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Essays on the restructuring of the electricity industry in the United States

Vladimir V. Starkov
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Essays on the Restructuring of the Electricity Industry in the United States

Vladimir V. Starkov

**Dissertation Submitted to the
College of Business and Economics
at West Virginia University
in partial fulfillment of the requirements
for the degree of**

**Doctor of Philosophy
in
Economics**

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ABSTRACT

Essays on the Restructuring of the Electricity Industry in the United States

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The dissertation explores several issues that arise from the restructuring of the American electricity industry. The first chapter discusses economic reasons for deregulation, describes possible deregulation scenarios, and outlines potential pitfalls of deregulation.

Chapter 2 develops a forecast of electricity prices after the transition to retail competition. The equilibrium prices are found from an n -firm Cournot model of oligopoly with non-uniform marginal costs. It is further shown how the translog cost function can be used in the traditional solution of the Cournot model. Major empirical findings include marginal cost functions of every producer and forecast prices of electricity after deregulation in all the NERC regions of the U.S.

Chapter 3 presents an application of the theory of real options to the sales of power plants by the U.S. electric utilities. Observations of the divestiture transaction prices make it possible to infer expected future prices of electricity. It is found that under plausible assumptions prices for electricity have to be rather high for the owners of power plants to earn an attractive rate of return on their investments. Power plants divested by electric utilities, in general, have commanded high prices in the market, and the utilities selling them received generous compensation.

Chapter 4 examines the question why some electric utilities are willing to sell generation plants, while affiliates of other utilities are buying them. The study applies a method of joint estimation of risk preferences and costs to assess the nature of attitudes toward risk of the U.S. electric utilities. The estimates of absolute risk aversion, relative risk aversion, and downside risk aversion are obtained. The absolute majority of electricity producers have been found risk averse with decreasing absolute risk aversion (DARA) and increasing relative risk aversion (IRRA). All the firms have been found averse to downside risk. It was determined that buyers of power plants have statistically lower degree of relative risk aversion than the sellers, as well as the rest of the firms.

Chapter 5 provides a summary of the dissertation and discusses the directions of possible future research.

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Chapter 1. Economic Arguments for Electricity Industry Deregulation

Currently the U.S. electric industry is undergoing some profound changes. Utilities tightly regulated in the recent past find themselves facing challenges of the open market environment. The exclusive right of utility companies to supply electric power was punctured in 1978 when the Public Utilities Regulatory Policy Act (PURPA) allowed entry of non-utility generators into the market and stipulated conditions for the independent producers to sell their power. The Energy Policy Act (EPAct) of 1992 opened yet another avenue towards the open market. It clarified the authority of the Federal Energy Regulatory Commission (FERC) to order owners of transmission lines (typically, large utilities) to provide non-discriminatory carrier service to all eligible generators for wholesale transactions. FERC Order 888 made such “open access” to transmission mandatory. The EPAct and Order 888 also set in motion the deregulation of retail distribution of power. More and more states have lifted the privilege of utilities to exclusively sell power at their designated areas and have given consumers the right to choose electric suppliers. Most of the current developments in the industry are directly related to the changes in retail competition.

The political will to reform the electric industry persists due to several reasons: generally successful experience of deregulation in other industries, broad dissatisfaction with price differences across utilities, and large potential gains to consumers should prices of power decrease. Economic research has provided a broad basis for the policy of deregulation. A comprehensive review of projections and outcomes of deregulation in several different industries provided by Winston (1993) concludes

that economic forecasts of gains from deregulation were generally accurate. Although Winston's study leaves out the market for electric power, a strong case against the existing regulation of electric industry has been built over the years.

After electricity rates fell by 40 percent between 1960 and 1970, the industry was hard hit by the energy crisis and general inflation of the 1970s (Edison Electric Institute, 1983, p.89). Motivated by rising prices of fuel as well as increases in construction and operating costs, utilities sought regulatory approval of rate hikes. However, regulators in many cases were unable or unwilling to process such requests expeditiously, and although the price of electricity doubled in real terms during the decade of the 1970s, revenues of electric utilities failed to keep up with soaring costs, which resulted in decline of the rate of return on utility capital. This fact was reflected in the 1984 Economic Report of the President. Both the increasing costs and regulatory lag pushed utilities to pay a premium on their debt obligations which further promoted the cost spiral. From the second half of the 1970s to early 1980s, ratings of many utilities bonds were downgraded. Many utility stocks were selling well below the book values (see, e.g., Males, 1984 and Navarro, 1981). Other problems electric utilities faced in the 1980s and 1990s included rising environmental costs, demand that fell short of projections, shortage of transmission capacity. This caused utilities to abandon many planned projects and some already underway (Berry, 1982). Shortages of electricity supply became a reality in certain regions of the country. Words such as blackouts, rolling brownouts, interruptible service became a part of the vocabulary in the decade of the 1970s. Ever since then, economists have pointed out several major flaws in the existing regulatory practices. One strand of research raises questions about the extent of scale economies that make the industry a natural monopoly and justify regulation. Another

branch argues that in absence of barriers to entry the monopoly power of utilities is not sustainable. Finally, some scholars argue that the issue of scale economies is irrelevant. As soon as bidding for the market takes place, competition will drive the prices down, at least for some consumers.

Some notable work on empirical estimation of economies of scale was done by Komiya (1962), Nerlove (1963), Christensen and Greene (1976), and Huettner and Landon (1978). The universal conclusion to which those authors generally agree is that large-scale generation is economically efficient. Zardkoohi (1986), however, cautioned that the range at which the decreasing returns to scale begin could be overstated by the past research because the existence of economies of scope and vertical integration on the firm level has not been controlled for. Such a statement is in agreement with the conclusion by the Edison Electric Institute that interconnection, power pooling, and coordination among interdependent utilities reduced the minimum efficient scale in the electric system from 8,000-10,000 MW to 1,600-3,000 MW (Edison Electric Institute, 1982). In addition, Stewart (1979) advanced an argument that in determining effective unit costs, load factor is even more important than scale economies. Interconnection and free wholesale market can keep the load factor relatively high. Thus the competition in generation among smaller units does not immediately imply a loss of productive efficiency.

The issue of economic efficiency of electric utilities was analyzed in detail by Pollitt (1995). He contrasted operating efficiency as an indicator of how close the firm is to the production frontier and allocative efficiency as the ability to achieve a cost-minimizing mix of inputs. Pollitt found that in terms of operating efficiency, privately-owned and publicly-owned utilities are on a par; however,

privately-owned utilities have lower costs than publicly-owned ones, which suggests that the direct government control has increased inefficiency. The observation that economies of scale are neither necessary nor sufficient to cause natural monopoly was stated by Baumol et al. (1982), which argues that only sunk costs give an incumbent firm the advantage needed to curb potential entrants. Thus, if entry is allowed to the market where firms operate without significant sunk costs, either an actual entry or the mere threat of it will keep prices well below the monopolistic level. Empirical evidence that contestable markets result in lower prices regardless of the issues of scale economies was collected by Primeaux (1975, 1986). He compared the costs of power producers in cities with two competing suppliers of power with the costs of monopoly suppliers in cities with similar characteristics. The conclusion was that even in a duopoly setting, competition causes firms to operate at lower average cost level than they would in the case of monopoly.

Having reached a broad consensus that regulation in its traditional form is deeply flawed, economists have proposed numerous alternatives. Weiss (1975) and later Joskow and Schmalensee (1983) proposed various scenarios of a deregulated market for electricity and analyzed implications of each scenario. However, even if the choice of a particular reform scenario is made, the question of how fast deregulation should proceed remains. Gordon (1986) expressed strong support for a swift and comprehensive reform as opposed to a gradual one because the latter presumes greater government involvement which can only aggravate the efficiency problem. It is worth noting that deregulation of the electricity industry has been embraced not only in this country, but by many nations across the world. The book edited by Gilbert and Kahn (1996), presents the experiences and lessons of

deregulation from the United Kingdom, Argentina, Brazil, Chile, Uruguay, Scandinavia, and the U.S.

Although deregulation in the electricity industry has gained wide support by now, it is important to understand that the move towards a competitive market is not a simple task because it involves a whole set of issues never addressed before. A comprehensive legal analysis of most of such issues is provided in Sidak and Spulber (1998). One of the problems arising in the course of deregulation is just compensation of utilities for the investments made according to regulatory orders that seemed to be warranted in the past, but now are considered irrecoverable under market conditions. The industry name for such expenses is “stranded costs”. Examples of stranded costs are losses from extra capacity that sits idle and contracts to purchase power from independent producers at prices well above current market prices. Another problem of deregulation is that there is no guarantee that once the market for power becomes unregulated, prices of electricity to consumers will go down. Instead, power producers, freed from regulatory oversight, may find themselves in a tight oligopoly which makes it feasible to charge prices above marginal cost. Such a situation is referred to as market power. A possibility of future market power must be of concern to those in charge of the regulatory reform because the loss of consumer surplus from rising prices may very well offset the intended benefits of deregulation.

This dissertation addresses several issues arising from restructuring of the U.S. electricity industry. The model developed in Chapter 2 provides a forecast of regional equilibrium prices of electric power after the transition to retail competition. The interaction among electric utilities is assumed to

be bounded by the present NERC regions within the U.S. The results of the Cournot model are contrasted with the Bertrand solution and regulated prices. It is found that Bertrand (competitive) prices are lower than average cost prices, but oligopoly prices are considerably higher. Predicted prices vary substantially among NERC regions, due to differences in average costs of producers and the industry concentration.

Chapter 3 presents an application of the theory of real options to the sales of power plants by the U.S. electric utilities. Observations of the divestiture transaction prices make it possible to infer expected future prices of electricity. It is found that under plausible assumptions, new power plant owners will have to rely on market power to earn attractive rates of return on their investment. In general, power plants commanded high prices, and electric utilities selling them were generously compensated.

Chapter 4 analyses the question whether risk attitudes drive divestiture in electricity industry. Observations of divestiture reveal that there are few newcomers in the market. The vast majority of the capacity sold is acquired by unregulated affiliates of electric utilities. The wide proliferation of sales in the environment that may not be characterized by asymmetry of information about the assets traded can plausibly be explained by different degrees of risk aversion of buyers and sellers. This study applies a method of joint estimation of risk preferences and costs to assess the nature of attitudes toward risk of the U.S. electric utilities. The estimates of absolute risk aversion, relative risk aversion, and downside risk aversion are obtained. The absolute majority of electricity producers have been found to be risk averse with decreasing absolute risk aversion (DARA) and

increasing relative risk aversion (IRRA). All the firms have been found to be averse to downside risk. It was determined that buyers of power plants have lower relative risk aversion than the rest of the firms the sample. The buyers also have lower degree of relative risk aversion than the sellers. This indicates that the buyers of power plants tend to be more aggressive than the plants' previous owners. Chapter 5 discusses some implications of the research and provides directions for future research.

Chapter 2. Oligopoly Model for Forecasting Market Prices of Electricity After Deregulation

2.1. Directions of Research on Market Power in the Electricity Industry.

Under traditional regulation, investor-owned electric utilities generally earn an adequate rate of return. While some might expect utilities to resist deregulation that creates a more competitive and uncertain environment, some utilities have embraced FERC Order 888 and the subsequent state-level movement toward competition in retail transactions. The support of deregulation by some power producers may arise from the fact that certain utilities believe they stand to gain from deregulation because the open market prices will be higher than the regulated rates they charge presently. Some other producers, however, stand to lose from the transition to competition.¹

This study develops a method to measure the amount of market power obtainable by U.S. electricity generators after deregulation of the retail electricity market. Interaction among electric utilities is assumed to occur within the regions set by the North American Electric Reliability Council (NERC).² The map of the NERC regions is shown in Figure 2.1. Holding companies and independent investor-owned electric utilities are assumed to be oligopoly players, while other producers in the region are considered price-takers. The proposed method is applied in all the ten existing NERC regions to estimate equilibrium market prices

¹ One most obvious distinction can be drawn between high- and low average cost producers.

² The reason to delineate markets for electricity according to the boundaries of the NERC regions is that physical constraints limit transfer of power between NERC regions. Also, in some regions, such as MAAC, the firms historically have operated as a common power pool.

Figure 2.1. Map of the U.S. NERC Regions

Source: North American Electric Reliability Council

for both Cournot and Bertrand cases of oligopoly. In addition, indexes of market power are calculated for every region.

While estimates of changes in market power after deregulation using *ex post* data are common in the academic literature, measurements of such changes *ex ante* are far less numerous. Studies by Green and Newbery (1992) and Green (1996, 1999) utilized a supply function method developed by Klemperer and Meyer (1989). Those studies modeled marginal costs of producers as either constant or linear. On the other hand, von der Fehr and Harbord (1993) argued that cost functions of the UK electricity producers are better represented by step functions. A stepwise function of

average variable costs was used by Rudkevich et al. (1998) to calculate instantaneous market clearing prices assuming interaction among n identical profit-maximizing firms. Andersson and Bergman (1995) used a conjectural variation model of oligopoly to find the equilibrium price and to explore the effect of firm size and the number of firms in the market on the price level. The analysis, applied to Swedish electric industry, showed that in Nash equilibrium unregulated prices will be higher than they are under regulation. Two papers by Borenstein et al. (1999a,b) presented assessments of potential market power in California and New Jersey electricity markets.

Although analysis in this essay has the same goal as that performed by Borenstein et al., this study is significantly different in many aspects. My inquiry has a broader scope that covers the entire territory of the U.S. While Borenstein et al. analyzed market power under different demand conditions at several periods of the year, my study presents annual averages of prices and indexes of market power. Like the model used by Borenstein et al., my model does not incorporate explicitly dynamic aspects of competition which, in particular, would include possibilities of entry and exit. As noted by Borenstein et al. (1999a,b), some dynamic aspects may be captured by using several different demand elasticities in the analysis. The short run is characterized by relatively price-inelastic demand, while in the long run demand is relatively price-elastic. While Borenstein et al. (1999a,b) modeled cost curves of generators using historical cost and capacity data, parameters of producers' marginal cost functions used in this study are derived from estimates of a translog total cost function. Borenstein et al. (1999a) forecast the price of electricity delivered to consumers. Such a forecast must account for the costs of transmission and distribution. They modeled those costs by adding a constant markup to the production costs of all the generators. This analysis, instead,

focuses on prices at generation sites. I do not attempt to model transmission effects because even with the competitive power generation, transmission will most likely remain regulated and thus transmission charges will be determined outside of the market realm.

Both Cournot and Bertrand outcomes of the oligopoly game are considered. A consideration that supports the Bertrand outcome in the market for electricity is the disincentive to raise prices above marginal cost that may come from implementing the market structure known as poolcos. In a poolco the right to make actual sales will be given to the suppliers who submit the lowest price bids. However, several studies have shown that the possibility of exercising market power in the poolco is also quite conceivable. One strategy described by Rudkevich et al. (1998) involves a cooperative agreement among producers to bid above marginal cost. Other way of achieving market power outlined by Newbery (1995) and Wolak and Patrick (1997) is deliberate withholding of low-cost units from operation and bidding the output of more expensive units to drive prices up. Empirical evidence of market power exercised by electricity producers in auction markets of England and Wales is found by Wolfram (1998). The sharp rise in prices observed in the California wholesale electricity market since the year 2000 may be due, at least partly, to the ability of the generation owners there to exercise market power.

2.2. The Model of Oligopoly Interaction.

Both Cournot and Bertrand models of oligopoly can be derived from a conjectural variation model

$$p^*(Y) + \frac{\eta_p}{\eta_Y} \left(1 + \frac{\eta_{Y_{-i}}}{\eta_{y_i}} \right) y_i = MC_i(y_i), \quad (2.1)$$

where p^* is the equilibrium price, Y is the total market supply, y_i is the supply by the firm i , Y_{-i} denotes total output of all firms except i , and MC represents the marginal cost.

This specification (2.1) embodies all the classical models of oligopoly as special cases depending on the value of the conjectural variation parameter $\frac{\eta_{Y_{-i}}}{\eta_{y_i}}$. The Cournot model is obtained when the conjectural variation is equal to 0. The Bertrand outcome follows when the conjectural variation is equal to -1.

A “competitive” outcome where producers price their output at marginal cost can be found by using Bertrand conjectures in the model (2.1). The supply condition for a single firm is then given by

$$p^*(Y) = MC_i(y_i) \quad (2.2)$$

The method of finding the Bertrand price used here is similar to the “competitive cost analysis” described by Torries (1998). Market supply is found by aggregating supply curves of producers, which are identical to the producers' marginal cost functions in a competitive market. Parameters of the marginal cost function are obtained by estimating a translog total cost function (see Appendix A for details). Market demand is set exogenously at its latest historical level (or, alternatively, at the level projected in the future). The market-clearing equilibrium price p^*_B will then be equal to the

marginal cost of the highest-marginal-cost producer(s) at the point where market supply is equal to market demand. A detailed description of the method used here to obtain the Bertrand prices is presented in Appendix B.

Alternatively, equation (2.1) solved for the Cournot case produces the reaction function of the firm³

$$p^*(Y) \left(1 + \frac{\mathbf{q}_i}{\mathbf{a}} \right) = MC_i(y_i), \quad (2.3)$$

where $\mathbf{a}$ is the price elasticity of market demand, and $\mathbf{q}$ is the market share of firm i , so that $\sum_i \mathbf{q}_i = 1$. The aggregate market supply is obtained by multiplying both sides of expression (2.3) by $\mathbf{q}$ and summing up across all the n firms,

$$\sum_i \mathbf{q}_i p^*(Y) \left(1 + \frac{\mathbf{q}_i}{\mathbf{a}} \right) = \sum_i \mathbf{q}_i MC_i(y_i). \quad (2.4)$$

After collecting terms and simplifying notation, equation (2.4) can be written as

$$p^* \left(1 + \frac{h}{\alpha} \right) = \sum_i \mathbf{q}_i MC_i(y_i), \quad (2.5)$$

where $h = \sum_i \mathbf{q}_i^2$ by analogy with the Herfindhal index of industry concentration expressed in decimals.

Substituting the definition of marginal cost, $MC_i = \frac{\mathcal{I}TC_i}{\mathcal{I}y_i}$, and the definition of market share,

$\mathbf{q} = y_i/Y$ into (2.5), a simple rearrangement leads to

³ See this result, e.g. in Varian (1992), p.290.

$$p^* = \frac{I}{Y} \left(\frac{\mathbf{a}}{\mathbf{a} + h} \right) \sum_i \mathbf{e}_i TC_i, \quad (2.6)$$

where $\mathbf{e}_i$ is the elasticity of total cost with respect to output of firm i .

Equation (2.6) allows us to estimate the equilibrium oligopoly price *ex ante*, that is, before the actual oligopoly game begins. The values of the parameters in equation (2.6) are found as follows. Market output Y is set exogenously at its latest historical level.⁴ The price elasticity of market demand $\mathbf{a}$ is taken to be within the range suggested by various expert evaluations. An analysis of the price sensitivity to the elasticity of demand is then conducted. The output-cost elasticities $\mathbf{e}_i$ are computed after estimating parameters of the translog total cost function (see Appendix A for the details on calculating $\mathbf{e}_i$'s).

To determine the equilibrium Cournot price p^*_C , a simulation study was conducted. Reaction functions of every producer in the oligopoly were calculated as the profit-maximizing response in output to a given market price. The algorithm starts with a certain exogenous price of electricity common for all producers, and firm-specific levels of output. Every firm then chooses its optimum output within its own capacity constraints, expecting its rivals to hold constant their levels of production. Summation across the firms gives a new level of the aggregate supply. The new market price is computed from the constant-elasticity demand function $p = \left(Y_D / k \right)^{-1/\mathbf{a}}$ where Y_D is the market demand, p is the price, $\mathbf{a}$ is the price elasticity of demand, and k is a parameter. The market

⁴ Alternatively, it could be set at the projected level of demand in the future.

clearing constraint is imposed, so that $Y_D = \sum_i y_i$. The new price level becomes an input for the next iteration of the output optimization. The iterations are terminated when the aggregate output changes by less than 1 percent. The additional assumptions used in the method are as follows. Like in Borenstein et al. (1999a,b), it is assumed that oligopoly supplies the residual demand after the supply of the price-taking firms is controlled for. The value of k is found using the assumption that the demand curve passes through the historical point of demand. Specifying the value of k , however, is problematic because under regulation there was no uniform market price of electricity, rather electricity rates were based on average costs of generation and regulated rates of return on capital. The problem of uniform initial price was mitigated by a sensitivity study when several different price levels were used to initialize the algorithm. The initial prices were selected based on the average prices faced by consumers in every NERC region as reported by the Energy Information Administration.⁵ The resulting oligopoly prices converged to a fairly narrow intervals. In most trials the variance of the oligopoly prices was smaller than the variance of the starting values.

2.3. Description of Data Used to Estimate the Model.

Because this research aims to forecast prices of electricity at generation sites, the cost data used pertain to the generation sector of electric utilities. The historical cost and output data were retrieved from the database on U.S. electric utilities published by Utility Data Institute (UDI). The information on utility bond ratings necessary for the estimation of the cost of capital was obtained from Moody's

⁵ Energy Information Administration. *Electric Power Annual*, 1998.

Electric Utilities Manual. The historical prices of utility stocks and returns on utility equity were found in the *Compustat* database. The price elasticity of market demand for electricity used here lies within the range suggested by various studies.⁶ A sensitivity analysis of the oligopoly equilibrium price with respect to the elasticity of demand was conducted. The results show that oligopoly prices rise substantially as the price-elasticity of demand decreases.

The empirical analysis covers all ten NERC regions. The time period of the observations is from 1982 to 1997. The sample consists of all electric producers in the U.S. classified as either independent investor-owned utilities or holding companies. Throughout the sample period, the structure of the industry is assumed to be fixed at the state it was at the end of 1998. Utilities that were parts of holding companies at the end of 1998 were treated as such from beginning to end of the sample period, and companies that by the end of 1998 were formed as result of mergers were treated as aggregated throughout the sample. Electric utilities that ceased operations before 1998 were excluded from the sample. The total number of electricity producing firms in the sample is 99. The panel data set therefore consists of 1584 observations. This sample is larger than that of most prior studies. For the definitions of variables used in the estimation and the methods by which they were obtained, see Appendix C. The mean sample values for the variables used in the study are reported in Table 2.1.

⁶ Several recent studies used the following absolute values of own-price demand elasticity. Borenstein et al. (1999a): 0.1, 0.4, and 1.0; Andersson and Bergman (1995): 0.3 and 0.6; Green and Newbery (1992) values range from 0.08 to 0.64. Estimates of long run demand-price elasticities by Halvorsen (1978) vary from 1.00 to 1.21 for residential sector and from 1.53 to 1.75 for industrial and commercial sectors. Short run demand elasticities of the residential sector lie between 0.48 and 0.78, and the short run estimate for the industrial sector is 0.42.

Table 2.1. Mean Sample Values of Variables
Period of Observations 1982 - 1997 ^a

Variable	Mean Value
Total Cost of Generation (\$ 000)	1,740,427
Net Generation (MWhr)	20,103,520
Price of Labor (\$ 000/employee)	56.789
Price of Fuel (\$/MMBTU)	199.76
Price of Capital (%)	6.26

a. All cost data are in constant 1996 dollars.

2.4. Analysis and Results.

At the first step, parameters of the total cost function are obtained by estimating the translog specification. The results of the translog function estimation are presented in Table 2.2. All the parameters of the translog cost functions are statistically significant, yet only four regional dummies are significant at the 5 percent level, and two more are significant at the 10 percent level. Monotonicity conditions were satisfied at every observation. Concavity conditions were not satisfied consistently, however, as noted in Thompson et al. (1996, p. 37), estimation of scale economies or other relationships between output level and cost is not affected by the incomplete satisfaction of the concavity restrictions. Although negative signs of coefficients for labor and fuel are contrary to those commonly reported, calculated elasticities of total cost with respect to labor and fuel are positive. The coefficients of the translog cost function were used to estimate firm-specific elasticities of total cost with respect to output as well as the parameters of firm-specific marginal cost functions. Then simulations were conducted to obtain Bertrand and Cournot prices.

Table 2.2. Coefficients of the Translog Cost Function ^a

Coefficient	Value	Standard Error
α_0	4.245*	0.382
b_L	-0.506*	0.028
b_F	-0.494*	0.027
b_K	2.000*	0.063
b_Y	0.393*	0.054
b_{LL}	0.608*	0.002
b_{FF}	0.568*	0.002
b_{KK}	2.012*	0.011
b_{YY}	0.031*	0.004
b_{LF}	0.418*	0.001
b_{LK}	-1.026*	0.004
b_{FK}	-0.986	0.004
g_L	-0.028*	0.002
g_F	0.013*	0.002
g_K	0.015	0.006
ECAR	0.032	0.025
ERCOT	0.349*	0.039
FRCC	0.038	0.043
MAAC	0.053**	0.028
MAIN	-0.123*	0.027
MAPP	-0.048**	0.029
NPCC	0.494*	0.024
SERC	0.010	0.036
SPP	0.087*	0.028

a. The estimated specification is

$$\ln TC = \alpha_0 + b_L \ln p_L + b_F \ln p_F + b_K \ln p_K + b_Y \ln y + \frac{1}{2} b_{LL} (\ln p_L)^2 + \frac{1}{2} b_{FF} (\ln p_F)^2 + \frac{1}{2} b_{KK} (\ln p_K)^2 + \frac{1}{2} b_{YY} (\ln y)^2 + b_{LF} \ln p_L \ln p_F + b_{LK} \ln p_L \ln p_K + b_{FK} \ln p_F \ln p_K + g_L \ln p_L \ln y + g_F \ln p_F \ln y + g_K \ln p_K \ln y + REGION DUMMIES$$

* Significant at 5 percent level.

** Significant at 10 percent level.

The results of the simulation study are presented in Table 2.3. The number of oligopoly players in every region shows the number of producers used in simulations. “Competitive” prices reported here represent those formed in an auction when producers bid prices equal to the marginal cost of generation, and every unit of output is sold at the market-clearing price. The “competitive prices” also correspond to those formed under the Bertrand model of oligopoly.

Table 2.3. Projected Prices of Electricity by NERC Region ^a

NERC Region	Number of Oligopoly Players	“Competitive Price” ^b , \$/MWhr	“Regulated Price” ^c , \$/MWhr	Cournot Price, \$/MWhr			
				Price Elasticity of Demand			
				-1.0	-0.7	-0.5	-0.3
ECAR	15	54.80	51.82	55.48 (2.83)	57.23 (3.83)	62.47 (2.02)	63.77 (9.98)
ERCOT	4	64.40	68.94	68.17 (1.14)	77.79 (2.07)	112.57 (59.26)	d
FRCC	3	79.70	96.50	94.14 (9.99)	90.14 (0.66)	90.12 (1.22)	d
MAAC	9	76.80	97.46	90.88 (3.29)	97.92 (2.69)	102.77 (3.48)	129.00 (14.62)
MAIN	10	55.50	70.76	69.87 (3.84)	79.30 (6.31)	82.73 (10.41)	99.58 (8.43)
MAPP	9	56.40	71.18	65.36 (3.25)	69.80 (2.71)	78.72 (5.52)	103.40 (14.86)
NPCC	20	121.30	111.86	142.70 (0.35)	157.58 (10.13)	154.46 (10.43)	177.60 (21.89)
SERC	9	60.10	69.44	69.78 (2.10)	71.65 (5.19)	78.55 (9.15)	108.43 (16.25)
SPP	5	63.10	76.44	69.78 (1.45)	73.86 (2.96)	76.78 (1.33)	83.22 (4.99)
WSCC	15	80.00	103.43	82.66 (2.44)	89.02 (2.85)	97.88 (9.81)	107.02 (16.13)

a. Standard errors in parentheses.

b. Computed according to the Bertrand model of oligopoly.

c. Computed as the means of producers’ average total costs weighted by producers’ market shares.

d. At these values of elasticity, prices grow explosively.

The Cournot prices were obtained by the iterative procedure described above. To initialize the algorithm, two different types of conditions were tested. One that used historical prices and historical output levels represents a simultaneous transition for all the producers from regulation to an open market regime. (Because regulated generators do not sell output at a uniform price, several initial prices were used). However, if different electric utilities face different timeframes for transition, a market power regime will be unlikely to develop until the transition is completed by all the producers. Thus in the short run producers may price their output at marginal cost. This scenario was modeled by using Bertrand price and output as a starting point for iterations. The results obtained with both initialization methods were very close. The means and standard deviations of the Cournot prices are presented in Table 2.3.

In general, the Cournot iteration method displays a robust convergence. The variance of the resulting prices was lower than the variance of the starting values in most trials. The Cournot prices were estimated for several different elasticities of market demand for electricity that lie within the range of expert evaluations, and tend to rise as price elasticity of demand decreases in absolute value. This is because an oligopoly can maintain higher prices in a market with less elastic demand. In addition to demand elasticity, oligopoly price is influenced by the sum of average total costs of producers.⁷ The higher the average total costs, the higher is the equilibrium price. This explains why regions with low concentration do not always get the lowest estimated oligopoly prices (e.g.

⁷ This can be seen mathematically when the definition of total market supply $Y = \sum_i y_i$ is substituted into the formula (2.6), which becomes $p^* = \left(\frac{\mathbf{a}}{\mathbf{a} + h} \right) \sum_i \mathbf{e}_i \frac{TC_i}{y_i}$.

NPCC). Table 2.3 also lists the “regulated prices”. They were calculated as average of producers’ average total costs weighted by respective market shares.

Comparison of the “regulated prices” with the “competitive prices” suggests that if the market for retail sales of electricity is perfectly competitive, the prices of electricity at generation sites will go down in every region except ECAR and NPCC. Secondly, comparison of the “regulated prices” and the Cournot prices suggests that in the case of producers exercising market power, retail competition will result in higher prices to consumers, although in several regions prices may go up only by a small margin (e.g. WSCC, SPP). It is worth a reminder that the model estimated in this study explicitly assumes away trade with other regions. Conditions that promote inter-regional power exchange may be desirable to assure that retail prices of electricity will not rise sharply due to market power.

Price markups are most often represented numerically with indices of market power. I provide the two most commonly used indexes of market power, the Lerner index and the Price-Cost Margin Index (PCMI). They are defined as follows

$$\text{Lerner Index} = \frac{\text{Actual Product Price} - \text{"Perfectly Competitive" Product Price}}{\text{Actual Product Price}} * 100\%, \quad (2.7)$$

$$\text{PCMI} = \frac{\text{Actual Product Price} - \text{"Perfectly Competitive" Product Price}}{\text{"Perfectly Competitive" Product Price}} * 100\%, \quad (2.8)$$

where the “perfectly competitive” price is equal to the marginal cost of the product.

Those indices summarized in Table 2.4 point to the conclusion that the degree of market power in different regions will vary substantially. According to the Department of Justice guidelines, a market can be considered competitive if prices do not exceed their “perfectly competitive” level by more than 5 percent.⁸ The results indicate that if electricity demand is unit- elastic, which approximates the long run, prices for power may approach their perfectly competitive level in several regions

Table 2.4. Projected Price Markups by NERC Region

NERC Region	Price Elasticity of Demand				Price Elasticity of Demand			
	-1.0	-0.7	-0.5	-0.3	-1.0	-0.7	-0.5	-0.3
	Lerner Index				PCMI			
ECAR	1.22	4.24	12.28	14.06	1.24	4.43	14.00	16.36
ERCOT	5.53	17.21	42.79	a	5.85	20.79	74.80	a
FRCC	15.34	11.58	11.55	a	18.12	13.10	13.07	a
MAAC	15.49	21.57	25.27	40.46	18.33	27.50	33.82	67.96
MAIN	20.56	30.01	32.91	44.26	25.89	42.88	49.06	79.42
MAPP	13.70	19.20	28.35	45.45	15.88	23.76	39.57	83.33
NPCC	14.99	23.02	21.47	31.70	17.64	29.91	27.34	46.41
SERC	13.87	16.12	23.49	44.57	16.10	19.22	30.70	80.42
SPP	9.57	14.57	17.82	24.18	10.58	17.05	21.68	31.89
WSCC	3.21	10.14	18.27	25.24	3.32	11.28	22.35	33.77

a. Omitted because at these values of elasticity concentration indexes grow explosively.

⁸ U.S. Department of Justice and Federal Trade Commission, “Statement Accompanying Release of Revised Merger Guidelines”, April 2, 1992.

(ECAR, WSCC), but most regions will experience a moderate degree of market power even in the long run.

The lowest across-the-board price markups are found in ECAR and WSCC, and the highest ones in ERCOT. The general trend is that, given the same price elasticity of demand, the larger markups are observed in the regions with smaller number of competitors. Markups are higher the more price-inelastic is demand for electricity.

There are reasons to believe that the forecast prices calculated according to the Cournot model reflect an upper bound of price rather than a mean outcome. First, there is broad agreement that the cost of power generation will go down as the industry is deregulated and competitive market incentives reduce overcapitalization and increase efficiency. Second, according to Green (1999), observations of the English spot market for electricity show that generators can raise prices well above marginal costs; however, if generators sell the entire volume of output forward, the incentive to raise prices above marginal cost disappears.⁹ If risk-averse generators seek to hedge their risk by entering forward contracts, prices of electricity may converge to marginal costs. Third, simulation results show that the low-cost generators operating at maximum capacity can supply enough power to meet baseload demand, while higher-cost utilities sell power only at peak demand. If, however, low-cost producers attempt to exercise market power by rising prices, the higher-cost producers

⁹ The market rules in California until the late 2000 did not allow electric utilities to enter long-term contracts, which left electric utilities without a mechanism to smooth the price spikes during the periods of particularly high demand.

will be able to compete for the baseload demand as well. If the low-cost producers prefer to protect their market share, their ability to exercise market power may be limited.

Recent developments in the deregulated markets for electricity deserve some additional comments.

1. Several deregulated electricity markets became a reality in the U.S., and the oldest two of them, California and PJM, have been operating for more than two years. The deregulated electricity markets are characterized by great price volatility.¹⁰ The annual duration curves for PJM show that from 80 to 90 percent of the time electricity prices stay at the level of short-run marginal costs, however, in the remaining 10 percent of time prices spike to levels that exceed costs of generation by huge margins.¹¹ This indicates that generators have only a very short time to earn return on capital. In California, wholesale electricity prices stayed at the level consistent with short-run marginal costs from the time the market was enacted up until the year 2000. Starting in May 2000, prices in California have soared to much higher levels, around \$100/MWh on average with peaks at \$750/MWh,¹² and did not go down ever since. The popular explanation of the price spiral was that the increase in demand for electricity overstretched the available generation capacity (for the past 12 years there were no capacity additions in California). However, California historically relied on imported electricity (as much as 25 percent of the power consumed in California comes out-of-state), and enjoyed relatively low prices. Another plausible explanation of the price shock is market

¹⁰ Farney (2001) reports that price volatility in the wholesale electricity markets usually ranges between 400 percent and 600 percent, and frequently spikes above 1000 percent.

¹¹ These data are available at http://www.pjm.com/pub/market_monitoring/indices/lmpfdi_monthly/index.html

¹² Source: archives posted at <http://www.caiso.com/surveillance/pricedata/>

power. California's electricity industry is highly concentrated. As much as 80 percent of the generation capacity there is owned by only 5 firms.¹³ The period of generally low prices from 1998 until early 2000 might have been used by California generators to learn the oligopoly game.

2. Divestiture of power plants changed the ownership of generation. In most cases, however, plants have been sold in large bundles, which had little effect on market concentration in the industry.

3. The new entry in power generation affects the supply curve of the market. Most of the capacity currently planned or built uses natural gas as a fuel. The generation cost of the new entry is thus sensitive to the price of the natural gas. While the operating cost of the new plants may not be the lowest in the system (running costs of hydro and some nuclear plants are lower), the new entry fits in the middle of the range, thereby "stretching" the flat portion of the supply curve. There is no way to draw a general conclusion on whether the new entry will lower the peaking price of power. On one hand, the new units increase the available low-cost capacity, on the other hand, the new units may be largely committed to supply the growing baseload demand for electricity.

¹³ According to Sioshansi (2000), the owners and their respective market shares are Pacific Gas & Electric, 24 percent, Los Angeles Department of Water and Power, 16 percent, AES Corp., 16 percent, Reliant Energy, 13 percent, Duke Energy, 11 percent.

2.5. Concluding Remarks.

This study describes a method of forecasting electricity prices in a market open to retail competition. The results of both Bertrand and Cournot types of oligopoly are presented. The paper shows how the translog cost function can be used in the Cournot model. The method is applied to estimate equilibrium prices in the markets for electric power bounded by existing NERC regions. The projected prices are compared with those under regulation. The degrees of market power are assessed for every market.

Results of the empirical study indicate that the predicted degree of market power varies from one NERC region to another. If the retail market for electricity operates as perfectly competitive, prices for electricity consumers will decrease from their regulated level in most regions. If producers are able to exercise market power as Cournot oligopolists, unregulated markets will result in prices higher than those under regulation. In the latter case, the price markups above the competitive level vary from one region to another. These markups are positively correlated with average total costs of producers and negatively correlated with price elasticity of market demand for electricity. The degree of market concentration has a lesser effect on the Cournot price because several mechanisms may limit the ability of low-cost producers to exercise market power.

Chapter 3. Divestiture Prices of Power Plants as Indicators of Expected Market Power

3.1. Background on Divestiture in the U.S. Electric Utility Industry.

The structure of the electric utility industry has been rapidly changing in the past few years. The federal and state legislative initiatives promoting the deregulation of the industry coincided with an unprecedented wave of mergers, acquisitions, and sales of power generation plants. The ongoing divestiture¹⁴ of generation assets by investor-owned utilities (IOUs) has a profound effect on the electricity industry as it changes from a business dominated by vertically integrated firms to one in which the market for power generation is separated from the market for power transmission. A 1999 report by the Energy Information Administration (EIA 1999, Chapter 6) shows that from late 1997 through September 1999 32 percent of the generation-owning IOUs had sold or were in the process of selling generation capacity. According to the report, the generation capacity involved in the transition represented 17 percent of the total capacity owned by the U.S. electric utilities at that time.

Industry observers indicate several factors that drive the divestiture. Legislation passed in some states (e.g. California, Connecticut, Maine, New Hampshire, Rhode Island) explicitly requires electric utilities to sell some or all of their power plants (EIA, 1999). Moreover, utilities may use the sales of generation assets to reduce the firm's risk. Many divestiture press releases indicate that

¹⁴ Divestiture conducted by electric utilities is defined here as the sale of assets to other companies or the transfer of assets to non-utility subsidiaries.

utilities use the sales proceeds to reduce their long-term debt as well as stranded costs.¹⁵ Additional risk factors that may arise from the lack of management's experience in competitive markets, uncertainty about future prices for electricity and prices of inputs for its generation, as well as the uncertainty about future regulatory policy (Douglas and Starkov, 2000). Another popular explanation of divestiture is that a bull market for power plants makes the opportunities to sell too attractive for electric utilities to forego.

For example, press releases summarized in EIA (1999) indicate that plants have been sold at multiples of their book values ranging from 1.5 for coal plants to 2.5 for hydro plants. Such information has fueled speculation that in order to recover their investments the buyers of the power plants must be expecting high prices of electricity in the future. The prices may be inflated, perhaps, by the market power of a few major owners of the generation. Contrary to that, the data collected by Douglas and Starkov (2000), based on the book values from FERC Form 1, show that the prices paid for most power plants do not exceed their book values by more than 30 percent (see Table 3.1). Nuclear plants are selling for much less than their book values.

Although widely used as a yardstick, book value is the sum of depreciated past capital expenditures, and as such is a poor measure of market valuation of generation assets. A better

¹⁵ Stranded cost can be defined as the value of past investments that will be irrecoverable under future market conditions. Generation-related stranded costs of electric utilities are typically twofold. One part of them comes from building excess generation capacity, another from the long-term obligations to buy power from independent power producers. The stranded costs associated with generation assets of a particular utility may be computed as the difference between the accounting value of those assets (book value) and their "fair market value".

Table 3.1. Characteristics of Plant Sales Occurred From 1997 Until August 2000

Fuel	Number of Plant Shares Sold	Average Capacity, MW	Average Price Per Kilowatt of Capacity	Market-to-Book Ratio
Hydropower	168	20	700.2	1.38
Coal	64	530	558.8	1.27
Gas	63	532	285.5	0.98
Geothermal	1	1354	157.3	0.25
Oil	55	202	478.1	1.39
Nuclear	8	453	75.9	0.07

Source: Book values from FERC Form 1, prices from press releases.

measure should take into account future expected profits. In this paper, the theory of real options is applied to infer what expected future prices of electricity would justify the amounts paid for power plants in divestiture sales. The findings are that under plausible assumptions new power plant owners will have to rely on market power to earn attractive rates of return on their investment. In general, power plants commanded high prices, and electric utilities selling them were generously compensated.

3.2. Mathematical Background on Dynamic Option Pricing.¹⁶

This section presents a mathematical derivation of the dynamic option pricing equation suitable to estimate threshold price of commodity that triggers investment in a project with

¹⁶ The material of this section draws on the monograph by Dixit and Pindyck (1994) who presented the elements of the method described below in various parts of their book. The idea that a production asset can be valued as a set of options was stated by Marcus and Modest (1984) and developed in detail by McDonald and Siegel (1985).

known investment value and known operating cost. We assume that investment commitment is irreversible, and the project value is uncertain. In this situation, an option to postpone investment to receive more information about future value of the project becomes valuable. We will show how the optimal investment rule can be found using contingent claims analysis under the assumptions of both infinite and finite lifetime of the project. Finally, we will provide comparative statics results.

We consider the following investment problem. An investor has the opportunity to purchase a plant that produces marketable output. The uncertain future demand for output translates into uncertainty about the future price of output. The investor can purchase the plant immediately, or postpone the investment. If the investor delays the decision to invest, more information about future prices will arrive, although the uncertainty about the prices in the periods ahead will *never* disappear. Delaying the investment results in missed profits from the plant's operation. Once made, however, the investment is irreversible, i.e. the investor cannot recover the funds spent if the project turns out to be less profitable than expected. Given these conditions, the option to invest in the project at any future date has some worth because of the uncertainty about the future value of the asset the holder of such an option can obtain.

We will assume that the plant in our problem is a price-taker in the market for output. The operating cost of the plant is a constant C . The plant's operation can be scheduled to avoid losses. If the price of output falls below C , the plant can be costlessly shut down. Once the price P is above C the plant can be costlessly restarted. There is no fixed cost of running the plant, i.e. when the plant is

shut down the cost of its ownership is zero. The shutdown or restart of the plant do not affect the value of the operating cost. Thus, the profit flow from the plant at any instant is given by

$$p = \max [P_t - C, 0]. \quad (3.1)$$

The price is a random variable that evolves continuously according to the geometric Brownian motion with drift.

$$dP = \mathbf{a}Pdt + \mathbf{s}Pdz \quad (3.2)$$

Equation (3.2) indicates that the expected value of P grows at the constant trend rate $\mathbf{a}$. The standard deviation of the growth rate is the constant $\mathbf{s}$, while dz is the random increment of the Wiener process.¹⁷ The changes in market price are specified as the nonstationary Wiener process, as opposed to a mean-reverting process, for analytical tractability of the model.¹⁸

In this framework, the valuation of the plant V is analogous to evaluation of a derivative asset whose value depends on P . The problem of valuation of the firm can be solved by either dynamic

¹⁷ The Wiener process is a continuous time version of a random walk. The random variable z evolving according to the Wiener process has the following properties.

a) At any i th time interval ahead of the current interval t , the probability distribution for z_{t+i} depends only on the current value of z_t , i.e. the Wiener process is a particular case of the Markov process.
b) The changes in z are normally distributed with variance that increases linearly with time. This can be written formally as $dz = \mathbf{e}\sqrt{dt}$, where $\mathbf{e} \sim N(0, 1)$.
c) The increments of the Wiener process are serially uncorrelated, i.e. the p.d.f. of dz_t at any time interval t is independent of any other non-overlapping time interval s . This property is implied by the condition $E[\mathbf{e}\mathbf{e}] = 0$ for $t \neq s$.

¹⁸ As indicated by McDonald and Siegel (1985) and Dixit and Pindyck (1994, p.78), if the underlying random variable in the model of this type is assumed to be mean-reverting, the analytical expression for the valuation formula is not obtainable.

programming or by contingent claims analysis. Both methods produce identical results. Here we will follow the method of contingent claims.¹⁹

Further, let us assume that the output of the plant is traded directly as an asset in financial markets.²⁰

As any marketable asset, the output of the plant will be held by investors only if it provides a rate of return sufficient to compensate the owner for the risk. Assume that the return on the output comes in the form of the expected price appreciation $\mathbf{a}$ and a rate of dividend $\mathbf{d}$.²¹ The total expected rate of return is denoted by $\mathbf{m} = \mathbf{a} + \mathbf{d}$. Assuming the exogenously given risk-free rate r , we can find $\mathbf{m}$ using the CAPM

$$\mathbf{m} = r + (r_m - r) \frac{\mathbf{s}}{\mathbf{s}_m} \mathbf{r}_{Pm}, \quad (3.3)$$

where r_m is the expected return on the market portfolio, $\mathbf{s}_m$ is the standard deviation of the return on the market portfolio, $\mathbf{s}$ is the standard deviation of the return on P , $\mathbf{r}_{Pm}$ is the

¹⁹ McDonald and Siegel (1985) pointed out that such a project can be presented as a set of European call options. Exercising such an option means paying C to receive P that is realized during that instant. The project can be valued at any instant using a Merton variation of the Black and Scholes formula (the stock pays a dividend at a rate $\mathbf{d}$ and then summing those values by integrating over time. The valuation formulas derived using dynamic programming or contingent claims analysis, however, are better suited for purposes of the present analysis.

²⁰ In fact, the contingent claims analysis applies as long as the financial markets allows us to construct a portfolio of assets whose composition is continuously adjusted so that the value of the portfolio is perfectly correlated with the stochastic process for P . This eliminates the requirement for the product to be traded directly in financial markets.

²¹ Such a dividend on a stored commodity is commonly called “convenience yield”.

coefficient of correlation between returns on P and the market portfolio m .

Now let us consider a portfolio that contains one unit of the plant valued at V and a short position of n units of the plant's output each priced at P . The value of n is chosen in such a way to make the portfolio riskless. If this portfolio is held for a short interval of time dt , it will pay the riskless return

$$r(V - nP)dt. \quad (3.4)$$

The total return on this portfolio consists of the "sure" dividend and the stochastic capital gain.²² The dividend comes in the form of the profit flow $\mathbf{p}t$ less the return that has to be paid to the holder of the long position of the portfolio $n\mathbf{d}Pdt$. The net dividend is therefore

$$(\mathbf{p} - n\mathbf{d}P)dt. \quad (3.5)$$

The (stochastic) capital gain of the portfolio accrued over the time interval dt is

$$dV - ndP. \quad (3.6)$$

Because V is a function of the random variable P that follows the stochastic process described by (3.2), the total differential of V according to Ito's lemma is

$$dV = \left[\mathbf{a} \frac{\mathcal{V}}{\mathcal{P}} + \frac{1}{2} \mathbf{s}^2 P^2 \frac{\mathcal{V}^2}{\mathcal{P}^2} \right] dt + \left[\mathbf{s} P \frac{\mathcal{V}}{\mathcal{P}} \right] dz. \quad (3.7)$$

²² As Dixit and Pindyck (1994) pointed out, rigorously speaking, the dividend $\mathbf{p}P$ can randomly change even in the short interval dt . However, that change is of magnitude dt^2 and it can be ignored.

Using (3.2) and (3.7), one can write the expression for the capital gains of the riskless portfolio as follows²³

$$\left[\alpha P \left(\frac{V}{P} - n \right) + \frac{1}{2} \sigma^2 P^2 \frac{V^2}{P^2} \right] dt + \left[\sigma P \left(\frac{V}{P} - n \right) \right] dz. \quad (3.8)$$

The riskless portfolio must be independent of the idiosyncratic risk, i.e. we need to choose n so that the term dz disappears from the equation (3.8). The value of n that makes the portfolio riskless is V/P .

The capital gain of the riskless portfolio is then specified as

$$\frac{1}{2} \sigma^2 P^2 \frac{V^2}{P^2} dt. \quad (3.9)$$

Combining (3.5) and (3.9), the total return on the portfolio is given by

$$\left[P - dP \frac{V}{P} + \frac{1}{2} \sigma^2 P^2 \frac{V^2}{P^2} \right] dt. \quad (3.10)$$

Equating this to the riskless return (3.4), we can obtain the differential equation for the value of the project

²³ The terms that go to zero faster than dt were ignored in the expression (3.8).

$$\frac{1}{2} \sigma^2 P^2 \frac{\mathcal{I}^2 V}{\mathcal{I} P^2} + (r - \delta P) \frac{\mathcal{I} V}{\mathcal{I} P} - rV + P = 0 \quad (3.11)$$

Equation (3.11) can be identified as a second-order differential equation linear in the dependent variable V and its first and second derivatives. Its general solution can be expressed as a linear combination of any two independent solutions. It can be seen by substitution that a function of the form $\mathcal{I} P^b$ satisfies the equation. Dixit and Pindyck (1994) show that the homogenous part of equation (3.11) has two independent solutions P^{b_1} and P^{b_2} , where b_1 and b_2 are the roots of the following quadratic equation

$$\frac{1}{2} \mathbf{s}^2 \mathbf{b}(\mathbf{b} - 1) + (r - \mathbf{d}\mathbf{b} - r = 0. \quad (3.12)$$

In terms of the parameters r , $\mathbf{s}$ and $\mathbf{d}$ the roots of equation (3.12) are

$$\mathbf{b}_1 = \frac{1}{2} - \frac{(r - \mathbf{d})}{\mathbf{s}^2} + \sqrt{\left[\frac{(r - \mathbf{d})}{\mathbf{s}^2} - \frac{1}{2} \right]^2 + \frac{2r}{\mathbf{s}^2}} \quad (3.13)$$

$$\mathbf{b}_2 = \frac{1}{2} - \frac{(r - \mathbf{d})}{\mathbf{s}^2} - \sqrt{\left[\frac{(r - \mathbf{d})}{\mathbf{s}^2} - \frac{1}{2} \right]^2 + \frac{2r}{\mathbf{s}^2}} \quad (3.14)$$

Moreover, the roots $\mathbf{b}_1$ and $\mathbf{b}_2$ satisfy the following conditions: $\mathbf{b}_1 > 1$, $\mathbf{b}_2 < 0$.²⁴

²⁴ This can be easily demonstrated mathematically. Denote $x = \frac{(r - \mathbf{d})}{\mathbf{s}^2} - \frac{1}{2}$. Then rewrite the expressions for $\mathbf{b}_1$

and $\mathbf{b}_2$ in terms of x as follows $\mathbf{b}_1 = 1 - x + \sqrt{x^2 + 2r/\mathbf{s}^2}$, $\mathbf{b}_2 = 1 - x - \sqrt{x^2 + 2r/\mathbf{s}^2}$. It is apparent that the expression under the radical is positive and greater than x , which renders $\mathbf{b}_1 > 1$ and $\mathbf{b}_2 < 0$.

The nonhomogenous part of equation (3.11), however, is defined differently depending on whether P is less or greater than C , so the equation has to be solved separately for the conditions $P < C$ and $P > C$.

In the region $P < C$ where $\pi(P) = 0$ the solution has the form

$$V(P) = kP^{b_1} + mP^{b_2}. \quad (3.15)$$

In the region $P > C$ the solution is

$$V(P) = aP^{b_1} + bP^{b_2} + P/d - C/r. \quad (3.16)$$

Consideration of limiting cases helps to impose additional restrictions on the solution. When $P \rightarrow 0$, the value of P must be in the region $P < C$, and the plant is shut down. The chance of P quickly rising above C is very small, so the expected present value of the profits must be close to zero, and so should the value of the project, i.e. $V(0) = 0$. For this condition to be satisfied, it must be that $m = 0$. Further, Dixit and Pindyck (1994, pp.181-182) demonstrate that when speculative bubbles are ruled out, the term a must be equal to zero. Thus, the solution becomes

$$V(P) = \begin{cases} kP^{b_1}, & \text{if } P < C \\ bP^{b_2} + P/d - C/r, & \text{if } P > C \end{cases} \quad (3.17)$$

Because the Brownian motion is a continuous stochastic process, at the point $P = C$ both equations must be equal. Additionally, the function $V(P)$ must be continuously differentiable across C .²⁵ Taking into account these conditions and evaluating the function and its derivative at C , we obtain

$$kC^{b_1} = bC^{b_2} + C/d - C/r \quad (3.18)$$

$$b_1 k C^{(b_1 - 1)} = b_2 b C^{(b_2 - 1)} + 1/d \quad (3.19)$$

The equations solved simultaneously yield the solution for the parameters b and k . Here we state only the expression for b , as only b is important for the further exposition

$$b = \frac{C^{(1-b_2)}}{(b_1 - b_2)} \left(\frac{b_1}{r} - \frac{b_1 - 1}{d} \right) \quad (3.20)$$

To create a parameter independent of C , we define q as

$$q = \frac{1}{b_1 - b_2} \left(\frac{b_1}{r} - \frac{b_1 - 1}{d} \right) \quad (3.21)$$

so that $b = qC^{(1-b_2)}$.

²⁵ A heuristic argument is provided in Dixit (1993, Section 3.8); for a rigorous proof, see Karatzas and Shreve (1988, Theorem 4.4.9)

Next, we turn to evaluating the option to invest in the project $F(P)$. This can be done using the contingent claims valuation again, only now the riskless portfolio will consist of one option to invest and a short position of $n = \frac{\mathcal{I}F}{\mathcal{I}P}$ units of the output. Using the same reasoning as above, we can derive the differential equation

$$\frac{1}{2} \sigma^2 P^2 \frac{\mathcal{I}^2 F}{\mathcal{I}P^2} + (r - \delta) P \frac{\mathcal{I}F}{\mathcal{I}P} - rF(P) = 0. \quad (3.22)$$

Unlike equation (3.11) for the value of the project, the equation (3.22) for the option value is homogenous. The reason for this is that the option has no dividend or profit flow. Thus, the solution of the equation (3.22) is a linear combination of any two linearly independent solutions, i.e.

$$F(P) = gP^{b_1} + hP^{b_2}. \quad (3.23)$$

Again, we can obtain additional information about the coefficients of equation (3.23) by considering the limiting behavior of $F(P)$. If the price is very low, it is very unlikely that it will soon rise to the level at which it will be optimal to invest in the project, i.e. to exercise the option, so the option to invest should be almost worthless when P is near zero. To ensure that the value of the option goes to zero as P goes to zero, the coefficient h of the negative power of P should be equal to zero.

On the other extreme, when the price reaches the threshold P^* at which it is optimal to invest in the project, an investor will acquire an asset of value $V(P^*)$ by paying the exercise price (the investment

cost of the project) I . At the exercise price P^* , the value of the option must equal the net value of the asset obtained as a result of exercising the option,²⁶ i.e.,

$$F(P^*) = V(P^*) - I. \quad (3.24)$$

Secondly, the theoretical considerations omitted here for brevity,²⁷ imply that functions $F(P)$ and $V(P)$ should meet tangentially at P^* , otherwise, one could do better by exercising the option at a different point. This can be written as

$$\frac{\partial F(P^*)}{\partial P} = \frac{\partial V(P^*)}{\partial P}. \quad (3.25)$$

Now we can apply conditions (3.24) and (3.25) to the functional forms for $V(P)$ and $F(P)$ just obtained noting, that the option to invest will be exercised only if the project makes a profit, i.e. when $P^* > C$.

$$g(P^*)^{b_1} = qC^{(b_1-1)}(P^*)^{b_2} + P/d - C/r - I \quad (3.26)$$

$$b_1 g(P^*)^{(b_1-1)} = b_2 qC^{(b_1-1)}(P^*)^{(b_2-1)} + 1/d \quad (3.27)$$

The solution of this system of equations for P^* is

²⁶ This is called a “value-matching condition” by Dixit and Pindyck.

²⁷ Dixit in Pindyck (1994) refer to this as a “smooth-pasting condition”. See Ch. 4, Appendix C of their book and references thereof for details.

$$(\mathbf{b}_1 - \mathbf{b}_2)qC^{(1 - \mathbf{b}_2)}(P^*)^{\mathbf{b}_2} + (\mathbf{b}_1 - 1)P^*/\mathbf{d} - \mathbf{b}_1(C/r + I) = 0, \quad (3.28)$$

which is the valuation equation used in the empirical analysis. The equation (3.28) solved numerically for P^* provides an estimate of the average price of electricity that buyers of power plants should expect to earn the market rate of return $\mathbf{m}$ on their investment.

Dixit and Pindyck (1994, p. 191) claim that the equation (3.28) has a unique positive solution for P^* that is larger than the full Marshallian cost (the sum of the operating cost plus return on invested capital), i.e.

$$P^* > C + rI. \quad (3.29)$$

We will use this property in the empirical part of the study to identify the region where the roots of the equation (3.28) can be found.

The assumption about the plant's infinite life can be relaxed. One can think of the project functioning a known number of periods T . The plant's operation cost and output remain constant during the lifetime. Dixit and Pindyck (1994, p.205) show that in this case the value of the expected profits has to be discounted at the rate $(1 - e^{-\mathbf{d}T})/\mathbf{d}$ rather than $1/\mathbf{d}$ for the infinite-lived project. The certain stream of the operating cost incurred during the finite lifetime of the plant has to be discounted at the

rate $(1 - e^{-rT})/r$ instead of $1/r$ in perpetuity. It is easily seen that the finite-life discounting is just a special case of the infinite-life discounting. The finite-life rates become identical to the perpetual ones when T is taken to infinity.

As before, the general form of the solution for the value of the asset will be different depending on the relationship between the price and the operating cost

$$V_T(P) = \begin{cases} k_T P^{b_1}, & \text{if } P < C \\ b_T P^{b_2} + \frac{P(1 - e^{-dT})}{d} - \frac{C(1 - e^{-rT})}{r}, & \text{if } P > C \end{cases} \quad (3.30)$$

Taking into the account that the function $V_T(P)$ is continuous and smooth at the point where $P = C$, we can write the conditions similar to (3.18) and (3.19) and obtain the value of b_T .

$$b_T = \frac{C^{(1-b_2)}}{(b_1 - b_2)} \left(\frac{b_1}{r} - \frac{(b_1 - 1)(1 - e^{-dT})}{d} \right) \quad (3.31)$$

Again, for convenience we define the parameter q_T so that $b_T = q_T C^{(1-b_2)}$.

We will assume that the option to invest gives the right to purchase a finite-lived project just once, in which case the option value will be expressed exactly as in the case of the perpetual project, i.e.,

$$F_T(P) = g_T P^{b_1}.$$

After combining the value-matching and the smooth-pasting conditions, the equation for the investment threshold P^* is found as

$$(\mathbf{b}_1 - \mathbf{b}_2)q_T C^{(1 - \mathbf{b}_2)} (P^*)^{\mathbf{b}_2} + (\mathbf{b}_1 - 1)(1 - e^{-d^T})P^*/\mathbf{d} - \mathbf{b}_1(C(1 - e^{-rT})/r + I) = 0, \quad (3.32)$$

which is finite-lifetime analog of expression (3.28) above.

Comparative statics

The algebraical expressions of total derivatives of P^* with respect to other variables of the equations (3.28) and (3.32) are given in the Appendix D. Rather than analyzing signs of these highly complex expressions, we will provide intuitive explanation of the effects of various parameters found in the expressions (3.28) and (3.32) on the threshold price P^* .

We begin by reiterating the fact that the stochastic price P and the stochastic value of the project $V(P)$ are positively related, as seen, for example, in the expressions (3.17). Because of this, comparative static results obtained for the value of the project hold for the price as well.

An increase in the risk-free rate r reduces the present value of the cost of the investment I but does not reduce its payoff discounted at the different rate $\mathbf{d}$ so the value of the option to invest $F(P^*)$ goes up as r rises. This increases the incentive to postpone investment. To induce immediate investment, the expected output price P^* must go down, i.e.

$$\frac{\partial P^*}{\partial d} < 0. \quad (3.33)$$

According to the definition of the dividend accrued the holders of the plant's output, $d = m - a$, an increase in d may be a result of increasing systematic risk of the plant's output m or a decrease in the expected rate of appreciation of the plant's output a . The greater the rate of the dividend accrued the holders of the plant's output d the more one forgoes by not exercising the option to invest in the asset. On the other hand, large d may indicate that the expected rate of increase of P is small, which will reduce the value of the claim on future production. Thus, there are two opposite effects of d on P^* . Dixit and Pindyck (1994, p. 193) claim that the sign of the combined effect is positive. This assertion is verified by scenario analysis in the empirical section of the essay.

$$\frac{\partial P^*}{\partial d} > 0. \quad (3.34)$$

The increased uncertainty about future prices, expressed as increased s , the greater the value of the investment opportunity. The value-matching condition $F(P^*) = V(P^*) - I$ implies that when the value of the option to purchase the asset $F(P^*)$ increases, the value of the asset $V(P^*)$ increases too, and so does the exercise price P^* . Therefore

$$\frac{\partial P^*}{\partial s} > 0. \quad (3.35)$$

The comparative statics results presented above have to be interpreted with caution because the parameters $\mathbf{d}$, r and $\mathbf{s}$ are interdependent. The return on the plant's output comes in the form of the expected price appreciation $\mathbf{a}$ and a rate of dividend $\mathbf{d}$. The total expected rate of return is $\mathbf{m} = \mathbf{a} + \mathbf{d}$. If the investors are to hold a claim on a firm's output, the total return on such an asset must be equal to the risk-adjusted market rate which can be found from the CAPM (Equation 3.3). The parameters $\mathbf{a}$, $\mathbf{d}$ and $\mathbf{s}$ are related in the following way

$$\mathbf{a} + \mathbf{d} = r + (r_m - r) \frac{\mathbf{s}}{\mathbf{s}_m} r_{Pm} \quad (3.36)$$

So, for example, an increase in $\mathbf{s}$ must be compensated by the increase in $\mathbf{d}$ if the other parameters remain constant. The sign of the change in P^* will then be ambiguous.

The effect of the investment cost I on the exercise price is easier to demonstrate when assuming away the operating cost. When $C = 0$ the value-matching and smooth-pasting conditions (3.26) and (3.27) become

$$g(P^*)^{b_I} = P/\mathbf{d} - I \quad (3.37)$$

and

$$b_I g(P^*)^{(b_I - 1)} = 1/\mathbf{d} \quad (3.38)$$

Solving for P^* , we obtain

$$P^* = \frac{b_l}{b_l - 1} d. \quad (3.39)$$

Since $b_l > 1$ and $d > 0$, it is apparent that

$$\mathcal{P}^* / \mathcal{I} > 0. \quad (3.40)$$

The presence of operating costs raises the hurdle that the price of the output must clear before the project becomes profitable. The optimum exercise price of the investment option must be higher the higher is the running cost, i.e.

$$\mathcal{P}^* / \mathcal{C} > 0. \quad (3.41)$$

As discussed above, a limited lifetime of the project implies a higher discount rate of the future profits compared to the infinite-life case. To compensate for the higher discount rate the expected profit must be higher, *ceteris paribus*. Thus the exercise price of the limited-time option will be higher than that of the perpetual option, or mathematically

$$\mathcal{P}^* / \mathcal{I} < 0. \quad (3.42)$$

3.3. Description of Data and Empirical Implementation of the Model.

The application of the theory of real options to power plant divestiture sales involves several assumptions. The dilemma faced by an investor who decides whether to purchase a power plant put up for sale by an electric utility is as follows. Making an irreversible investment before the beginning of a full fledged market for electricity represents a risk because uncertain future prices for electricity make future profits uncertain and, ultimately, the future market value of the plant is uncertain as well. If the investment is postponed, more information about the future will arrive, although the uncertainty will never be resolved completely. At the same time, the delayed investment results in missed profits from the plant's operation. The plant can be shut down at times when the market price for output is lower than the plant's operating cost. When a rational investor decides to invest in such a project, it must be that the expected future value of the plant allows the investment to earn a return on capital equal to or above the competitive risk-adjusted rate. When the price of output is a dominant source of uncertainty about future profits,²⁸ the expected future value of the project is directly linked with the expected future price.

Further, we assume that the operating costs in real terms and output of the plants in the future will remain where they were on average for the last 3 years of observations available (1995 to 1997).

²⁸ The option pricing model used here implies constant production cost. In reality, the cost of plant's operation is subject to variation according to conditions in input markets, while the output price is determined by supply and demand conditions for electricity. The largest component of a power plant's operating cost is the cost of fuel. Observations of wholesale electricity market in the U.S. reveal that volatility of electricity prices is much greater than the volatility of fuel prices (see, for example, Farney, 2001). It is the *difference* between the price and cost that drives the model, and as Dixit and Pindyck (1994, Chapter 2.5) suggest, treating the more volatile variable of the two as the only source of uncertainty provides a good first approximation.

If costs of power generation will fall in the future, as expected by some observers (e.g. EIA, 1999), the estimates of the threshold electricity price P^* will be biased upward. As a reference case, we use the model based on the assumption of infinite lifetime of the plants. Its results are contrasted with those obtained from the finite-lifetime model.

Complications for the empirical analysis arise from the fact that most power plants are sold in bundles containing a diverse mix of generation assets in terms of their type, vintage, and capacity. Most utilities own a large number of plants and sell them in bundles consisting of several plants. Only 18 sales made through the end of August 2000 consisted of a single plant. The largest bundle to date contained 74 hydroelectric plants sold by Niagara Mohawk to Orion Power Holdings. “Bundled” sales present a dilemma as to whether sales should be analyzed by transaction or by plant. If the sample is constructed by plant, individual characteristics of plants such as type, vintage, production cost, capacity, historical generation can be taken into account. The threshold price of electricity P^* computed on the per-plant basis applies to a homogenous asset. The drawback of the per-plant analysis is that the price paid for the bundle has to be arbitrarily divided among the various plants. The method of assigning the plant price according to its capacity applied here²⁹ does not capture the individual characteristics of plants that must affect their market value. The sample constructed on the per-transaction basis is free of arbitrary price assignments. Yet, as the composition of each bundle is unique, the valuation model will produce estimates of price thresholds that are less reliable for forecasting purposes. The application of the finite-lifetime formula has some

²⁹ This method of assigning price to individual plants is the same as used by the EIA.

arbitrary assumptions as to the time of capital recovery, rather than lifetimes of individual plants that are supported by engineering considerations. To circumvent the described dilemma, two separate samples have been constructed. One sample contains observations by plant, the other has observations by transaction.

The formulas for the optimum investment threshold given by (3.28) and (3.32) make it possible to infer what expected future prices of electricity would justify the prices paid for divested power plants.

$$(\mathbf{b}_1 - \mathbf{b}_2)qC^{(1-b_2)}(P^*)^{b_2} + (\mathbf{b}_1 - 1)P^*/\mathbf{d} - \mathbf{b}_1(C/r + I) = 0 \quad (3.28)$$

$$(\mathbf{b}_1 - \mathbf{b}_2)q_T C^{(1-b_2)}(P^*)^{b_2} + (\mathbf{b}_1 - 1)(1 - e^{-dT})P^*/\mathbf{d} - \mathbf{b}_1(C(1 - e^{-rT})/r + I) = 0 \quad (3.32)$$

The derivation of both equations was based on the premise that quantity of output produced is one unit per period. Thus, to convert the variables observed as annual totals to those measured in unit-per-period, they have to be divided by the annual output which converts all measurement units to the per-megawatt-hour basis.

In the plant-level sample, C is the plant's operating cost per megawatt-hour of output averaged over the last three years of observations. The per-unit investment cost of the plant I was constructed by multiplying the price per megawatt of capacity paid in the transaction by the capacity of the plant and dividing the product by the annual generation obtained as three-year average of historical observations.

In the transaction-level sample, C was found as the sum of operating costs across all the plants in the bundle divided by the aggregated output of the plants in the bundle. The value of I was found as the price paid per bundle divided by the combined average annual generation of the plants in the bundle. The plots of the functions (3.28) and (3.32) in terms of P look like asymmetric U-shaped curves. In the range of $P \geq 0$ these curves may have zero, one, or two roots. The roots that fall below the operating cost can be disregarded because the plant will turn out zero profit whenever $P^* < C$. This means that the constraint $P^* \geq C$ can be imposed when calculating the roots of the equations (3.28) and (3.32). The equations (3.28) and (3.32) were numerically solved for P^* using *Mathematica* software. An example of such a program is given in the Appendix E.

In addition to bundled sales, the ownership structure of the plants sold presents a challenge for analysis. Certain large plants are jointly owned by several utilities. Investors often wish to obtain the entire plant or the majority of its shares. On some occasions, all the utilities sell their shares at the same time (a large coal plant, Centralia, divided among eight utilities was sold to TransAlta in one piece). At some other times, different owners sell their shares at different times (e.g., Southern California Edison and Sierra Pacific Resources sold their shares of Mohave to AES Corp. in May 2000, while the Los Angeles Department of Water and Power announced its willingness to sell its share in August 2000). Still other utilities never sell their stake in the plants (an example is a large coal-fired plant Conemaugh divided among as many as nine utilities: GPU Inc. sold its 16.45 percent share to Sith Energy in November 1998; Potomac Electric Power sold its 9.72 percent share to Allegheny Energy and PPL Global in May 2000; the remaining utilities did not announce their intention to sell).

For the purposes of this paper, the transaction is defined as a deal between one seller and one buyer. For example, a large plant owned by four utilities sold to one buyer would constitute four transactions, a utility selling 20 plants at one time to two investors each getting 10 plants would count as two transactions. A sale of a plant is defined as a transfer of one plant share in one transaction. In the example above, a plant owned by four utilities sold to one buyer would count as sales of four plant shares with the capacity and generation assigned according to the shares of the owners.

The data set used in this study contains information on transaction dates and prices, as well as characteristics of the plants sold. The information about divestiture transactions was obtained from various sources, including the EIA *Electric Power Monthly*, trade press, company press releases and company Web sites. The plant-level data come from a database published by the Utility Data Institute that includes the information from the FERC Form 1 and other official sources. The data set explicitly excludes sales of nuclear plants. Considerations of decommissioning costs, costs of fuel, considerable regulatory uncertainty, etc., that usually accompany nuclear plant transactions may distort the price signal greatly. We have collected information on 66 transactions of non-nuclear plants involving 350 plants that occurred until August 2000. The earliest transaction in the set is the sale of coal-fired Fort Martin plant in West Virginia by Duquesne Light to AYP Energy in October 1996; the latest is the purchase of the Danskammer Point and Roseton coal-fired plants in New York by Dynergy, Inc. from three utilities: Central Hudson Gas & Electric, Niagara Mohawk, and Consolidated Edison. The total capacity sold exceeds 84.2 gigawatts. About half of this capacity (44 percent) was sold in the years 1999 and 2000. Steam-powered plants dominate the capacity

volume: 113 steam plants account for 70.5 gigawatt of capacity or 83.7 percent of the total capacity sold. The group of hydro plants has the largest count: 168 hydro-powered plants have been sold with total capacity of 4.2 gigawatt. The third group of plants, gas turbines, consists of 70 plants with total capacity of 9.5 gigawatt.

3.4. Results of the Analysis and Their Discussion.

The two samples grouped by transaction and by plant were analyzed, each under the two different scenarios of infinite and finite time for capital recovery. Both equations (3.28) and (3.32) have been solved under the same set of assumptions about parameters, namely: risk-free rate of return r is 5 percent, rate of return on capital d is 10 percent, standard deviation of future electricity price s is 0.2. The baseline number for s is supported by consideration of standard deviations of the daily returns on prices of several major power marketers during August 2000 (Tables 3.2 and 3.3). The use of daily returns on price to derive price volatility in practical applications is advocated by Farney (2001). According to Farney, the volatility of the S&P 500 index is about 20 percent. The scenario analysis was conducted to analyze the effects of changes in r , d and s on the magnitude of P^* .

In the model with the finite-lived assets, the following assumptions on lifetimes were used. When analyzing transactions, the asset lifetime represents the recovery time of the invested capital. The capital recovery periods considered in this case were 20, 35, and 50 years. In the analysis by plant, lifetimes of different plants were assigned individually. The remaining

Table 3.2. Next Day PowerTrax Index, Weighted Average, August 2000, \$/MWh

	1	2	3	4	7	8	9	10	11	14	15	16	17	18	21	22	23	24	25	28	29	30	31
Cinergy	42	44	44	37	57	86	107	51	36	58	55	34	27	25	25	24	25	31	38	49	66	53	64
Entergy	50	49	49	48	65	95	116	90	77	84	86	66	76	88	65	54	65	60	73	78	116	78	89
TVA	48	48	53	47	67	99	120	72	49	64	61	49	53	52	37	35	30	40	49	61	71	60	68
ComEd	43	42	43	36	53	75	88	49	33	54	57	34	27	25	26	24	26	29	37	48	55	48	55
PJM West	54	54	57	42	57	93	125	62	43	50	46	38	32	29	27	25	25	29	34	45	54	46	49
Palo Verde	483	518	388	257	212	198	200	169	129	223	231	218	229	166	209	210	218	224	195	220	217	181	102
Mid Columbia	457	479	321	200	185	168	163	145	125	199	195	177	196	163	191	197	223	255	231	230	226	159	100
COB	437	485	336	207	181	171	169	151	123	200	194	180	197	159	190	199	221	239	231	227	225	165	101
Four Corners	n.a.	525	n.a.	263	228	199	208	n.a.	n.a.	n.a.	n.a.	220	225	n.a.	n.a.	206	224	n.a.	n.a.	n.a.	214	n.a.	n.a.
Mead	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	195	n.a.	n.a.	n.a.	n.a.
NP 15	362	335	n.a.	165	157	139	131	n.a.	113	172	173	178	196	151	n.a.	n.a.	172	n.a.	181	188	n.a.	159	99
SP 15	390	387	352	233	186	180	179	154	120	200	210	194	207	150	187	191	201	218	187	210	208	168	99

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Source: *Restructuring Today*, September 5, 2000, US Publishing

Table 3.3. Rates of change from the previous day price (a.k.a. returns) and their standard deviations, August 2000

	2	3	4	7	8	9	10	11	14	15	16	17	18	21	22	23	24	25	28	29	30	31	<i>s</i>
Cinergy	0.05	0.00	-0.16	0.54	0.51	0.24	-0.52	-0.29	0.61	-0.05	-0.38	-0.21	-0.07	0.00	-0.04	0.04	0.24	0.23	0.29	0.35	-0.20	0.21	0.30
Entergy	-0.02	0.00	-0.02	0.35	0.46	0.22	-0.22	-0.14	0.09	0.02	-0.23	0.15	0.16	-0.26	-0.17	0.20	-0.08	0.22	0.07	0.49	-0.33	0.14	0.23
TVA	0.00	0.10	-0.11	0.43	0.48	0.21	-0.40	-0.32	0.31	-0.05	-0.20	0.08	-0.02	-0.29	-0.05	-0.14	0.33	0.23	0.24	0.16	-0.15	0.13	0.24
ComEd	-0.02	0.02	-0.16	0.47	0.42	0.17	-0.44	-0.33	0.64	0.06	-0.40	-0.21	-0.07	0.04	-0.08	0.08	0.12	0.28	0.30	0.15	-0.13	0.15	0.27
PJM West	0.00	0.06	-0.26	0.36	0.63	0.34	-0.50	-0.31	0.16	-0.08	-0.17	-0.16	-0.09	-0.07	-0.07	0.00	0.16	0.17	0.32	0.20	-0.15	0.07	0.26
Palo Verde	0.07	-0.25	-0.34	-0.18	-0.07	0.01	-0.16	-0.24	0.73	0.04	-0.06	0.05	-0.28	0.26	0.00	0.04	0.03	-0.13	0.13	-0.01	-0.17	-0.44	0.24
Mid Columbia	0.05	-0.33	-0.38	-0.08	-0.09	-0.03	-0.11	-0.14	0.59	-0.02	-0.09	0.11	-0.17	0.17	0.03	0.13	0.14	-0.09	0.00	-0.02	-0.30	-0.37	0.21
COB	0.11	-0.31	-0.38	-0.13	-0.06	-0.01	-0.11	-0.19	0.63	-0.03	-0.07	0.09	-0.19	0.19	0.05	0.11	0.08	-0.03	-0.02	-0.01	-0.27	-0.39	0.22
Four Corners				-0.13	-0.13	0.05						0.02				0.09							0.10
Mead																							
NP 15	-0.07			-0.05	-0.11	-0.06			0.52	0.01	0.03	0.10	-0.23						0.04			-0.38	0.22
SP 15	-0.01	-0.09	-0.34	-0.20	-0.03	-0.01	-0.14	-0.22	0.67	0.05	-0.08	0.07	-0.28	0.25	0.02	0.05	0.08	-0.14	0.12	-0.01	-0.19	-0.41	0.22

lifetime of each plant was computed by subtracting the number of years the plant was in service prior to the sale from the useful lifetime of the plant. There were two sets of assumptions about useful plant lifetimes. In the “short-lived” scenario, useful life of a steam plant was 50 years, a gas turbine would last 35 years, and a hydro plant would work for 100 years. The “long-lived” scenario allowed 75 years of useful life for steam plants, 50 years for gas turbines, and 150 years for hydro plants.

The values of P^* that solve the equations represent the lower bound of the price of electricity that will make the transaction attractive for the investor in the sense of earning the return on capital comparable with other assets on the market with similar risk. Every observation has a unique value of P^* . The resulting values of P^* should be comparable in magnitude with the operating costs of the plants. Because different types of plants tend to have vastly different operating costs, the plant sample was further divided into three subsamples: steam plants, gas turbines, and hydroelectric plants. Generally, gas turbines have the highest operating costs, while hydro plants have the lowest.

The descriptive statistics of the distributions of P^* for the infinite-horizon model are summarized in Table 3.4. The same results are presented in the form of histograms in the Figures 3.1 and 3.2. The Table 3.4 reflects the results of scenario analysis to study the effect of variations in the parameters r , d and s on the threshold price P^* . Within the range of values tested, the following results were obtained

$$\frac{\partial P^*}{\partial s} > 0, \quad \frac{\partial P^*}{\partial d} > 0, \quad \frac{\partial P^*}{\partial r} < 0, \quad \frac{\partial P^*}{\partial C} > 0, \quad \frac{\partial P^*}{\partial I} > 0, \quad \frac{\partial P^*}{\partial T} < 0.$$

**Table 3.4. Descriptive Statistics of Distribution of P^*
Infinite-Lifetime Model**

	Threshold Electricity Price P^* , \$/MWhr					Number of observations
	Mean	Median	Min	Max	Std. Deviation	
Scenario 1: Risk-free rate of return $r = 5\%$, Risk-adjusted return on capital $d = 10\%$, Standard deviation of future electricity price $s = 0.2$						
Analysis by Transaction	109.88	74.06	21.64	1347.50	176.62	66
Analysis by Plant	231.20	50.25	12.69	5370.24	546.57	353
• Steam Plants	84.97	54.69	27.77	1305.25	139.28	115
• Gas Turbines	914.01	666.67	17.50	5370.24	942.48	70
• Hydro Plants	46.80	29.37	12.69	647.10	71.09	168
Scenario 2: Risk-free rate of return $r = 5\%$, Risk-adjusted return on capital $d = 5\%$, Standard deviation of future electricity price $s = 0.2$						
Analysis by Transaction	91.14	59.28	16.07	1001.56	136.37	66
Analysis by Plant	175.23	42.74	9.22	3906.21	403.84	353
• Steam Plants	69.74	46.37	23.97	967.40	105.35	115
• Gas Turbines	680.61	491.83	12.88	3906.21	693.79	70
• Hydro Plants	36.79	22.67	9.22	482.87	56.64	168
Scenario 3: Risk-free rate of return $r = 5\%$, Risk-adjusted return on capital $d = 10\%$, Standard deviation of future electricity price $s = 0.4$						
Analysis by Transaction	140.20	91.55	31.24	1943.70	247.01	66
Analysis by Plant	327.40	61.39	18.54	7955.68	797.49	353
• Steam Plants	109.57	68.39	33.58	1889.22	197.72	115
• Gas Turbines	1318.6	970.94	25.57	7955.68	1382.84	70
• Hydro Plants	63.50	40.68	18.54	929.01	95.81	168
Scenario 4: Risk-free rate of return $r = 3\%$, Risk-adjusted return on capital $d = 10\%$, Standard deviation of future electricity price $s = 0.2$						
Analysis by Transaction	112.83	75.93	22.05	1373.11	180.26	66
Analysis by Plant	235.30	51.71	12.81	5414.98	553.25	353
• Steam Plants	87.43	56.51	28.62	1328.59	142.29	115
• Gas Turbines	927.82	676.56	17.76	5414.98	952.23	70
• Hydro Plants	47.97	30.13	12.81	660.33	72.87	168

**Figure 3.1. Distribution of P^* in the Infinite-Time Framework
Analysis by Transaction**

Assumptions:

Risk-free rate of return 5 percent

Risk-adjusted return on capital 10 percent

Standard deviation of electricity price 0.2

**Figure 3.2. Distribution of P^* in the Infinite-Time Framework
Analysis by Plant Share**

Assumptions:

Risk-free rate of return 5 percent

Risk-adjusted return on capital 10 percent

Standard deviation of electricity price 0.2

These empirical findings are consistent with mathematical results described above.

The data of the Table 3.4 show that although the means of the by-plant samples are generally higher than the mean of the samples grouped by transactions, there are vast differences among the sub-groups of plants: samples that include hydro- and steam plants have means lower than the means of the corresponding by-transaction samples. The medians of both by-transaction and by-plant distributions are smaller than the respective means, which indicates that the majority of the points are below the mean, i.e. the distributions are skewed to the right. The distribution of roots in the by-transaction sample has fewer outliers than that of the by-plant sample. This indicates that the variability of the transaction prices per unit of generation is lower than that of the prices per plant. This makes sense because the transactions contain diverse mix of assets.

The descriptive statistics of the finite-horizon model are presented in Tables 3.5 and 3.6. The distributions of P^* are presented graphically in the Figures 3.3. and 3.4. Table 3.5 summarizes the analysis by transaction, and the Table 3.6 presents the results of the analysis by plant. The number of solutions found with the finite-horizon model decreases in comparison to the infinite-horizon model. The shorter is the time horizon the more this effect is pronounced. Graphically, decrease in value of T shifts up the U-shaped graph of the function (3.32) in terms of P^* . For certain observations, this leads to absence of roots. The analysis of scenarios with different lifetimes shows that individual observations display inverse relationship between the magnitude of P^* and the length of the time period, exactly

**Table 3.5. Descriptive Statistics of Distribution of P^*
Finite-Lifetime Model. Transaction-Level Sample**

Capital Recovery Time	Threshold Electricity Price P^* , \$/MWhr					Number of solutions
	Mean	Median	Min	Max	Std. Deviation	
20 years	168.68	71.34	20.86	1296.24	357.07	12
35 years	90.30	50.71	20.56	1279.03	211.62	34
50 years	97.01	62.55	20.98	1305.80	178.65	55

Assumptions used in calculations:

Risk-free rate of return 5 percent

Risk-adjusted return on capital 10 percent

Standard deviation of future electricity price 0.2

**Table 3.6. Descriptive Statistics of Distribution of P^*
Finite-Lifetime Model. Plant-Level Sample**

	Threshold Electricity Price P^* , \$/MWhr					Number of solutions
	Mean	Median	Min	Max	Std. Deviation	
Scenario 1 Lifetimes: Steam Plant 50 yrs, GT Plant 35 yrs, Hydro Plant 100 yrs						
All Plants in Sample	420.55	33.20	11.94	8444.71	1078.24	233
• Steam Plants	143.52	50.92	18.90	1824.97	349.72	27
• Gas Turbines	1627.59	1070.13	25.77	8444.71	1754.63	54
• Hydro Plants	40.94	26.13	11.94	619.85	69.14	152
Scenario 2 Lifetimes: Steam Plant 75 yrs, GT Plant 50 yrs, Hydro Plant 150 yrs						
All Plants in Sample	228.85	41.45	12.54	5275.51	549.59	317
• Steam Plants	78.85	43.82	22.14	1240.85	146.43	86
• Gas Turbines	907.87	655.97	17.19	5275.51	940.81	64
• Hydro Plants	45.88	29.06	12.54	644.16	70.64	167

Assumptions used in calculations:

Risk-free rate of return 5 percent

Risk-adjusted return on capital 10 percent

Standard deviation of future electricity price 0.2

**Figure 3.3. Distribution of P^* in the Finite-Time Framework
Analysis by Transaction**

Assumptions:

Risk-free rate of return 5 percent

Risk-adjusted return on capital 10 percent

Standard deviation of electricity price 0.2

Investment capital recovery time 50 years

**Figure 3.4. Distribution of P^* in the Finite-Time Framework
Analysis by Plant Share**

Assumptions:

Risk-free rate of return 5 percent

Risk-adjusted return on capital 10 percent

Standard deviation of electricity price 0.2

Lifetime of the steam plant 50 years

Lifetime of the hydro plant 100 years

Lifetime of the gas-turbine plant 35 years

as implied by the comparative statics. All the distributions remain skewed to the right which means that the majority of points fall below the mean.

In the transaction-level sample, the number of observations that satisfy the finite-time model (3.32) varies from 18 to 83 percent depending on the assumed capital recovery time. In the plant-level sample, with its own set of lifetime assumptions, the number of observations that satisfy the finite-lifetime model is from 66 to 90 percent. Within the plant-level sample, the smallest percentage of solutions was found for the steam plants, the largest – for hydro-powered plants. The lower explanatory power of the finite-life model may be due to the fact that investors view power plants as virtually infinitely-lived. As Ellerman (1998) points out, it appears to be a wide-spread phenomenon that operation of power plants is extended far beyond the limits formerly considered useful lives. This is especially evident for coal- and gas-fired steam plants. Ellerman cites three causes of such a phenomenon. One is the difficulty to obtain a new plant license due to environmental regulation and active opposition of communities. Second, with the deregulation of electricity generation, recovery of the capital invested in new power plants is no longer guaranteed, which increases the risk of investment in new generation capacity, and thereby raises the cost of capital invested in new plants. The third, and most fundamental cause, according to Ellerman, is that recent improvements in diagnostics and declining maintenance costs substantially prolong the useful life of power plants. Given that maintenance costs increase at a decreasing rate as the plant ages, a certain level of maintenance may extend the lifetime of the plant indefinitely.

The values of the observation-specific lower bound of the electricity price that would make the sale attractive to the investors provide insight on whether the investors' spending is consistent with expectations of competitive electricity pricing or market power in the future deregulated environment. Using the calculated values of P^* and the observed operating costs C , we computed observation-specific price-cost margin indexes (PCMI). The PCMI data for the results of the infinite-lifetime model are presented in Table 3.7, and those for the finite-lifetime model are shown in Tables 3.8 and 3.9. These results suggest that owners of power plants have to keep quite high price-cost markups to earn a market rate of return of their investments.

Overall, the expectations of electricity prices expressed through the prices paid for power plants appear to be rather high. These prices are equivalent or greater than the oligopoly prices derived under conditions of inelastic demand in highly concentrated markets (see, for example, Table 2.3). The price-cost margins calculated using the option-pricing framework, generally, exceed those found under the oligopoly model (see Table 2.4). The finding of high expected electricity prices may have several possible explanations.

“The winner’s curse” of power plant buyers. Almost all power plants were sold through sealed-bid auctions. The prices observed in divestiture sales thus represent the highest bids, and not average willingness to pay across investors.

Power plant buyers may have overlooked values of their options. Instead of option pricing

**Table 3.7. Descriptive Statistics of Distribution of Price-Cost Margins
Infinite-Lifetime Model**

	Price-Cost Margin, %				
	Mean	Median	Min	Max	Std. Deviation
Scenario 1: Risk-free rate of return $r = 5\%$, Risk-adjusted return on capital $d = 10\%$, Standard deviation of future electricity price $s = 0.2$					
Analysis by Transaction	160.83	115.39	23.21	881.52	156.51
Analysis by Plant	361.51	223.88	23.21	1842.30	344.19
• Steam Plants	127.70	110.70	23.21	540.13	77.43
• Gas Turbines	521.39	518.79	67.23	1463.81	278.92
• Hydro Plants	454.94	298.37	57.24	1842.30	395.38
Scenario 2: Risk-free rate of return $r = 5\%$, Risk-adjusted return on capital $d = 5\%$, Standard deviation of future electricity price $s = 0.2$					
Analysis by Transaction	112.40	81.05	16.65	611.84	108.27
Analysis by Plant	251.43	155.89	16.65	1284.06	239.02
• Steam Plants	89.46	77.80	16.65	374.43	53.52
• Gas Turbines	361.90	359.64	47.61	1018.80	193.53
• Hydro Plants	316.28	207.24	40.63	1284.06	274.91
Scenario 3: Risk-free rate of return $r = 5\%$, Risk-adjusted return on capital $d = 10\%$, Standard deviation of future electricity price $s = 0.4$					
Analysis by Transaction	240.06	168.92	32.58	1359.58	241.82
Analysis by Plant	550.62	334.94	32.58	2859.90	535.39
• Steam Plants	188.45	161.82	32.58	826.52	118.59
• Gas Turbines	798.31	793.23	96.73	2269.02	434.12
• Hydro Plants	695.33	450.17	81.98	2859.90	616.12
Scenario 4: Risk-free rate of return $r = 3\%$, Risk-adjusted return on capital $d = 10\%$, Standard deviation of future electricity price $s = 0.2$					
Analysis by Transaction	167.88	122.41	25.01	887.77	158.17
Analysis by Plant	369.42	234.20	25.01	1821.55	341.83
• Steam Plants	134.63	117.51	25.01	551.57	79.44
• Gas Turbines	530.73	530.40	71.87	1454.99	277.22
• Hydro Plants	462.92	309.83	61.30	1821.55	390.60

**Table 3.8. Descriptive Statistics of Distribution of Price-Cost Margins
Finite-Lifetime Model. Transaction-Level Sample**

Capital Recovery Time	Price-Cost Margin, %					Number of solutions
	Mean	Median	Min	Max	Std. Deviation	
20 years	377.59	352.47	64.98	919.96	253.48	12
35 years	179.63	98.97	13.69	863.45	196.87	34
50 years	152.57	101.96	19.58	865.75	168.42	55

Assumptions used in calculations:
Risk-free rate of return 5 percent
Risk-adjusted return on capital 10 percent
Standard deviation of future electricity price 0.2

**Table 3.9. Descriptive Statistics of Distribution of Price-Cost Margins
Finite-Lifetime Model. Plant-Level Sample**

	Price-Cost Margin, %				
	Mean	Median	Min	Max	Std. Deviation
Scenario 1 Lifetimes: Steam Plant 50 yrs, GT Plant 35 yrs, Hydro Plant 100 yrs					
All Plants in Sample	562.42	364.28	26.62	5091.63	572.81
• Steam Plants	167.22	115.03	60.14	795.01	151.78
• Gas Turbines	1054.86	948.53	51.58	5091.63	759.37
• Hydro Plants	457.68	296.96	26.62	1833.49	414.55
Scenario 2 Lifetimes: Steam Plant 75 yrs, GT Plant 50 yrs, Hydro Plant 150 yrs					
All Plants in Sample	375.54	249.39	14.85	1841.14	362.27
• Steam Plants	106.96	80.24	14.85	508.54	88.30
• Gas Turbines	539.57	530.04	50.74	1643.64	298.79
• Hydro Plants	450.99	293.34	46.91	1841.14	398.10

Assumptions used in calculations:
Risk-free rate of return 5 percent
Risk-adjusted return on capital 10 percent
Standard deviation of future electricity price 0.2

model, power plant buyers may have used a discounted cash flow approach to arrive at the value of their price bids. The discounted cash flow analysis implies that the price paid for the plant should be less or equal the discounted present value of future operating profits. The future price of the electricity in the discounted cash flow model is assumed to always take its forecast value with no variation around the forecast. The discounted cash flow approach ignores the fact that investment in real assets is at least partially irreversible, i.e. the some or all the cost of investment becomes sunk once the investment is made. In addition, discounted cash flow model ignores the opportunity to learn more about future postponing the decision to invest. The dynamic option pricing theory, however, predicts that the *investment cost must always be lower than expected value of the project*. This is evident, for example, from the equation (3.24).

$$I = V(P^*) - F(P^*). \quad (3.24)$$

According to the boundary conditions, the value of the option to invest is zero only when the product is worthless, i.e. $F(0) = 0$. Application of the cash flow framework would induce investors to pay for power plants more than is warranted by the option pricing model. The higher is the investment cost I the higher must be the expected electricity price P^* in order to allow the investor to earn the market rate of return.

Imperfect observations and measurement of divestiture. During divestiture, electric utilities transfer more than just power plants' capital. New owners receive the land under the plants, the assembled workforce, fuel stock, etc. The plant sites per se are valuable because their ownership

allows to build new capacity and upgrade the existing one without incurring the costs (environmental, regulatory, etc.) associated with siting a greenfield plant. Therefore, the prices paid for power plants in divestiture transactions may reflect the value of plants' sites in addition to plants' capital. The measurement of investment cost may itself be biased due to the method used to "unbundle" plants in the transaction. A better method of calculating prices of different types of plants is a subject of further research.

Expectation of market power. Both economic theory³⁰ and current events in certain markets (e.g. California) show that flawed restructuring of the electric utilities may result in prices well above marginal costs of electricity producers. It is plausible that the buyers of power plants may have expected to exercise market power.

3.5. Concluding Remarks.

This study presents an application of the theory of real options to the process of power plant divestiture in the electricity industry in the United States. Observations of prices paid for plants already sold allow us to infer the future prices of electricity that would justify the transaction prices.

Two specifications were developed. One assumes an infinite lifetime of the production assets, the

³⁰ The literature on market power in electricity industry is ample. Far from being exhaustive, the following references highlight some prominent recent work. Papers by Green and Newbery (1992) and Rudkevich et al. (1998) analyze market power due to strategic interaction in oligopoly. Market power arising due to special nature of electricity networks is addressed in Cardell et al. (1997), Wolfram (1998) and Borenstein et al. (2000).

other assigns certain finite life span to the power plants or a finite time for investment capital recovery. These specifications were tested on two different samples. One contained observations per transaction, another observations per plant. The study shows that prices for electricity have to be rather high for the owners of power plants to earn an attractive rate of return on their investments. Power plants divested by electric utilities, in general, have commanded high prices in the market, and the utilities selling them received generous compensation. The hypothesis of the bullish market for power plants thus found some empirical support.

Chapter 4. Attitudes Toward Risk and Divestiture in the Electricity Industry

4.1. Review of Research on Risk Attitudes of Business Entities.

The ongoing deregulation of the electricity industry in the U.S. has brought about some profound changes. The recent federal and state legislative initiatives promoting the deregulation of the electricity industry have coincided with an unprecedented wave of mergers, acquisitions, and sales of power generation plants. The divestiture³¹ of generation assets by many electric utilities converts them from vertically integrated firms to providers of power transmission and distribution services. Some other electric utilities, however, have kept their power plants by transferring them to their unregulated affiliates, and some have purchased more plants through the affiliates. In fact, newcomers are rare in the emerging market for power plants. A report by the Energy Information Administration says that about 82 percent of the capacity sold is acquired by unregulated subsidiaries of electric utilities (EIA, 1999, Chapter 6).

Without taking risk preferences into account, such wide proliferation of the market for power plants presents a puzzle. If both buyers and sellers are industry insiders with long experience in the business, both parties should have symmetrical information about the traded assets and the same expectations about the future. In other words, both the buyer and the seller should converge on the same expected net present value for a given power plant. Why would the trade take place? Risk

³¹ Divestiture conducted by electric utilities is understood as the sale of assets to other companies or the transfer of assets to unregulated affiliates.

preference theory suggests that if neither party can fully diversify the risk, the asymmetry of risk preferences will drive such a transaction.³² The deregulated market for power generation makes power plants risky assets because of uncertain future profits and long horizons of capital recovery. If the sellers of the power plants prefer to receive lump sums of cash now rather than an uncertain return later, they must be more risk averse than the buyers who are ready to pay to obtain the risky assets.

Estimates of risk aversion are derived from observations about asset allocation decisions and budgets of investors. Much of the empirical research conducted in this field deals with financial decisions made by individuals. A study by Friend and Blume (1975) analyzed survey data on household asset holdings; in contrast, a more recent work by Dalal and Arshanapalli (1993) empirically estimated the demand for financial assets by the U.S. households. Bartunek and Chowdhury (1997) estimate a representative investor's risk aversion coefficient using option prices data. Several authors have attempted to analyze properties of a utility function of business entities. An application of the von Neumann-Morgenstern utility framework to risk in the petroleum industry was suggested by Cozzolino (1977). Walls and Dyer (1996) have extended the analysis to measure the risk aversion coefficient of firms in the petroleum industry. Analysis of risk attitudes is extensively studied in the literature on agricultural production. Saha et al. (1994) provide a good review of the research conducted to date along with the extension of research in the field. One of the most

³² There may be other reasons why equally informed parties will trade, e.g. to raise liquidity, or because different parties have different time horizons, etc. We assume that these reasons are of secondary importance in the market for power plants.

complete empirical studies of preferences along with the production process is found in Chavas and Holt (1996) where the analysis is applied to farmers' decisions on acreage allocation between corn and soybeans.

The estimation of risk attitudes in the electric utility industry described in this paper, appears to be the first study of its kind. We present the method of joint estimation of utility and cost functions that allows us to determine the nature of risk preferences of the U.S. electricity producers. We present estimates of the coefficients of absolute risk aversion, relative risk aversion, and downside risk aversion. These estimates indicate that the vast majority of electric utilities are risk averse. Decreasing absolute risk aversion is observed for all the firms. Majority of the firms have an increasing relative risk aversion. In addition, electric utilities are found to be averse to downside risk. The estimates of risk attitudes obtained here allow us to test a set of hypotheses about the relationship between the divestiture of power plants and risk preferences. The tests indicate that relative risk aversion of power plant buyers is lower than that of the rest of the firms in the sample. In addition, we find that buyers' relative risk aversion is significantly lower than that of sellers.

4.2. The Method of Joint Estimation of Utility and Cost Functions.

The unique characteristic of the electricity market is that the amount of electricity demanded must be matched by the supply instantaneously to ensure reliability of the entire network. At the same time, electricity demand is highly cyclical. A typical electric utility faces daily patterns of high and low

demand, as well as seasonal trends. To cope with the volatile load profile, most electric utilities build different types of power plants which generally can be referred to as baseload and peaking units. The baseload plants usually require large initial capital outlays but have relatively low operating costs (e.g. coal-fired, hydro- and nuclear plants). These plants usually have large capacity and significant startup and shutdown costs, this is why they are rarely used to follow the demand profile. Peak-load plants, on the other hand, have relatively low capital investments but high operating costs due to the type of fuel used (e.g. gas- and petroleum-fired turbines). These plants are typically shut down when demand is low, and are restarted when demand is high. From the investment perspective, the different generation technologies available to electric utilities can be viewed as different types of assets. These two kinds of generation capacity are sensitive to different types of risk. Uncertainty about future prices of electricity heavily affects the riskiness of the large capital cost plants, while the fluctuations in fuel prices affect the riskiness of plants with high fuel costs. Most large electric utilities diversify their investments by building both types of capacity.

The electricity industry uses a vast variety of technologies to generate electricity, ranging from renewable resources to nuclear power. In 1999 67.8 percent of all the electricity in the United States was produced by fossil fuel powered plants, 22.8 percent of the total output was generated by nuclear plants, and 9.4 percent by hydroelectric plants and plants using other types of renewable sources of energy (EIA, 2000). From the investment perspective, the different generation technologies available to electric utilities can be viewed as those that are capital-intensive (most of the baseload plants fall into this category), and those that are fuel-intensive (this category is

comprised of typical peak-load plants).³³ These two kinds of generation capacity are sensitive to different types of risk. Uncertainty about future prices of electricity heavily affects the riskiness of the large capital cost plants, while the fluctuations in fuel prices affect the riskiness of plants with high fuel costs. Most large electric utilities diversify their investments by building both types of capacity. Thus the framework of portfolio theory can be applied to investment decisions made by electric utilities.

In its most general form, an investment portfolio can be represented by just two assets with different degrees of risk and different expected return. The investor's decision about the proportion of each of the two assets in the portfolio is based on *relative* magnitude of risk and returns of the assets, so a risk-free asset is not necessary to the analysis (Arrow, 1971). The method outlined below allows us to estimate risk preferences by solving the problem of portfolio allocation between two types of generation plants.

We assume that a representative utility has two types of plants that differ in magnitude of initial capital outlays and operating costs per unit of capacity. The index variable for the type of the plant is i . Let x_i stand for the total capacity of all the plants of the type i owned by an electric utility firm. The annual output per megawatt of capacity type i will be denoted by y_i . Therefore the total output of all the plants of the type i be found as $x_i y_i$. An uncertain demand for electricity makes the output

³³ The power plants of new type known as combined-cycle gas turbines are typically built as baseload units. The capital cost of these plants is usually much lower than that of the steam-powered baseload plants. Yet, the fuel cost of the combined-cycle plants is comparable to that of the gas turbines. This is why the combined-cycle plants in this study are attributed to the fuel-intensive technology.

per megawatt of capacity random. The firm, however, can choose the type and the capacity of the plants in its portfolio of generation assets.

Because the firm's decisionmakers can allocate capital between the two types of plants, they will maximize an expected utility of a von Neumann-Morgenstern type with respect to the choice between the two types of generation assets

$$\text{Max}_{\mathbf{x}} E[U(\mathbf{x}, \mathbf{y}, \mathbf{a})] \quad (4.1)$$

where $\mathbf{y}$ is the vector of random variables, and $\mathbf{a}$ is the vector of parameters describing preferences.

The firm's choice between the two types of capacity will be constrained by the cost function

$$g(\mathbf{x}, \mathbf{y}, \mathbf{b}) = 0 \quad (4.2)$$

where $\mathbf{x}' = (x_1, x_2)$ is a vector of allocational choices made by the firm, $\mathbf{y}' = (y_1, y_2)$ is a vector of random output, and $\mathbf{b}$ is the vector of parameters.

The Lagrangian of the described problem is

$$L = E[U(\mathbf{x}, \mathbf{y}, \mathbf{a})] + \mathbf{m}g(\mathbf{x}, \mathbf{y}, \mathbf{b}) \quad (4.3)$$

The first-order conditions are

$$\frac{\partial L}{\partial x_1} = E\left[\frac{\partial U}{\partial x_1}\right] + \mathbf{m}\frac{\partial g}{\partial x_1} = 0 \quad (4.4.a)$$

$$\frac{\partial L}{\partial x_2} = E\left[\frac{\partial U}{\partial x_2}\right] + \mathbf{m}\frac{\partial g}{\partial x_2} = 0 \quad (4.4.b)$$

$$\frac{\mathcal{U}L}{\mathcal{U}\mathbf{m}} = g(\mathbf{x}, \mathbf{y}, \mathbf{b}) = 0 \quad (4.4.c)$$

Solving (4.4.b) for $\mathbf{m}$ and substituting the result into (4.4.a) reduces the number of equations in the system to two.

$$\Theta = E \left[\frac{\mathcal{U}U}{\mathcal{U}x_1} \right] - \frac{\mathcal{U}g}{\mathcal{U}x_1} \left(\frac{\mathcal{U}g}{\mathcal{U}x_2} \right)^{-1} E \left[\frac{\mathcal{U}U}{\mathcal{U}x_2} \right] = 0 \quad (4.5.a)$$

$$\Theta = g(\mathbf{x}, \mathbf{y}, \mathbf{b}) = 0 \quad (4.5.b)$$

To make the model empirically tractable, a parametric specification of the system (4.5) is needed. The value of expected utility depends on expected profit, which, in turn, is determined by the allocational decision $\mathbf{x}$ and random output $\mathbf{y}$; i.e. the profit can be defined as $\mathbf{p} = (\mathbf{x}, \mathbf{y})$. If profit $\mathbf{p}$ is distributed on the interval $[l, h]$, the utility function can be specified as

$$u(\mathbf{p}, \mathbf{a}) = \int_l^h \exp(\mathbf{a}_0 + \mathbf{a}_1 z + \mathbf{a}_2 z^2) dz \quad (4.6)$$

where z is a dummy of integration, and $\mathbf{a}$'s are parameters to be estimated.

The above specification of the utility function follows that of Chavas and Holt (1996). This form of the utility function is very flexible. While the marginal utility of income is always positive ($\mathcal{U}u/\mathcal{U}\mathbf{p} = \exp(\mathbf{a}_0 + \mathbf{a}_1 \mathbf{p} + \mathbf{a}_2 \mathbf{p}^2) > 0$), the function can accommodate different types of attitudes towards risk. The attitudes toward risk are measured by the sign of the second derivative of the utility function. For example, $\mathcal{U}^2 u / \mathcal{U}\mathbf{p}^2 < 0$ implies risk-averse behavior, $\mathcal{U}^2 u / \mathcal{U}\mathbf{p}^2 = 0$ implies risk-

neutrality, and $\mathcal{I}^2 u / \mathcal{I} p^2 > 0$ implies risk-seeking behavior. The measure of absolute risk aversion

according to Arrow and Pratt is $\mathbf{I} = -(\mathcal{I}^2 u) / (\mathcal{I} p^2) / (\mathcal{I} u) / (\mathcal{I} p)$. In this case $\mathbf{I} = -(\mathbf{a}_1 + 2\mathbf{a}_2 p)$. A

positive value of $\mathbf{I}$ indicates risk-averse behavior, a risk-neutral behavior is characterized by $\mathbf{I} = 0$,

and a negative $\mathbf{I}$ would indicate a risk-loving behavior. The derivative the coefficient of absolute risk

aversion with respect to profit is $\mathbf{j} = \partial \mathbf{I} / \partial p = -2\mathbf{a}_2$. Depending on the sign and magnitude of $\mathbf{a}_2$,

the derivative $\mathbf{j}$ can be negative, zero or positive. Negative $\mathbf{j}$ indicates a decreasing absolute risk

aversion (DARA), positive $\mathbf{j}$ is observed in the case of increasing absolute risk aversion (IARA),

and $\mathbf{j} = 0$ shows constant absolute risk aversion (CARA). Such a specification of the utility function

provides a basis to evaluate empirically the nature of preferences displayed by electric utilities.

Let c_i be the annual operating cost per megawatt-hour of capacity type i , and p be the (regulated)

price of a megawatt-hour of electricity during the given year. Then the annual profit from operating

both types of plants can be expressed as³⁴

$$\mathbf{p}(\mathbf{x}, \mathbf{y}) = (p - c_1)x_1y_1 + (p - c_2)x_2y_2. \quad (4.7)$$

A proposed parametric specification of the cost function is

³⁴ The data used to measure c_1 and c_2 in the profit function are variable operating costs. Thus $\mathbf{p}$ in the expression (4.7) measures producer surplus.

$$g(x, y, \mathbf{b}) = \mathbf{b}_0 + \mathbf{b}_1 x_1 y_1 + \mathbf{b}_2 x_2 y_2 - TC \quad (4.8)$$

where TC is a total cost of generation in the plants of both types, which includes both the operating cost and the cost of capital. This specification is conformable with the linear treatment of costs in the profit function above. The coefficients $\mathbf{b}_1$ and $\mathbf{b}_2$ here represent marginal costs of capital-intensive and fuel-intensive technologies respectively, and $\mathbf{b}_0$ represents the total fixed cost. Thus, the estimates of $\mathbf{b}_0$, $\mathbf{b}_1$, and $\mathbf{b}_2$ are expected to be positive.

Given the parametric specification of the utility, cost and profit functions, we can obtain some useful derivatives

$$\frac{\mathcal{I}_u}{\mathcal{I}_p} = \exp(\mathbf{a}_0 + \mathbf{a}_1 p + \mathbf{a}_2 p^2) = A_0 \exp(\mathbf{a}_1 p + \mathbf{a}_2 p^2) \quad (4.9)$$

$$\frac{\mathcal{I}_g}{\mathcal{I}_{k_1}} = \mathbf{b}_1 y_1 \quad (4.10)$$

$$\frac{\mathcal{I}_g}{\mathcal{I}_{k_2}} = \mathbf{b}_2 y_2 \quad (4.11)$$

$$\frac{\mathcal{I}_p}{\mathcal{I}_{k_1}} = (p - c_1) y_1 \quad (4.12)$$

$$\frac{\mathcal{I}_p}{\mathcal{I}_{k_2}} = (p - c_2) y_2 \quad (4.13)$$

The equations (4.9 – 4.13) can now be used to obtain the parametric specification of the model described by the system of equations (4.5)

$$\Theta_1 = A_0(p - c_1)E[y_1 \exp(\mathbf{a}_1\mathbf{p} + \mathbf{a}_2\mathbf{p}^2)] - A_0 \frac{\mathbf{b}_1 y_1}{\mathbf{b}_2 y_2} (p - c_2)E[y_2 \exp(\mathbf{a}_1\mathbf{p} + \mathbf{a}_2\mathbf{p}^2)] = 0, \quad (4.14.a)$$

$$\Theta_2 = \mathbf{b}_0 + \mathbf{b}_1 x_1 y_1 + \mathbf{b}_2 x_2 y_2 - TC = 0. \quad (4.14.b)$$

For the purposes of econometric estimation, we append the right-hand side of each equation of the system (4.14) with error terms that are assumed to have zero mean. Further, we assume that the error terms are independently and identically distributed across the observations, but there is dependence between the error term of equation for Θ_1 and that of equation for Θ_2 due to interdependence between the profit and the cost functions. The assumption of cross-equation dependence makes it necessary to estimate the equations (4.14) as a system of simultaneous equations. We used a full-information likelihood (FIML) method to perform the estimation.³⁵ To evaluate the expectation operator in the equation (4.14.a) we used numerical Monte Carlo integration.

4.3. Empirical Implementation of the Method.

Assessment of Risk Preferences and Hypothesis Test.

The data for the analysis come from a database published by the Utility Data Institute that includes the information from the FERC Form 1 and other official sources. The electric utilities are assumed

³⁵ For details, refer to Appendix F.

to allocate their investment funds between steam-propelled plants and gas turbines. Utilities that rely heavily on other types of generation were excluded from the sample.³⁶ The data set was organized as a panel containing 80 electric producers observed over 11 years from 1986 to 1996. Because of missing data, the number of actual observations was 831. Descriptive statistics of the data set are presented in Table 4.1.

The presence of the exponential form in one of the model equations required data scaling for the estimation algorithm to work. The estimation algorithm was implemented using the Gauss Maxlik routine with embedded algorithm for Monte Carlo integration. The parameter estimates are presented in Table 4.2. All but one of them are significantly different from zero at the 5 percent level.

As expected, the signs of $\mathbf{b}_0$, $\mathbf{b}_1$, and $\mathbf{b}_2$ are positive. After adjustment for scaling, the magnitude of $\mathbf{b}_1$ becomes 21.0, which is the estimate of the marginal cost of capital-intensive generation in dollars per megawatt-hour. The estimate of the marginal cost of fuel-intensive generation is 71.7\$/MWh obtained by re-scaling the coefficient $\mathbf{b}_2$.

The estimates of $\mathbf{a}_1$ and $\mathbf{a}_2$ can be used to obtain several characteristics of the utility of the firms in the electricity industry. The sign of the second derivative of the utility function with respect to profit reveals a general attitude toward risk. Given the specification of the utility function, the second

³⁶ The excluded utilities generated more than 5 percent of their total output on plants powered by hydro energy and other renewable resources.

**Table 4.1. Descriptive Statistics of the Data Set
Period of Observations 1986 - 1996**

Variable	Symbol	Mean	Standard Deviation
Residential Price of Electricity, \$/MWhr	p	84.417	22.015
Average Production Cost of Steam Plants, \$/MWhr	c_1	26.435	11.824
Average Production Cost of Gas Turbine Plants, \$/MWhr	c_2	355.778	1527.79
Installed Capacity of Steam Plants, MW $\times 10^{-3}$	x_1	3.160	3.121
Installed Capacity of Gas Turbine Plants, MW $\times 10^{-3}$	x_2	0.481	0.645
Annual Generation of Steam Plants per Capacity Unit, (MWhr/MW) $\times 10^{-3}$	y_1	3.829	1.217
Annual Generation of Gas Turbine Plants per Capacity Unit, (MWhr/MW) $\times 10^{-3}$	y_2	0.369	1.001
Annual Total Profit from Generation, billions of dollars	p	0.691	0.699
Annual Total Cost of Generation, billions of dollars	TC	0.289	0.281

Number of observations is 831

Table 4.2. Estimates of the Model Coefficients

Coefficient	Estimate	Asymptotic Standard Error	Asymptotic t-value	Approximate p-value
A_0	1.805×10^{-5}	2.057×10^{-6}	8.78	< 0.0001
$\mathbf{a}_1$	-15.828	2.336	-6.78	< 0.0001
$\mathbf{a}_2$	3.351	18.233	0.18	0.8542
$\mathbf{b}_0$	0.017326	0.00410	4.22	< 0.0001
$\mathbf{b}_1$	0.021036	0.000237	88.90	< 0.0001
$\mathbf{b}_2$	0.071698	0.00691	10.37	< 0.0001

derivative is

$$\frac{\mathcal{U}^2 u}{\mathcal{U} p^2} = (\mathbf{a}_1 + 2\mathbf{a}_2 p) \exp(\mathbf{a}_0 + \mathbf{a}_1 p + \mathbf{a}_2 p^2) = A_0(\mathbf{a}_1 + 2\mathbf{a}_2 p) \exp(\mathbf{a}_1 p + \mathbf{a}_2 p^2).$$

Evaluated at the sample mean of profits of 0.69 billion dollars, this derivative is equal to

-1.801×10^{-8} . The negative sign of the second derivative indicates that the electricity producers are *risk averse*. In the framework of the choice between the two types of generation facilities risk aversion means that the electricity producers dislike risk exposure, and diversify their generation portfolios by building both types of capacity, i.e. both capital- and fuel-intensive plants.

The risk aversion of electricity firms can further be characterized with the following measures:

1. The coefficient of absolute risk aversion $\mathbf{I} = -\frac{(\mathcal{U}^2 u)/(\mathcal{U} p^2)}{(\mathcal{U} u)/(\mathcal{U} p)} = -(\mathbf{a}_1 + 2\mathbf{a}_2 p)$
2. The coefficient of relative risk aversion $\mathbf{r} = \mathbf{pI} = -p \frac{(\mathcal{U}^2 u)/(\mathcal{U} p^2)}{(\mathcal{U} u)/(\mathcal{U} p)} = -(\mathbf{a}_1 p + 2\mathbf{a}_2 p^2)$
3. The coefficient of downside risk aversion $\mathbf{d} = \frac{(\mathcal{U}^3 u)/(\mathcal{U} p^3)}{(\mathcal{U} u)/(\mathcal{U} p)} = 2\mathbf{a}_2 + (\mathbf{a}_1 + 2\mathbf{a}_2 p)^2$

The estimates of the risk characteristics of the electricity industry evaluated at the mean of the sample are reported in Table 4.3.

**Table 4.3. Characteristics of Risk Aversion of the Firms
in the U.S. Electricity Industry**

	Mean of the Sample	Asymptotic Standard Deviation ^a
Coefficient of Absolute Risk Aversion, <i>I</i>	10.369	10.858
Coefficient of Relative Risk Aversion, <i>r</i>	3.675	7.476
Coefficient of Downside Risk Aversion, <i>d</i>	139.777	189.134

a. Evaluated at the mean of the sample of profits, $\bar{p} = 0.69$ billion dollars. See Appendix G for details.

All the risk measures calculated are significantly different from zero. The coefficient of absolute risk aversion ***I*** evaluated at the mean of the sample is positive. A higher value of absolute risk aversion implies stronger aversion to risk. Because the absolute risk aversion coefficient is a function of profit, each firm in the sample has a different estimate of ***I***. The range of estimates in the sample varies from -4.75 to 15.27. Seventy-six companies out of 80 (or 95 percent) have positive estimated ***I***, which indicates that those companies are *risk averse*. The negative estimates of ***I*** for the remaining four companies in the sample can be interpreted as a risk-loving behavior.

The derivative of the coefficient of absolute risk aversion with respect to profit $\frac{dI}{dp} = -2a_2$ = -6.702 is negative, hence the electric utilities must have a *decreasing absolute risk aversion* (DARA). As discussed above, neither of the two types of plants owned by utilities can be viewed as riskless, thus the reduction of overall risk can only be achieved by diversification of the generation between the capital- and fuel-intensive plants. The finding of decreasing absolute risk aversion of the

electric utilities implies that firms with higher total generation capacity tend to be *less* diversified than those with lower total generation capacity.

The coefficient of relative risk aversion r helps to compare attitudes toward risk among entities with different magnitudes of profit. Just as with absolute risk aversion, relative risk aversion coefficients are firm-specific. The estimates of relative risk aversion are positive for all but four companies.³⁷ The range of values of r is from -23.86 to 8.04. The derivative of the coefficient of relative risk aversion with respect to profit evaluated at the sample mean of profit is $n = \frac{dr}{dp} = -(a_1 + 4a_2\bar{p}) = 6.579$. The fact that n is positive and significantly different from zero indicates that the electric utilities have an *increasing relative risk aversion* (IRRA). One interpretation of this finding may be that large firms have lower risk exposure. The range of values of n in the sample is from -21.43 to 15.69. Eleven firms in the sample, or 14 percent, have negative estimated n which indicates decreasing relative risk aversion.

The coefficient of downside risk aversion d is positive which indicates aversion to the distribution of profits skewed to the left, or *aversion to downside risk*. The latter result is in accord with theory because DARA preferences imply a positive sign of the third derivative of the utility function with respect to profits (Pratt, 1964; Menezes et al., 1980). The aversion to downside risk indicates aversion to losses. The range of d values in the sample is from 20.83 to 240.16. There are no firms

³⁷ These are the same five companies that were found to possess an increasing absolute risk aversion.

with negative downside risk aversion. This result is in agreement with the conclusion of Menezes et al. (1980) that both risk avoiders and risk takers can be downside risk averse.

Because this study appears to be the first to estimate the risk preferences of electric utilities, direct comparison of the results with other studies of the industry is not possible. Yet the estimators can be compared with those obtained in the studies of risk preferences of other agents. Using the model of acreage decisions between corn and soybeans by U.S. farmers, Chavas and Holt (1996) find an Arrow-Pratt absolute risk aversion coefficient of 12.17, a relative risk aversion coefficient of 6.07, and a downside risk aversion coefficient equal to 157.27. In his study of Kansas wheat producers Saha (1997) finds the absolute risk aversion of small firms to be around 0.90 and the absolute risk aversion of large firms around 0.53. The summary provided by Saha et al. (1994) shows that different studies of agriculture report coefficients of absolute risk aversion ranging from 0 to 14.75, and coefficients of relative risk aversion ranging from 0 to 18.80. Friend and Blume (1975) conclude that the coefficient of relative risk aversion of the U.S. households is larger than unity, and is, probably, in the neighborhood of 2.0. Dalal and Arshanapalli (1993) provide an estimate of the relative risk aversion coefficient of about 1.3. Hansen and Singleton (1983) used a framework of production-exchange economy in which identical agents seek to maximize the intertemporal utility of consumption. This study puts the estimate of relative risk aversion between zero and two. Wolf and Pohlman (1983) estimated risk preferences of a dealer of U.S. government bonds based on its bids. Their assessment of the coefficient of absolute risk aversion is 4. Using time series data on the value of the assets insured by various economic agents in the U.S., Szpiro (1986) estimated the degree of relative risk aversion to be between 1.2 and 1.8. In the study of oil exploration industry, Walls and

Dyer (1996) calculated the risk aversion coefficient of ARCO, a major oil company, during the period from 1983 to 1990. The reported values of the risk aversion coefficient are ranging from 30 to 44 when the monetary values are expressed in billions of dollars. Compared to the existing empirical estimates of risk preferences, the risk aversion of electric utilities found in this study may be considered moderate.

With the estimates of risk aversion in hand, the hypothesis on whether the divestiture process is driven by the risk attitudes of the participants can be tested. More specifically, we can pose the following questions:

- a) Do the sellers display higher relative risk aversion than the rest of the firms in the sample?
- b) Is the relative risk aversion of the buyers lower than that of the rest of the firms?
- c) Is the relative risk aversion of the sellers larger than that of the buyers?

The answers to these questions can be obtained by testing whether the means of the two samples under consideration are statistically different.

The information on agents participating in the market for generation assets was collected jointly by Professor Stratford Douglas, graduate research assistant Matthew McPherson, and the author. The time span of those observations extends from late 1996 until the end of August 2000. The divestiture data were matched with the sample of the utilities used for the estimation of risk preferences. Out of the 80 firms in the sample, forty-one have been involved in transactions with

power plants. Twenty-three firms have sold their power plants either directly or through their holding companies. Twenty-two firms have bought plants through their affiliates. Four firms in the sample conducted both sales and purchases.

The procedure of testing whether two samples of random variables belong to distributions with the same means and variances is outlined in Hogg and Craig (1995, p. 277). The two samples of normally distributed random variables X and Y contain n and m observations respectively. The means of the samples are $\bar{X}$ and $\bar{Y}$, and the variances are s_1^2 and s_2^2 . If the two samples are drawn from the same normal distribution, the respective first and second moments of the two populations must be the same. This can be tested with the following statistic

$$T = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{ns_1^2 + ms_2^2}{n+m-2} \left(\frac{1}{n} + \frac{1}{m} \right)}}, \quad (4.15)$$

which has a t -distribution with $n + m - 2$ degrees of freedom.

The results of this test are presented in Table 4.4. The firms that acted as both buyers and sellers were excluded from the samples.

The results of the hypotheses test show statistical evidence that the buyers of power plants have lower relative risk aversion than the rest of the firms the sample, as well as indicate that the buyers

Table 4.4. Comparison of Risk Attitudes Among Different Groups of Firms

Relative Risk Aversion, Sample Average		Samples Compared	Degrees of Freedom	T Values for the Sample Mean Comparison
Whole Sample	3.675	Sellers vs. Rest	74	0.94
Sellers	4.884	Buyers vs. Rest	74	-3.09*
Buyers	1.198	Buyers vs. Sellers	35	-1.81*

* significant at 5 percent level

have lower degree of relative risk aversion than the sellers. However, there is no evidence that the sellers of power plants have higher than the rest relative risk aversion.

4.4. Concluding Remarks.

This study applies a method of joint estimation of risk preferences and costs to assess the nature of attitudes toward risk of U.S. electric utilities. The two generation assets with different risk characteristics considered here are capital-intensive plants and fuel-intensive plants. The allocation of capital between these two assets by electric utilities was studied to reveal the risk aversion characteristics of the firms' decisionmakers. A full-information maximum likelihood method (FIML) was used to jointly estimate utility and cost functions of the U.S. electricity producers. We estimate the coefficients of absolute risk aversion, relative risk aversion, and downside risk aversion. The vast majority of the electricity producers (94 percent) have been found to be *risk averse*. Utility functions of all the producers exhibit *decreasing absolute risk aversion* (DARA). Majority of the firms (86 percent) have an *increasing relative risk aversion* (IRRA). All the firms have been

found averse to downside risk. A set of hypotheses on whether the divestiture of power plants is driven by risk preferences was tested. The relative risk aversion of the buyers of power plants was found to be significantly lower than that of the rest of the firms in the sample. In addition, the relative risk aversion of the buyers was found to be lower than the relative risk aversion of the sellers. This indicates that the buyers of power plants tend to be more aggressive than the plants' previous owners.

Chapter 5. Conclusions and Perspectives of Future Research

This dissertation explores several issues that arise from the restructuring of the American electricity industry. The introductory chapter discusses economic reasons for deregulation, describes possible deregulation scenarios, and outlines potential pitfalls of deregulation.

The first essay describes a method of forecasting electricity prices based on an oligopoly model. The method is applied to estimate equilibrium prices in the markets for electric power bounded by existing NERC regions. The projected oligopoly prices are compared with those under competitive scenario and those that prevailed under regulation. The results of the study indicate that the predicted degree of market power varies from one NERC region to another. If the retail market for electricity operates as perfectly competitive, prices for electricity consumers will decrease from their regulated level in most regions. If producers are able to exercise market power as Cournot oligopolists, unregulated markets will result in prices higher than those under regulation. The price markups are positively correlated with average total costs of producers and negatively correlated with price elasticity of market demand for electricity. The degree of market concentration has a lesser effect on the Cournot price because several mechanisms may limit the ability of low-cost producers to exercise market power. Future research on this topic may incorporate a different definition of the market. Recent developments show that electricity markets are forming within the boundaries of ISO (Independent System Operator) or RTO (Regional Transmission Organizations)

that are smaller than whole NERC regions. The market boundaries usually coincide with physical constraints imposed by relatively “thin” interconnections. Trade across market boundaries may be considered as well, as long as the limits of transmission are taken into account. Future research should also account for the changed ownership structure of the electricity industry. Recent mergers and divestitures have dramatically changed the faces of the game, although, arguably, their impact on the industry concentration was only minor. Another desirable aspect of future research would be incorporating some dynamics into the model. On the supply side, the new entry should be explicitly considered. The new entrants will most likely fit in the middle of the market supply curve which will make the flat portion of this curve longer. On the demand side, growth in demand should be accounted for. The market prices in such a model will be impacted by the relative rates of growth in both supply and demand in addition to the factors already considered.

In the second essay of the dissertation, the theory of real options is applied to calculate expected future prices of electricity that would justify the amounts paid for power plants in divestiture sales. The study finds that under plausible assumptions new power plant owners will have to rely on market power to earn attractive rates of return on their investment. In general, power plants commanded high prices, and electric utilities selling them were generously compensated. Future research may focus on improving the methodology of measuring investment cost of individual plants sold in “bundles”. In addition, a study of stock prices of both buyers and sellers may be conducted to find whether transactions of power plants have generated excess returns.³⁸ The results of such

³⁸ I thank Jon Vilasuso for this suggestion.

study may reveal ex ante market expectations of future profits, and as such, they could be used to forecast electricity prices.

The third essay of the dissertation applies a method of joint estimation of risk preferences and costs to assess the nature of risk attitudes of the U.S. electric utilities. To reveal the risk aversion characteristics of the firms' decisionmakers, the allocation of capital between capital-intensive and fuel-intensive plants was studied. The estimates of the coefficients of absolute risk aversion, relative risk aversion, and downside risk aversion were obtained. The vast majority of the electricity producers (94 percent) have been found to be *risk averse*. Utility functions of all the producers exhibit *decreasing absolute risk aversion* (DARA). Majority of the firms (86 percent) have an *increasing relative risk aversion* (IRRA). All the firms have been found averse to downside risk. A set of hypotheses on whether the divestiture of power plants is driven by risk preferences was tested. The relative risk aversion of the buyers of power plants was found to be significantly lower than that of the rest of the firms in the sample. In addition, the relative risk aversion of the buyers was found to be higher than the relative risk aversion of the sellers. This indicates that the buyers of power plants tend to be more aggressive than their previous owners. Future research may use an alternative empirical methodology, e.g. generalized method of moments or GMM,³⁹ or a different form of utility function⁴⁰ to test robustness of the results. In addition, more insight may be obtained by analyzing firm-specific parameters of risk aversion.

³⁹ This comment was suggested by Jon Vilasuso.

⁴⁰ This suggestion is due to Ronald Balvers.

Appendix A

Estimation of the Translog Total Cost Function

Output cost elasticities can be obtained by estimating parameters of the total cost function. The translog cost function developed by Christensen and Greene (1976) has the advantage of not imposing *a priori* restrictions on the elasticity of substitution between the factors of production. I begin with the following specification.

$$\begin{aligned}
 \ln TC = & \alpha_0 + \mathbf{b}_L \ln p_L + \mathbf{b}_F \ln p_F + \mathbf{b}_K \ln p_K + \mathbf{b}_Y \ln y \\
 & + \frac{1}{2} \mathbf{b}_{LL} (\ln p_L)^2 + \frac{1}{2} \mathbf{b}_{FF} (\ln p_F)^2 + \frac{1}{2} \mathbf{b}_{KK} (\ln p_K)^2 + \frac{1}{2} \mathbf{b}_{YY} (\ln y)^2 \\
 & + \mathbf{b}_{LF} \ln p_L \ln p_F + \mathbf{b}_{LK} \ln p_L \ln p_K + \mathbf{b}_{FK} \ln p_F \ln p_K \\
 & + \mathbf{g}_L \ln p_L \ln y + \mathbf{g}_F \ln p_F \ln y + \mathbf{g}_K \ln p_K \ln y
 \end{aligned} \tag{A.1}$$

Here subscripts L , F , K are used for labor, fuel and capital respectively; p_X denotes price of the input X ; y denotes output.

To obtain efficient estimates, the above specification is commonly estimated simultaneously with the input cost share equations. The usual restrictions of homogeneity of degree one in input prices and symmetry of input price cross-effects are imposed as well. This leads to the system of equations

$$\begin{aligned}
 \ln \frac{TC}{p_K} = & \alpha_0 + \mathbf{b}_L \ln \frac{p_L}{p_K} + \mathbf{b}_F \ln \frac{p_F}{p_K} + \mathbf{b}_Y \ln y \\
 & + \frac{1}{2} \mathbf{b}_{LL} \left(\ln \frac{p_L}{p_K} \right)^2 + \frac{1}{2} \mathbf{b}_{FF} \left(\ln \frac{p_F}{p_K} \right)^2 + \frac{1}{2} \mathbf{b}_{YY} (\ln y)^2
 \end{aligned}$$

$$+\mathbf{b}_{LF} \ln \frac{p_L}{p_K} \ln \frac{p_F}{p_K} + \mathbf{g}_{L'} \ln \frac{p_L}{p_K} \ln y + \mathbf{g}_{F'} \ln \frac{p_F}{p_K} \ln y \quad (\text{A.2.a})$$

$$s_L = \mathbf{b}_L + \mathbf{b}_{LL} \ln \frac{p_L}{p_K} + \mathbf{b}_{LF} \ln \frac{p_F}{p_K} + \mathbf{g}_L \ln y \quad (\text{A.2.b})$$

$$s_F = \mathbf{b}_F + \mathbf{b}_{FF} \ln \frac{p_F}{p_K} + \mathbf{b}_{LF} \ln \frac{p_L}{p_K} + \mathbf{g}_F \ln y \quad (\text{A.2.c})$$

Equations (A.2.a-c) are estimated as a seemingly unrelated system with the usual restrictions of homogeneity of degree one in input prices and symmetry of input price cross-products.

The elasticities of total cost with respect to output are then calculated for each firm at the means of observations according to the following formula

$$\mathbf{e}_i = \frac{\mathcal{J} \ln TC_i}{\mathcal{J} \ln y_i} = \mathbf{b}_Y + \mathbf{b}_{YY} \ln y_i + \mathbf{g}_L \ln \left(\frac{p_L}{p_K} \right)_i + \mathbf{g}_F \ln \left(\frac{p_F}{p_K} \right)_i \quad (\text{A.3})$$

Appendix B

The Method of Calculating Bertrand Prices.

The method uses the additivity of the market supply function and the fact that marginal cost functions of producers represent their supply curves in a competitive market. Marginal costs of producers can be obtained using the relationship established by formula (A.3)

$$\mathbf{e}_i = \frac{\mathcal{I} \ln TC_i}{\mathcal{I} \ln y_i} = \frac{\partial TC_i}{\partial y_i} \frac{y_i}{TC_i} = \frac{MC_i}{ATC_i}, \quad (\text{B.1})$$

where ATC_i denotes average total cost of the firm i .

Substituting the expression for $\mathbf{e}_i$ from (A.3) into (B.1), marginal cost is expressed as

$$MC_i(y_i) = ATC_i \left(\mathbf{b}_Y + \mathbf{b}_{YY} \ln y_i + \mathbf{g}_{YL} \ln \left(\frac{p_L}{p_K} \right)_i + \mathbf{g}_{YF} \ln \left(\frac{p_F}{p_K} \right)_i \right). \quad (\text{B.2})$$

Thus, after the coefficients of the total cost function (A.2) are estimated, marginal cost functions for every producer can be computed according to the formula (B.2), assuming that average total cost remains constant within the range of output considered.

In the Bertrand case, every firm will choose to produce the amount of output at which marginal cost is equal to the market price. Thus, the supply curve of a firm in Bertrand oligopoly can be written as

$$\ln y_i = \frac{1}{\mathbf{b}_{YY}} \left[\frac{p'}{ATC_i} - \mathbf{b}_Y - \mathbf{g}_{YL} \ln \left(\frac{p_L}{p_K} \right)_i - \mathbf{g}_{YF} \ln \left(\frac{p_F}{p_K} \right)_i \right], \quad (\text{B.3})$$

where p' is the market price.

Total market supply is found by adding up the supplies of all producers. By varying p' exogenously a point can be found at which the total market supply is equal to the certain level of demand set at its historical or projected value. This price p^*_B is a market equilibrium price in the Bertrand model. The price p^*_B will be equal to the marginal cost of the highest-marginal-cost producer(s) in the market. In a broad enough market it will likely be the case that some producers will have marginal cost lower than p^*_B at any level of output. In the Bertrand oligopoly those infra-marginal (baseload) producers will operate at full capacity, and earn an economic rent. Some other producers' marginal cost may be lower than p^*_B at some range of output, and exceed p^*_B in another range of output. Those producers will be marginal (or peaking) producers. The output of marginal producers will fluctuate greatly depending on instantaneous demand. Marginal firms will only use a part of their capacity at a time. The most expensive generators of the marginal firms will earn zero economic profit at any given time period. Yet other producers may have marginal cost exceeding p^*_B at all levels of output. Such producers will make economic losses and eventually will be phased out of the market.

Appendix C

Description of Data Used to Estimate the Translog Total Cost Function

The variables for the translog cost function estimation were obtained as follows.

TC - Total Cost. Combined cost of operation and maintenance, fuel, and labor, thousands of dollars per year.

p_L - Price of labor. Annual salaries and wages per employee, thousands of dollars.

p_F - Price of fuel, dollars per BTU.

y - Output, MMWhr/year

p_K - Price of capital.

Estimated as a weighted average of prices of debt and equity. The nominal price of long-term debt is taken equal to yield on utility bonds issued during the period under consideration. Because the overall rating of the firm's debt affects its cost of borrowing, rating of newly issued bonds was matched with the rating of the particular utility's debt.

The price of equity was obtained according to the model of dividend growth (see, e.g., Morin, 1984). The price of equity capital at the time period t is expressed as

$$p_{Kt} = (D_{t+1}/p_{St}) + g,$$

where D_{t+1} is an expected dividend per share, $D_{t+1} = D_t(1+g)$, p_{St} is current period stock price, g is an expected rate of dividend growth (obtained as an estimate from the exponential growth model based on 20-year historical rates of growth). Using the expression for D_{t+1} , p_{Kt} can be presented as

$$p_{Kt} = D_t(1+g)/p_{St} + g,$$

where D_t/p_{St} is the yield in period t .

All nominal variables were converted into real values expressed in constant 1996 dollars.

Appendix D

Algebraical Expressions of Total Derivatives of the Threshold Price P^* With Respect to Various Parameters of the Valuation Equations

Total derivative $dP^*/d\mathbf{S} =$

$$\begin{aligned}
& \left| \frac{c}{\sigma^2} \frac{d}{p} \right|^{0.5 + \left\{ \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2}}{\sigma^2} + \frac{r}{\sigma^2} \right\} \sigma^2} \left| \frac{p}{k} \left[\frac{2H-d+rL}{\sigma^3} + \frac{-2d^2+4dr-2r^2-1. d \sigma^2-1. r \sigma^2}{\left\{ \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2}}{\sigma^2} \right\} \sigma^5} \right] \right| \\
& \left| \frac{c}{\sigma^2} \right|^{0.5 + \left\{ \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2}}{\sigma^2} + \frac{-d+r}{\sigma^2} \right\} \sigma^2} \left| \frac{p}{\sigma^2} \right|^{0.5 - \left\{ \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2}}{\sigma^2} + \frac{d-r}{\sigma^2} \right\} \sigma^2} \\
& \left| J - \frac{1}{d} + \frac{1}{r} N \right| \left| \frac{2H-d+rL}{\sigma^3} + \frac{-2d^2+4dr-2r^2-1. d \sigma^2-1. r \sigma^2}{\left\{ \frac{-0.5 + \frac{-d+r}{\sigma^2} N^2 + \frac{2r}{\sigma^2}}{\sigma^2} \right\} \sigma^5} \right| + \\
& \left| J_i + \frac{c}{r} N \right| \left| \frac{2H-d+rL}{\sigma^3} + \frac{-2d^2+4dr-2r^2-1. d \sigma^2-1. r \sigma^2}{\left\{ \frac{-0.5 + \frac{-d+r}{\sigma^2} N^2 + \frac{2r}{\sigma^2}}{\sigma^2} \right\} \sigma^5} \right| + \\
& \left| \frac{c}{2} \right|^{0.5 + \left\{ \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2}}{\sigma^2} - \frac{d}{\sigma^2} + \frac{r}{\sigma^2} \right\} \sigma^2} \left| \frac{p}{\sigma^2} \right|^{0.5 - \left\{ \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2}}{\sigma^2} + \frac{d}{\sigma^2} - \frac{r}{\sigma^2} \right\} \sigma^2} \\
& \left\{ \frac{\left\{ \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2}}{\sigma^2} \right\} \sigma^2 \log @ c D}{\left\{ \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2}}{\sigma^2} \right\} \sigma^2} \right\} \\
& \left| \frac{H1. d^2 - 2. dr + 1. r^2 + 1. d \sigma^2 + 1. r \sigma^2 + 0.25 \sigma^4 L}{2} \right|^{0.5 + \left\{ \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2}}{\sigma^2} - \frac{d}{\sigma^2} + \frac{r}{\sigma^2} \right\} \sigma^2} \left| \frac{p}{\sigma^2} \right|^{0.5 - \left\{ \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2}}{\sigma^2} + \frac{d}{\sigma^2} - \frac{r}{\sigma^2} \right\} \sigma^2} \\
& \left\{ \frac{\left\{ \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2}}{\sigma^2} \right\} \sigma^2 \log @ p D}{\left\{ \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2}}{\sigma^2} \right\} \sigma^2} \right\} \\
& \left| \frac{H1. d^2 - 2. dr + 1. r^2 + 1. d \sigma^2 + 1. r \sigma^2 + 0.25 \sigma^4 L}{2} \right|^{yy} \\
& \left| \frac{c}{k} \right| \left| \frac{d}{p} \right|^{0.5 + \left\{ \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2}}{\sigma^2} + \frac{r}{\sigma^2} \right\} \sigma^2} \left| \frac{p}{\sigma^2} \right|^{0.5 + \left\{ \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2}}{\sigma^2} + \frac{r}{\sigma^2} \right\} \sigma^2} \left| \frac{d}{p} \right| \left| \frac{c}{\sigma^2} \right| \\
& \left| \frac{1. d - 1. r + 1.}{k} \right| \left| \frac{-0.5 + 1.}{k} \right| \left\{ \frac{\left\{ \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2}}{\sigma^2} \right\} \sigma^2}{\left\{ \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2}}{\sigma^2} \right\} \sigma^2} \right\} \left\{ \frac{Y}{\sigma^2} \right\} \left\{ \frac{Y}{\sigma^2} \right\}
\end{aligned}$$

$$\begin{aligned} & \left| \frac{d}{\sigma^2} \right|_p^{0.5} \$ \left| \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2} + \frac{r}{\sigma^2}}{\sigma^2} \right|_{\sigma^2}^2 \\ & \left| \frac{c}{k} \left| \frac{0.5 + \$ \frac{J - 0.5 + \frac{-d+r}{\sigma^2} N^2 + \frac{2r}{\sigma^2} + \frac{d \cdot r}{\sigma^2} Y}{r^2} - \frac{1}{\sigma^2} + \frac{-d \cdot r + 0.5 \sigma^2}{\$ \frac{J - 0.5 + \frac{-d+r}{\sigma^2} N^2 + \frac{2r}{\sigma^2} \sigma^4} + \frac{d \cdot r}{\sigma^2}} \right| \right|_k^Y \\ & \left| \frac{J_i + \frac{c}{r} N}{k} \left| \frac{1}{\sigma^2} + \frac{-d + r + 0.5 \sigma^2}{\$ \frac{J - 0.5 + \frac{-d+r}{\sigma^2} N^2 + \frac{2r}{\sigma^2} \sigma^4} \right| - \frac{1}{k} \left| \frac{0.5 + \$ \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2} + \frac{-d \cdot r}{\sigma^2}}{\sigma^2} \right|_k^Y \right|_k^Y \\ & \left| \frac{p^{0.5} \$ \left| \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2} + \frac{d \cdot r}{\sigma^2}}{k} \right|_k^{-d^3 + d^2} \left| \frac{r}{k} + \left| \frac{-1. - \$ \left| \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2}}{\sigma^2} \right| \sigma^2}{\sigma^2} \right|_k^Y \right|_k^Y + \right. \\ & \left. r^2 \left| \frac{r}{k} + \left| \frac{-0.5 + \$ \left| \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2}}{\sigma^2} \right| \sigma^2}{\sigma^2} \right|_k^Y \right|_k^Y + \right. \\ & \left. d \left| \frac{r^2 - 0.5 r \sigma^2}{k} + \left| \frac{-0.25 - 0.5 \$ \left| \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2}}{\sigma^2} \right| \sigma^4}{\left\{ \left\{ \left\{ \right\} \right\} \right\}} \right|_k^Y \right|_k^Y \right. \\ & \left| \frac{d r^2 \$ \left| \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2}}{\sigma^2} \right| \sigma^4}{k} - c^{0.5} \$ \left| \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2} + \frac{-d \cdot r}{\sigma^2}}{\sigma^2} \right|_k^Y \right|_k^Y \\ & \left| \frac{p^{0.5} \$ \left| \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2} + \frac{d \cdot r}{\sigma^2}}{\sigma^2} \right|_k^Y}{k} \right|_k^Y \\ & \left| \frac{-0.5 + \$ \frac{J - 0.5 + \frac{-d+r}{\sigma^2} N^2 + \frac{2r}{\sigma^2} + \frac{d \cdot r}{\sigma^2}}{d} + \frac{0.5 + \$ \frac{J - 0.5 + \frac{-d+r}{\sigma^2} N^2 + \frac{2r}{\sigma^2} + \frac{d \cdot r}{\sigma^2}}{r} \right|_k^Y \\ & \left| \frac{1}{\sigma^2} + \frac{-d + r + 0.5 \sigma^2}{\$ \frac{J - 0.5 + \frac{-d+r}{\sigma^2} N^2 + \frac{2r}{\sigma^2} \sigma^4} \right|_k^Y \text{Log@cD} \\ & \left| \frac{0.5 + \$ \left| \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2} + \frac{d \cdot r}{\sigma^2}}{\sigma^2} \right|_k^Y}{p^{0.5} \$ \left| \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2} + \frac{d \cdot r}{\sigma^2}}{\sigma^2} \right|_k^Y} \right|_k^Y \\ & \left| \frac{-0.5 + \$ \frac{J - 0.5 + \frac{-d+r}{\sigma^2} N^2 + \frac{2r}{\sigma^2} + \frac{d \cdot r}{\sigma^2}}{d} + \frac{0.5 + \$ \frac{J - 0.5 + \frac{-d+r}{\sigma^2} N^2 + \frac{2r}{\sigma^2} + \frac{d \cdot r}{\sigma^2}}{r} \right|_k^Y \\ & \left| \frac{1}{\sigma^2} + \frac{d - r - 0.5 \sigma^2}{\$ \frac{J - 0.5 + \frac{-d+r}{\sigma^2} N^2 + \frac{2r}{\sigma^2} \sigma^4} \right|_k^Y \text{Log@pD} \right|_k^Y \\ & \left| \frac{1}{k} \left| \frac{d}{\sigma^2} \right|_p^{0.5} \$ \left| \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2} + \frac{r}{\sigma^2}}{\sigma^2} \right|_{\sigma^2}^2 - 1 \cdot c^{0.5} \$ \left| \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2} + \frac{r}{\sigma^2}}{\sigma^2} \right|_p^{0.5} \frac{d}{\sigma^2} \right|_k^Y \\ & \left| \frac{1}{k} \left| \frac{d - 1 \cdot r + \left| \frac{-0.5 + 1 \cdot \$ \left| \frac{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2}}{\sigma^2} \right| \sigma^2}{\sigma^2} \right|_k^Y \right|_k^Y \right|_k^Y \end{aligned}$$

Total derivative $dP^*/d\mathbf{d} =$

$$\begin{aligned}
 & \left| \frac{d}{\sigma^2} \right|_k^i p^{0.5 + \$ \left| \frac{-d+r}{\sigma^2} \right|_k^i \left\{ \frac{y^2}{\sigma^2} + \frac{2r}{\sigma^2} + \frac{r}{\sigma^2} \right\} \sigma^2} \\
 & \left| \frac{p \left| \frac{-0.5 + \$ \left| \frac{-d+r}{\sigma^2} \right|_k^i \left\{ \frac{y^2}{\sigma^2} + \frac{2r}{\sigma^2} + \frac{d-r}{\sigma^2} \right\} \right|_k^i}{d^2} - \frac{p \left| \frac{1 + \frac{d-r+0.5 \sigma^2}{\$ \left| \frac{-0.5 + \frac{-d+r}{\sigma^2} \right|_k^i \left\{ \frac{y^2}{\sigma^2} + \frac{2r}{\sigma^2} \right\} \sigma^2} \right|_k^i}{d \sigma^2} \right|_k^i}{d^2} + \right. \\
 & \left. \frac{\left| \frac{1 + \frac{d-r+0.5 \sigma^2}{\$ \left| \frac{-0.5 + \frac{-d+r}{\sigma^2} \right|_k^i \left\{ \frac{y^2}{\sigma^2} + \frac{2r}{\sigma^2} \right\} \sigma^2} \right|_k^i}{\sigma^2} - c^{0.5 + \$ \left| \frac{-0.5 + \frac{-d+r}{\sigma^2} \right|_k^i \left\{ \frac{y^2}{\sigma^2} + \frac{2r}{\sigma^2} + \frac{d-r}{\sigma^2} \right\}} \right|_k^i}{\sigma^2} - \frac{p^{0.5 + \$ \left| \frac{-0.5 + \frac{-d+r}{\sigma^2} \right|_k^i \left\{ \frac{y^2}{\sigma^2} + \frac{2r}{\sigma^2} + \frac{d-r}{\sigma^2} \right\}} \left| \frac{-0.5 + \$ \left| \frac{-d+r}{\sigma^2} \right|_k^i \left\{ \frac{y^2}{\sigma^2} + \frac{2r}{\sigma^2} + \frac{d-r}{\sigma^2} \right\}}{d^2} \right|_k^i}{\sigma^2} + \right. \\
 & \left. \frac{1 + \frac{d-r+0.5 \sigma^2}{\$ \left| \frac{-0.5 + \frac{-d+r}{\sigma^2} \right|_k^i \left\{ \frac{y^2}{\sigma^2} + \frac{2r}{\sigma^2} \right\} \sigma^2}}{d \sigma^2} + \frac{1 + \frac{d-r+0.5 \sigma^2}{\$ \left| \frac{-0.5 + \frac{-d+r}{\sigma^2} \right|_k^i \left\{ \frac{y^2}{\sigma^2} + \frac{2r}{\sigma^2} \right\} \sigma^2}}{r \sigma^2} \right|_k^i}{\sigma^2} - \frac{1}{\sigma^2} \right|_k^i \\
 & \left| \frac{1}{\sigma^2} \right|_k^i c^{0.5 + \$ \left| \frac{-0.5 + \frac{-d+r}{\sigma^2} \right|_k^i \left\{ \frac{y^2}{\sigma^2} + \frac{2r}{\sigma^2} + \frac{d-r}{\sigma^2} \right\}} p^{0.5 + \$ \left| \frac{-0.5 + \frac{-d+r}{\sigma^2} \right|_k^i \left\{ \frac{y^2}{\sigma^2} + \frac{2r}{\sigma^2} + \frac{d-r}{\sigma^2} \right\}} \\
 & \left| \frac{-0.5 + \$ \left| \frac{-d+r}{\sigma^2} \right|_k^i \left\{ \frac{y^2}{\sigma^2} + \frac{2r}{\sigma^2} + \frac{d-r}{\sigma^2} \right\}}{d} + \frac{0.5 + \$ \left| \frac{-d+r}{\sigma^2} \right|_k^i \left\{ \frac{y^2}{\sigma^2} + \frac{2r}{\sigma^2} + \frac{d-r}{\sigma^2} \right\}}{r} \right|_k^i \\
 & \left| \frac{-1 + \frac{d-r+0.5 \sigma^2}{\$ \left| \frac{-d+r}{\sigma^2} \right|_k^i \left\{ \frac{y^2}{\sigma^2} + \frac{2r}{\sigma^2} \right\} \sigma^2}}{\sigma^2} \right|_k^i \text{Log} @ c D \left| \frac{y}{\sigma^2} \right|_k^i \\
 & \left| \frac{1}{\sigma^2} \right|_k^i c^{0.5 + \$ \left| \frac{-0.5 + \frac{-d+r}{\sigma^2} \right|_k^i \left\{ \frac{y^2}{\sigma^2} + \frac{2r}{\sigma^2} + \frac{d-r}{\sigma^2} \right\}} p^{0.5 + \$ \left| \frac{-0.5 + \frac{-d+r}{\sigma^2} \right|_k^i \left\{ \frac{y^2}{\sigma^2} + \frac{2r}{\sigma^2} + \frac{d-r}{\sigma^2} \right\}} \\
 & \left| \frac{-0.5 + \$ \left| \frac{-d+r}{\sigma^2} \right|_k^i \left\{ \frac{y^2}{\sigma^2} + \frac{2r}{\sigma^2} + \frac{d-r}{\sigma^2} \right\}}{d} + \frac{0.5 + \$ \left| \frac{-d+r}{\sigma^2} \right|_k^i \left\{ \frac{y^2}{\sigma^2} + \frac{2r}{\sigma^2} + \frac{d-r}{\sigma^2} \right\}}{r} \right|_k^i \\
 & \left| \frac{1 + \frac{-0.5 + \frac{-d+r}{\sigma^2}}{\$ \left| \frac{-d+r}{\sigma^2} \right|_k^i \left\{ \frac{y^2}{\sigma^2} + \frac{2r}{\sigma^2} \right\} \sigma^2}}{\sigma^2} \right|_k^i \text{Log} @ p D \left| \frac{y}{\sigma^2} \right|_k^i \\
 & \left| \frac{1}{\sigma^2} \right|_k^i \left| \frac{d}{\sigma^2} \right|_k^i p^{0.5 + \$ \left| \frac{-0.5 + \frac{-d+r}{\sigma^2} \right|_k^i \left\{ \frac{y^2}{\sigma^2} + \frac{2r}{\sigma^2} + \frac{r}{\sigma^2} \right\}} - 1 \cdot c^{0.5 + \$ \left| \frac{-0.5 + \frac{-d+r}{\sigma^2} \right|_k^i \left\{ \frac{y^2}{\sigma^2} + \frac{2r}{\sigma^2} + \frac{r}{\sigma^2} \right\}} p \left| \frac{d}{\sigma^2} \right|_k^i \\
 & \left| \frac{1 \cdot d - 1 \cdot r + \left| \frac{-0.5 + 1 \cdot \$ \left| \frac{-d+r}{\sigma^2} \right|_k^i \left\{ \frac{y^2}{\sigma^2} + \frac{2r}{\sigma^2} \right\} \sigma^2}{\sigma^2} \right|_k^i}{\sigma^2} \right|_k^i
 \end{aligned}$$

Total derivative $dP^*/dC =$

$$\begin{aligned}
 & - \left[\frac{\partial}{\partial k} p^{0.5} \right] \left[\frac{\partial}{\partial k} 1 \cdot c \right] \left[\frac{\partial}{\partial k} \left\{ \sqrt{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2} + \frac{r}{\sigma^2}} \right\} p^{0.5 + \frac{d}{\sigma^2}} - 1 \cdot c \right] \left[\frac{\partial}{\partial k} \left\{ \sqrt{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2} + \frac{r}{\sigma^2}} \right\} p^{0.5 + \frac{d}{\sigma^2}} \right] \\
 & \left[\frac{\partial}{\partial k} 1 \cdot d - 1 \cdot r + \left[\frac{\partial}{\partial k} 0.5 + 1 \cdot \left\{ \sqrt{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2} + \frac{r}{\sigma^2}} \right\} \sigma^2 \right] \right] Y \\
 & \left[\frac{\partial}{\partial k} c^{0.5} \right] \left[\frac{\partial}{\partial k} 1 \cdot c \right] \left[\frac{\partial}{\partial k} \left\{ \sqrt{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2} + \frac{r}{\sigma^2}} \right\} p^{0.5 + \frac{d}{\sigma^2}} - 1 \cdot c \right] \left[\frac{\partial}{\partial k} \left\{ \sqrt{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2} + \frac{r}{\sigma^2}} \right\} p^{0.5 + \frac{d}{\sigma^2}} \right] \\
 & \left[\frac{\partial}{\partial k} r \right] \left[\frac{\partial}{\partial k} 1 \cdot d - 1 \cdot r + \left[\frac{\partial}{\partial k} 0.5 + 1 \cdot \left\{ \sqrt{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2} + \frac{r}{\sigma^2}} \right\} \sigma^2 \right] \right] Y
 \end{aligned}$$

Total derivative $dP^*/dI =$

$$\begin{aligned}
 & \left[\frac{\partial}{\partial k} \frac{d}{\sigma^2} p^{0.5} \right] \left[\frac{\partial}{\partial k} \left\{ \sqrt{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2} + \frac{r}{\sigma^2}} \right\} \right] \left[\frac{\partial}{\partial k} 0.5 + \left\{ \sqrt{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2} + \frac{r}{\sigma^2}} \right\} - \frac{-d+r}{\sigma^2} \right] \left[\frac{\partial}{\partial k} \left\{ \sqrt{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2} + \frac{r}{\sigma^2}} \right\} \sigma^2 \right] Y \\
 & \left[\frac{\partial}{\partial k} 1 \cdot c \right] \left[\frac{\partial}{\partial k} \left\{ \sqrt{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2} + \frac{r}{\sigma^2}} \right\} p^{0.5 + \frac{d}{\sigma^2}} - 1 \cdot c \right] \left[\frac{\partial}{\partial k} \left\{ \sqrt{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2} + \frac{r}{\sigma^2}} \right\} p^{0.5 + \frac{d}{\sigma^2}} \right] \\
 & \left[\frac{\partial}{\partial k} 1 \cdot d - 1 \cdot r + \left[\frac{\partial}{\partial k} 0.5 + 1 \cdot \left\{ \sqrt{-0.5 + \frac{-d+r}{\sigma^2} Y^2 + \frac{2r}{\sigma^2} + \frac{r}{\sigma^2}} \right\} \sigma^2 \right] \right] Y
 \end{aligned}$$

Finite-time model. Total derivative $dP^*/dT=$

$$\begin{aligned}
 & \left| -\tilde{a}^{-dt} p \right|_k - 0.5 + \$ \left| -0.5 + \frac{-d+r}{\sigma^2} \gamma^2 + \frac{2r}{\sigma^2} + \frac{d-r}{\sigma^2} \gamma \right|_k + c \left| -0.5 + \frac{-d+r}{\sigma^2} \gamma^2 + \frac{2r}{\sigma^2} + \frac{d-r}{\sigma^2} \gamma \right|_k \\
 & \tilde{a}^{-dt} p \left| -0.5 + \frac{-d+r}{\sigma^2} \gamma^2 + \frac{2r}{\sigma^2} + \frac{d-r}{\sigma^2} \gamma \right|_k - 0.5 + \$ \left| -0.5 + \frac{-d+r}{\sigma^2} \gamma^2 + \frac{2r}{\sigma^2} + \frac{d-r}{\sigma^2} \gamma \right|_k + \\
 & c \tilde{a}^{-rt} \left| 0.5 + \$ \left| -0.5 + \frac{-d+r}{\sigma^2} \gamma^2 + \frac{2r}{\sigma^2} + \frac{d-r}{\sigma^2} \gamma \right|_k \right|_k " \\
 & \left| c \right|_k \left| -0.5 + \frac{-d+r}{\sigma^2} \gamma^2 + \frac{2r}{\sigma^2} + \frac{d-r}{\sigma^2} \gamma \right|_k - 0.5 + \$ \left| -0.5 + \frac{-d+r}{\sigma^2} \gamma^2 + \frac{2r}{\sigma^2} + \frac{d-r}{\sigma^2} \gamma \right|_k \\
 & \left| - \frac{H1 - \tilde{a}^{-dt} L}{d} \right|_k - 0.5 + \$ \left| -0.5 + \frac{-d+r}{\sigma^2} N^2 + \frac{2r}{\sigma^2} + \frac{d-r}{\sigma^2} \gamma \right|_k + \frac{0.5 + \$ \left| -0.5 + \frac{-d+r}{\sigma^2} N^2 + \frac{2r}{\sigma^2} + \frac{d-r}{\sigma^2} \gamma \right|_k}{r} \\
 & \left| 0.5 - \$ \left| -0.5 + \frac{-d+r}{\sigma^2} \gamma^2 + \frac{2r}{\sigma^2} + \frac{d-r}{\sigma^2} \gamma \right|_k \right|_k + \\
 & \frac{H1 - \tilde{a}^{-dt} L}{d} \left| -0.5 + \$ \left| -0.5 + \frac{-d+r}{\sigma^2} N^2 + \frac{2r}{\sigma^2} + \frac{d-r}{\sigma^2} \gamma \right|_k \right|_k
 \end{aligned}$$

Appendix E

A Sample *Mathematica* Program for Computing the Threshold Prices P^*

```

c = 12.845;
i = 218.192;
d = 0.1;
r = 0.05;
sigma = 0.2;

x1 = H - 0.5 + Hr - dL • Hsigma^2LL^2 + H2 • r • Hsigma^2LL •• N;
x2 = Sqrt[x1D] •• N;
x3 = 0.5 - Hr - dL • Hsigma^2L •• N;

"Beta 1"
b1 = x3 + x2 •• N

"Beta 2"
b2 = x3 - x2 •• N

"Q"
b3 = HHb1 • rL - Hb1 - 1L • dL • Hb1 - b2L •• N

h := Hc • rL + i;
s@p_D := Hb1 - b2L • b3 • Hc^H1 - b2LL • Hp^b2L - b1 • h + Hb1 - 1L • p • d •• N

FindRoot@s@pD Š 0, 8p, c<D

```

Appendix F

The Form of the Log-Likelihood Function Used for Estimating Risk Attitudes

A log-likelihood function of a system of equations with two independent variables and error terms that are jointly normally distributed can be written as

$$L(\mathbf{y}, \mathbf{g}) = -\frac{T}{2} \{2\ln(2\mathbf{p}) + \ln|\mathbf{\hat{U}}|\} - \frac{1}{2} \sum_{t=1}^T [\mathbf{\hat{I}}_t' \mathbf{\hat{U}}^{-1} \mathbf{\hat{I}}_t] \quad (\text{E.1})$$

where T is the number of observations, $\mathbf{n}$ is a vector of error terms, and $\mathbf{W}$ is a variance-covariance matrix of the random variables y_1 and y_2 . The log-likelihood of the sample is a function of the random variables given by the vector $\mathbf{y} = (\mathbf{y}_{1t}', \mathbf{y}_{2t}')$. The parameters to be estimated are represented by the vector $\mathbf{g} = (\mathbf{a}, \mathbf{b})$. Further, we define $\mathbf{n}$ and $\mathbf{W}$ in the following way: $\mathbf{n} = \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \end{bmatrix}$,

$\mathbf{W} = \begin{bmatrix} \mathbf{s}_1^2 & \mathbf{s}_{12} \\ \mathbf{s}_{12} & \mathbf{s}_2^2 \end{bmatrix}$. Using these definitions, we can rewrite the log-likelihood function in the explicit

form

$$L(\mathbf{y}, \mathbf{g}) = -T \ln(2\mathbf{p}) - \frac{T}{2} \ln(\mathbf{s}_1^2 \mathbf{s}_2^2 - \mathbf{s}_{12}^2) - \frac{1}{\mathbf{s}_1^2 \mathbf{s}_2^2 - \mathbf{s}_{12}^2} \left[\frac{\mathbf{s}_2^2}{2} \sum_{t=1}^T \mathbf{e}_{1t}^2 - \mathbf{s}_{12} \sum_{t=1}^T \mathbf{e}_{1t} \mathbf{e}_{2t} + \frac{\mathbf{s}_1^2}{2} \sum_{t=1}^T \mathbf{e}_{2t}^2 \right] \quad (\text{E.2})$$

Because the function $L(\mathbf{y}, \mathbf{g})$ is nonlinear, the maximum likelihood estimator $\mathbf{g}^*$ is obtained by using a numerical algorithm to maximize (E.2) with respect to $\mathbf{g}$. As shown by Amemiya (1985, p. 114), under fairly general conditions, the estimator $\mathbf{g}^*$ is consistent and asymptotically normally distributed.

Appendix G

Estimating the Variance of the Coefficients of Risk Aversion

The formulas relating risk aversion coefficients to the estimated parameters $\mathbf{a}_1$ and $\mathbf{a}_2$ and the profit $\mathbf{p}$ are

$$\mathbf{l} = -(\mathbf{a}_1 + 2\mathbf{a}_2\mathbf{p})$$

$$\mathbf{r} = -(\mathbf{a}_1\mathbf{p} + 2\mathbf{a}_2\mathbf{p}^2)$$

$$\mathbf{d} = 2\mathbf{a}_2 + (\mathbf{a}_1 + 2\mathbf{a}_2\mathbf{p})^2$$

Variance of $\mathbf{l}$

$$\text{Var}(\lambda) = \text{Var}(\alpha) + 2\pi\text{Cov}(\alpha, \alpha) + \pi^2\text{Var}(\alpha)$$

Variance of $\mathbf{r}$

$$\text{Var}(\rho) = \pi^2\text{Var}(\alpha) + 2\pi^3\text{Cov}(\alpha, \alpha) + \pi^4\text{Var}(\alpha)$$

Variance of $\mathbf{d}$

First, we present δ as $\delta = 2\alpha + \lambda^2$. Then $\text{Var}(\delta)$ becomes

$$\text{Var}(\delta) = \text{Var}(2\alpha + \lambda^2) = 4\text{Var}(\alpha) + 4\text{Cov}(\alpha, \lambda^2) + \text{Var}(\lambda^2)$$

A linear Taylor series approximation is useful to find the $\text{Var}(\lambda^2)$

$$\text{Var}[g(X)] = \{E[g_X(X)]\}^2\text{Var}(X),$$

so that

$$\text{Var}(\lambda^2) = (2\lambda)^2\text{Var}(\lambda) = 4\lambda^2\text{Var}(\lambda) = 4\lambda^2 [\text{Var}(\alpha) + 2\pi\text{Cov}(\alpha, \alpha) + \pi^2\text{Var}(\alpha)]$$

To find the $\text{Cov}(\alpha, \lambda^2)$ we apply a bivariate extension of the Stein's lemma

$$\text{Cov}[X, h(Y, Z)] = E[h_Y(Y, Z)]\text{Cov}(X, Y) + E[h_Z(Y, Z)]\text{Cov}(X, Z).$$

Thus

$$\begin{aligned}\text{Cov}(\alpha_2, \lambda^2) &= \text{Cov}[\alpha_2, (-\alpha_1 - \alpha_2\pi)^2] = \text{Cov}[\alpha_2, (\alpha_1^2 + 2\alpha_1\alpha_2\pi + \alpha_2^2\pi^2)] = \\ &= \text{E}(2\alpha_1 + 2\alpha_2\pi)\text{Cov}(\alpha_1, \alpha_2) + \text{E}(2\alpha_1\pi + 2\alpha_2\pi^2)\text{Var}(\alpha_2) \\ &= 2(\alpha_1 + \alpha_2\pi)[\text{Cov}(\alpha_1, \alpha_2) + \pi\text{Var}(\alpha_2)] = -2\lambda[\text{Cov}(\alpha_1, \alpha_2) + \pi\text{Var}(\alpha_2)]\end{aligned}$$

Summing up,

$$\begin{aligned}\text{Var}(\mathfrak{J}) &= 4\text{Var}(\alpha_2) - 8\lambda[\text{Cov}(\alpha_1, \alpha_2) + \pi\text{Var}(\alpha_2)] + \\ &4\lambda^2[\text{Var}(\alpha_1) + 2\pi\text{Cov}(\alpha_1, \alpha_2) + \pi^2\text{Var}(\alpha_2)] = \\ &4\lambda^2\text{Var}(\alpha_1) + \text{Cov}(\alpha_1, \alpha_2)[8\pi\lambda^2 - 8\lambda] + \text{Var}(\alpha_2)[4\pi^2\lambda^2 - 8\pi\lambda + 4]\end{aligned}$$

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