressive model is a special case of the linear filter model. This can be seen by eliminating z_{t-1} from the right hand side of equation (2.5) by substituting

$$\tilde{z}_{t-1} = \Phi_1 \tilde{z}_{t-2} + \Phi_2 \tilde{z}_{t-3} + \dots + \Phi_p \tilde{z}_{t-p-1} + a_{t-1} \quad (2.8)$$

Similarly, further substitution eventually yields an infinite series in the a 's. Symbolically it can be written as

$$\Phi(B)\tilde{z}_t = a_t \quad \text{or} \quad \tilde{z}_t = \psi(B)a_t \quad \text{with} \quad \psi(B) = \Phi^{-1}(B)$$

Autoregressive process can be stationary or non-stationary. For the process to be stationary, the ϕ 's must be chosen such that they are the respective weights form a convergent series.

2.2 MOVING AVERAGE MODELS

The autoregressive model expresses the deviation of the process as a finite weighted sum of p previous deviations of the process plus a random shock. Another kind of model, of great practical importance in the representation of observed time series, is the finite moving average process. Here z_t is made linearly dependent on a finite number, q of previous a 's. Thus,

$$\tilde{z}_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \dots - \theta_q a_{t-q} \quad (2.9)$$

is called a moving average (MA) process of order q . The name “moving average” is somewhat misleading because the weights which multiply the a 's need not total unity nor need they be positive. If a moving average operator of order q is defined by

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q \quad (2.10)$$

then the moving average model may be written as

$$\tilde{z}_t = \theta(B)a_t$$

it contains $q+2$ unknown parameters $\mu, \phi_1, \dots, \phi_q, \sigma_a^2$ which in practice can be estimated from the data.

2.2.1 MIXED AUTOREGRESSIVE-MOVING AVERAGE MODELS

To achieve greater flexibility in fitting of actual time series, it is sometimes advantageous to include both autoregressive and moving average terms in the model. This leads to the mixed autoregressive-moving average model

$$\tilde{z}_t = \Phi_1 \tilde{z}_{t-1} + \dots + \Phi_p \tilde{z}_{t-p} + a_t - \theta_1 a_{t-1} - \dots - \theta_q a_{t-q} \quad (2.11)$$

which is equivalent to

$$\Phi(B) \tilde{z}_t = \theta(B) a_t$$

which employs $p+q+2$ unknown parameters μ ; $\phi_1 \dots \phi_p$; $\theta_1 \dots \theta_q$; σ_a^2 that are estimated from the data.

In practice, it is frequently true that adequate representation of actually occurring stationery time series can be obtained with autoregressive, moving average, or mixed models, in which p and q are not greater than 2 and often less than 2.

2.2.2 NONSTATIONARY MODELS

Many series encountered in practice exhibit nonstationary behavior and in particular do not vary about a fixed mean. Such series may nevertheless exhibit homogeneous behavior. In particular, although the general level about which fluctuations are occurring may be different at different times, the broad behavior of the series, when differences in level are allowed for, may be similar. Such behavior may be represented by a generalized autoregressive operator $\Phi(B)$, in which one or more of the zeroes of the polynomial $\Phi(B)$ (i.e. one or more of the roots of the equation $\Phi(B) = 0$) is unity. Thus the operator $\Phi(B)$ can be written as $\Phi(B) = \Phi(B)(1-B)^d$ where $\Phi(B)$ is a stationary operator. Thus, a general model that can represent homogeneous nonstationary behavior is of the form

$$\Phi(B) z_t = \Phi(B)(1-B)^d z_t = \theta(B) a_t \quad (2.12)$$

$$\text{or } \Phi(B) w_t = \theta(B) a_t$$

$$\text{where } w_t = \nabla^d z_t$$

Homogeneous nonstationary behavior can therefore be represented as a model which calls for the d'th difference of the process to be stationary. In practice d is usually 0, 1, or at most 2.

The process defined by the above two equations provides a powerful model for describing stationary and nonstationary time series and is called an autoregressive integrated moving average (ARIMA) process, of order (p,d,q). The process is defined by

$$w_t = \phi_1 w_{t-1} + \dots + \phi_p w_{t-p} + a_t - \theta_1 a_{t-1} - \dots - \theta_q a_{t-q} \quad \text{where} \quad w_t = \nabla^d z_t \quad (2.12a)$$

If w_t is replaced by $z_t - \mu$, when $d=0$, the above model will include the stationary mixed model as a special case and the pure autoregressive model and the pure moving average model. The relationship which is inverse to the relationship in equation (2.12a) is $z_t = S^d w_t$ where S is the summation operator and is defined by

$$S w_t = \sum_{j=0}^{\infty} w_{t-j} = w_t + w_{t-1} + w_{t-2} + \dots \quad (2.13)$$

thus the ARIMA process can be generated from white noise by means of three filtering operations as indicated by the block diagram of figure 2.2. The first filter has input a_t , transfer function $\theta(B)$ and output e_t , where

$$e_t = a_t - \theta_1 a_{t-1} - \dots - \theta_q a_{t-q} = \theta(B) a_t$$

Figure 2.2 Block Diagram for ARIMA model

The second filter in the above diagram has input e_t , transfer function $\phi^{-1}(B)$ and output w_t according to

$$w_t = \phi_1 w_{t-1} + \dots + \phi_p w_{t-p} + e_t = \phi^{-1}(B)e_t \quad (2.14)$$

Finally, the third filter has input w_t and output z_t , and has transfer function S^d .

2.3 TRANSFER FUNCTION MODELS

An important type of dynamic relationship between a continuous input and a continuous output, is that in which the deviations of input X and of output Y , from appropriate mean values, are related by a differential equation of the form

$$(1 + \Xi_1 D + \dots + \Xi_R D^R)Y(t) = (H_0 + H_1 D + \dots + H_S D^S)X(t - \tau) \quad (2.15)$$

Where D is the differential operator d/dt , the Ξ 's and H 's are unknown parameters and τ is a parameter which measures the dead-time or pure delay between input and output. In a similar way, for discrete data, the transfer between output Y and input X , each measured at equispaced times can be represented by the difference equation

$$(1 + \xi_1 \nabla + \dots + \xi_r \nabla^r)Y_t = g(1 + \eta_1 \nabla + \dots + \eta_s \nabla^s)X_{t-b} \quad (2.16)$$

In which the differential operator D is replaced by the difference operator ∇ . An equation of the above form containing only a few parameters may often be used as an approximation to a dynamic relationship, whose true nature is more complex. This linear model can be written in terms of past values of input and output by substituting $B=1-\nabla$,

$$(1 - \partial_1 B - \dots - \partial_r B^r)Y_t = (\omega_0 - \omega_1 B - \dots - \omega_s B^s)X_{t-b} = (\omega_0 B^b - \omega_1 B^{b+1} - \dots - \omega_s B^{b+s})X$$

or

$$\partial(B)Y_t = \omega(B)B^b X_t = \Omega(B)X_t \quad (2.17)$$

alternatively, it can also be said that the output Y_t and the input X_t are linked by a linear filter

$$Y_t = v_0 X_t + v_1 X_{t-1} + v_2 X_{t-2} + \dots = v(B)X_t \quad (2.18)$$

for which the transfer function

$$v(B) = v_0 + v_1 B + v_2 B^2 + \dots$$

can be expressed as a ratio of two polynomials

$$v(B) = \Omega(B) / \partial(B) = \partial^{-1}(B)\Omega(B) \quad (2.19)$$

the linear filter is said to be stable if the above series converges for $|B| \leq 1$. The series of weights, which appear in the transfer function, is called the impulse response function.

2.3.1 PARSIMONY

Mathematical models contain certain constants or parameters whose values must be estimated from the data. It is important, in practice, to employ the smallest number of parameters for adequate representation. The central role played by the principle of parsimony can be seen to be evident from the illustration that follows. Here a dynamic model of the following form can be considered [Box and Jenkins, 1976]

$$Y_t = (\eta_0 + \eta_1 \nabla + \eta_2 \nabla^2 + \dots + \eta_s \nabla^s) X_t \quad (2.20)$$

Which can also be represented as

$$(1 + \xi \nabla) Y_t = X_t$$

The second model here contains only one parameter ξ but, for s sufficiently large, could be represented approximately by the previous model. Because of experimental error, the relationship between the coefficients in the fitted equation may be misinterpreted. Thus, a relationship like the previous equation could be developed unnecessarily, which will contain s parameters. This could lead to poor estimation of the output Y_t for given values of the input. It is necessary that the objective should be to obtain adequate but parsimonious models. Forecasting and control procedures could be seriously deficient if these models were either inadequate or unnecessarily prodigal in the use of parameters. Care and effort is needed in selecting the model. The process of selection is necessarily iterative; that is, it is a process of evolution, adaptation, or trial and error.

In practice many data series contain observations at irregular times whereas most forecasting methods are restricted to the case of equal time intervals between data points. The whole range of forecasting methods available in the literature from elementary smoothing methods to sophisticated Box Jenkins Analysis is almost entirely restricted in producing forecasts from data series with a fixed time interval between observations. Newbold (1981), in his reviews of the forecasting literature, deals entirely with such methods. In practice, however, data often occur intermittently.

Firstly, the frequency of reporting of a series may change. Equivalently older data may be aggregated to give a less frequent reporting interval when summary statistics are compiled. Secondly, the data may naturally occur at irregular time intervals. Thirdly, occasional data points may simply be unavailable in an otherwise regularly spaced series.

This does not often occur in published time series. Interpolation can also be used when data points are missing but not in the first two cases described above.

2.4 EXPONENTIAL SMOOTHING FOR IRREGULARLY SPACED DATA

A time series x_i occurring at regularly spaced times $i=0\dots t$ can be “exponentially smoothed” by taking a weighted average of all the data points with a weighted factor $(1 - a)^{t-i}$ for x_i , $0 < a < 1$. A sequential method of calculating this weighted average is the standard equation for single exponential smoothing, Brown (1959):

$$\bar{x}_t = (1 - a)\bar{x}_{t-1} + ax_t \quad (2.21)$$

The extension of (2.21) to the case of data at irregular times is obtained by extending the weighted average upon which (2.21) is based:

$$\bar{X}_n = A_n V_n, \quad (2.22)$$

Where the weighted average,

$$A_n = \sum_{i=-\infty}^n (1 - a)^{t_n - t_i} X_i \quad (2.23)$$

And the normalizing factor,

$$\frac{1}{V_n} = \sum_{i=-\infty}^n (1 - a)^{t_n - t_i} \quad (2.24)$$

The sequential form of (2.23) and (2.24) is:

$$A_n = b_n A_{n-1} + X_n, \quad (2.25)$$

$$V_n = V_{n-1} / (b_n + V_{n-1}), \quad (2.26)$$

where

$$b_n = (1 - a)^{t_n - t_{n-1}} \quad (2.27)$$

Substituting (2.25) into (2.22) we obtain:

$$\bar{X}_n = (1 - V_n) \bar{X}_{n-1} + V_n X_n \quad (2.28)$$

The extension of single exponential smoothing for data with irregular time spacing therefore consists of (2.26), (2.27) and (2.28) with initial conditions

$$\bar{X}_0 = X_0 \quad (2.29)$$

$$V_0 = 1 - (1 - a)^q \quad (2.30)$$

The initial condition, (2.30), is an approximation obtained by assuming all data earlier than time t_0 occurred at the same time spacing, q , which is taken to be the average time spacing of the data. It is important to note that the extended form (2.28) and the regular form (1) correspond in the sense of following the basic principle

new estimate = (1 – parameter) * previous estimate + parameter * new information.

In the extended form (2.28) the parameter changes reflecting changes in time spacing.

2.5 HOLT'S METHOD FOR REGULARLY SPACED DATA

This consists of fitting a trend line $l_t - m_t (t - i)$ to the data where the estimates of l_t and m_t are obtained sequentially from single exponential smoothing equations based on above relation. In the case of irregularly spaced data the following line can be fitted:

$$L_n - M_n(t_n - t_i) \quad (2.31)$$

In the data X_I using (2.31) as follows. At time, t_n the previous estimate of L_n is

$$l_{n-1} + (t_n - t_{n-1})M_{n-1} \quad \text{and the new information is } X_n.$$

The linearity of the approximation (2.32) implies that the slope does not change. Thus, the previous estimate of L_n is M_{n-1} . The new information about M_n is $(L_n - L_{n-1})/(t_n - t_{n-1})$.

This leads to the two equations:

$$L_n = (1 - V_n)[L_{n-1} + (t_n - t_{n-1})M_{n-1}] + V_n X_n \quad (2.32)$$

$$M_n = (1 - U_n)M_{n-1} + U_n(L_n - L_{n-1})/(t_n - t_{n-1}) \quad (2.33)$$

The derivations of this result follows ad hoc arguments rather than a strict mathematical proof, and this corresponds directly with the derivation given by Holt (1960). The parameter V_n is updated by (2.26) and (2.27) and U_n by the corresponding equations:

$$U_n = U_{n-1} / (d_n + U_{n-1}) \quad (2.34)$$

$$d_n = (1 - c)^{t_n - t_{n-1}} \quad (2.35)$$

The extended Holt's method therefore maintains one of the primary advantages of the regular Holt's method, viz. the flexibility afforded by having two smoothing parameters:

$$0 < a < 1, \quad 0 < c < 1.$$

Gilchrist (1976) recommends that the use of least squares regression on the first few data points as a means of obtaining initial conditions on the level and slope in Holt's method. We use the same approach with irregularly spaced data taking initial conditions for (2.32) and (2.33) from the slope B of the regression line and its intercept a at time t_0 :

$$L_0 = A \quad (2.36)$$

$$M_0 = B \quad (2.37)$$

The initial condition on U_n corresponds to (2.30)

$$U_0 = 1 - (1 - c)^q \quad (2.38)$$

The sequential nature of this forecasting procedure renders it highly efficient in terms of computer storage and time. The number of variables to be stored is 2 as compared to 5 for the regular Holt's method. The computation time is dependent on equations (2.27) and (2.35). If the time intervals between data points are integer multiples of a base value, then these equations can be evaluated by table look up. This is the case when the frequency of reporting changes, e.g. from quarterly to monthly, or when there are occasional observations missing in an otherwise regular series. If, however the data occur at completely irregular times then (2.27) and (2.35) must be evaluated by exponentiation, which is much more time consuming. The other variable contributor to computation time is $t_n - t_{n-1}$ in (2.32) and (2.33). In the case of changed reporting frequency or missing data in a regular series, $t_n - t_{n-1}$ can be scaled to be 1 for much of the analysis. This accelerates the computation of (2.32) and (2.33).

The literature on time series analysis with missing observations offers a broad range of methods that differ in their theoretical level, numerical complexity, and general applicability. From a practical point of view, extensions of the classical methods of exponential smoothing to the case of missing observations (or more generally, to the case of irregularly observed data) seem to be very important. The exponential smoothing approaches, including Holt's and Holt-Winters methods, constitute now the basic equipment of practical time series analysis.

Wright (1986) has suggested such extensions for simple exponential smoothing and for Holt's method, which give good practical results in spite of their apparent numerical simplicity, as already discussed above. In this research this approach can be extended to the Holt-Winters method for seasonal data with missing observations. Although this extension, like Wright's, can be based on ad hoc arguments, it is expected to work well in practice. It can be treated in an acceptable way similar to seasonal time series even with 50% missing observations. Moreover, some theoretical investigations comparing the suggested procedure with the optimal results obtained by interpolating missing observations in the corresponding seasonal ARIMA models have been done.

2.6 EXTENSION OF THE HOLT-WINTERS METHOD

Let us consider observations $y_{t_1}, y_{t_2}, \dots$ published at irregularly spaced times $t_1 < t_2 < \dots < t_n$ (in general these times need not be integers). In this case of the simple exponential smoothing and Holt's method for nonseasonal irregularly spaced data, as already discussed, Wright's (1986) argument consist of application of recursive scheme of the type:

Estimate for time $t_n = V_{t_n} * \text{new info at time } t_n + (1 - V_{t_n}) * \text{estimate for time } t_n \text{ at time } t_{n-1}$

$$V_{t_n} = \frac{V_{t_{n-1}}}{b_{t_n} + V_{t_{n-1}}}, \quad b_{t_n} = (1 - \alpha)^{t_n - t_{n-1}}$$

Where the smoothing coefficient V_{t_n} is also calculated as

And the constant α would be used as the smoothing coefficient in the classical case of regularly observed data. For instance, in the case of simple exponential smoothing the formula (2.39) can be used as

$$\hat{y}_{t_n} = V_{t_n} y_{t_n} + (1 - V_{t_n}) \hat{y}_{t_{n-1}} \quad (2.40)$$

Where the R.H.S value is the smoothed value for time t_i . The non-recursive form

$$y_{t_n} = V_{t_n} \sum_{i=1}^n (1 - \alpha)^{t_n - t_i} y_{t_i},$$

$$V_{t_n} = \frac{1}{\sum_{i=1}^n (1 - \alpha)^{t_n - t_i}} \quad (2.41)$$

In the formulae (2.40) and (2.41) shows that it is the weighted average of observations with decreasing exponential weights, reflecting the irregular character of observations. The schemes (2.39) and (2.40) can be used to extend the Holt-Winters method. Let p denote the seasonal length. Further times $t_1 < t_2 < \dots < t_n$ correspond to time points chosen from a regular seasonal pattern. It can also be assumed that for each of p seasonal periods at least one observation is present in an initial portion of data. If the observations are completely missing for a seasonal period, then it is impossible to estimate the seasonal index for this seasonal period.

Let the symbols S_t , T_t , and I_t , denote the estimated level, trend and seasonal index at time t , respectively. It is sufficient to give formulas of the extended Holt-Winters method for the additive case only since, then their form for the multiplicative case will be

$$S_{t_n} = V_{t_n} (y_{t_n} - I_{t_n}) + (1 - V_{t_n}) [S_{t_{n-1}} + (t_n - t_{n-1}) T_{t_{n-1}}] \quad (2.42)$$

immediate.

$$T_{t_n} = U_{t_n} \frac{S_{t_n} - S_{t_{n-1}}}{t_n - t_{n-1}} + (1 - U_{t_n}) T_{t_{n-1}} \quad (2.43)$$

$$I_{t_n} = W_{t_n} (y_{t_n} - S_{t_n}) + (1 - W_{t_n}) I_{t_n} \quad (2.44)$$

$$V_{t_n} = \frac{V_{t_{n-1}}}{b_{t_n} + V_{t_{n-1}}}, \quad b_{t_n} = (1 - \alpha)^{t_n - t_{n-1}} \quad (2.45)$$

$$U_{t_n} = \frac{U_{t_{n-1}}}{d_{t_n} + V_{t_{n-1}}}, \quad d_{t_n} = (1 - \gamma)^{t_n - t_{n-1}} \quad (2.46)$$

$$W_{t_n} = \frac{W_{t_n}^{\wedge}}{f_{t_n} + W_{t_n}^{\wedge}}, \quad f_{t_n} = (1 - \delta)^{(t_n - t_n^{\wedge}) / p} \quad (2.47)$$

Where t_n^* denotes the largest value among t_{n-1} , t_{n-2} , such that the time t_n^* corresponds to the same seasonal period as time t_n . The constants α, γ, δ ($0 < \alpha, \gamma, \delta < 1$) would be used as the smoothing coefficients by the classical Holt-Winters method in the case without missing observations.

The recursive formulas (2.43), (2.44), (2.46) and (2.47) for the level and trend correspond to the ones of the Holt's method extended by Wright (1986) and as already discussed earlier. The estimation of the seasonal indices according to (2.45), (2.47) can be explained as follows: the estimated seasonal index I_{t_n} combines the new observation adjusted by the estimated trend S_{t_n} with the most recent estimate $I_{t_n}^{\wedge}$ of the seasonal index for the same seasonal period.

The estimated level, trend, and seasonal indices enable us to calculate the

$$\hat{y}_{t_{n+m}}(t_n) = S_{t_n} + mT_{t_n} + I_{(t_{n+m})}^{\wedge} \quad \text{for } m = 1, 2, \dots \quad (2.48)$$

predicted values for m steps ahead.

the smoothed values

$$\hat{y}_{t_n} = S_{t_n} + I_{t_n}$$

and the interpolated values

$$\hat{y}_t = \hat{y}_t(t_{n-1}) \quad \text{for } t = t_{n-1} + 1, \dots, t_n \quad (2.49)$$

where $(t_n + m)^*$ in equation 2.48 denotes the largest value among $t_n, t_{n-1}, \dots$ such that the time $(t_n + m)^*$ corresponds to the same seasonal period as the time $t_n + m$. The initial values $S_0, T_0, I_i, V_0, U_0, W_i$ ($i = -p+1, -p+2, \dots, 0$), which are necessary to start the recursive formulas (2.46), (2.49) at time $t_0 = 0$, can be calculated following the practical recommendation by Chatfield and Yar (1988) for the classical Holt-Winters method:

$$T_0 = \frac{\bar{y}_{k_1} - \bar{y}_{k_0}}{(k_1 - k_0)p} \quad (2.49a)$$

$$S_0 = \bar{y}_{k_0} - \left(k_0 p - \frac{p-1}{2} \right) T_0 \quad (2.49b)$$

$$I_i = \frac{1}{k(i)} \sum_k I_{i+kp} \quad (2.49c)$$

for $i = -p+1, -p+2, \dots, 0$

$$V_o = 1 - (1 - \alpha)^q, \quad U_0 = 1 - (1 - \gamma)^q$$

$$W_i = 1 - (1 - \partial)^{Q_i} \quad \text{for } i = -p+1, -p+2, \dots, 0, \quad (2.49d)$$

where

$$I_{i+kp} = y_{i+kp} - \left[\bar{y}_k + \left(i + \frac{p-1}{2} \right) T_0 \right] \quad (2.49e)$$

and the following notation is used:

k_0 is such that the k_0 th season is the first nonempty season containing at least one observation;

k_I is minimal ($k_I \geq k_0 + 1$) such that the first k_I seasons contain at least one observation for each of the p seasonal periods;

the sum in equation 2.52 is over all such k that the observations y_{i+kp} ($k = 1, \dots, k_I$) are present in the first k_I seasons and $k(i)$ is the number of summands in this sum;

q is the average time spacing of the data (i.e., every q th observation is at our disposal on average)

Q_i is the average time spacing in the framework of the i th seasonal period.

Since the assessment of the average time spacing q at Q_i may be complicated, for the sake of simplicity it can be assumed that $q = Q_{-p+1} = Q_{-p+2} = \dots = Q_0 = 1$ in the initial values. This assumption does not affect the forecasting model and seems to work well in practice.

2.7 TRACKING SIGNAL TESTS

Having developed statistics for estimating the mean and variance of forecast errors, the next step is to use these results to periodically monitor and control the forecasting process. Generally, a major concern is that the forecasts be unbiased, i.e., that the expected value of forecast error be zero. To test this hypothesis, the deviation of the estimate of the expected forecast error from zero can be measured, relative to the natural variability of this statistic about its true mean. If this deviation is large, the hypothesis would be rejected.

2.7.1 TESTS BASED ON THE CUMULATIVE FORECAST ERROR

Suppose $Y(T)$ is the sum of errors and this is used as the estimate of the expected error. At any time T , $Y(T)$ will undoubtedly differ from zero. There are two possible explanations: (1) The forecasts are unbiased and the deviation from zero is due to random variation; and (2) the forecasts are biased and the deviation is due to a combination of bias and random variation. To distinguish between the two cases, a measure of the natural random variation in the statistic $Y(T)$ about its mean is needed. The standard deviation of $Y(T)$, σ_y , provides this information. Assuming that σ_y can be estimated, it can be concluded that the forecast is biased if $|Y(T)|$ is greater than some multiple K_1 of this estimate of the standard deviation, that is if

$$\left| \frac{Y(T)}{\hat{\sigma}_y} \right| > K_1 \quad 2.50$$

To estimate σ_y a relationship between σ_y^2 and σ_ϵ^2 is sought, in which the estimate of the latter can be used to obtain that of the former. Brown (refer), gives the following results for direct smoothing of constant and linear processes:

$$\sigma_y^2 = \frac{1}{1 - \beta^2} \sigma_\epsilon^2 \quad \text{constant} \quad 2.51$$

$$\sigma_y^2 = \frac{2 - 2\alpha^2 + \alpha^4}{4 - \alpha - 4\alpha^2 + 2\alpha^3} \frac{1}{1 - \beta^2} \sigma_\epsilon^2 \quad \text{linear} \quad 2.52$$

In general, for direct smoothing with an n -parameter forecasting model, brown found through simulation studies that the following relationship is approximately correct:

$$\sigma_y^2 = \frac{1}{1 - \beta^{2n}} \sigma_\epsilon^2 \quad 2.53$$

Normally, the decision rule given by equation 2.50 is operated using the mean absolute deviation, Δ rather than σ_Y . Equation 2.54 expresses the relation between estimate of the error variance and Δ , so this can be substituted in equation 2.53 and then the result in equation 2.50 to obtain

$$\hat{\sigma}_Y \approx [c_1(1 - \beta^{2n})]^{1/2} (1.25\hat{\Delta}) \quad 2.54$$

$$\left| \frac{Y(T)}{\hat{\Delta}(T)} \right| > \frac{1.25K_1}{[c_1(1 - \beta^{2n})]^{1/2}} \equiv K_2 \quad 2.55$$

The ratio $Y(T)/\Delta(T)$ is called the tracking signal, computed at time T . Each period the absolute value of the tracking signal is compared with the constant K_2 to test the hypothesis that the expected forecast error is zero. Since K_1 is chosen to be 2 or 3, it turns out that K_2 is typically between 4 and 6.

2.7.2 TESTS BASED ON THE SMOOTHED FORECAST ERROR

It is also possible to develop a tracking signal test based on the smoothed error statistic $Q(T)$. The hypothesis of an unbiased forecast would be rejected if,

$$\left| \frac{Q(T)}{\hat{\Delta}(T)} \right| > K_3 \quad 2.56$$

It is clear that the above ratio can never exceed 1.0. the proper value of K_3 is usually between 0.2 and 0.5. For a constant model, where the level of time series is estimated by exponential smoothing, Brown has shown that

$$\sigma_Q^2 = \frac{\alpha}{(1 + \beta)^2} \sigma_\epsilon^2 \quad 2.57$$

If the tracking signal test is of the form $|Q(T)/\sigma_Q| > K_1$, where K_1 is usually between 2 and 5, substitution will yield

$$\left| \frac{Q(T)}{\hat{\sigma}_e} \right| > \frac{\sqrt{\alpha} K_1}{1 + \beta}$$

$$\left| \frac{Q(T)}{\hat{\Delta}(T)} \right| > \frac{1.25\sqrt{\alpha}}{1 + \beta} K_1 \equiv K_3 \quad 2.58$$

Although the result in equation 2.57 is for a constant model, it is a good approximation for other models. From simulation runs using a seasonal model similar to equation 2.57 Brown found that $\sigma_Q \approx \alpha^{0.5} \Delta$.

The smoothed error form of the tracking signal appears to have several advantages relative to the cumulative error version. Suppose that there was a large, but random, error in the forecast for a particular time period. This would increase both the smoothed error and the cumulative error, but assume that the increase is not quite enough to generate an out-of-control signal. Now suppose that several periods go by with small errors averaging zero. The cumulative error remains at its large value, but the smoothed error decreases toward zero. At some point of time, a second large, random error with the same sign as the previous one occurs. The cumulative error tracking signal would incorrectly generate an out-of-control signal, but the smoothed error version would not.

As a second example, suppose that the cumulative error tracking signal was just less than the control limit, and a period having zero forecast error occurs. The cumulative error would not change, but the mean absolute deviation estimate would become smaller. The net effect would be to increase the value of tracking signal, perhaps above the control limit. If the smoothed error tracking signal had been employed, the forecast would have caused the smoothed error to decrease along with the mean absolute deviation, resulting in little change in the tracking signal.

3. RESEARCH METHODOLOGY

The literature on time series analysis with missing observations offers a broad range of methods that differ in their theoretical level, numerical complexity, and general applicability. The exponential smoothing approaches, including the Holt's and Holt-Winters methods constitute the basic equipment of practical time series analysis. This research presents a procedure for interpolating, smoothing, and predicting in seasonal time series with missing observation. The approach suggested by Wright (1986) for the Holt's method for non-seasonal data published at irregular time intervals is extended to the Holt-Winters method in the seasonal case. Numerical examples demonstrate the procedure.

3.1 ASSUMPTIONS

The fraction non-conforming of a process has usually been assumed to be constant throughout the quality control literature. This is often not the case in the real world situations. In this research, the variation in fraction non-conforming is recognized and applied to the decision making criteria. The initial value in all the time series is generated using Beta distribution. The density function of the Beta distribution varies between 0 and 1 and the value of the fraction non-conforming varies between the same limits. Further values for fraction non-conforming are generated assuming the process to be Autoregressive or Moving Average or Autoregressive-Moving Average. This assumption is more realistic than the traditional approach.

Another assumption used to generate the values of fraction non-conforming is that the error terms are normally distributed with mean zero and a certain variance. Three smoothing constants are used to develop the model and the values of these constants are assumed to be $\alpha=0.4$, $\gamma=0.1$, $\delta=0.4$, where α is the coefficient of level factor, γ is the coefficient of trend factor and δ is the coefficient of seasonal factor in consideration as explained before. These values are "guestimates" of the smoothing coefficients as recommended by Chatfield and Yar, 1988. For performing the tracking signal analysis, values of α (smoothing constant) and β are assumed to be between 0.1 and 0.3 and 0.7 and 0.95 respectively.

3.2 SYMBOLS AND NOTATIONS

The symbols and notations that were employed in the research are summarized below:

y_t	Observation at time t
S_t	Level index at time t
T_t	Trend index ant time t
I_t	Seasonal index at time t
α, γ, δ	Smoothing constants, $0 < \alpha, \gamma, \delta < 1$
V_t	Coefficient of level index at time t
U_t	Coefficient of trend index at time t
W_t	Coefficient of seasonal index at time t
q	Average time spacing
Q_1	Average time spacing in the base seasons (assumed to be 1)

3.3 CALCULATION OF THE INITIAL VALUES

The initial values of the level, trend, and seasonal indices S_0 , T_0 , and I_i and the coefficients of these indices which are necessary to start the recursive formulae for calculation of the respective values at time $t = 0$, are calculated following the practical recommendation by Chatfield and Yar (1988) for the classical Holt-Winters method. Since the assessment of the average time spacing, q , and Q_i is complicated, for the sake of simplicity, it can be assumed that $q = Q_{-p+1} = Q_{-p+2} = \dots = Q_0 = 1$ in the initial values of equation 3.5 (i.e., $V_0 = \alpha$, $U_0 = \gamma$, $W_{-p+1} = W_{-p+2} = \dots = W_0 = \delta$). The formulae used are:

$$T_0 = \frac{\bar{y}_{k_1} - \bar{y}_{k_0}}{(k_1 - k_0)p} \quad 3.1$$

$$S_0 = \bar{y}_{k_0} - \left(k_0 p - \frac{p-1}{2} \right) T_0 \quad 3.2$$

$$I_i = \frac{1}{k(i)} \sum_k I_{i+kp} \quad 3.3$$

where,

$$I_{i+kp} = y_{i+kp} \left[\bar{y}_k + \left(i + \frac{p-1}{2} \right) T_0 \right] \quad 3.4$$

$$V_0 = 1 - (1 - \alpha)^q; \quad U_0 = 1 - (1 - \gamma)^q$$

$$W_i = 1 - (1 - \delta)^{Q_i} \text{ for } i = -p+1, -p+2, \dots, 0. \quad 3.5$$

and the following notation is used:

k_0 is such that the k_0 th season is the first non-empty season containing at least one observation;

k_I is minimal ($k_I \geq k_0+1$) such that the first k_I seasons contain at least one observation for each of the p seasonal periods;

the sum in equation 3.3 is over all such k that the observation y_{i+kp} ($k = 1, \dots, k_I$) are present in the first k_I seasons and $k(i)$ is the number of summands in this sum;

q is the average time spacing of the data.

3.4 METHOD ADOPTED TO DEVELOP THE MODEL

The model development started with generating the data. This was done by assuming the error terms to be normally distributed with mean zero and a range of values for variance. Further, the following steps were employed to generate the data and develop the final model:

3.4.1 DATA GENERATION AND MODEL DEVELOPMENT

1. The mean and variance of Beta distribution are functions of the parameters α and β .

For a given set of mean and variance of Beta distribution, the values of the parameters α and β were calculated using the following relations:

$$mean = \alpha / (\alpha + \beta)$$

$$variance = \alpha\beta / (\alpha + \beta)^2 (\alpha + \beta + 1)$$

2. For a given combination of the parameters α and β , a random number was generated from the Beta distribution to represent the value of fraction non-conforming for the first period. Now, the data was assumed to be collected over four days with twelve observations each day and some of the observations missing. This initially required 48 data points. This was done by developing different processes like AR(1), AR(2),

MA(1), MA(2), ARMA(1,1), ARMA(1,2), ARMA(2,1), and ARMA(2,2). These processes were generated using the following relations:

$$\text{AR}(p) \text{ process : } y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + \varepsilon_t$$

$$\text{MA}(q) \text{ process : } y_t = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} + \dots - \theta_q \varepsilon_{t-q}$$

$$\text{ARMA}(p,q) \text{ process : } y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \dots - \theta_q \varepsilon_{t-q} + \varepsilon_t$$

where $\{\phi\}$ and $\{\theta\}$ are constants and different values are assumed for different models.

3. Another assumption here was of the number of missing observations. After the data was generated, different number of random numbers (integers) between 7 and 24 were generated. This was done so that some of the values could be assumed to be missing from the data. The minimum number of missing observations thus assumed was 7 and the maximum number was 24. So after getting done with the first three steps, the required data for model development was available.
4. After all the data was generated, the next step now was to start the actual model development procedure. For this the first step was to calculate the initial values of the various indices and their respective coefficients. This was done using equations 3.1 through 3.5.
5. The initial values thus obtained were then substituted in recursive equations 2.42 through 2.47 to get the level, trend and seasonal indices for all the periods. Values of smoothing coefficients were assumed to be $\alpha = 0.4$, $\gamma = 0.1$, and $\delta = 0.4$ respectively for all the models.
6. On calculating all these values for the various indices, the smoothed values, the interpolated values and the forecasted values (m periods ahead) could be calculated. This was done using equations 2.48 and 2.49. The predicted and the smoothed and

interpolated values were then presented in the spreadsheet and graphical inspections were done for all the models, which indicated that the method gave practically acceptable results. This conclusion was also supported by the analysis of the corresponding residuals and further by doing the tracking signal analysis.

3.4.2 TRACKING SIGNAL ANALYSIS

1. As already discussed in the second chapter, to perform the tracking signal analysis, three variables are required. These variables are Q , Y and Δ . The assumption made during performing tracking signal analysis was that the model is correct and the initial parameter estimates are unbiased. So initial values of $Q(0) = 0$ and $Y(0) = 0$. Since there was some random variation assumed in the process so the initial value of $\Delta(0)$ has to be greater than 0.
2. The first step was to calculate the errors, and this was the difference between the actual value and the predicted value for any time period. Next step was to initialize the parameters. As already discussed, $Q(0) = 0$ and $Y(0) = 0$. To calculate $\Delta(0)$, equation 2.55 was used. The value of c_1 can be obtained from Table 1 in Appendix 1, once a proper value of β is assumed. Generally, value of β is assumed to be between 0.75 and 0.95.
3. After the initial values for Q , Y and Δ were calculated, the next step was to calculate the same values for all the remaining periods. This was done using equations 2.56 through 2.58.
4. Since the Holt-Winters method takes into consideration seasonal factor for forecasting, the ratio $Q(T) / \Delta(T)$ was then calculated and assuming $K_I = 2$, the value

of K_3 was calculated using equation 2.60. Examining the values, it could be concluded that the values remained well in control. This was expected, since the data for percent non-conforming were generated using random number generators.

Example 1:

In this example, a mean fraction non-conforming of 0.12 and a variance of 0.01 was considered. For simplicity sake and also to analyze all the models on the same basis, these values of mean and variance were considered for all the models developed.

1. Considering a mean of 0.12 and variance of 0.01.

$$\text{Therefore, } 0.12 = \alpha / (\alpha + \beta)$$

$$\text{and } 0.01 = \alpha\beta / (\alpha + \beta)^2 (\alpha + \beta + 1)$$

Solving the two equations above, we get $\alpha = 10.45$ and $\beta = 76.6$

2. A random number generated from the Beta distribution using the given values of α and β is 0.1504, corresponding to a fraction non-conforming of $p = 15.04\%$.
3. Error terms were generated using random number generator in Excel. This was done by generating 48 error terms corresponding the 48 periods into consideration. The mean of these error terms was assumed to be zero and variance was assumed to be 0.01.
4. An AR(1) process was then developed using the equation

$$y_t = \phi_1 y_{t-1} + \varepsilon_t$$

Where, y_t is the observation for the current period t ,

y_{t-1} is the observation for the previous period

ε_t is the error term associated with the current period

ϕ_I is the smoothing constant used for the AR(1) process and its value was assumed to be 0.65.

5. After the data generation was complete, a random number between 7 and 24 was generated. This was done to eliminate some of the generated values for fraction non-conforming, since the model considers missing values. A random number generated thus was 16, which meant 12 values out of the 48 values generated were to be considered missing. The next step was to determine which values were to be eliminated. This was done again by generating 16 random numbers between 1 and 48 and the corresponding 12 values were eliminated. One assumption for generating 48 data points was that inspection was made 12 times a day, once every two hours, and data for four days was generated.
6. The above 5 steps gave the final data, which could now be used further for model development. The first step in model development was to determine the base seasons on which all the initial calculations could be performed. As already discussed, the first base season had to be chosen such that it would be the first non-empty season containing at least one observation. In this model, the first base season was season one (day one) and the second was season two (day two). Season two was the second base season, since the second base season had to be such that it contained observations for at least all those periods, which the first base season did not.
7. The averages of the values for the first and the second base seasons were then calculated and used for determining the initial values of the level and trend factors. The initial values for level and trend factors were calculated using equations 3.1 and 3.2 and these values were 0.91241 and -0.00181 respectively. Calculations for initial

values of S_t and T_t are shown below and calculations for I_t are shown in Table 2 of Appendix 1. Initial values for seasonal factor were calculated using equations 3.3 and 3.4.

$$T_0 = \frac{(0.693/12) - (0.945/12)}{(2-1)12} = -0.00181$$

$$S_0 = 0.057 - (12 - 5.5) * (-0.00181) = 0.09421$$

8. After all the initializing was done, the next step was to calculate the values for trend, level and seasonal factors for all the periods. This was done using equations 2.42 through 2.47. For periods in which "data was not available" the factor y_t was not considered and the negative sign after y_t in equations 2.42 and 2.47 was removed. Sample calculations for period 3 in season one are shown below:

$$S_3 = V_3(y_3 - I_3^*) + (1 - V_3)[S_2 + (t_3 - t_2)T_2] = 0.4(0.015) + 0.6[0.099 + (1)(-0.00088)] = 0.054$$

$$T_3 = U_3 \frac{S_3 - S_2}{t_3 - t_2} + (1 - U_3)T_2 = 0.1 \frac{0.054 - 0.099}{3 - 2} + 0.9(-0.00088) = -0.0052$$

9. The sum of level and seasonal factors for a particular period gave the predicted value of fraction non-conforming for that period and m period ahead forecast was obtained using equations 2.48 and 2.49. calculations for predicted as well as smoothed and interpolated values for the third period in the first season are shown below:

Predicted value for third period in first season :

$$\hat{y}_3 = S_3 + I_3 = 0.054 + 0.0151 = 0.0691 = 6.91\%$$

Smoothed and Interpolated value for third period in first season (forecast made one period before)

$$\hat{y}_3(2) = S_3 + mT_3 + I_3^* \quad (\text{where } m \text{ is the time lag between the period for which forecast is}$$

done and the period at which forecast is made

$$= 0.054 + 1(-0.0052) + 0.0151 = 0.0638 = 6.38\%$$

10. The predicted values for $m = 1, \dots, 12$ are given in the third column of Table 3.1 and

Figure 3.1, while the smoothed and interpolated values are given in fourth column of

Table 3.1 and in Figure 3.2.

Periods	12 values missing	$Y_{t-1}^{\wedge}$	$Y_t^{\wedge}$	Periods	12 values missing	$Y_{t-1}^{\wedge}$	$Y_t^{\wedge}$
1	0.1504	0.135395	0.135556	25			0.092282
2	0.115854555	0.121499	0.122386	26	0.061867439	0.064584	0.06626
3			0.069368	27	0.089388621	0.075749	0.075931
4	0.111620009	0.095189	0.099016	28			0.094451
5	0.109561231	0.098367	0.101273	29	0.069609018	0.071548	0.073217
6	0.091609763	0.083507	0.085763	30	0.079455412	0.073443	0.07457
7	0.080298013	0.07383	0.075559	31	0.072211875	0.064794	0.065134
8	0.086905232	0.083329	0.084827	32	0.091401322	0.08337	0.082748
9	0.082045314	0.078236	0.079446	33			0.058001
10			0.07368	34	0.097444077	0.080191	0.080112
11	0.06590498	0.056521	0.059726	35	0.097171636	0.087642	0.086362
12	0.059514731	0.051604	0.054222	36	0.095165771	0.088506	0.086232
13	0.067934571	0.069682	0.072845	37	0.090989548	0.097729	0.096194
14			0.064092	38	0.104474331	0.10335	0.101483
15	0.074920565	0.057862	0.059634	39	0.087662312	0.092757	0.091294
16			0.085058	40			0.115443
17			0.074216	41	0.089752895	0.089818	0.090152
18	0.088947398	0.073664	0.074896	42	0.090064921	0.088967	0.089205
19	0.071689058	0.063863	0.064271	43			0.063943
20	0.08238714	0.077249	0.077066	44			0.054299
21	0.069388444	0.068785	0.068503	45			0.028767
22	0.08181357	0.080294	0.079714	46	0.110560216	0.082702	0.083841
23	0.084202241	0.082507	0.081644	47	0.092698776	0.080792	0.080585
24	0.071793781	0.072923	0.072092	48	0.091477781	0.082054	0.080643
						$Y_{48+m}^{\wedge}(48)$	
				49		0.089902	
				50		0.091743	
				51		0.085132	
				52		0.109667	
				53		0.103617	
				54		0.103461	
				55		0.088125	
				56		0.097876	
				57		0.08379	
				58		0.12252	
				59		0.108353	
				60		0.097576	

Table 3.1. Forecasting fraction defective for AR(1) process, 12 observations missing

Figure 3.1 Generated vs. Predicted Values

Figure 3.2. Generated vs. Forecasted values

Period	Errors	Error Sq.	Delta(t)	Y(T)	Q(T)	Q(T)/Delta(T)	>0.42?
1	0.009505	9.03361E-05	0.007866	0.009505	0.001045	0.13292022	n
2	-0.01274	0.000162431	0.008354	-0.00324	-0.00033	-0.03992753	n
4	0.008509	7.2402E-05	0.008369	0.005269	0.000551	0.06580311	n
5	0.003494	1.22048E-05	0.007882	0.008762	0.000845	0.10721196	n
6	0.000823	6.77208E-07	0.007176	0.009585	0.000843	0.11745072	n
7	-0.00029	8.27853E-08	0.006487	0.009297	0.00073	0.11249428	n
8	-0.00262	6.83891E-06	0.0061	0.006682	0.000395	0.06479815	n
9	-0.00182	3.29789E-06	0.005671	0.004866	0.000174	0.03070236	n
11	0.004814	2.31764E-05	0.005586	0.00968	0.000638	0.11424526	n
12	0.003809	1.45095E-05	0.005408	0.01349	0.000955	0.17663399	n
13	-0.00542	2.94116E-05	0.005409	0.008066	0.000317	0.05867137	n
15	0.014582	0.000212621	0.006327	0.022648	0.001744	0.27562531	n
18	-0.01458	0.000212701	0.007152	0.008064	0.000111	0.01551803	n
19	0.006612	4.3714E-05	0.007098	0.014675	0.000761	0.10721588	n
20	0.003633	1.31985E-05	0.006752	0.018308	0.001048	0.15525411	n
21	-0.00104	1.07347E-06	0.00618	0.017272	0.00084	0.13588698	n
22	-0.00043	1.81475E-07	0.005605	0.016846	0.000713	0.1272536	n
23	0.000365	1.33436E-07	0.005081	0.017211	0.000678	0.13352821	n
24	-0.00249	6.21254E-06	0.004822	0.014719	0.000361	0.07493651	n
26	-0.00351	1.23335E-05	0.004691	0.011207	-2.6E-05	-0.00553801	n
27	0.012719	0.000161785	0.005494	0.023926	0.001249	0.22726553	n
29	-0.00202	4.07795E-06	0.005146	0.021907	0.000922	0.17910915	n
30	0.005559	3.09002E-05	0.005188	0.027466	0.001385	0.26707101	n
31	0.006851	4.69374E-05	0.005354	0.034317	0.001932	0.360858	n
32	-0.00734	5.38041E-05	0.005552	0.026982	0.001005	0.18106945	n
34	0.016842	0.000283645	0.006681	0.043824	0.002589	0.38750669	n
35	-0.00924	8.54104E-05	0.006937	0.034582	0.001406	0.20266102	n
36	0.006157	3.79058E-05	0.006859	0.040739	0.001881	0.27423048	n
37	-0.00745	5.54608E-05	0.006918	0.033291	0.000948	0.13705855	n
38	0.000432	1.86717E-07	0.006269	0.033724	0.000897	0.14300627	n
39	-0.00579	3.35138E-05	0.006221	0.027934	0.000228	0.03664629	n
41	5.16E-05	2.66463E-09	0.005604	0.027986	0.00021	0.03753336	n
42	0.000967	9.35623E-07	0.005141	0.028953	0.000286	0.05564362	y
46	0.028431	0.000808303	0.00747	0.057384	0.003101	0.41508094	n
47	0.012234	0.000149682	0.007946	0.069618	0.004014	0.50513964	y
48	0.009372	8.78277E-05	0.008089	0.07899	0.00455	0.56247477	y
	SSE	0.002757916					
	SSE/34	8.11152E-05					
	K3	0.42					
	Delta(0)	0.007683507					

Table A.1.1 Tracking Signal Analysis { AR(1) process, 12 observations missing }

3.4.3 Analysis of the developed models

Various models were developed using the similar procedures and results were presented in similar form. The different models developed differed in their way of data generation, viz., data was generated using AR(1), AR(2), MA(1), MA(2), ARMA(1,1), ARMA(1,2), ARMA(2,1) and ARMA(2,2) and also in the number of "missing observations". The maximum number of missing observations considered was 23 (i.e., 47.9%). The value of Root Mean Squared Forecast Error and graphical inspections of predicted as well as smoothed and interpolated values indicated that the method gave acceptable results. This conclusion is also supported by the analysis of tracking signal and also the corresponding residual time series.

4. ANALYSIS OF RESULTS

The approach to forecasting work in process quality that was considered was Holt-Winters method. The variable under consideration was fraction non-conforming and an assumption of some of the values being missing was made. The initial values of fraction non-conforming were generated for given values of mean and variance of Beta distribution and then adding the normally distributed error terms (with zero mean and given variance) to these values. Further values of fraction non-conforming were generated using various processes like Autoregressive, Autoregressive Moving Average process. Considering fraction non-conforming as the variable of interest, forecasting models using Holt-Winters method were developed and compared for effectiveness based on the errors and error variance obtained. The analysis of forecast errors was done using the method of tracking signals. The number of missing observations in each model was varied and the developed models gave acceptable results with as many as 40% of the observations missing. The results obtained for various processes with different number of missing observations were tabulated.

Almost all the models gave similar results irrespective of the process used or followed for data generation although some of the models gave somewhat varying results depending on the number of missing observations. It was seen in all the models that the coefficients of the trend and level factors remained constant throughout the forecasting process. This is attributed to the fact that these coefficients are smoothed based on the previous period in the same season. Further, it was observed that the estimated level factor adjusted based on level factor for the previous period as well as based on the most recent seasonal factor corresponding to the same seasonal period in which the observed percent non-conforming was not missing. It was also seen

that the coefficient of seasonal factor combined the new observation adjusted by the estimated trend factor with the most recent estimate of the seasonal factor for the same seasonal period. The trend in all the models was observed to be constant but the seasonal index seemed to vary in a manner such that it was high during the first six periods and low during the last six periods of a season. Further, to avoid complexity, average time spacing in data was considered to be one period so the initial values of the coefficients of level and trend factors came out to be same as the smoothing constants used.

When the tracking signal exceeds the control limit for three or more successive periods, this seems to give a strong indication that something is wrong with the forecasting system. The action to be taken depends upon the likely cause of the out-of-control condition. The possible causes of variation that were observed were:

1. The out-of-control point is caused by random variation alone. In this case no action is to be taken.
2. The parameter estimates used in the forecasting model are not accurate because the true parameter values have changed over time. The model form is still correct, but improved estimates of the model parameters are required.

4.1 RESULTS FOR AR (1) PROCESS

The values of fraction non-conforming were generated using the Autoregressive "AR(1)" process in which values for a certain period are obtained by regressing over the values in the previous period. The estimated and forecasted values of percent non-conforming were obtained using the Holt-Winters method as described in chapter 2. The data for AR(1) process was generated by assuming a mean of 0.1 and the value of ϕ_1 was assumed to be 0.65. Data was

generated for 48 periods and 12 values of the total values generated were assumed to be missing. The results showed that unbiased forecasts were obtained for almost all the periods using the method and forecasts for 2 periods out of the total 48 were biased. It could also be seen that the level and trend of the forecasted values were similar to those of the generated data, which confirmed that the method gave acceptable results. Further, for periods where the forecast was biased, it could be seen that either data for the previous period was missing which caused the forecast for the following period to be biased or the period was preceded by a period where the forecast was biased. This bias in the forecast can be justified by the fact that the data was generated using "Autoregressive" process and hence if forecast for a particular period was biased then there was a high probability of the forecast for the following period being biased.

The seasonal index for any period is one of the important factors that decide the forecast and the estimated values for that particular period. Since the forecast and estimation for a period include the seasonal index as one of the major factors any variation in the seasonal index could affect these values and thus cause the forecast to be biased and go out of acceptable limits. So one major reason for the forecast being biased was that data for the same seasonal period in one of the preceding seasons was missing which caused variation in the value of the seasonal index of that period thus giving a biased forecast.

The results which show the generated values and the estimated as well as forecasted values for data generated using AR(1) process have been listed in table 4.1.1 and also in the following sections. The number of missing observations for each of the models is different and it can be seen that this method gives acceptable results with as many as 40% of the total observations missing. The results have been tabulated and graphs showing the generated data and the estimated as well as smoothed and interpolated values against time are shown for different

number of observations missing. Analysis of tracking signal, which shows whether the forecast for a particular period is biased or unbiased is presented in Appendix 1.

4.2 RESULTS FOR AR(2) PROCESS

The values of fraction non-conforming were generated using the AR(2) process in which values for a certain period are obtained by regressing over the values in the preceding two periods. The estimated and forecasted values of percent non-conforming were obtained using the Holt-Winters method as described in chapter 2. The data for AR(2) process was generated by assuming a mean of 0.1 and the values of ϕ_1 and ϕ_2 were assumed to be 0.45 and 0.54 respectively. Data was generated for 48 periods and 16 values of the total values generated were assumed to be missing. Results obtained by using this method of data generation were similar to those obtained for AR(1) process. The results have been tabulated and graphs showing the generated data and the estimated as well as smoothed and interpolated values against time are shown for different number of observations missing. Analysis of tracking signal, which shows whether the forecast for a particular period is biased or unbiased is presented in Appendix 1.

Period	12 obs. Missing	Prediction	Forecast	Period	12 obs. Missing	Prediction	Forecast
1	0.1504	0.135556	0.140895	31	0.072211875	0.065641	0.066304
2	0.115854555	0.124366	0.129293	32	0.091401322	0.083369	0.084991
3		0.077024		33		0.058535	
4	0.111620009	0.103123	0.104854	34	0.097444077	0.080384	0.081124
5	0.109561231	0.105738	0.108067	35	0.097171636	0.086297	0.088319
6	0.091609763	0.090327	0.093031	36	0.095165771	0.086457	0.089649
7	0.080298013	0.080083	0.083081	37	0.090989548	0.09682	0.099504
8	0.086905232	0.089244	0.092282	38	0.104474331	0.102413	0.105595
9	0.082045314	0.083729	0.086884	39	0.087662312	0.092079	0.095089
10		0.083342		40		0.118391	
11	0.06590498	0.06373	0.064333	41	0.089752895	0.090462	0.091721
12	0.059514731	0.057995	0.058827	42	0.090064921	0.089829	0.09124
13	0.067934571	0.076351	0.076331	43		0.064676	
14		0.070109		44		0.052666	
15	0.074920565	0.061775	0.062307	45		0.026299	
16		0.087643		46	0.110560216	0.083098	0.082279
17		0.074957		47	0.092698776	0.079869	0.080393
18	0.088947398	0.075031	0.075278	48	0.091477781	0.080254	0.082078
19	0.071689058	0.064894	0.06592	49			0.090037
20	0.08238714	0.077982	0.079601	50			0.092479
21	0.069388444	0.069602	0.071359	51			0.085695
22	0.08181357	0.081452	0.083436	52			0.111344
23	0.084202241	0.082519	0.084889	53			0.105227
24	0.071793781	0.073215	0.075663	54			0.104897
25		0.094615		55			0.090009
26	0.061867439	0.067072	0.066857	56			0.099973
27	0.089388621	0.076684	0.077859	57			0.08668
28		0.096102		58			0.127765
29	0.069609018	0.073248	0.072738	59			0.112923
30	0.079455412	0.074954	0.074894	60			0.102137

Table 4.1.1 Forecasting fraction non-conforming for data generated using AR(1) process (12 observations missing)

Figure 4.1.1(a) Generated vs. Predicted values, AR(1) process, 12 observations missing

Figure 4.1.1(b) Generated vs. Forecasted values, AR(1) process, 12 observations missing

Periods	16 values missing	Estimate	Forecast	Periods	16 values missing	Estimate	Forecast
1	0.1504	0.135556	0.155395	31	0.072211875	0.061885	0.06173
2	0.115854555	0.129586	0.147899	32	0.091401322	0.079206	0.080406
3		0.095576	0.108301	33		0.050163	0.049018
4	0.111620009	0.112218	0.124876	34	0.097444077	0.07904	0.07994
5	0.109561231	0.115129	0.127169	35	0.097171636	0.086924	0.088962
6	0.091609763	0.099457	0.110625	36	0.095165771	0.086877	0.089835
7	0.080298013	0.090158	0.100231	37	0.090989548	0.094224	0.096724
8	0.086905232	0.096286	0.105316	38	0.104474331	0.102473	0.105195
9	0.082045314	0.090111	0.098245	39	0.087662312	0.091089	0.093431
10		0.09646	0.100827	40		0.109488	0.109606
11	0.06590498	0.065264	0.069702	41	0.089752895	0.087614	0.087969
12	0.059514731	0.06359	0.067575	42	0.090064921	0.088746	0.089248
13	0.067934571	0.080843	0.083394	43		0.075103	0.073685
14		0.07452	0.07534	44		0.062335	0.059978
15	0.074920565	0.063658	0.066075	45		0.030815	0.027154
16		0.085558	0.086494	46	0.110560216	0.085318	0.084462
17		0.069765	0.069632	47	0.092698776	0.082091	0.082413
18	0.088947398	0.072847	0.074503	48		0.056488	0.054922
19		0.044025	0.043123	49			0.05256
20	0.08238714	0.065059	0.066081	50			0.054519
21	0.069388444	0.061792	0.063658	51			0.042654
22	0.08181357	0.079012	0.081276	52			0.057701
23		0.052534	0.051821	53			0.05345
24	0.071793781	0.057601	0.058465	54			0.051196
25		0.071084	0.070565	55			0.043167
26	0.061867439	0.061432	0.060975	56			0.045528
27	0.089388621	0.072766	0.074156	57			0.023101
28		0.085693	0.085721	58			0.06201
29	0.069609018	0.070095	0.070054	59			0.046286
30		0.057296	0.055994	60			0.037689

Table 4.1.2 Forecasting fraction non-conforming for data generated using AR(1) process (16 observations missing)

Figure 4.1.2(a) Generated vs. Predicted values, AR(1) process, 16 observations missing

Figure 4.1.2(b) Generated vs. Forecasted values, AR(1) process, 16 observations missing

Period	18 obs. Missing	Estimate	Forecast	Period	18 obs. Missing	Estimate	Forecast
1		0.050186	0.068939	31	0.072211875	0.065481	0.066833
2	0.115854555	0.107284	0.126989	32		0.059242	0.05838
3		0.065522	0.08107	33		-0.03038	-0.03523
4	0.111620009	0.105506	0.121733	34	0.097444077	0.069515	0.072771
5	0.109561231	0.112386	0.128299	35	0.097171636	0.075077	0.080787
6		0.096299	0.108443	36		0.043297	0.046462
7	0.080298013	0.078276	0.090644	37	0.090989548	0.069834	0.076
8	0.086905232	0.091532	0.103387	38	0.104474331	0.104505	0.110667
9	0.082045314	0.061018	0.075208	39	0.087662312	0.087532	0.093708
10		0.124418	0.13351	40		0.116055	0.120104
11	0.06590498	0.077448	0.085257	41	0.089752895	0.090594	0.094549
12	0.059514731	0.06882	0.075595	42	0.090064921	0.090866	0.094732
13	0.067934571	0.066873	0.073799	43		0.067801	0.068925
14		0.118421	0.122941	44		0.06475	0.064766
15	0.074920565	0.071747	0.076717	45		0.007744	0.005192
16		0.107351	0.110228	46	0.110560216	0.08967	0.089439
17		0.084982	0.086239	47	0.092698776	0.078504	0.07985
18	0.088947398	0.080672	0.083103	48		0.05524	0.055067
19	0.071689058	0.068912	0.071652	49			0.036358
20	0.08238714	0.080027	0.083029	50			0.064725
21		-0.00629	-0.00879	51			0.041272
22	0.08181357	0.066579	0.066239	52			0.063978
23	0.084202241	0.066999	0.068569	53			0.055977
24	0.071793781	0.064309	0.066712	54			0.05301
25		0.036758	0.035477	55			0.036875
26	0.061867439	0.06349	0.061979	56			0.051251
27	0.089388621	0.065327	0.066489	57			0.003508
28		0.087317	0.087437	58			0.089645
29	0.069609018	0.070548	0.070535	59			0.062765
30	0.079455412	0.073893	0.074498	60			0.053169

Table 4.1.3 Forecasting fraction non-conforming for data generated using AR(1) process (18 observations missing)

Figure 4.1.3(a) Generated vs. Predicted values, AR(1) process, 18 observations missing

Figure 4.1.3(b) Generated vs. Forecasted values, AR(1) process, 18 observations missing

Period	11 values missing	Estimate	Forecast	Period	11 values missing	Estimate	Forecast
1	0.1642	0.162503	0.16544	31	0.11586343	0.106995	0.115427
2	0.1565	0.157247	0.1601	32	0.119512935	0.105943	0.115883
3	0.168425324	0.168757	0.171574	33	0.118884858	0.112204	0.122886
4		0.145005	0.146763	34		0.082601	0.089223
5	0.159723293	0.136135	0.140514	35	0.123152657	0.100262	0.109428
6	0.155129697	0.138934	0.145113	36	0.122215973	0.106982	0.11784
7	0.161908339	0.152652	0.15986	37	0.137968606	0.125489	0.137734
8	0.154414864	0.146455	0.154547	38		0.098164	0.106157
9	0.147746925	0.147105	0.150268	39	0.111964215	0.107014	0.115709
10	0.139014334	0.136783	0.140194	40	0.129786853	0.121201	0.131115
11	0.129620493	0.130856	0.134129	41		0.106477	0.114268
12		0.107313	0.109812	42	0.127637061	0.11241	0.121892
13	0.127747695	0.105771	0.110712	43		0.102182	0.109879
14		0.06781	0.06878	44	0.130162518	0.110161	0.120081
15	0.140545734	0.10758	0.112212	45	0.146261316	0.12804	0.139984
16	0.141163464	0.124733	0.131696	46	0.129851493	0.122716	0.136019
17		0.10054	0.104468	47	0.13845949	0.139247	0.147463
18	0.123496584	0.103044	0.109244	48	0.131770323	0.132435	0.140577
19	0.135975827	0.120456	0.12838	49			0.139801
20	0.116196971	0.108849	0.11759	50			0.130379
21	0.131808538	0.121991	0.131823	51			0.162865
22		0.085932	0.091169	52			0.169321
23	0.11556988	0.092192	0.100027	53			0.166315
24	0.107286341	0.098385	0.107482	54			0.173415
25	0.101427086	0.10222	0.106229	55			0.168077
26	0.123238537	0.112383	0.122931	56			0.175904
27		0.123847	0.131185	57			0.176758
28		0.093723	0.099209	58			0.159041
29	0.108545276	0.097417	0.104481	59			0.176286
30	0.09555355	0.092112	0.099558	60			0.170138

Table 4.2.1 Forecasting fraction non-conforming for data generated using AR(2) process (11) observations missing

Figure 4.2.1(a) Generated vs. Predicted values, AR(2) process, 11 observations missing

Figure 4.2.1(b) Generated vs. Forecasted values, AR(2) process, 11 observations missing

Period	13 values missing	Estimate	Forecast	Period	13 values missing	Estimate	Forecast
1	0.1642	0.162503	0.17044	31		0.092914	0.098979
2	0.1565	0.157247	0.1651	32	0.119512935	0.097494	0.106005
3	0.168425324	0.168757	0.176574	33	0.118884858	0.106994	0.116826
4	0.170067154	0.170107	0.177919	34		0.074644	0.080647
5	0.159723293	0.160108	0.167878	35	0.123152657	0.098089	0.106877
6	0.155129697	0.154539	0.162374	36		0.065109	0.071076
7	0.161908339	0.162611	0.170368	37	0.137968606	0.106497	0.115961
8	0.154414864	0.152629	0.160584	38		0.068802	0.075421
9	0.147746925	0.15076	0.158381	39	0.111964215	0.100805	0.109007
10	0.139014334	0.138781	0.146427	40		0.076426	0.083274
11	0.129620493	0.131779	0.139186	41		0.049977	0.05571
12		0.107877	0.109483	42	0.127637061	0.0965	0.105693
13	0.127747695	0.105595	0.109662	43		0.078233	0.085916
14		0.066829	0.066971	44	0.130162518	0.105645	0.116053
15	0.140545734	0.10703	0.110896	45	0.146261316	0.125102	0.13786
16	0.141163464	0.11992	0.126147	46	0.129851493	0.121361	0.135737
17		0.080218	0.082855	47		0.125096	0.135745
18	0.123496584	0.096074	0.101758	48	0.131770323	0.117989	0.130593
19	0.135975827	0.117443	0.125186	49			0.132431
20	0.116196971	0.107884	0.116551	50			0.109255
21	0.131808538	0.121989	0.131747	51			0.14848
22		0.08637	0.091385	52			0.133921
23	0.11556988	0.092209	0.099819	53			0.125815
24	0.107286341	0.098503	0.107359	54			0.151302
25	0.101427086	0.102266	0.111029	55			0.13954
26	0.123238537	0.112302	0.122616	56			0.156629
27		0.123932	0.131065	57			0.154771
28	0.100639538	0.099475	0.106738	58			0.134704
29	0.108545276	0.098914	0.107543	59			0.151574
30	0.09555355	0.097503	0.105916	60			0.149234

Table 4.2.2 Forecasting fraction non-conforming for data generated using AR(2) process (13 observations missing)

Figure 4.2.2(a) Generated vs. Predicted values, AR(2) process, 13 observations missing

Figure 4.2.2(b) Generated vs. Predicted values, AR(2) process, 13 observations missing

Period	16 values missing	Estimate	Forecast	Period	16 values missing	Estimate	Forecast
1	0.1642	0.162503	0.17044	31		0.082534	0.093865
2	0.1565	0.157247	0.1651	32	0.119512935	0.095457	0.10946
3	0.168425324	0.168757	0.176574	33	0.118884858	0.105576	0.121058
4		0.145005	0.151763	34		0.072276	0.084149
5	0.159723293	0.136135	0.145514	35	0.123152657	0.097923	0.112599
6	0.155129697	0.138934	0.150113	36		0.065251	0.077112
7	0.161908339	0.152652	0.16486	37	0.137968606	0.113547	0.128872
8	0.154414864	0.146455	0.154547	38		0.083404	0.096286
9	0.147746925	0.147105	0.155268	39	0.111964215	0.102606	0.116816
10	0.139014334	0.136783	0.145194	40		0.087555	0.100339
11	0.129620493	0.130856	0.139129	41		0.078506	0.090784
12		0.107313	0.114812	42	0.127637061	0.105052	0.11984
13	0.127747695	0.105771	0.115712	43		0.087064	0.100361
14		0.06781	0.07378	44	0.130162518	0.109171	0.124801
15	0.140545734	0.10758	0.117212	45	0.146261316	0.126776	0.14457
16	0.141163464	0.124733	0.136696	46	0.129851493	0.121524	0.140905
17		0.10054	0.109468	47		0.125228	0.140946
18	0.123496584	0.103044	0.114244	48	0.131770323	0.117984	0.135658
19	0.135975827	0.120456	0.13338	49			0.138797
20	0.116196971	0.108849	0.11759	50			0.120772
21	0.131808538	0.121991	0.131823	51			0.146147
22		0.085932	0.096169	52			0.137483
23	0.11556988	0.092192	0.105027	53			0.146681
24	0.107286341	0.098385	0.107482	54			0.156563
25		0.089996	0.100685	55			0.144516
26	0.123238537	0.102084	0.115774	56			0.159525
27		0.10388	0.115309	57			0.157522
28		0.08098	0.091164	58			0.136288
29	0.108545276	0.094765	0.106904	59			0.152936
30	0.09555355	0.090107	0.097851	60			0.150068

Table 4.2.3 Forecasting fraction non-conforming for data generated using AR(2) process (16 observations missing)

Figure 4.2.3(a) Generated vs. Predicted values, AR(2) process, 16 observations missing

Figure 4.2.3(b) Generated vs. Predicted values, AR(2) process, 16 observations missing

Period	18 values missing	Estimate	Forecast	Period	18 values missing	Estimate	Forecast
1	0.1642	0.162503	0.17044	31		0.083852	0.090932
2	0.1565	0.157247	0.1651	32	0.119512935	0.121127	0.128028
3	0.168425324	0.168757	0.176574	33	0.118884858	0.103584	0.112184
4		0.145005	0.146763	34		0.058293	0.064869
5	0.159723293	0.136135	0.140514	35	0.123152657	0.096683	0.106201
6	0.155129697	0.138934	0.145113	36		0.060219	0.067552
7	0.161908339	0.152652	0.15986	37	0.137968606	0.112911	0.123798
8		0.104173	0.105746	38		0.081438	0.09005
9	0.147746925	0.121293	0.125806	39	0.111964215	0.102509	0.112462
10	0.139014334	0.119982	0.141609	40		0.087688	0.096231
11	0.129620493	0.120133	0.132814	41		0.078546	0.086531
12		0.091803	0.094449	42	0.127637061	0.105151	0.115633
13	0.127747695	0.101836	0.107361	43		0.088056	0.096971
14		0.062791	0.064583	44	0.130162518	0.128511	0.137609
15	0.140545734	0.106586	0.112151	45		0.094463	0.103411
16	0.141163464	0.124529	0.142454	46	0.129851493	0.113665	0.130696
17		0.100984	0.105853	47		0.106883	0.117103
18	0.123496584	0.103497	0.125588	48	0.131770323	0.114065	0.126796
19	0.135975827	0.121048	0.129798	49			0.129754
20	0.116196971	0.139247	0.144728	50			0.109225
21	0.131808538	0.112954	0.120529	51			0.134651
22		0.069924	0.074733	52			0.125501
23	0.11556988	0.088767	0.096554	53			0.134
24	0.107286341	0.094746	0.114312	54			0.143713
25		0.08693	0.093483	55			0.132081
26	0.123238537	0.101393	0.126046	56			0.186982
27		0.104223	0.111661	57			0.157166
28		0.082062	0.088226	58			0.150217
29	0.108545276	0.095231	0.113284	59			0.151383
30	0.09555355	0.090685	0.099278	60			0.146836

Table 4.2.4 Forecasting fraction non-conforming for data generated using AR(2) process (18 observations missing)

Figure 4.2.4(a) Generated vs. Predicted values, AR(2) process, 18 observations missing

Figure 4.2.4(b) Generated vs. Forecasted values, AR(2) process, 18 observations missing

Period	10 obs. Missing	Estimate	Forecast	Period	10 obs. Missing	Estimate	Forecast
1	0.1465	0.148913	0.147766	31	0.138325026	0.127773	0.13203
2	0.130053586	0.131088	0.134826	32		0.10357	0.104703
3	0.138678662	0.138381	0.142153	33	0.15460471	0.126486	0.130744
4	0.157103537	0.158063	0.161728	34	0.162430559	0.140254	0.166976
5	0.153766381	0.151151	0.155106	35	0.162554885	0.1518	0.159717
6		0.12782	0.1271	36	0.164281013	0.155009	0.168956
7	0.138591559	0.120463	0.121758	37	0.169349983	0.166638	0.175886
8	0.137380734	0.125169	0.137821	38	0.173767257	0.168429	0.17827
9	0.118681376	0.112171	0.120546	39		0.149667	0.153432
10	0.11673186	0.113595	0.117319	40	0.158672836	0.146661	0.151761
11		0.105432	0.104881	41		0.125563	0.127104
12	0.107904413	0.08996	0.091402	42	0.156703571	0.150393	0.154148
13	0.129791784	0.117269	0.120103	43	0.166318687	0.152965	0.168203
14	0.113370163	0.105398	0.114118	44	0.165603794	0.157548	0.164321
15	0.119406005	0.11495	0.119165	45	0.180469625	0.174806	0.182208
16		0.120469	0.121509	46	0.172775322	0.169365	0.177146
17	0.128191506	0.114073	0.116681	47		0.148605	0.1516
18	0.133870786	0.123948	0.137964	48	0.164728272	0.144262	0.149531
19		0.105905	0.106886	49			0.135555
20		0.071059	0.070174	50			0.127436
21	0.117405078	0.088133	0.090501	51			0.128409
22	0.122978165	0.102304	0.106969	52			0.150022
23	0.126881878	0.11885	0.129654	53			0.14333
24	0.124074717	0.117707	0.124219	54			0.171104
25	0.116652999	0.122151	0.128052	55			0.160352
26	0.127978247	0.120842	0.127536	56			0.149623
27	0.136740953	0.132031	0.139248	57			0.159085
28	0.14215751	0.144401	0.1513	58			0.147722
29	0.139708315	0.143327	0.149823	59			0.143592
30		0.130482	0.133243	60			0.147495

Table 4.3.1 Forecasting fraction non-conforming for data generated using ARMA(1,1) process (10 observations missing)

Figure 4.3.1(a) Generated vs. Predicted values, ARMA(1,1) process, 10 observations missing

Period	11 obs. Missing	Estimate	Forecast	Period	11 obs. Missing	Estimate	Forecast
1	0.1465	0.148913	0.153766	31	0.138325026	0.125428	0.13113
2	0.130053586	0.131088	0.135826	32		0.099397	0.102157
3	0.138678662	0.138381	0.143153	33	0.15460471	0.125729	0.131697
4	0.157103537	0.158063	0.162728	34	0.162430559	0.140056	0.14851
5	0.153766381	0.151151	0.156106	35	0.162554885	0.151944	0.161577
6		0.12782	0.1281	36	0.164281013	0.155353	0.165978
7	0.138591559	0.120463	0.122758	37	0.169349983	0.167088	0.177964
8	0.137380734	0.125169	0.128821	38	0.173767257	0.168633	0.180237
9	0.118681376	0.112171	0.116546	39		0.167786	0.173698
10	0.11673186	0.113595	0.118319	40	0.158672836	0.153778	0.160233
11		0.105432	0.105881	41		0.13265	0.135727
12	0.107904413	0.08996	0.092402	42	0.156703571	0.150766	0.155926
13	0.129791784	0.117269	0.121103	43	0.166318687	0.154769	0.161212
14	0.113370163	0.105398	0.110118	44	0.165603794	0.157667	0.165623
15	0.119406005	0.11495	0.120165	45	0.180469625	0.175176	0.183719
16		0.120469	0.122509	46	0.172775322	0.169504	0.178411
17	0.128191506	0.114073	0.117681	47		0.148682	0.15279
18	0.133870786	0.123948	0.128964	48	0.164728272	0.144255	0.150638
19		0.105905	0.107886	49			0.135587
20		0.071059	0.071174	50			0.125355
21	0.117405078	0.088133	0.091501	51			0.142059
22	0.122978165	0.102304	0.107969	52			0.157241
23	0.126881878	0.11885	0.125654	53			0.149556
24	0.124074717	0.117707	0.125219	54			0.170417
25	0.116652999	0.122151	0.129052	55			0.162809
26		0.091493	0.094228	56			0.150531
27	0.136740953	0.112396	0.117836	57			0.160586
28	0.14215751	0.13439	0.140932	58			0.149119
29	0.139708315	0.13517	0.142216	59			0.144967
30		0.118684	0.122557	60			0.148855

Table 4.3.2 Forecasting fraction non-conforming for data generated using ARMA(1,1) process (11 observations missing)

Figure 4.3.2(a) Generated vs. Predicted values, ARMA(1,1) process, 11 observations missing

Figure 4.3.2(b) Generated vs. Forecasted values, ARMA(1,1) process, 11 observations missing

Period	13 obs. Missing	Estimate	Forecast	Period	13 obs. Missing	Estimate	Forecast
1	0.1465	0.148913	0.157766	31	0.138325026	0.126992	0.136631
2	0.130053586	0.131088	0.139826	32		0.102395	0.108961
3	0.138678662	0.138381	0.147153	33	0.15460471	0.126365	0.136069
4	0.157103537	0.158063	0.166728	34		0.08855	0.095122
5	0.153766381	0.151151	0.160106	35	0.162554885	0.129892	0.140093
6		0.12782	0.1321	36	0.164281013	0.140886	0.153687
7	0.138591559	0.120463	0.126758	37	0.169349983	0.161298	0.175242
8	0.137380734	0.125169	0.132821	38	0.173767257	0.16533	0.180211
9	0.118681376	0.112171	0.120546	39		0.144685	0.15422
10	0.11673186	0.113595	0.122319	40	0.158672836	0.146437	0.157332
11		0.105432	0.109881	41		0.124612	0.132189
12	0.107904413	0.08996	0.096402	42	0.156703571	0.150334	0.160146
13	0.129791784	0.117269	0.125103	43	0.166318687	0.15355	0.16478
14	0.113370163	0.105398	0.114118	44	0.165603794	0.157776	0.170498
15	0.119406005	0.11495	0.124165	45	0.180469625	0.175411	0.188695
16		0.120469	0.126509	46	0.172775322	0.166445	0.180627
17	0.128191506	0.114073	0.121681	47		0.172603	0.182248
18	0.133870786	0.123948	0.132964	48		0.123025	0.129353
19		0.105905	0.111886	49			0.099181
20		0.071059	0.075174	50			0.086223
21	0.117405078	0.088133	0.095501	51			0.078405
22	0.122978165	0.102304	0.111969	52			0.095425
23	0.126881878	0.11885	0.129654	53			0.082307
24	0.124074717	0.117707	0.129219	54			0.10434
25		0.117401	0.125281	55			0.089884
26	0.127978247	0.108881	0.118883	56			0.074299
27	0.136740953	0.124246	0.135637	57			0.07992
28	0.14215751	0.140509	0.152134	58			0.05502
29	0.139708315	0.140247	0.151811	59			0.071473
30		0.126227	0.134259	60			0.0526

Table 4.3.3 Forecasting fraction non-conforming for data generated using ARMA(1,1) process (13 observations missing)

Figure 4.3.3(a) Generated vs. Predicted values, ARMA(1,1) process, 13 observations missing

Figure 4.3.3(b) Generated vs. Forecasted values, ARMA(1,1) process, 13 observations missing

Period	18 obs. Missing	Estimate	Forecast	Period	18 obs. Missing	Estimate	Forecast
1	0.1465	0.148913	0.147766	31	0.138325026	0.122751	0.139308
2	0.130053586	0.131088	0.146826	32		0.09291	0.107023
3		0.112233	0.12266	33	0.15460471	0.124695	0.142131
4	0.157103537	0.136899	0.149571	34	0.162430559	0.142296	0.16197
5	0.153766381	0.140084	0.154277	35	0.162554885	0.150348	0.171379
6		0.104523	0.115149	36		0.119525	0.135473
7	0.138591559	0.114237	0.127569	37	0.169349983	0.147506	0.165882
8	0.137380734	0.121447	0.13655	38	0.173767257	0.153985	0.174559
9	0.118681376	0.1101	0.126157	39		0.125542	0.141629
10		0.096718	0.108542	40	0.158672836	0.144779	0.16241
11		0.061353	0.071001	41		0.130784	0.145823
12	0.107904413	0.073166	0.086674	42		0.107998	0.121616
13	0.129791784	0.112027	0.127509	43	0.166318687	0.140805	0.157258
14	0.113370163	0.102486	0.119178	44	0.165603794	0.151588	0.17071
15	0.119406005	0.113532	0.131056	45		0.137346	0.153669
16		0.137517	0.152398	46	0.172775322	0.151856	0.170503
17		0.104615	0.117688	47		0.118637	0.134101
18	0.133870786	0.116238	0.131812	48	0.164728272	0.145583	0.164694
19		0.096569	0.109948	49			0.157298
20		0.067176	0.07911	50			0.142531
21	0.117405078	0.087543	0.102796	51			0.130571
22	0.122978165	0.108145	0.125501	52			0.164312
23	0.126881878	0.118083	0.136688	53			0.176466
24		0.087311	0.101895	54			0.15927
25	0.116652999	0.10468	0.120595	55			0.183165
26	0.127978247	0.109085	0.127099	56			0.169907
27	0.136740953	0.123703	0.143166	57			0.166851
28	0.14215751	0.144794	0.163884	58			0.1777
29		0.135974	0.152042	59			0.15768
30		0.105533	0.119881	60			0.170909

Table 4.3.4 Forecasting fraction non-conforming for data generated using ARMA(1,1) process (18 observations missing)

Figure 4.3.4(a) Generated vs. Predicted values, ARMA(1,1) process, 18 observations missing

Figure 4.3.4(b) Generated vs. Predicted values, ARMA(1,1) process, 18 observations missing

Period	11 obs. Missing	Estimate	Forecast	Period	11 obs. Missing	Estimate	Forecast
1	0.066460039	0.070415	0.069831	31	0.057862093	0.055498	0.062382
2	0.068406027	0.066673	0.066281	32	0.072386077	0.067588	0.075004
3	0.069022682	0.071403	0.070747	33	0.066056977	0.065613	0.073079
4	0.060061276	0.057793	0.057389	34		0.044668	0.049475
5	0.059611554	0.061685	0.061051	35	0.070092187	0.06055	0.066417
6		0.048523	0.045204	36	0.083032668	0.080482	0.086711
7	0.061069648	0.047798	0.050953	37	0.077154064	0.071375	0.078246
8	0.063367129	0.054482	0.060125	38	0.045250608	0.049604	0.055991
9	0.077372599	0.068289	0.074942	39	0.072075606	0.06868	0.075444
10		0.040029	0.043566	40		0.04246	0.046803
11	0.078609831	0.061984	0.067368	41	0.032493901	0.036643	0.040525
12	0.095618202	0.088144	0.094359	42		0.024588	0.028062
13	0.086649656	0.077237	0.084498	43	0.040396009	0.030819	0.035358
14		0.055647	0.06016	44		0.027134	0.031244
15	0.086791981	0.071725	0.077912	45	0.034510139	0.02885	0.033589
16		0.035754	0.039407	46	0.042137827	0.033651	0.039593
17	0.077484967	0.058338	0.064118	47	0.055724682	0.052387	0.0587
18	0.064368993	0.058361	0.064993	48	0.045733141	0.053361	0.058827
19		0.046773	0.05118	49			0.042156
20	0.059921581	0.049146	0.054751	50			0.023817
21	0.05451498	0.05132	0.057279	51			0.0382
22	0.055040064	0.048739	0.055593	52			0.011286
23	0.061028263	0.063222	0.069832	53			0.034813
24		0.080415	0.086186	54			0.025971
25	0.073778102	0.066507	0.073086	55			0.025998
26	0.068166136	0.06315	0.070442	56			0.021755
27	0.068164771	0.07102	0.077994	57			0.024308
28		0.04252	0.04688	58			0.017798
29	0.072138127	0.060195	0.065882	59			0.032348
30	0.06517116	0.05742	0.063968	60			0.040951

Table 4.4.1 Forecasting fraction non-conforming for data generated using ARMA(1,2) process (11 observations missing)

Figure 4.4.1(a) Generated vs. Predicted values, ARMA(1,2) process, 11 observations missing

Figure 4.4.1(b) Generated vs. Forecasted values, ARMA(1,2) process, 11 observations missing

Period	15 obs. Missing	Estimate	Forecast	Period	15 obs. Missing	Estimate	Forecast
1	0.066460039	0.070415	0.069831	31	0.057862093	0.05417	0.062805
2	0.068406027	0.066673	0.066281	32		0.049323	0.056168
3	0.069022682	0.071403	0.070747	33	0.066056977	0.055578	0.063587
4	0.060061276	0.057793	0.057389	34		0.029816	0.035873
5	0.059611554	0.061685	0.061051	35	0.070092187	0.056641	0.064193
6		0.048523	0.053204	36	0.083032668	0.078697	0.086864
7	0.061069648	0.047798	0.053953	37		0.053361	0.060432
8	0.063367129	0.054482	0.061625	38	0.045250608	0.041975	0.049671
9	0.077372599	0.068289	0.076442	39	0.072075606	0.067336	0.075558
10		0.040029	0.045066	40		0.032859	0.039048
11	0.078609831	0.061984	0.068868	41	0.032493901	0.035782	0.041605
12	0.095618202	0.088144	0.095859	42		0.02338	0.029035
13	0.086649656	0.077237	0.085998	43	0.040396009	0.030999	0.037698
14		0.055647	0.06166	44		0.024014	0.030178
15	0.086791981	0.071725	0.079412	45		0.024481	0.030658
16		0.035754	0.040907	46	0.042137827	0.0316	0.039272
17	0.077484967	0.058338	0.065618	47	0.055724682	0.051837	0.05994
18	0.064368993	0.058361	0.066493	48	0.045733141	0.052922	0.060227
19		0.046773	0.05268	49			0.035898
20	0.059921581	0.049146	0.056251	50			0.025079
21	0.05451498	0.05132	0.058779	51			0.044784
22	0.055040064	0.048739	0.057093	52			0.009898
23	0.061028263	0.063222	0.071332	53			0.037418
24		0.080415	0.087686	54			0.02717
25	0.073778102	0.066507	0.074586	55			0.027005
26		0.04608	0.052062	56			0.021723
27	0.068164771	0.059897	0.066798	57			0.027532
28		0.024057	0.029149	58			0.020757
29	0.072138127	0.055574	0.062506	59			0.036306
30	0.06517116	0.054556	0.062667	60			0.044581

Table 4.4.2 Forecasting fraction non-conforming for data generated using ARMA(1,2) process (15 observations missing)

Figure 4.4.2(a) Generated vs. Predicted values, ARMA(1,2) process, 15 observations missing

Figure 4.4.2(b) Generated vs. Predicted values, ARMA(1,2) process, 15 observations missing

Period	17 obs. Missing	Estimate	Forecast	Period	17 obs. Missing	Estimate	Forecast
1	0.066460039	0.070415	0.069831	31	0.057862093	0.046241	0.053812
2	0.068406027	0.066673	0.071281	32		0.034259	0.040758
3	0.069022682	0.071403	0.070747	33	0.066056977	0.051579	0.059687
4	0.060061276	0.057793	0.057389	34		0.0243	0.030717
5	0.059611554	0.061685	0.061051	35	0.070092187	0.055352	0.063408
6		0.048523		36	0.083032668	0.078234	0.08697
7	0.061069648	0.047798	0.053953	37		0.049869	0.057332
8	0.063367129	0.054482	0.061625	38	0.045250608	0.041598	0.049757
9	0.077372599	0.068289	0.076442	39	0.072075606	0.068429	0.076994
10		0.040029	0.045066	40		0.031387	0.037978
11	0.078609831	0.061984	0.068868	41	0.032493901	0.036125	0.042312
12	0.095618202	0.088144	0.095859	42		0.015584	0.02134
13	0.086649656	0.077237	0.085998	43	0.040396009	0.033125	0.039689
14		0.055647	0.06166	44		0.018935	0.025186
15	0.086791981	0.071725	0.079412	45		0.024857	0.031356
16		0.035754	0.040907	46	0.042137827	0.031445	0.03946
17	0.077484967	0.058338	0.065618	47	0.055724682	0.05215	0.060563
18	0.064368993	0.058361	0.066493	48	0.045733141	0.053101	0.060695
19		0.046773	0.05268	49			0.031115
20	0.059921581	0.049146	0.056251	50			0.026134
21	0.05451498	0.05132	0.058779	51			0.048349
22	0.055040064	0.048739	0.057093	52			0.01083
23	0.061028263	0.063222	0.071332	53			0.039855
24		0.080415	0.087686	54			0.021604
25		0.048313	0.054268	55			0.036329
26		0.024625	0.029417	56			0.023812
27	0.068164771	0.05442	0.06074	57			0.032577
28		0.015144	0.020043	58			0.023601
29	0.072138127	0.053407	0.060387	59			0.039914
30		0.027893	0.033816	60			0.048232

Table 4.4.3 Forecasting fraction non-conforming for data generated using ARMA(1,2) process (17 observations missing)

Figure 4.4.3(a) Generated vs. Predicted values, ARMA(1,2) process, 17 observations missing

Figure 4.4.3(b) Generated vs. Forecasted values, ARMA(1,2) process, 17 observations missing

Period	18 obs. Missing	Estimate	Forecast	Period	18 obs. Missing	Estimate	Forecast
1	0.066460039	0.070415	0.069831	31	0.057862093	0.046241	0.052812
2	0.068406027	0.066673	0.073281	32		0.034259	0.039758
3	0.069022682	0.071403	0.077747	33	0.066056977	0.051579	0.058687
4	0.060061276	0.057793	0.064389	34		0.0243	0.029717
5	0.059611554	0.061685	0.068051	35	0.070092187	0.055352	0.062408
6		0.048523	0.052204	36	0.083032668	0.078234	0.08597
7	0.061069648	0.047798	0.052953	37		0.049869	0.056332
8	0.063367129	0.054482	0.060625	38	0.045250608	0.041598	0.048757
9	0.077372599	0.068289	0.075442	39	0.072075606	0.068429	0.075994
10		0.040029	0.044066	40		0.031387	0.036978
11	0.078609831	0.061984	0.067868	41	0.032493901	0.036125	0.041312
12	0.095618202	0.088144	0.094859	42		0.015584	0.02034
13	0.086649656	0.077237	0.084998	43	0.040396009	0.033125	0.038689
14		0.055647	0.06066	44		0.018935	0.024186
15	0.086791981	0.071725	0.078412	45		0.024857	0.030356
16		0.035754	0.039907	46	0.042137827	0.031445	0.03846
17	0.077484967	0.058338	0.064618	47	0.055724682	0.05215	0.059563
18	0.064368993	0.058361	0.065493	48		0.067391	0.075101
19		0.046773	0.05168	49			0.043396
20	0.059921581	0.049146	0.055251	50			0.039532
21	0.05451498	0.05132	0.057779	51			0.062863
22	0.055040064	0.048739	0.056093	52			0.026461
23	0.061028263	0.063222	0.070332	53			0.056603
24		0.080415	0.086686	54			0.039468
25		0.048313	0.053268	55			0.05531
26		0.024625	0.028417	56			0.043909
27	0.068164771	0.05442	0.05974	57			0.05379
28		0.015144	0.019043	58			0.045931
29	0.072138127	0.053407	0.059387	59			0.06336
30		0.027893	0.032816	60			0.077707

Table 4.4.4 Forecasting fraction non-conforming for data generated using ARMA(1,2) process (18 observations missing)

Figure 4.4.4(a) Generated vs. Predicted values, ARMA(1,2) process, 18 observations missing

Figure 4.4.4(b) Forecasted vs. Predicted values, ARMA(1,2) process, 18 observations missing

<i>Period</i>	<i>11 obs.</i>	<i>Missing</i>	<i>Estimate</i>	<i>Forecast</i>	<i>Period</i>	<i>11 obs.</i>	<i>Missing</i>	<i>Estimate</i>	<i>Forecast</i>
1	0.101417725		0.091112	0.097957	31			0.042256	0.049574
2	0.104934133		0.109469	0.11581	32	0.052658789		0.038402	0.047305
3			0.016617	0.015801	33	0.070499177		0.059791	0.069884
4	0.116023022		0.074885	0.078639	34	0.054464728		0.043861	0.055131
5	0.080922007		0.071516	0.076315	35			0.04343	0.051514
6	0.099886633		0.093919	0.099381	36	0.0620485		0.056242	0.065149
7	0.094682663		0.080712	0.087727	37	0.049443655		0.041285	0.051099
8	0.07593737		0.070697	0.078294	38	0.062172423		0.0554	0.065967
9	0.090285713		0.087723	0.095605	39			0.017781	0.024943
10			0.030924	0.033445	40	0.042668614		0.030299	0.038835
11	0.056614679		0.039837	0.044222	41	0.022828031		0.025768	0.033978
12	0.067695322		0.060475	0.065662	42			0.019874	0.026797
13	0.044159983		0.033847	0.04018	43	0.023460506		0.019995	0.027409
14			0.03987	0.043707	44			0.006119	0.012711
15	0.037047144		0.014802	0.021794	45	0.022340746		0.01731	0.024462
16	0.027258606		0.036536	0.042497	46	0.051637085		0.027037	0.036922
17			0.023539	0.027609	47	0.046641543		0.03957	0.050458
18			0.026992	0.031002	48	0.037209459		0.044729	0.054781
19	0.037437339		0.022684	0.028333	49				0.026864
20	0.033519587		0.019885	0.027049	50				0.033693
21	0.047304722		0.03911	0.047185	51				0.009177
22	0.032965677		0.023242	0.032696	52				0.031266
23	0.038172224		0.039422	0.048737	53				0.031517
24			0.050894	0.058483	54				0.049418
25	0.052574234		0.035393	0.044891	55				0.038668
26	0.041473637		0.041936	0.051368	56				0.030719
27	0.068509549		0.046154	0.05807	57				0.044714
28	0.041733999		0.051267	0.062124	58				0.030008
29	0.074192064		0.06937	0.08091	59				0.032679
30	0.034715071		0.049303	0.058774	60				0.045356

Table 4.6.2 Forecasting fraction non-conforming for data generated using ARMA(2,2) process (11 observations missing)

Figure 4.5.1(a) Generated vs. Predicted values, ARMA(2,2) process, 11 observations missing

Figure 4.5.1(b) Generated vs. Forecasted values, ARMA(2,2) process, 11 observations missing

Period	14 obs. Missing	Estimate	Forecast	Period	14 obs. Missing	Estimate	Forecast
1	0.101417725	0.091112	0.100457	31		0.020511	0.02934
2	0.104934133	0.109469	0.11831	32	0.052658789	0.032455	0.043529
3		0.016617	0.018301	33	0.070499177	0.056105	0.068778
4	0.116023022	0.074885	0.081139	34		0.00636	0.015887
5	0.080922007	0.071516	0.078815	35		-0.00156	0.006927
6	0.099886633	0.093919	0.101881	36	0.0620485	0.046588	0.057263
7	0.094682663	0.080712	0.090227	37	0.049443655	0.033627	0.04606
8	0.07593737	0.070697	0.080794	38	0.062172423	0.050848	0.064539
9	0.090285713	0.087723	0.098105	39		0.011933	0.022499
10		0.030924	0.035945	40	0.042668614	0.02912	0.041191
11	0.056614679	0.039837	0.046722	41	0.022828031	0.023205	0.035204
12	0.067695322	0.060475	0.068162	42		0.036986	0.048087
13	0.044159983	0.033847	0.04268	43	0.023460506	0.022386	0.03364
14		0.03987	0.046207	44		0.016118	0.026451
15	0.037047144	0.014802	0.024294	45	0.022340746	0.020601	0.031128
16	0.027258606	0.036536	0.044997	46	0.051637085	0.031474	0.04486
17		0.023539	0.030109	47		0.023537	0.034875
18		0.026992	0.033502	48	0.037209459	0.039144	0.050266
19	0.037437339	0.022684	0.030833	49			0.060345
20	0.033519587	0.019885	0.029549	50			0.063674
21	0.047304722	0.03911	0.049685	51			0.034694
22	0.032965677	0.023242	0.035196	52			0.056139
23	0.038172224	0.039422	0.051237	53			0.049995
24		0.050894	0.060983	54			0.070646
25	0.052574234	0.035393	0.047391	55			0.055769
26	0.041473637	0.041936	0.053868	56			0.053052
27	0.068509549	0.046154	0.06057	57			0.061916
28	0.041733999	0.051267	0.064624	58			0.044784
29		0.051081	0.061507	59			0.055074
30	0.034715071	0.038088	0.048035	60			0.072615

Table 4.5.2 Forecasting fraction non-conforming for data generated using ARMA(2,2) process (14 observations missing)

Figure 4.5.2(a) Generated vs. Predicted values, ARMA(2,2) process, 14 observations missing

Figure 4.5.2(b) Generated vs. Forecasted values, ARMA(2,2) process, 14 observations missing

Period	12 obs. Missing	Estimate	Forecast	Period	12 obs. Missing	Estimate	Forecast
1	0.101417725	0.091112	0.097957	31		0.042256	0.049574
2	0.104934133	0.109469	0.11581	32	0.052658789	0.038402	0.047305
3		0.016617	0.015801	33	0.070499177	0.059791	0.069884
4	0.116023022	0.074885	0.078639	34	0.054464728	0.043861	0.055131
5	0.080922007	0.071516	0.076315	35		0.04343	0.051514
6	0.099886633	0.093919	0.099381	36	0.0620485	0.056242	0.065149
7	0.094682663	0.080712	0.087727	37	0.049443655	0.041285	0.051099
8	0.07593737	0.070697	0.078294	38	0.062172423	0.0554	0.065967
9	0.090285713	0.087723	0.095605	39		0.017781	0.024943
10		0.030924	0.033445	40	0.042668614	0.030299	0.038835
11	0.056614679	0.039837	0.044222	41	0.022828031	0.025768	0.033978
12	0.067695322	0.060475	0.065662	42		0.019874	0.026797
13	0.044159983	0.033847	0.04018	43	0.023460506	0.019995	0.027409
14		0.03987	0.043707	44		0.006119	0.012711
15	0.037047144	0.014802	0.021794	45	0.022340746	0.01731	0.024462
16	0.027258606	0.036536	0.042497	46	0.051637085	0.027037	0.036922
17		0.023539	0.027609	47		0.017872	0.025955
18		0.026992	0.031002	48	0.037209459	0.03362	0.042101
19	0.037437339	0.022684	0.028333	49			0.031779
20	0.033519587	0.019885	0.027049	50			0.037037
21	0.047304722	0.03911	0.047185	51			0.01095
22	0.032965677	0.023242	0.032696	52			0.031468
23	0.038172224	0.039422	0.048737	53			0.030148
24		0.050894	0.058483	54			0.046478
25	0.052574234	0.035393	0.044891	55			0.034157
26	0.041473637	0.041936	0.051368	56			0.024637
27	0.068509549	0.046154	0.05807	57			0.037061
28	0.041733999	0.051267	0.062124	58			0.020784
29	0.074192064	0.06937	0.08091	59			0.013911
30	0.034715071	0.049303	0.058774	60			0.040397

Table 4.5.3 Forecasting fraction non-conforming for data generated using ARMA(2,2) process (12 observations missing)

Figure 4.5.3(a) Generated vs. Predicted values, ARMA(2,2) process, 12 observations missing

Figure 4.5.3(a) Generated vs. Forecasted values, ARMA(2,2) process, 12 observations missing

5. CONCLUSIONS AND FUTURE RESEARCH

5.1 CONCLUSIONS

The results using Holt-Winters method for data generated using Autoregressive, Moving Average and Autoregressive Moving Average process illustrate that the method gives acceptable results even with some of the observations missing. This is further confirmed by performing the tracking signal analysis which shows that for almost all the methods, only a few number of forecasts are biased. The maximum number of missing observations considered is 18 which is almost 40% of the total number of observations. Thus based on the results obtained, it can be concluded that the Holt-Winters method for missing observations is good for a substantial number of observations missing.

It can be concluded from the data generated and the forecast observed that work-in-process quality is affected by the pattern of seasonality. This seasonal pattern is fairly predictable using the Holt-Winters method, unless otherwise influenced by some extraordinary conditions. For making the forecast of percent non-conforming, the past historical data is analyzed in this research to explore a pattern of fluctuation, which has been observed to be a good guide for the future forecasts. Curves plotted after generating the forecasts indicate reduced fluctuations; the curves are smoothed and show seasonal regularity. In other words, when an average fraction non-confirming is estimated using Holt-Winters method, the effects of WIP quality fluctuations are smoothened. An important decision, which has to be taken in time, is that of adjusting the process with the fluctuations in the WIP quality. This method can be applied to scenarios where there is change in demand over season and also where the level of the demand changes over time simultaneously. Practically, this method has been applied in several instances with appropriate

results, although the assumption of missing observations was not considered in all of these cases. The use of additive and multiplicative versions of this method has been made for forecasting for incoming calls to telemarketing centers for the purposes of planning and budgeting. Another instance of the use of this method has been for forecasting monthly claims for the Workers Compensation Board of the University of British Columbia. This was done with the aim of developing software for future use by non-expert staff.

5.2 FUTURE RESEARCH

There is not much research done specifically in the field of quality control with the assumption of variable percent non-conforming. This approach provides a much more realistic estimate for the percent non-conforming than assuming it to be a constant value. The research opportunities using this approach are endless. The effectiveness of the different approaches to inspection can be compared using many distributions (including beta) for percent non-conforming. Also, the effectiveness of various sampling plans can be compared.

This research uses Autoregressive, Moving Average and Autoregressive Moving Average process for generating values of fraction non-conforming for all the periods except the first one. Moreover, investigations comparing the used research methodology can be applied to obtain optimal results by interpolating missing observations in corresponding seasonal Autoregressive Integrated Moving Average (ARIMA) process. This research deals only with additive models, so similar model could be developed considering multiplicative models and the same approach could be applied to those.

Another major issue on work-in-process quality that can be targeted is corrective control of process conditions for achieving a target quality. So, a causal forecasting model that will be

based on multiple regression models can be used to predict a WIP quality under the current process conditions. This will require to find the most significant process variables affecting quality, set of process conditions and quality indices of WIP. Further, the most stable and corrective process conditions for achieving a target quality can also be found out using this approach. A computer model to implement these systems can also be developed. Most statistical time series forecasting methods are based upon some assumptions on the data generating process. This research presents a similar approach in which assumptions are made regarding the data generation. These assumptions, however, may not be necessarily satisfied in practical situations. So robustness properties of some forecasting methods for seasonal time series could be investigated. These investigations might include whether the various methods have reasonably good forecasting performances for a wide class of time series to which the methods are likely to be used.

References

1. Cipra, Trujillo, and Rubio, "*Holt-Winters Method with Missing Observations*", Management Science, 41 (1991), 174-178.
2. Wright. David J, "*Forecasting Data Published at Irregular Time Intervals*", Management Science, 36 (1986), 499-510.
3. Montgomery. Douglas C, "*Forecasting and Time Series Analysis*", McGraw-Hill Book Company, New York, 1990.
4. Box. G. E. P., and G. M. Jenkins, "*Time Series Analysis, Forecasting and Control*", Revised Edition, Holden-Day, San Francisco, CA, 1976.
5. Chatfield. C, and M. Yar, "*Holt-Winters Forecasting: Some Practical Issues*", The Statistician, 37 (1988), 129-140.
6. Gilchrist. W, "*Statistical Forecasting*", J. Wiley, New York, 1976.
7. Newbold. P, "*Some Recent Developments in Time Series Analysis*", International Statistical Review, 49 (1981), 53-66.
8. Brown, R. G., "*Smoothing, Forecasting and Prediction of Discrete Time Series*", Prentice-Hall, Englewood Cliffs, N.J., 1963.
9. Croston, J. D., "*Forecasting for Intermittent Demand*", Oper. Res. Quart., 23 (1972), 289-303.
10. Dunsmuir, W., "*Estimation for Stationery Time Series When Data are Irregularly Spaced or Missing*", Applied Time Series Analysis, II, Academic Press, New York, 1981, 609-648.
11. Jones, R.H., "*Maximum Likelihood Fitting of ARMA Models to Time Series with Missing Observations*", Technometrics, 22 (1980), 389-395.

12. Rao, A.G., “*A Comment on Forecasting for Intermittent Demand*”, Oper. Res. Quart., 24 (1973), 639-652.
13. Chunhang Chen, “*Robustness Properties of some Forecasting Methods for Seasonal Time Series: A Monte Carlo Study*”, International Journal of Forecasting, Vol. 13 (2), (Jan.1997), 269-280.
14. Bianchi, L., Jarrett, J., Choudhary, R., “*Improving Forecasting for Telemarketing Centers by ARIMA Modeling with Intervention*”, International Journal of Forecasting, Vol. 14 (4), (Oct.1998), 497-504.
15. Lawton, R., “*How Should Additive Holt-Winters Estimates be Corrected?*”, International Journal of Forecasting, Vol. 14 (3), (Sep.1998), 393-403.
16. Kapalka, B., Puterman, M., “*Forecasting Workers Compensation Claims*”, 1996, <http://www.informs.org/Conf/WA96/TALKS/WE28.4.html>

Period	Errors	Error Sq.	Delta(t)	Y(T)	Q(T)	Q(T)/Delta(T)
1	0.009505	9.03361E-05	0.007866	0.009505	0.001045	0.13292022
2	-0.01274	0.000162431	0.008354	-0.00324	-0.00033	-0.03992753
4	0.008509	7.2402E-05	0.008369	0.005269	0.000551	0.06580311
5	0.003494	1.22048E-05	0.007882	0.008762	0.000845	0.10721196
6	0.000823	6.77208E-07	0.007176	0.009585	0.000843	0.11745072
7	-0.00029	8.27853E-08	0.006487	0.009297	0.00073	0.11249428
8	-0.00262	6.83891E-06	0.0061	0.006682	0.000395	0.06479815
9	-0.00182	3.29789E-06	0.005671	0.004866	0.000174	0.03070236
11	0.004814	2.31764E-05	0.005586	0.00968	0.000638	0.11424526
12	0.003809	1.45095E-05	0.005408	0.01349	0.000955	0.17663399
13	-0.00542	2.94116E-05	0.005409	0.008066	0.000317	0.05867137
15	0.014582	0.000212621	0.006327	0.022648	0.001744	0.27562531
18	-0.01458	0.000212701	0.007152	0.008064	0.000111	0.01551803
19	0.006612	4.3714E-05	0.007098	0.014675	0.000761	0.10721588
20	0.003633	1.31985E-05	0.006752	0.018308	0.001048	0.15525411
21	-0.00104	1.07347E-06	0.00618	0.017272	0.00084	0.13588698
22	-0.00043	1.81475E-07	0.005605	0.016846	0.000713	0.1272536
23	0.000365	1.33436E-07	0.005081	0.017211	0.000678	0.13352821
24	-0.00249	6.21254E-06	0.004822	0.014719	0.000361	0.07493651
26	-0.00351	1.23335E-05	0.004691	0.011207	-2.6E-05	-0.00553801
27	0.012719	0.000161785	0.005494	0.023926	0.001249	0.22726553
29	-0.00202	4.07795E-06	0.005146	0.021907	0.000922	0.17910915
30	0.005559	3.09002E-05	0.005188	0.027466	0.001385	0.26707101
31	0.006851	4.69374E-05	0.005354	0.034317	0.001932	0.360858
32	-0.00734	5.38041E-05	0.005552	0.026982	0.001005	0.18106945
34	0.016842	0.000283645	0.006681	0.043824	0.002589	0.38750669
35	-0.00924	8.54104E-05	0.006937	0.034582	0.001406	0.20266102
36	0.006157	3.79058E-05	0.006859	0.040739	0.001881	0.27423048
37	-0.00745	5.54608E-05	0.006918	0.033291	0.000948	0.13705855
38	0.000432	1.86717E-07	0.006269	0.033724	0.000897	0.14300627
39	-0.00579	3.35138E-05	0.006221	0.027934	0.000228	0.03664629
41	5.16E-05	2.66463E-09	0.005604	0.027986	0.00021	0.0375336
42	0.000967	9.35623E-07	0.005141	0.028953	0.000286	0.05564362
46	0.028431	0.000808303	0.00747	0.057384	0.003101	0.41508094
47	0.012234	0.000149682	0.007946	0.069618	0.004014	0.50513964
48	0.009372	8.78277E-05	0.008089	0.07899	0.00455	0.56247477
	SSE	0.002757916				
	SSE/34	8.11152E-05				
	K3	0.42				
	Delta(0)	0.007683507				

Table A.1.1 Tracking Signal Analysis { AR(1) process, 12 observations missing }

Period	Errors	Error Sq.	Delta(t)	Y(T)	Q(T)	Q(T)/Delta(T)
1	-0.005	2.49547E-05	0.011086	-0.005	-0.0005	-0.04506083
2	-0.03204	0.001026872	0.013182	-0.03704	-0.00365	-0.27720341
4	-0.01326	0.000175731	0.013189	-0.0503	-0.00461	-0.34985008
5	-0.01761	0.000310024	0.013631	-0.0679	-0.00591	-0.43383043
6	-0.01902	0.00036159	0.01417	-0.08692	-0.00722	-0.50980995
7	-0.01993	0.000397305	0.014746	-0.10685	-0.00849	-0.5760705
8	-0.01841	0.000338961	0.015112	-0.12526	-0.00949	-0.62771627
9	-0.0162	0.000262444	0.015221	-0.14146	-0.01016	-0.66733899
11	-0.0038	1.44168E-05	0.014079	-0.14526	-0.00952	-0.67631064
12	-0.00806	6.49675E-05	0.013477	-0.15332	-0.00938	-0.69566978
13	-0.01546	0.000238989	0.013675	-0.16878	-0.00998	-0.73007327
15	0.008846	7.82468E-05	0.013192	-0.15993	-0.0081	-0.61406729
18	0.014445	0.000208646	0.013317	-0.14549	-0.00585	-0.43899954
20	0.016306	0.000265877	0.013616	-0.12918	-0.00363	-0.26667663
21	0.00573	3.28326E-05	0.012828	-0.12345	-0.0027	-0.21009551
22	0.000538	2.89E-07	0.011599	-0.12292	-0.00237	-0.20448681
24	0.013329	0.000177663	0.011772	-0.10959	-0.0008	-0.06810317
26	0.000893	7.96818E-07	0.010684	-0.10869	-0.00063	-0.05917899
27	0.015233	0.000232038	0.011139	-0.09346	0.000954	0.08566995
29	-0.00045	1.98365E-07	0.010069	-0.09391	0.000814	0.08086787
31	0.010482	0.00010988	0.010111	-0.08342	0.001781	0.17616027
32	0.010995	0.00012089	0.010199	-0.07243	0.002702	0.26497338
34	-0.0175	0.000306408	0.01093	-0.08993	0.000682	0.06237957
35	0.00821	6.74024E-05	0.010658	-0.08172	0.001435	0.13460719
36	0.00533	2.84124E-05	0.010125	-0.07639	0.001824	0.18016636
37	-0.00573	3.28789E-05	0.009686	-0.08213	0.001068	0.11030046
38	-0.00072	5.1948E-07	0.008789	-0.08285	0.000889	0.10119567
39	-0.00577	3.32783E-05	0.008487	-0.08862	0.000224	0.02634812
41	0.001784	3.18347E-06	0.007817	-0.08683	0.00038	0.0485718
42	0.000817	6.67363E-07	0.007117	-0.08602	0.000423	0.0594928
46	0.026098	0.00068113	0.009015	-0.05992	0.002991	0.33176681
47	0.010285	0.000105787	0.009142	-0.04963	0.00372	0.40694589
	SSE	0.005703281				
	SSE/30	0.000190109				
	Delta(0)	0.011762785				

Table A.1.2 Tracking Signal Analysis { AR(1) process, 16 observations missing }

Period	Errors	Error Sq.	Delta(t)	Y(T)	Q(T)	Q(T)/Delta(T)
2	-0.01113	0.000123971	0.009451	-0.01113	-0.00111	-0.11781302
4	-0.01011	0.00010227	0.009517	-0.02125	-0.00201	-0.21155541
5	-0.01874	0.000351121	0.010439	-0.03999	-0.00369	-0.35308163
7	-0.01035	0.000107037	0.01043	-0.05033	-0.00435	-0.41725295
8	-0.01648	0.000271633	0.011035	-0.06681	-0.00556	-0.50428956
9	0.006837	4.67455E-05	0.010615	-0.05998	-0.00432	-0.40740022
11	-0.01935	0.0003745	0.011489	-0.07933	-0.00583	-0.50721891
12	-0.01608	0.00025859	0.011948	-0.09541	-0.00685	-0.57354189
13	-0.00586	3.43895E-05	0.01134	-0.10127	-0.00675	-0.59559602
15	-0.0018	3.2282E-06	0.010385	-0.10307	-0.00626	-0.60259243
18	0.005844	3.41554E-05	0.009931	-0.09722	-0.00505	-0.50828426
19	3.71E-05	1.37447E-09	0.008942	-0.09719	-0.00454	-0.50765891
20	-0.00064	4.1162E-07	0.008112	-0.09783	-0.00415	-0.51155292
22	0.015575	0.000242582	0.008858	-0.08225	-0.00218	-0.24577985
23	0.015633	0.000244383	0.009536	-0.06662	-0.0004	-0.04154514
24	0.005082	2.58292E-05	0.00909	-0.06154	0.000152	0.01668636
26	-0.00011	1.24324E-08	0.008192	-0.06165	0.000125	0.01530262
27	0.0229	0.0005244	0.009663	-0.03875	0.002403	0.24865761
29	-0.00193	3.70922E-06	0.008889	-0.04068	0.00197	0.22160479
30	0.000957	9.16724E-07	0.008096	-0.03972	0.001869	0.23081009
31	0.005379	2.89326E-05	0.007824	-0.03434	0.00222	0.28368777
34	0.002467	6.08775E-06	0.007289	-0.03187	0.002244	0.30793584
35	0.016385	0.000268457	0.008198	-0.01549	0.003658	0.4462471
37	0.01499	0.000224701	0.008878	-0.0005	0.004792	0.53975036
38	-0.00619	3.83531E-05	0.008609	-0.00669	0.003693	0.42898738
39	-0.00605	3.65561E-05	0.008353	-0.01274	0.002719	0.32555001
41	-0.0048	2.3007E-05	0.007997	-0.01753	0.001968	0.2460457
42	-0.00467	2.17808E-05	0.007664	-0.0222	0.001304	0.17016909
46	0.021121	0.000446101	0.00901	-0.00108	0.003286	0.36470044
47	0.012849	0.000165095	0.009394	0.011769	0.004242	0.45159789
	SSE	0.004008957				
	SSE/28	0.000117911				
	Delta(0)	0.009263704				

Table A.1.3 Tracking Signal Analysis { AR(1) process, 18 observations missing }

Period	Error	error Sq.	delta(T)	Y(T)	Q(T)	Q(T)/Delta(T)
1	-0.00124	1.53637E-06	0.007104	-0.00124	-0.00012	-0.01744907
2	-0.0036	1.29611E-05	0.006753	-0.00484	-0.00047	-0.06982916
3	-0.00315	9.91478E-06	0.006393	-0.00799	-0.00074	-0.11564499
5	0.019209	0.000368993	0.007674	0.011221	0.001256	0.16360308
6	0.010017	0.000100332	0.007909	0.021237	0.002132	0.26953584
7	0.002049	4.19772E-06	0.007323	0.023286	0.002123	0.28997383
8	-0.00013	1.7398E-08	0.006604	0.023154	0.001898	0.28739719
9	-0.00252	6.35581E-06	0.006195	0.020633	0.001456	0.23500889
10	-0.00118	1.39121E-06	0.005694	0.019454	0.001192	0.20942486
11	-0.00451	2.03288E-05	0.005575	0.014945	0.000622	0.1116176
13	0.017035	0.000290201	0.006721	0.03198	0.002264	0.33678197
15	-0.02833	0.000802786	0.008882	0.003647	-0.0008	-0.08962758
16	0.009468	8.96379E-05	0.008941	0.013114	0.00023	0.02575445
18	0.014252	0.000203129	0.009472	0.027367	0.001632	0.17234534
19	0.007595	5.76907E-05	0.009284	0.034962	0.002229	0.24005422
20	-0.00139	1.94066E-06	0.008495	0.033569	0.001867	0.21971963
21	-1.4E-05	2.03444E-10	0.007647	0.033555	0.001679	0.21949213
23	0.015543	0.000241591	0.008437	0.049098	0.003065	0.36328559
24	-0.0002	3.82438E-08	0.007613	0.048903	0.002739	0.35978348
25	-0.0048	2.30541E-05	0.007332	0.044101	0.001985	0.27073073
26	0.000307	9.4283E-08	0.006629	0.044408	0.001817	0.27410865
29	0.004064	1.65192E-05	0.006373	0.048472	0.002042	0.32040499
30	-0.004	1.60363E-05	0.006136	0.044468	0.001437	0.23422916
31	0.000436	1.90278E-07	0.005566	0.044904	0.001337	0.24023067
32	0.00363	1.31789E-05	0.005372	0.048534	0.001566	0.29157122
33	-0.004	1.60108E-05	0.005235	0.044533	0.00101	0.19285453
35	0.013725	0.00018837	0.006084	0.058258	0.002281	0.37493199
36	0.004376	1.91464E-05	0.005913	0.062634	0.002491	0.421185
37	0.000235	5.51736E-08	0.005345	0.062868	0.002265	0.42372842
39	-0.00375	1.40269E-05	0.005185	0.059123	0.001664	0.32089797
40	-0.00133	1.76338E-06	0.0048	0.057795	0.001365	0.28435295
42	0.005745	3.30021E-05	0.004894	0.06354	0.001803	0.36835452
44	0.010082	0.000101642	0.005413	0.073622	0.002631	0.48600006
45	0.006278	3.94084E-05	0.005499	0.079899	0.002995	0.54467326
46	-0.00617	3.80409E-05	0.005566	0.073732	0.002079	0.37351476
47	-0.009	8.10599E-05	0.00591	0.064728	0.000971	0.16427145
48	-0.00881	7.75491E-05	0.0062	0.055922	-6.9E-06	-0.00110731
	SSE	0.00289219				
	SSE/35	8.2634E-05				
	Delta(0)	0.007755107				

Table A.2.1 Tracking Signal Analysis { AR(2) process, 11 observations missing }

Period	Error	error Sq.	delta(T)	Y(T)	Q(T)	Q(T)/Delta(T)
1	-0.00624	3.89314E-05	0.010045	-0.00624	-0.00062	-0.06211701
2	-0.0086	7.39627E-05	0.0099	-0.01484	-0.00142	-0.14358871
3	-0.00815	6.64025E-05	0.009725	-0.02299	-0.00209	-0.21534808
4	-0.00785	6.16523E-05	0.009538	-0.03084	-0.00267	-0.27994363
5	-0.00815	6.64968E-05	0.009399	-0.03899	-0.00322	-0.34241232
6	-0.00724	5.24812E-05	0.009184	-0.04624	-0.00362	-0.3942833
7	-0.00846	7.15717E-05	0.009112	-0.0547	-0.0041	-0.45052341
8	-0.00617	3.8058E-05	0.008817	-0.06087	-0.00431	-0.48896797
9	-0.01063	0.000113078	0.008999	-0.0715	-0.00494	-0.54935507
10	-0.00741	5.49535E-05	0.00884	-0.07892	-0.00519	-0.58714366
11	-0.00957	9.1498E-05	0.008913	-0.08848	-0.00563	-0.63145201
13	0.018086	0.0003271	0.00983	-0.07039	-0.00326	-0.33129248
15	0.02965	0.000879119	0.011812	-0.04074	3.4E-05	0.00287754
16	0.015017	0.0002255	0.012133	-0.02573	0.001532	0.12629227
18	0.021739	0.000472563	0.013093	-0.00399	0.003553	0.27135303
19	0.01079	0.000116424	0.012863	0.0068	0.004277	0.33247538
20	-0.00035	1.25283E-07	0.011612	0.006446	0.003814	0.32841378
21	6.13E-05	3.76345E-09	0.010457	0.006508	0.003438	0.32880777
23	0.015751	0.000248085	0.010986	0.022258	0.00467	0.42503444
24	-7.3E-05	5.33468E-09	0.009895	0.022185	0.004195	0.42398256
25	-0.0096	9.21956E-05	0.009866	0.012584	0.002816	0.28539218
26	0.000622	3.87167E-07	0.008941	0.013206	0.002596	0.29036514
28	-0.0061	3.71894E-05	0.008657	0.007107	0.001727	0.19946735
29	0.001002	1.00451E-06	0.007892	0.00811	0.001654	0.20963436
30	-0.01036	0.000107375	0.008139	-0.00225	0.000453	0.05562195
32	0.013508	0.000182453	0.008676	0.011255	0.001758	0.20265891
33	0.002059	4.23765E-06	0.008014	0.013314	0.001788	0.22314068
35	0.016276	0.000264894	0.00884	0.029589	0.003237	0.36617067
37	-0.02201	0.000484334	0.010157	0.007582	0.000712	0.07014943
39	0.002957	8.74637E-06	0.009437	0.010539	0.000937	0.09929025
42	0.021944	0.000481558	0.010688	0.032483	0.003038	0.28422985
44	0.01411	0.000199085	0.01103	0.046593	0.004145	0.37579406
45	0.008401	7.05797E-05	0.010767	0.054994	0.004571	0.42449933
46	-0.00589	3.46366E-05	0.010279	0.049109	0.003525	0.34293715
48	0.001177	1.38624E-06	0.009369	0.050286	0.00329	0.35119467
	SSE	0.004968076				
	SSE/33	0.000150548				
	DELTA(0)	0.010467563				

Table A.2.2 Tracking Signal Analysis { AR(2) process, 13 observations missing }

Period	Error	error Sq.	delta(T)	Y(T)	Q(T)	Q(T)/Delta(T)
1	-0.00624	3.89314E-05	0.007096	-0.00624	-0.00062	-0.08792541
2	-0.0086	7.39627E-05	0.007247	-0.01484	-0.00142	-0.19616698
3	-0.00815	6.64025E-05	0.007337	-0.02299	-0.00209	-0.28544469
5	0.014209	0.000201901	0.008024	-0.00878	-0.00046	-0.05781828
6	0.005017	2.51659E-05	0.007723	-0.00376	8.41E-05	0.01088989
7	-0.00295	8.70938E-06	0.007246	-0.00671	-0.00022	-0.03028081
8	-0.00013	1.7398E-08	0.006535	-0.00685	-0.00021	-0.03223815
9	-0.00752	5.65665E-05	0.006633	-0.01437	-0.00094	-0.14196504
10	-0.00618	3.81862E-05	0.006588	-0.02055	-0.00147	-0.22244813
11	-0.00951	9.04162E-05	0.00688	-0.03006	-0.00227	-0.32991128
13	0.012035	0.000144848	0.007396	-0.01802	-0.00084	-0.11348706
15	0.023333	0.000544451	0.008989	0.005314	0.001578	0.17553757
16	0.004468	1.99606E-05	0.008537	0.009781	0.001867	0.21868365
18	0.009252	8.56057E-05	0.008609	0.019034	0.002605	0.30265657
19	0.002595	6.73631E-06	0.008007	0.021629	0.002604	0.32525959
20	-0.00139	1.94066E-06	0.007346	0.020236	0.002205	0.30012762
21	-1.4E-05	2.03444E-10	0.006613	0.020222	0.001983	0.29984719
23	0.010543	0.000111159	0.007006	0.030765	0.002839	0.40521428
24	-0.0002	3.82438E-08	0.006325	0.030569	0.002535	0.40086942
26	-0.00746	5.57233E-05	0.006439	0.023105	0.001535	0.23846027
29	0.001642	2.6951E-06	0.005959	0.024746	0.001546	0.25943999
30	-0.0023	5.27848E-06	0.005593	0.022449	0.001162	0.20770408
32	0.010052	0.000101052	0.006039	0.032501	0.002051	0.33959162
33	-0.00217	4.72304E-06	0.005652	0.030328	0.001628	0.28808584
35	0.010554	0.000111378	0.006142	0.040882	0.002521	0.41040272
37	0.009096	8.27444E-05	0.006438	0.049978	0.003178	0.49371021
39	-0.00485	2.3537E-05	0.006279	0.045127	0.002375	0.37830208
42	0.007797	6.07948E-05	0.006431	0.052924	0.002918	0.45367827
44	0.005361	2.87455E-05	0.006324	0.058285	0.003162	0.49999504
45	0.001691	2.85956E-06	0.005861	0.059976	0.003015	0.51442187
46	-0.01105	0.000122182	0.00638	0.048923	0.001608	0.25204339
48	-0.00389	1.51104E-05	0.006131	0.045035	0.001059	0.17265749
	SSE	0.002131823				
	SSE/30	7.10608E-05				
	Delta(0)	0.007191565				

Table A.2.3 Tracking Signal Analysis { AR(2) process, 16 observations missing }

Period	Error	error Sq.	delta(T)	Y(T)	Q(T)	Q(T)/Delta(T)
1	-0.00624	3.89314E-05	0.010513	-0.00624	-0.00062	-0.05935239
2	-0.0086	7.39627E-05	0.010321	-0.01484	-0.00142	-0.13773055
3	-0.00815	6.64025E-05	0.010104	-0.02299	-0.00209	-0.20727082
5	0.019209	0.000368993	0.011015	-0.00378	3.61E-05	0.00327351
6	0.010017	0.000100332	0.010915	0.006237	0.001034	0.09474332
7	0.002049	4.19772E-06	0.010028	0.008286	0.001136	0.11323833
9	0.021941	0.000481415	0.01122	0.030227	0.003216	0.28665564
10	-0.00259	6.73305E-06	0.010357	0.027633	0.002635	0.25442031
11	-0.00319	1.02016E-05	0.009641	0.024439	0.002052	0.21286125
13	0.020387	0.00041562	0.010715	0.044825	0.003886	0.36262031
15	0.028395	0.00080625	0.012483	0.07322	0.006336	0.50759923
16	-0.00129	1.6651E-06	0.011364	0.071929	0.005574	0.49048035
18	-0.00209	4.37376E-06	0.010437	0.069838	0.004807	0.46061346
19	0.006178	3.81678E-05	0.010011	0.076016	0.004944	0.49390071
20	-0.02853	0.000814002	0.011863	0.047485	0.001597	0.1346099
21	0.011279	0.000127217	0.011804	0.058764	0.002565	0.21729716
23	0.019016	0.000361597	0.012526	0.07778	0.00421	0.33612319
24	-0.00703	4.93553E-05	0.011976	0.070755	0.003087	0.25774096
26	-0.00281	7.87921E-06	0.011059	0.067948	0.002497	0.22581613
29	-0.00474	2.24543E-05	0.010427	0.063209	0.001774	0.1701067
30	-0.00372	1.38733E-05	0.009756	0.059485	0.001224	0.12543607
32	-0.00851	7.25018E-05	0.009632	0.05097	0.00025	0.02594951
33	0.0067	4.48966E-05	0.009339	0.05767	0.000895	0.09583412
35	0.016952	0.000287373	0.0101	0.074622	0.002501	0.24758497
37	0.014171	0.00020081	0.010507	0.088793	0.003668	0.34905837
39	-0.0005	2.48064E-07	0.009507	0.088295	0.003251	0.34199046
42	0.012004	0.000144086	0.009756	0.100299	0.004126	0.42294871
44	-0.00745	5.54497E-05	0.009525	0.092852	0.002969	0.31170835
46	-0.00084	7.13719E-07	0.008657	0.092007	0.002588	0.29890797
48	0.004974	2.47408E-05	0.008289	0.096981	0.002826	0.34097928
	SSE	0.004644443				
	SSE/28	0.000165873				
	Delta(0)	0.010987432				

Table A.2.4 Tracking Signal Analysis { AR(2) process, 18 observations missing }

Period	Error	Error Sq.	Delta(T)	Y(T)	Q(T)	Q(T)/Delta(T)
1	-0.00127	1.60321E-06	0.007375	-0.00127	-0.00013	-0.01716851
2	-0.00477	2.27779E-05	0.007115	-0.00604	-0.00059	-0.083097364
3	-0.00347	1.20694E-05	0.006751	-0.00951	-0.00088	-0.130283782
4	-0.00462	2.13858E-05	0.006538	-0.01414	-0.00125	-0.191800149
5	-0.00134	1.79484E-06	0.006018	-0.01548	-0.00126	-0.209791433
7	0.016834	0.000283384	0.0071	0.001357	0.000547	0.077056031
8	-0.00044	1.93798E-07	0.006434	0.000917	0.000448	0.069686473
9	-0.00186	3.47711E-06	0.005977	-0.00095	0.000217	0.036314127
10	-0.00059	3.44716E-07	0.005438	-0.00154	0.000137	0.025125258
12	0.016502	0.000272325	0.006544	0.014967	0.001773	0.270948755
13	0.009689	9.38736E-05	0.006859	0.024656	0.002565	0.373934986
14	-0.00075	5.58832E-07	0.006248	0.023908	0.002234	0.357495581
15	0.000241	5.82389E-08	0.005647	0.02415	0.002034	0.360241321
17	0.01151	0.000132482	0.006233	0.03566	0.002982	0.478374418
18	-0.00409	1.67504E-05	0.006019	0.031567	0.002274	0.377854976
21	0.026904	0.000723836	0.008108	0.058471	0.004737	0.584302123
22	0.016009	0.000256288	0.008898	0.07448	0.005865	0.659093897
23	-0.00277	7.68598E-06	0.008285	0.071708	0.005001	0.603579064
24	-0.00014	2.08141E-08	0.007471	0.071564	0.004486	0.600482532
25	-0.0114	0.000129946	0.007864	0.060164	0.002898	0.368484295
26	0.000443	1.95852E-07	0.007122	0.060607	0.002652	0.3724085
27	-0.00251	6.28769E-06	0.00666	0.058099	0.002136	0.320740248
28	-0.00914	8.35894E-05	0.006909	0.048957	0.001008	0.145958281
29	-0.01012	0.000102314	0.007229	0.038842	-0.0001	-0.014380614
31	0.006295	3.96222E-05	0.007136	0.045136	0.000536	0.075098885
33	0.023861	0.000569351	0.008808	0.068997	0.002868	0.325645709
34	-0.00455	2.06618E-05	0.008382	0.064452	0.002127	0.253757263
35	0.002838	8.05558E-06	0.007828	0.06729	0.002198	0.280815136
36	-0.00467	2.18528E-05	0.007512	0.062615	0.001511	0.20111464
37	-0.00654	4.27219E-05	0.007415	0.056079	0.000706	0.095235386
38	-0.0045	2.02781E-05	0.007124	0.051576	0.000185	0.026001196
40	0.006912	4.77748E-05	0.007102	0.058488	0.000858	0.120788306
42	0.002556	6.53158E-06	0.006648	0.061044	0.001028	0.154589038
43	-0.00188	3.55061E-06	0.006171	0.059159	0.000736	0.119336283
44	0.001283	1.64591E-06	0.005683	0.060442	0.000791	0.139218604
45	-0.00174	3.02148E-06	0.005288	0.058704	0.000538	0.101771944
46	-0.00437	1.91017E-05	0.005196	0.054333	4.73E-05	0.009104699
48	0.015197	0.000230949	0.006196	0.06953	0.001562	0.252125095
	SSE	0.003208362				
	SSE/36	8.91212E-05				
	DELTA(0)	0.008053763				

Table A.3.1 Tracking Signal Analysis{ARMA(1,1) process, 10 observations missing }

Period	Error	Error Sq.	Delta(T)	Y(T)	Q(T)	Q(T)/Delta(T)
1	-0.007266178	5.27973E-05	0.006976	-0.00727	-0.00073	-0.104166353
2	-0.005772623	3.33232E-05	0.006855	-0.01304	-0.00123	-0.179602005
3	-0.004474102	2.00176E-05	0.006617	-0.01751	-0.00156	-0.23507223
4	-0.005624483	3.16348E-05	0.006518	-0.02314	-0.00196	-0.301080274
5	-0.002339717	5.47428E-06	0.0061	-0.02548	-0.002	-0.327887779
7	0.01583402	0.000250716	0.007073	-0.00964	-0.00022	-0.030638492
8	0.008559775	7.32697E-05	0.007222	-0.00108	0.000661	0.091514993
9	0.002135298	4.5595E-06	0.006713	0.001052	0.000808	0.120410683
10	-0.001587126	2.51897E-06	0.006201	-0.00054	0.000569	0.09173313
12	0.015502284	0.000240321	0.007131	0.014967	0.002062	0.289185744
13	0.008688839	7.54959E-05	0.007287	0.023656	0.002725	0.373944707
14	0.003252449	1.05784E-05	0.006883	0.026908	0.002778	0.403526657
15	-0.000758673	5.75584E-07	0.006271	0.02615	0.002424	0.386546181
17	-0.010510085	0.000110462	0.006695	0.01564	0.001131	0.168872615
18	0.004907278	2.40814E-05	0.006516	0.020547	0.001508	0.231465725
21	2.58642E-05	6.68958E-10	0.005867	0.020573	0.00136	0.231804528
22	0.01500901	0.00022527	0.006781	0.035582	0.002725	0.401831348
23	0.00122764	1.5071E-06	0.006226	0.036809	0.002575	0.413626313
24	-0.001144271	1.30936E-06	0.005718	0.035665	0.002203	0.385335638
25	-0.012399371	0.000153744	0.006386	0.023266	0.000743	0.116345884
27	0.018904852	0.000357393	0.007638	0.042171	0.002559	0.335066637
28	0.001225708	1.50236E-06	0.006997	0.043396	0.002426	0.346715445
29	-0.002507755	6.28884E-06	0.006548	0.040889	0.001932	0.295136281
31	0.007195508	5.17753E-05	0.006612	0.048084	0.002459	0.371837808
33	0.000265979	7.0745E-08	0.005978	0.04835	0.002239	0.374632784
34	0.013920721	0.000193786	0.006772	0.062271	0.003408	0.503183251
35	0.000977714	9.55924E-07	0.006193	0.063249	0.003165	0.511027127
36	-0.001696866	2.87935E-06	0.005743	0.061552	0.002678	0.466381905
37	-0.008614462	7.4209E-05	0.00603	0.052937	0.001549	0.256902056
38	-0.006469709	4.18571E-05	0.006074	0.046468	0.000747	0.123027072
40	-0.001560658	2.43565E-06	0.005623	0.044907	0.000516	0.091856547
42	0.000777524	6.04543E-07	0.005138	0.045684	0.000543	0.105598544
43	0.005106622	2.60776E-05	0.005135	0.050791	0.000999	0.194542325
44	-1.87147E-05	3.5024E-10	0.004623	0.050772	0.000897	0.194058804
45	-0.003249453	1.05589E-05	0.004486	0.047523	0.000483	0.107568147
46	-0.00563538	3.17575E-05	0.004601	0.041887	-0.00013	-0.028088393
48	0.014090234	0.000198535	0.00555	0.055978	0.001293	0.232924021
	SSE	0.002318345				
	SSE/35	6.62384E-05				
	DELTA(0)	0.00694326				

Table A.3.2 Tracking Signal Analysis{ARMA(1,1) process, 11 observations missing}

Period	Error	Error Sq.	Delta(T)	Y(T)	Q(T)	Q(T)/Delta(T)
1	-0.01127	0.000126927	0.008468	-0.01127	-0.00113	-0.133041823
2	-0.00977	9.55042E-05	0.008599	-0.02104	-0.00199	-0.231574836
3	-0.00847	7.18104E-05	0.008586	-0.02951	-0.00264	-0.307414605
4	-0.00962	9.26307E-05	0.00869	-0.03914	-0.00334	-0.384121073
5	-0.00634	4.0192E-05	0.008455	-0.04548	-0.00364	-0.430301079
7	0.011834	0.000140044	0.008793	-0.03364	-0.00209	-0.237801584
8	0.00456	2.07915E-05	0.00837	-0.02908	-0.00143	-0.170365514
9	-0.00186	3.47711E-06	0.007719	-0.03095	-0.00147	-0.190407072
10	-0.00559	3.1216E-05	0.007506	-0.03654	-0.00188	-0.250670516
12	0.011502	0.000132303	0.007906	-0.02503	-0.00054	-0.068701724
13	0.004689	2.19852E-05	0.007584	-0.02034	-2E-05	-0.002627468
14	-0.00075	5.58832E-07	0.0069	-0.02109	-9.3E-05	-0.013432734
15	-0.00476	2.2645E-05	0.006686	-0.02585	-0.00056	-0.083649708
17	0.00651	4.23812E-05	0.006668	-0.01934	0.000148	0.022141543
18	0.000907	8.23154E-07	0.006092	-0.01843	0.000224	0.03670391
21	0.021904	0.000479794	0.007674	0.003471	0.002392	0.311678142
22	0.011009	0.000121198	0.008007	0.01448	0.003253	0.40631617
23	-0.00277	7.68598E-06	0.007484	0.011708	0.002651	0.354218107
24	-0.00514	2.64635E-05	0.00725	0.006564	0.001871	0.258124618
26	0.009095	8.27175E-05	0.007434	0.015659	0.002594	0.348884843
27	0.001104	1.21956E-06	0.006801	0.016763	0.002445	0.359457229
28	-0.00998	9.95247E-05	0.007119	0.006787	0.001203	0.168942081
29	-0.0121	0.000146482	0.007617	-0.00532	-0.00013	-0.016792562
31	0.001694	2.87001E-06	0.007025	-0.00362	5.43E-05	0.007728411
33	0.018536	0.000343581	0.008176	0.014914	0.001902	0.232689385
35	0.022462	0.000504522	0.009605	0.037375	0.003958	0.412136378
36	0.010594	0.000112223	0.009703	0.047969	0.004622	0.476315502
37	-0.00589	3.47126E-05	0.009322	0.042077	0.003571	0.383011012
38	-0.00644	4.15201E-05	0.009034	0.035634	0.002569	0.284370272
40	0.001341	1.79765E-06	0.008265	0.036974	0.002446	0.295979332
42	-0.00344	1.18482E-05	0.007783	0.033532	0.001857	0.238661125
43	0.001539	2.36707E-06	0.007158	0.035071	0.001826	0.255024513
44	-0.00489	2.39534E-05	0.006932	0.030177	0.001154	0.166414379
45	-0.00823	6.76551E-05	0.007061	0.021951	0.000216	0.030544813
46	-0.00785	6.16447E-05	0.00714	0.0141	-0.00059	-0.082773778
	SSE	0.003017069				
	SSE/33	9.14263E-05				
	DELTA(0)	0.008157256				

Table A.3.3 Tracking Signal Analysis{ARMA(1,1) process, 13 observations missing}

Period	Error	Error Sq.	Delta(T)	Y(T)	Q(T)	Q(T)/Delta(T)
1	-0.00127	1.60321E-06	0.007187	-0.00127	-0.00013	-0.017617064
2	-0.01677	0.000281321	0.008146	-0.01804	-0.00179	-0.21989561
4	0.007533	5.67429E-05	0.008084	-0.01051	-0.00086	-0.106230602
5	-0.00051	2.60276E-07	0.007327	-0.01102	-0.00082	-0.112453801
7	0.011022	0.000121491	0.007697	6.1E-06	0.000361	0.046861254
8	0.00083	6.8964E-07	0.00701	0.000837	0.000408	0.058152773
9	-0.00748	5.58818E-05	0.007056	-0.00664	-0.00038	-0.053944353
12	0.02123	0.000450721	0.008474	0.014591	0.00178	0.210108016
13	0.002282	5.20906E-06	0.007855	0.016874	0.001831	0.23305983
14	-0.00581	3.3728E-05	0.00765	0.011066	0.001067	0.139450754
15	-0.01165	0.000135729	0.00805	-0.00058	-0.0002	-0.025454597
18	0.002059	4.23958E-06	0.007451	0.001475	2.15E-05	0.002883273
21	0.014609	0.000213432	0.008167	0.016084	0.00148	0.1812547
22	-0.00252	6.36588E-06	0.007602	0.013561	0.00108	0.142051446
23	-0.00981	9.61555E-05	0.007823	0.003755	-8.7E-06	-0.001105872
25	-0.00394	1.55383E-05	0.007435	-0.00019	-0.0004	-0.05406729
26	0.000879	7.72547E-07	0.006779	0.000692	-0.00027	-0.040400717
27	-0.00643	4.12865E-05	0.006744	-0.00573	-0.00089	-0.131831901
28	-0.02173	0.000472024	0.008242	-0.02746	-0.00297	-0.360684143
31	-0.00098	9.65373E-07	0.007516	-0.02844	-0.00277	-0.369041619
33	0.012473	0.000155585	0.008012	-0.01597	-0.00125	-0.155897701
34	0.000461	2.12454E-07	0.007257	-0.01551	-0.00108	-0.148555694
35	-0.00882	7.78617E-05	0.007413	-0.02433	-0.00185	-0.249900337
37	0.003468	1.20299E-05	0.007019	-0.02086	-0.00132	-0.188136102
38	-0.00079	6.26232E-07	0.006396	-0.02165	-0.00127	-0.1981807
40	-0.00374	1.39653E-05	0.00613	-0.02539	-0.00151	-0.247059896
43	0.009061	8.20993E-05	0.006423	-0.01633	-0.00046	-0.071146379
44	-0.00511	2.60753E-05	0.006292	-0.02144	-0.00092	-0.146534049
46	0.002272	5.16257E-06	0.00589	-0.01916	-0.0006	-0.102302719
48	3.47E-05	1.20468E-09	0.005304	-0.01913	-0.00054	-0.101581411
	SSE	0.002367776				
	SSE/28	8.45634E-05				
	DELTA(0)	0.007845121				

Table A.3.4 Tracking Signal Analysis{ARMA(1,1) process, 18 observations missing}

Period	Error	Error Sq.	Delta(T)	Y(T)	Q(T)	Q(T)/Delta(T)
1	-0.003371379	1.13662E-05	0.004268	-0.00337	-0.000337138	-0.07898444
2	0.002124533	4.51364E-06	0.004054	-0.00125	-9.09708E-05	-0.022439652
3	-0.00172442	2.97362E-06	0.003821	-0.00297	-0.000254316	-0.066556304
4	0.002672247	7.1409E-06	0.003706	-0.0003	3.83405E-05	0.010345026
5	-0.001439163	2.07119E-06	0.003479	-0.00174	-0.00010941	-0.031444311
7	0.010116298	0.000102339	0.004143	0.008378	0.000913161	0.220402032
8	0.003241692	1.05086E-05	0.004053	0.01162	0.001146014	0.282756048
9	0.002430894	5.90925E-06	0.003891	0.014051	0.001274502	0.327567996
11	0.000115847	1.34205E-08	0.003513	0.014167	0.001158637	0.329785255
12	0.001259603	1.5866E-06	0.003288	0.015426	0.001168733	0.355461072
13	0.002152153	4.63176E-06	0.003174	0.017578	0.001267075	0.399159568
15	0.000156342	2.44429E-08	0.002873	0.017735	0.001156002	0.402429715
17	9.85123E-05	9.70467E-09	0.002595	0.017833	0.001050253	0.4046981
18	-0.000623901	3.89252E-07	0.002398	0.017209	0.000882838	0.368151717
20	0.00517085	2.67377E-05	0.002675	0.02238	0.001311639	0.490275665
21	-0.002764253	7.6411E-06	0.002684	0.019616	0.00090405	0.336803715
22	-0.000552734	3.05515E-07	0.002471	0.019063	0.000758371	0.306901635
23	-0.008803789	7.75067E-05	0.003104	0.010259	-0.000197845	-0.06373186
25	0.000691678	4.78418E-07	0.002863	0.010951	-0.000108892	-0.038033531
26	-0.002275442	5.17764E-06	0.002804	0.008676	-0.000325547	-0.116088573
27	-0.009829437	9.66178E-05	0.003507	-0.00115	-0.001275936	-0.363844715
29	0.006256252	3.91407E-05	0.003782	0.005102	-0.000522717	-0.138220715
30	0.001202789	1.4467E-06	0.003524	0.006305	-0.000350167	-0.099370196
31	-0.004519619	2.0427E-05	0.003623	0.001786	-0.000767112	-0.211708357
32	-0.002618157	6.85475E-06	0.003523	-0.00083	-0.000952217	-0.270292641
33	-0.007021941	4.93077E-05	0.003873	-0.00785	-0.001559189	-0.402598585
35	0.003675521	1.35095E-05	0.003853	-0.00418	-0.001035718	-0.268802377
36	-0.0036781	1.35284E-05	0.003836	-0.00786	-0.001299956	-0.338919909
37	-0.001091831	1.19209E-06	0.003561	-0.00895	-0.001279144	-0.359187949
38	-0.010740062	0.000115349	0.004279	-0.01969	-0.002225236	-0.520024792
39	-0.003368859	1.13492E-05	0.004188	-0.02306	-0.002339598	-0.558633704
41	-0.008030842	6.44944E-05	0.004572	-0.03109	-0.002908722	-0.636154982
43	0.005038209	2.53835E-05	0.004619	-0.02605	-0.002114029	-0.457687632
45	0.000921287	8.4877E-07	0.004249	-0.02513	-0.001810498	-0.426082663
46	0.002544335	6.47364E-06	0.004079	-0.02258	-0.001375014	-0.337121851
47	-0.002975391	8.85295E-06	0.003968	-0.02556	-0.001535052	-0.386823065
48	-0.013094056	0.000171454	0.004881	-0.03865	-0.002690952	-0.551319962
	SSE	0.000917555				
	SSE/35	2.62159E-05				
	Delta(0)	0.00436808				

Table A.4.1 Tracking Signal Analysis {ARMA(1,2) process, 11 observations missing}

Period	Error	Error Sq.	Delta(T)	Y(T)	Q(T)	Q(T)/Delta(T)
1	-0.00337	1.13662E-05	0.004405	-0.00337	-0.000337138	-0.076538032
2	0.002125	4.51364E-06	0.004177	-0.00125	-9.09708E-05	-0.021779975
3	-0.00172	2.97362E-06	0.003932	-0.00297	-0.000254316	-0.064685512
4	0.002672	7.1409E-06	0.003806	-0.0003	3.83405E-05	0.010074662
5	-0.00144	2.07119E-06	0.003569	-0.00174	-0.00010941	-0.030655661
7	0.007116	5.06417E-05	0.003924	0.005378	0.000613161	0.156270224
8	0.001742	3.03349E-06	0.003706	0.00712	0.000726014	0.19592776
9	0.000931	8.66564E-07	0.003428	0.008051	0.000746502	0.217762463
11	0.009742	9.48993E-05	0.004059	0.017792	0.001646015	0.405480847
12	-0.00024	5.77908E-08	0.003678	0.017552	0.001457373	0.396293289
13	0.000652	4.25304E-07	0.003375	0.018204	0.001376851	0.407958831
15	0.000646	4.17179E-07	0.003102	0.01885	0.001303756	0.420285959
17	0.00235	5.52191E-06	0.003027	0.0212	0.001408368	0.465291694
18	-0.00212	4.51095E-06	0.002937	0.019076	0.001055141	0.359312579
20	0.003671	1.34751E-05	0.00301	0.022747	0.001316712	0.437448143
21	-0.00426	1.81839E-05	0.003135	0.018483	0.000758615	0.241950847
22	-0.00205	4.21372E-06	0.003027	0.01643	0.00047748	0.157732982
23	-0.0103	0.000106168	0.003755	0.006126	-0.000600647	-0.159967348
25	-0.00081	6.53385E-07	0.00346	0.005318	-0.000621414	-0.179591216
27	0.001367	1.86855E-06	0.003251	0.006685	-0.000422578	-0.129990394
29	0.009632	9.27794E-05	0.003889	0.016317	0.0005829	0.149885386
30	0.002504	6.26988E-06	0.00375	0.018821	0.000775008	0.206642555
31	-0.00494	2.44311E-05	0.00387	0.013878	0.000203229	0.052517852
33	0.00247	6.09934E-06	0.00373	0.016348	0.000429874	0.115256933
35	0.005899	3.48004E-05	0.003947	0.022247	0.000976806	0.247502353
36	-0.00383	1.46811E-05	0.003935	0.018415	0.000495965	0.126034777
38	-0.00442	1.95357E-05	0.003984	0.013995	4.37695E-06	0.001098736
39	-0.00348	1.21286E-05	0.003934	0.010513	-0.000344322	-0.087535247
41	-0.00911	8.30113E-05	0.004451	0.001402	-0.001220995	-0.274302298
43	0.002698	7.27932E-06	0.004276	0.0041	-0.000829093	-0.193896879
46	0.002866	8.21365E-06	0.004135	0.006966	-0.000459589	-0.111147518
47	-0.00422	1.77705E-05	0.004143	0.00275	-0.000835181	-0.201588283
48	-0.01449	0.000210076	0.005178	-0.01174	-0.002201063	-0.425071164
	SSE	0.000870079				
	SSE/31	2.80671E-05				
	Delta(0)	0.004519671				

Table A.4.2 Tracking Signal Analysis{ARMA(1,2) process, 15 observations missing}

Period	Error	Error Sq.	Delta(T)	Y(T)	Q(T)	Q(T)/Delta(T)
1	-0.00337	1.13662E-05	0.005226	-0.00337	-0.00034	-0.064506021
2	-0.00288	8.26831E-06	0.004991	-0.00625	-0.00059	-0.118398812
3	-0.00172	2.97362E-06	0.004665	-0.00797	-0.0007	-0.150989597
4	0.002672	7.1409E-06	0.004465	-0.0053	-0.00037	-0.082110815
5	-0.00144	2.07119E-06	0.004163	-0.00674	-0.00047	-0.113844099
7	0.007116	5.06417E-05	0.004458	0.000378	0.000285	0.063952806
8	0.001742	3.03349E-06	0.004187	0.00212	0.000431	0.102894781
9	0.000931	8.66564E-07	0.003861	0.003051	0.000481	0.124524482
11	0.009742	9.48993E-05	0.004449	0.012792	0.001407	0.316220106
12	-0.00024	5.77908E-08	0.004028	0.012552	0.001242	0.308364994
13	0.000652	4.25304E-07	0.003691	0.013204	0.001183	0.320586809
15	0.000785	6.15471E-07	0.0034	0.013989	0.001143	0.336263913
17	0.011867	0.000140831	0.004247	0.025856	0.002216	0.521743104
18	-0.00212	4.51095E-06	0.004034	0.023732	0.001782	0.441631176
20	0.003671	1.34751E-05	0.003998	0.027403	0.001971	0.492898508
21	-0.00426	1.81839E-05	0.004025	0.023139	0.001347	0.334721289
22	-0.00205	4.21372E-06	0.003827	0.021086	0.001007	0.263137992
23	-0.0103	0.000106168	0.004475	0.010782	-0.00012	-0.027695846
27	0.007425	5.5128E-05	0.00477	0.018207	0.000631	0.132269383
29	0.011751	0.000138086	0.005468	0.029958	0.001743	0.318743493
31	0.00405	1.64012E-05	0.005326	0.034008	0.001974	0.370542334
33	0.00637	4.0583E-05	0.005431	0.040378	0.002413	0.444380138
35	0.006684	4.46808E-05	0.005556	0.047063	0.00284	0.51122492
36	-0.00394	1.55011E-05	0.005394	0.043125	0.002163	0.400923301
38	-0.00451	2.03074E-05	0.005305	0.038619	0.001496	0.281930513
39	-0.00492	2.41887E-05	0.005267	0.033701	0.000854	0.162220151
41	-0.00982	9.64001E-05	0.005722	0.023882	-0.00021	-0.037209161
43	0.000707	4.99894E-07	0.00522	0.024589	-0.00012	-0.023161551
46	0.002677	7.16885E-06	0.004966	0.027267	0.000159	0.032002225
47	-0.00484	2.34126E-05	0.004953	0.022428	-0.00034	-0.068808385
48	-0.01496	0.000223861	0.005954	0.007466	-0.0018	-0.302801828
	SSE	0.001175961				
	SSE/29	4.05504E-05				
	Delta(0)	0.005432577				

Table A.4.3 Tracking Signal Analysis {ARMA(1,2) process, 17 observations missing}

Period	Error	Error Sq.	Delta(T)	Y(T)	Q(T)	Q(T)/Delta(T)
1	-0.00337	1.13662E-05	0.005512	-0.00337	-0.00034	-0.061165489
2	-0.00488	2.37702E-05	0.005448	-0.00825	-0.00079	-0.145178752
3	-0.00872	7.61155E-05	0.005776	-0.01697	-0.00158	-0.274299001
4	-0.00433	1.87294E-05	0.005631	-0.0213	-0.00186	-0.33007278
5	-0.00844	7.12195E-05	0.005912	-0.02974	-0.00252	-0.425704535
7	0.008116	6.58743E-05	0.006132	-0.02162	-0.00145	-0.237008303
8	0.002742	7.51688E-06	0.005793	-0.01888	-0.00103	-0.178466106
9	0.001931	3.72835E-06	0.005407	-0.01695	-0.00074	-0.136382014
11	0.010742	0.000115383	0.00594	-0.00621	0.00041	0.069099686
12	0.00076	5.76997E-07	0.005422	-0.00545	0.000445	0.082140332
13	0.001652	2.72961E-06	0.005045	-0.0038	0.000566	0.112196527
15	0.00838	7.02231E-05	0.005379	0.004584	0.001347	0.250511693
17	0.012867	0.000165565	0.006128	0.017451	0.002499	0.407893469
18	-0.00112	1.26315E-06	0.005627	0.016327	0.002137	0.379774532
20	0.004671	2.18168E-05	0.005532	0.020998	0.00239	0.432145623
21	-0.00326	1.06553E-05	0.005305	0.017734	0.001825	0.344021727
22	-0.00105	1.10825E-06	0.00488	0.016681	0.001537	0.315026046
23	-0.0093	8.65605E-05	0.005322	0.007377	0.000453	0.085140563
27	0.008425	7.09776E-05	0.005632	0.015802	0.00125	0.221984022
29	0.012751	0.000162588	0.006344	0.028553	0.0024	0.378354291
31	0.00505	2.55009E-05	0.006215	0.033603	0.002665	0.428866203
33	0.00737	5.43239E-05	0.00633	0.040974	0.003136	0.495363642
35	0.007684	5.90496E-05	0.006466	0.048658	0.003591	0.555338219
36	-0.00294	8.6268E-06	0.006113	0.045721	0.002938	0.480606964
38	-0.00351	1.22946E-05	0.005852	0.042214	0.002293	0.391896508
39	-0.00392	1.53523E-05	0.005659	0.038296	0.001672	0.295521114
41	-0.00882	7.77634E-05	0.005975	0.029478	0.000623	0.104311781
43	0.001707	2.91396E-06	0.005548	0.031185	0.000732	0.1318706
46	0.003677	1.35238E-05	0.005361	0.034862	0.001026	0.191421862
47	-0.00384	1.47353E-05	0.005209	0.031024	0.00054	0.103618173
	SSE	0.001271852				
	SSE/28	4.54233E-05				
	Delta(0)	0.005749733				

Table A.4.4 Tracking Signal Analysis {ARMA(1,2) process, 18 observations missing}

Period	Error	Error Sq.	DELTA(T)	Y(T)	Q(T)	Q(T)/Delta(T)
1	-0.0003	9.18865E-08	0.005426	-0.0003	-3.03128E-05	-0.005586531
3	0.009729	9.46617E-05	0.005856	0.009426	0.000945661	0.161475175
4	-0.0008	6.44469E-07	0.005351	0.008624	0.000770816	0.144050132
5	-0.00396	1.56969E-05	0.005212	0.004662	0.000297541	0.057086351
8	0.011562	0.000133688	0.005847	0.016224	0.00142402	0.243541395
9	0.002735	7.48095E-06	0.005536	0.018959	0.001555132	0.280915634
10	-0.00239	5.72966E-06	0.005222	0.016565	0.001160251	0.222197512
11	-0.00276	7.61247E-06	0.004975	0.013806	0.000768319	0.154422101
14	0.007934	6.29559E-05	0.005271	0.021741	0.001484935	0.281699155
15	-0.00324	1.05075E-05	0.005068	0.018499	0.001012288	0.199726618
16	-1.4E-05	1.84194E-10	0.004563	0.018486	0.000909702	0.199369773
18	0.004961	2.46112E-05	0.004603	0.023447	0.001314829	0.285664854
19	-0.00137	1.8692E-06	0.004279	0.022079	0.001046627	0.244587859
21	0.005464	2.98553E-05	0.004398	0.027543	0.001488364	0.338446819
22	0.000248	6.1297E-08	0.003983	0.027791	0.001364286	0.342559402
23	-0.00736	5.41073E-05	0.00432	0.020435	0.000492281	0.113955553
24	-0.00682	4.64962E-05	0.00457	0.013616	-0.000238828	-0.052261978
25	-0.00981	9.62816E-05	0.005094	0.003804	-0.001196177	-0.234817318
29	0.017984	0.000323426	0.006383	0.021788	0.000721846	0.113087553
30	-0.00376	1.41075E-05	0.00612	0.018032	0.000274062	0.04477872
32	0.005297	2.80607E-05	0.006038	0.023329	0.00077638	0.128581104
35	0.009379	8.79639E-05	0.006372	0.032708	0.001636632	0.256841898
37	0.002712	7.35626E-06	0.006006	0.035421	0.001744193	0.290401257
38	-0.00271	7.36679E-06	0.005677	0.032706	0.001298356	0.228706441
39	-0.00598	3.57542E-05	0.005707	0.026727	0.000570572	0.099973939
40	-0.0088	7.73595E-05	0.006016	0.017931	-0.000366028	-0.060842153
42	-0.00676	4.57091E-05	0.006091	0.011171	-0.00100551	-0.165094585
45	0.009821	9.6455E-05	0.006464	0.020992	7.71557E-05	0.011937005
46	0.004361	1.90216E-05	0.006253	0.025353	0.000505578	0.080849018
48	0.006918	4.78613E-05	0.00632	0.032271	0.001146839	0.181466473
	SSE	0.001382794				
	SSE/28	4.93855E-05				
	DELTA(0)	0.005995261				

Table A.5.1 Tracking Signal Analysis {ARMA(2,1) process, 18 observations missing}

Period	Error	Error Sq.	DELTA(T)	Y(T)	Q(T)	Q(T)/Delta(T)
1	0.003461	1.19774E-05	0.009117	0.003461	0.000346	0.037960229
2	-0.01088	0.000118283	0.009293	-0.00741	-0.00078	-0.083515869
4	0.037384	0.001397559	0.012102	0.029969	0.00304	0.251190342
5	0.004607	2.12199E-05	0.011352	0.034576	0.003197	0.281575004
6	0.000505	2.55418E-07	0.010268	0.035081	0.002927	0.28511117
7	0.006956	4.83825E-05	0.009937	0.042037	0.00333	0.335154679
8	-0.00236	5.55462E-06	0.009179	0.03968	0.002762	0.300871257
9	-0.00532	2.8292E-05	0.008793	0.034361	0.001954	0.222175965
11	0.012393	0.000153581	0.009153	0.046754	0.002997	0.327494377
12	0.002033	4.13327E-06	0.008441	0.048787	0.002901	0.34369255
13	0.00398	1.5839E-05	0.007995	0.052766	0.003009	0.376364509
15	0.015253	0.000232658	0.00872	0.06802	0.004233	0.485446046
16	-0.01524	0.000232206	0.009372	0.052781	0.002286	0.243927284
19	0.009104	8.28845E-05	0.009345	0.061885	0.002968	0.317582217
20	0.00647	4.18645E-05	0.009058	0.068356	0.003318	0.366329014
21	0.00012	1.44053E-08	0.008164	0.068476	0.002998	0.367260585
22	0.00027	7.28336E-08	0.007375	0.068746	0.002726	0.369576099
23	-0.01056	0.000111617	0.007694	0.058181	0.001396	0.1815078
25	0.007683	5.90307E-05	0.007693	0.065864	0.002025	0.263255888
26	-0.00989	9.79013E-05	0.007913	0.055969	0.000833	0.105293633
27	0.010439	0.000108981	0.008165	0.066409	0.001794	0.219679968
28	-0.02039	0.000415762	0.009388	0.046018	-0.00042	-0.045229093
29	-0.00672	4.51353E-05	0.009121	0.0393	-0.00105	-0.115554942
30	-0.02406	0.000578855	0.010615	0.015241	-0.00335	-0.316021615
32	0.005354	2.86662E-05	0.010089	0.020595	-0.00248	-0.246180648
33	0.000615	3.78798E-07	0.009141	0.02121	-0.00217	-0.237790487
34	-0.00067	4.44271E-07	0.008294	0.020544	-0.00202	-0.243915926
36	-0.0031	9.61514E-06	0.007775	0.017443	-0.00213	-0.274071516
37	-0.00166	2.7402E-06	0.007163	0.015788	-0.00208	-0.290848252
38	-0.00379	1.43962E-05	0.006826	0.011993	-0.00225	-0.330267323
40	0.003833	1.46939E-05	0.006527	0.015827	-0.00165	-0.252136811
41	-0.01115	0.000124313	0.006989	0.004677	-0.0026	-0.371445323
43	-0.00395	1.55935E-05	0.006685	0.000728	-0.00273	-0.408574936
45	-0.00212	4.49832E-06	0.006228	-0.00139	-0.00267	-0.428714118
46	0.014715	0.000216546	0.007077	0.013323	-0.00093	-0.131644031
47	-0.00382	1.4567E-05	0.006751	0.009506	-0.00122	-0.180735509
48	-0.01757	0.000308753	0.007833	-0.00807	-0.00286	-0.364512917
	SSE	0.004567263				
	SSE/35	0.000130493				
	DELTA(0)	0.009745462				

Table A.6.1 Tracking Signal Analysis {ARMA(2,2) process, 11 observations missing}

Period	Error	Error Sq.	DELTA(T)	Y(T)	Q(T)	Q(T)/Delta(T)
1	0.000961	9.23201E-07	0.008628	0.000961	9.60834E-05	0.011135608
2	-0.01338	0.000178911	0.009103	-0.01241	-0.001251103	-0.137435339
4	0.034884	0.001216889	0.011681	0.022469	0.002362401	0.202238174
5	0.002107	4.43738E-06	0.010724	0.024576	0.002336812	0.217908854
6	-0.00199	3.97847E-06	0.009851	0.022581	0.00190367	0.193248588
7	0.004456	1.98537E-05	0.009311	0.027037	0.002158878	0.231853904
8	-0.00486	2.35887E-05	0.008866	0.02218	0.001457308	0.164371907
9	-0.00782	6.11372E-05	0.008761	0.014361	0.000529674	0.060456626
11	0.009893	9.7867E-05	0.008874	0.024254	0.001465985	0.165192839
12	-0.00047	2.18047E-07	0.008034	0.023787	0.001272691	0.158420155
13	0.00148	2.18987E-06	0.007378	0.025266	0.001293404	0.175299319
15	0.012753	0.000162642	0.007916	0.03802	0.002439375	0.308167495
16	-0.01774	0.000314647	0.008898	0.020281	0.000421607	0.047382259
19	0.006604	4.3614E-05	0.008669	0.026885	0.001039856	0.119956469
20	0.00397	1.57631E-05	0.008199	0.030856	0.001332898	0.162572825
21	-0.00238	5.66429E-06	0.007617	0.028476	0.000961611	0.126247039
22	-0.00223	4.97345E-06	0.007078	0.026246	0.000642437	0.090762559
23	-0.01306	0.000170691	0.007677	0.013181	-0.000728296	-0.094868617
25	0.005183	2.6865E-05	0.007428	0.018364	-0.000137151	-0.018465331
26	-0.01239	0.000153624	0.007924	0.005969	-0.001362887	-0.171990233
27	0.007939	6.30343E-05	0.007926	0.013909	-0.000432657	-0.054588907
28	-0.02289	0.000523964	0.009422	-0.00898	-0.002678417	-0.284267063
30	-0.01332	0.000177409	0.009812	-0.0223	-0.003742526	-0.381426595
32	0.00913	8.33587E-05	0.009744	-0.01317	-0.002455264	-0.251983838
33	0.001721	2.96173E-06	0.008941	-0.01145	-0.00203764	-0.227886815
36	0.004785	2.28972E-05	0.008526	-0.00666	-0.001355366	-0.158971871
37	0.003384	1.14496E-05	0.008012	-0.00328	-0.000881456	-0.110022327
38	-0.00237	5.60259E-06	0.007447	-0.00565	-0.001030009	-0.138309109
40	0.001478	2.1836E-06	0.00685	-0.00417	-0.000779238	-0.113753919
41	-0.01238	0.000153172	0.007403	-0.01655	-0.001938943	-0.261919687
43	-0.01018	0.000103628	0.007681	-0.02673	-0.002763029	-0.359745359
45	-0.00879	7.72111E-05	0.007791	-0.03551	-0.003365425	-0.431954234
46	0.006777	4.59242E-05	0.00769	-0.02874	-0.002351208	-0.305760006
48	-0.01306	0.000170481	0.008226	-0.04179	-0.003421771	-0.415948491
	SSE	0.003951757				
	SSE/32	0.000123492				
	DELTA(0)	0.00948044				

Table A.6.2 Tracking Signal Analysis {ARMA(2,2) process, 14 observations missing}

Period	Error	Error Sq.	DELTA(T)	Y(T)	Q(T)	Q(T)/Delta(T)
1	0.003461	1.19774E-05	0.008948	0.003461	0.000346	0.038675292
2	-0.01088	0.000118283	0.009141	-0.00741	-0.00078	-0.084901898
4	0.037384	0.001397559	0.011965	0.029969	0.00304	0.254056642
5	0.004607	2.12199E-05	0.01123	0.034576	0.003197	0.284656217
6	0.000505	2.55418E-07	0.010157	0.035081	0.002927	0.288215554
7	0.006956	4.83825E-05	0.009837	0.042037	0.00333	0.338545913
8	-0.00236	5.55462E-06	0.009089	0.03968	0.002762	0.303836656
9	-0.00532	2.8292E-05	0.008712	0.034361	0.001954	0.224232045
11	0.012393	0.000153581	0.00908	0.046754	0.002997	0.33011146
12	0.002033	4.13327E-06	0.008375	0.048787	0.002901	0.346372407
13	0.00398	1.5839E-05	0.007936	0.052766	0.003009	0.379151946
15	0.015253	0.000232658	0.008668	0.06802	0.004233	0.488408661
16	-0.01524	0.000232206	0.009325	0.052781	0.002286	0.245172665
19	0.009104	8.28845E-05	0.009303	0.061885	0.002968	0.319044963
20	0.00647	4.18645E-05	0.009019	0.068356	0.003318	0.36789524
21	0.00012	1.44053E-08	0.008129	0.068476	0.002998	0.368828476
22	0.00027	7.28336E-08	0.007343	0.068746	0.002726	0.371148077
23	-0.01056	0.000111617	0.007666	0.058181	0.001396	0.182173433
25	0.007683	5.90307E-05	0.007667	0.065864	0.002025	0.26412457
26	-0.00989	9.79013E-05	0.00789	0.055969	0.000833	0.105597506
27	0.010439	0.000108981	0.008145	0.066409	0.001794	0.220232697
28	-0.02039	0.000415762	0.00937	0.046018	-0.00042	-0.045318127
29	-0.00672	4.51353E-05	0.009104	0.0393	-0.00105	-0.115765628
30	-0.02406	0.000578855	0.0106	0.015241	-0.00335	-0.31646702
32	0.005354	2.86662E-05	0.010075	0.020595	-0.00248	-0.24650918
33	0.000615	3.78798E-07	0.009129	0.02121	-0.00217	-0.238105683
34	-0.00067	4.44271E-07	0.008283	0.020544	-0.00202	-0.24423664
36	-0.0031	9.61514E-06	0.007765	0.017443	-0.00213	-0.274417489
37	-0.00166	2.7402E-06	0.007154	0.015788	-0.00208	-0.291206907
38	-0.00379	1.43962E-05	0.006818	0.011993	-0.00225	-0.330651923
40	0.003833	1.46939E-05	0.006519	0.015827	-0.00165	-0.252413163
41	-0.01115	0.000124313	0.006982	0.004677	-0.0026	-0.371787433
43	-0.00395	1.55935E-05	0.006679	0.000728	-0.00273	-0.408928995
45	-0.00212	4.49832E-06	0.006223	-0.00139	-0.00267	-0.429072969
46	0.014715	0.000216546	0.007073	0.013323	-0.00093	-0.131731295
48	-0.00489	2.39296E-05	0.006854	0.008431	-0.00133	-0.193696871
	SSE	0.004267874				
	SSE/34	0.000125526				
	DELTA(0)	0.009558169				

Table A.6.3 Tracking Signal Analysis {ARMA(2,2) process, 12 observations missing}