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Modeling and controls for a laser glass cutting machine workcell robot

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MODELING AND CONTROLS FOR A LASER GLASS CUTTING MACHINE WORKCELL ROBOT

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Thesis submitted to the
College of Engineering and Mineral Resources
at West Virginia University
in partial fulfillment of the requirements
for the degree of

Master of Science
in
Mechanical Engineering

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Abstract

Modeling and Controls for a Laser Glass Cutting Machine Workcell Robot

Asif M. Mohammad

The SCARA (Selective Compliance Assembly Robot Arm) -Type Nimbl loader junior robot includes a four-degree of freedom robot manipulator arm. This thesis deals with three main areas of robot behavior: kinematics, dynamics and controls for the purpose of improving the performance of the Laser Glass Cutting Machine (LGCM) workcell. The thesis deals with developing kinematic and dynamic models of this robot arm and their application to task planning of this robot. In the kinematic analysis, both direct and inverse kinematic solutions, are performed. Using algebraic approach on direct kinematics, the inverse kinematic solution for the manipulator arm is derived.

The Lagrange-Euler (L-E) technique is used to derive the dynamic model of the Nimbl loader junior robot arm. The L-E method is the most systematic and organized method of solving for the dynamic behavior of the manipulator. The computer software program MATLAB is used to generate symbolic dynamic equations for the manipulator with as many as three degrees of freedom. Lagrange-Euler formulation is used to obtain the dynamic equations of the manipulator. Based on these dynamic equations and actuator dynamics, an integrated joint dynamic model was developed.

Task planning is done for SCARA-Type Nimbl loader robot, in which the robot starts from pick position to place position. A 4-3-4 joint trajectory was generated for pick-to-place path for the robot.

By applying the PID control technique to the integrated joint dynamic model, an independent joint control scheme was derived using a classical approach. An experimental verification study was done to prove the theoretical model of Work Cell robot.

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List of Symbols

S_i	$\sin \theta_i$ for i^{th} Joint
C_i	$\cos \theta_i$ for i^{th} joint
D_{ik}	Inertia terms
g	gravitational constant
h_{ijk}	Coriolis and centrifugal terms
I_{xx}, I_{yy}, I_{zz}	moments of inertia about coordinate frame axes
I_{xy}, I_{xz}, I_{yz}	products of inertia with respect to coordinate frame axes
J_i	pseudo-inertia matrix for i^{th} link
m_i	mass of i^{th} link
n_x, n_y, n_z	elements of orientation vector
o_x, o_y, o_z	elements of sliding vector
a_x, a_y, a_z	elements of approach vector
p_x, p_y, p_z	elements of position vector
L_i	length of link i
H	height of robot post
Q_j	differentiation matrix
T_{goal}	general position of end-effector plate with respect to fixed base frame
Tr	trace of a matrix
K	kinetic energy

P	potential energy
U_{ij}	partial derivative of a transformation matrix
$D(\theta)$	acceleration related symmetric matrix
h_1, h_2, h_3	coriolis and centrifugal terms
c_1, c_2, c_3	gravitational terms
$V_a(t)$	applied voltage
$I_a(t)$	armature current
K_b	back electromotive force constant
K_t	motor torque constant
J_m	inertia of motor rotor about motor axis
B_{e_i}	effective coefficient of friction at joint i
J_e	effective inertia
m_3	mass of link 3
K_1	stiffness
$B_T + K_2$	friction coefficient
θ_i	joint angle from x_{i-1} axis to x_i axis about z_{i-1} axis
$\dot{\theta}_i$	angular velocity of ith joint
$\ddot{\theta}_i$	angular acceleration of ith joint
τ_i	input torque required at ith joint
ω_n	undamped natural frequency
ξ	damping ratio

K_p, K_d, K_i	PID controller gains
$E(s)$	error actuating signal
$\theta_L(s)$	actual angular displacement
$\theta^d_L(s)$	desired angular displacement

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Chapter 1

Introduction

1.1 OVERVIEW OF LASER GLASS CUTTING MACHINE

The Laser Glass Cutting Machine (LGCM) is a new development in the field of automation to perform a glass-cutting operation, especially after hot glassware is removed from the mold. The traditional crack-off approach for glass cutting in the cold shop requires high skill and is a time-consuming job. Laser cutting is an emerging technology for improved product quality and reduced material cost in hand glass industries, thus minimizing waste. This technology also reduces energy usage and hazards to workers, since exposure to sharp edges and broken pieces are minimized. Hand glass manufacturers can use this enhanced technology since the speed and efficiency it offers can increase throughput and reduce costs [1]. A SCARA-Type robot work cell is used in conjunction with a laser to perform the above-mentioned task. This work cell handles the hot glassware to be cut with precision, and the laser performs the cut (see Figure 1.1). After the cut is made, the task of the SCARA-Type robot is to put the finished glassware on a turntable. This movement of the robot should be precise to place the glassware exactly into the position to be cut by the laser.

1.2 NOMENCLATURE OF THE WORKCELL

The Laser Glass Cutting Machine consists of several components. These components are used to accomplish the task of making a cut in the glassware. Figure 1.2 shows all the components of the LGCM. The bold words in the figure show the components on which this thesis is based.

Fig 1.1 Schematic Diagram of Laser Glass Cutting Machine

1.3 DESCRIPTION OF SCARA-TYPE NIMBL ROBOT

The Selective Compliance Assembly Robot Arm (SCARA) is a relatively desirable robot configuration for certain industrial applications. One version of the SCARA robot arm is a SCARA-Type four-degree-of-freedom mechanism, with four links connected by three revolute joints. A conventional SCARA robot has two revolute joints and one translation motion joint. It is also called as RRP type robot. This robot is called as SCARA-Type robot since it is modified version of conventional SCARA robot. Instead of RRP type robot it is RRR robot i.e. it has three revolute joints. The modified version is made to behave as a SCARA robot by adding an extra link to third joint so that the extra link forms a parallelogram with the third link so that the end-effector moves in vertical plane like a prismatic joint. The first two revolute joints move links 1 and 2 in the horizontal plane and the third revolute joint moves a parallelogram link in the vertical plane (Figure 1.3 [2]) so that the motion is like a translational motion. These robot arms are often used for simple three-dimensional motion that uses a gripper as an end-effector, such as pick-and-place or assembly line sorting.

Fig 1.2 Components of the Laser Glass Cutting Machine

The bold word in Figure 1.2 shows the components on which this thesis is based.

Fig 1.3 Side-view drawing of SCARA-Type Robot [2]

1.4 LITERATURE REVIEW AND BACKGROUND

The SCARA-Type Nimbl Robot is a three revolute joint (RRR) robot. This robot is classified as a SCARA-Type robot with none of its motion prismatic, as the revolute motion of the third joint is set perpendicular to the revolute motions of the other two joints. The revolute motion of the third joint is in the vertical plane. This is clear from Figure 1.3 [2]. This makes the kinematic mechanism of the SCARA-Type robot very interesting. For the three main links of the SCARA-Type Nimbl Junior Robot, the Denavit-Hartenberg [4] 4x4 notation to describe the links in terms of the coordinate frames will be utilized.

The work in this thesis is based mostly on the formulations presented by C.S.G.Lee, et al [4]. The formulation of the dynamic equations of motion is done using Lagrange-Euler (L-E) formulation [4]. The L-E method presents a systematic and an organized way of determining the dynamic equations of motion, but the computation involved in the L-E method makes it unsuitable for real-time control purposes [4]. The advantage of using the L-E formulation is that it provides explicit equations for the robot

dynamics and can be utilized to analyze and design advanced joint-variable space control strategies [4].

1.5 RESEARCH OBJECTIVES

The eight main objectives of this research are:

- 1) Develop a kinematic model for the SCARA-Type robot.
- 2) Determine the inverse kinematics solution for the SCARA-Type robot to determine the joint angles for a specified robot task.
- 3) Determine the dynamic equations of motion of this SCARA-Type robot using Lagrange-Euler formulation.
- 4) Develop workspace trajectory motion in terms of joint space motion.
- 5) Develop dynamic model for actuator.
- 6) Develop integrated joint dynamic model for the SCARA-Type robot.
- 7) Develop closed-loop controller using a classical approach.
- 8) Perform experimental verification study of the SCARA-Type robot.

Chapter 2

Kinematics of SCARA-Type Robot

2.1 OBJECTIVE

A mechanical manipulator is a closed or open kinematic chain mechanism consisting of a sequence of N rigid links. Each pair of robot links is connected by revolute or prismatic joints. Each joint is driven by an actuator directly or via a mechanical linkage mechanism and each joint constitutes one degree of freedom. The last link is connected to a terminal device, called as end-effector, for manipulating an object. The motion of the end-effector is usually specified in terms of Cartesian space coordinates, but typically the control of robot position is performed in terms of joint space coordinates. The kinematic analysis of mechanical manipulators deals with the study of the relation between these two space coordinates and consist of two problems:

1. Given the joint positions of the manipulator, determine the position and orientation of the manipulator end-effector with respect to the Cartesian coordinate system at the manipulator base (base coordinate frame).
2. Given manipulator end-effector position and orientation with respect to a base coordinate frame, determine the joint positions required.

The position and orientation of the robot end-effector with respect to the base coordinate frame can be determined by means of homogeneous transformation [4]. This transformation is calculated by a series of rotations and translations about the manipulator links and is known as forward transformation. In case of second problem above, the elements of the forward transformation matrices are known, and the problem becomes one of solving the equations of the forward transformation for the joint angles. This is referred to as inverse transformation.

The mechanical manipulator is usually composed of an arm and a wrist attached to an end-effector. The arm typically has three DOF, which is the minimum number of DOF required to place the end-effector at any position in the workspace. Thus the arm is major positioning mechanism. In this thesis, only the kinematics analysis of the arm will be presented since the main objective is to design a positional controller.

The objective of this chapter is to determine the forward kinematics and inverse kinematics of the SCARA-Type robot. The forward kinematics consists in part of finding the position and orientation of the end-effector of the manipulator given particular joint angles θ_i and the geometric link parameters; i.e. what is the position and orientation of the end-effector of the robot with respect to a reference coordinate system. Inverse kinematics is important in determining the joint angles for a given end-effector location.

2.2 SPECIFIED LGCM TASK

The Laser Glass Cutting Machine (LGCM) is a new development in the field of glassware making technology. In the traditional process, molten glass from the batch tank is removed using a blowpipe. After the glass is blown into the required shape using a mold, the blown glassware is annealed and sent to cold shop. In the cold shop, the moil is removed from the glassware by crack-off and then the surface of the glass product is ground and polished to remove sharp edges [1]. Existing glass crack-off methods often result in considerable loss of glassware and require subsequent finishing of the edges, which is labor intensive and can result in further glass product losses [1]. Depending on the glassware piece, glass properties, and worker skill level, losses can be as high as 80 percent in some product lines and often-average 40 percent of total scrap for some products [1].

The laser-enhanced cutting and finishing method will dramatically decrease waste, which will improve productivity in the manufacture of the hand blown glass.

The LGCM Plant Prototype includes a lower gripper and turntable as shown in Figure 2.1. The function of the robot is to handle the glassware at high temperature in the hot shop i.e., to place the hot glassware in the correct position in front of laser. The task of the laser is to cut and polish the molded glassware, while the task of the robot is to move the glassware in front of laser, rotate the glassware while the cut is being performed and placing back the glassware on the turntable. After the cut is completed with glassware edges polished, the task of the robot is to then place the glassware back on the turntable. The robot arm, lower gripper and turntable together with the robot controller are known as a workcell.

Fig. 2.1 Schematic of LGCM Plant Prototype Workcell

2.3 METHODOLOGY

The Denavit-Hartenberg representation [4,6] was used to develop the overall homogeneous transformation matrices for the robot arm. A goal matrix, which denotes the position and orientation that the endeffector should obtain, was defined. Then, an inverse kinematic [4,6] model was developed using matrix manipulations [7]. The goal matrix input for this model was the position and orientation of the endeffector in Cartesian coordinates as well as unit directional vectors n , o and a . The output of this model was the angles of the three joints for the links to achieve the goal position in the robot workspace.

Analysis was done using MATLAB (Appendix A)

The homogeneous transformation matrices which are based from the coordinate frames of the robot links are entered as code into Matlab. The equations for the joint angles of the robot were calculated mathematically from the joint angle equations.

2.4 KINEMATIC ANALYSIS OF SCARA-TYPE ROBOT

Robot arm kinematics [5] involve the geometry of robot arm motions with respect to a fixed reference coordinate system as a function of time without regard to the forces and moments that cause the motions. In a kinematic analysis [5], distinction is made between direct kinematics and inverse kinematics.

The direct kinematics [5] involves finding the location (position and orientation) of the end-effector given particular joint angles θ_i , whereas the inverse kinematics involves finding the joint angles for a given location of the end-effector with respect to a fixed reference coordinate system.

Inverse kinematics [4, 6] is important in trajectory planning [4,15]. In planning a movement of the manipulator (a component of the robot as shown in Figure 1.2), one is primarily interested in the Cartesian position of the end-effector with respect to the Cartesian space of the manipulator base. At a given end-effector position in Cartesian space, one must solve for the corresponding set of robot joint angles. This is inverse kinematics.

In this thesis, the main objective is to derive the inverse kinematic solution for a kinematic chain mechanism of a SCARA-Type robot arm. Kinematic description of spatial linkages based on Denavit-Hartenberg notation [4,6] will be introduced. The assignment of coordinate frames and the definition of transformation matrices, which describe the relationship between two coordinate frames of the links, will also be presented. Once the transformation matrices are specified, the solution of the inverse kinematics can be derived.

2.5 HOMOGENEOUS TRANSFORMATION MATRICES

A 3x3 rotation matrix [4] can describe the orientation of a link-attached frame with respect to the frame of the robot's previous link. The matrix representation of rotation of coordinate frame with respect to coordinate frame of previous frame is represented by vectors $\hat{i}, \hat{j}, \hat{k}$ and the translation of the origin of the link coordinate frame with respect to the origin of the coordinate frame of the previous link is represented by

position vector $\bar{p}$. Cartesian coordinates are used to represent a link's position vector in 3-D space. The rotation matrix and position vector are then combined to form a 4x4 homogeneous transformation [4] matrix that defines the translational and rotational position of the link-attached coordinate frame with respect to the reference coordinate frame. Denavit and Hartenberg [4, 6] first introduced this application of a 4x4 homogeneous transformation matrix to a rigid link. The representation of the robot arm link can be expressed as the product of four homogeneous transformations [5] relating each link's coordinate frame to the previous link's coordinate frame. This mathematically expressed relationship is known as an A matrix, as shown in Equation (2.1) [4].

$$A = \begin{bmatrix} R_{3 \times 3} & P_{3 \times 1} \\ 0 & 1 \end{bmatrix} \quad (2.1)$$

This matrix defines the rotation matrix $R_{3 \times 3}$ and the position vector $P_{3 \times 1}$ expressed in homogeneous coordinates from one coordinate frame to the coordinate frame of the previous link frame of the link.

2.6 DENAVIT-HARTENBERG REPRESENTATION

To describe the transitional or rotational relationship between adjacent links, Denavit-Hartenberg (D-H) [4,6] proposed a matrix method of systematically establishing a coordinate system (body-attached frames) to each rigid link (see Figure 2.2 [7]). The D-H representation results in a 4 x 4 homogeneous transformation matrix representing each rigid link's coordinate system with respect to the previous link. Thus, through sequential transformations, the end-effector frame expressed in end-effector coordinates (X_E, Y_E, Z_E) can be expressed in base coordinates (X_O, Y_O, Z_O). According to the D-H representation, an orthogonal Cartesian coordinate system (X_i, Y_i, Z_i) can be established for each rigid link at its joint axis, where $i = 1, 2, \dots, n$ (n = number of degrees of freedom). Thus, for this SCARA-Type robot in the LGCM, the location of the end-effector can be described in terms of four link coordinate frames, namely (x_0, y_0, z_0), (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3).

Fig 2.2 Coordinate Frames for $i-1$, i and $i+1$ Rigid Links, and Associated Link-Joint Parameters [7]

- θ_i - angle from x_{i-1} axis to x_i about z_{i-1} axis using right-hand rule.
- d_i - distance from origin of $(i-1)^{th}$ coordinate frame to intersection of z_{i-1} axis with x_i axis along z_{i-1} axis.
- a_i - distance from intersection of z_{i-1} axis with x_i axis to intersection of z_i axis with x_i axis.
- α_i - angle from z_{i-1} axis to z_i axis about x_i axis using right-hand rule.

2.7 SCARA-TYPE ROBOT COORDINATE FRAME SYSTEM

Fig. 2.3 Coordinate Frame Assignments and Associated Link-Joint Parameters for SCARA-Type Robot

$\theta_1, \theta_2, \theta_3, \theta_4$ and θ_5 are the angles of rotation for joint 1, joint 2, joint 3, joint 4, and joint 5 respectively. Joint 1 and joint 2 of the robot rotate in the horizontal plane, joint 3 and joint 4 rotate in vertical plane and joint 5 rotates in horizontal plane. The lengths of link 4 and link 5 are assumed to be zero so that the end-effector can be considered at the end of link 3. For the cut to be perfect, the SCARA-Type robot should handle the glassware such that the end-effector always rotates in the horizontal plane i.e. the end-effector should always face down. To achieve this, the sum of angles θ_3 and θ_4 is equal to 90 degrees i.e. $\theta_3 + \theta_4 = 90$. This assumption will always make the end-effector rotate in horizontal plane. To simplify the calculations, the rotation of joint 5 is fixed i.e. $\theta_5 = 0$.

2.8 KINEMATIC SOLUTION OF SCARA-TYPE ROBOT

2.8.1 Direct Kinematic Model

Direct kinematic [4, 6] modeling involves computing basic rotation matrices as discussed in Section 2.2 using D-H representation. Direct kinematic modeling is also used to find homogeneous transformation matrix ${}^{i-1}A_i$ between link i and link $i-1$. After determining the homogeneous transformation matrices, the overall homogenous transformation matrix 0T_E is calculated.

Table 2.1 Link and Joint Parameters of SCARA-Type robot

I	a_i	α_i	d_i	θ_i
1	L1	0	H	θ_1
2	L2	-90°	0	θ_2
3	L3	0	0	θ_3

Dimensions of the Links:

$$L1 = 1.70 \text{ feet.}$$

$$L2 = 0.72 \text{ feet.}$$

$$L3 = 1.27 \text{ feet.}$$

$$H = 5.00 \text{ feet.}$$

In case of a link i with a revolute joint, d_i , L_i , α_i are the three link-joint parameters that remain constant, while θ_i is the joint variable that changes when link i rotates with respect to Z_{i-1} of the link $i-1$ frame [4].

The next step is to determine the homogenous transformation matrix 0T_i which specifies the location of the i^{th} coordinate frame with respect to the fixed base coordinate frame. The matrix representation [4] which accomplishes this is the product of the successive coordinates transformation matrices of ${}^{i-1}A_i$ matrices. ${}^{i-1}A_i$ refers to the 4x4 homogeneous transformation matrix which establishes the position and orientation of the link i with respect to link $i-1$ frame. Thus, the 0T_n matrix can be expressed as shown in Equation (2.2) [4].

$${}^0T_n = {}^0A_1 {}^1A_2 \dots {}^{n-1}A_n \quad (2.2)$$

To determine the 4x4 ${}^{i-1}A_i$ homogenous transformation matrices [4, pg 41], the following general form of matrix for a rotation link is used:

$${}^{i-1}A_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & LiC\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & LiS\theta_i \\ 0 & S\alpha_i & C\alpha_i & di \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.3)$$

The basic matrices for this SCARA-Type robot are 0A_1 , 1A_2 , 2A_3 , 3A_4 and 4A_E and which are computed as:

$${}^0A_1 = \begin{bmatrix} C1 & -S1 & 0 & L1C1 \\ S1 & C1 & 0 & L1S1 \\ 0 & 0 & 1 & H \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1A_2 = \begin{bmatrix} C2 & 0 & -S2 & L2C2 \\ S2 & 0 & C2 & L2S2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2A_3 = \begin{bmatrix} 0 & -S3 & C3 & L3C3 \\ 0 & C3 & S3 & L3S3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^3A_4 = \begin{bmatrix} C4 & -S4 & 0 & 0 \\ S4 & C4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^4A_E = \begin{bmatrix} 0 & -S5 & C5 & 0 \\ 0 & -C5 & -S5 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

since θ_5 is assumed to be zero for simplification, 4A_E can be rewritten as:

$${}^4A_E = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where S_i represents $\sin \theta_i$,

C_i represents $\cos \theta_i$,

L_i represents the length of link i

Also,

$${}^1A_0 = {}^0A_1^{-1}$$

$${}^2A_1 = {}^1A_2^{-1}$$

$${}^3A_2 = {}^2A_3^{-1}$$

$${}^4A_3 = {}^3A_4^{-1}$$

Thus, the overall homogeneous transformation matrix 0T_E is:

$${}^0T_E = {}^0A_1 * {}^1A_2 * {}^2A_3 * {}^3A_4 * {}^4A_E \quad (2.4)$$

$${}^0T_E = \begin{bmatrix} -S12, & C12, & 0, & 1/2*L3*\cos(\theta_1+\theta_2-\theta_3)+1/2*L3*\cos(\theta_1+\theta_2+\theta_3) \\ +L2*C12+L1*C1 & & & \\ C12, & S12, & 0, & 1/2*L3*\sin(\theta_1+\theta_2+\theta_3)+1/2*L3*\sin(\theta_1+\theta_2-\theta_3) \\ +L2*S12+L1*S1 & & & \\ 0, & 0, & -1, & -L3*\sin(\theta_3)+H \\ 0, & 0, & 0, & 1 \end{bmatrix}$$

2.8.2 INVERSE KINEMATIC SOLUTION

An inverse kinematic solution is necessary when one considers moving the end-effector through a specified motion. For this SCARA-Type robot, a specified end-effector location ${}^0T_{\text{goal}}$ is equal to the product of four homogeneous transformation matrices 0T_E shown in Equation (2.4) [4].

$${}^0T_{\text{goal}} = \begin{bmatrix} nx & ox & ax & px \\ ny & oy & ay & py \\ nz & oz & az & pz \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.5)$$

Thus, the goal matrix is equal to the product of four homogeneous transformation matrices.

$${}^0T_E = {}^0T_{\text{goal}} \quad (2.6)$$

where ${}^0T_{\text{goal}}$ is the desired position and orientation of the end-effector.

Equation (2.4) is also called the direct kinematic equation for this robot arm. For the inverse kinematic solution, the transformation matrix 0T_E , which relates the E^{th}

coordinate frame to 0th coordinate frame, is given for a certain position and orientation of the end-effector [6]. Thus, the elements of the right-hand-side matrix of Equation (2.4) are known but the joint angles of the left-hand-side matrix are not known.

The technique by Paul [4] is applied to solve for joint angle variables from Equation (2.6). This technique is based on equating matrices. Each successive variable is isolated from the right-hand side of Equation (2.4) by pre-multiplying successive inverse matrices on both sides. Knowing that ${}^iA_{i-1} = ({}^{i-1}A_i)^{-1}$, three equations are obtained as shown in Equations (2.7), (2.8) and (2.9):

$${}^1A_0 * {}^0T_{\text{goal}} = {}^1A_2 * {}^2A_3 * {}^3A_4 * {}^4A_E \quad (2.7)$$

$${}^0T_{\text{goal}} * {}^EA_4 * {}^4A_3 * {}^3A_2 = {}^0A_1 * {}^1A_2 \quad (2.8)$$

$${}^1A_0 * {}^0T_{\text{goal}} * {}^EA_4 * {}^4A_3 * {}^3A_2 = {}^1A_2 \quad (2.9)$$

By equating corresponding elements of these equated matrices in Equations (2.7), (2.8), and (2.9), the robot joint angles are determined.

Also, ${}^0T_E = {}^0T_{\text{goal}}$ when the arm matrix matches the goal matrix.

To solve for θ_1 , consider Equation (2.7). The left-hand side of Equation (2.7) is:

$$\begin{bmatrix} C1 & S1 & 0 & -L1 \\ -S1 & C1 & 0 & 0 \\ 0 & 0 & 1 & -H \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} nx & ox & ax & px \\ ny & oy & ay & py \\ nz & oz & az & pz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} nx*C1+ny*S1, & ox*C1+oy*S1, & ax*C1+ay*S1, & px*C1+py*S1-L1 \\ nx*-S1+ny*C1, & ox*-S1+oy*C1, & ax*-S1+ay*C1, & px*-S1+py*C1 \\ nz, & oz, & az, & pz-H \\ 0, & 0, & 0, & 1 \end{bmatrix}$$

and the right-hand side of the Equation (2.7) is,

$$\begin{bmatrix} -S2, & C2, & 0, & C2 * (L3 * C3 + L2) \\ C2, & S2, & 0, & S2 * (L3 * C3 + L2) \\ 0, & C3, & -1, & -L3 * S3 \\ 0, & 0, & 0, & 1 \end{bmatrix}$$

Now, equating the (3,4) elements and the (3,2) elements in Equation (2.7) yields,

$$-S3 * L3 = pz - H$$

Thus,

$$S3 = \frac{H - pz}{L3}$$

Hence from the Pythagoras theorem,

$$\tan \theta_3 = \frac{H - pz}{(L3^2 - (H - pz)^2)^{0.5}}$$

Therefore,

$$\theta_3 = \text{Atan2}\left(\frac{H - pz}{(L3^2 - (H - pz)^2)^{0.5}}\right) \quad (2.10)$$

From Equation (2.8),

$${}^0T_{\text{goal}} * {}^E A_4 * {}^4 A_3 * {}^3 A_2 = {}^0 A_1 * {}^1 A_2 \quad (2.8)$$

The left-hand side of Equation (2.8) is:

$$= \begin{bmatrix} ox, & ax, & nx, & -ax * L3 * S3 - ox * L3 * C3 + px \\ oy, & ay, & ny, & -ay * L3 * S3 - oy * L3 * C3 + py \\ oz, & az, & nz, & -az * L3 * S3 - L3 * oz * C3 + pz \\ 0, & 0, & 0, & 1 \end{bmatrix}$$

while the right-hand side of Equation (2.8) is:

$$= \begin{bmatrix} C12, & 0, & -S12, & L2 * C12 + L1 * C1 \\ S12, & 0, & C12, & L2 * S12 + L1 * S1 \\ 0, & -1, & 0, & H \\ 0, & 0, & 0, & 1 \end{bmatrix}$$

Equating the (1, 4) elements and (2, 4) elements in Equation (2.8) yields

$$L2 * C12 + L1 * C1 = -ax * L3 * S3 - ox * L3 * C3 + px, \text{ and} \quad (2.11)$$

$$L2*S12 + L1S1 = -ay*L3*S3-oy*L3*C3+py \quad (2.12)$$

Equating the (1, 1) and (2, 1) elements in Equation (2.8) yields,

$$C12 = ox, \text{ and} \quad (2.13)$$

$$S12 = oy \quad (2.14)$$

Substituting Equations (2.13) and (2.14) in Equations (2.11) and (2.12) yields,

$$\theta_1 = \text{Atan2} \left[\frac{-L3*ay*(H-pz)-L3*oy*(L3^2-(H-pz)^2)^{0.5}+py*L3-L2*L3*oy}{-L3*ax*(H-pz)-L3*ox*(L3^2-(H-pz)^2)^{0.5}+px*L3-L2*L3*ox} \right] \quad (2.15)$$

To solve for θ_2 , Equation (2.7) is used:

$${}^1A_0 * {}^0T_{\text{goal}} = {}^1A_2 * {}^2A_3 * {}^3A_4 * {}^4A_E \quad (2.7)$$

The right-hand side of Equation (2.7) is:

$$\begin{bmatrix} -S2, & C2, & 0, & C2*(L3*C3+L2) \\ C2, & S2, & 0, & S2*(L3*C3+L2) \\ 0, & C3, & -1, & -L3*S3 \\ 0, & 0, & 0, & 1 \end{bmatrix}$$

The left-hand side of the Equation (2.7) is:

$$\begin{bmatrix} nx*C1+ny*S1, & ox*C1+oy*S1, & ax*C1+ay*S1, & px*C1+py*S1-L1 \\ nx*-S1+ny*C1, & ox*-S1+oy*C1, & ax*-S1+ay*C1, & px*-S1+py*C1 \\ nz, & oz, & az, & pz-H \\ 0, & 0, & 0, & 1 \end{bmatrix}$$

Equating the (1,4) and (2,4) elements in Equation (2.7) yields,

$$S2*(L3*C3+L2) = -S1*px +C1*py, \text{ and} \quad (2.16)$$

$$C2*(L3*C3+L2) = C1*px +S1*py-L1 \quad (2.17)$$

Dividing Equation (2.16) by (2.17) and simplifying then yields,

$$\theta_2 = \text{Atan2} \left[\frac{-S1*px +C1*py}{C1*px +S1*py - L1} \right] \quad (2.18)$$

Verification of inverse kinematic model is shown in Appendix A.

2.9 SUMMARY

In this chapter, the procedure for obtaining the inverse kinematic solution for the LGCM robot was presented. Starting by introducing the coordinate frames assignments and homogeneous transformations, the equations for the inverse kinematic solution for this SCARA-Type robot were obtained. Kinematics validation study was done to verify different positions of end-effector in the workspace.

Chapter 3

Dynamics of SCARA-Type Robot

3.1 METHODOLOGY

The dynamic equations of motion are used to develop the dynamic model [5] for the arm. These dynamic equations of motion for three joints, which give the torques required by the actuators to perform the motion, are developed using Lagrange-Euler (L-E) formulation [4]. The simulation for torques vs time is done using Excel. The input to this dynamic model is the angle, velocity and acceleration of each joint. The output will be the torque required by the actuator to move the joint at any given time of the pick-to-place trajectory.

Analysis using Matlab (Appendix D) and EXCEL

The torques for the three joints of the SCARA-Type robot were derived using the L-E formulation. Matlab produces results using the inertia, Coriolis and centrifugal and gravitational terms. The equations for torque for the three joints were also derived using Matlab. Excel was used to plot the torque equations.

3.2 LAGRANGE-EULER FORMULATION

The equations of motion of the manipulator can be derived through direct application of the L-E formulation [4, 8] to conservative systems. The Denavit-Hartenberg [4, 6] matrix representation is used to describe the spatial displacement between adjacent link coordinate frames to obtain link kinematic information. The L-E formulation is utilized to find the dynamics equations of motion for this SCARA-Type robot. The L-E formulation coupled with the Denavit-Hartenberg link-joint parameters representation provided a systematic approach to and description of the equations of motion [4, 8].

The Lagrange-Euler equation is:

$$\frac{d}{dt} \left[\frac{\partial L}{\partial \dot{q}_i} \right] - \frac{\partial L}{\partial q_i} = \tau_i \quad i = 1, 2, 3, \dots, n \quad (3.1)$$

where,

L = Lagrangian function = Kinetic energy K – Potential energy P

K= Total kinetic energy of the robot arm

P = Total potential energy of the robot arm

q_i = Generalized coordinates of the robot

$\dot{q}_i$ = First time derivative of the generalized coordinates of the robot arm

τ_i = Generalized force (or torque) applied to the system at joint i to drive link i.

The Kinetic energy K of the system is [4]:

$$K = \frac{1}{2} \sum_{i=1}^n \sum_{p=1}^i \sum_{r=1}^i \left[\text{Tr } U_{ip} \left(\int {}^i r_i {}^i r_i^T dm \right) U_{ir}^T \dot{q}_p \dot{q}_r \right] \quad (3.2)$$

$$K = \frac{1}{2} \sum_{i=1}^n \sum_{p=1}^i \sum_{r=1}^i \left[\text{Tr } (U_{ip} J_i U_{ir}^T) \dot{q}_p \dot{q}_r \right] \quad (3.3)$$

where: $J_i = \int {}^i r_i {}^i r_i^T dm$

$$U_{ij} = \begin{cases} {}^0 A_{j-1} Q_j {}^{j-1} A_i & \text{---> } j \leq i \\ 0 & \text{---> } j > i \end{cases}$$

The matrix Q_j is:

$$Q_j = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (\text{for revolute joints})$$

The partial derivative of ${}^{j-1} A_j$ with respect to q_i can be determined by pre-multiplying ${}^{j-1} A_j$ with Q_j matrix.

The matrix U_{ij} is the rate of change of the points (${}^i r_i$) on the link i relative to the base coordinate frame [4]. It is constant for all points on link i and independent of mass distribution of link i . Also $\dot{q}_i$ is independent of the mass distribution of the link i .

Matrix J_i is a pseudo-inertia matrix [4]. The moments of inertia of the link's [4,9] are determined for the three links of the LGCM robot and are used to calculate the J_i for each of the three links.

The potential energy P of the robot arm is given by [4]:

$$P = \sum_{i=1}^n -m_i g ({}^o A_i {}^i r_i) \quad (3.4)$$

where,

g = Gravitational constant

m_i = Mass of i th link

${}^i r_i$ = Fixed point in link i and expressed in homogeneous coordinates with respect to the i th link coordinate frame.

3.3 EQUATIONS OF MOTION FOR THE MANIPULATOR

The Lagrangian function ($L = K - P$) for the robot is [4]:

$$L = \frac{1}{2} \sum_{i=1}^n \sum_{p=1}^i \sum_{r=1}^i [\text{Tr} (U_{ip} J_i U_{ir}^T) \dot{q}_p \dot{q}_r] + \sum_{i=1}^n m_i g ({}^o A_i {}^i r_i) \quad (3.5)$$

Applying the Lagrangian function to the Lagrange-Euler formulation of the robot arm yields the generalized torque τ_i for the joint i actuator to drive the i th link of the manipulator. That is,

$$\begin{aligned} \tau_i &= \frac{d}{dt} \left[\frac{\partial L}{\partial \dot{q}_i} \right] - \frac{\partial L}{\partial q_i} \quad i = 1, 2, 3, \dots, n \\ &= \sum_{j=i}^n \sum_{k=1}^j [\text{Tr} (U_{jk} J_i U_{ji}^T) \ddot{q}_k] + \sum_{j=i}^n \sum_{k=1}^j \sum_{m=1}^j [\text{Tr} (U_{jkm} J_i U_{ji}^T) \dot{q}_k \dot{q}_m] \\ &\quad - \sum_{j=i}^n m_j g U_{ij}^j \bar{r}_j \end{aligned} \quad (3.6)$$

Equation (3.6) can also be written as:

$$\tau_i = \sum_{k=1}^n D_{ik} \ddot{q}_k + \sum_{k=1}^n \sum_{m=1}^n h_{ikm} \dot{q}_k \dot{q}_m + c_i$$

or, in matrix form:

$$\tau(t) = D[\theta(t)] \ddot{\theta}(t) + h[\theta(t), \dot{\theta}(t)] + c[\theta(t)]$$

Here,

$D(\theta)$ = an $n \times n$ inertial acceleration-related matrix whose elements D_{ik} are

$$D_{ik} = \sum_{j=\max(i,k)}^n \text{Tr}(U_{jk} J_i U_{ji}^T) \quad i, k = 1, 2 \dots n$$

Also,

$h(\dot{\theta}_i, \theta_i)$ = an $n \times 1$ non-linear Coriolis and centrifugal force vector whose elements h_i are,

$$h_i = \sum_{k=1}^n \sum_{m=1}^n h_{ikm} \dot{q}_k \dot{q}_m \quad i = 1, 2 \dots n$$

with,

$$h_{ikm} = \sum_{j=\max(i,k,m)}^n \text{Tr}(U_{jkm} J_i U_{ji}^T) \quad i, k, m = 1, 2 \dots, n$$

and,

$c(\theta)$ = an $n \times 1$ gravity loading force vector whose elements c_i are

$$c_i = \sum_{j=i}^n (-m_j g U_{ij}^j \bar{r}_j) \quad i = 1, 2 \dots, n$$

where ${}^j \bar{r}_j$ is a fixed point in link j and expressed in homogeneous coordinates with respect to the j^{th} link coordinate frame.

The coefficient D_{ik} is related to the acceleration of the joint variable. For the case of $i=k$, the D_{ii} is related to the acceleration of the joint i where the driving torque τ_i acts. In the case $i \neq k$, D_{ik} is related to the reaction torque induced by acceleration of joint k acting at joint i [8].

The coefficient h_{ikm} [4] is related to the velocity of the two associated joint variables. Indices (k, m) refer to the velocities of joints k and m, whose dynamic interaction induces a torque at joint i.

Coefficient c_i [4] represents the gravity loading terms due to the weight of the links.

For this three-axis robot arm with rotary joints, the terms that form the acceleration-related symmetric matrix $D(\theta)$ are:

$$D(\theta) = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{12} & D_{22} & D_{23} \\ D_{13} & D_{23} & D_{33} \end{bmatrix}$$

The inertia terms (D_{ij} 's) of the acceleration-related symmetric matrix are obtained using Matlab and are presented in Appendix B1.

The velocity-related matrix is:

$$h(\theta, \dot{\theta}) = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}$$

The Coriolis and Centrifugal terms (h_i 's) are obtained using Matlab and the results are presented in Appendix B2.

The gravity-related matrix is given by:

$$c(\theta) = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

The gravitational terms (c_i 's) are obtained using Matlab and the results are presented in Appendix B3.

3.4 DETERMINATION OF THE J MATRIX

From Equation (3.2) [4], it can be seen that the term inside the bracket, $\int {}^i \mathbf{r}_i {}^i \mathbf{r}_i^T dm$, is the inertia of all points on link i, hence,

$$\mathbf{J}_i = \int {}^i \mathbf{r}_i {}^i \mathbf{r}_i^T dm = \begin{bmatrix} \int x_i^2 dm & \int x_i y_i dm & \int x_i z_i dm & \int x_i dm \\ \int x_i y_i dm & \int y_i^2 dm & \int y_i z_i dm & \int y_i dm \\ \int x_i z_i dm & \int y_i z_i dm & \int z_i^2 dm & \int z_i dm \\ \int x_i dm & \int y_i dm & \int z_i dm & \int dm \end{bmatrix} \quad (3.7)$$

where ${}^i \mathbf{r}_i = (x_i, y_i, z_i, 1)$. If we use inertia tensor I_{ij} which is defined as [1]:

$$I_{ij} = \int \left[\delta_{ij} \left(\sum_k x_k^2 \right) - x_i x_j \right] dm$$

where the indices i, j, k indicates principal axes of ith coordinate frame and δ_{ij} is called Kronecker delta, then $\mathbf{J}_i$ can be expressed in inertia tensor as [4]:

$$\mathbf{J}_i = \begin{bmatrix} (-I_{xx} + I_{yy} + I_{zz})/2 & I_{xy} & I_{xz} & m_i \bar{x}_i \\ I_{xy} & (I_{xx} - I_{yy} + I_{zz})/2 & I_{yz} & m_i \bar{y}_i \\ I_{xz} & I_{yz} & (I_{xx} + I_{yy} - I_{zz})/2 & m_i \bar{z}_i \\ m_i \bar{x}_i & m_i \bar{y}_i & m_i \bar{z}_i & m_i \end{bmatrix}$$

where,

I_{xx} = Moment of inertia of the ith link about the x-axis with respect to the i^{th} coordinate frame,

I_{yy} = Moment of inertia of the ith link about the y-axis with respect to the i^{th} coordinate frame,

I_{zz} = Moment of inertia of the ith link about the z-axis with respect to the i^{th} coordinate frame,

I_{xy}, I_{xz}, I_{yz} = Products of inertia for i^{th} link,

m_i = Mass of i^{th} link,

$\bar{x}_i$ = Distance from center of mass of the i^{th} link to which coordinate frame in the direction of the x-axis

$\bar{y}_i$ = Distance from center of mass of the i^{th} link to which coordinate frame in the direction of the y-axis

$\bar{z}_i$ = Distance from center of mass of the i^{th} link to which coordinate frame in the direction of the z-axis

The exact values of the moments of inertia cannot be determined exactly since the robot is not able to be dismantled. Therefore, estimated values have been determined by geometric approximation of the links of the SCARA-Type robot.

Moments of Inertia for Link 1:

Mass = 0.981 lbm.

Length = 1.70 ft.

Width = 0.42 ft.

Thickness = 0.26ft.

The moments of inertia of link 1 are:

$$I_{zz} = 9.6\text{E-}4 \text{ lb-ft}^2$$

$$I_{yy} = 9.37\text{E-}4 \text{ lb-ft}^2$$

$$I_{xx} = 1.96\text{E-}5 \text{ lb-ft}^2$$

Moments of Inertia for Link 2:

Mass = 1.44E-4 lbm.

Length = 0.72 ft.

Width = 0.30 ft.

Thickness = 0.131 ft.

The moments of inertia of link 2 are:

$$I_{zz} = 2.6\text{E-}5 \text{ lb-ft}^2$$

$$I_{yy} = 2.5\text{E-}5 \text{ lb-ft}^2$$

$$I_{xx} = 1.28\text{E-}6 \text{ lb-ft}^2$$

Moments of Inertia for actuator on Link 2:

The actuator in joint 2 of the robot has been modeled as a cylinder. The mass and dimensions of the cylinder are given below.

$$\text{Mass} = 0.513 \text{ lbm.}$$

$$\text{Height} = 0.68 \text{ ft.}$$

$$\text{Radius} = 0.18 \text{ ft.}$$

The moments of inertia of the actuator for link 2 are:

$$I_{zz} = 2.27\text{E-}5 \text{ lb-ft}^2$$

$$I_{yy} = 7.35\text{E-}5 \text{ lb-ft}^2$$

$$I_{xx} = 7.37\text{E-}5 \text{ lb-ft}^2$$

Total moments of inertia of link 2 are:

$$I_{zz} = 2.6\text{E-}5 \text{ lb-ft}^2 + 2.28\text{E-}5 \text{ lb-ft}^2 = 4.8\text{E-}5 \text{ lb-ft}^2$$

$$I_{yy} = 2.5\text{E-}5 \text{ lb-ft}^2 + 7.35\text{E-}5 \text{ lb-ft}^2 = 9.8\text{E-}5 \text{ lb-ft}^2$$

$$I_{xx} = 1.28\text{E-}6 \text{ lb-ft}^2 + 7.37\text{E-}5 \text{ slug-ft}^2 = 7.5\text{E-}5 \text{ lb-ft}^2$$

Moments of Inertia for Link 3:

$$\text{Mass} = 1.4\text{E-}4 \text{ lbm.}$$

$$\text{Length} = 1.27 \text{ ft.}$$

$$\text{Width} = 0.164 \text{ ft.}$$

$$\text{Thickness} = 0.131 \text{ ft.}$$

$$I_{zz} = 7.53\text{E-}5 \text{ lb-ft}^2$$

$$I_{yy} = 7.5\text{E-}5 \text{ lb-ft}^2$$

$$I_{xx} = 5.12\text{E-}7 \text{ lb-ft}^2$$

Moments of Inertia for actuator on Link 3:

The actuator in joint 3 of the robot has been modeled as a cylinder. The mass and dimensions are given below.

$$\text{Mass} = 0.611 \text{ lbm.}$$

$$\text{Height} = 0.78 \text{ ft.}$$

Radius = 0.18 ft.

The moments of inertia of the actuator for link 3 are:

$$I_{zz} = 6.62\text{E-}6 \text{ lb-ft}^2$$

$$I_{yy} = 3.71\text{E-}5 \text{ lb-ft}^2$$

$$I_{xx} = 3.71\text{E-}5 \text{ lb-ft}^2$$

The moments of inertia of the supporting link forming a parallelogram, as shown in Figure 2.3, with link 3 is:

Mass = 0.0014 lbm.

Length = 1.14 ft.

Width = 0.098 ft.

Thickness = 0.098 ft.

$$I_{zz} = 2.44\text{E-}5 \text{ lb-ft}^2$$

$$I_{yy} = 3.71\text{E-}5 \text{ lb-ft}^2$$

$$I_{xx} = 1.27\text{E-}5 \text{ lb-ft}^2$$

Total moments of inertia of Link 3 including actuator and supporting link are:

$$I_{zz} = 7.53\text{E-}5 \text{ lb-ft}^2 + 6.62\text{E-}6 \text{ lb-ft}^2 + 2.44\text{E-}5 \text{ lb-ft}^2 = 1.06\text{E-}4 \text{ lb-ft}^2$$

$$I_{yy} = 7.5\text{E-}5 \text{ lb-ft}^2 + 3.71\text{E-}5 \text{ lb-ft}^2 + 3.71\text{E-}5 \text{ lb-ft}^2 = 1.5\text{E-}4 \text{ lb-ft}^2$$

$$I_{xx} = 5.12\text{E-}7 \text{ lb-ft}^2 + 3.71\text{E-}5 \text{ lb-ft}^2 + 1.27\text{E-}5 \text{ lb-ft}^2 = 5\text{E-}5 \text{ lb-ft}^2$$

3.5 TORQUE EQUATIONS FOR THE SCARA-TYPE ROBOT

From the Lagrange-Euler formulation [1,5], the torque equations for each of the joints of the SCARA-type robot are given as below.

$$\tau_1 = D_{11}\ddot{\theta}_1 + D_{12}\ddot{\theta}_2 + D_{13}\ddot{\theta}_3 + h_1 + c_1 \quad (3.8)$$

$$\tau_2 = D_{12}\ddot{\theta}_1 + D_{22}\ddot{\theta}_2 + D_{23}\ddot{\theta}_3 + h_2 + c_2 \quad (3.9)$$

$$\tau_3 = D_{13}\ddot{\theta}_1 + D_{23}\ddot{\theta}_2 + D_{33}\ddot{\theta}_3 + h_3 + c_3 \quad (3.10)$$

More detailed equations of the torques for the three joints of the SCARA-Type robot are shown in Appendix C.

3.6 SUMMARY

In this chapter, the formulation for obtaining the dynamics for the LGCM robot is presented. Starting with equations of motion for the manipulator and the moments of inertia of the links, the torque equations for this SCARA-Type robot are obtained.

Chapter 4

Trajectory Planning

4.1 INTRODUCTION

Trajectory planning [4,15] is defining the movement of the robot arm through an end-effector path by interpolating the desired path by a set of polynomial functions and generating a sequence of time-based set points for the control of the manipulator from the start point to the end point. Path start and end points can be expressed in either joint or Cartesian coordinates [4]. Usually the path is specified in Cartesian coordinates because it is easier to visualize the correct end-effector configuration in Cartesian coordinates than in joint coordinates. Planning of the trajectory in joint coordinates has some advantages. The first one is that the trajectory is planned directly in terms of the controlled motion variables [4, pg. 151]. Real time trajectory planning can be done. The disadvantage is that the end-effector motion is hard to visualize [4, pg.149]. The preferred method in which the planned motion can be approached is that the trajectory be planned in Cartesian coordinates and then the inverse kinematic solution is used to convert the Cartesian trajectory to corresponding joint angles at each point. To plan the trajectory in Cartesian coordinates, the path has to be divided into segments and desired path continuity constraints have to be maintained [4, pg.155]. Planning and execution of a defined task which would be expected to be performed in industry is task planning. In this chapter, Section 4.3 introduces the task that was planned for the ‘pick-to-place’ studies for the SCARA-Type robot. The chapter then goes through the trajectory planning.

4.2 METHODOLOGY

Trajectory planning for the manipulator is done using Excel. The 4-3-4 trajectory planning [4, pg.157, 15] is done mathematically in Excel to obtain three equations for three segments of the trajectory. The simulation of the trajectories is done in Excel as presented in Table 4.1.

Analysis using EXCEL

EXCEL is used to generate the required path of the trajectory. The lift-off and set-down positions are the inputs to determine the required trajectories for the three joints.

Table 4.1 Trajectory Planning Using Excel

Theta0	Theta1	Theta2	Theta3
70.00	65.00	25.00	20.00
Delta1 (Theta1-Theta0)	Delta2 (Theta2-Theta1)	Delta3 (Theta3-Theta2)	
-5.00	-40.00	-5.00	
t1 (time for first segment of trajectory)	t2 (time for second segment of trajectory)	t3 (time for third segment of trajectory)	
0.50	1.00	0.50	
F	G		
-70.00	10.50		
Zeta			
-6.67			
v1	A1		
-26.67	-80.00		
v2	A2		
-26.67	80.00		

Table 4.1 cont. Trajectory Planning Using Excel

Time (sec)	Angle (degrees)	Velocity (deg/sec)	Acceleration (deg/sec^2)
0.00	70.00	0.00	0.00
0.10	69.95	-1.49	-28.80
0.20	69.62	-5.55	-51.20
0.30	68.78	-11.52	-67.20
0.40	67.27	-18.77	-76.80
0.50	65.00	-26.67	-80.00
0.60	61.96	-33.87	-64.00
0.70	58.28	-39.47	-48.00
0.80	54.12	-43.47	-32.00
0.90	49.64	-45.87	-16.00
1.00	45.00	-46.67	0.00
1.10	40.36	-45.87	16.00
1.20	35.88	-43.47	32.00

1.30	31.72	-39.47	48.00
1.40	28.04	-33.87	64.00
1.50	25.00	-26.67	80.00
1.60	22.73	-18.77	76.80
1.70	21.22	-11.52	67.20
1.80	20.38	-5.55	51.20
1.90	20.05	-1.49	28.80
2.00	20.00	0.00	0.00

4.3 TASK PLANNING FOR THE SCARA-TYPE ROBOT

A pick-to-place task for the SCARA-Type robot has been defined. The robot arm moves the end-effector from the soft home position to the glassware pick position to pickup the glassware. The robot arm then moves the glassware to the place position in front of the laser gun for the cut in the glassware to be performed by the laser. After the cut is made, the robot arm moves the glassware back to the pick position and the robot arm moves back to the soft home position. The robot end-effector follows a minimum time trajectory [15] to accomplish its pick-to-place task. Theoretical trajectory planning is done for its task from pick position to place position of the glassware.

4.4 TRAJECTORY PLANNING

4.4.1 4-3-4 Trajectory Planning

In planning a 4-3-4 trajectory [4, pg.157] for a joint motion, the first segment is a fourth-degree polynomial and specifies the trajectory from initial position to lift-off position as shown in Figure 4.1 [4, pg.155]. The second segment is the third-degree polynomial and specifies the path trajectory from lift-off position to set-down position as shown in Figure 4.1 [4, pg.155]. The set-down position to final position as shown in Figure 4.1 is specified using a fourth-degree polynomial. The planning of the 4-3-4 trajectory is done using the following procedure. A normalized time variable, t (0, 1), is introduced to simplify the procedure to compute the 4-3-4 trajectory equations [4, 15]. The three polynomial equations are follows:

$$h_1(t) = a_{14}t^4 + a_{13}t^3 + a_{12}t^2 + a_{11}t + a_{10} \text{ (first segment: } 0 \leq t \leq t_1 \text{)} \quad (4.1)$$

$$h_2(t) = a_{23}t^3 + a_{22}t^2 + a_{21}t + a_{20} \text{ (second segment: } t_1 \leq t \leq t_2 \text{)} \quad (4.2)$$

$$h_3(t) = a_{34}t^4 + a_{33}t^3 + a_{32}t^2 + a_{31}t + a_{30} \quad (\text{third segment: } t_2 \leq t \leq t_3) \quad (4.3)$$

where t is the normalized time.

The following 14 boundary conditions are specified in order to determine the 14 coefficient of these equations [4].

1. Initial position (at time t_0) = θ_0
2. Magnitude of the initial velocity = v_0
3. Magnitude of the initial acceleration = a_0
4. Lift-off position (at time t_1) = θ_1
5. Continuity in position (at time t_1) $\theta(t_1^-) = \theta(t_1^+)$
6. Continuity in velocity (at time t_1) $\dot{\theta}(t_1^-) = \dot{\theta}(t_1^+)$
7. Continuity in acceleration (at time t_1) $\ddot{\theta}(t_1^-) = \ddot{\theta}(t_1^+)$
8. Set down position (at time t_2) = θ_2
9. Continuity in position (at time t_2) $\theta(t_2^-) = \theta(t_2^+)$
10. Continuity in velocity (at time t_2) $\dot{\theta}(t_2^-) = \dot{\theta}(t_2^+)$
11. Continuity in acceleration (at time t_2) $\ddot{\theta}(t_2^-) = \ddot{\theta}(t_2^+)$
12. Final position (at time t_3) = θ_3
13. Magnitude of final velocity = v_3
14. Magnitude of final acceleration = a_3

4.1. Trajectory Planning for Joint Motion

The resultant polynomial equations for the defined 4-3-4 trajectory are given as follows [4, pg.158]:

First trajectory segment:

$$h_1(t) = (\delta_1 - v_0 t_1 - a_0 t_1^2 / 2 - \sigma) t^4 + \sigma t^3 + (a_0 t_1^2 / 2) t^2 + (v_0 t_1) t + \theta_0$$

where,

$$\delta_1 = \theta_1 - \theta_0$$

$\sigma = f/g$, for which

$$f = 2\delta_1(4 + 2t_3/t_2 + 2t_3/t_1 + 3t_2/t_1) - (\delta_2 t_1/t_2)(3 + t_3/t_2) + 2\delta_3 t_1/t_3 -$$

$$v_0(6 + 6t_2/t_1 + 4t_3/t_1 + 3t_3/t_2) - v_3 t_1 - a_0 t_1 t_3(5/3 + t_1/t_2 + 2t_1/t_3 + 5t_2/2t_3) +$$

$$a_3 t_1 t_3$$

$$g = t_3/t_2 + 2t_3/t_1 + 2 + 3t_2/t_1$$

Second trajectory segment:

$$h_2(t) = (\delta_2 - v_1 t_2 - a_1 t_2^2 / 2) t^3 + (a_1 t_2^2 / 2) t^2 + (v_1 t_2) t + \theta_1$$

where,

$$\delta_2 = \theta_2 - \theta_1$$

$$v_1 = 4\delta_1/t_1 - 3v_0 - a_0 t_1 - \sigma/t_1$$

$$a_1 = 12\delta_1/t_1^2 - 12v_0/t_1 - 5a_0 - 6\sigma/t_1^2$$

Third trajectory segment:

$$h_3(t) = (9\delta_3 - 4v_2t_3 - a_2t_3^2/2 - 5v_3t_3 + a_3t_3^2/2)t^4 + (-8\delta_3 + 5v_3t_3 - a_3t_3^2/2 + 3v_2t_3)t^3 + (a_2t_3^2/2)t^2 + (v_2t_3)t + \theta_2$$

where,

$$\delta_3 = \theta_3 - \theta_2$$

$$v_2 = 3\delta_2/t_2 - 2v_1 - a_1t_2/2$$

$$a_2 = 6\delta_2/t_2^2 - 6v_1/t_2 - 2a_1t_2$$

Figure 4.2 Top View of Work Envelop of the Robot [3]

4.5 RESULTS OF TRAJECTORY PLANNING

The 4-3-4 trajectory equations for the motion of three joints to complete the specified pick-to-place task (Figure 4.2 [3]) of the end-effector were determined. The

trajectory equations for the three segments of the pick-to-place motion for joint 1 were determined as:

$$h1 = -10.04*t^4 - 7.54*t^3 + 70 \quad (\text{first segment})$$

$$h2 = 30.2521*(t-0.5)^3 - 45.3782*(t-0.5)^2 - 24.8739*(t-0.5) + 65 \quad (\text{second segment})$$

$$h3 = 10.04*(t-1.5)^4 - 35.6714*(t-1.5)^3 + 45.37815*(t-1.5)^2 - 24.8739*(t-1.5) + 25 \quad (\text{third segment})$$

where t is the actual time associated with the robot traversing the trajectory. These equations have been used to generate the desired trajectory.

4.6 INVERSE KINEMATIC VERIFICATION

Inverse kinematic verification of the SCARA-Type robot was presented in Chapter 2. In this section the solution will be used to determine the joint angles θ_1 , θ_2 , θ_3 and $\theta_4 = (90-\theta_3)$ for the pick-to-place trajectory presented in Section 4.3. The first step for this process is to convert the path into corresponding position and orientations of the end-effector. The pick and place position in the workspace has the following positions and orientations:

Cartesian components of the Pick Position (p) and Orientation (n, o, a) of the robot end-effector with respect to the fixed base coordinate frame are:

$P_x = 2.46 \text{ feet}$	$P_y = 1.92 \text{ feet}$	$P_z = 4.56 \text{ feet}$
$n_x = -0.1736$	$n_y = 0.9848$	$n_z = 0$
$o_x = 0.9848$	$o_y = 0.1736$	$o_z = 0$
$a_x = 0$	$a_y = 0$	$a_z = -1$

Place position and orientation of the robot with respect to the fixed base are:

$P_x = 3.54 \text{ feet}$	$P_y = 0.24 \text{ feet}$	$P_z = 4.78 \text{ feet}$
$n_x = 0.1736$	$n_y = 0.9848$	$n_z = 0$
$o_x = 0.9848$	$o_y = -0.1736$	$o_z = 0$
$a_x = 0$	$a_y = 0$	$a_z = -1$

The intermediate points are a function of the path and follow the same related path with respect to time since the position and orientation of the pick and place are known. The inverse kinematic solution is applied to convert the Cartesian coordinates to joint coordinates. Applying inverse kinematic solution to previously mentioned Cartesian coordinates gives corresponding joint coordinates. Hence, the corresponding values for joint coordinate system for pick and place positions are calculated as:

Pick position:

Joint 1 = 70 degrees

Joint 2 = -60 degrees

Joint 3 = 20 degrees

Place Position:

Joint 1 = 20 degrees

Joint 2 = -30 degrees

Joint 3 = 10 degrees

The joint angle variation of the three joints for the pick-to-place task to be initially accomplished in 2 sec (arbitrary choice) is shown in Table 4.2.

Table 4.2: Joint Angle Variation of Three Joints for Pick-to-Place Task

Time (sec)	θ_1 (joint 1) degrees	θ_2 (joint 2) degrees	θ_3 (joint 3) degrees
0	70.00	-60.00	20.00
0.1	69.95	-59.93	19.97
0.2	69.62	-59.49	19.78
0.3	68.78	-58.49	19.37
0.4	67.27	-56.93	18.74
0.5	65.00	-55.00	18.00
0.6	61.96	-53.00	17.30
0.7	58.28	-51.00	16.67
0.8	54.12	-49.00	16.09
0.9	49.64	-47.00	15.54
1.0	45.00	-45.00	15.00

Table 4.2 (cont.): Joint Angle Variation of Three Joints for Pick-to-Place Task

1.1	40.36	-43.00	14.46
1.2	35.88	-41.00	13.91
1.3	31.72	-39.00	13.33
1.4	28.04	-37.00	12.70
1.5	25.00	-35.00	12.00
1.6	22.73	-33.07	11.26
1.7	21.22	-31.51	10.63
1.8	20.38	-30.51	10.22
1.9	20.05	-30.07	10.03
2.0	20.00	-30.00	10.00

Using the inverse kinematic solution the corresponding Cartesian positions of the end-effector are computed, which are shown in Table 4.3.

Table 4.3: Cartesian Position for the corresponding Joint Angles

Time (sec)	Px	Py	Pz
0	2.4658	1.9297	4.5656
0.1	2.4673	1.93	4.5663
0.2	2.4772	1.9305	4.5702
0.3	2.5026	1.9274	4.5789
0.4	2.5483	1.9131	4.592
0.5	2.617	1.8755	4.6075
0.6	2.7081	1.8014	4.6222
0.7	2.8148	1.6915	4.6356
0.8	2.9289	1.5506	4.6481
0.9	3.0424	1.3849	4.6598
1.0	3.1488	1.2021	4.6713
1.1	3.2431	1.0111	4.6828
1.2	3.3224	0.82209	4.6947
1.3	3.3861	0.64597	4.7072
1.4	3.4355	0.49405	4.7209
1.5	3.4732	0.37771	4.736
1.6	3.5016	0.304	4.752
1.7	3.5213	0.26367	4.7656
1.8	3.5327	0.24549	4.7747
1.9	3.5375	0.23989	4.7788
2.0	3.5382	0.23922	4.7795

where, Px, Py, and Pz are Cartesian coordinate for corresponding joint angles.

Figures 4.3 to 4.5 show the joint angles, which are required for the end-effector to traverse the specified pick-to-place motion.

Figure 4.3 Motion of θ_1 with time for the Pick-to-Place Task

Figure 4.4 Motion of θ_2 with time for the Pick-to-Place Task

Figure 4.5 Motion of θ_3 with time for the Pick-to-Place Task

4.7 INVERSE DYNAMIC VERIFICATION

This section determines the torques required at the joints using the dynamic equations [4] determined in Chapter 3. The angular velocities and angular accelerations of each joint are required to determine the torques. To achieve this, Matlab and Excel are used. There are three equations, which calculate the torques for the joints. This code first uses inverse kinematic solution to determine the joint angles for the SCARA-Type robot. Thereafter, the joint velocities and joint accelerations are determined by differentiating the joint angles with respect to time. Figures 4.6 to 4.8 show the angular positions, velocities and accelerations of the three joints. Figures 4.9, 4.10 & 4.12 show the comparison of real torque and independent torque required by the three joints of the robot. The real torque characteristic shown in Figure 4.10 does not follow the angular acceleration characteristic of joint 2. Real torque characteristic of Joint 2 is shown in Figure 4.11. From the Figure 4.11 it can be seen that the inertia term of the link 2 is dominated by the inertias and coriolis & centrifugal terms of link 1 and link 3 since link 2 is small (0.7 feet) compared to link 1 (1.7 feet) and link 3 (1.27 feet). The angular

positions, velocities and accelerations are in radians, radians per second and radians per second square, respectively.

The velocity for the first joint shown in Figure 4.6 shows that the angular velocity for joint 1 is negative, which was expected, because the first joint angle decreases to produce a trajectory. Figure 4.7 shows that the angular velocity for joint 2 increases rapidly and decreases rapidly to zero velocity. The velocity for the third joint shown in Figure 4.8 shows that the angular velocity is negative and it first decreases rapidly then becomes constant and finally increases rapidly again to zero velocity. For all the three joints, the angular velocity goes through a smooth angular motion transition at the lift-off and the set-down positions, which were specified.

The angular acceleration for joint 1 in Figure 4.6 shows that there is a rapid deceleration at the beginning of the path and then there is a constant acceleration during the major part of the trajectory, finally at the end it decelerates to come to a stop. The same trend is followed for the angular acceleration of the second joint shown in Figure 4.7, except that it accelerates at the beginning and then decelerates during the actual trajectory, finally at the end it accelerates to come to a stop. Figure 4.8 for joint 3 shows that the angular acceleration decelerates and accelerates rapidly and it is smooth during its course of the trajectory. It then accelerates and decelerates to come to zero.

The real and independent torque curves for all three joints for the SCARA-Type robot are shown in Figures 4.9, 4.10 and 4.12. From Figure 4.9 it is seen that the real torque required for joint 1 at time $t_1 = 0.5$ sec is -24.62 lb-ft. The angular acceleration of joint 1 at time $t_1 = 0.5$ sec is -1.4 rad/sec^2 . Hence the inertia for link 1 can be calculated from $\text{Torque} = \text{Inertia} * \text{Angular Acceleration}$. The inertia calculated from the equation mentioned is $-24.62/-1.4 = 17.6 \text{ lb-ft}^2$. From the geometry of the robot the inertia of link 1, which incorporates the effects of link 2 and link 3 is 13.4 lb-ft^2 . Independent torque from Figure 4.9 at time $t_1 = 0.5$ sec is -11.58 lb-ft. Angular acceleration is -1.4 rad/sec^2 . Inertia calculated for independent torque is $-11.58/-1.4 = 8.21 \text{ lb-ft}^2$. From the geometry of the robot, the inertia of link 1, independent of link 2 and link 3, is 8.3 lb-ft^2 . The input torque required at joint 1 shows that the torque follows the same trend as the acceleration, which indicates that the major portion of the torque consists of acceleration-

related terms. The input torque required at joint 2 varies from +8.29 lb-ft to -9.75 lb-ft. The torque for the second joint is dominated by the inertia and coriolis & centrifugal terms of link 1 and link 3.

The torque curve for the third joint is shown in Figure 4.12. There is a decrease in of magnitude of torque. This is because the third link moves in the vertical plane to accomplish this task and hence it is dominated by the gravitational term. Input torque required for the third joint follows the acceleration, but it starts from 11.70 lb-ft instead of zero because of the acceleration due to gravity term, which always exists throughout its motion.

It can be studied from Figure 4.6 that the robot will go through small jerk of 2.79 rad/sec³ at time $t = 0.5$ sec and $t = 1.5$ sec. For designing the trajectory of the robot the 4-3-4 trajectory planning method was used. This method doesn't apply boundary condition on the jerk. It applies boundary conditions on position, velocities and acceleration and makes them continuous between each segment of the trajectory. The jerk is small and can be eliminated by applying an extra boundary condition on jerk. This can be achieved by planning the trajectory using 5-4-5 trajectory planning method.

The links of the SCARA-Type robot are rigid as defined in Chapter 2. The torques calculated from the dynamic equations of motion for the SCARA-Type robot, is for rigid links. If the links and joints of the work cell robot are flexible, the torque requirement at the joint will increase. If only the link is rigid and not the joint, the change in torque requirement will not be significant unless the length of the link is considerably large. The torque requirement changes considerably if the joint is flexible. The actuator inside the flexible joint has to counteract the torque generated by jerk, due to flexible joint. But in reality the links and joints of the robots are made rigid to make the robot system more stable to perform a task with optimal torque requirements at the joints.

Figure 4.6 Joint 1 Position, Velocity and Acceleration for Pick-to-Place Task

Figure 4.7 Joint 2 Position, Velocity and Acceleration for Pick-to-Place Task

Figure 4.8 Joint 3 Position, Velocity and Acceleration for Pick-to-Place Task

Figure 4.9 Joint 1 input torque required for Pick-to-Place Task

Figure 4.10 Joint 2 input torque required for Pick-to-Place Task

Figure 4.11 Joint 2 real torque characteristics for Pick-to-Place Task

Figure 4.12 Joint 3 input torque (Link 1 & 2 are fixed) required for Pick-to-Place Task

Chapter 5

Transfer Function Model for SCARA-Type Robot

5.1 METHODOLOGY

The integrated model [8] for the DC motor and manipulator was developed using kinematic and dynamic analysis of motor and manipulator. The input for this model is the desired angle for the joints and the output is the actual angle achieved by the joint. Matlab was used to simulate the output of the models of three joints for the step input.

5.2 ACTUATOR DYNAMIC MODEL

Three actuators located in the three joints of the robot drive the SCARA-Type robot. The schematic diagram of an actuator is shown in Figure 5.1 [4, pg.206]. These actuators are permanent magnet DC servomotors [13] with armature current control.

Figure 5.1 Armature Controlled DC Motor [4]

For an armature controlled DC motor [8, 14], the actuator dynamics can be represented as follows:

$$V_a(t) = R_a I_a(t) + L_a \frac{d}{dt} [I_a(t)] + K_b \omega_m(t) \quad (5.1)$$

where,

$V_a(t)$ = Applied Voltage

$I_a(t)$ = Armature Current

L_a = Armature Inductance

R_a = Armature Resistance

K_b = Back Electromotive Force (emf) constant

$\omega_m(t)$ = Angular Velocity of motor shaft

For an armature controlled DC motor, the electrical time constant $\tau = L_a / R_a$ is very small. Since the electrical time constant of the motor is much smaller than the mechanical time constant, $\tau = L_a / R_a$ can be neglected. Thus, Equation 5.1 can be written as:

$$V_a(t) = R_a I_a(t) + K_b \omega_m(t)$$

When the armature controlled DC motor is operated in its linear range, the generated torque is proportional to the armature current [13]. The relation between the generated torque and armature current is written as [4]:

$$T_m(t) = K_t I_a(t) \quad (5.2)$$

where,

$T_m(t)$ = Torque generated by DC motor

K_t = Motor torque constant

Now, substituting $I_a(t)$ from Equation (5.2) into Equation (5.1) gives,

$$V_a(t) = \frac{R_a}{K_t} T_m(t) + K_b \omega_m(t) \quad (5.3)$$

Equation (5.3) represents the armature controlled DC motor dynamics. When an actuator, through gear reduction drives a manipulator link, the torque generated by an actuator must be equal to the torque required to overcome the actuator inertia and friction and the load from the manipulator link. This can be written as

$$T_m(t) = J_m \dot{\omega}_m(t) + B_v \omega_m(t) + T_d(t) \quad (5.4)$$

where,

J_m = Inertia of motor rotor about motor axis

B_v = Combined friction coefficient of motor about motor axis

$\dot{\omega}_m(t)$ = Angular acceleration of motor shaft

$T_d(t)$ = Load torque on motor about motor axis

If a harmonic drive with speed reduction ratio equal to n is used, then,

$$n \Rightarrow \gg 1$$

Hence, for manipulator link i ,

$T_i^L(t)$ = Load on motor from manipulator link i about joint i .

$\dot{\theta}_i(t)$ = Angular velocity of manipulator link i about joint i .

$\ddot{\theta}_i(t)$ = Angular acceleration of the manipulator link i about joint i .

Then,

$$T_d(t) = \frac{T_i^L(t)}{n} \quad (5.5)$$

$$\omega_m(t) = n \dot{\theta}_i(t) \quad (5.6)$$

$$\dot{\omega}_m(t) = n \ddot{\theta}_i(t) \quad (5.7)$$

Substituting Equations (5.5), (5.6) and (5.7) into Equations (5.3) and (5.4) gives

$$T_i^L(t) = \frac{nV_a K_t}{R_a} - \frac{K_b K_t}{R_a} n^2 \dot{\theta}_i - J_m n^2 \ddot{\theta}_i - B_v n^2 \dot{\theta}_i \quad (5.8)$$

Equation (5.8) shows the relationship between the load $T_i^L(t)$ from the manipulator link and link angular acceleration and angular velocity. This equation is used in developing the integrated model for joint dynamics in Section 5.3.

5.3 INTEGRATED JOINT DYNAMIC MODEL FOR THE SCARA-TYPE ROBOT

In this section the combined dynamics for both manipulator link i and link i actuator, called the integrated joint dynamic model [8], is derived. The dynamic equation of manipulator link i can be written as:

$$T_i = D_{ii} \ddot{\theta}_i + h_i + c_i + \sum_{j=1}^3 D_{ij} \ddot{\theta}_j \quad (5.9)$$

Let the coefficient of friction at joint i be equal to B_i . Then the torque T_i required at joint i must overcome the frictional torque $B_i \dot{\theta}_i$ due to this friction as well. Equation (5.9), when joint friction is considered, becomes

$$T_i = D_{ii} \ddot{\theta}_i + B_i \dot{\theta}_i + \sum_{j=1}^3 D_{ij} \ddot{\theta}_j + h_i + c_i \quad (5.10)$$

Let ,

$$T_{c_i} = \sum_{j=1}^3 D_{ij} \ddot{\theta}_j + h_i + c_i$$

where,

T_{c_i} = Effective coupling torque from all other links to joint i .

B_i = Coefficient of friction at joint i .

Hence, Equation (5.10) can be written as:

$$T_i = D_{ii} \ddot{\theta}_i + B_i \dot{\theta}_i + T_{c_i} \quad (5.11)$$

The T_i in Equation (5.11) represents the joint torque required at joint i to drive link i at velocity $\dot{\theta}_i$ and acceleration $\ddot{\theta}_i$. T_i can also be viewed as the load T_L from link i to the actuator at joint i . By ignoring effective coupling torque from all other links to joint i term (T_{c_i}), Equations (5.8) and (5.11) yields,

$$D_{ii} \ddot{\theta}_i + B_i \dot{\theta}_i = \frac{V_a K_t}{n R_a} - \frac{K_b K_t}{n^2 R_a} \dot{\theta}_i - \frac{J_m}{n^2} \ddot{\theta}_i - \frac{B_v}{n^2} \dot{\theta}_i \quad (5.12)$$

Combining terms, Equation (5.12) can be written as,

$$\ddot{\theta}_i [D_{ii} n^2 + J_m] + \dot{\theta}_i [B_i n^2 + B_v + \frac{K_b K_t}{R_a}] = \frac{n V_a K_t}{R_a} \quad (5.13)$$

Simplifying coefficients by defining effective coefficient J_{e_i} and B_{e_i} as:

$$J_{e_i} = \frac{D_{ii}}{n^2} + J_m$$

$$B_{e_i} = \frac{B_i}{n^2} + B_v$$

Now, Equation (5.13) can be rewritten as:

$$J_{e_i} \ddot{\theta}_i + B_{e_i} \dot{\theta}_i + \frac{K_b K_t}{R_a} \dot{\theta}_i = \frac{n K_t}{R_a} V_{a_i} \quad (5.14)$$

where,

$J_{e_i} = J_{eff}$ = Effective inertia of joint i

$B_{e_i} = B_{eff}$ = Effective coefficient of friction at joint i

T_{c_i} = Effective coupling torque from other links to joint i

V_{a_i} = Voltage applied to actuator at joint i

n = Harmonic drive speed ratio

$\ddot{\theta}_i$ = Angular acceleration of link i

$\dot{\theta}_i$ = Angular velocity of link i

The output of the integrated transfer function model of the joint i is the angular position of the joint, $\theta_L(s)$. The relation between the angular position $\theta_L(s)$ to the armature voltage $V_a(s)$ is obtained by taking the Laplace Transform of Equation (5.14), i.e.,

$$\frac{\theta_L(s)}{V_a(s)} = \frac{K_t}{ns(sR_a J_{eff} + R_a B_{eff} + K_t K_b)} \quad (5.15)$$

The relation between the angular velocity $\dot{\theta}_m(s)$ to the armature voltage $V_a(s)$ is given by:

$$\frac{\dot{\theta}_m(s)}{V_a(s)} = \frac{K_t}{sR_a J_{eff} + R_a B_{eff} + K_t K_b} \quad (5.16)$$

Equation (5.15) is the open-loop transfer function of the single joint relating the angular displacement to the applied voltage of the joint. The block diagram is shown in Figure 5.2 [4, pg.210].

Figure 5.2 Block Diagram of Integrated Joint Dynamic Model for One Link

The motors in the joints of the SCARA-Type robot are SM series servomotors made by Parker Hannifin Corporation and all the three joints have similar actuators. The technical characteristics of the actuators are shown in Table 5.1.

Table 5.1 Actuator Characteristics to Compute Open & Closed Loop Transfer Function Model for SCARA-Type Robot

Resistance (R_a)	9.65 ohms
Inductance (L_a)	4.08 mH
Viscous Damping (B_v)	3.78E-3 Nm/Krpm
Rotor Inertia (J_m)	1.3E-4 kg-m ²
Voltage Constant (K_b)	0.484 volts/rad/sec
Torque Constant (K_t)	0.48 Nm/Amp DC
Effective Inertia ($J_{eff_1}, J_{eff_2}, J_{eff_3}$)	4.072 kg-m ² , 0.4344 kg-m ² , 0.116 kg-m ²
Effective Friction (B_{eff})	3.81E-3 Nm/Krpm

For deriving the transfer function model for the SCARA-Type robot the moment of inertias [4, 9] of the links play an important role. The moments of inertia of the actuators and the links are added to get the effective moments of inertia to derive the

transfer function model for the three joints. The inertias of three links are shown in Table 5.2.

Table 5.2 Moments of Inertia of the Three Links

Link	Inertias
1.	0.031 slug-ft ² or 0.35 kg-m ²
2.	1.55*10 ⁻³ slug-ft ² or 0.01 kg-m ²
3.	3.11*10 ⁻³ slug-ft ² or 0.0358 kg-m ²

5.4 OPEN-LOOP TRANSFER FUNCTION MODEL FOR THE THREE JOINTS OF THE SCARA-TYPE ROBOT

The open-loop transfer function [10,11] for the integrated joint dynamics for joint 1, whose output is the angular velocity of the motor, is derived by substituting the constants in Table 5.1 and 5.2 in Equation (5.16) to obtain:

$$G_1(s) = \frac{\dot{\theta}_m(s)}{V_a(s)} = \frac{0.0096}{39s + 0.27} \quad (5.17)$$

The step response [11, 14] for this transfer function is shown in Figure 5.3.

The coefficient “39” in Equation (5.17) represents a case when the robot is fully stretched out and hence the moments of inertia of link 2 and link 3 are added to the moment of inertia of link 1 to calculate the coefficient “39”. The coefficient changes with the change in the angles of joint 2 and joint 3. The coefficient has to be changed for different angles of joint 2 and joint 3 to maintain controlled trajectory.

The moment of inertia at joint 1 is calculated for the case when the robot is operating in minimum workspace volume. The open-loop transfer function for the integrated joint dynamics for joint 1, with the robot working in minimum workspace volume is:

$$G_1(s) = \frac{\dot{\theta}_m(s)}{V_a(s)} = \frac{0.0096}{17s + 0.27} \quad (5.18)$$

The relative stability and the transient performance of the system are directly related to the location of roots of the characteristic equation in s-plane. Often it is required to adjust system parameters in order to obtain suitable root locations. Hence it is

worthwhile to determine the roots of the characteristic equation [4, 11, 12] of a given system to study the behavior of the system.

The open loop transfer function of the system is:

$$G(s) = \frac{\dot{\theta}_m(s)}{V_a(s)} = \frac{nK_t}{sR_a J_{eff} + R_a B_{eff} + K_t K_b}$$

Figure 5.3 Open-Loop Step Response for Integrated Dynamic Model of Link-Actuator 1

Let the system have a unity feedback. Therefore the characteristic equation for joint 1 is written as,

$$1 + G_1(s) = 0;$$

where,

$$G_1(s) = \frac{0.0096}{39s + 0.27}$$

Therefore the characteristic equation for joint 1 (with gain $K = 1$) is:

$$1 + \frac{0.0096}{39s + 0.27} = 0;$$

$$39s + 0.2796 = 0;$$

$$s = -0.00717;$$

The root-locus plot using Matlab is shown in Figure 5.4

Figure 5.4 Open-Loop Root Locus for Integrated Dynamic Model of Link-Actuator 1

From the root-locus plot [11, 14] of the open-loop transfer function (Equation (5.17)) as shown in Figure 5.4, the pole is determined as:

$$s = -0.00716$$

The graphical representation of the frequency response method is called as Bode plot [11, 14]. Bode plot of the system gives the steady state response of the system for

sinusoidal input signal. It was tedious to locate the poles and zeros using frequency response method therefore Bode plot was introduced by H.W.Bode to get graphical representation of poles and zeros [14].

Bode plot for joint 1 with gain ($K = 1$) is shown in Figure 5.5.

Figure 5.5 Open-Loop Bode Plot for Integrated Dynamic Model of Link-Actuator 1

The gain margin and phase margin [11] of the open-loop transfer function model for Joint 1 ($K = 1$) are,

$Gm = \text{inf db}$

$Pm = \text{inf degrees}$

The open-loop transfer function model for joint 2 is computed using values shown in Table 5.1 and Table 5.2 as:

$$G_2(s) = \frac{\dot{\theta}_m(s)}{V_a(s)} = \frac{0.0096}{4.2s + 0.27} \quad (5.19)$$

The step response of the transfer function model for joint 2 is shown in Figure 5.6

Figure 5.6 Open-Loop Step Response for Integrated Dynamic Model of Link-Actuator 2

The characteristic equation for joint 2 (with gain $k = 1$) is:

$$1 + G_2(s) = 0;$$

$$G_2(s) = \frac{0.0096}{4.2s + 0.27}$$

Therefore the characteristic equation for joint 2 is:

$$1 + \frac{0.0096}{4.2s + 0.27} = 0;$$

$$4.2s + 0.2796 = 0;$$

$$s = -0.066;$$

Figure 5.7 Open Loop Root Locus for Integrated Dynamic model of Link-Actuator2

From the root-locus plot of the open loop transfer function as shown in Figure 5.7, the pole is determined as:

$$s = -0.066$$

Bode plot for joint 2 is shown in Figure 5.8 with gain $K = 1$.

Figure 5.8 Open-Loop Bode Plot for Integrated Dynamic Model of Link-Actuator 2

The open-loop transfer function for joint 3 is computed using Equations from (5.1) to (5.14). As mentioned earlier that link 3 moves in vertical plane. Therefore, to derive the open-loop transfer function for joint 3 of the robot the acceleration due to gravity factor has to be considered. As seen from Figure 5.9 the torque for the link 3 at joint 3 is [12]:

$$T_3(L) = m_3 * g * Lc * \cos(\theta) + m_3 * Lc^2 * \ddot{\theta} + B_L * \dot{\theta} \quad (5.20)$$

Equation (5.20) is simplified for calculation and written as:

$$T_3(L) = m_3 * g * Lc * \theta_L + m_3 * Lc^2 * \ddot{\theta}_L + B_T * \dot{\theta}_L \quad (5.21)$$

The motor torque referred to motor shaft is given as:

$$T_m(t) = J_m \ddot{\theta}_m + B_m \dot{\theta}_m$$

Also,

$$T^*_L = n^* T_3(L);$$

Hence the net torque becomes:

$$T = T_m(t) + T^*_L$$

After simplification the open-loop transfer function for joint 3 is calculated as:

$$\frac{\theta_L(s)}{V_a(s)} = \frac{nK_t}{s^2[J_{eff}R_a + n^2m_3L_c^2R_a] + s[B_mR_a + n^2B_LR_a + K_tK_b] + n^2m_3gL_cR_a} \quad (5.22)$$

where,

$$m_3 = \text{mass of link 3} = 2.1 \text{ kg}$$

$$g = \text{acceleration due to gravity} = 9.8 \text{ m/sec}^2$$

$$n = \text{gear ratio} = 1/50$$

$$L_c = L_3/2 = 0.39/2 = 0.195 \text{ m}$$

$$J_{eff} = 0.1159 \text{ kg-m}^2$$

Substituting the values from Table 5.1 in Equation (5.22) and simplifying, the open-loop transfer function for joint 3 is computed as:

$$G_3(s) = \frac{\theta_L(s)}{V_a(s)} = \frac{0.0096}{1.12s^2 + 0.27s + 0.016} \quad (5.23)$$

Figure 5.9 Open-Loop Step Response for Integrated Dynamic Model of Link-Actuator 3

The root-locus plot for open-loop transfer function for joint 3 is:

Figure 5.10 Open Loop Root Locus for Integrated Dynamic Model of Link-Actuator 3

The roots of the open-loop transfer function from the root-locus plot are determined as:

$$s = -0.1361$$

$$s = -0.1049$$

The Bode plot of the open-loop transfer function for joint 3 is shown in Figure 5.11

Figure 5.11 Open-Loop Bode Plot for Integrated Dynamic Model of Link-Actuator 3

5.5 TRANSFER FUNCTION MODEL FOR INDEPENDENT LINK-JOINT 3

Joint 3 of the robot moves the third link of the robot in vertical plane. Therefore, link 3 can be treated as free link moving in the vertical plane under the influence of acceleration due to gravity as shown in Figure 5.12.

Figure 5.12 Link-Joint 3 Motion of Robot

The governing equation for link 3 is written as:

$$T_3 = m_3 * g * Lc * \cos(\theta) + m_3 * Lc^2 * \ddot{\theta} + B_T * \dot{\theta} \quad (5.24)$$

The control law equation for joint 3 can be written as:

$$m_3 * Lc^2 * \ddot{\theta} + (B_T + K_2) \dot{\theta} + K_1 * \theta = 0; \quad (5.25)$$

By applying Laplace Transform to Equation (5.25), it can be written as:

$$[s^2 * m_3 * Lc^2 + (B_T + K_2) s + K_1] * \theta = 0; \quad (5.26)$$

where:

B_T = Rotational Friction Coefficient;

L_3 = Length of Link 3;

$Lc = L_3 / 2$;

K_1 = Stiffness;

$B_T + K_2$ = Friction Coefficient;

To control the position of link 3, K_1 and K_2 can be adjusted.

Therefore the transfer function model for link-joint 3 is:

$$\frac{\theta_3(s)}{T(s)} = \frac{1}{s^2 * (0.3) * (0.2)^2 + (3.78 * 10^{-3} + K_2) * s + K_1}$$

Appropriate gain values were chosen to make torque proportional to angle.

Hence, the transfer function model for link-joint 3 becomes:

$$\frac{\theta_3(s)}{T(s)} = \frac{1}{0.012 * s^2 + 0.061 * s + 0.1} \quad (5.27)$$

The step response for the Equation (5.27) is shown in Figure 5.13

Figure 5.13 Step Response for Independent Link-Joint 3 Model

Figure 5.14 Root Locus Plot for Independent Link-Joint 3 Model

From the root locus plot of the transfer function model of Equation (5.25) as shown in Figure 5.14, the poles are determined as:

$$s_1 = -2.5417 + 1.3687i$$

$$s_2 = -2.5417 - 1.3687i$$

The link-joint 3 system is stable since the root locus plot shows that the roots lie in the left hand side of s-plane.

Bode plot is shown in Figure 5.15.

Figure 5.15 Bode Plot for Independent Link-Joint 3 Model

The independent link-joint 3 system position can be controlled by changing the values of K_1 and K_2 .

Chapter 6

Controller Design

6.1 METHODOLOGY

PID controllers [10, 11, 14] were designed using classical control theory to achieve reasonable gain and phase margin, over-damped system and zero steady state error.

6.2 CONTROLLER DESIGNS FOR JOINTS OF SCARA-TYPE ROBOT

The goal for this section is to design controllers and derive transfer functions for the three joints of the SCARA-Type robot and to analyze the performance of a controller designed for the joint dynamic model for the SCARA-Type robot. The controllers will be developed based on the desired operating behavior (pick-to-place task) of the joints and physical characteristics of the three links of the robot.

The controller can be classified as many types but some of the four main types of controllers [8] are: Proportional (P), Proportional plus Integral (PI), Proportional plus Derivative (PD) and Proportional plus Integral plus Derivative (PID). A PID controller is designed and analyzed for each joint of the SCARA-Type robot.

Each joint of the robot can be treated independently but the inertias of the other joints are included to design a controller for a particular link-joint.

6.3 PROPORTIONAL, DERIVATIVE AND INTEGRAL (PID) CONTROLLER FOR A SINGLE JOINT

The purpose of the three PID controllers is to servo the three actuators in the joints of the robot manipulator so that the actual angular displacement of the joint will track a desired angular displacement specified by a planned pick-to-place trajectory. The trajectory derivation and description is given in Chapter 4. The designing of a PID controller and then deriving the transfer function for the closed-loop system is based on using the error signal between the desired and actual angular positions of the joint to actuate an appropriate voltage. The applied voltage $V_a(t)$ to the motor is linearly

proportional to the error between the desired and actual angular displacements, error in angular velocities and error in integral of angular displacements of the joint as shown in Equation (6.1).

$$V_a(t) = \frac{K_p[\theta_L^d(t) - \theta_L(t)] + K_d \frac{d}{dt}[\theta_L^d(t) - \theta_L(t)] + K_i \int [\theta_L^d(t) - \theta_L(t)] dt}{n} \quad (6.1)$$

where K_p is the proportional feedback gain, K_d is the derivative feedback gain and K_i is integral feedback gain in volts per radian, volts per radian per sec and volts per radian-sec. Also $e(t) = [\theta_L^d(t) - \theta_L(t)]$, (defined as the system error), $\dot{e}(t) = [\dot{\theta}_L^d(t) - \dot{\theta}_L(t)]$ and $\int e(t) dt = \int [\theta_L^d(t) - \theta_L(t)] dt$. Speed ratio is n , which is included in Equation (6.1) to compute the applied voltage, referred to the motor shaft speed. By adding a PID controller and unity negative feedback, the open-loop link-actuator system is converted into a closed-loop control system.

Taking the Laplace Transform of Equation (6.1), yields:

$$V_a(s) = \frac{K_p E(s) + K_d s E(s) + K_i \frac{E(s)}{s}}{n} \quad (6.2)$$

Substituting $V_a(s)$ into Equation (5.16) yields an open-loop transfer function $G_o(s)$ for single link relating the error actuating signal $E(s)$ to the actual displacement of the joint:

$$G_o(s) = \frac{\theta_L(s)}{E(s)} = \frac{nK_t}{s(sR_a J_{eff} + R_a B_{eff} + K_t K_b)} * \frac{1}{n} * \frac{K_p s + K_i + K_d s^2}{s}$$

One can obtain the closed-loop transfer function $T_c(s)$ relating the actual angular displacement $\theta_L(s)$ to desired angular displacement $\theta_L^d(s)$, with some simple algebraic manipulations.

The closed-loop system with PID controller is shown in Figure 6.1 [1].

Figure 6.1 Closed-Loop System with PID Controller

From Figure 6.1, the overall transfer function for a single joint of the robot from $\theta_L^d(s)$ to $\theta_L(s)$ is shown in Figure 6.2

Figure 6.2 Transfer Function of Single Joint Closed-Loop System with PID Controller

Therefore the closed-loop transfer function is written as:

$$T_c(s) = \frac{\theta_L(s)}{\theta_L^d(s)}$$

$$= \frac{K_t (K_d s^2 + K_p s + K_i) / R_a J_{eff}}{s^3 + [(R_a B_{eff} + K_t K_b + K_t K_d) / R_a J_{eff}] s^2 + K_t K_p s / R_a J_{eff} + K_t K_i / R_a J_{eff}} \quad (6.3)$$

Equation (6.3) shows that adding the PID controller for the single link-joint results in a third-order system. Controlling the amount of proportional or derivative or integral action makes it possible to design the control system to achieve desired transient performance characteristics. In order to change the system response time, positional feedback gain K_p is changed, to reduce the steady state error of the system, the integral of

the positional error (K_i) is added and some damping is incorporated into the system by adding a derivative of positional error (K_d).

Simplifying Equation (6.3), the closed-loop transfer function $T_c(s)$ relating the actual angular displacement $\theta_L(s)$ to desired angular displacement $\theta^d_L(s)$, is given as:

$$\begin{aligned} T_c(s) &= \frac{\theta_L(s)}{\theta^d_L(s)} \\ &= \frac{K_t (K_d s^2 + K_p s + K_i)}{R_a J_{\text{eff}} s^3 + [(R_a B_{\text{eff}} + K_t K_b + K_t K_d)] s^2 + K_t K_p s + K_t K_i} \end{aligned} \quad (6.4)$$

6.4 CONTROLLERS FOR THE THREE LINK-JOINTS OF THE SCARA-TYPE ROBOT

A PID controller is designed for each link-joint of the workcell robot. The performance of the closed-loop control system is based on several criteria's like fast rise time, small or zero steady state error, response damping and fast settling time [14]. The performance of a closed-loop control system is based on its natural un-damped frequency ω_n and damping ratio ξ . For the system to have a good performance, it should have a critically damped or an over-damped behavior, which requires the system damping ratio to be greater than or equal to unity.

Using the transfer function from Equation (6.4), the error for the system is given by:

$$\begin{aligned} E(s) &= \theta^d_L(s) - \theta_L(s) \\ &= \frac{R_a J_{\text{eff}} s^3 + (R_a B_{\text{eff}} + K_t K_b) s^2}{R_a J_{\text{eff}} s^3 + [(R_a B_{\text{eff}} + K_t K_b + K_t K_d)] s^2 + K_t K_p s + K_t K_i} \theta^d_L(s) \\ &= F(s) \theta^d_L(s) \end{aligned} \quad (6.5)$$

For the system it is expected that the steady state error becomes zero for a step input in $\theta_L(t)$. Therefore, using Final Value Theorem:

$$\lim_{s \rightarrow 0} s E(s) = \frac{0}{K_t K_i} \quad (6.6)$$

From Equation (6.6) it can be shown that the steady state error can be made equal to zero as long as K_i and K_t are not zeroes. Also, if $K_i = 0$, the steady state error goes to infinity. So any gain can be chosen except $K_i = 0$, as long as steady state error is concerned.

6.4.1 Controller for Link-Joint 1

The main design principle on which the controller is designed is that the system should be critically damped or over-damped. The general form $G_c(s)$ for a PID controller is:

$$G_c(s) = K_d s + K_p + \frac{K_i}{s} \quad (6.7)$$

Design of the controller is based on choosing the appropriate values of the position feedback gain K_p , velocity feedback gain K_d and integral gain K_i . The proportionality, derivative and integral gain values for link-joint 1 for stable system behavior are:

$$K_p = 1500 \text{ volts/rad}$$

$$K_d = 1330 \text{ volts/rad/sec}$$

$$K_i = 0.4 \text{ volts/rad-sec}$$

The proportionality, derivative and integral gain values for joint 1 were chosen after many simulations. The simulation was done using Matlab. A code is written in Matlab where the variables are the proportionality, derivative and integral gains. The study was done for the step response behavior of the system by changing the variables and finally a stable system was achieved for a particular combination of the proportionality, derivative and integral gains. The Matlab code for this simulation is shown in Appendix G.

Hence, the PID controller $G_{c1}(s)$ for joint 1 is:

$$G_{CI}(s) = \frac{1330s^2 + 1500s + 0.4}{s} \quad (6.8)$$

The overall closed-loop transfer function $T_{CI}(s)$ for joint 1 relating the actual angular displacement $\theta_L(s)$ to desired angular displacement $\theta^d_L(s)$ with the work cell robot operating in maximum workspace volume is:

$$\frac{\theta_L(s)}{\theta^d_L(s)} = \frac{0.48(K_d s^2 + K_p s + K_i)}{39s^3 + (0.27 + 0.48K_d)s^2 + 0.48K_p s + 0.48K_i} \quad (6.9)$$

$$\text{or, } T_{CI}(s) = \frac{\theta_L(s)}{\theta^d_L(s)} = \frac{638.4s^2 + 720s + 0.192}{39s^3 + 638.7s^2 + 720s + 0.192} \quad (6.10)$$

The unit step response of the closed-loop transfer function of the link-joint 1, which is determined using Matlab, is shown in Figure 6.3. It can be seen from Figure 6.3 that the link-actuator system rise time is 0.114 sec, and its settling time is 1.21 seconds and the system is stable.

Figure 6.3 Step Response for Joint 1

The stability for the system is determined using both Bode plot and root locus plot for the system. The stability margins (gain and phase margin) for the joint 1 is determined using Bode plot for the system as shown in Figure 6.4. The stability margins, i.e. gain margin and phase margin for the joint 1 are determined as:

Phase Margin = 159 degrees

Gain Margin = infinity

The stability criteria for a system from the Bode plot is that the phase margin should always be greater than -180 degrees and gain margin should always be positive. From the stability margin values determined from the Bode plot, the system for joint 1 is stable for given controller [11].

The root locus plot of the system is shown in Figure 6.5. According to the control definitions, for the system to be stable, the poles and zeros should lie on the left-hand plane. It can be seen from the plot that the poles and zeros of the system lie on the left hand side of the s-plane. Hence, the system is stable for the given controller.

Figure 6.4 Bode Diagram for Stability Margin of Joint 1

Figure 6.5 Root Locus Plot for Joint 1

The poles of the link-actuator system for joint 1 are determined using Matlab as:

$$s_1 = -15.16;$$

$$s_2 = -1.22;$$

$$s_3 = -0.000313;$$

The overall closed-loop transfer function $T_{cl}(s)$ for joint 1 relating the actual angular displacement $\theta_L(s)$ to desired angular displacement $\theta^d_L(s)$ with the work cell robot operating in minimum workspace volume with same controller is:

$$\frac{\theta_L(s)}{\theta^d_L(s)} = \frac{0.48(K_d s^2 + K_p s + K_i)}{17s^3 + (0.27 + 0.48K_d)s^2 + 0.48K_p s + 0.48K_i} \quad (6.11)$$

$$\text{or, } T_{cl}(s) = \frac{\theta_L(s)}{\theta^d_L(s)} = \frac{638.4s^2 + 720s + 0.192}{17s^3 + 638.7s^2 + 720s + 0.192} \quad (6.12)$$

Figure 6.6 Step Response for Joint 1 with Robot Operating in Minimum Workspace Volume

The unit step response of the closed-loop transfer function of the link-joint 1 with the robot operating in minimum workspace volume is shown in Figure 6.6. The link-actuator system rise time is 0.02 sec, and its settling time is 0.03 seconds and the system is stable. Hence it can be concluded that same PID controller can be used for both cases.

6.4.2 Controller for Link-Joint 2

A PID controller is designed for joint 2 based on the same principles as discussed in designing controller for joint 1. The proportional feedback gain K_p , derivative feedback gain K_d and integral feedback gain K_i for the joint 2 are determined as:

$$K_p = 140 \text{ volts/rad}$$

$$K_d = 160 \text{ volts/rad/sec}$$

$$K_i = 0.4 \text{ volts/rad-sec}$$

Hence, the PID controller $G_{c2}(s)$ for link-joint 2 is:

$$G_{c2}(s) = \frac{160s^2 + 140s + 0.4}{s} \quad (6.13)$$

The overall closed-loop transfer function $T_{c2}(s)$ of joint 2 relating the actual angular displacement $\theta_L(s)$ to desired angular displacement $\theta^d_L(s)$ is:

$$\frac{\theta_L(s)}{\theta^d_L(s)} = \frac{0.48(K_d s^2 + K_p s + K_i)}{4.2s^3 + (0.27 + 0.48K_d)s^2 + 0.48K_p s + 0.48K_i} \quad (6.14)$$

$$\text{or, } T_{c2}(s) = \frac{\theta_L(s)}{\theta^d_L(s)} = \frac{76.8s^2 + 67.2s + 0.192}{4.2s^3 + 77.07s^2 + 67.2s + 0.192} \quad (6.15)$$

The unit step response of the closed-loop transfer function of the system is shown in Figure 6.7. It is evident from Figure 6.7 that the link-actuator system rise time is 0.107 sec, its settling time is 1.04 seconds and the system is stable.

The stability margins for the system are determined using Bode plot and root-locus plot for the system. The Bode plot for the system is shown in Figure 6.8. The stability margins, i.e. gain margin and phase margin for the joint 1 is determined as:

Phase Margin = 163.1 degrees

Gain Margin = infinity

The stability criteria for a system from the Bode plot is that the phase margin should be always be greater than -180 degrees and gain margin should always be positive. From the stability margin values determined from the Bode plot, the system for joint 2 is stable for the given controller.

Figure 6.7 Step Response for Joint 2

The root locus plot of the system is shown in Figure 6.9. It can be seen from the plot that the poles and zeros of the system lie in the left hand side of the s-plane. Hence, the system for joint 2 is stable for the given controller.

The poles of the link-actuator system for joint 2 are determined using Matlab as:

$$s_1 = -17.43;$$

$$s_2 = -0.91;$$

$$s_3 = -0.0029;$$

Figure 6.8 Bode Diagram for Stability Margin of Joint 2

Figure 6.9 Root Locus Plot for Joint 2

6.4.3 Controller for joint 3

A PID controller is designed for joint 3 based on the same principles as discussed in designing controller for joint 1 and joint 2. The proportional feedback gain K_p , derivative feedback gain K_d and integral feedback gain K_i for the joint 3 are determined as:

$$K_p = 100 \text{ volts/rad}$$

$$K_d = 90 \text{ volts/rad/sec}$$

$$K_i = 10 \text{ volts/rad-sec}$$

Hence, the PID controller $G_{C3}(s)$ for joint 3 is:

$$G_{C3}(s) = \frac{90s^2 + 100s + 10}{s} \quad (6.16)$$

The overall closed-loop transfer function $T_{C3}(s)$ of joint 3 relating the actual angular displacement $\theta_L(s)$ to desired angular displacement $\theta^d_L(s)$ is:

$$\frac{\theta_L(s)}{\theta^d_L(s)} = \frac{K_t(K_d s^2 + K_p s + K_i)}{s^3[J_m R_a + n^2 m_3 L_c^2 R_a] + s^2[B_m R_a + n^2 B_l R_a + K_t K_b + K_d K_t] + s[K_t K_p] + n^2 m_3 g L_c R_a + K_t K_i} \quad (6.17)$$

$$\text{or, } T_{C3}(s) = \frac{\theta_L(s)}{\theta^d_L(s)} = \frac{43.2s^2 + 48s + 4.8}{1.12s^3 + 43.47s^2 + 48s + 4.816} \quad (6.16)$$

The unit step response of the closed-loop transfer function of the system is shown in Figure 6.10. It can be seen from Figure 6.10 that the link-actuator system rise time is 0.053 seconds, its settling time is 0.38 seconds and the system is stable.

Figure 6.10 Step Response for Joint 3

The stability margins for the system are determined using Bode plot and root-locus plot for the system. The Bode plot for the system is shown in Figure 6.11. The stability margins, i.e. gain margin and phase margin for the joint 3 is determined as:

Phase Margin = 167.8 degrees

Gain Margin = infinity

The stability criteria for a system from the Bode plot is that the phase margin should be always greater than -180 degrees and gain margin should always be positive [11]. From the stability margin values determined from the Bode plot, the system for joint 3 is stable for the given controller.

The root locus plot of the system is shown in Figure 6.12. It is known from the plot that the poles and zeros of the system lie on the left hand side of the s-plane. According to the control definitions, for the system to be stable, the poles and zeros should lie on the left hand plane. Hence, the system for joint 3 is stable for the designed controller.

Figure 6.11 Bode Diagram for Stability Margin of Joint 3

Figure 6.12 Root Locus Plot for Joint 3

The poles of the system for joint 3 are determined using Matlab as:

$$s_1 = -37.67;$$

$$s_2 = -1.02;$$

$$s_3 = -0.11;$$

6.5 SUMMARY

The controlled joint motion of the three joints allows the robot manipulator to perform the pick-to-place task in minimum time depending on the values of the gains for the PID controllers. The time can be changed depending on the task to be performed by the robot. The pick-to-place task was accomplished in 2 sec as described in Chapter 4. This time can be changed depending on the task. PID controllers for three joints were designed to achieve desired angle in minimum time.

Chapter 7

Experimental Verification Study of SCARA-Type Robot

7.1 EXPERIMENTAL RESULTS

Experiments were carried on the LGCM workcell robot to verify the theoretical kinematic model. The primary goals of this experimental verification study were to determine the position of the end-effector given the joint angles of the three joints.

The SCARA-Type robot, placed at the Davis-Lynch glass plant in Star City, WV, was studied with various joint angles for a specified task and the results were compared with the values obtained from the theoretical model.

A pick-to-place routine was run to verify the theoretical model for the SCARA-Type robot. The robot was run with different set of pick and place positions as compared to pick and place positions defined in Chapter 4. This was done to cross verify the model. The motion of the robot was programmed such that the robot picked up the glassware at the pick position and placed the glassware at the place position in front of the laser.

Experiments were carried out using the robot controller. The robot is first homed so that it determines the exact position within the workspace. The robot arm was moved using the built-in program with the robot controller. The pick position and place position were defined in the workspace of the robot arm as shown in Figure 7.1. The controller program was run and the Cartesian coordinates were noted from the computer monitor for the corresponding joint angles of the robot arm. A timer was used to record the time it took for the robot to move from the pick position to the place position and it was noted that the robot took 3.2 seconds to complete the task.

The pick position as shown in Figure 7.1, defined for the SCARA-Type robot inside the workspace is:

$$\theta_1 = 30.3 \text{ degrees}$$

$$\theta_2 = 44.88 \text{ degrees}$$

$$\theta_3 = -25.46 \text{ degrees}$$

The corresponding Cartesian coordinates for the pick position are:

$$p_x = 1.8734 \text{ ft}; p_y = 2.5891 \text{ ft}; p_z = -0.53 \text{ ft}$$

The joint angles for the place position, as shown in Figure 7.1, for the robot arm inside the workspace, are:

$$\theta_1 = -5.03 \text{ degrees}$$

$$\theta_2 = 0 \text{ degrees}$$

$$\theta_3 = -31.75 \text{ degrees}$$

The corresponding Cartesian coordinates for the place position are:

$$p_x = 3.3549 \text{ ft}; p_y = -0.2898 \text{ ft}; p_z = -0.6386 \text{ ft}$$

Figure 7.1 Defined Pick-to- Place Positions for Experimental Verification Study

The above joint and Cartesian values, which were obtained from experimentally positioning the robot, were compared with the values obtained from the theoretical kinematic model. The joint angles obtained experimentally were substituted in the theoretical inverse kinematic model of the robot and the corresponding end-effector position was calculated.

The Cartesian coordinates calculated with the theoretical kinematic model for the pickup position are:

$$p_x = 1.9452 \text{ ft}; p_y = 2.6623; p_z = -0.5459;$$

And the place position coordinates are calculated as:

$$p_x = 3.4865 \text{ ft}; p_y = -0.3069 \text{ ft}; p_z = -0.6683 \text{ ft};$$

The time was noted for the robot to complete the task from pick position to place position. It was noted that the time taken for the robot to complete the task was 3.2 seconds. From the theoretical controller, which was designed in Chapter 6, it was determined that it takes 1.21 seconds for the joint 1 to complete its path, and 1.04 second for joint 2 and 1.01 for joint 3 to complete their paths, which is equivalent to 3.26 seconds for complete task. The gains of the PID controllers can be changed to match the real time with theoretical time to complete the task.

There was a 3-4% error in calculating the exact Cartesian coordinate positions. This is due to the error in measuring the exact length of the three links of the robot.

Three PID controllers were used for the SCARA-Type robot to control the path. There were three PID controllers for the three actuators in the joints of the robot. Accordingly, three theoretical PID controllers were designed for each joint using a classical approach. These theoretical controllers were designed considering the kinematics and dynamics of the robot. The designed controller can behave like an actual controller by adjusting the values of the gain of the PID controller. The results of the PID controllers for three joints were discussed in Chapter 6.

A 4-3-4 approach was used to plan a trajectory for pick-to-place task for the robot. The actual trajectory for the robot at Davis-Lynch Glass Plant to perform the pick-to-place task was determined as straight line trajectory. When compared to the actual trajectory, the trajectory derived in Chapter 4 was not a Straight line trajectory. While planning the trajectory, it was taken care that the path for pick-to-place task follows the straight line as close as possible. Figure 7.2 shows the theoretical trajectory in Cartesian coordinate system.

Figure 7.2 Pick-to-Place Trajectory in Cartesian Coordinate System

7.2 KINEMATICS VALIDATION STUDY

The goal of this section is to determine whether the robot should reach the desired position in the workspace for an input of a desired position to the robot in Cartesian coordinates. In this study the joint angles and the Cartesian coordinates of the end-effector of the robot in the workspace are already defined. Validation study of kinematics of the SCARA-Type robot was performed for different positions of the end-effector in the workspace. Now the desired position in Cartesian coordinates system in the workcell is input to the robot and the joint angles obtained were compared to the known values of the joint angles. Figure 7.3 shows the workcell of the robot with laser gun and turn table. The pick position of the robot is defined on the turntable where the glassware is available to be picked up and the place position of the robot is defined in front of the laser gun where the glassware is rotated by the robot and the cut made by the laser gun. Cases 1 to 8 in this chapter indicates the results of the validation study which

shows that using the end-effector position as input to the inverse kinematics of the robot, the corresponding joint angles are determined for various positions inside the workspace. Figure 7.4 and 7.5 shows all the defined positions in the workspace. The soft home position shown in Figure 7.4 is defined as the position where the robot starts before reaching the pick position. A hard home position of the robot is defined as a position inside the workspace, which is set by the user to which the robot reverts when the power supply is removed.

Figure 7.3 Top View of the WorkCell

($L1 = 1.7$ ft, $L2 = 0.72$ ft, $L3 = 1.27$ ft, $L4 = 0$, $L5 = 0$, $H = 5$ ft)

□ **Work Space Limits: X-Y Plane**

- Max: $L1 + L2 + L3 = 3.69$ ft
- Min: $L1 - L2 - L3 = -0.29$ ft $\Rightarrow 0$ ft (Not physically possible)

- Work Space Limits: X, Y - Z Plane
 - Max: $H + L3 = 6.27$ ft
 - Min: $H - L3 = 3.73$ ft

Figure 7.4 Path of Robot End-Effector (In $P_y - P_x$)

Figure 7.5 Path of Robot End-Effector (In Pz – R)

Case 1: Pick Position (1)

Given Kinematics:

Theta1 = 55 deg, Theta2 = 28 deg, Theta3 = 25 deg, Theta4 = 65, Theta5 = 0;

Computed Inverse Kinematics:

$$T_{0E} = \begin{bmatrix} -0.9925 & 0.1219 & 0 & 1.2031 \\ 0.1219 & 0.9925 & 0 & 3.2496 \\ 0 & 0 & -1 & 4.4633 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

we know that:

$$\theta_1 = \text{Atan2} \left[\frac{-L3*ay*(H-pz)-L3*oy*(L3^2-(H-pz)^2)^{0.5}+py*L3-L2*L3*oy}{-L3*ax*(H-pz)-L3*ox*(L3^2-(H-pz)^2)^{0.5}+px*L3-L2*L3*ox} \right] \quad (7.1)$$

$$\theta_2 = \text{Atan2} \left[\frac{-S1*px+C1*py}{C1*px+S1*py-L1} \right] \quad (7.2)$$

$$\theta_3 = \text{Atan2} \left(\frac{H - Pz}{(L3^2 - (H - pz)^2)^{0.5}} \right) \quad (7.3)$$

$$\theta_4 = 90 - \theta_3;$$

$$\theta_4 = 90 - \text{Atan2} \left(\frac{H - Pz}{(L3^2 - (H - pz)^2)^{0.5}} \right) \quad (7.4)$$

$$\theta_5 = 0;$$

Substituting the computed inverse kinematics values of the end-effector in the inverse kinematic equations to compute the joint angles of the robot. The results are computed using Excel.

Px	Py	Pz	Theta1	Theta3	Theta2	Theta4	Theta5
1.2031	3.2496	4.4633	55.00	25.00	28.00	65.00	0
nx	ny	nz					
-0.9925	0.1219	0					
ox	oy	oz		H			
0.1219	0.9925	0		5.00			
ax	ay	az					
0	0	-1					
L1	L2	L3					
1.7	0.72	1.27					

Case 2: Intermediate Position (2)

Theta1 = 42 deg, Theta2 = 22 deg, Theta3 = 18 deg, Theta4 = 72, Theta5 = 0;

Computed Inverse Kinematics:

$$T_{0E} = \begin{bmatrix} -0.8988 & 0.4384 & 0 & 2.1085 \\ 0.4384 & 0.8988 & 0 & 2.8703 \\ 0 & 0 & -1 & 4.6075 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Px	Py	Pz	Theta1	Theta3	Theta2	Theta4	Theta5
2.1085	2.8703	4.6075	42.00	18.00	22.00	72.00	0
nx	ny	nz					
-0.8988	0.4384	0					
ox	oy	oz		H			
0.4384	0.8988	0		5.00			
ax	ay	az					
0	0	-1					
L1	L2	L3					
1.7	0.72	1.27					

Case 3: Intermediate Position (3)

Theta1 = 23 deg, Theta2 = 15 deg, Theta3 = 13 deg, Theta4 = 77, Theta5 = 0;

Computed Inverse Kinematics:

$$T0E = \begin{bmatrix} -0.6157 & 0.7880 & 0 & 3.1073 \\ 0.7880 & 0.6157 & 0 & 1.8694 \\ 0 & 0 & -1 & 4.7143 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Px	Py	Pz	Theta1	Theta3	Theta2	Theta4	Theta5
3.1073	1.8694	4.7143	23.00	13.00	15.00	77.00	0
nx	ny	nz					
-0.6157	0.788	0					
ox	oy	oz		H			
0.788	0.6157	0		5.00			
ax	ay	az					
0	0	-1					
L1	L2	L3					
1.7	0.72	1.27					

Case 4: Place Position (4)

Theta1 = -5 deg, Theta2 = 9.5 deg, Theta3 = 10 deg, Theta4 = 80, Theta5 = 0;

Computed Inverse Kinematics:

$$T_{0E} = \begin{bmatrix} -0.0785 & 0.9969 & 0 & 3.6582 \\ 0.9969 & 0.0785 & 0 & 0.0065 \\ 0 & 0 & -1 & 4.7795 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Px	Py	Pz	Theta1	Theta3	Theta2	Theta4	Theta5
3.6582	0.0065	4.7795	-5.00	10.00	9.50	80.00	0
nx	ny	nz					
-0.0785	0.9969	0					
ox	oy	oz		H			
0.9969	0.0785	0		5.00			
ax	ay	az					
0	0	-1					
L1	L2	L3					
1.7	0.72	1.27					

Case 5: Soft Home Position (5)

Theta1 = 62 deg, Theta2 = 35 deg, Theta3 = 0 deg, Theta4 = 90, Theta5 = 0;

Computed Inverse Kinematics:

$$T_{0E} = \begin{bmatrix} -0.9925 & -0.1219 & 0 & 0.5556 \\ -0.1219 & 0.9925 & 0 & 3.4762 \\ 0 & 0 & -1 & 5.0000 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Px	Py	Pz	Theta1	Theta3	Theta2	Theta4	Theta5
0.5556	3.4762	5	62.00	0.00	35.00	90.00	0.00
nx	ny	nz					
-0.9925	-0.1219	0					

ox	oy	oz		H			
-0.1219	0.9925	0		5.00			
ax	ay	az					
0	0	-1					
L1	L2	L3					
1.7	0.72	1.27					

Case 6: Hard Home Position (6)

Theta1 = 50 deg, Theta2 = 72 deg, Theta3 = 25 deg, Theta4 = 65, Theta5 = 0;

Computed Inverse Kinematics:

$$T_{0E} = \begin{bmatrix} -0.8480 & 0.5299 & 0 & 0.1013 \\ -0.5299 & 0.8480 & 0 & 2.8890 \\ 0 & 0 & -1 & 4.4633 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Px	Py	Pz	Theta1	Theta3	Theta2	Theta4	Theta5
0.1013	2.889	4.4633	50.00	25.00	72.00	65.00	0.00
nx	ny	nz					
-0.848	-0.5299	0					
ox	oy	oz		H			
-0.5299	0.848	0		5.00			
ax	ay	az					
0	0	-1					
L1	L2	L3					
1.7	0.72	1.27					

Case 7: Arbitrary Position (7)

Theta1 = -11 deg, Theta2 = 68 deg, Theta3 = 25 deg, Theta4 = 65, Theta5 = 0;

Computed Inverse Kinematics:

$$T_{0E} = \begin{bmatrix} -0.8387 & 0.5446 & 0 & 2.6878 \\ 0.5446 & 0.8387 & 0 & 1.2448 \\ 0 & 0 & -1 & 4.4633 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Px	Py	Pz	Theta1	Theta3	Theta2	Theta4	Theta5
2.6878	1.2448	4.4633	-11.00	25.00	68.00	65.00	0
nx	ny	nz					
-0.8387	0.5446	0					
ox	oy	oz		H			
0.5446	0.8387	0		5.00			
ax	ay	az					
0	0	-1					
L1	L2	L3					
1.7	0.72	1.27					

Case 8: Arbitrary Position (8)

Theta1 = 20 deg, Theta2 = 6 deg, Theta3 = 11 deg, Theta4 = 79, Theta5 = 0;

Computed Inverse Kinematics:

$$T_{0E} = \begin{bmatrix} -0.4384 & 0.8988 & 0 & 3.3651 \\ 0.8988 & 0.4384 & 0 & 1.4436 \\ 0 & 0 & -1 & 4.7577 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Px	Py	Pz	Theta1	Theta3	Theta2	Theta4	Theta5
3.3651	1.4436	4.7577	20.00	11.00	6.00	79.00	0
nx	ny	nz					
-0.4384	0.8988	0					
ox	oy	oz		H			
0.8988	0.4384	0		5.00			
ax	ay	az					
0	0	-1					

L1	L2	L3					
1.7	0.72	1.27					

7.3 KINEMATIC STUDY OF SCARA-TYPE ROBOT

The SCARA-Type robot was studied by changing the length of link 2 to improve the kinematics of the robot. This study was done by varying the length of link 2 and calculating the maximum acceleration achieved by the end-effector.

Figure 7.6 Change in acceleration of end-effector with the change in length of Link 2

Figure 7.6 illustrates that the maximum acceleration occurs when the length of link 2 is changed to 0.4 feet. The original length of the link 2 is 0.72 feet. A Study was carried on to find whether the change in length would save money to manufacture the robot with same kinematics. It can be seen from Figure 7.6 that if link 2 is eliminated, the performance will remain the same i.e. the maximum acceleration achieved by the end-effector remains the same.

7.4 DYNAMIC STUDY OF SCARA-TYPE ROBOT

Dynamics study of the SCARA-Type robot was conducted to see the effects on torque characteristic of the actuator in joint 2 by selecting different materials for link 2. The study was done by selecting different materials like steel, aluminum alloy and titanium alloy for link 2. The original system of SCARA-Type robot was constructed using aluminum. The study shows that steel requires maximum torque at the actuator in joint 2 and aluminum alloy requires less torque at actuator in joint 2 as illustrated in Figure 7.7.

Figure 7.7 Change in torque requirements at actuator 2 with the change in material of Link 2

7.5 WORKSPACE STUDY OF SCARA-TYPE ROBOT

The other factor, which was studied, is the workspace. The changes in workspace are studied by changing the length of link 2 as shown in Figure 7.8. Studies were first done by keeping the original lengths of the link 2 and then the length of link 2 was changed to 0.3 feet. Finally, link 2 was eliminated completely and the effects on

workspace were seen. Figure 7.8 will show how the workspace varies by changing the length of link 2. The side view and top view of the robot are shown in Figure 7.9 and Figure 7.10 respectively.

Figure 7.8 Work Envelop of Robot for Varying Length of Link 2.

Figure 7.9 Side View of the Work Envelop

Figure 7.10 Top View of the Work Envelop

$$\theta_1 = 0 \text{ degrees}$$

$$\theta_2 = \text{N/A}$$

$\theta_3 = 0$ degrees (Extreme position for θ_3)

$\theta_3 = 180$ degrees (Extreme position for θ_3)

$\theta_3 = 90$ degrees (Desired position for θ_3)

The above studies indicate that any change in the length of the links will change the workspace of the robot. Kinematic study of the robot shows that a length of 0.3 feet can be chosen to have the same kinematics. The change in the length of link 2 from 0.7 feet to 0.3 feet will reduce the workspace. It can be seen from Figure 7.8 that the length of 0.3 feet will save money, but with a little lose of workspace. Hence a balance has to be drawn between choosing the length of link 2 and the workspace of the robot. By eliminating link 2 completely, money can be saved but there is a considerable lose of workspace. It can be concluded that a thin margin can be drawn between length of link 2 and the workspace and it is seen that a length of 0.3 feet will bring the benefits.

7.6 SUMMARY

The purpose of this study was to verify the theoretical kinematic model of the SCARA-Type robot. It was verified that the theoretical kinematic model of the SCARA-Type robot behaves in the same way as the actual robotic system in the glass plant at Davis-Lynch Glass with a 3-4% error. It was also studied experimentally at Davis-Lynch Glass Plant that a better theoretical controller was designed to perform the pick-to-place task. A theoretical study was done on the SCARA-Type robot by changing the length and material of the link 2.

Chapter 8

Conclusions and Contributions

8.1 CONTRIBUTIONS

The main goal of this thesis was to develop a model of the workcell robot and to design joint control schemes for position control of the workcell robot model using a classical approach that would improve the performance of the Laser Glass Cutting Machine workcell. It was discussed in Chapter 7 that 0.3 ft for link 2 will make the work cell robot to behave the same way it does with length of 0.72 ft. The only disadvantage for length of link 2 being 0.3 ft is the constraint to manufacture such a small link and installing with other links. In which case the actuator should be very small or it has to be driven by an external actuator.

Listed below are the key contributions for this thesis towards achieving the above-mentioned goal.

1. A kinematic model of the manipulator arm was developed. This arm is a modified SCARA robot arm. The link coordinate systems for the robot were defined in order to develop kinematics of the robot arm. Based on the direct kinematics of the robot arm, an inverse kinematic description was developed.
2. A dynamic model of the manipulator was developed. To further assess the behavior of the manipulator arm, the dynamic equations for the robot were derived which yields the torque equations for the three joints. These dynamic equations, which were based on direct kinematics equations, were used to develop the dynamic model of the manipulator arm. Transfer functions for each of the robot joints were developed.
3. A model for the actuator motor was developed. An electro-mechanical model of DC motor was developed in Section 4.5. Key control parameters were identified to develop theoretical model of DC motor. A transfer function for the actuator motor was developed.
4. A PID controller was developed for each joint of the three closed-loop models for the manipulator arm using a classical approach. The performance of the position

- control of the manipulator was analyzed using Bode plots and root locus plots. The PID controller was designed to achieve specified gain and phase margins, over-damped system and zero steady state error.
5. An experimental verification study of the integrated joint model of the manipulator arm and joint actuator was performed. It was determined that the controlled dynamic model behaves about the same way as the actual system.
 6. Kinematic study of the SCARA-Type robot was done by changing the length of link 2. It was determined that the performance (maximum acceleration achieved by end-effector) of the robot remains the same even after eliminating link 2 completely.
 7. Dynamic study of the SCARA-Type robot was done by changing the element type of the robot. The robot structure was constructed with aluminum. Different materials were selected and a study was done on the torque required by joint 2. It was determined that steel requires maximum torque at joint 2 and an aluminum alloy requires the least torque at joint 2.

8.2 FUTURE RESEARCH AND RECOMMENDATIONS

The main objective of this thesis was to develop a controller for the SCARA-Type robot using a classical approach that improved the operational performance of the LGCM workcell. Further research could be done to design a controller using state-space approach.

For the dynamics model, the cumbersome Lagrange-Euler method involving 4x4 homogeneous matrices has been used. This particular method of calculating the torques is not efficient as far a control of the robot arm motion is concerned. The recursive Newton-Euler method could be used for determining the joint torques. Also, the inertias were approximated to determine this torques, since the robot was not dismantled. For better values of torques, the exact values of the inertias have to be determined.

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Appendix A

Inverse Kinematic Model Verification

For the pick position of the specified task, $\theta_1 = 60$ deg, $\theta_2 = 60$ deg, $\theta_3 = -60$ deg,
 $\theta_5 = 0$ deg.

$$T_{\text{goal}} = {}^0T_E \quad (\text{A.1})$$

$$\begin{bmatrix} nx & ox & ax & px \\ ny & oy & ay & py \\ nz & oz & az & pz \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -0.8660 & -0.5000 & 0.0000 & 0.1000 \\ -0.5000 & -0.8660 & 0.0000 & 2.6000 \\ 0.0000 & 0.0000 & -1.000 & 6.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix} \quad (\text{A.2})$$

where Equation (2.4) has been used to compute 0T_E

Equation the two matrices of Equation (A.2) yields,

$$\theta_1 = \text{Atan2} \left[\frac{-L_3 \cdot ay \cdot (H - pz) - L_3 \cdot oy \cdot (L_3^2 - (H - pz)^2)^{0.5} + py \cdot L_3 - L_2 \cdot L_3 \cdot oy}{-L_3 \cdot ax \cdot (H - pz) - L_3 \cdot ox \cdot (L_3^2 - (H - pz)^2)^{0.5} + px \cdot L_3 - L_2 \cdot L_3 \cdot ox} \right] \quad (\text{A.3})$$

$$\theta_2 = \text{Atan2} \left[\frac{-S_1 \cdot px + C_1 \cdot py}{C_1 \cdot px + S_1 \cdot py - L_1} \right] \quad (\text{A.4})$$

$$\theta_3 = \text{Atan2} \left[\frac{H - pz}{(L_3^2 - (H - pz)^2)^{0.5}} \right] \quad (\text{A.5})$$

$$\theta_4 = 90 - \theta_3;$$

$$\theta_4 = 90 - \text{Atan2} \left(\frac{H - Pz}{(L_3^2 - (H - pz)^2)^{0.5}} \right); \quad (\text{A.6})$$

$$\theta_5 = 0; \quad (\text{A.7})$$

$px = 0.1000$ ft, $py = 2.6000$ ft, $pz = 6.0000$ ft, $nx = -0.8660$, $ny = -0.5000$, $nz = 0$,
 $ox = -0.5000$, $oy = -0.8660$, $oz = 0.0000$, $oy = 0.7500$, $ax = 0.0000$, $ay = 0.0000$,
 $az = -1.0000$;

Substituting the above values in Equation (A.3) to verify for θ_1 :

$$\theta_1 = \text{Atan2} \left[\frac{-L3*ay*(H-pz)-L3*oy*(L3^2-(H-pz)^2)^{0.5}+py*L3-L2*L3*oy}{-L3*ax*(H-pz)-L3*ox*(L3^2-(H-pz)^2)^{0.5}+px*L3-L2*L3*ox} \right] \quad (\text{A.3})$$

$$\theta_1 = \text{Atan2} \left[\frac{-1.27*0*(5.0-6.0)-1.27*(-0.8660)(1.27^2-(5.0-6.0)^2)^{0.5}+2.6*1.27-0.72*1.27*(-0.8660)}{-1.27*0*(5.0-6.0)-1.27*(-0.5000)(1.27^2-(5.0-6.0)^2)^{0.5}+0.1*1.27-0.72*1.27*(-0.5000)} \right]$$

Hence;

$$\theta_1 = 60.00 \text{ degrees}$$

Substituting the above values in Equation (A.5) to verify for θ_3 :

$$\theta_3 = \text{Atan2} \left[\frac{H - pz}{(L3^2 - (H - pz)^2)^{0.5}} \right] \quad (\text{A.5})$$

$$\theta_3 = \text{Atan2} \left[\frac{5.0 - 6.0}{(1.27^2 - (5.0 - 6.0)^2)^{0.5}} \right]$$

$$\theta_3 = \text{Atan2} [-1.732]$$

Hence;

$$\theta_3 = -59.999 \text{ degrees}$$

Substituting the above values in Equation (A.4) to verify for θ_2 :

$$\theta_2 = \text{Atan2} \left[\frac{-S1*px + C1*py}{C1*px + S1*py - L1} \right] \quad (\text{A.4})$$

$$\theta_2 = \text{Atan2} \left[\frac{-\sin(60)*0.1000 + \cos(60)*2.6000}{\cos(60)*0.1000 + \sin(60)*2.6000 - 1.7} \right]$$

$$\theta_2 = \text{Atan2} [1.7384]$$

Hence;

$$\theta_2 = 60.09 \text{ degrees}$$

Therefore the inverse Kinematic Model has been verified for one location.

Appendix B

Calculation of Inertia, Coriolis and Centrifugal and Gravitational terms

B.1 Inertial terms

$$D11 = 0.1065 * \cos(\theta_2) + 0.0526 * \cos(\theta_3) + 0.3433 + 0.0636 * \cos(\theta_3)^2 + 0.1241 * \cos(\theta_2) * \cos(\theta_3)$$

$$D12 = D21 = 0.0533 * \cos(\theta_2) + 0.0526 * \cos(\theta_3) + 0.0636 * \cos(\theta_3)^2 + 0.0621 * \cos(\theta_2) * \cos(\theta_3) + 0.0315$$

$$D13 = D31 = -0.0621 * \sin(\theta_2) * \sin(\theta_3)$$

$$D22 = 0.0526 * \cos(\theta_3) + 0.0636 * \cos(\theta_3)^2 + 0.0315$$

$$D23 = D32 = 0$$

$$D33 = 0.0639$$

B.2 Coriolis and Centrifugal terms

$$\begin{aligned} h1 = & -0.0621 * \theta_2^2 * \cos(\theta_3) * \sin(\theta_2) - 0.0621 * \theta_3^2 * \sin(\theta_3) * \sin(\theta_2) - 0.1065 * \theta_1 D * \theta_2 D \\ & * \sin(\theta_2) - 0.0698 * \theta_1 D^2 * \sin(\theta_1) - 0.1241 * \theta_1 D^2 * \cos(\theta_3) * \cos(\theta_2) * \sin(\theta_1) - 0.0526 * \theta_1 D^2 * \\ & * \cos(\theta_3) * \sin(\theta_1) + 0.0621 * \theta_1 D^2 * \cos(\theta_3) * \sin(\theta_2) - 0.0636 * \theta_1 D^2 * \cos(\theta_3)^2 * \sin(\theta_1) - \\ & 0.0533 * \theta_2 D^2 * \sin(\theta_2) - 0.0490 * \theta_1 D^2 * \cos(\theta_2) * \sin(\theta_1) - 0.0621 * \theta_1 D * \theta_3 D * \sin(\theta_3) \\ & * \cos(\theta_2) + 0.0245 * \theta_1 D^2 * \sin(\theta_2) - 0.1273 * \theta_2 D * \theta_3 D * \sin(\theta_3) * \cos(\theta_3) - 0.0636 * \theta_1 D * \theta_3 D \\ & * \sin(\theta_3) * \cos(\theta_3) - 0.0263 * \theta_1 D * \theta_3 D * \sin(\theta_3) - 0.1241 * \theta_1 D * \theta_2 D * \cos(\theta_3) * \sin(\theta_2) - \\ & 0.1241 * \theta_2 D * \theta_3 D * \sin(\theta_3) * \cos(\theta_2) - 0.0526 * \theta_2 D * \theta_3 D * \sin(\theta_3) \end{aligned}$$

$$\begin{aligned} h2 = & -0.0526 * \theta_2 D * \theta_3 D * \sin(\theta_3) + 0.0621 * \theta_1 D^2 * \sin(\theta_2) * \cos(\theta_3) + 0.0245 * \theta_1 D^2 * \cos(\theta_1) \\ & * \sin(\theta_2) - 0.0526 * \theta_1 D^2 * \cos(\theta_3) * \sin(\theta_1) - 0.0263 * \theta_1 D * \theta_3 D * \sin(\theta_3) - 0.0636 * \theta_1 D^2 * \\ & \sin(\theta_1) * \cos(\theta_3)^2 + 0.0621 * \theta_1 D^2 * \cos(\theta_1) * \sin(\theta_2) * \cos(\theta_3) - 0.0245 * \theta_1 D^2 * \cos(\theta_2) * \\ & \sin(\theta_1) - 0.0636 * \theta_1 D * \theta_3 D * \sin(\theta_3) * \cos(\theta_3) - 0.1273 * \theta_2 D * \theta_3 D * \sin(\theta_3) * \cos(\theta_3) + 0.0533 \\ & * \theta_1 D^2 * \sin(\theta_2) - 0.0621 * \theta_1 D^2 * \cos(\theta_2) * \cos(\theta_3) * \sin(\theta_1) - 0.0120 * \theta_1 D^2 * \sin(\theta_1) \end{aligned}$$

$$\begin{aligned}
h_3 = & 0.0621*\theta_1 D^2*\sin(\theta_3)*\cos(\theta_1)*\cos(\theta_2)+0.0263*\theta_2 D^2*\sin(\theta_3)+ 0.0636*\theta_1 D^2 * \\
& \sin(\theta_3)*\cos(\theta_1)*\cos(\theta_3)+0.0621*\theta_1 D^2*\sin(\theta_3)*\cos(\theta_2)+0.1273*\theta_1 D*\theta_2 D*\sin(\theta_3)* \\
& \cos(\theta_3)+0.0263*\theta_1 D^2*\sin(\theta_3)*\cos(\theta_1)+0.0621*\theta_1 D^2*\sin(\theta_3)*\sin(\theta_2)*\sin(\theta_1)+0.0526 \\
& *\theta_1 D*\theta_2 D*\sin(\theta_3)+0.0636*\theta_2 D^2*\sin(\theta_3)*\cos(\theta_3)+0.0639*\theta_3 D^2
\end{aligned}$$

where,

$$\theta_1 D = \dot{\theta}_1 ; \theta_2 D = \dot{\theta}_2 ; \theta_3 D = \dot{\theta}_3$$

B.3 Gravitational terms

$$c_1 = 0$$

$$c_2 = 0$$

$$c_3 = -0.4084*\cos(\theta_3)$$

Appendix C

Torque Equations for the Joints

T1 =

$$\begin{aligned}
 & -0.0189*\theta_2^2*\cos(\theta_2)+0.0245*\theta_1^2*\sin(\theta_2)+0.0189*\theta_1^2*\cos(\theta_2)-0.0189*\theta_2\ddot{\theta}_1 \\
 & * \sin(\theta_2)+0.0533*\theta_2\ddot{\theta}_1*\cos(\theta_2)-0.0533*\theta_2^2*\sin(\theta_2)-0.0716*\theta_1^2*\sin(\theta_1)-0.0377 \\
 & * \theta_1\dot{\theta}_1*\sin(\theta_2)+0.0141*\theta_3\ddot{\theta}_1*\sin(\theta_3)-0.0336*\theta_1\dot{\theta}_1*\theta_3\dot{\theta}_1*\cos(\theta_3)*\sin(\theta_3)+0.0141 \\
 & * \theta_3^2*\sin(\theta_3)-0.0864*\theta_2\dot{\theta}_1*\theta_3\dot{\theta}_1*\sin(\theta_3)*\cos(\theta_2)+0.0366*\theta_2\ddot{\theta}_1*\cos(\theta_3)+0.3451 \\
 & * \theta_1\ddot{\theta}_1+0.0432*\theta_2\ddot{\theta}_1*\cos(\theta_2)*\cos(\theta_3)+0.0336*\theta_2\ddot{\theta}_1*\cos(\theta_3)^2+0.0333*\theta_2\ddot{\theta}_1- \\
 & 0.0673*\theta_2\dot{\theta}_1*\theta_3\dot{\theta}_1*\cos(\theta_3)*\sin(\theta_3)+0.1065*\theta_1\ddot{\theta}_1*\cos(\theta_2)-0.0432*\theta_1\dot{\theta}_1*\theta_3\dot{\theta}_1*\sin(\theta_3) \\
 & * \cos(\theta_2)-0.0864*\theta_1\dot{\theta}_1*\theta_2\dot{\theta}_1*\sin(\theta_2)*\cos(\theta_3)-0.1065*\theta_1\dot{\theta}_1*\theta_2\dot{\theta}_1*\sin(\theta_2)-0.0377*\theta_1\dot{\theta}_1 \\
 & * \theta_2\dot{\theta}_1*\cos(\theta_2)+0.0366*\theta_1\ddot{\theta}_1*\cos(\theta_3)-0.0490*\theta_1^2*\cos(\theta_2)*\sin(\theta_1)+0.0864*\theta_1\ddot{\theta}_1 \\
 & * \cos(\theta_2)*\cos(\theta_3)-0.0432*\theta_2^2*\sin(\theta_2)*\cos(\theta_3)+0.0336*\theta_1\ddot{\theta}_1*\cos(\theta_3)^2+ \\
 & 0.0377*\theta_1^2*\sin(\theta_2)*\sin(\theta_1)-0.0864*\theta_1^2*\cos(\theta_2)*\cos(\theta_3)*\sin(\theta_1)+0.0432 \\
 & * \theta_1^2*\sin(\theta_2)*\cos(\theta_3)-0.0432*\theta_3\ddot{\theta}_1*\sin(\theta_3)*\sin(\theta_2)-0.0432*\theta_3^2*\sin(\theta_3) \\
 & * \sin(\theta_2)-0.0366*\theta_1^2*\cos(\theta_3)*\sin(\theta_1)-0.0336*\theta_1^2*\cos(\theta_3)^2*\sin(\theta_1)+0.0015 \\
 & * \theta_1\dot{\theta}_1*\theta_3\dot{\theta}_1*\cos(\theta_3)-0.0366*\theta_2\dot{\theta}_1*\theta_3\dot{\theta}_1*\sin(\theta_3)-0.0183*\theta_1\dot{\theta}_1*\theta_3\dot{\theta}_1*\sin(\theta_3)
 \end{aligned}$$

T2 =

$$\begin{aligned}
 & 0.0141*\theta_3\ddot{\theta}_1*\sin(\theta_3)+0.0336*\theta_2\ddot{\theta}_1*\cos(\theta_3)^2+0.0333*\theta_1\ddot{\theta}_1+0.0432*\theta_1\ddot{\theta}_1 \\
 & * \cos(\theta_2)*\cos(\theta_3)-0.0366*\theta_2\dot{\theta}_1*\theta_3\dot{\theta}_1*\sin(\theta_3)+0.0333*\theta_2\ddot{\theta}_1-0.0138*\theta_1^2*\sin(\theta_1)- \\
 & 0.0673*\theta_2\dot{\theta}_1*\theta_3\dot{\theta}_1*\cos(\theta_3)*\sin(\theta_3)+0.0366*\theta_1\ddot{\theta}_1*\cos(\theta_3)+0.0336*\theta_1\ddot{\theta}_1*\cos(\theta_3)^2 \\
 & +0.0141*\theta_3^2*\sin(\theta_3)+0.0015*\theta_1\dot{\theta}_1*\theta_3\dot{\theta}_1*\cos(\theta_3)-0.0183*\theta_1\dot{\theta}_1*\theta_3\dot{\theta}_1*\sin(\theta_3)- \\
 & 0.0336*\theta_1\dot{\theta}_1*\theta_3\dot{\theta}_1*\cos(\theta_3)*\sin(\theta_3)+0.0189*\theta_1^2*\cos(\theta_2)+0.0432*\theta_1^2*\cos(\theta_1) \\
 & * \sin(\theta_2)*\cos(\theta_3)-0.0432*\theta_1^2*\cos(\theta_2)*\cos(\theta_3)*\sin(\theta_1)+0.0245*\theta_1^2*\cos(\theta_1) \\
 & * \sin(\theta_2)+0.0432*\theta_1^2*\sin(\theta_2)*\cos(\theta_3)+0.0189*\theta_1^2*\cos(\theta_1)*\cos(\theta_2)-0.0245 \\
 & * \theta_1^2*\cos(\theta_2)*\sin(\theta_1)+0.0189*\theta_1^2*\sin(\theta_2)*\sin(\theta_1)-0.0336*\theta_1^2*\sin(\theta_1) \\
 & * \cos(\theta_3)^2+0.0533*\theta_1^2*\sin(\theta_2)-0.0189*\theta_1\ddot{\theta}_1*\sin(\theta_2)-0.0366*\theta_1^2*\cos(\theta_3) \\
 & * \sin(\theta_1)+0.0533*\theta_1\ddot{\theta}_1*\cos(\theta_2)+0.0366*\theta_2\ddot{\theta}_1*\cos(\theta_3)
 \end{aligned}$$

T3 =

$$\begin{aligned}
 & -0.8169*\cos(\theta_3)+0.0339*\theta_3\ddot{\theta}_1+0.0339*\theta_3^2+0.0141*\theta_1\ddot{\theta}_1*\sin(\theta_3)+0.0141* \\
 & \theta_2\ddot{\theta}_1*\sin(\theta_3)-0.0432*\theta_1\ddot{\theta}_1*\sin(\theta_3)*\sin(\theta_2)+0.0183*\theta_2^2*\sin(\theta_3)+0.0432*\theta_1^2 \\
 & * \sin(\theta_3)*\cos(\theta_2)-0.0141*\theta_1^2*\sin(\theta_3)*\sin(\theta_1)+0.0336*\theta_2^2*\cos(\theta_3)*\sin(\theta_3)+ \\
 & 0.0183*\theta_1^2*\cos(\theta_1)*\sin(\theta_3)+0.0432*\theta_1^2*\sin(\theta_3)*\sin(\theta_2)*\sin(\theta_1)+0.0673*\theta_1\dot{\theta}_1 \\
 & * \theta_2\dot{\theta}_1*\cos(\theta_3)*\sin(\theta_3)+0.0432*\theta_1^2*\sin(\theta_3)*\cos(\theta_1)*\cos(\theta_2)+0.0336*\theta_1^2* \\
 & \cos(\theta_3)*\cos(\theta_1)*\sin(\theta_3)+0.0366*\theta_1\dot{\theta}_1*\theta_2\dot{\theta}_1*\sin(\theta_3)
 \end{aligned}$$

where,

$$\theta_1\dot{\theta}_1 = \dot{\theta}_1; \theta_2\dot{\theta}_1 = \dot{\theta}_2; \theta_3\dot{\theta}_1 = \dot{\theta}_3$$

$$\theta_1\ddot{\theta}_1 = \ddot{\theta}_1; \theta_2\ddot{\theta}_1 = \ddot{\theta}_2; \theta_3\ddot{\theta}_1 = \ddot{\theta}_3$$

Appendix D

Matlab Code to Calculate Kinematics, Inverse Kinematics, Dynamics and Torques for the SCARA-Type Robot

```
syms L1 L2 L3 q1 q2 q3 H nx ox ax px ny oy ay py nz oz az pz m1 m2 m3 g q11 q22
q33 q111 q222 q333 ;
Q = [0 -1 0 0;1 0 0 0;0 0 0 0;0 0 0 0];
L1=1.7; L2=0.72; L3=1.27; H=5; m1=0.98; m2=0.513; m3=0.61; g=32.16; %All units
in lb and feet.
```

```
%theta1=62, theta2=35, theta3=0, theta5=0;
% q3 + q4 = 90 degrees.; q4 = 90 - q3.
%q1= (theta1*pi/180), q2=(theta2*pi/180),q3=(theta3*pi/180),q5=(theta5*pi/180);
```

```
% Homogeneous Transformation Matrices
```

```
A01=[cos(q1), -sin(q1), 0, L1*cos(q1); sin(q1), cos(q1), 0, L1*sin(q1); 0, 0, 1, H; 0, 0, 0,
1];
A12=[cos(q2),0,-sin(q2),L2*cos(q2); sin(q2),0,cos(q2),L2*sin(q2);0,-1,0,0; 0,0,0,1];
A23=[cos(q3),-sin(q3),0,L3*cos(q3); sin(q3),cos(q3),0,L3*sin(q3);0,0,1,0; 0,0,0,1];
A34=[sin(q3),-cos(q3),0,0;cos(q3),sin(q3),0,0;0,0,1,0;0,0,0,1];
A4E=[0,-sin(q5),cos(q5),0;0,-cos(q5),-sin(q5),0;1,0,0,0;0,0,0,1];
Tgoal=[nx ox ax px; ny oy ay py; nz oz az pz; 0 0 0 1];
```

```
T0E=A01*A12*A23*A34*A4E;
```

```
% Tgoal = T0E
```

```
A02=A01*A12;
A13=A12*A23;
A10=inv(A01);
A21=inv(A12);
A31=inv(A13);
A32=inv(A23);
A03=A01*A12*A23;
A0E=A01*A12*A23*A34*A4E;
A43=inv(A34);
AE4=inv(A4E);
```

```
px=0.1725; py=2.6457;pz=6.0999; nx=-0.8660; ny=-0.5000; nz=0; ox=-0.5000;
oy=0.8660; oz=0; ax=0; ay=0; az=-1;
```

```
%Q1=atan(-L3*ay*(H-pz)-L3*oy*(L3^2-(H-pz)^2)^0.5+py*L3-L2*L3*oy)/(-L3*ax*(H-
pz)-L3*ox*(L3^2-(H-pz)^2)^0.5+px*L3-L2*L3*ox);
%Theta1=(Q1*180)/pi;
```

```
%Q3 = asin((H-pz)/(L3));
%Theta3=(Q3*180)/pi;
```

```
%Q2=atan((L3*sin(Theta3)*sin(Theta1)*ax-L3*sin(Theta3)*cos(Theta1)*ay+
L3*cos(Theta3)*sin(Theta1)*ox-L3*cos(Theta3)*cos(Theta1)*oy-px*sin(Theta1)
+py*cos(Theta1))/(-L3*sin(Theta3)*cos(Theta1)*ax-L3*sin(Theta3)*sin(Theta1)*ay-
L3*cos(Theta3)*cos(Theta1)*ox-L3*cos(Theta3)*sin(Theta1)*oy+px*cos(Theta1)
+py*sin(Theta1)-L1));
%Theta2=(Q2*180)/pi;
```

```
U11=Q*A01;
U21=Q*A02;
U31=Q*A03;
UE1=Q*A0E;
```

```
U12=A01*Q;
U22=A01*Q*A12;
U32=A01*Q*A13;
```

```
U13=A02*Q*A21;
U23=A02*Q;
U33=A02*Q*A23;
```

```
% Moment of Inertias
```

```
J1=[0.944,0,0,0.3528;0,0.01445,0,0;0,0,0.00555,0;0.3528,0,0,0.98];
J2=[0.03755,0,0,0.1848;0,0.01285,0,0;0,0,0.06535,0;0.1848,0,0,0.5134];
J3=[0.1063,0,0,0.2194;0,0.0038,0,0;0,0,0.1082,0;0.2194,0,0,0.61];
```

```
U11t=transpose(U11);
U21t=transpose(U21);
U31t=transpose(U31);
U22t=transpose(U22);
U32t=transpose(U32);
U33t=transpose(U33);
```

```
% Inertial Matrices
```

$D11 = \text{trace}(U11 * J1 * U11t) + \text{trace}(U21 * J2 * U21t) + \text{trace}(U31 * J3 * U31t);$
 $D12 = \text{trace}(U22 * J2 * U21t) + \text{trace}(U32 * J3 * U31t);$
 $D13 = \text{trace}(U33 * J3 * U31t);$
 $D22 = \text{trace}(U22 * J2 * U22t) + \text{trace}(U32 * J3 * U32t);$
 $D23 = \text{trace}(U33 * J3 * U32t);$
 $D33 = \text{trace}(U33 * J3 * U33t);$
 $D21 = \text{trace}(U22 * J2 * U21t) + \text{trace}(U32 * J3 * U31t);$
 $D31 = \text{trace}(U33 * J3 * U31t);$
 $D32 = \text{trace}(U33 * J3 * U32t);$
 $D = [D11, D12, D13; D12, D22, D23; D13, D23, D33];$

$U111 = Q * Q * A01;$
 $U211 = Q * Q * A02;$
 $U212 = Q * A01 * Q * A12;$
 $U222 = A01 * Q * Q * A12;$
 $U221 = Q * A01 * Q * A12;$
 $U333 = A02 * Q * Q * A23;$
 $U331 = Q * A02 * Q * A21;$
 $U311 = Q * Q * A01 * A03;$
 $U321 = Q * A01 * Q * A13;$
 $U312 = Q * A01 * Q * A13;$
 $U322 = A01 * Q * Q * A13;$
 $U332 = A01 * Q * A12 * Q * A23;$
 $U313 = Q * A02 * Q * A23;$
 $U323 = A01 * Q * A12 * Q * A23;$
 $U333 = A02 * Q * A23;$

% Calculation of coefficients hikm

$h111 = \text{trace}(U111 * J1 * \text{transpose}(U11)) + \text{trace}(U211 * J2 * \text{transpose}(U21)) + \text{trace}(U311 * J3 * \text{transpose}(U31));$
 $h121 = \text{trace}(U221 * J2 * \text{transpose}(U21)) + \text{trace}(U321 * J3 * \text{transpose}(U31));$
 $h131 = \text{trace}(U331 * J3 * \text{transpose}(U31));$
 $h112 = \text{trace}(U212 * J2 * \text{transpose}(U21)) + \text{trace}(U312 * J3 * \text{transpose}(U31));$
 $h122 = \text{trace}(U222 * J2 * \text{transpose}(U21)) + \text{trace}(U322 * J3 * \text{transpose}(U31));$
 $h132 = \text{trace}(U332 * J3 * \text{transpose}(U31));$
 $h113 = \text{trace}(U313 * J3 * \text{transpose}(U31));$
 $h123 = \text{trace}(U323 * J3 * \text{transpose}(U31));$
 $h133 = \text{trace}(U333 * J3 * \text{transpose}(U31));$
 $h211 = \text{trace}(U211 * J2 * U22t) + \text{trace}(U311 * J3 * U32t);$
 $h221 = \text{trace}(U221 * J2 * U22t) + \text{trace}(U321 * J3 * U32t);$
 $h212 = \text{trace}(U212 * J2 * U22t) + \text{trace}(U312 * J3 * U32t);$
 $h222 = \text{trace}(U222 * J2 * U22t) + \text{trace}(U322 * J3 * U32t);$
 $h233 = \text{trace}(U333 * J3 * U32t);$
 $h231 = \text{trace}(U331 * J3 * U32t);$
 $h213 = \text{trace}(U313 * J3 * U32t);$

```

h232=trace(U332*J3*U32t);
h223=trace(U323*J3*U32t);
h311=trace(U311*J3*U33t);
h321=trace(U321*J3*U33t);
h331=trace(U331*J3*U33t);
h312=trace(U312*J3*U33t);
h322=trace(U322*J3*U33t);
h332=trace(U332*J3*U33t);
h313=trace(U313*J3*U33t);
h323=trace(U323*J3*U33t);
h333=trace(U333*J3*U33t);

```

```

% Calculation of Coriolis and centrifugal force vector
h(theta,theta dot)

```

```

h1=h111*q11^2+h121*q11*q22+h131*q11*q33+h112*q11*q22+h122*q22^2+h132*q2
2*q33+h113*q11*q33+h123*q22*q33+h133*q33^2;
h2=h211*q11^2+(h221+h212)*q11*q22+h222*q22^2+h233*q33^2+(h231+h213)*q11*
q33+(h232+h223)*q22*q33;
h3=h311*q11^2+(h321+h312)*q11*q22+h322*q22^2+(h331+h313)*q11*q33+(h332+h
323)*q22*q33+h333*q33^2;
h=transpose([h1,h2,h3]);

```

```

% Calculation of gravity loading force vector

```

```

c(theta)
G=[0,0,-g,0];          g = 32.16 ft/sec^2
r1=[-L1/2,0,0,1];
R1=transpose(r1);
r2=[-L2/2,0,0,1];
R2=transpose(r2);
r3=[-L3/2,0,0,1];
R3=transpose(r3);
c1=-m1*G*U11*R1-m2*G*U21*R2-m3*G*U31*R3;
c2=-m2*G*U22*R2-m3*G*U32*R3;
c3=-m3*G*U33*R3;
c= transpose([c1,c2,c3]);

```

```

% Calculation of torque vector

```

```

T1=D11*q111+D12*q222+D13*q333+h1+c1;
T2=D12*q111+D22*q222+D23*q333+h2+c2;
T3=D13*q111+D23*q222+D33*q333+h3+c3;

```

Appendix E

Technical Characteristics of the SCARA-Type Robot

1. Motion Range	Axis 1	± 110 degrees
	Axis 2	± 140 degrees
	Axis 3	+20 degrees and -10 degrees
2. Speed Range	Axis 1	± 240 degrees/sec
	Axis 2	± 240 degrees/sec
	Axis 3	± 40 degrees/sec
3. Input Voltage: 115 VAC		

Appendix F

Technical Characteristics of the Joint Actuators

The motors in the joints of the SCARA-Type robot are SM series servomotors made by Parker Hannifin Corporation.

1. Resistance (R_a)	9.65 ohms
2. Inductance (L_a)	4.08 mH
3. Viscous Damping (B_v)	3.78E-3 Nm/Krpm
4. Rotor Inertia (J_m)	1.3E-4 kg-m ²
5. Voltage Constant (K_b)	0.484 Volts/rad/sec
6. Torque Constant (K_t)	0.48 Nm/Amp DC

Appendix G

A) Matlab Code to determine closed-loop transfer function for Joint 1 with PID controller

```
syms kp ki kd ;
kp=1500;
ki=0.4;
kd=1330;
num=[0.48*kd 0.48*kp 0.48*ki];
den=[39 0.27+0.48*kd 0.48*kp 0.48*ki];
G=tf(num,den);
step(G)
margin(G)
rlocus(G)
```

B) Matlab Code to determine closed-loop transfer function for Joint 2 with PID controller

```
syms kp ki kd;
kp=140;
ki=0.4;
kd=160;
num=[0.48*kd 0.48*kp 0.48*ki];
den=[4.2 0.27+0.48*kd 0.48*kp 0.48*ki];
G=tf(num,den);
step(G)
margin(G)
rlocus(G)
```

C) Matlab Code to determine closed-loop transfer function for Joint 3 with PID controller

```
syms kp ki kd s;
kp=10;
ki=15;
kd=0.9;
num=[0.48*kd 0.48*kp 0.48*ki];
den=[0.0016 0.27+0.48*kd 0.48*kp 0.48*ki+0.016];
G=tf(num,den);
step(G)
margin(G)
rlocus(G)
```

Appendix H

Simulink Model for the Three Joints

Figure H.1 Controller Design for Joint 1

Figure H.2 Controller Design for Joint 2

Figure H.3 Controller Design for Joint 3

Vita

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