2001

Large -eddy simulation of a three-dimensional compressible tornado vortex

Jianjun Xia
West Virginia University

Follow this and additional works at: <https://researchrepository.wvu.edu/etd>

Recommended Citation

Xia, Jianjun, "Large -eddy simulation of a three-dimensional compressible tornado vortex" (2001).
Graduate Theses, Dissertations, and Problem Reports. 1448.
<https://researchrepository.wvu.edu/etd/1448>

This Dissertation is protected by copyright and/or related rights. It has been brought to you by the The Research Repository @ WVU with permission from the rights-holder(s). You are free to use this Dissertation in any way that is permitted by the copyright and related rights legislation that applies to your use. For other uses you must obtain permission from the rights-holder(s) directly, unless additional rights are indicated by a Creative Commons license in the record and/ or on the work itself. This Dissertation has been accepted for inclusion in WVU Graduate Theses, Dissertations, and Problem Reports collection by an authorized administrator of The Research Repository @ WVU. For more information, please contact researchrepository@mail.wvu.edu.

LARGE-EDDY SIMULATION OF A THREE- DIMENSIONAL COMPRESSIBLE TORNADO VORTEX

JIANJUN XIA

Dissertation submitted to the
College of Engineering and Mineral Resources
at West Virginia University
in partial fulfillment of the requirements
for the degree of

Doctor of Philosophy
in
Mechanical Engineering

William Steve Lewellen, Ph.D., Chair
David C. Lewellen, Ph.D.
John M. Kuhlman, Ph.D.
Donald D. Gray, Ph.D.
Ian Christie, Ph.D.

Department of Mechanical & Aerospace Engineering

Morgantown, West Virginia
2001

Keywords: Computational Fluid Dynamics, Large-Eddy Simulation,
Compressible, Tornado Vortex

Copyright © 2001 Jianjun Xia

ABSTRACT

Large-Eddy Simulation of a Three-Dimensional Compressible Tornado Vortex

Jianjun Xia

Large-Eddy simulation (LES) has become a very useful tool for investigating tornadoes, one of the more spectacular and destructive phenomena of nature. A new three-dimensional, unsteady, compressible model is generated to determine how significant the differences between compressible and incompressible LES simulations may be in some extremely violent tornadoes. In particular, this study seeks to determine how high the Mach number within the tornado may become before significant changes occur due to compressibility, and what the major effects of these changes may be expected to be.

After developing and verifying the compressible LES model, three different patterns of tornadic corner flows cataloged by local swirl ratio are simulated under quasi-steady conditions for different Mach numbers. Simulation comparisons have demonstrated that the compressibility effects are different for different corner flow structures. At peak average Mach numbers less than approximately 0.5, the compressibility effects are not very significant and may be accounted for to leading order by an appropriate isentropic transformation applied to the incompressible results. As the maximum Mach number is increased to more than 1.0, the compressibility effects for low-swirl-ratio corner flows are dramatic, with significant increase in peak vertical velocity and the height of the vortex breakdown above the surface. The effects are much weaker for medium swirl conditions, and expected to be still weaker for high swirl corner flow where the effects are essentially limited to influencing the secondary vortices. In general, compressibility effects would not change the basic dynamics of tornadic corner flows even if Mach numbers greater than one are achieved.

This study also shows that during the sharp temporal overshoot in near-surface intensity that can sometimes occur during a tornado's evolution, the maximum pressure drop will tend to be restricted by supersonic velocities, and thus limit the intensification of the overshoot. There are no apparent physical barriers to transonic speeds occurring within real tornadoes on rare occasions, however it is believed that most tornado dynamics are not significantly impacted by Mach number effects. There may be occasions when the maximum velocity within a tornado is limited by sonic conditions for brief periods but it is a soft limit which allows modest supersonic velocities that would be difficult to observe in an actual tornado.

Dedication

To

my parents : Shi-Jie Xia, Guan-Yin Zhou

wife : Wen-Hui Zhang

& daughter : Amanda Yuan Xia

Acknowledgements

I am deeply indebted to my advisors, Dr. William Steve Lewellen and Dr. David C. Lewellen, for their support, encouragement and patience. Their suggestions, supervision and guidance have been invaluable to the completion of this dissertation, and will long be treasured and greatly appreciated. No words in this world are adequate to express my gratitude.

With respect and gratitude, I wish to thank Dr. John M. Kuhlman, Dr. Donald D. Gray, and Dr. Ian Christie for agreeing to be on the committee. Their assistance and critical comments to the research work and dissertation have been very helpful to the accomplishment of this dissertation.

Acknowledgments are also due to my parents Shi-Jie Xia and Guan-Yin Zhou, wife Wen-Hui Zhang and daughter Amanda Yuan Xia, who have given me so much love and devotion. I sincerely thank them for providing the motivation and inspiration that made this endeavor all worthwhile.

This research was supported by National Science Foundation Grant 9317599 with S. P. Nelson as technical monitor. I would like to thank the American Meteorological Society for permission to reprint Appendix A.

Table of Contents

Title	i
Abstract	ii
Dedication	iii
Acknowledgements	iv
Table of Contents	v
List of Tables.....	viii
List of Figures	ix
Nomenclature	xv
Chapter 1: Introduction	1
1.1 The tornado vortex.....	1
1.2 Tornado research	5
1.3 Outline of the dissertation.....	6
Chapter 2: Literature Review and Research Background	8
2.1 Different research approaches	8
2.2 Previous work	10
2.3 Potential importance of compressible effects	16
2.4 Connection between incompressible and compressible results	18
Chapter 3: Description of the Compressible Large-Eddy Simulation Model	20
3.1 Introduction.....	20
3.2 Governing equations.....	20
3.2.1 Compressible Navier-Stokes equations	20
3.2.2 Definition of Π and θ	22
3.2.3 Density-based method	22
3.3 Subgrid scale model.....	24
3.4 Full equation set.....	25
3.5 Numerical scheme	25
3.6 Boundary and initial conditions.....	27

Chapter 4: Verification of the Compressible Large-Eddy Simulation Model	29
4.1 Introduction.....	29
4.2 One-dimensional shock and expansion wave case	29
4.2.1 Introduction.....	29
4.2.2 Isentropic cases	31
4.2.3 Cases with the initial temperature uniform.....	36
4.3 Instability tests in a two-dimensional purely swirling vortex.....	41
4.3.1 Introduction.....	41
4.3.2 Low Mach number case.....	41
4.3.3 High Mach number case	42
4.4 Simulation of a 3D tornado vortex at a very low Mach number	44
4.4.1 Introduction.....	44
4.4.2 Simulation results	45
4.5 Conclusions of the chapter.....	47
Chapter 5: Tornado Vortex Simulations with Quasi-Steady Conditions	49
5.1 Basic behaviors of tornadic corner flows	49
5.2 Simulations of tornado vortex flows with a low swirl ratio	54
5.2.1 Simulation results at a small Mach number.....	55
5.2.2 Simulation results at a medium Mach number	57
5.2.3 Simulation results at a large Mach number	58
5.2.4 Section summary	64
5.3 Simulations of tornado vortex flows with a medium swirl ratio	66
5.4 Simulations of tornado vortex flows with a high swirl ratio	72
5.5 Conclusions of the chapter.....	77
Chapter 6: Simulations of a Transient Tornado Vortex	83
6.1 Introduction.....	83
6.2 Comparison of incompressible and compressible simulation results	84
6.3 Conclusions of the chapter.....	94

Chapter 7: Summary and Discussion	95
7.1 Conclusions from previous chapters.....	95
7.2 Recommendations and future work	96
 Bibliography.....	 98
Appendix A: The Influence of a Local Swirl Ratio on Tornado Intensification near the Surface.....	 105
Appendix B: Nonaxisymmetric, Unsteady Tornadic Corner Flows	124
Curriculum Vitae.....	129

List of Tables

Table	Caption	Page
4-2-1	Comparison of simulation results and analytical values ($T_0 = 300$ K)	37
5-1-1	Summary of the cases simulated in Chapter 5	53
5-5-1	Summary of simulation results plotted in the following figures	77
5-5-2	Summary of reference values	77

List of Figures

Figure	Caption	Page
1-1-1	Schematic of a tornado flow (from <i>Whipple</i> 1982)	1
1-1-2	Conceptual model of the flow regimes associated with a tornado (from <i>Wurman et al.</i> 1996)	2
1-1-3	Schematic of different types of corner flow for increasing swirl (from <i>Davies-Jones</i> 1986)	3
2-2-1	Summary of the surface intensification of the vortex for a set of 50 simulations as measured by the ratios $V_{\max}/V_c$ and $\sqrt{\Delta P_{\min}/\Delta P_c}$ versus S_c (from <i>Lewellen et al.</i> 2000a)	13
2-2-2	Instantaneous horizontal cross section of simulated tornado showing perturbation pressure and normalized vertical velocity for the first high swirl case discussed in the text (from <i>Lewellen et al.</i> 2000a)	14
2-2-3	Instantaneous horizontal cross section of simulated tornado showing perturbation pressure and normalized vertical velocity for the second high swirl case discussed in the text (from <i>Lewellen et al.</i> 2000a)	15
2-3-1	Relative intensification as a function of time for two cases described in <i>Lewellen et al.</i> (2000b)	17
2-4-1	Comparison of perturbation pressure ΔP normalized by $\rho_s V_s^2$ along radius at a certain vertical level with different Mach numbers	19
4-2-1	Arrangement of a shock tube (from <i>Oosthuizen and Carscallen</i> 1997)	30
4-2-2	Waves generated in a shock tube following rupture of diaphragm (from <i>Oosthuizen and Carscallen</i> 1997)	30
4-2-3	Pressure distribution of simulation results at 1.0 second, starting from $P_1/P_2 = 150 \text{ kPa}/100 \text{ kPa}$, $\theta = 300 \text{ K}$	32
4-2-4(a)	Velocity distribution of simulation results at 1.0 second with 400 grid points, starting from $P_1/P_2 = 150 \text{ kPa}/100 \text{ kPa}$, $\theta = 300 \text{ K}$	32
4-2-4(b)	Velocity distribution of simulation results at 1.0 second with 200 grid points, starting from $P_1/P_2 = 150 \text{ kPa}/100 \text{ kPa}$, $\theta = 300 \text{ K}$	33

4-2-4(c)	Velocity distribution of simulation results at 1.0 second with 100 grid points, starting from $P_1/P_2 = 150 \text{ kPa}/100 \text{ kPa}$, $\theta = 300 \text{ K}$	33
4-2-5	Temperature distribution of simulation results at 1.0 second, starting from $P_1/P_2 = 150 \text{ kPa}/100 \text{ kPa}$, $\theta = 300 \text{ K}$	34
4-2-6	Pressure distribution of simulation results at 0.7 second, starting from $P_1/P_2 = 300 \text{ kPa}/30 \text{ kPa}$, $\theta = 300 \text{ K}$	35
4-2-7	Velocity distribution of simulation results at 0.7 second, starting from $P_1/P_2 = 300 \text{ kPa}/30 \text{ kPa}$, $\theta = 300 \text{ K}$	35
4-2-8	Temperature distribution of simulation results at 0.7 second, starting from $P_1/P_2 = 300 \text{ kPa}/30 \text{ kPa}$, $\theta = 300 \text{ K}$	36
4-2-9	Pressure distribution of simulation results at 1.0 second, starting from $P_1/P_2 = 150 \text{ kPa}/100 \text{ kPa}$, $T_0 = 300 \text{ K}$	37
4-2-10	Velocity distribution of simulation results at 1.0 second, starting from $P_1/P_2 = 150 \text{ kPa}/100 \text{ kPa}$, $T_0 = 300 \text{ K}$	38
4-2-11	Potential temperature θ distribution of simulation results at 1.0 second, starting from $P_1/P_2 = 150 \text{ kPa}/100 \text{ kPa}$, $T_0 = 300 \text{ K}$	38
4-2-12	Temperature distribution of simulation results at 1.0 second, starting from $P_1/P_2 = 150 \text{ kPa}/100 \text{ kPa}$, $T_0 = 300 \text{ K}$	39
4-2-13	Pressure distribution of simulation results at 0.7 second, starting from $P_1/P_2 = 300 \text{ kPa}/30 \text{ kPa}$, $T_0 = 300 \text{ K}$	39
4-2-14	Velocity distribution of simulation results at 0.7 second, starting from $P_1/P_2 = 300 \text{ kPa}/30 \text{ kPa}$, $T_0 = 300 \text{ K}$	40
4-2-15	Temperature distribution of simulation results at 0.7 second, starting from $P_1/P_2 = 300 \text{ kPa}/30 \text{ kPa}$, $T_0 = 300 \text{ K}$	40
4-3-1	Instantaneous perturbation density, $\Delta\rho$ ($\text{kg}\cdot\text{m}^{-3}$), on a horizontal slice after 300 seconds, where the maximum Mach number is 0.03	42
4-3-2	Instantaneous horizontal velocity, V_{xy} ($\text{m}\cdot\text{s}^{-1}$), on a horizontal slice after 300 seconds, where the maximum Mach number is 0.03	43

4-3-3	Instantaneous perturbation density, $\Delta\rho$ ($\text{kg}\cdot\text{m}^{-3}$), on a horizontal slice after 300 seconds, where the maximum Mach number is 0.52	43
4-3-4	Instantaneous horizontal velocity, V_{xy} ($\text{m}\cdot\text{s}^{-1}$), on a horizontal slice after 300 seconds, where the maximum Mach number is 0.52	44
4-4-1	Results from the incompressible LES code in a three-dimensional tornado vortex at $M = 0$	46
4-4-2	Results from the compressible LES code in a three-dimensional tornado vortex, where the instantaneous maximum Mach number is 0.18	47
5-1-1	Instantaneous isosurfaces of perturbation pressure, ΔP (Pa), within a $300\text{ m} \times 300\text{ m} \times 300\text{ m}$ domain, at (a) high swirl ratio, (b) medium swirl ratio, and (c) low swirl ratio	50
5-2-1	Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for the corner flow of an incompressible low-swirl tornado vortex	55
5-2-2	Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for the corner flow of a compressible low-swirl tornado vortex at $M_{\max} = 0.37$	56
5-2-3	Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for the corner flow of a compressible low-swirl tornado vortex at $M_{\max} = 0.70$	57
5-2-4	Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for the corner flow of a compressible low-swirl tornado vortex at $M_{\max} = 1.24$	58
5-2-5	Zoom in Figures 5-2-1(a) and 5-2-4(a). (a) swirl velocity at $M = 0$, (b) product of density and swirl velocity at $M_{\max} = 1.24$, (c) swirl velocity at $M_{\max} = 1.24$	59
5-2-6	Zoom in Figures 5-2-1(b) and 5-2-4(b). (a) vertical velocity at $M = 0$, (b) product of density and vertical velocity at $M_{\max} = 1.24$, (c) vertical velocity at $M_{\max} = 1.24$	60
5-2-7	Zoom in Figures 5-2-1(c) and 5-2-4(c). (a) radial velocity at $M = 0$, (b) product of density and radial velocity at $M_{\max} = 1.24$, (c) radial	61

	velocity at $M_{\max} = 1.24$	
5-2-8	Instantaneous vertical cross section of an incompressible low-swirl tornado vortex showing normalized swirl velocity and magnitude of the velocity vector in the r - z plane	62
5-2-9	Instantaneous vertical cross section of a compressible low-swirl tornado vortex at large Mach number showing normalized swirl velocity and magnitude of the velocity vector in the r - z plane	63
5-2-10	Instantaneous vertical cross section of compressible low-swirl tornado vortex at large Mach number showing local Mach number and normalized vertical velocity in the r - z plane	64
5-2-11	In a low-swirl ratio tornado vortex, axisymmetric time-averaged distribution of $\Delta P / \rho_s V_s^2$ versus height along the centerline	66
5-3-1	Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for the corner flow of an incompressible medium-swirl tornado vortex	67
5-3-2	Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for the corner flow of a compressible medium-swirl tornado vortex at $M_{\max} = 0.40$	68
5-3-3	Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for the corner flow of a compressible medium-swirl tornado vortex at $M_{\max} = 0.67$	69
5-3-4	Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for the corner flow of a compressible medium-swirl tornado vortex at $M_{\max} = 1.04$	70
5-3-5	In a medium-swirl ratio tornado vortex, axisymmetric time-averaged distribution of $\Delta P / \rho_s V_s^2$ versus height along the centerline	71
5-4-1	Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for the corner flow of an incompressible high-swirl tornado vortex	72
5-4-2	Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for the corner flow of a compressible high-swirl tornado vortex at $M_{\max} = 0.34$	73

5-4-3	Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for the corner flow of a compressible high-swirl tornado vortex at $M_{\max} = 0.64$	74
5-4-4	Instantaneous horizontal cross section of a compressible high-swirl tornado where the highest Mach number is found at the height of $0.113 r_c$, showing normalized perturbation pressure and vertical velocity	75
5-4-5	In a high-swirl ratio tornado vortex, axisymmetric time-averaged distribution of $\Delta P / \rho_s V_s^2$ along radius at the same height where $\Delta P_{\min}$ occurs in the incompressible case ($z = 14.59$ m)	76
5-5-1	Summary of the ratio $\sqrt{\Delta P_{\min} / \Delta P_c}$ versus S_c for the incompressible and compressible results at different Mach numbers	78
5-5-2	Summary of the ratio $W_{\max} / V_c$ versus S_c for the incompressible and compressible results at different Mach numbers	79
5-5-3	Summary of the ratio $V_{t\max} / V_c$ versus S_c for the incompressible and compressible results at different Mach numbers	80
5-5-4	Summary of the ratio $W_{\max} / V_{t\max}$ versus S_c for the incompressible and compressible results at different Mach numbers	81
5-5-5	Summary of the ratio $(\rho W)_{\max} / (\rho V_t)_{\max}$ versus S_c for the incompressible and compressible results at different Mach numbers	81
6-1-1	Vertical cross section of azimuthally averaged fields for the time-dependent case at $t = 0$	84
6-2-1	Vertical cross section of azimuthally averaged fields for the time-dependent case at 80 seconds	85
6-2-2	Vertical cross section of azimuthally averaged fields for the time-dependent case at 96 seconds	86
6-2-3	Vertical cross section of azimuthally averaged fields for the time-dependent case when the peak pressure drop occurs, around 108 seconds	86
6-2-4	Vertical cross section of azimuthally averaged fields for the time-dependent case at 140 seconds	87

6-2-5	Vertical cross section of azimuthally averaged fields for the time-dependent case at 170 seconds	87
6-2-6	Applying the incompressible LES model, vertical cross section of azimuthally averaged fields for the same case at 100 seconds, when the peak pressure drop occurs	88
6-2-7	Instantaneous isosurface contours of perturbation pressure, ΔP (Pa), within a $300 \text{ m} \times 300 \text{ m} \times 300 \text{ m}$ domain at 100 seconds when the peak pressure drop occurs, using the incompressible LES model	89
6-2-8	Instantaneous isosurface contours of perturbation pressure, ΔP (Pa), within a $300 \text{ m} \times 300 \text{ m} \times 300 \text{ m}$ domain at 108 seconds when the peak pressure drop occurs, using the compressible LES model	90
6-2-9	Normalized minimum pressure ($P_{\min}/P_0$) as a function of time (t/t^*) for the incompressible and compressible results during the temporal overshoot	91
6-2-10	Comparison of maximum vertical mass flow, $(\rho W)_{\max}$ ($\text{kg} \cdot \text{m}^{-2} \cdot \text{s}^{-1}$), at different time from the incompressible and compressible results	92
6-2-11	Comparison of maximum vertical velocity, $W_{\max}$ ($\text{m} \cdot \text{s}^{-1}$), at different time from the incompressible and compressible results	92
6-2-12	Instantaneous maximum local Mach numbers at different time from the compressible results	93

Nomenclature

English letter symbols

a	speed of sound, [$\text{m}\cdot\text{s}^{-1}$];
a_c	average horizontal convergence, [s^{-1}];
c_1	constant in Equation (3-3-5);
C_p	specific heat at constant pressure for the atmosphere, [$\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$];
C_v	specific heat at constant volume for the atmosphere, [$\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$];
e	internal energy per unit mass, [$\text{J}\cdot\text{kg}^{-1}$];
E	total energy per unit mass, [$\text{J}\cdot\text{kg}^{-1}$];
$\vec{f}_e$	external forces vector, [$\text{m}\cdot\text{s}^{-2}$];
g	acceleration of gravity, [$\text{m}\cdot\text{s}^{-2}$];
h	height of the domain, [m];
$\underline{\mathbf{I}}$	unit tensor;
k	thermal conductivity coefficient, [$\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$];
K	subgrid diffusivity for scalars, [$\text{m}^2\cdot\text{s}^{-1}$];
M	Mach number;
P	pressure, [Pa];
P_0	constant reference pressure, [Pa];
ΔP	perturbation pressure, [Pa];
ΔP_c	minimum average perturbation pressure in the quasi-cylindrical region above the surface layer, [Pa];
q^2	two times subgrid turbulent kinetic energy, [$\text{m}^2\cdot\text{s}^{-2}$];
q_t^2	total velocity variance, [$\text{m}^2\cdot\text{s}^{-2}$];
Q_H	heat source per unit volume, [$\text{W}\cdot\text{m}^{-3}$];
r	radius, [m];
r_0	radius of the domain, [m];
r_c	core radius in the quasi-cylindrically symmetric region of the vortex well above the surface, [m];

R_d	gas constant for dry air, [$\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$];
s	entropy per unit mass, [$\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$];
S	domain-size swirl ratio;
S_c	local swirl ratio;
S_c^*	local swirl ratio, when quasi-steady peak intensification is achieved;
S_H	constant in Equation (3-3-3);
S_m	constant in Equation (3-3-4);
t	time, [s];
Δt	time step, [s];
T	absolute temperature, [K];
T_0	initial uniform temperature in section 4.2.3, [K];
u_i	one component of velocity, [$\text{m}\cdot\text{s}^{-1}$];
U_i	product of density and velocity, [$\text{kg}\cdot\text{m}^{-2}\cdot\text{s}^{-1}$];
$\vec{V}$	velocity vector, [$\text{m}\cdot\text{s}^{-1}$];
V_c	maximum swirl velocity in the quasi-cylindrical core flow, [$\text{m}\cdot\text{s}^{-1}$];
V_r	radial velocity, [$\text{m}\cdot\text{s}^{-1}$];
V_s	reference swirl velocity, [$\text{m}\cdot\text{s}^{-1}$];
V_t	swirl velocity, [$\text{m}\cdot\text{s}^{-1}$];
V_{rz}	velocity in the radial-vertical plane, [$\text{m}\cdot\text{s}^{-1}$];
V_{xy}	horizontal velocity, [$\text{m}\cdot\text{s}^{-1}$];
W	vertical velocity, [$\text{m}\cdot\text{s}^{-1}$];
W_f	work of external forces per unit volume, [$\text{W}\cdot\text{m}^{-3}$];
x_j	length scale in one dimension, [m];
$\Delta x, \Delta y, \Delta z$	spatial mesh size in x , y and z directions respectively, [m];
z	height above the surface, [m],
z_0	effective surface roughness, [m].

Greek letter symbols

Γ	local angular momentum about central axis, [$\text{m}^2 \cdot \text{s}^{-1}$];
Γ_∞	angular momentum flowing radially into the domain, [$\text{m}^2 \cdot \text{s}^{-1}$];
Π	nondimensional Exner pressure function;
θ	potential temperature, [K];
ρ	density of the fluid, [$\text{kg} \cdot \text{m}^{-3}$];
$\Delta\rho$	perturbation density, [$\text{kg} \cdot \text{m}^{-3}$];
Y	total depleted angular momentum flux, [$\text{m}^5 \cdot \text{s}^{-2}$];
$\underline{\tau}$	viscous shear stress tensor, [$\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-2}$];
$\underline{\sigma}$	total internal stress tensor, [$\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-2}$];
μ	dynamic viscosity of the fluid, [$\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$];
ν	subgrid diffusivity for velocities, [$\text{m}^2 \cdot \text{s}^{-1}$];
γ	ratio of specific heats at constant pressure and constant volume, respectively;
Λ	dissipation length scale in the subgrid model, [m];
ψ	stream function, [$\text{kg} \cdot \text{s}^{-1}$].

Superscripts

n	time step number;
$*$	identified value.

Subscripts

∞	infinite away from the domain;
c	tornado vortex core well above the surface;
s	reference point;
max	maximum value;
min	minimum value;
i, j, k	component in three dimensions;
0	initial value;
$1,2$	two sides of diaphragm in a shock tube.

Abbreviations

<i>BRN</i>	Bulk Richardson Number;
<i>CAPE</i>	Convective Available Potential Energy;
<i>CFD</i>	Computational Fluid Dynamics;
<i>DOW</i>	Doppler On Wheels;
<i>LES</i>	Large-Eddy Simulation;
<i>PPM</i>	Piecewise Parabolic Method;
<i>TKE</i>	Turbulence Kinetic Energy;
<i>TVC</i>	Tornado Vortex Chamber.

Mathematics symbols

$\otimes$	tensor product of two vectors;
$\langle \rangle$	both a time and an axisymmetric average.

Chapter 1: Introduction

1.1 The tornado vortex

In the Glossary of Meteorology (*Huschke* 1959) a tornado was defined as: “A violently rotating column of air, pendant from a cumulonimbus cloud, and nearly always observable as a ‘funnel cloud’ or tuba.” It is one of the more spectacular and destructive phenomena of nature, and tornadoes are one of the most challenging subjects in atmospheric science because, being on the ferocious and elusive side, they do not exactly lend themselves to intimate study.

Figure 1-1-1: Schematic of a tornado flow (from *Whipple* 1982)

The essential elements of a tornado flow are illustrated in Figure 1-1-1, taken from *Whipple* (1982). The flow spirals radially inward into a core flow, which is basically a swirling rising plume but may include a downward jet along the axis. The radial flow is greatly intensified in the surface layer. The whole flow is driven by a thunderstorm, and feeds through a rotating wall cloud. Via the tornado vortex, a significant fraction of the

potential energy of the parent storm is converted into wind kinetic energy very close to the surface where it can cause great damage.

Figure 1-1-2: Conceptual model of the flow regimes associated with a tornado: region I, outer flow; region II, core; region III, corner; region IV, surface boundary layer; region V, convective plume. (from *Wurman et al.* 1996)

Although still not well understood, a widely accepted conceptual model of the flow regions in and around a tornado, as shown in Figure 1-1-2 (from *Wurman et al.* 1996), has been developed on the basis of numerous theoretical, numerical, and observational studies (e.g., *Lewellen* 1976, 1993; *Church et al.* 1979; *Snow* 1982; *Rotunno and Klemp* 1995; *Davies-Jones* 1986; *Rotunno* 1986; and *Komurasaki et al.* 1999). The tornado flow is often divided into five regions. “The vortex of the tornado is embedded in a swirling, rising outer-flow region (region I), which usually is associated with the parent thunderstorm mesocyclone on a larger scale (radius 5 to 10 km) and with the tornado cyclone on a somewhat smaller scale (radius 1 to 3 km). The flow approximately conserves angular momentum in this region. The core region of the tornado (region II) is thought to be nearly axisymmetric and is characterized by flow that spirals radially inward and upward. This region extends out to about radius of the maximum tangential winds. The winds in this regime are approximately in cyclostrophic balance (a balance between the horizontal pressure gradient force and the centrifugal force), which helps

suppress turbulence and radial flow. Cyclostrophic pressure deficits associated with stronger rotation in the lower levels than in the upper levels can accelerate the movement of air downward along the central axis. The corner-flow region in the tornado (region III) is characterized by intense, frictionally induced, radial inflow that must turn abruptly upward, owing to vertical pressure gradients, before reaching the axis. In the surface boundary layer region (region IV), friction produced by the lower boundary (the Earth's surface) retards the rotating flow. This disrupts the cyclostrophic wind balance, and flow accelerates radially inward toward the axis of rotation. Finally, the convective plume region (region V) is where angular momentum concentrated in the lower levels of the tornado circulation is transported outward by divergence or turbulence. ” (Wurman *et al.* 1996).

Figure 1-1-3: Schematic of different types of corner flow for increasing swirl. (a) very weak swirl so that the boundary layer separates, (b) one-cell vortex, (c) vortex breakdown above the surface, (d) two-cell vortex with downdraft penetrating to the surface, and (e) multiple vortices rotating about the annulus separating the two cells. (from *Davies-Jones* 1986)

When watching any appreciable sample of the videotape records of actual tornadoes, one may notice that there are many different types of tornadic corner flows, not only in the range of sizes and intensities, but in the different structures encountered. And this region is generally the site of the largest velocities, lowest pressures, sharpest velocity gradients, and greatest damage potential in the entire flow. Categorizing and understanding this range of structures has been a long-standing goal of tornado research.

Previous laboratory (e.g., *Ward 1972; Church et al. 1979; and Snow 1982*) and axisymmetric modeling studies (*Lewellen 1962, 1993; and Davies-Jones 1973, 1986*) have shown that one of the primary variables governing the tornado vortex is a domain-scale swirl ratio of the flow through the vortex, which can be defined as

$$S \equiv \frac{\Gamma_{\infty}}{a_c r_0 h}, \quad (1-1-1)$$

where Γ_{∞} is the angular momentum flowing radially into the domain, a_c the average horizontal convergence in the domain (i.e., $\partial W / \partial z$), r_0 the radius of the domain, and h the height of the inflow layer. S can also be interpreted as the ratio of swirl velocity to flow-through velocity for the whole vortex inhabiting the domain.

S has often been used to categorize the patterns of vortex corner flow. By varying this ratio a range of distinct behaviors in the vortex corner flow region can be realized. Figure 1-1-3, taken from *Davies-Jones (1986)*, shows a schematic plot of the variation of the corner flow with swirl ratio. If $S = 0$, there is no enhanced flow in the boundary layer. Instead, there is a deficit within the boundary layer, which can lead to local separation in the presence of the slight adverse pressure gradient resulting from the stagnating radial velocity above the boundary layer. Relatively little swirl is required to keep the flow attached as in Figure 1-1-3(b). As S is increased, the ratio of the volume of the flow passing through the boundary layer to that above the boundary layer increases, and the center jet is intensified. At a larger value of S , which depends on both the inflow and the top outflow conditions, the conditions for Figure 1-1-3(c) are encountered. Here the central erupting jet is stronger than can be supported throughout the core flow by the top boundary conditions. The result is a sharp transition in the flow, a type of vortex

breakdown, at some distance above the ground. At still higher values of S , the boundary layer eruption occurs in an annulus around the center, and the flow pattern of Figure 1-1-3(d) appears. When instabilities occur in the annular region of concentrated vorticity, coherent vortices may rotate around the primary vortex as in Figure 1-1-3(e).

1.2 Tornado research

Remarkable advances have been achieved in the understanding of the structure and dynamics of tornadoes and tornadic storms. This progress can be attributed to the improvements in numerical simulations of clouds and storms; to the development of Doppler radars, wind profilers, lightning ground-strike location detectors, and automated surface observing systems; to the application of multispectral satellite data; and to the deployment of mobile storm-intercept teams with means to make quantitative observations.

Among several tornado research methods (such as observing networks of surface stations, conventional radar, Doppler radar and satellites, surface observation and laboratory experiments), numerical simulations play an active role and have made significant advances, due to several advantages: no risks, lower time and money expenditures, high resolution, and ease of changing of boundary conditions, etc.

Owing to the great increase in computational hardware, fully three-dimensional, unsteady simulations are now easier than steady state, axisymmetric calculations were twenty years ago. However, it is still not possible to do a direct simulation of turbulence at full-scale Reynolds numbers, so some subgrid closure model is still required.

A model which explicitly simulates the large eddies but parameterizes the small eddies is called a large-eddy simulation (LES) model. For three-dimensional turbulence this type of model must be time-dependent and should have a grid size which falls within the inertial subrange (*Moeng* 1984). This represents one way of at least partially circumventing the problems associated with modeling turbulent transport within the tornado, that is, to provide sufficient resolution to simulate the largest eddies responsible for the principle transport within the flow. The more resolution which can be

incorporated into the simulation, the less importance the subgrid model assumes.

A three-dimensional incompressible numerical cloud model has been used to simulate the tornado vortex. Previous work (e.g., *Lewellen and Lewellen* 1996; *Lewellen et al.* 1997; *Lewellen and Lewellen* 1998; and *Lewellen et al.* 2000a, 2000b) indicates that this is a reasonable methodology. In some extreme cases, however, theoretical analysis (*Fiedler and Rotunno* 1986) estimates that the maximum swirl velocity may exceed $100 \sim 150 \text{ m}\cdot\text{s}^{-1}$. Some recent radar observations [e.g., Doppler On Wheels (DOW) radar measurement group at University of Oklahoma, <http://bigmac.metr.ou.edu/dow/image>] support this possibility, as discussed in section 2.2 later. At these conditions, the potential importance of compressibility effects of the atmosphere to tornado vortex dynamics is an open and challenging question. It is not even known whether tornado wind speeds could locally become supersonic, or whether this would have any dramatic effect on the tornado structure. Therefore it is desirable to set up a three-dimensional compressible model for further tornado research.

1.3 Outline of the dissertation

The object of this research is to explore the characteristics of an extremely violent tornado vortex, by extending the present high-resolution, three-dimensional, unsteady incompressible model to represent the compressible continuity equation for high Mach numbers which might arise in a tornado. Using compressible and incompressible models separately, calculation results for high Mach number cases will be compared. During this research, the following three questions will be addressed: In a violent tornado (where the maximum Mach number may likely approach 0.5), are the compressibility effects important? If not, when do they become important? What are the most important compressible effects? It is hoped that more will be learned about the special conditions which allow a mesocyclone to spawn a dangerous tornado and further the understanding of the near surface dynamics of a tornado vortex evolving within a mesocyclone.

The remainder of this dissertation is organized in the following manner. Chapter 2 summarizes previous work and discusses the potential importance of compressible effects and the desirability of developing a three-dimensional unsteady compressible model. In

Chapter 3, a set of density-based compressible Navier-Stokes equations are derived and listed. A one-dimensional expansion wave and shock case is used in Chapter 4 to check the compressible effects of the compressible code, while a two-dimensional purely swirling vortex is employed to test the stability of the code at both low and high Mach numbers. In the last section of Chapter 4, the incompressible and compressible LES codes are applied in a three-dimensional tornadic corner flow with very low Mach number. In Chapter 5, three-dimensional high resolution tornado vortices at different swirl ratios, which present different types of tornadic corner flows, are simulated for quasi-steady conditions at different Mach numbers. An evolving tornado with a sudden increase in intensity is simulated in Chapter 6. Chapter 7 summarizes and discusses the dissertation, and some recommendations of future work are listed.

Chapter 2: Literature Review and Research Background

2.1 Different research approaches

During the past thirty years, remarkable advances have been made in the understanding of the structure and dynamics of tornadoes and tornadic storms. This knowledge has led to improvements in prediction capability, procedures for issue and dissemination of warnings, and the practice of hazard mitigation. This progress can be attributed to the development of Doppler radars, wind profilers, lightning ground-strike location detectors, and automated surface observing systems; to the deployment of mobile storm-intercept teams with means to make quantitative observations; to the enhancement of computer resources and numerical models; and to improved understanding of how structures fail when subjected to tornadoes.

However, neither the development nor structure of the tornado has yet been well understood. Up to this time, direct measurements of the tornado's kinetic and thermodynamic fields are nonexistent. Only limited and approximate inferences of the tornado's velocity field are available, being derived from photogrametric analysis and dual Doppler radar. To better understand tornado dynamics, one must appeal to laboratory experiments, theoretical models, and field studies of other geophysical vortices.

As early as 1811 (*Tannehill* 1950) there were some reports of the occurrence of tornadoes within landfalling tropical cyclones. However, as noted by *McCaul* (1993), “the widespread rain, low clouds, and generally poor visibility attending most landfalling tropical cyclones has hindered direct observations of these tornadoes and impeded progress in understanding the character of their parent convective storms.” In the early studies, most of investigators focused on their temporal and spatial patterns of occurrence within the parent hurricane. A good description about field observations of tornadic storms may be found in *Davies-Jones* (1988). After 1980s, progress has been made regarding the specific characteristics of the tornadoes (e.g., *Stiegler and Fujita* 1982; *McCaul* 1987; and *Fujita* 1993). This progress can be attributed to the development of Doppler radar, and it has become the primary sensor for observing tornadic storms. In the

foreseeable future Doppler radar observations will remain the principal basis for issuing severe weather warnings. “It is important to note that tornado flow patterns are not well resolved in Doppler radar measurements. Therefore kinematic and thermodynamic properties fostering tornado genesis and dissipation necessarily are deduced from observations of the larger-scale storm flow” (*Brandes 1993*).

In the laboratory, tornado-like vortices are created in a tornado vortex chamber (TVC). From necessity the background flow conditions are considerably simplified, as it does not seem feasible to incorporate in a laboratory model all the factors (dynamic, thermodynamic, and microphysical) that exist in the thunderstorm environment. “A compelling reason for taking the laboratory approach is that it brings the classic attributes of experiments in physical science, namely, precision, control, and repeatability, to complex and transient atmospheric phenomenon.” (*Church and Snow 1993*) Some good reviews about this approach may be found in *Davies-Jones (1976)*, *Maxworthy (1982)*, *Snow (1982)*, *Wilkins (1988)*, and *Church and Snow (1993)*. Laboratory simulation has improved understanding of the tornado's behavior. However laboratory results suffer from relative low Reynolds number and problems with instrumental technique.

A number of relatively recent reviews of the dynamics of the tornado vortex are available in the literature (*Lewellen 1976*; *Smith and Leslie 1979*; *Snow 1982*; *Bengtsson and Lighthill 1982*; *Lugt 1983*; *Snow 1984*; *Davies-Jones 1986*; *Kessler 1986*; *Rotunno 1986*; *Snow 1987*; *Houze 1993*; *Lewellen 1993*; *Rotunno 1993*; and *Davies-Jones 1995*). Theoretical studies by numerical simulation have been used to investigate triggering mechanisms, development and structure of tornadoes. Two-dimensional numerical simulations have been presented by *Chi (1977)*, *Leslie (1977)*, *Lewellen and Teske (1977)*, *Rotunno (1977, 1979)*, *Proctor (1979)*, *Lewellen and Sheng (1979, 1980)*, *Leslie and Smith (1982)*, *Howells and Smith (1983)*, *Wilson and Rotunno (1986)*, *Howells et al. (1988)*, *Walko (1988)*, and *McClellan et al. (1990)*. Three-dimensional simulations have been presented by *Rotunno (1982, 1984)*, *Walko (1990, 1993)*, *Wicker and Wilhelmson (1990)*, and *Trapp and Fiedler (1993)*.

There are few numerical simulations of tornadoes that include the parent clouds. *Grasso and Cotton* (1995) initialized a three dimensional horizontally homogeneous gridded domain with a special sounding taken near the May 20, 1977 Del City, OK tornadic thunderstorm. With a total of three two-way interactive grids, a weak tornado was simulated along with the parent thunderstorm. *Grasso* (1996) simulated a tornado with the maximum velocity of $60 \text{ m}\cdot\text{s}^{-1}$. The model included six levels of two-way nested grids. The simulation did produce a condensation funnel well below the cloud base but did not extend to the surface. *Wicker* (1990) and *Wicker and Wilhelmson* (1990, 1993 and 1995) simulated a weak tornado and the parent thunderstorm. Their results demonstrate that parcels passing through the forward flanking downdraft gain positive vertical vorticity from tilting of the horizontal vorticity. Horizontal convergence then amplifies the vertical vorticity as the parcel ascends. *Fiedler* (1995) compared the solution of a tornado vortex uncoupled from its parent storm in a simple numerical model with a coupled case and found profound differences, while manifesting doubts about the utility of modeling tornadoes isolated from a parent storm. The focus of these simulations was on the tornadogenesis. These simulations could not reproduce the small-scale phenomenology of tornadoes, such as multiple vortices or vortex break down.

Various environmental parameters such as Convective Available Potential Energy (CAPE, *Moncirieff and Miller* 1976; and *Weisman and Klemp* 1982), vertical shear and Bulk Richardson Number (BRN, *Weisman and Klemp* 1982, 1984), helicity (*Lilly* 1986; and *Davies-Jones et al.* 1990) and streamwise vorticity (*Davies-Jones* 1984) have been employed to describe the tornadic potential of severe thunderstorms.

2.2 Previous work

In an early work, *Lewellen and Sheng* (1980) attempted to simulate fully turbulent tornadic flows. Toward this end they developed an axisymmetric vortex model with a second order turbulence closure scheme. Their results show that the resulting vortex flow is sensitive to the swirl ratio and sensitive to the surface roughness length assumed at the ground. They also performed a sensitivity experiment where the turbulent fluxes were parameterized using the eddy viscosity method. The main differences in the resulting

simulations were that the maximum horizontal velocity in the model was 11% smaller than in the fully turbulent model, and that this maximum velocity was at a radius 50% larger than the fully turbulent model's radius of maximum velocity.

Lewellen et al. (1997) described an incompressible LES code, used extensively for studies of the atmospheric boundary layer, and simulation procedures, then demonstrated that: (1) “when the grid spacing is small enough, the time averaged velocity and pressure distributions in the corner flow region show little sensitivity to either finer resolution or modified subgrid turbulence model”; (2) “the turbulence in the vortex corner flow region can be dominated by highly chaotic secondary vortices, revolving around the main vortex, whose location and strength are tied to the location and strength of the largest velocity gradients in the mean flow”; (3) “the radial turbulent transport of angular momentum near the center is predominantly inward near the surface, thus enhancing the surface intensification of the velocities in the tornado”.

Only two sets of physically distinct boundary conditions were presented in *Lewellen et al.* (1997): medium swirl conditions with and without translation relative to the surface. *Lewellen and Lewellen* (1996) extended these results to show some of the sensitivity of the tornado's low level dynamics to the domain swirl ratio (again both with and without translation).

Previous laboratory and modeling studies have shown that the domain swirl ratio S [defined in Equation (1-1-1)] is one of the primary variables governing the structure of a tornado vortex. However many other physical parameters also strongly affect the structure of the central vortex corner flow, so that flows which share the same large scale swirl ratio can in fact have much different corner flow structure. *Lewellen et al.* (2000a) (reprinted in Appendix A) present a more complete analysis of the sensitivity of a tornado's dynamical features to domain swirl ratio, the effective surface roughness, the tornado translation speed, the low level inflow structure, and the upper core structure. In that work it is shown that an appropriately defined local corner flow swirl ratio is a better indicator of the intensification of the vortex close to the surface than the domain swirl ratio. It also avoids the difficulty of determining the appropriate radial and vertical scales

to use in Equation (1-1-1), which can be problematic when Equation (1-1-1) is applied to unconfined flows. This local swirl ratio is defined as:

$$S_c \equiv r_c \Gamma_\infty^2 / Y \quad (2-2-1)$$

Here r_c is the core radius in the quasi-cylindrically symmetric region of the vortex well above the surface, defined as

$$r_c = \Gamma_\infty / V_c \quad (2-2-2)$$

(where V_c is the maximum swirl velocity in the quasi-cylindrical core flow, i.e., region II of Figure 1-1-1) because it provides a more robust measure of the core radius than taking r_c as the radius at which V_c occurs. Y is the total depleted angular momentum flux (i.e., the volume flux weighted by the difference between Γ_∞ and the local Γ) flowing from the surface layer into the corner flow region of the vortex. S_c can also be written as

$$S_c = \frac{\Gamma_\infty / r_c}{Y / (\Gamma_\infty r_c^2)}, \quad (2-2-3)$$

the ratio of a characteristic swirl velocity to a characteristic flow-through velocity for the surface-layer/corner/core flow.

While a single dimensionless ratio is certainly inadequate to completely describe the corner flow, *Lewellen et al.* (2000a) showed that the dominant effects of many physical variations on the corner flow are largely captured by their effects on the corner flow swirl ratio. For example, Figure 2-2-1 shows two normalized variables, maximum swirl velocity ($V_{\max}/V_c$) and minimum perturbation pressure ($\sqrt{\Delta P_{\min}/\Delta P_c}$), as a function of swirl that provide measures of the surface intensification of the tornado relative to the core flow above. There is noticeable scatter, but both ratios show a distinct peaking of the intensification for S_c around 1.2 (identified as S_c^*).

Figure 2-2-1: Summary of the surface intensification of the vortex for a set of 50 simulations as measured by the ratios $V_{\max}/V_c$ (*) and $\sqrt{\Delta P_{\min}/\Delta P_c}$ (O) versus S_c . (from Lewellen *et al.* 2000a)

Another important result of that study is that modest differences in the near surface inflow layer alone can mean the difference between strong, little, or no intensification of the vortex at the surface. Lewellen and Lewellen (1998) further emphasize this result and speculate on its relevance to the mesocyclone scale, e.g., for the Garden City, Kansas tornado of 16 May 1995 which was observed during VORTEX 95 (Wakimoto and Liu 1998). The difference between the Garden City mesocyclone which produced a tornado and similar low level mesocyclones which have been observed not to form a tornado may be associated with such small differences in the surface inflow.

In addition, Lewellen *et al.* (2000a) showed some tornado vortex features, such as: secondary vortex structure, translation effects, and more complex corner flows, not characterized by swirl ratio alone. For example, Figures 2-2-2 and 2-2-3 show the instantaneous pressure and vertical velocity contours for two cases on a horizontal slice through the corner flow. In both cases, $S_c = 6.6$, but the flows outside of the corner flow

region differ chiefly in the angular momentum gradients in the upper core region. The former case, with the stronger Γ gradients, displays strong secondary vortices rotating around the main vortex as evidenced by the minimum pressure in their cores. In contrast, Figure 2-2-3 is dominated by the central vortex supported by a lower pressure aloft, with no strong secondary vortices in evidence.

Figure 2-2-2: Instantaneous horizontal cross section of simulated tornado showing perturbation pressure (solid contour in units of ρV_c^2) and normalized vertical velocity (color) for the first high swirl case discussed in the text. (from *Lewellen et al. 2000a*)

Figure 2-2-3: Instantaneous horizontal cross section of simulated tornado showing perturbation pressure (solid contour in units of ρV_c^2) and normalized vertical velocity (color) for the second high swirl case discussed in the text. (from *Lewellen et al. 2000a*)

In a recent study of nonaxisymmetric and/or time varying corner flows, *Lewellen et al. (2000b)* (reprinted in Appendix B) found different mechanisms for enhancing the near surface tornado intensification relative to the vortex strength far from the surface, by changing the inflow conditions near surface and introducing a large surface translation speed. Both results reinforce one of the main conclusions in *Lewellen et al. (2000a)*: the critical role that the near surface inflow plays in tornado vortex intensification.

The compressible equations have been used previously to simulate so-called “supercell” tornadic storms (*Wilhelmson and Klemp* 1978; *Weisman and Klemp* 1982, 1984; *Klemp and Rotunno* 1983; *Wicker and Wilhelmson* 1995; *Wicker et al.* 1997; and *Vergara* 1997). However the calculation regions were too large and the resolutions were too coarse for a tornado’s length and time scale to catch the characteristics of tornadoes in detail.

Up to this time, a high-resolution, three-dimensional, unsteady compressible model has not been used to simulate a tornado-scale vortex.

2.3 Potential importance of compressible effects

To explain the maximum wind speeds observed in tornadoes *Fiedler and Rotunno* (1986) showed that the most intense vortices occur when the swirl ratio S is less than one and friction is significant enough to alter the cyclostrophic balance in the flow. The flow in and around the vortex is divided into two regions, an inviscid region above the boundary layer and the boundary layer itself. Using *Lilly’s* (1969) estimated maximum wind speed of $65 \text{ m}\cdot\text{s}^{-1}$ as the tangential velocity in the inviscid region (near cloud base), they estimated that the tangential velocity near the ground could exceed that in the inviscid region by a factor of 1.7. Further, the vertical velocity in the axial jet can be twice as large as the maximum tangential velocity in their model, indicating that vertical motion near the ground could exceed $200 \text{ m}\cdot\text{s}^{-1}$ in extreme cases.

The Fujita scale F5 damage rating, which occurs in the most violent tornadoes, is officially estimated as nominal wind speeds in the range of 117 to $142 \text{ m}\cdot\text{s}^{-1}$ (*Fujita* 1971). The Doppler On Wheels (DOW) radar measurement group at the University of Oklahoma (*Wurman et al.* 1996) has captured some violent (F4 and F5) tornadoes, and some results are posted on their web site (<http://bigmac.metr.ou.edu/dow/image>). On May 30th 1998, a violent tornado destroyed most of Spencer SD. According to DOW data, the maximum wind speed was $115 \text{ m}\cdot\text{s}^{-1}$. While recently, on May 3rd 1999, the maximum measured wind speed was $142 \text{ m}\cdot\text{s}^{-1}$ in a tornado passing through New Castle, Moore, and southern Oklahoma City. One should note that these are not instantaneous point

measurements, but include both time and space averaging. The actual peak velocities will be higher.

Figure 2-3-1 from *Lewellen et al.* (2000b) shows relative intensification as a function of time for two simulated cases, in which the near surface inflow conditions were abruptly changed. In case 1, the near surface intensification in a period is far greater than anything found for the quasi-steady state cases of Figure 2-2-1, though this is not true for case 2. In this period of time, although it is short, the minimum pressure is significantly dropped, high Mach numbers may be reached, and therefore compressible effects in the flow may become important.

Figure 2-3-1: Relative intensification as a function of time for two cases described in *Lewellen et al.* (2000b). $P_{\min}$ is the minimum pressure drop within the domain; P^* the average minimum pressure drop near the top of the domain for the low swirl quasi-steady state; and $t^* = r_d^2 / \Gamma_\infty$ represents a characteristic time scale for the low swirl surface layer flow to be exhausted after the lateral boundary condition at radius r_d is changed.

2.4 Connection between incompressible and compressible results

In order to have a direct relation between the incompressible model and the compressible, two parameters are defined in section 3.2.2: the nondimensional Exner pressure function Π , and the potential temperature θ .

As long as θ is constant (i.e., isentropic flow) and the Mach number is low ($M^2 \ll 1$), the compressible continuity equation may be simplified to $\nabla \cdot \vec{V} = 0$, which is the incompressible continuity equation. Moreover, looking ahead to Equation (3-2-13) it is possible to replace $\frac{1}{\rho} \frac{\partial P}{\partial x_i}$ by $C_p \theta \frac{\partial \Pi}{\partial x_i}$, so that the momentum equation for compressible flows is of the same form as that for incompressible flows with $C_p \theta \Pi$ replacing P/ρ . That means, one can recalculate the perturbation pressure due to the vortex dynamics, $\Delta P_c = P_c - P_0$ (for stagnation pressure P_0), in terms of the corresponding incompressible result ΔP_{in} , when θ is constant and $M \ll 1$, as

$$\Delta P_c = P_0 \left(\Pi^{\frac{C_p}{R_d}} - 1 \right) = P_0 \left[\left(1 + \frac{\Delta P_{in}}{\rho_{in} C_p \theta} \right)^{\frac{C_p}{R_d}} - 1 \right], \quad (2-4-1)$$

This approximation gives a chance to estimate the pressure difference from our previous simulation results when the Mach number is increased. Since the speed of sound is given by

$$a = \sqrt{C_p R_d \Pi \theta / C_v}, \quad (2-4-2)$$

where C_v is the specific heat at constant volume for the atmosphere, different Mach numbers may be obtained when θ is changed.

For different peak Mach numbers, Figure 2-4-1 compares the distributions of perturbation pressure ΔP normalized by the combination of density and velocity far away from the corner region (i.e., $\rho_s V_s^2$ at a large reference radius), at the vertical level of peak pressure drop occurring versus radius in the calculation domain from a low-swirl-ratio case performed in *Lewellen et al.* (2000a), while still assuming that $\nabla \cdot \vec{V} = 0$.

Figure 2-4-1 shows that at a Mach number of 0.61, the maximum difference is around 15%; while if the Mach number is increased to 1.06, the maximum difference is about 40%; and as it is reduced to 0.19, the maximum difference is less than 5%. That is, the maximum difference increases as the Mach number increases. This estimate does not include any error introduced by using $\nabla \cdot \vec{V} = 0$ instead of the full compressible continuity equation.

Figure 2-4-1: Comparison of perturbation pressure ΔP normalized by $\rho_s V_s^2$ along radius at a certain vertical level with different maximum Mach numbers.

The above discussions suggest that: the incompressible model is good enough to simulate tornadoes as long as the maximum wind velocity is less than about 100 m s^{-1} , i.e., Mach number is smaller than 0.3. But when the Mach number is increased to moderately high, e.g., 0.5, a new three-dimensional, unsteady, compressible model may be needed to determine how significant the differences between compressible and incompressible models may be in some extremely violent tornadoes.

Chapter 3: Description of the Compressible Large-Eddy Simulation Model

In this chapter a detailed description of the modified compressible large-eddy simulation (LES) model will be given. A short general introduction in the next section will be followed by a description of the governing equations and density-based method in section 3.2 and a specification of the subgrid model in section 3.3. The full equation set is given in section 3.4. Finally section 3.5 discusses the numerical characteristics of the finite difference scheme of the equation set.

3.1 Introduction

In principle, it is possible to obtain an exact solution of the time evolution of the flow if the Navier-Stokes equations (which govern the turbulent motions in the atmospheric boundary layer) are solved on all scales. But covering the whole range from small dissipative eddies, as small as 1 mm, to 1 km requires more grid points [$O(10^{18})$ in three dimensions] than it is feasible to employ on any computer, now or in the near future. In general, however, turbulent flows distinguish themselves by the specific structure of the large scale eddies because those large scale eddies are sensitive to their particular environment (for example, geometry, stratification and especially to the buoyancy forcing). The behavior of the smaller scale eddies is generally assumed to be more independent and thus statistically similar in turbulent flow. An LES model is a three dimensional model in which the large scale eddies in a turbulent flow are explicitly calculated while motions on a scale smaller than the filter size are only parameterized. Therefore, it is generally believed that by choosing the grid spacing small enough, the turbulent flow can be described with sufficient detail and accuracy.

3.2 Governing equations

3.2.1 Compressible Navier-Stokes equations

The Navier-Stokes equations are a differential form of the conservation laws which govern fluid motion. The equation of mass conservation, known as the continuity equation, is

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0. \quad (3-2-1)$$

where ρ is the density of the fluid and $\vec{V}$ is the velocity vector.

The momentum equation is

$$\frac{\partial}{\partial t}(\rho \vec{V}) + \nabla \cdot (\rho \vec{V} \otimes \vec{V} + P \underline{\underline{I}} - \underline{\underline{\tau}}) = \rho \vec{f}_e, \quad (3-2-2)$$

where $\underline{\underline{I}}$ is the unit tensor, $\otimes$ denotes the tensor product of two vectors, $\vec{f}_e$ is the external force vector, $\underline{\underline{\tau}}$ is the viscous shear stress tensor, equals to

$$\underline{\underline{\tau}} = \mu \left[\nabla \vec{V} + (\nabla \vec{V})^T \right] - \frac{2}{3} \mu (\nabla \cdot \vec{V}) \underline{\underline{I}}, \quad (3-2-3)$$

and μ is the dynamic viscosity of the fluid.

The equation of energy conservation is

$$\frac{\partial}{\partial t}(\rho E) + \nabla \cdot (\rho \vec{V} E) = \nabla \cdot (k \nabla T) + \nabla \cdot (\underline{\underline{\sigma}} \cdot \vec{V}) + W_f + Q_H \quad (3-2-4)$$

where E is the total energy per unit mass

$$E = e + \frac{\vec{V}^2}{2}, \quad (3-2-5)$$

e is the internal energy per unit mass, k is the thermal conductivity coefficient, T is the absolute temperature, the total internal stress tensor $\underline{\underline{\sigma}}$ is taken to be

$$\underline{\underline{\sigma}} = -P \underline{\underline{I}} + \underline{\underline{\tau}}, \quad (3-2-6)$$

and W_f is the work of external forces per unit volume

$$W_f = \rho \vec{f}_e \cdot \vec{V}. \quad (3-2-7)$$

In this research $\vec{f}_e$ and W_f may be neglected as discussed later, and the heat source per unit volume Q_H equals 0.

Finally, the thermodynamic relation between the entropy per unit mass s and e is

$$T ds = de + P d\left(\frac{1}{\rho}\right), \quad (3-2-8)$$

and the equation of state is

$$P = \rho R_d T. \quad (3-2-9)$$

3.2.2 Definition of Π and θ

In order to have a direct relation between the incompressible model and the compressible, two parameters are defined. One is the nondimensional Exner pressure function, Π , defined as

$$\Pi \equiv \left(\frac{P}{P_0} \right)^{\frac{R_d}{C_p}}. \quad (3-2-10)$$

Here, P is the pressure, P_0 a constant reference pressure usually taken to be 1000 millibars, C_p the specific heat at constant pressure for the atmosphere, and R_d the gas constant for dry air. The other is the potential temperature, θ , defined as

$$\theta \equiv T \left(\frac{P_0}{P} \right)^{\frac{R_d}{C_p}}, \quad (3-2-11)$$

so θ is simply related to Π by

$$\theta = T/\Pi. \quad (3-2-12)$$

From Equations (3-2-9), (3-2-10) and (3-2-11), it is easy to derive

$$\frac{1}{\rho} \frac{\partial P}{\partial x_i} = C_p \theta \frac{\partial \Pi}{\partial x_i}. \quad (3-2-13)$$

Then by substituting Equations (3-2-9) and (3-2-13), the entropy equation

$$ds = C_p \frac{dT}{T} - R_d \frac{dP}{P} \quad (3-2-14)$$

may be rewritten as

$$ds = C_p \frac{d\theta}{\theta}. \quad (3-2-15)$$

Equation (3-2-15) indicates that the flow is isentropic when θ is constant. This means, in particular, that θ is conserved under adiabatic compression or expansion of a fluid parcel.

3.2.3 Density-based method

For the most part, the computational methods available for solving the unsteady Navier-Stokes equations can be classified as either density-based or pressure-based methods. The difference between the two methodologies depends on whether pressure or

density is used as the primary variable for mass conservation. These two methodologies have distinct advantages or disadvantages depending on the nature of the flow field.

The pressure-based method was the first choice in the present study since it is close in implementation to the incompressible model and less code modification is involved. Unfortunately at the discrete level there is no finite difference scheme which keeps the mass and momentum exactly conserved and this led to subtle numerical instability in some test cases. Then it was necessary to modify the momentum equation to be density-based. Density-based methods are accurate and robust for high speed (compressible) flow situations. However, density-based methods are difficult to apply to low-Mach number (incompressible) flows due to the stiff eigenvalues of the resulting system. The decoupling that occurs between pressure and density is to blame for the stiffness.

Before developing the density-based momentum equation, the relationship between density-based and pressure-based methods are obtained by deriving Π in terms of density ρ . Substituting Equations (3-2-9) and (3-2-12) into Equation (3-2-10), the definition of Π ,

$$\Pi = \left(\frac{\rho R_d \theta \Pi}{\rho_0 R_d \theta_0 \Pi_0} \right)^{\frac{R_d}{C_p}}, \quad (3-2-16)$$

with $\Pi_0 = 1.0$, $R_d = C_p - C_v$, $\gamma = C_p / C_v$, one may have

$$\Pi = \left(\frac{\rho \theta}{\rho_0 \theta_0} \right)^{\gamma-1}. \quad (3-2-17)$$

Then the state equation, Equation (3-2-9), may be rewritten as

$$P = \frac{R_d}{(\rho_0 \theta_0)^{\gamma-1}} (\rho \theta)^\gamma. \quad (3-2-18)$$

Taking the derivative of both sides of Equation (3-2-18), one may obtain

$$\frac{\partial P}{\partial x_i} = R_d \gamma \left(\frac{\rho \theta}{\rho_0 \theta_0} \right)^{\gamma-1} \frac{\partial(\rho \theta)}{\partial x_i} \quad (3-2-19)$$

which is the density-based form used in the momentum equation.

3.3 Subgrid scale model

In compressible turbulence, away from shocks, compressibility effects decrease with a decrease in scale, so that a compressible large-eddy simulation using an incompressible subgrid model may be justified if the grid scale is chosen small enough (*Lesieur and Comte 1997*).

The subgrid model employed (*Sykes and Henn 1989*; and *Lewellen et al. 2000a*), utilizes the turbulent kinetic energy (TKE) equation in the form ($q^2 = \overline{u_i u_i} = 2 \times \text{TKE}$) and is similar to that of *Deardorff (1980)* and *Moeng (1984)*. Here it is repeated briefly, and more details can be found in the appendix in *Lewellen et al. (2000a)* (Appendix A of this dissertation).

The subgrid energy equation is written as

$$\frac{\partial q^2}{\partial t} + u_j \frac{\partial q^2}{\partial x_j} = 2\tau_{ij} \frac{\partial u_i}{\partial x_j} + \frac{\partial}{\partial x_i} \left(K \frac{\partial q^2}{\partial x_i} \right) - \frac{q^3}{4\Lambda} \quad (3-3-1)$$

where Λ is the dissipation scale, which is associated with the filter lengthscale, K the subgrid diffusivity, and τ_{ij} the modeled flux terms for momentum, i.e.,

$$\tau_{ij} = \nu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \delta_{ij} \frac{q^2}{3}, \quad (3-3-2)$$

where

$$K = S_H q \Lambda, \quad (3-3-3)$$

$$\nu = S_m q \Lambda. \quad (3-3-4)$$

The constant values of S_H and S_m are 1/3 and 0.25 respectively.

Finally, the dissipation length scale, Λ , is specified as in Appendix A to complete the model description. In practice, Λ includes some rotational damping effects and is determined locally from the relation

$$\Lambda = \min[c_1 \max(\Delta x, \Delta y, \Delta z), 0.65z, 0.5q/\xi], \quad (3-3-5)$$

where c_1 is a constant and equals 0.25 typically, $\Delta x, \Delta y, \Delta z$ are grid sizes, z is the normal distance from the wall, and

$$\xi^2 = \sum_{i=1}^3 \left[2\nabla\rho \cdot \frac{\partial \vec{V}}{\partial x_i} \left(\nabla\rho \cdot \frac{\partial \vec{V}}{\partial x_i} - \nabla\rho \cdot \nabla u_i \right) / (\nabla\rho \cdot \nabla\rho) \right]. \quad (3-3-6)$$

The diagnostic variable Λ should be thought of (up to a constant factor) as the characteristic size of the eddies comprising the subgrid scale turbulence. In general it is assumed that the subgrid turbulence cascades down from the resolved turbulence so that its scale is related to the grid spacing, giving the first term on the right hand side of Equation (3-3-5). The next term in Equation (3-3-5) represents the reduction in eddy length scale as the distance z to the wall becomes small. The last term in Equation (3-3-5) limits the subgrid turbulence length scale in the presence of strong rotation.

3.4 Full equation set

When Equations (3-2-1), (3-2-2) and (3-2-4) are averaged over an ensemble of realizations the equations may be written as

$$\frac{\partial \rho}{\partial t} + \frac{\partial U_i}{\partial x_i} - \frac{\partial}{\partial x_j} \left(K \frac{\partial \rho}{\partial x_j} \right) = 0, \quad (3-4-1)$$

where $U_i = \rho u_i$,

$$\frac{\partial U_i}{\partial t} + \frac{\partial (U_i u_j)}{\partial x_j} = -\gamma \mathcal{R}_d \left(\frac{\rho \theta}{\rho_0 \theta_0} \right)^{\gamma-1} \frac{\partial (\rho \theta)}{\partial x_i} - \rho g_i + \frac{\partial \tau_{ij}}{\partial x_j}, \quad (3-4-2)$$

$$\frac{\partial (\rho \theta)}{\partial t} + \frac{\partial (U_j \theta)}{\partial x_j} = \rho \left[\frac{\partial}{\partial x_j} \left(K \frac{\partial \theta}{\partial x_j} \right) - \frac{q^3}{4C_p \Lambda} \right]. \quad (3-4-3)$$

Equations (3-4-1) to (3-4-3), and (3-3-1) to (3-3-6) are the full equation set.

3.5 Numerical scheme

As stated in the previous section, a density-base momentum equation is employed in this compressible LES model. Although compressibility effects are expected in some regions, density is close to 1.0 in most of the simulation domain. To increase the numerical accuracy, perturbation density $\Delta\rho$ is used, defined as

$$\Delta\rho = \rho - \rho_0, \quad (3-5-1)$$

where $\rho_0 = 1$, but the product of density and velocity, U_i , which is the physical mass flow, is kept as a variable. So Equation (3-4-1) is rewritten as

$$\frac{\partial \Delta \rho}{\partial t} + \frac{\partial U_i}{\partial x_i} - \frac{\partial}{\partial x_j} \left(K \frac{\partial \Delta \rho}{\partial x_j} \right) = 0. \quad (3-5-2)$$

In this research, the effect of gravity may be neglected since the height of simulation domain is less than 2 km. So can the buoyancy force in tornadic corner flows. In other words, the flows are isentropic and θ keeps constant. Finally the momentum equation is

$$\frac{\partial U_i}{\partial t} + \frac{\partial (U_i u_j)}{\partial x_j} = -\gamma \mathcal{R}_d \theta (\Delta \rho + 1)^{\gamma-1} \frac{\partial \Delta \rho}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j}. \quad (3-5-3)$$

In general the numerical model is a standard, second-order accurate, finite difference model, using the advection scheme of *Piacsek and Williams* (1970) on a staggered grid stretched in three dimensions and leapfrog time-differencing, which is second-order accurate in time. A small amount (2% typically) of mixing is applied to remove the time-splitting instability of the leapfrog method. In order to insure that the finite difference scheme conserves mass and momentum exactly, following *Morinishi et al.* (1998), the divergence form has been adopted to discretize the advection term, $\partial(v_i v_j)/\partial x_j$. For example, the term $\partial(U u_j)/\partial x_j$ in Equation (3-5-3) is discretized as

$$\begin{aligned} \frac{\partial (U u_j)}{\partial x_j} &= \frac{\partial (U u)}{\partial x} + \frac{\partial (U v)}{\partial y} + \frac{\partial (U w)}{\partial z} \\ &\cong \frac{1}{\Delta x_i} \left[\frac{1}{2} (U_{i+1,j,k}^n + U_{i,j,k}^n) \cdot \frac{1}{2} (u_{i+1,j,k}^n + u_{i,j,k}^n) - \frac{1}{2} (U_{i,j,k}^n + U_{i-1,j,k}^n) \cdot \frac{1}{2} (u_{i,j,k}^n + u_{i-1,j,k}^n) \right] \\ &\quad + \frac{1}{\Delta y_j} \left[\frac{1}{2} (U_{i,j+1,k}^n + U_{i,j,k}^n) \cdot \frac{1}{2} (v_{i+\frac{1}{2},j+\frac{1}{2},k}^n + v_{i-\frac{1}{2},j+\frac{1}{2},k}^n) - \frac{1}{2} (U_{i,j,k}^n + U_{i,j-1,k}^n) \cdot \frac{1}{2} (v_{i+\frac{1}{2},j-\frac{1}{2},k}^n + v_{i-\frac{1}{2},j-\frac{1}{2},k}^n) \right] \\ &\quad + \frac{1}{\Delta z_k} \left[\frac{1}{2} (U_{i,j,k+1}^n + U_{i,j,k}^n) \cdot \frac{1}{2} (w_{i+\frac{1}{2},j,k+\frac{1}{2}}^n + w_{i-\frac{1}{2},j,k+\frac{1}{2}}^n) - \frac{1}{2} (U_{i,j,k}^n + U_{i,j,k-1}^n) \cdot \frac{1}{2} (w_{i+\frac{1}{2},j,k-\frac{1}{2}}^n + w_{i-\frac{1}{2},j,k-\frac{1}{2}}^n) \right] \end{aligned} \quad (3-5-4)$$

where $U = \rho u$, u , v and w are velocity components in three dimensions respectively, superscripts denote time and subscripts denote spatial grid points.

In the energy equation (3-4-3), the Piecewise Parabolic Method (PPM) (*Colella and Woodward 1984*), a numerical technique developed in astrophysics for modeling fluid flows with strong shocks and discontinuities, is adapted for treating sharp gradients of potential temperature θ .

To maintain the stability of the numerical scheme, the Courant condition is modified in terms of sound speed to limit the time step, i.e., time step Δt is chosen so that at each grid point

$$\Delta t < \min \left(\frac{\Delta x_i}{|u_i| + a} \right) \quad (3-5-5)$$

where a is the speed of sound. The grid is stretched in three dimensions to have more resolution where the research is focused on.

3.6 Boundary and initial conditions

As in *Lewellen et al. (1997)*, relatively simple boundary conditions that are representative of the conditions that can occur on approximately a 1 km^3 domain within tornadic corner flows are provided. Axisymmetric boundary conditions are applied to the horizontal boundaries, since the flow on this small domain is expected to be nearly axisymmetric on the average, and because these are the simplest realistic boundary conditions to apply. The circulation $2\pi \Gamma = 2\pi V_t r$ is held constant in the inflow at the horizontal boundaries (usually $\Gamma = 10^4 \text{ m}^2 \cdot \text{s}^{-1}$), except in a surface layer, and the average horizontal convergence a_c is also held constant, except in the surface layer. Those values are within the range of values obtained on low-level domains of order 1 km^3 in both the thunderstorm simulations by *Wicker and Wilhelmson (1995)* and *Grasso and Cotton (1995)*.

The horizontal boundary conditions imposed various versions of a turbulent surface layer below 100 m. The tangential velocity on the horizontal boundaries was sometimes taken to have a vertical variation proportional to $\ln(z/z_0)$ below 100 m, as expected in a fully turbulent surface layer; the effective surface roughness, z_0 , has been set equal to 0.2 m. The radial velocity within this surface layer was given a modified logarithmic

distribution following *Lewellen* (1977, 1981), with the maximum inward radial velocity at approximately 30 m height. This surface layer specification roughly approximates what would be obtained if the domain were doubled with the same values of circulation and convergence, and the surface layer started with zero thickness at this larger radius. In the cases shown in Chapter 5, the horizontal boundary conditions were always imposed on a larger domain for a coarse grid simulation first, and the results obtained from this used for setting the boundary conditions on the refined grid simulation.

It is desired to simulate the dynamics of the tornado vortex within the lowest few hundred meters, but place the top boundary condition much higher to allow for adjustment between the region of interest and the top where the boundary condition is applied. At the top boundary, the velocity distribution was specified with a zero slope condition for horizontal components $\left(\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = 0 \right)$ and specified normal component W (generally constant across the boundary). And the total mass flux is kept constant. To achieve a high-swirl ratio vortex, a disk of down-flow at the top of the domain was sometimes imposed.

In most cases, quasi-steady state conditions are sought for the above boundary conditions, so the initial conditions are not critical. For computational efficiency, the flow is allowed to spin up on a relatively coarse grid first before refining the inner mesh.

Chapter 4: Verification of the Compressible Large-Eddy Simulation Model

4.1 Introduction

After development of the compressible large-eddy simulation (LES) model and modification of the code, it is necessary to verify it in different ways. In this chapter, accuracy of the compressible LES code will be checked in three types of cases. First a one-dimensional shock and expansion wave case is used to check the compressible characteristics. Simulation results are compared with the analytical data in several conditions. In the next section, the compressible code is applied to simulate a two-dimensional purely swirling vortex, where its two-dimensional stability in low and high Mach number cases is checked. Then results from the compressible and incompressible codes are compared in a three-dimensional tornado vortex at very low Mach number where they should closely match each other.

Although the verification cases are one-, two- and three-dimensional respectively, the modified compressible LES code was always tested in three dimensions. The validation processes shown here might seem to be easy and straight forward, however a lot of work, which includes some failed tries, were actually involved. For example, at first the pressure-based momentum equation was adopted since it was the closest to the incompressible LES code. Unfortunately it could not pass the second test, so that the density-based method is employed and more modification of the code was needed. Another example is Equation (3-5-4). It is finally used after some other forms failed.

4.2 One-dimensional shock and expansion wave case

4.2.1 Introduction

Probably the most important and significant characteristics of a compressible flow are its shock and expansion waves where the Mach number is larger than 1. A so-called “shock tube” can simultaneously generate unsteady shock waves and expansion. In its simplest form, this consists of a long tube of constant area divided into two sections by a diaphragm which is typically made from a thin sheet of metal which often has grooves cut into it to ensure that it can be easily and cleanly broken. The tube contains a high

pressure gas on one side of the diaphragm and a low pressure gas on the other side of the diaphragm, as shown in Figure 4-2-1 (following *Oosthuizen and Carscallen* 1997).

Figure 4-2-1: Arrangement of a shock tube. (from *Oosthuizen and Carscallen* 1997)

When the diaphragm is broken either by mechanical means or by increasing the pressure on the high pressure side of the diaphragm, a shock wave propagates into the low pressure section and an expansion wave propagates into the high pressure section as illustrated in Figure 4-2-2.

Figure 4-2-2: Waves generated in a shock tube following rupture of diaphragm. (from *Oosthuizen and Carscallen* 1997)

Between the shock wave and the expansion wave a region of uniform velocity is generated that can be used for many different types of experimental studies. The flow in a shock tube only lasts for a short period of time, of course, because the waves are reflected off the ends of the tube. However, the velocity that is generated in a shock tube is determined by noting that the velocity and pressure behind the shock wave must be equal to the velocity and pressure behind the expansion wave. The shock wave increases the temperature of the gas whereas the expansion wave decreases the temperature of the gas. If the temperatures in the high and low pressure sections of the tube are initially the same, it follows that there will not be a uniform temperature between the shock wave and the expansion wave.

In this one-dimensional simple case, for any given ratio of high pressure and low pressure (P_1/P_2) at the initial time, the velocity, pressure and temperature between the shock and expansion waves can be calculated analytically, so that they may be used to check with the simulation results, which makes it a good benchmark test for the compressible code.

Although it is a one-dimensional simple case, the three-dimensional compressible LES code is applied. A large domain and fine grids are employed in only one dimension, and very small domain and coarse grids in the other two dimensions with slip boundary conditions.

4.2.2 Isentropic cases

In the first case, the pressure difference across the diaphragm is relatively small (the high and low pressure are 150 kPa and 100 kPa), while the potential temperature is kept as constant ($\theta = 300$ K), i.e. the flow is isentropic (since $ds = C_p d\theta/\theta$).

A typical set of simulation results obtained with 400 grid points in 1000 meters length are shown in Figures 4-2-3, 4-2-4a and 4-2-5 one second after the diaphragm is ruptured. The shock wave, the expansion wave, and the division between the flow traversed by the shock wave and by the expansion wave will be noted. The figures also show the values of the pressure, velocity and temperature obtained from analysis.

Quantitatively comparing the analytical values and simulation results, they agree very well. The average pressure, velocity and temperature from simulation are 122.804 kPa, 52.940 m·s⁻¹ and 331.95 K respectively, while the analytical results are 122.828 kPa, 52.916 m·s⁻¹ and 332.03 K. At this time, the expansion wave travels to -311.25 meters according to the analytical calculation, while the simulation result is -308.75 meters; the position of the shock wave is 391.25 meters in analysis and 388.75 meters in simulation.

It will be seen that there are over shoots and under shoots at the tail of the expansion and shock waves. This is a well known shortcoming of the second order central finite difference scheme employed for the advection terms. As expected,

increasing the number of grid points improves that, as shown in Figures 4-2-4 (a), (b) and (c). The sharp gradient in the shock wave is better captured when a larger number of grid points are used. Four hundred uniform grid points ($\Delta x_i = 2.5$ m) are employed in 1 km domain for all simulations, except for the resolution tests in Figures 4-2-4(b) and (c).

Figure 4-2-3: Pressure distribution of simulation results at 1.0 second, starting from $P_1/P_2 = 150$ kPa/100 kPa, $\theta = 300$ K.

Figure 4-2-4(a): Velocity distribution of simulation results at 1.0 second with 400 grid points, starting from $P_1/P_2 = 150$ kPa/100 kPa, $\theta = 300$ K.

Figure 4-2-4(b): Velocity distribution of simulation results at 1.0 second with 200 grid points, starting from $P_1/P_2 = 150 \text{ kPa}/100 \text{ kPa}$, $\theta = 300 \text{ K}$.

Figure 4-2-4(c): Velocity distribution of simulation results at 1.0 second with 100 grid points, starting from $P_1/P_2 = 150 \text{ kPa}/100 \text{ kPa}$, $\theta = 300 \text{ K}$.

Figure 4-2-5: Temperature distribution of simulation results at 1.0 second, starting from $P_1/P_2 = 150 \text{ kPa}/100 \text{ kPa}$, $\theta = 300 \text{ K}$.

Another typical set of results, for a high pressure difference ($P_1 = 300 \text{ kPa}$, $P_2 = 30 \text{ kPa}$ with 400 grid points), is shown in Figures 4-2-6 to 4-2-8 at 0.7 seconds after the diaphragm is ruptured. Again quantitative comparison between the simulation results and analytical values shows excellent agreement. The average pressure, velocity and temperature from the simulation are 101.57 kPa , $297.50 \text{ m}\cdot\text{s}^{-1}$ and 314.45 K respectively, while the analytical results are 102.68 kPa , $294.79 \text{ m}\cdot\text{s}^{-1}$ and 315.57 K . At this time, the expansion wave travels to -41.25 meters according to the analytical calculation, while the simulation result is -38.75 meters; the position of the shock wave is 358.75 meters in the analysis and 351.25 meters in the simulation.

Compared with the previous case, it may be noted that the shock wave is captured better. This is principally due to higher dissipation in this higher Mach number case, which “smears out” the large fluctuations.

Figure 4-2-6: Pressure distribution of simulation results at 0.7 second, starting from $P_1/P_2 = 300 \text{ kPa}/30 \text{ kPa}$, $\theta = 300 \text{ K}$.

Figure 4-2-7: Velocity distribution of simulation results at 0.7 second, starting from $P_1/P_2 = 300 \text{ kPa}/30 \text{ kPa}$, $\theta = 300 \text{ K}$.

Figure 4-2-8: Temperature distribution of simulation results at 0.7 second, starting from $P_1/P_2 = 300 \text{ kPa}/30 \text{ kPa}$, $\theta = 300 \text{ K}$.

4.2.3 Cases with the initial temperature uniform

In practice it is easy to set up the experiments in a laboratory when the initial temperature is uniform on both sides of the diagram. In this case the potential temperature θ is not constant anymore, so that it can be used to check the temperature solver part of the compressible code.

Similar to the previous section, two types of cases with different initial pressure ratio ($P_1/P_2 = 150 \text{ kPa}/100 \text{ kPa}$ and $300 \text{ kPa}/30 \text{ kPa}$) and uniform temperature ($T_0 = 300 \text{ K}$), have been performed. The simulation results and analytical values are compared in Table 4-2-1, which shows they agree with each other excellently. They are also shown in Figures 4-2-9 to 4-2-15.

The piecewise parabolic method (PPM) is employed in the temperature solver (Colella and Woodward 1984). One may see from Figure 4-2-11 that its performance is quite good across the interface where the potential temperature jumps. Shown in Figures 4-2-12 and 4-2-15, the temperature overshoots and undershoots at the expansion and shock waves actually come from pressure and velocity. Meanwhile there are tiny

overshoots and undershoots of pressure and velocity across the interface because of the large temperature jumps, but away from the interface they remain constant. Note that in Figure 4-2-12 the over and undershoots of temperature are not from PPM advection θ (as seen in Figure 4-2-11), but from the advection of ρ , because T is a function of ρ and θ .

It may be noticed that in the high pressure ratio case ($P_1/P_2 = 300 \text{ kPa}/30 \text{ kPa}$) there is a temperature difference behind the shock when the data is compared between the simulation and analysis, shown in Figure 4-2-15. The reason for that is the significant entropy change that occurs across the shock once the Mach number of the shock is too high, while the flow is still assumed as isentropic.

Table 4-2-1: Comparison of simulation results and analytical values ($T_0 = 300 \text{ K}$)

	$P_1/P_2 = 150 \text{ kPa}/100 \text{ kPa}$		$P_1/P_2 = 300 \text{ kPa}/30 \text{ kPa}$	
	analysis	simulation	analysis	simulation
pressure (kPa)	122.111	122.074	85.459	84.945
velocity ($\text{m}\cdot\text{s}^{-1}$)	50.299	50.305	285.28	286.93
left side temperature (K)	282.91	282.84	209.73	209.29
right side temperature (K)	317.65	317.54	417.69	403.79

Figure 4-2-9: Pressure distribution of simulation results at 1.0 second, starting from $P_1/P_2 = 150 \text{ kPa}/100 \text{ kPa}$, $T_0 = 300 \text{ K}$.

Figure 4-2-10: Velocity distribution of simulation results at 1.0 second, starting from $P_1/P_2 = 150 \text{ kPa}/100 \text{ kPa}$, $T_0 = 300 \text{ K}$.

Figure 4-2-11: Potential temperature θ distribution of simulation results at 1.0 second, starting from $P_1/P_2 = 150 \text{ kPa}/100 \text{ kPa}$, $T_0 = 300 \text{ K}$.

Figure 4-2-12: Temperature distribution of simulation results at 1.0 second, starting from $P_1/P_2 = 150 \text{ kPa}/100 \text{ kPa}$, $T_0 = 300 \text{ K}$.

Figure 4-2-13: Pressure distribution of simulation results at 0.7 second, starting from $P_1/P_2 = 300 \text{ kPa}/30 \text{ kPa}$, $T_0 = 300 \text{ K}$.

Figure 4-2-14: Velocity distribution of simulation results at 0.7 second, starting from $P_1/P_2 = 300 \text{ kPa}/30 \text{ kPa}$, $T_0 = 300 \text{ K}$.

Figure 4-2-15: Temperature distribution of simulation results at 0.7 second, starting from $P_1/P_2 = 300 \text{ kPa}/30 \text{ kPa}$, $T_0 = 300 \text{ K}$.

4.3 Instability tests in a two-dimensional purely swirling vortex

4.3.1 Introduction

Before doing any numerical simulations, one must make sure the numerical scheme is stable. Therefore numerical stability of the compressible code is another important characteristic to check. A two-dimensional purely swirling vortex, whose vertical and radial velocities are zero and swirl velocity is a function of radius, is a good test object, since the analytical solutions may be employed as its initial conditions. If the numerical scheme is unstable, any fluctuations will be amplified with time. This is a sensitive test particularly for slowly growing instabilities, because the same flow conditions may be followed for long times. Two cases are simulated separately to verify the code's stability for low and high Mach numbers, where its behavior could be different.

Similar to the previous one-dimensional case, the three-dimensional compressible LES code is applied with large domain and fine grids in two dimensions, and very small domain and coarse grids in the third dimension. The instantaneous incompressible results after a long simulation time are transferred to be the initial values in the compressible simulations, and the axisymmetric boundary conditions are kept unchanged.

4.3.2 Low Mach number case

First a low Mach number case is simulated within a $1 \text{ km} \times 1 \text{ km}$ domain. At the side boundaries, angular momentum $\Gamma_\infty = 5 \times 10^3 \text{ m}^2 \cdot \text{s}^{-1}$, and speed of sound $a_\infty = 347 \text{ m} \cdot \text{s}^{-1}$. The grid is uniform, $\Delta x = \Delta y = 20 \text{ m}$.

Figure 4-3-1 shows the instantaneous perturbation density, $\Delta\rho$ (i.e. the density minus the stagnation density), on a horizontal slice after 300 seconds, where the maximum Mach number is 0.03. The shape of its contour lines is as smooth as initially given, after a long time and tens of thousands of steps. In Figure 4-3-2, the horizontal velocity, V_{xy} , on the same horizontal slice is shown at the same time. There is low level turbulence, as seen in incompressible results. Since a Rankine vortex is used as its initial conditions, not surprisingly there are some wiggles in the contour lines due to small

perturbation of vertical and radial velocities in this quasi-two-dimensional simulation, but they stay at the same level and never grow in magnitude.

4.3.3 High Mach number case

Similar to the low Mach number case, a purely swirling vortex with high Mach number is set up to check the behavior of the code. This time angular momentum Γ_∞ is increased to $1 \times 10^5 \text{ m}^2 \cdot \text{s}^{-1}$, and speed of sound a_∞ is still $347 \text{ m} \cdot \text{s}^{-1}$ at the side boundaries. The same uniform grids are employed, $\Delta x = \Delta y = 20 \text{ m}$.

The contour lines of perturbation density are still very smooth after a long time when the maximum Mach number is increased to 0.52, as shown in Figure 4-3-3. Again because of local violation of continuity from the initial conditions, in Figure 4-3-4, bigger wiggles are shown in the contour lines of the horizontal velocity, while the same level is observed during the long time simulation.

Figure 4-3-1: Instantaneous perturbation density, $\Delta\rho$ ($\text{kg} \cdot \text{m}^{-3}$), on a horizontal slice after 300 seconds, where the maximum Mach number is 0.03.

Figure 4-3-2: Instantaneous horizontal velocity, V_{xy} ($\text{m}\cdot\text{s}^{-1}$), on a horizontal slice after 300 seconds, where the maximum Mach number is 0.03.

Figure 4-3-3: Instantaneous perturbation density, $\Delta\rho$ ($\text{kg}\cdot\text{m}^{-3}$), on a horizontal slice after 300 seconds, where the maximum Mach number is 0.52.

Figure 4-3-4: Instantaneous horizontal velocity, V_{xy} ($\text{m}\cdot\text{s}^{-1}$), on a horizontal slice after 300 seconds, where the maximum Mach number is 0.52.

4.4 Simulation of a three-dimensional tornado vortex at a very low Mach number

4.4.1 Introduction

When the Mach number of the flow is very low, for example less than 0.1, the effects of density change may be ignored, and it may safely be considered as incompressible. In other words, the incompressible continuity equation, $\partial u_i / \partial x_i = 0$, used in the incompressible code is approximately equal to the compressible continuity equation, $\partial \rho / \partial t + \partial(\rho u_i) / \partial x_i = 0$, used in the compressible code. Therefore the simulation results from both codes should be close to each other.

The last test case is the simulation of a three-dimensional tornado vortex at very low Mach number, using the incompressible code and compressible one separately, and comparing their results. It is simulated in a $2 \text{ km} \times 2 \text{ km} \times 1 \text{ km}$ domain, with angular momentum $\Gamma_\infty = 5 \times 10^3 \text{ m}^2 \cdot \text{s}^{-1}$, average horizontal convergence $a_c = 0.005 \text{ s}^{-1}$, and speed

of sound $a_\infty = 347 \text{ m}\cdot\text{s}^{-1}$. The grids are stretched in three dimensions and the finest grid spacing is 15 m.

One might think this is a very simple test, but on the contrary, two reasons make it the toughest. One: it is difficult for a compressible code to simulate an incompressible flow, i.e., a flow with very low Mach number; the other: large speed of sound for low Mach number forces the time step very small, so it takes a much longer time to collect the average.

4.4.2 Simulation results

Both the incompressible and compressible LES codes are applied to simulate a three-dimensional tornado vortex, whose instantaneous maximum Mach number is 0.18. In Figures 4-4-1 and 4-4-2 axisymmetric time averages have been plotted for several flow variables from the two codes to illustrate some of the general features of a tornado vortex flow. The figures show a radial-vertical cross section of the flow. Figures a-c show, respectively, contours of the swirl velocity (V), the magnitude of the velocity in the radial-vertical plane (V_{rz}), and perturbation pressure (ΔP) (i.e. the pressure minus the hydrostatic component). In figures d-f are shown, respectively, contours of the angular momentum of the flow about the center line (Γ), the stream function (ψ) indicating the direction and integrated mass flux of the flow, that is,

$$\begin{aligned}\psi &= \int_0^z \rho V_r(r, z') r \cdot dz' \\ &= \int_0^r -\rho W(r', z) r' \cdot dr'\end{aligned}\tag{4-4-1}$$

and total velocity variance

$$q_t^2 = \overline{u_i u_i} - \bar{u}_i \bar{u}_i + q^2.\tag{4-4-2}$$

As can be seen in these figures, they are very close when the results from the incompressible and compressible codes are compared. Note that each is based on a limited time average over a turbulent flow so that some of the differences remaining might be absent with longer time averages.

Figure 4-4-1: Results from the incompressible LES code in a three-dimensional tornado vortex at $M = 0$. (a) swirl velocity, (b) velocity in the radial-vertical plane, (c) perturbation pressure, (d) angular momentum, (e) stream function, (f) total velocity variance.

Figure 4-4-2: Results from the compressible LES code in a three-dimensional tornado vortex, where the instantaneous maximum Mach number is 0.18. (a) swirl velocity, (b) velocity in the radial-vertical plane, (c) perturbation pressure, (d) angular momentum, (e) stream function, (f) total velocity variance.

4.5 Conclusions of the chapter

In this chapter the compressible code has been verified in different ways. In the one-dimensional shock tube, it captures the shock and expansion waves and improves its performance as more grid points are employed in the simulation domain. An excellent agreement is achieved between the simulation results and analytical predictions when sufficient grid points are used. The code's numerical stability is checked for two-dimensional purely swirling vortices at low and high Mach number. Finally it has been

verified in a three-dimensional tornado vortex with small Mach number that the incompressible and compressible results are very close to each other. For all simulations, the modified compressible LES code is numerically stable, and approximately 20% faster than the incompressible LES code in each step with around one million grid points. However, usually more simulation time is needed due to smaller time step (Δt), when the speed of sound is considered in Equation (3-5-5). This case was simulated on a SGI 10000 machine with 512 M RAM. In this simulation, 184 seconds of simulation time required 467 thousand steps and a total clock time of 422 hours.

For some violent tornadoes simulated in the later chapters, the Mach numbers are small in most of the regions and the flow may be considered as incompressible, while the maximum Mach number might be greater than 0.5 in the corner regions where the compressible effects are most interesting. This requires the code be able to perform well in both incompressible and compressible conditions. All of the test cases indicate that the compressible code satisfies this requirement and is robust and accurate, so that it is a good tool to study the compressible effects in tornado vortices.

Chapter 5: Tornado Vortex Simulations with Quasi-Steady Conditions

The corner flow in a tornado vortex, where the surface layer inflow turns into the vortex core, is generally the site of the highest velocities, shears, and pressure drops in the entire flow. This is one of the main reasons it is of critical interest. The other main reason is simple: people live just above the surface where it can cause great damage. In this chapter the research is focused on three types of corner flows with quasi-steady conditions, as simulated with the modified compressible large-eddy simulation (LES) code described in Chapter 3.

5.1 Basic behaviors of tornadic corner flows

In the surface layer outside of the core underneath a swirling flow, the magnitude of the pressure is to a good approximation equal to that set by the cyclostrophic balance in the cylindrically symmetric region above, since the vertical pressure gradient tends to be small near the surface. This implies a strong radial pressure gradient since this is required to balance the strongly swirling flow. On the other hand, the swirl velocity near the surface necessarily drops to zero at least in a thin layer due to surface friction and perhaps in a deeper layer depending on the time development of the vortex. The resulting deviation from cyclostrophic balance in the surface layer drives a strong radial inflow which can overshoot its equilibrium point in the corner flow before turning up into the core flow. This radial overshoot brings angular momentum levels to smaller radii than elsewhere in the flow leading to larger swirl velocities and lower pressures.

The swirl ratio is easy to control in both laboratory and numerical simulation, but it becomes somewhat ambiguous in the real tornado vortex because of its dependence on domain geometry. Some results of laboratory investigations of the characteristics of tornado-like vortices as a function of swirl ratio may be found in the literature (e.g., *Church et al.* 1979).

Figure 5-1-1: Instantaneous isosurfaces of perturbation pressure, ΔP (Pa), within a $300 \text{ m} \times 300 \text{ m} \times 300 \text{ m}$ domain, at (a) high swirl ratio, (b) medium swirl ratio, and (c) low swirl ratio. The perturbation pressure levels in Pascals are labeled in the plots.

Figure 5-1-1 shows instantaneous isosurfaces of perturbation pressure within a $300 \text{ m} \times 300 \text{ m} \times 300 \text{ m}$ domain for different corner flow swirl ratios, S_c . The structures of the corner flow are significantly different as S_c is decreased. In Figure 5-1-1(a), the “high-swirl” corner flow has five or six secondary vortices rotating around the main vortex at

this particular sample time. As discussed in *Lewellen et al. (1997)*, the secondary vortices provide a net angular momentum transport directed radially inward into the central recirculating flow. In the “low-swirl” corner flow shown in Figure 5-1-1(c), the most extreme velocities are pushed much closer to the center than in the “high-swirl” corner flow. The radial velocity in this case converges all the way to the axis to form an intense central vertical jet driven upward by an intense low pressure minimum above the surface. At the top of this surface jet is an abrupt vortex breakdown where the vortex core diameter increases dramatically. Between these two extreme cases, the corner flow with medium swirl ratio is shown in Figure 5-1-1(b). In this chapter, all three different types of corner flows are considered since the compressible effects might be different in each type.

As in *Lewellen et al. (2000a)* and repeated in Chapter 2, the local swirl ratio is defined in compressible tornado vortex flows,

$$S_c \equiv \frac{\rho_\infty \Gamma^* r^*}{M^*} = \frac{\rho_\infty \Gamma_\infty^3}{V_c Y^*}. \quad (5-1-1)$$

Y^* is calculated approximately as

$$\begin{aligned} Y^* &\approx 2\pi \int_0^{z_1} -\langle \rho V_r(r_1, z) [\Gamma_\infty - \Gamma(r_1, z)] \rangle r_1 \cdot dz \\ &\approx 2\pi \int_0^{r_2} \langle \rho W(r, z_2) [\Gamma_\infty - \Gamma(r, z_2)] \rangle r \cdot dr \end{aligned}, \quad (5-1-2)$$

where r_1 is a radius close to, but outside of, the corner flow, z_1 is a height safely above the surface layer, z_2 is a height just above the corner flow, r_2 is a radius safely outside of the upper-core region, and the angular brackets denote both a time and an axisymmetric average. In this chapter S_c is adopted to categorize the patterns of compressible vortex corner flows at different Mach numbers.

Axisymmetric time-independent outer boundary conditions are imposed in the far-field of a larger domain than might seem necessary. When the flow reaches a quasi-steady state, the mean flow can be obtained by taking a time average that is axisymmetric about a known center line, even though the instantaneous flow is asymmetric, fully three-dimensional, and unsteady in time. As long as the characteristic timescale of the corner

flow itself is short compared to the timescales on which the larger-scale flow (which sets our boundary conditions) changes appreciably, the quasi-steady approximation is, in general, reasonable.

In order to have confidence in the results, in general a finer numerical resolution has been obtained in each case until there is evidence that the results are not sensitive to the grid spacing, which means the time-averaged results are independent of details of the small-scale resolution. The simulations of high-swirl-ratio tornado corner flows are a little different. In some cases, secondary vortices rotating around the main vortex are strongly dependent on grid resolutions. After refining the grids, slightly different axisymmetric time-averaged results will be obtained due to different secondary vortex structures. The further regriding is stopped after the main corner flow structure has been confidently achieved by comparing different resolutions.

In order to investigate the compressible effect in a tornado vortex flow, the speed of sound (a) has been decreased by adopting a smaller potential temperature (θ), i.e.,

$$a = \sqrt{\gamma P / \rho} = \sqrt{\gamma R_d \theta \left(\frac{\rho \theta}{\rho_0 \theta_0} \right)^{\gamma-1}}. \quad (5-1-3)$$

Thus, the flow is normalized to a larger Mach number, while the other parameters are kept unchanged. However θ is kept constant in the whole calculation domain. When it is regrided from the previous smaller Mach number case, the mass flow ($\rho \vec{V}$) is strictly reserved at the boundaries. Meanwhile the approximate isentropic transformation is employed to determine the densities. That is, it is assumed that at the boundaries

$$C_p \theta_1 \Delta \Pi_1 = C_p \theta_2 \Delta \Pi_2. \quad (5-1-4)$$

Combining with Equation (3-2-17), the relation between Π and ρ , gives

$$\rho_2 = \left[\left(\frac{\theta_1}{\theta_2} \right) \rho_1^{\gamma-1} + \left(1 - \frac{\theta_1}{\theta_2} \right) \right]^{\frac{1}{\gamma-1}}. \quad (5-1-5)$$

The velocities are changed according to densities. Keeping the same boundary conditions outside the corner flow region while the Mach number is increased, one is able to compare the compressible effects due to different Mach numbers within the corner flow.

All of the quasi-steady tornado vortex flows simulated in this chapter are summarized in Table 5-1-1, in which a_c is average horizontal convergence and a_∞ speed of sound at infinity. In all cases $\Gamma_\infty = 1 \times 10^4 \text{ m}^2 \cdot \text{s}^{-1}$. To achieve a high-swirl ratio vortex, in simulations 2528, 2522 and 2539, a disk of down-flow with radius of 100m and vertical velocity of $-2.5 \text{ m} \cdot \text{s}^{-1}$ was imposed at the top boundary.

It should be noted that at the beginning an incompressible LES simulation with coarse grids (usually 20 m) was started from a Rankine vortex in a large domain (typically 1 km^3) for each series of S_c . After the quasi-steady condition had been reached, it was regrided to a smaller domain with finer grids. The time-averaged fields from the previous simulation were employed as boundary conditions, and the instantaneous profiles at the last second as initial conditions separately. For incompressible cases this was found to give the same corner flow results for different simulation domain sizes (Lewellen and Lewellen 1996; Lewellen et al. 1997; Lewellen and Lewellen 1998; and Lewellen et al. 2000a, b). Therefore in the same series, incompressible and compressible runs have the same boundary conditions, so that the compressibility effects are able to be shown in the corner flow due to different Mach numbers. As will be seen, this did not work out as well as expected since it proved difficult to match conditions exactly.

Table 5-1-1: Summary of the cases simulated in Chapter 5.

Run #	type	S_c	a_c (s^{-1})	a_∞ ($m \cdot s^{-1}$)	domain (km)	finest grid (m)
2538	①	low	0.04	∞	0.4×0.4×0.3	2
2531	②			1002	0.4×0.4×0.3	
2535				549	0.4×0.4×0.3	
2515				347	0.4×0.4×0.3	
2533	①	medium	0.0125	∞	1×1×1	4
2532	②			347	0.9×0.9×1	
2534				245	0.9×0.9×1	
2529				174	0.9×0.9×1	
2528	①	high	0.01	∞	1×1×0.6	5
2522	②			174	0.6×0.6×0.4	
2539				90	0.6×0.6×0.4	

①: incompressible simulations, ②: compressible simulations.

5.2 Simulations of tornado vortex flows with a low swirl ratio

Using the incompressible LES model, a tornado vortex with low swirl ratio was simulated. The results are shown in Figure 5-2-1, where normalized axisymmetric time averages of several flow variables are plotted. The figures show a radial-vertical cross section of the flow about the corner flow region of interest. In Figures 5-2-1(a) to (f) are shown, respectively, contours of the swirl velocity (V_t), total velocity variance (q_t^2), vertical velocity (W), radial velocity (V_r), perturbation pressure (ΔP), angular momentum (Γ) and streamfunction (ψ). They are normalized by a characteristic velocity scale V_c , which is the maximum swirl velocity in the upper core region, a characteristic length scale $r_c \equiv \Gamma_\infty/V_c$, and the density at V_c as characteristic density ρ_c .

In this typical low-swirl-ratio tornado corner flow, the most extreme velocities in the corner flow are pushed close to the centerline. The radial velocity converges all the way to the axis to form an intense central vertical jet driven upward by an intense low pressure minimum above the surface. At the top of this surface jet is an abrupt vortex breakdown where the vortex core diameter increases dramatically. The radius where the peak swirl velocity occurs is more than a factor of four larger above the breakdown than it is below. The broad region of large TKE in Figure 5-2-1(a) is primarily a manifestation of the unsteadiness in the height of the breakdown and some wobble in the very narrow core jet upstream of the breakdown. More details of this incompressible low-swirl-ratio tornado corner flow simulation and discussions may be found in *Lewellen et al. (2000a)*, i.e., Appendix A.

Figure 5-2-1: Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for the corner flow of an incompressible low-swirl tornado vortex. (a) swirl velocity (solid contours) together with highest levels of total velocity variance (dashed, contour level is 0.1), (b) vertical velocity, (c) radial velocity, (d) perturbation pressure, (e) angular momentum, (f) streamfunction.

5.2.1 Simulation results at a small Mach number

Applying the compressible LES model the compressible tornado vortex with low swirl ratio is first simulated when the Mach number is small. In this case, the maximum Mach number is 0.37 in the axisymmetric time-averaged field. Figure 5-2-2 shows the simulation results in the same form as that in Figure 5-2-1.

The comparison of Figures 5-2-1 and 5-2-2 indicates that the compressible effects are small in the low-swirl tornado corner flow at low Mach number. The basic flow structure is the same in the two cases and quantitative differences are small, possibly within the accuracy to which the comparison can be made (given the limited time averages and the difficulties in exactly matching boundary conditions in the two cases).

Figure 5-2-2: Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for the corner flow of a compressible low-swirl tornado vortex at $M_{\max} = 0.37$. (a) swirl velocity (solid contours) together with highest levels of total velocity variance (dashed, contour level is 0.1), (b) vertical velocity, (c) radial velocity, (d) perturbation pressure, (e) angular momentum, (f) streamfunction.

5.2.2 Simulation results at a medium Mach number

Figure 5-2-3 shows the simulation results when the maximum Mach number is 0.70 in the axisymmetric time-averaged field. Again the similarity to the incompressible results (Figure 5-2-1) indicates the compressible effect is still not very significant in this case. A close inspection shows a modest increase in the height of the intense vertical velocity below the vortex breakdown.

Figure 5-2-3: Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for the corner flow of a compressible low-swirl tornado vortex at $M_{\max} = 0.70$. (a) swirl velocity (solid contours) together with highest levels of total velocity variance (dashed, contour level is 0.1), (b) vertical velocity, (c) radial velocity, (d) perturbation pressure, (e) angular momentum, (f) streamfunction.

5.2.3 Simulation results at a large Mach number

A further decrease in the speed of sound allows a higher Mach number to be achieved in the corner flow. Figure 5-2-4 shows the simulation results when the maximum Mach number is 1.24 in the axisymmetric time-averaged field. This time the compressible effect is more significant.

Figure 5-2-4: Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for the corner flow of a compressible low-swirl tornado vortex at $M_{\max} = 1.24$. (a) swirl velocity (solid contours) together with highest levels of total velocity variance (dashed, contour level is 0.1), (b) vertical velocity, (c) radial velocity, (d) perturbation pressure, (e) angular momentum, (f) streamfunction.

Probably the most impressive phenomena in this case is the much higher and stronger surface jet than that in the incompressible case. The main reason for that is the compressible flow must conserve the mass flux in the corner flow region while the density drops dramatically (for example, the minimum value of $0.378 \text{ kg}\cdot\text{m}^{-3}$ was observed in an instantaneous field) at high Mach number, so the volume flux has to be increased correspondingly. This increased volume flux is achieved partly by an increase in vertical velocity and partly by an increase in the lower core size. Far from the corner region where the density drop is small, the results are close to the incompressible ones.

In order to clearly compare the details of the incompressible and compressible results in the corner flow region, Figures (a), (b) and (c) in 5-2-1 and 5-2-4 have been zoomed in and shown in Figures 5-2-5, 5-2-6 and 5-2-7 respectively.

Comparing swirl velocity distributions shown in Figures 5-2-5(a) and (c), the peak value in the compressible flow is a little larger than that in the incompressible one, and located at greater height and radius. The peak value of the product of density and swirl velocity in Figure 5-2-5(b) is smaller, showing the big density drop there.

Figure 5-2-5: Zoom in Figures 5-2-1(a) and 5-2-4(a). (a) swirl velocity at $M = 0$, (b) product of density and swirl velocity at $M_{\max} = 1.24$, (c) swirl velocity at $M_{\max} = 1.24$.

Similar results are shown in Figure 5-2-6, where vertical velocities and the product of density and vertical velocity are compared. Meanwhile it clearly shows that the height of vortex breakdown at high Mach number is about 3 times that at $M = 0$. While only a modest increase of around 5% occurred in $V_{t\max}$ as seen in Figure 5-2-5, $W_{\max}$ was increased by approximately 40%.

Figure 5-2-6: Zoom in Figures 5-2-1(b) and 5-2-4(b). (a) vertical velocity at $M = 0$, (b) product of density and vertical velocity at $M_{\max} = 1.24$, (c) vertical velocity at $M_{\max} = 1.24$.

In incompressible and compressible simulation, the locations of the maximum negative radial velocities, which indicate convergent flow shown in Figure 5-2-7(a) and (c), are close, so are the peak values. Associated with the vortex breakdown the peak positive radial velocity in compressible flow occurs much higher and further from the center, while its value is smaller due to larger divergent flow areas. In Figure 5-2-7(b) the distribution of the product of density and radial velocity at $M_{\max} = 1.24$ has the similar structure as that of radial velocity alone, and because of the compressibility effect its value is smaller. There is not near as big a difference between 5.2.7 b and c as there is between 5.2.6 b and c because the maximum radial velocity occurs well off the axis where the lowest density occurs.

Figure 5-2-7: Zoom in Figures 5-2-1(c) and 5-2-4(c). (a) radial velocity at $M = 0$, (b) product of density and radial velocity at $M_{\max} = 1.24$, (c) radial velocity at $M_{\max} = 1.24$.

In Figures 5-2-8 and 5-2-9 instantaneous tangential velocities in the low-swirl tornado vortices at $M = 0$ and $M_{\max} = 1.24$ are compared. Here the increase in the height of the breakdown at high Mach number is even more striking. One can easily observe small eddies of turbulent flows in the corner flow region, which have been smoothed out in the axisymmetric time-averaged plots.

To let readers have a feeling how high Mach numbers could reach, Figure 5-2-10 shows instantaneous local Mach numbers in contours in a vertical plane, together with vertical velocities in color. The maximum Mach number reaches as high as 2.0 at the particular time shown, compared with 1.24 in the axisymmetric time-averaged field. It is somewhat surprising that there is no evidence of shocks in this flow with the deceleration from supersonic conditions occurring nearly as smooth as the acceleration. Part of the explanation may be that this code does not employ a shock capturing scheme so the resulting smearing of any shock is hard to distinguish in the presence of the strong gradients in the same region. A shock may be co-located with the abrupt vortex breakdown present in the subsonic case, so there is no dramatic change occurring when the flow becomes supersonic, in contrast to most familiar supersonic flows.

Figure 5-2-8: Instantaneous vertical cross section of an incompressible low-swirl tornado vortex showing normalized swirl velocity (color) and magnitude of the velocity vector (arrows, interpolated onto a uniform grid for clarity; maximum length corresponds to $V_{rz}/V_c = 3.36$) in the r - z plane.

Figure 5-2-9: Instantaneous vertical cross section of a compressible low-swirl tornado vortex at large Mach number showing normalized swirl velocity (color) and magnitude of the velocity vector (arrows, interpolated onto a uniform grid for clarity; maximum length corresponds to $V_{rz}/V_c = 3.60$) in the r - z plane.

Figure 5-2-10: Instantaneous vertical cross section of compressible low-swirl tornado vortex at large Mach number showing local Mach number (solid contours) and normalized vertical velocity (color) in the r - z plane.

5.2.4 Section summary

As discussed in Chapter 3, the pressure gradient in the momentum equation may be formally replaced with the gradient of the Exner function, Π , so that for constant potential temperature, θ , any incompressible solution may be interpreted in terms of an

isentropic approximation neglecting any changes in the continuity equation. This provides another means to compare the incompressible and compressible results, and it is possible to see what level of approximation is associated with neglecting compressibility effects only in the continuity equation for different Mach numbers.

For the low-swirl-ratio tornado, Figure 5-2-11 shows this comparison of incompressible ($M = 0$), compressible ($M_{\max} = 0.37, 0.70$ and 1.24) results and the isentropic model transformed from incompressible results for the separate Mach numbers. The parameter, ΔP , is the pressure change between the local average pressure and that at the reference location (the radius r_s is 177.92 m and height z_s is 271.61 m separately) normalized by $\rho_s V_s^2$ (density times the swirl velocity square at the reference location). Listed in Table 5-5-2 (page 79), the values of ρ_s and V_s are very close in every case.

In low-swirl-ratio tornado corner flows, the minimum perturbation pressure, $\Delta P_{\min}$, usually occurs near the centerline, so the distribution of ΔP is shown as a function of height at the vortex centerline in Figure 5-2-11. Somewhat surprisingly, the largest normalized pressure drop occurs for the compressible run with modest maximum Mach number. Both the isentropic transformations from the incompressible result and the results from the compressible runs show qualitatively similar decrease in pressure drop with increasing Mach number, but there is no obvious reason for the relatively large difference between the incompressible result and the low Mach number result. This discrepancy was not noticeable by comparing the respective pressure plots in Figure 5-2-1 and Figure 5-2-2. It is probably a result of the sharp peaking of the low-level $\Delta P_{\min}$ at the critical value of corner flow swirl ratio, S_c^* (approximately 1.2) that was seen in Figure 2-2-1. In this current series of low-swirl-ratio vortex simulations the local swirl ratio S_c is very close to S_c^* , and a very small change in boundary conditions would be enough to have the incompressible minimum pressure drop in Figure 5-2-11 modified to values close to that for the low Mach number case. It is also possible for the density decrease associated with increasing Mach number to cause a slight effective shift in S_c .

that increases the pressure drop in this region of great sensitivity to S_c . Unfortunately, the boundary conditions may not have been matched precisely enough to completely limit the changes in this series of test to compressible effects.

One can see that the height where $\Delta P_{\min}$ locates and the depth of the sharply reduced pressure region increases as the Mach number is increased. This is correlated with the increase of the vortex breakdown height. Above the breakdown the compressibility effect is small and the values of ΔP are close to each other at different Mach numbers.

Figure 5-2-11: In a low-swirl ratio tornado vortex, axisymmetric time-averaged distribution of $\Delta P / \rho_s V_s^2$ versus height along the centerline.

5.3 Simulations of tornado vortex flows with a medium swirl ratio

As for low-swirl ratio vortex flows shown in the previous section, Figures 5-3-1 to 5-3-4 compare the medium-swirl ratio corner flow structures by showing seven normalized parameters in the radial-vertical plane, as the peak average Mach number increases from 0 to more than 1.0. In general they have relatively similar distributions and values, and unlike in the low-swirl ratio corner flow compressible effects are modest in medium-swirl ratio vortex flows even at relatively high Mach numbers.

Figure 5-3-1: Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for the corner flow of an incompressible medium-swirl tornado vortex. (a) swirl velocity (solid contours) together with highest levels of total velocity variance (dashed, contour level is 0.1), (b) vertical velocity, (c) radial velocity, (d) perturbation pressure, (e) angular momentum, (f) streamfunction.

Figure 5-3-2: Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for the corner flow of a compressible medium-swirl tornado vortex at $M_{\max} = 0.40$.

(a) swirl velocity (solid contours) together with highest levels of total velocity variance (dashed, contour level is 0.1), (b) vertical velocity, (c) radial velocity, (d) perturbation pressure, (e) angular momentum, (f) streamfunction.

Figure 5-3-3: Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for the corner flow of a compressible medium-swirl tornado vortex at $M_{\max} = 0.67$.

(a) swirl velocity (solid contours) together with highest levels of total velocity variance (dashed, contour level is 0.1), (b) vertical velocity, (c) radial velocity, (d) perturbation pressure, (e) angular momentum, (f) streamfunction.

Figure 5-3-4: Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for the corner flow of a compressible medium-swirl tornado vortex at $M_{\max} = 1.04$. (a) swirl velocity (solid contours) together with highest levels of total velocity variance (dashed, contour level is 0.1), (b) vertical velocity, (c) radial velocity, (d) perturbation pressure, (e) angular momentum, (f) streamfunction.

In this series of medium-swirl-ratio vortex simulations, S_c is around 3. Referring to Figure 2-2-1 the value of $\Delta P_{\min}$ is relatively flat when the swirl ratio is much larger than S_c^* . Therefore in Figure 5-3-5 the change of $\Delta P_{\min}$ at different Mach numbers should be mainly due to the compressibility effects. Similar to Figure 5-2-11, ΔP is normalized by $\rho_s V_s^2$ at $r_s = 401.85$ m and $z_s = 875.00$ m. The values of $\rho_s V_s^2$ are also listed in Table 5-5-2.

Figure 5-3-5 does not show a clear trend with respect to increasing Mach number. The transformed curves show that increasing the Mach number is expected to yield less of a pressure drop along the centerline, because of the somewhat lower density reducing the outward inertia. However, there also appears to be a tendency, in the contour plots for W in Figures 5-3-1 to 5-3-4, for the compressible flow pattern in the corner flow to increase the vertical velocity near r and z equal to zero; i.e., there is a tendency for the volume flux to increase to compensate for the lowered density, lowering the effective corner flow swirl ratio as Mach number is increased. This latter effect apparently overrides the expected lower pressure drop due to lower density at some heights and Mach numbers. Perhaps the most important observation from this figure is that even at the highest Mach number the change within the normalized pressure drop in Figure 5-3-5 is not particularly significant.

It should be noted that although the normalized centerline pressure drop is more often reduced as the Mach number is increased, the absolute pressure drop in the atmosphere is expected to increase, since the V_s needs to be increased to achieve the higher Mach number when the speed of sound is held constant.

Figure 5-3-5: In a medium-swirl ratio tornado vortex, axisymmetric time-averaged distribution of $\Delta P / \rho_s V_s^2$ versus height along the centerline.

5.4 Simulations of tornado vortex flows with a high swirl ratio

To compare the high-swirl ratio corner flow structures as Mach number is increased from 0 to as high as 0.64, seven normalized parameters in the radial-vertical plane are shown in Figures 5-4-1 to 5-4-3. Like the medium-swirl ratio corner flow case, they have similar distributions and values, so compressible effects are not very significant in the high-swirl ratio tornado vortex even at Mach number around 0.6 at least for mean flow quantities.

Figure 5-4-1: Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for the corner flow of an incompressible high-swirl tornado vortex. (a) swirl velocity (solid contours) together with highest levels of total velocity variance (dashed, contour level is 0.1), (b) vertical velocity, (c) radial velocity, (d) perturbation pressure, (e) angular momentum, (f) streamfunction.

Figure 5-4-2: Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for the corner flow of a compressible high-swirl tornado vortex at $M_{\max} = 0.34$. (a) swirl velocity (solid contours) together with highest levels of total velocity variance (dashed, contour level is 0.1), (b) vertical velocity, (c) radial velocity, (d) perturbation pressure, (e) angular momentum, (f) streamfunction.

Figure 5-4-3: Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for the corner flow of a compressible high-swirl tornado vortex at $M_{\max} = 0.64$. (a) swirl velocity (solid contours) together with highest levels of total velocity variance (dashed, contour level is 0.1), (b) vertical velocity, (c) radial velocity, (d) perturbation pressure, (e) angular momentum, (f) streamfunction.

One of the most important features in a high swirl tornado vortex is its secondary vortex structures. Figure 5-4-4 shows normalized instantaneous perturbation pressure as solid contours and vertical velocities in color in a horizontal cross section where the highest Mach number is found at the height of $0.113 r_c$ in a high-swirl tornado vortex simulation. At this time, the instantaneous peak Mach number in the strongest secondary structure is 1.155. Here five secondary vortices, three relatively strong and two weak, are rotating around the main vortex. The qualitative structure observed is indistinguishable

from that seen in the incompressible simulations. The potential for transonic flow conditions being reached in the secondary vortices was suggested by *Fiedler* (1996), who used an axisymmetric, laminar model to argue that speeds in a tornado need not be limited to the speed of sound.

Figure 5-4-4: Instantaneous horizontal cross section of a compressible high-swirl tornado where the highest Mach numbers is found at the height of $0.113 r_c$, showing normalized perturbation pressure (solid contours) and vertical velocity (color).

The contour plots show little variation in pressure occurring with height along the centerline with the minimum pressure occurring off the centerline near where the secondary vortices occur. In Figure 5-4-5 the normalized pressure changes in the high-swirl-ratio tornado vortices are shown as a function of radius at the same height where the minimum pressure drop occurs in the incompressible case ($z = 14.59$ m). The reference point is located at $r_s = 283.34$ m and $z_s = 337.02$ m. The values of ρ_s and V_s may be found in Table 5-2-2.

As the Mach number is increased, a clear trend in the centerline pressure may be seen in Figure 5-4-5 with the data having a slightly larger reduced pressure drop than indicated by the transformed incompressible result. In general the variation in average pressure drop is not very significant, even for the $M = 0.64$ case where transonic velocities are occurring in the instantaneous secondary vortices.

Figure 5-4-5: In a high-swirl ratio tornado vortex, axisymmetric time-averaged distribution of $\Delta P / \rho_s V_s^2$ along radius at the same height where $\Delta P_{\min}$ occurs in the incompressible case ($z = 14.59$ m).

5.5 Conclusions of the chapter

In this chapter tornado vortices with three types of corner flows based on different values of swirl ratio are simulated for increasing Mach numbers. In order to compare the compressibility effects in the three different types of vortices, five normalized parameters are summarized in Table 5-5-1 and plotted in Figures 5-5-1 to 5-5-5.

Table 5-5-1: Summary of simulation results plotted in the following figures

Run #	S_c	$\frac{V_c}{a_\infty}$	$\sqrt{\frac{\Delta P_{\min}}{\Delta P_c}}$	$\frac{W_{\max}}{V_c}$	$\frac{V_{\max}}{V_c}$	$\frac{W_{\max}}{V_{\max}}$	$\frac{(\rho W)_{\max}}{(\rho V)_{\max}}$
2538	1.355	0	1.859	2.709	2.091	1.296	1.296
2531	1.409	0.104	2.158	3.323	2.238	1.485	1.433
2535	1.405	0.190	2.063	3.332	2.252	1.480	1.274
2515	1.486	0.280	1.825	3.679	2.116	1.739	1.072
2533	2.890	0	1.107	0.684	1.518	0.450	0.450
2532	3.022	0.237	1.091	0.943	1.540	0.612	0.570
2534	2.893	0.340	1.199	1.358	1.532	0.886	0.711
2529	2.766	0.495	1.094	1.408	1.468	0.959	0.612
2528	6.827	0	1.038	0.520	1.348	0.386	0.386
2522	7.159	0.245	1.026	0.509	1.302	0.391	0.389
2539	6.800	0.458	1.041	0.499	1.311	0.381	0.376

Table 5-5-2: Summary of reference values

Run #	r_c (m)	ρ_c (kg·m ⁻³)	ΔP_c (Pa)	r_s (m)	z_s (m)	ρ_s (kg·m ⁻³)	V_s (m·s ⁻¹)	$M_{\max}$	$\rho_{\min}$ (kg·m ⁻³)
2538	98.04	1	-17093	177.92	271.61	1	52.36	0	1
2531	96.26	0.900	-15461			0.908	55.20	0.37	0.832
2535	95.99	0.885	-15365			0.911	55.14	0.70	0.672
2515	102.80	0.854	-15544			0.917	55.15	1.24	0.407
2533	126.60	1	-13676	401.85	875.00	1	25.02	0	1
2532	121.49	0.917	-13824			0.977	24.64	0.40	0.835
2534	119.70	0.855	-13544			0.980	24.84	0.67	0.627
2529	116.46	0.715	-12444			0.988	24.64	1.04	0.415
2528	238.26	1	-1812	283.34	337.02	1	34.38	0	1
2522	234.81	0.950	-1732			0.979	35.20	0.34	0.917
2539	243.44	0.904	-1573			1.02	34.79	0.64	0.795

Shown in Figure 5-5-1, the minimum perturbation pressure in the corner flows, $\Delta P_{\min}$, has the same definition as in the previous sections (e.g. in Figure 5-2-11), while here it is normalized by ΔP_c to represent a measure of near surface intensification. ΔP_c is defined as the minimum perturbation pressure in the quasi-cylindrical region above the surface boundary layer. It shows that in three types of vortices there is relatively little change of swirl ratio [as defined in Equation (5-1-1)] at different Mach numbers. In all cases the pressure intensification is within the scatter shown in Figure 2-2-1 for a large number of incompressible runs with a variety of boundary conditions. In most cases the normalized values of $\Delta P_{\min}$ decrease as the Mach number is increased, but there are exceptions. This is particularly true for the low-swirl-ratio vortex since here S_c is so close to the critical value, S_c^* , that even a small change of S_c due to compressibility effects can change $\Delta P_{\min}$ as shown in Figure 2-2-1. For example, the minimum pressure drops at $M_{\max} = 0.37$ and $M_{\max} = 0.70$ is larger than that at $M = 0$.

Figure 5-5-1: Summary of the ratio $\sqrt{\Delta P_{\min}/\Delta P_c}$ versus S_c for the incompressible and compressible results at different Mach numbers.

The low level radial inflow in the low-swirl-ratio vortex converges all the way to the axis to form an intense central vertical jet driven upward by an intense low pressure minimum above the surface, then an abrupt vortex breakdown occurs at the top of this jet. Including the density drop in the continuity equation requires either an increase in the vertical velocity within the vortex core, and/or an increase in the core size. Figure 5-5-2 shows the maximum vertical velocity $W_{\max}$ normalized by V_c for different M and S_c , while Figure 5-5-3 shows the normalized maximum swirl velocity $V_{t\max}$.

In Figure 5-5-2 the large density drop in the corner flow forces $W_{\max}$ to increase dramatically in order to keep the mass flux constant when the Mach number is more than 1 and the compressibility effects are significant. The relatively large change in $W_{\max}$ even at low March number, particularly for low swirl, is probably due to an effective shift in S_c from either compressibility effects or slight unintended changes in boundary conditions from run to run.

Figure 5-5-2: Summary of the ratio $W_{\max}/V_c$ versus S_c for the incompressible and compressible results at different Mach numbers.

In contrast, Figure 5-5-3 shows that $V_{t\max}$ is almost unchanged in all three types of vortices as the Mach number is increased. One of the explanations is that vertical velocity is a component of velocity through the boundaries of the conservation region, while swirl velocity is not. Therefore density change affects $W_{\max}$ much more significantly than $V_{t\max}$.

Figure 5-5-3: Summary of the ratio $V_{t\max}/V_c$ versus S_c for the incompressible and compressible results at different Mach numbers.

To check the peak values of vertical and swirl velocities easily, the ratio of $W_{\max}$ and $V_{t\max}$ is plotted in Figure 5-5-4 for the three types of vortices at different Mach numbers. Figures 5-5-4 and 5-5-2 are very similar.

As swirl velocities are increased with increasing Mach number, the pressure and density decreases, leading to an increase in W to maintain the constant mass flow. To investigate the compressibility effects for density, the ratio of $(\rho W)_{\max}$ and $(\rho V_t)_{\max}$ is shown in Figure 5-5-5 for three types of vortices at different Mach numbers. The range in this ratio for low swirl is only a little less than that in Figure 5-5-4, but the order has a tendency to be reversed (except for the $M = 0$ case) by the reduction in the density ratio

between the radius where $W_{\max}$ occurs and where $V_{t\max}$ occurs. The ratio shows less variation for the medium swirl case and is essentially unchanged for high swirl.

Figure 5-5-4: Summary of the ratio $W_{\max}/V_{t\max}$ versus S_c for the incompressible and compressible results at different Mach numbers.

Figure 5-5-5: Summary of the ratio $(\rho W)_{\max}/(\rho V_t)_{\max}$ versus S_c for the incompressible and compressible results at different Mach numbers.

Based on the above results and analysis of this chapter, one may conclude that compressibility effects would not change the basic dynamics of tornadic corner flows even if Mach numbers greater than one are achieved. The most dramatic effects occur for low swirl ratios with a significant increase in the maximum vertical velocity and in the height of the vortex breakdown above the surface possible. The effects for medium and high swirl corner flows are not as large with the effect on high swirl corner flow essentially limited to influencing the secondary vortices.

At peak average Mach numbers less than approximately 0.5, the compressibility effects are not very significant and may be estimated for by an appropriate isentropic transformation applied to the incompressible results. This approximation leaves the velocity field unchanged and the reduced density yields an estimate of the reduction in pressure drop within the tornado. Thus it misses the structural changes in the velocity field by any volume flow variations forced by the decreasing density as the Mach number is increased. The compressible simulations suggests that these velocity modifications can override the expected isentropic effects on pressure drop at some points in space for certain combinations of S_c and M .

Chapter 6: Simulations of a Transient Tornado Vortex

6.1 Introduction

In the previous chapter, the compressible large-eddy simulation (LES) model has been applied to study the corner flow for quasi-steady conditions with axisymmetric boundary conditions. Here the study is extended to time-dependent boundary conditions in order to address tornadogenesis and variability.

A potentially important effect of compressibility occurs under conditions where a temporal overshoot can occur in the evolution of the tornadic corner flow. In *Lewellen et al.* (2000b), case 1 started from a very low swirl corner flow which has only a weak vortex at low levels. It was generated from a high swirl flow by enhancing the near-surface, radial inflow. If such a very low swirl flow has its low level, low swirl flow suddenly blocked (for example by a cold downdraft reaching the surface and wrapping around the central flow), then the temporal evolution of the corner flow from very low to high swirl produces an incompressible temporal overshoot that is much more intense than occurs for the quasi-steady state flow. The process is repeated here by using both the incompressible and the compressible LES models, where the finest grid is 2 m and the effective surface roughness (z_0) is 0.02 m. To highlight the compressible effects, the speed of sound for the compressible simulation is taken as $174 \text{ m}\cdot\text{s}^{-1}$. The solution started from a very low swirl corner flow ($S_c = 0.8$) achieved by including lower swirl inflow through the side boundaries in a layer above the surface. The layer of inflow was abruptly shut off at the initial time, then the simulation followed until it reached its new quasi-steady state, which has a high swirl ratio ($S_c = 12$).

The initial condition is shown in Figure 6-1-1, which includes angular momentum Γ , perturbation pressure ΔP , swirl velocity V_t and vertical velocity W in the vertical cross section of azimuthally averaged fields. As shown, the simulation domain is $2 \text{ km} \times 2 \text{ km} \times 2 \text{ km}$.

Figure 6-1-1: Vertical cross section of azimuthally averaged fields for the time-dependent case at $t = 0$. (a) angular momentum (solid line) and perturbation pressure (dash line), (b) swirl velocity, (c) vertical velocity.

6.2 Comparison of incompressible and compressible simulation results

The whole process may be divided into four phases, as shown in Figures 6-2-1 to 6-2-5 from the compressible LES model, which have the same format as Figure 6-1-1 but show the flow conditions in a smaller domain (500 m in radius and 1 km in height) at different times.

In the first phase, about the first 80 seconds, the low swirl fluid is exhausted from the surface layer through the top while the core flow changes little. It is shown in Figure 6-2-1.

Shown in Figure 6-2-2, once most of the low swirl fluid is exhausted from the surface layer there is still a lot of inertia in the fluid rushing up the core, which pulls the core in at lower levels spinning up the flow. This continues until the flow reaches the point of maximum intensification in the inner corner with an intense surface jet, and above it a narrow downdraft driven by low pressure near the surface, as shown in Figure

6-2-3. At this time, around 108 seconds, the instantaneous minimum perturbation pressure has dropped from the initial -1.59 kPa to a peak drop of -19.16 kPa. That is the second phase of the process.

Subsequently, in the third phase as shown in Figure 6-2-4 at 140 seconds, the narrow downdraft reaches the surface giving rise to a medium swirl corner flow at the surface which is still intense and persists for a while.

In the last phase a wider downdraft follows, opening up the core to give a high swirl corner, which finally forces the lower core bigger than the higher core. Figure 6-2-5 shows that at 170 seconds.

Figure 6-2-1: Vertical cross section of azimuthally averaged fields for the time-dependent case at 80 seconds. (a) angular momentum (solid line) and perturbation pressure (dash line), (b) swirl velocity, (c) vertical velocity.

Figure 6-2-2: Vertical cross section of azimuthally averaged fields for the time-dependent case at 96 seconds. (a) angular momentum (solid line) and perturbation pressure (dash line), (b) swirl velocity, (c) vertical velocity.

Figure 6-2-3: Vertical cross section of azimuthally averaged fields for the time-dependent case when the peak pressure drop occurs, around 108 seconds. (a) angular momentum (solid line) and perturbation pressure (dash line), (b) swirl velocity, (c) vertical velocity.

In general, the whole process is quite similar to the results using the incompressible LES model. For example, Figure 6-2-6 shows the distributions of four same parameters in the vertical cross section of azimuthally averaged fields at 100 seconds, when the peak pressure drop occurs. Comparing it with Figure 6-2-3, one may see that the distributions of angular momentum are close, and the peak swirl and vertical velocity locate at the same positions.

Figure 6-2-6: Applying the incompressible LES model, vertical cross section of azimuthally averaged fields for the same case at 100 seconds, when the peak pressure drop occurs. (a) angular momentum (solid line) and perturbation pressure (dash line), (b) swirl velocity, (c) vertical velocity.

Figures 6-2-7 and 6-2-8 show the instantaneous isosurface contour levels of perturbation pressure within a $300 \text{ m} \times 300 \text{ m} \times 300 \text{ m}$ domain from the incompressible and compressible simulations separately. Both are at the time of peak pressure drop occurring. Even in the instantaneous fields their structures are similar, while the peak values of the pressure drop are significantly different, about -32.90 kPa in the incompressible results, -19.16 kPa in the compressible ones.

Figure 6-2-7: Instantaneous isosurface contours of perturbation pressure, ΔP (Pa), within a $300 \text{ m} \times 300 \text{ m} \times 300 \text{ m}$ domain at 100 second when the peak pressure drop occurs, using the incompressible LES model.

Figure 6-2-8: Instantaneous isosurface contours of perturbation pressure, ΔP (Pa), within a $300 \text{ m} \times 300 \text{ m} \times 300 \text{ m}$ domain at 108 seconds when the peak pressure drop occurs, using the compressible LES model.

In order to compare the results using the incompressible and compressible LES model quantitatively, Figure 6-2-9 shows the minimum pressure ($P_{\min}$) normalized by the outside total pressure (P_0) as a function of time (t/t^*). As in Appendix B, $t^* = r_d^2 / \Gamma_\infty$ represents a characteristic time scale for the low swirl surface layer flow to be exhausted after the lateral boundary condition at radius r_d is changed. A much deeper drop of $P_{\min}$ may be observed from the incompressible result, compared with the compressible one, and its slope is sharper.

Figure 6-2-9: Normalized minimum pressure ($P_{\min}/P_0$) as a function of time (t/t^*) for the incompressible and compressible results during the temporal overshoot.

The results indicate that $P_{\min}$ in the incompressible flow is much lower than was the case for a coarser grid (8 m finest grid) simulation (not shown here) indicating that it is quite sensitive to grid resolution. As long as the flow is strictly incompressible, the principal limit on how small the local vortex core can become appears to be supplied by the grid resolution. In the incompressible LES code, perturbation pressure is calculated and it knows nothing about the outside total pressure, i.e., arbitrary P_0 may be applied and will not change the solution at all. That is the reason why negative $P_{\min}$ values occur in the incompressible results shown in Figure 6-2-9.

In the compressible flow, however, the decreasing density appears to impose a limit on minimum vortex core size, as the Mach number exceeds one. This limit on the minimum radius in turn constitutes a limit on the minimum pressure. The density in compressible flows is always positive and can never reach zero or negative. This feature limits the minimum pressure and the quite broad minimum pressure is evidence of transonic choking.

Figure 6-2-10: Comparison of maximum vertical mass flow, $(\rho W)_{\max}$ ($\text{kg}\cdot\text{m}^2\cdot\text{s}^{-1}$), at different time from the incompressible and compressible results.

Figure 6-2-11: Comparison of maximum vertical velocity, $W_{\max}$ ($\text{m}\cdot\text{s}^{-1}$), at different time from the incompressible and compressible results.

Figures 6-2-10 and 6-2-11 show the maximum vertical mass flow $(\rho W)_{\max}$ and maximum vertical velocity $(W_{\max})$ during the process. From these pictures one may see that the pattern of the corner flow is changed from low local swirl ratio to high local swirl ratio through the process. The results using the two different models are close as long as the maximum Mach number is less than approximately 1. With low swirl ratio at high Mach number, the maximum vertical velocity from the compressible results is significantly larger than that from the incompressible one, while the maximum vertical mass flow $(\rho W)_{\max}$ is significantly smaller. This strong restriction on the mass flow is an essential feature of transonic choking.

Figure 6-2-12: Instantaneous maximum local Mach numbers at different time from the compressible results.

The instantaneous maximum local Mach numbers from the compressible results are plotted in Figure 6-2-12. It shows that a value of almost 2.0 has been reached. The dynamics of transonic swirling flow are quite different from that of simple one-dimensional transonic flow, with Mach numbers permitted to exceed 1 within the minimum flow cross section as shown by *Lewellen et al.* (1969). Their analysis shows that the maximum Mach number occurring in transonic swirling flow depends on the

detailed distribution of angular momentum and total pressure across the streamlines flowing through the minimum cross section and that a value of 2 is well within an expected reasonable range.

6.3 Conclusions of the chapter

In this chapter, a time-dependent tornado vortex (case 1 in *Lewellen et al. 2000b*) is simulated using the compressible LES model and compared with the corresponding incompressible results. Starting from a quasi-steady very low swirl ratio corner flow, it sweeps through medium swirl ratio to high swirl ratio until it reaches quasi-steady state again. The case not only could take place in the nature, for example, as a cold downdraft surrounds the boundary layer of the tornado vortex suddenly, but also is a good example to show different patterns of tornado vortices in one process.

Comparing the simulation results from the incompressible and compressible LES models, it shows that the whole processes are close to each other except where $P_{\min}$ approaches zero. The great reduction in pressure induces corresponding decreases in density and temperature that in turn force increases in vertical velocity and Mach number. During this temporal overshoots, the Mach number was raised as high as 2 as the pressure drop is restricted by a local swirl modified choking effect.

Those simulation results are consistent with the conclusions in the previous chapter and suggest that there are no apparent physical barriers to transonic speeds occurring within real tornadoes, at least on rare occasions. If supersonic velocities do occur it will tend to restrict the maximum pressure drop. Of course, this occurrence would be extremely difficult to observe in an actual tornado.

Chapter 7: Summary and Discussion

7.1 Conclusions from previous chapters

In Chapter 4 the three-dimensional, unsteady, compressible LES model developed in Chapter 3 has been verified. Three types of cases are used. The one-dimensional shock tube is simple and good to check the mass conservation of the code. In that case it captures the shock and expansion waves, and the performance improves as more grid points are employed in the simulation domain. Compared with analytical predictions an excellent agreement is achieved when sufficient grid points are used. The code's numerical stability is validated for two-dimensional purely swirling vortices at low and high Mach numbers. Finally a three-dimensional tornado vortex is simulated with a small Mach number. The incompressible and compressible results are very close to each other, which indicates that the modified compressible LES code performs well in incompressible conditions, too. This feature is required since the Mach number is still small in most of the regions and the flow may be considered as incompressible even in some extremely violent tornado vortices where the maximum Mach number might be greater than 0.5 in the corner regions. All of the test cases indicate that the modified three-dimensional compressible LES code is robust and accurate, so that it is a good tool to study the compressibility effects in tornado vortices.

Different tornadoes may have different flow patterns. Usually the basic behaviors of tornado corner flows are cataloged in three types: low-swirl, medium-swirl and high-swirl. In Chapter 5 the three types of tornado vortices with quasi-steady conditions are simulated respectively as the Mach number is increased to investigate the significance of compressibility effects. It is found that the effects are different among them. At peak average Mach numbers less than approximately 0.5, the compressibility effects are not very significant and may be accounted for to leading order by an appropriate isentropic transformation applied to the incompressible results. As the maximum Mach number is increased to more than 1.0, the compressibility effects for low-swirl-ratio corner flows are dramatic with significant increase of vertical velocities and the height of the vortex breakdown above the surface. The effects are much weaker for medium swirl, and are expected to be still weaker for high swirl corner flow where the effects are essentially

limited to influencing the secondary vortices. In general, compressibility effects would not change the basic dynamics of tornadic corner flows even if Mach numbers greater than one are achieved.

In Chapter 6, a time-dependent tornado vortex (essentially case 1 in *Lewellen et al.* 2000b) is simulated using the compressible LES model. Starting from a quasi-steady low swirl ratio corner flow, this flow sweeps through medium swirl ratio to high swirl ratio until it reaches quasi-steady state again. This case not only might occur in nature, for example when a cold downdraft surrounds the boundary layer of the tornado vortex suddenly, but also provides a good example of how transonic velocities might be reached in a temporal overshoot within a tornado vortex.

Comparing the simulation results from the incompressible and compressible LES models shows that the whole processes are generally similar except where $P_{\min}$ approaches zero. The great reduction in pressure induces corresponding decreases in density and temperature that in turn force increases in vertical velocity and Mach number. During this temporal overshoot, the Mach number was raised as high as 2, as the pressure drop is restricted by a local swirl modified choking effect and thus limits the intensification of the overshoot.

There are no apparent physical barriers to transonic speeds occurring within real tornadoes on rare occasions, but it is believed that most tornado dynamics are not significantly impacted by Mach number effects. There may be occasions when the maximum velocity within a tornado is limited by sonic conditions for brief periods but it is a soft limit which allows modest supersonic velocities and would be difficult to observe in an actual tornado.

7.2 Recommendations and future work

- Variable θ

In the numerical simulations of this dissertation constant potential temperature (θ) has been employed because the temperature difference is not believed to be significant in tornado corner flows where pressure gradients tend to dominate buoyancy effects. When

simulation domains are large, on the order of the thunderstorm scale, big temperature differences are expected and buoyancy effects may be significant in driving the vortex flow, therefore variable θ should be needed in the governing equations of these larger scale simulations. In fact the θ equation (3-4-3) is included in the modified compressible code and has been validated successfully in section 4-2-3 with initial uniform temperature in a “shock tube”.

- Improve code efficiency

Large computer resources are still necessary in LES, and sometimes it is time-intensive. Instead of storing and computing the whole domain for pressure as in the incompressible code, density is stored and computed locally as well as other variables in the modified compressible code. This should allow the efficiency of the code to be improved. Moreover this feature should make it much easier to apply the code in a parallel calculation that would reduce the computational time required significantly.

- Height of vortex breakdown

Vortex breakdown is one of the most interesting issues in tornado vortex investigation. In low-swirl-ratio tornado corner flow, the height of the vortex breakdown increases dramatically as the Mach number is increased. Can it be quantitatively predicted? It seems that it is related to $r_c \frac{W_{\max}}{V_{t \max}}$, and S_c may be involved. Currently there are not enough cases to determine the quantitative relationship.

- The transient case

The transient case shown in Chapter 6 may be the most representative example of where compressibility effects become important to a real tornado vortex in nature. A better understanding of this case will help in the investigation of the physics of vortex dynamics. The current research in the transient case is still preliminary, and further studies are needed. For example, the maximum pressure drop tends to increase with higher resolution at high Mach numbers. Under what conditions does this continue until transonic velocities limit the overshoot?

Bibliography

- Bengtsson, L., and J. Lighthill (Eds.) (1982): "Intense Atmospheric Vortices", Springer-Verlag, New York.
- Brandes, E. A. (1993): "Tornadic thunderstorm characteristic determined with Doppler radar", in *The Tornado: Its Structure, Dynamics, Prediction, and Hazards*, edited by C. Church, D. Burgess, C. Doswell, and R. Davies-Jones, Geophysical Monograph **79**, American Geophysical Union, 143-159.
- Chi, J. (1977): "Numerical analysis of turbulent end-wall boundary layers of intense vortices", *Journal of Fluid Mechanics*, **82**, 209-222.
- Church, C. R., J. T. Snow, G. L. Baker, and E. M. Agee (1979): "Characteristics of tornado-like vortices as a function of swirl ratio: a laboratory investigation", *Journal of the Atmospheric Sciences*, **36**, 1755-1766.
- Church, C. R., and J. T. Snow (1993): "Laboratory models of tornadoes", in *The Tornado: Its Structure, Dynamics, Prediction, and Hazards*, edited by C. Church, D. Burgess, C. Doswell, and R. Davies-Jones, Geophysical Monograph **79**, American Geophysical Union, 277-295.
- Colella, P., and P. R. Woodward (1984): "The piecewise parabolic method (PPM) for gas-dynamical simulations", *Journal of Computational Physics*, **54**, 174-201.
- Davies-Jones, R. P. (1973): "The dependence of core radius on swirl ratio in a tornado simulator", *Journal of the Atmospheric Sciences*, **30**, 1427-1430.
- Davies-Jones, R. P. (1976): "Laboratory simulations of tornadoes", in *Proceedings of the Symposium on Tornadoes: Assessment of Knowledge and Implications of Man*, American Meteorological Society, Boston, MA, 151-173.
- Davies-Jones, R. P. (1984): "Streamwise vorticity: the origin of updraft rotation in supercell storms", *Journal of the Atmospheric Sciences*, **41**, 2991-3006.
- Davies-Jones, R. P. (1986): "Tornado dynamics", in *Thunderstorm Morphology and Dynamics*, 2nd ed., edited by E. Kessler, University of Oklahoma Press, Norman, OK, 197-236.
- Davies-Jones, R. P. (1988): "Tornado interception with mobile teams", in *Instruments and Techniques for Thunderstorm Observation and Analysis*, 2nd edition, University of Oklahoma Press, Norman, 23-32.
- Davies-Jones, R. P., D. Burgess, and M. Foster (1990): "Test of helicity as a tornado forecast parameter", in *Preprints, 16th Conference on Severe Local Storms*, American Meteorological Society, Boston, MA, 588-592.
- Davies-Jones, R. P. (1995): "Tornadoes", *Scientific American*, **261**(8), 48-57.
- Deardorff, J. W. (1980): "Stratocumulus-capped mixed layers derived from a three-dimensional model", *Boundary-Layer Meteorology*, **18**, 495-527.

- Fiedler, B. H., and R. A. Rotunno (1986): "A theory for the maximum windspeeds in tornado-like vortices", *Journal of the Atmospheric Sciences*, **43**, 2328-2340.
- Fiedler, B. H. (1995): "On modeling tornadoes in isolation from the parent storm", *Atmosphere-Ocean*, **33**(3), 501-512.
- Fiedler, B. H. (1996): "The sonic speed limit of tornadoes", in *Proceedings of the AMS 18th Conference on Severe Local Storms*, 19-23 February, San Francisco, CA, 385-386.
- Fujita, T. T. (1971): "Proposed characterization of tornadoes and hurricanes by area and intensity", *SMRP Research Paper* **91**, University of Chicago, Chicago, IL, 42 pp.
- Fujita, T. T. (1993): "Plainfield tornado of August 28, 1990", in *The Tornado: Its Structure, Dynamics, Prediction, and Hazards*, edited by C. Church, D. Burgess, C. Doswell, and R. Davies-Jones, Geophysical Monograph **79**, American Geophysical Union, 1-17.
- Grasso, L. D., and W. R. Cotton (1995): "Numerical simulation of a tornado vortex", *Journal of the Atmospheric Sciences*, **52**, 1192-1203.
- Grasso, L. D. (1996): "Numerical simulation of the May 15 and April 26, 1991 Thunderstorms", *Ph.D. Dissertation*, Department of Atmospheric Science, Colorado State University, 151 pp.
- Howells, P., and R. K. Smith (1983): "Numerical simulation of tornadolike vortices, I: Vortex evolution", *Geophys. Astrophys. Fluid Dyn.*, **27**, 253-284.
- Howells, P., R. A. Rotunno, and R. K. Smith (1988): "A comparative study of atmospheric and laboratory-analogue numerical tornado vortex models", *Quarterly Journal of the Royal Meteorological Society*, **114**, 801-822.
- Houze, R. A. Jr. (1993): "Cloud Dynamics", *International Geophysics* vol. **5**, Academic Press Inc., 573 pp.
- Huschke, R. E. (1959): "Glossary of Meteorology", American Meteorological Society, Boston, MA, 50.
- Kessler, E. (Ed.) (1986): "Thunderstorm morphology and dynamics", University of Oklahoma Press, 2nd edition.
- Klemp, J. B., and R. A. Rotunno (1983): "A study of the tornadic region within a supercell thunderstorm", *Journal of the Atmospheric Sciences*, **40**, 359-377.
- Komurasaki, S., T. Kawamura, and K. Kuwahara (1999): "Formation of a tornado and its breakdown", *AIAA paper* 99-3766, **A99-33717**, Norfolk, VA.
- Lesieur, M., and P. Comte (1997): "Large-eddy simulations of compressible turbulent flows", *AGARD-R-819*, 4-1~ 4-39.
- Leslie, F. W. (1977): "Surface roughness effects on suction vortex formations: A laboratory simulation", *Journal of the Atmospheric Sciences*, **34**, 1022-1027.

- Leslie, F. W., and R. K. Smith (1982): "Numerical studies of tornado structure and genesis", in *Intense Atmospheric Vortices*, edited by L. Bengtsson and J. Lighthill, Spring-Verlag, New York, 205-211.
- Lewellen, D. C., and W. S. Lewellen (1996): "Large eddy simulations of a tornado's interaction with the surface", in *Proceedings of the AMS 18th Conference On Severe Local Storms*, San Francisco, CA, February, 392-395.
- Lewellen, D. C., and W. S. Lewellen (1998): "Importance of surface layer inflow to tornado vortex intensity", in *Proceedings of the AMS 19th Conference On Severe Local Storms*, Minneapolis, MN, September, 14-19.
- Lewellen, D. C., W.S. Lewellen, and J. Xia (2000a): "The influence of a local swirl ratio on tornado intensification near the surface", *Journal of the Atmospheric Sciences*, **57**, 527-544.
- Lewellen, D. C., W. S. Lewellen, and J. Xia (2000b): "Nonaxisymmetric, unsteady tornadic corner flows", in *Proceedings of the AMS 20th Conference on Severe Local Storms*, Orlando, FL, 11-15 September, 265-268.
- Lewellen, W. S. (1962): "A solution for three-dimensional vortex flows with strong circulation", *Journal of Fluid Mechanics*, **14**, 420-432.
- Lewellen, W. S., W. J. Burns, and H. J. Strickland (1969): "Transonic swirling flow", *AIAA Journal*, **7**, 1290-1297.
- Lewellen, W. S. (1976): "Theoretical models of the tornado vortex", in *Proceedings, Symposium on Tornadoes: Assessment of Knowledge and Implications for Man*, edited by R. E. Peterson, Texas Technology University, Lubbock, 107-143.
- Lewellen, W. S. (1977): "Use of invariant modeling", in *Handbook of Turbulence*, Plenum, 237-280.
- Lewellen, W. S., and M. E. Teske (1977): "Turbulent transport model of low-level winds in a tornado", in *Preprints, 10th Conference On Severe Local Storms*, American Meteorological Society, Boston, MA, 291-298.
- Lewellen, W. S., and Y. P. Sheng (1979): "Influence of surface conditions on tornado wind distributions", in *Preprints, 11th Conference on Severe Local Storms*, American Meteorological Society, Boston, MA, 375-381.
- Lewellen, W. S., and Y. P. Sheng (1980): "Modeling tornado dynamics", *Rep. NUREG/CR-2585*, U.S. Nuclear Regulatory, Commission, Washington, D.C., 226 pp.
- Lewellen, W. S. (1981): "Modeling the lowest 1 km of the atmosphere", *AGARD-AG-267*, NATO, NTIS No. ADA 111413, 81 pp.
- Lewellen, W. S. (1993): "Tornado Vortex Theory", in *The Tornado: Its Structure, Dynamics, Prediction, and Hazards*, edited by C. Church, D. Burgess, C. Doswell, and R. Davies-Jones, Geophysical Monograph **79**, American Geophysical Union, 19-39.

- Lewellen, W. S., D. C. Lewellen, and R. I. Sykes (1997): "Large-eddy simulation of a tornado's interaction with the surface", *Journal of the Atmospheric Sciences*, **54**, 581-605.
- Lilly, D. K. (1969): "Tornado dynamics", *NCAR Manuscript 69-117*. National Center for Atmospheric Research, Boulder, CO, 37 pp.
- Lilly, D. K. (1986): "The structure, energetics and propagation of rotating convective storms, II, Helicity and storm stabilization", *Journal of the Atmospheric Sciences*, **43**, 126-140.
- Lugt, H. J. (1983): "Vortex Flow in Nature and Technology", Wiley-Interscience, New York, 297 pp.
- Maxworthy, T. (1982): "The laboratory modeling of atmospheric vortices: A critical review", in *Intense Atmospheric Vortices*, edited by L. Bengtsson and J. Lighthill, Springer-Verlag, New York, 229-246.
- McCaul, E. W. Jr. (1987): "Observations of the Hurricane 'Danny' tornado outbreak of 16 August 1985", *Monthly Weather Review*, **115**, 1206-1223.
- McCaul, E. W. Jr. (1993): "Observations and simulations of hurricane-spawned tornadic storms", in *The Tornado: Its Structure, Dynamics, Prediction, and Hazards*, edited by C. Church, D. Burgess, C. Doswell, and R. Davies-Jones, Geophysical Monograph **79**, American Geophysical Union, 119-142.
- McClellan, T. M., M. E. Akridge, and J. T. Snow (1990): "Streamlines and vortex lines in numerically-simulated tornado vortices", in *Proceedings of the AMS 16th Conference On Severe Local Storms*, Boston, MA, 572-577.
- Moeng, C.-H. (1984): "A large-eddy-simulation model for the study of planetary boundary-layer turbulence", *Journal of the Atmospheric Sciences*, **41**, 2052-2062.
- Moncirieff, M. W., and M. J. Miller (1976): "The dynamics and simulation of tropical cumulonimbus and squall lines", *Quarterly Journal of the Royal Meteorological Society*, **102**, 373-394.
- Morinishi, Y., T. S. Lund, O. V. Vasilyev, and P. Moin (1998): "Fully conservative higher order finite difference schemes for incompressible flow", *Journal of Computational Physics*, **143**, 90-124.
- Oosthuizen, P. H., and W. E. Carscallen (1997): "Compressible Fluid Flow", McGraw-Hill, 171-173.
- Piacsek, S. A., and G. P. Williams (1970): "Conservative properties of convection difference schemes", *Journal of Computational Physics*, **6**, 392-405.
- Proctor, F. H. (1979): "The dynamic and thermal structure of an evolving tornado", in *Proceedings of the AMS 11th Conference On Severe Local Storms*, Boston, MA, 389-396.

- Rotunno, R. A. (1977): "Numerical simulation of a laboratory vortex", *Journal of the Atmospheric Sciences*, **34**, 1942-1956.
- Rotunno, R. A. (1979): "A study in tornado-like vortex dynamics", *Journal of the Atmospheric Sciences*, **36**, 140-155.
- Rotunno, R. A. (1982): "A numerical simulation of multiple vortices", in *Intense Atmospheric Vortices*, edited by L. Bengtsson and J. Lighthill, Springer-Verlag, New York, 215-228.
- Rotunno, R. A. (1984): "An investigation of a three-dimensional asymmetric vortex", *Journal of the Atmospheric Sciences*, **41**, 283-298.
- Rotunno, R. A., and J. B. Klemp (1985): "On the rotation and propagation of simulated supercell thunderstorms", *Journal of the Atmospheric Sciences*, **42**, 271-292.
- Rotunno, R. A. (1986): "Tornadoes and tornadogenesis", in *Mesoscale Meteorology and Forecasting*, edited by P. S. Ray, American Meteorological Society, Boston, MA, 414-436.
- Rotunno, R. A. (1993): "Supercell thunderstorm modeling and theory", in *The Tornado: Its Structure, Dynamics, Prediction, and Hazards*, edited by C. Church, D. Burgess, C. Doswell, and R. Davies-Jones, Geophysical Monograph **79**, American Geophysical Union, 57-74.
- Smith, R. K., and L. M. Leslie (1979): "A tornadogenesis in a rotating thunderstorm", *Quarterly Journal of the Royal Meteorological Society*, **105**, 107-127.
- Snow, J. T. (1982): "A review of recent advances in tornado vortex dynamics", *Reviews of Geophysics*, **20**, 953-964.
- Snow, J. T. (1984): "The tornado", *Scientific American*, **250**(4), 86-96.
- Snow, J. T. (1987): "Atmospheric columnar vortices", *Reviews of Geophysics*, **25**, 371-385.
- Stiegler, D. J., and T. T. Fujita (1982): "A detailed analysis of the San Marcos, Texas, tornado induced by Hurricane 'Allen' on 10 August 1980", in *Preprints, 12th Conference on Severe Local Storms*, American Meteorological Society, Boston, MA, 371-374.
- Sykes, R. I., and D. S. Henn (1989): "Large-eddy simulation of turbulent sheared convection", *Journal of the Atmospheric Sciences*, **46**, 1106-1118.
- Tannehill, I. R. (1950): "Hurricanes", Princeton University Press, Princeton, NJ, 303 pp.
- Trapp, R. J., and B. H. Fielder (1993): "Numerical simulation of tornadolike vortices in asymmetric flow", in *The Tornado: Its Structure, Dynamics, Prediction, and Hazards*, edited by C. Church, D. Burgess, C. Doswell, and R. Davies-Jones, Geophysical Monograph **79**, American Geophysical Union, 49-54.
- Vergara, J. A. (1997): "Numerical simulation of a tornado with a fully compressible cloud model", *Ph.D. Dissertation*, University of Maryland.

- Wakimoto, R. M., and C. Liu (1998): "The garden city, Kansas, storm during VORTEX 95. Part II: the wall cloud and tornado", *Monthly Weather Review*, **126**, 393-408.
- Walko, R. L. (1988): "Plausibility of substantial dry adiabatic subsidence in a tornado core", *Journal of the Atmospheric Sciences*, **45**, 2251-2267.
- Walko, R. L. (1990): "Generation of tornado-like vortices in nonaxisymmetric environments", in *Proceedings of the AMS 16th Conference On Severe Local Storms*, Boston, MA, 583-587.
- Walko, R. L. (1993): "Tornado spin-up beneath a convective cell: required basic structure of the near-field boundary layer winds", in *The Tornado: Its Structure, Dynamics, Prediction, and Hazards*, edited by C. Church, D. Burgess, C. Doswell, and R. Davies-Jones, Geophysical Monograph **79**, American Geophysical Union, 89-95.
- Ward, N. B. (1972): "The exploration of certain features of tornado dynamics using a laboratory model", *Journal of the Atmospheric Sciences*, **29**, 1149-1204.
- Weisman, M. L., and J. B. Klemp (1982): "The dependence of numerically simulated convective storms on vertical wind shear and buoyancy", *Monthly Weather Review*, **110**, 504-520.
- Weisman, M. L., and J. B. Klemp (1984): "The structure and classification of numerically simulated convective storms in directionally varying wind shears", *Monthly Weather Review*, **112**, 2479-2498.
- Whipple, A. B. C. (1982): "Storm", vol. **4**, *Planet Earth*, Time-Life Books, New York.
- Wicker, L. J. (1990): "A numerical simulation of a tornado-scale vortex in a three-dimensional cloud model", *Ph.D. Dissertation*, University of Illinois at Urbana-Champaign, 264 pp.
- Wicker, L. J., and R. B. Wilhelmson (1990): "Numerical simulation of a tornado-like vortex in a high resolution three dimensional cloud model", in *Preprints, 16th Conference on Severe Local Storms*, American Meteorological Society, Boston, MA, 263-268.
- Wicker, L. J., and R. B. Wilhelmson (1993): "Numerical simulation of tornadogenesis within a supercell thunderstorm", in *The Tornado: Its Structure, Dynamics, Prediction, and Hazards*, edited by C. Church, D. Burgess, C. Doswell, and R. Davies-Jones, Geophysical Monograph **79**, American Geophysical Union, 75-88.
- Wicker, L. J., and R. B. Wilhelmson (1995): "Simulation and analysis of tornado development and decay within a three-dimensional supercell thunderstorm", *Journal of the Atmospheric Sciences*, **52**, 2675-2702.
- Wicker, L. J., M. P. Kay, and M. P. Foster (1997): "STORMTIPE-95: A convective storm forecast experiment", *Weather and Forecasting*, **12**(3), 427-436.
- Wilhelmson, R. B., and J. B. Klemp (1978): "A numerical study of storm splitting that leads to long-lived storms", *Journal of the Atmospheric Sciences*, **35**, 1974-1986.

- Wilkins, E. M. (1988): "Influence of Neil Ward's simulator on tornado research", *CIMMS Report*, University of Oklahoma, **87**, 64 pp.
- Wilson, T., and R. A. Rotunno (1986): "Numerical simulation of a laminar end-wall vortex and boundary layer", *Physics of Fluids*, **29**, 3993-4005.
- Wurman, J., J. M. Straka, and E. N. Rasmussen (1996): "Fine-Scale Doppler Radar Observations of Tornadoes", *Science*, **272**(21), 1774-1777.

Appendix A

The Influence of a Local Swirl Ratio on Tornado Intensification near the Surface^{*}

D.C. Lewellen, W. S. Lewellen, and J. Xia

Journal of the Atmospheric Sciences, Volume **57**, pp. 527-544

^{*} The Reprint Permission has been granted by the American Meteorological Society, the copyright holder.
Correction: Equation (A4) should be

$$\xi^2 = 2\nabla p \cdot \frac{dV}{dx_i} \left(\nabla p \cdot \frac{dV}{dx_i} - \nabla p \cdot \nabla V_i \right) / (\nabla p \cdot \nabla p), \quad (\text{A4})$$

The Influence of a Local Swirl Ratio on Tornado Intensification near the Surface

D. C. LEWELLEN, W. S. LEWELLEN, AND J. XIA

Department of Mechanical and Aerospace Engineering, West Virginia University, Morgantown, West Virginia

(Manuscript received 1 June 1998, in final form 2 March 1999)

ABSTRACT

The results of high-resolution, fully three-dimensional, unsteady simulations of the interaction of a tornado with the surface are presented. The goal is to explore some of the range of structures that should be expected to occur in nature within the tornadic “corner flow”—that region where the central vortex meets the surface. The most important physical variables considered are the tornado-scale circulation and horizontal convergence, the effective surface roughness, the tornado translation speed, the low-level inflow structure, and the upper-core structure. A key ingredient of the corner flow dynamics is the radial influx of fluid in the surface layer with low angular momentum relative to that of the fluid in the main vortex above it. This low swirl fluid arises initially from outside or below the larger-scale vortex or through frictional loss of angular momentum to the surface and forms much of the vortex core flow after it exits the corner flow region. Changes in the surface layer inflow or upper-core structure can dramatically affect the level of intensification and turbulent structure in the corner flow even when the swirl ratio of the tornado vortex as a whole is unchanged. The authors define a local corner flow swirl ratio, S_c , based on the total flux of low angular momentum fluid through the corner flow and show that it parameterizes the leading effects on the corner flow of changes to the flow conditions immediately outside of the corner flow. As S_c decreases, the low-level vortex intensity rises to a maximal level where mean swirl velocities near the surface reach 2.5 times the maximum mean swirl velocity aloft; further decreases force a transition to a much weaker low-level tornado vortex. This sensitivity suggests that differences in the near-surface inflow layer may be a critical factor in determining whether an existing supercell low-level mesocyclone spawns a tornado or not.

1. Introduction

Viewing any appreciable sample of the videotape records of actual tornadoes, one is impressed by the wide variety of flows that are evidenced, not only in the range of sizes and intensities, but in the different structures encountered. For example, some tornadoes display thin, relatively smooth, almost elegant single funnels on the ground, while others are composed of many highly turbulent secondary vortices revolving about large central cores. Categorizing and understanding this range of structures has been a long-standing goal of tornado research. Controlled laboratory and numerical experiments of confined tornado-like vortices have produced similar ranges of flow structures when a single parameter such as the swirl angle of the incoming flow is varied. Correlating these studies with actual tornadoes in the field is more problematic; here the flow is highly turbulent, and many physical parameters are involved. More important, physical processes on a broad range of

length scales from a few meters in the surface layer to many kilometers on the storm scale are all important and strongly coupled with each other. Consequently one finds in the field that storms with seemingly quite similar structure on the kilometer scale may give rise to tornadoes interacting with the surface with quite different structures, or even to no tornado at all.

In broad caricature we can identify three length scales of direct importance to the tornado structure: the storm scale of tens of kilometers, which ultimately drives the entire flow; the outer tornado scale of a few kilometers in which the flow may be considered a converging, swirling plume;¹ and the inner tornado scale of tens to several hundreds of meters characterizing the tornado core, boundary layer, and corner flow regions of the flow. In this work we focus on the last of these regimes and, in particular, on the tornadic corner flow where the boundary layer flow transitions into the core flow. As the place where the tornado vortex meets the ground,

Corresponding author address: Dr. David C. Lewellen, Department of Mechanical and Aerospace Engineering, West Virginia University, P.O. Box 6106, Morgantown, WV 26506-6106.
E-mail: dlewell@wvu.edu

¹ The outer tornado scale may be identified with the mesocyclone when the tornado spins up directly from the mesocyclone core; when the tornado is better characterized as a secondary vortex within the mesocyclone, these become distinct scales with the mesocyclone scale lying in between the storm and outer tornado scales.

this region is of particular relevance, typically being the scene of the largest velocities, lowest pressures, sharpest velocity gradients, and greatest damage potential in the entire flow. In concentrating on this part of the flow we knowingly set aside the important question of tornadogenesis on the larger scale, that is, of how the storm-scale flow gives rise to and maintains the swirling, converging plume on the few kilometer scale that makes the occurrence of an intense tornadic vortex possible. Instead, we focus on which properties of the velocity and pressure fields immediately outside of the corner flow region most strongly govern the resulting corner flow structure and on how this dependence might best be parameterized. To do this we consider a larger domain than might seem necessary, imposing a variety of flows on the outer tornado scale as our far-field boundary conditions, in order to achieve a physically reasonable range of flow fields immediately bounding the corner flow.

Some researchers have emphasized an inviscid nature of the corner flow as it provides for a rapid transition between the boundary layer and core flows (e.g., Burggraf et al. 1971; Wilson and Rotunno 1986). Inviscid dynamics may be used to explain part of the transition, but we also expect turbulence to play an important role in the flow dynamics in this region where the tornado vortex interacts with the surface. Lewellen et al. (1997, referred to as LLS97 hereafter) presented a large eddy simulation of turbulent transport in the tornado vortex for one set of physical boundary conditions. The results in the corner flow were shown to be relatively independent of grid resolution and subgrid model modifications (for sufficiently fine grid resolution), providing a strong indication that our simulations are properly resolving the most important turbulent eddies in the corner flow. In the present work we utilize this tool for a number of sets of realistic physical conditions in order to address the sensitivity of the vortex flow structure to changes in a variety of physical parameters. Our goals coincide in part with previous laboratory work using tornado vortex chambers, with the advantage in our numerical study that we can explore a wider range of boundary conditions and physical variations, obtain more detailed velocity and pressure measurements with better control over our averaging procedures, and simulate the flows at the higher Reynolds numbers characteristic of the actual atmospheric flow.

In the following section we present a brief description of the tornadic corner flow, using some of our recent simulation results as illustrations. Previous laboratory (e.g., Ward 1972; Snow 1982; Church et al. 1979) and axisymmetric (Lewellen 1962, 1993; Davies-Jones 1973, 1986) modeling studies have shown that one of the primary variables governing the tornado vortex is the swirl ratio of the flow through the vortex. While particular definitions vary in detail, this amounts to the ratio of a typical swirl velocity to a typical flow-through velocity for the converging, swirling plume of the outer

tornado-scale flow. By varying this ratio, a range of distinct behaviors in the vortex corner flow region can be realized; however, a given behavior regime is not uniquely determined by any specific value of this ratio. As we will illustrate, other physical parameters also strongly affect the structure of the central vortex corner flow, so that flows that share the same large-scale swirl ratio can produce different corner flow structures.

In order to better categorize the types of corner flows encountered, we will define a local swirl ratio specific to the inner tornado scale. While it is natural to consider the tornado on the few-kilometer scale as a converging plume with large angular momentum, the surface-corner-core flow itself is better characterized as a low angular momentum jet in a background of roughly constant angular momentum. This is close in spirit to the similarity solutions studied by Long (1958), Burggraf and Foster (1977), and Shtern and Hussain (1993), although they did not include any horizontal convergence of the outer flow as we will here. The corner flow swirl ratio we will define is, in essence, a ratio of typical swirl to flow-through velocities for this body of low angular momentum fluid making up the boundary layer and core regions of the flow. The swirl ratio on the outer tornado scale strongly influences the corner flow swirl ratio, but does not uniquely specify it.

A single dimensionless ratio is certainly inadequate to completely describe the corner flow; nonetheless, we have found that the dominant effects on the corner flow of many physical variables—such as surface roughness, low-level inflow structure, the large-scale swirl ratio, or the properties of the upper core of the vortex—are largely captured by their effects on the corner flow swirl ratio. In section 2 we present examples of tornado vortices exhibiting corner flow behavior characteristic of different swirl ratios and then present a definition for the corner flow swirl ratio. In section 3 we describe our dataset of simulated tornadic vortices under different physical conditions and plot some selected time averaged results from this dataset versus the new swirl ratio. The numerical model and simulation procedures are as described in LLS97 with one addition: a modification of the subgrid model to incorporate rotational damping effects, which is described in the appendix. In section 4 we discuss some important features of the corner flow structure that are only partly governed by the corner flow swirl ratio, in particular, the nature of the turbulent fluctuations and the strength and character of secondary vortices. We close in section 5 with a summary and discussion of future work.

2. A local swirl ratio for categorizing vortex corner flow dynamics

a. Basic corner flow dynamics

In order to simplify the theoretical discussion of the interaction of the tornado vortex with the surface, as

FIG. 1. Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for the corner flow region of a sample high-swirl tornado vortex. (a) Swirl velocity (solid contours) together with highest levels of total velocity variance (dashed, contour level is 0.1), (b) magnitude of the velocity vector in the r - z plane, (c) perturbation pressure, (d) angular momentum, (e) streamfunction, (f) depleted angular momentum streamfunction.

well as the analysis of our numerical data, let us initially restrict the class of flows that we consider to ones with axisymmetric, time-independent outer boundary conditions. In these cases the mean flow can be obtained by taking a time average and is guaranteed to be axisymmetric about a known center line, even though the instantaneous flow is asymmetric, fully three-dimensional, and unsteady in time. When we discuss the effects of the translation of the tornado on the corner flow (among other variations considered in both sections 3 and 4 below) we will necessarily relax the axisymmetric requirement. The quasi-steady approximation is a good one in our study of the corner flow as long as the characteristic timescale of the corner flow itself is short compared to the timescales on which the larger-scale flow

(which sets our boundary conditions) changes appreciably. The former timescale is typically of order 10 s (given corner flow length scales of tens to a few hundred meters and velocities of tens of meters per second), while the latter is of order a few minutes, so that this is, in general, a reasonable approximation. The flow conditions on the kilometer scale do change appreciably as a storm system evolves, however, so that a given tornado may be expected to display different corner flow structures over the course of its lifetime.

1) A HIGH-SWIRL CORNER FLOW

In Fig. 1 we have plotted normalized axisymmetric time averages of several flow variables from one of our

simulations to illustrate some of the general features of a tornado vortex corner flow. The figures show a radial-vertical cross section of the flow about the corner flow region of interest. This represents a small portion of the domain in which the simulation was actually performed; in particular, the inflow and outflow boundary conditions were initially set far from the corner flow itself. In Figs. 1a–c are shown, respectively, contours of the swirl velocity (V), the magnitude of the velocity in the radial-vertical plane (V_{rz}), and the perturbation pressure (p) (i.e., the pressure minus the hydrostatic component). Figures 1d–f show, respectively, contours of the angular momentum of the flow about the center line (Γ), the streamfunction (ψ) indicating the direction and integrated volume flux of the flow, and a depleted angular momentum streamfunction ($\psi_{\delta\Gamma}$) indicating the direction and integrated flux of the flow of fluid with low angular momentum. The latter will be defined precisely in section 2b below when we discuss quantitative measures of the corner flow. The tornado vortex structure is often discussed in terms of four distinct regions of the flow, which are readily identified in Fig. 1d: an outer region in which Γ is constant (or slowly varying) with value Γ_∞ , a surface layer with sharp vertical Γ gradients and nearly zero vertical velocity, an upper-core region with strong radial Γ gradients and nearly zero radial velocity, and the corner flow region proper where the flow transitions from horizontal to vertical and all velocity components are significant. In the outer- and upper-core regions the flow is in approximate cyclostrophic balance and, consequently, is approximately cylindrically symmetric: any appreciable vertical gradients in the swirl field would give rise to vertical pressure gradients driving a vertical flow that would tend to eliminate the deviation from cylindrical symmetry. We have adopted the maximum swirl velocity in this upper-core region, V_c , as a characteristic velocity scale and $r_c \equiv \Gamma_\infty/V_c$ as a characteristic length scale for the purpose of presenting Fig. 1 (and subsequent plots) in nondimensional form. As an example of reasonable physical values for this case, we could take $V_c = 75 \text{ m s}^{-1}$, and $\Gamma_\infty = 15\,000 \text{ m}^2 \text{ s}^{-1}$, giving $r_c = 200 \text{ m}$ and a core pressure drop of approximately 70 mb.

In the surface layer the pressure is to a good approximation equal to that set by the cyclostrophic balance in the cylindrically symmetric region above, since the vertical pressure gradient tends to be small near the surface. The swirl velocity, however, necessarily drops to zero at the surface due to surface friction, so that the flow in the surface layer is out of cyclostrophic balance. The radial pressure gradient, now exceeding the centripetal acceleration needed to turn the swirling flow, drives the radial velocity acceleration apparent in Fig. 1b. In the corner flow this radial surface jet, converging toward $r = 0$, is forced to turn into an (in this case) annular vertical jet (Fig. 1b), but not before overshooting the radial point at which the lowered Γ in the surface layer equals that in the cylindrically symmetric core

FIG. 2. Instantaneous horizontal cross section of simulated tornado from Fig. 1 at a height of $0.2 r_c$, showing perturbation pressure (solid contours in units of ρV_c^2) and normalized vertical velocity (grayscale). The dashed contour indicates a normalized vertical velocity level of -0.2 .

region above (Fig. 1d). It is this inertial overshoot that leads to the highest swirl velocities generally being found near the surface within the corner flow (Fig. 1a), as well as the highest radial and vertical velocities (Fig. 1b) and lowest pressure drop (Fig. 1c). In Fig. 1a we have also indicated the region of largest turbulent kinetic energy, which coincides with the region of sharpest velocity gradients, again within the corner flow.

As can be seen in Figs. 1d–f, the Γ contours, mass flux streamlines, and depleted Γ streamlines are all nearly parallel with each other, showing that advection by the mean flow is the dominant transport mechanism for both the mass and angular momentum fluxes. A more careful inspection within the corner flow region itself shows that the contours are not exactly parallel here: there is significant turbulent angular momentum transport due to the secondary vortices as described in LLS97. Figure 2 shows the instantaneous pressure and vertical velocity contours on a horizontal slice through the corner flow at a height of $0.2 r_c$, within the region of large turbulent kinetic energy (TKE) indicated in Fig. 1a. At this particular sample time, seven secondary vortices rotating about the main vortex are clearly evident. The strong up–down vertical velocity couplet associated with each secondary vortex is largely due to its tilt. As discussed in LLS97, the secondary vortices provide a net angular momentum transport directed radially inward into the central recirculating flow.

FIG. 3. Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for the corner flow region of a sample low-swirl tornado vortex. (a) Swirl velocity (solid contours) together with highest levels of total velocity variance (dashed, contour level is 0.2), (b) angular momentum, (c) depleted angular momentum streamfunction.

2) A LOW-SWIRL CORNER FLOW

The vortex of Fig. 1 illustrates what is considered a “high-swirl” corner flow behavior. Figures 3 and 4, in contrast, show normalized time-averaged axisymmetric contours exhibiting a “low-swirl” corner flow. Figure 3a–c show contours of nondimensionalized swirl velocity, angular momentum, and depleted angular momentum streamfunction. In this case the most extreme velocities in the corner flow are pushed much closer to $r = 0$ (relative to r_c) than in the high-swirl example. In order to better illustrate what is happening in this case we zoom in to the scale of the inner corner flow in Fig. 4, showing the swirl velocity, velocity magnitude in the r – z plane, and perturbation pressure. The radial velocity in this case converges all the way to the axis to form an intense central vertical jet driven upward by an intense low pressure minimum above the surface. At the top of this surface jet is an abrupt vortex breakdown where the vortex core diameter increases dramatically. The radius where the peak swirl velocity occurs is more than a factor of 4 larger above the breakdown than it is below. The instantaneous view of this breakdown given in Fig. 5 shows that the turbulence in this low-swirl case is quite different from that in the high-swirl case of Fig. 2. For this case the resolved turbulence is primarily confined within and downstream of the breakdown. The broader region of large TKE in Fig. 4a is primarily a manifestation of the unsteadiness in the height of the breakdown and some wobble in the very narrow core jet upstream of the breakdown. The qual-

itative features distinguishing these high- and low-swirl cases in Figs. 4 and 1 are similar to those exhibited previously in laboratory flows (e.g., Church et al. 1979) and axisymmetric simulations (e.g., Lewellen and Sheng 1980). The primary difference here is the ability to also exhibit the character of the turbulence, which, particularly in the high-swirl case, will strongly affect the potential damage on the surface.

3) A VERY LOW SWIRL CORNER FLOW

In addition to high-, low-, and medium-swirl corner flows (such as illustrated above and in LLS97, respectively), studies of corner flows in tornado vortex chambers also identify a very low swirl regime. As the swirl ratio is dropped starting from a low-swirl vortex, the vortex breakdown is observed to occur progressively higher above the surface and its magnitude to progressively weaken [see, e.g., the review of Davies-Jones (1986)]. We exhibit axisymmetric time-averaged contours from a simulation with such a very low swirl corner flow behavior in Fig. 6. There is now no increase in the tangential velocity near the surface because the radial inflow separates from the surface well outside of the upper-core radius. The negative radial pressure gradient induced by the decelerating radial inflow in the surface layer is now large enough to overcome the swirl-induced positive pressure gradient even outside of the upper-core radius (cf. the nearly flat pressure contours near the surface in Fig. 6c). The resulting slight adverse

FIG. 4. The inner corner flow for the low-swirl vortex of Fig. 3. (a) Swirl velocity (solid contours) together with highest levels of total velocity variance (dashed, contour level is 0.5), (b) magnitude of the velocity vector in the r - z plane, (c) perturbation pressure.

pressure gradient is sufficient to force the surface layer flow to separate from the surface outside of r_c as seen in Fig. 6b. This effectively prevents any vortex intensification near the surface. What weak intensification that does occur is located approximately a core radius or more above the surface. About the only signatures

for this intensification remaining in Fig. 6 are the weak central pressure minimum toward the top of the figure and an almost imperceptible increase in core size and drop in peak swirl velocity above this level.

While this example fits the picture of a very low swirl corner flow coming from the laboratory studies, we have achieved it here in quite different fashion than is usually encountered in tornado vortex chambers. We could have taken the conditions of the low-swirl example in Fig. 3 and reduced the swirl ratio of the larger-scale flow by either increasing the overall horizontal convergence, a_c , or decreasing the incoming Γ level, in order to achieve this corner flow behavior. Instead, the simulation of Fig. 6 has the same domain size and imposed Γ and a_c on the inflow boundaries away from the surface as the high-swirl example of Fig. 1; that is, the outer tornado-scale flow would be characterized as having high swirl. The very low swirl corner flow evidenced in Fig. 6 was achieved by including a layer of zero swirl fluid near the surface in the outer inflow boundary conditions. In our previous two examples, frictional losses to the surface from the base of the large-scale vortex produced the low angular momentum fluid in the surface layer. This is not the only possible source of low-swirl fluid, however. The low-swirl fluid in the surface layer can be transported over relatively large distances, with its possible origins including preexisting low angular momentum fluid either initially outside of or below the outer-scale tornado vortex. Such conditions might arise, for example, from any evolutionary process for the mesocyclone that does not create a uniformly swirling flow all the way to the surface. As the example of Fig. 6 illustrates, changes in the flow within tens of meters of the surface can dramatically affect the intensity and structure of the corner flow even if those changes are

FIG. 5. Instantaneous vertical cross section of simulated tornado from Fig. 4 showing normalized absolute swirl velocity (grayscale) and magnitude of the velocity vector in the r - z plane (arrows, interpolated onto a uniform grid for clarity; maximum length corresponds to $V_c/V_c = 3.8$).

FIG. 6. Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for a very low swirl vortex. (a) Swirl velocity (solid contours) together with highest levels of total velocity variance (dashed, contour level is 0.2), (b) magnitude of the velocity vector in the r - z plane, (c) perturbation pressure (dashed) and angular momentum (solid).

imposed many hundreds of meters away from the vortex center.

If, as is sometimes argued, we consider the core flow to be an extension of the surface layer and vortex breakdown to be somewhat analogous to a boundary layer separation (cf. Hall 1972), then the corner flows in Figs. 1, 3, and 6 differ by where the separation of the strong surface layer-core flow occurs relative to the upper-core radius. This flow separates from the surface at a radius that is a sizable fraction of r_c for “high swirl” (Fig. 1), at smaller radius for medium swirl, after turning the corner to form the central core flow for “low swirl” (Fig. 3), and finally well above the surface for “very low swirl” (Fig. 6). It is this sort of continuous change in structure that we would hope to quantitatively parameterize with an appropriately defined corner flow swirl ratio, independent of whether that change was achieved through changing the large-scale swirl conditions, low-level inflow conditions, surface roughness, upper-core structure, etc.

b. Defining a local corner flow swirl ratio

We consider the three most important dimensional parameters affecting the corner flow to be a characteristic angular momentum, mass flux, and upper-core size for the surface layer-core flow, denoted Γ^* , $\mathcal{M}^*$, and r^* , respectively. From these we can form a dimensionless swirl ratio,

$$S_c \equiv \Gamma^* r^* / \mathcal{M}^*. \quad (1)$$

Obviously there is some arbitrariness in defining Γ^* , $\mathcal{M}^*$, and r^* . For our purposes we would like definitions that can be easily and robustly computed from our simulations; other choices might prove more convenient or accessible for laboratory or field measurements. By “robust” we mean that the values measured for these quantities, which should all be measurable outside of the corner flow region itself, should not be overly sensitive to the particular choice of radius or height at which they are measured. We take Γ^* to scale with Γ_∞ , the angular momentum level immediately outside of the surface and core regions. For r^* we use r_c as we defined it above for nondimensionalizing our figures. This choice agrees with the definition of core radius often used for some idealized core profiles such as the Rankine vortex but is by no means unique. Different measures of the core radius are distinct to the extent that the shapes of the Γ profiles across the core differ, a level of detail that can be important (as we will illustrate in section 4) but that is impossible to incorporate into a single parameter. Figure 7 shows profiles of Γ/Γ_∞ and V/V_c in the upper core versus r/r_c from a representative sample of our simulations to illustrate the range of variation encountered. It illustrates as well why we have not chosen to use the radius at which the swirl velocity is a maximum for our definition of core radius, as is often done. In each case shown there exists a broad range of radii over which V is within 80% of V_c ; modest changes in the Γ profile can then shift the location of the absolute peak by more than a factor of 2 in radius.

FIG. 7. Axisymmetric, time-averaged radial profiles in the quasi-cylindrically symmetric upper cores of five sample simulated tornado vortices. (a) Normalized swirl velocity V/V_c , (b) normalized angular momentum Γ/Γ_∞ .

The swirl velocity profile sometimes even has more than one local maximum.

In choosing $\mathcal{M}^*$ we must distinguish between mass flow associated with the surface layer and core versus that in the outer converging swirling flow. Ideally the definition of $\mathcal{M}^*$ should also be based only on conserved or nearly conserved quantities, so that the flux can be measured at some radius before entering, or at some height after exiting, the corner flow region with the result being relatively insensitive to the precise radial or vertical position chosen.

Away from the boundaries both the mean mass flux and the mean angular momentum flux through any region are conserved in a flow that is steady and axisymmetric in the mean. We take the density to be constant within our domain of interest. In the region just outside of the surface and core flows, Γ is nearly constant as well, so that in this region the fluxes are carried together in simple fashion. This is not the case within the surface-corner-core flow; a distinguishing feature of this region is that Γ is necessarily strongly varying. An appropriate combination of the two fluxes, the flux of $\Gamma_d \equiv \Gamma_\infty -$

Γ (“depleted angular momentum”), is then to a good approximation nonvanishing within our domain only within the surface-corner-core flow, thus providing a useful conserved signature for these regions. The conservation equation,

$$\nabla \cdot \langle \mathbf{V} \Gamma_d \rangle = \frac{\partial \langle w \Gamma_d \rangle}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} (r \langle u \Gamma_d \rangle) = 0, \quad (2)$$

can be derived from the azimuthal momentum and continuity equations. Here w and u are the axial and radial velocity components, and the angular brackets denote both a time and an axisymmetric average. The bracketed terms include both the mean and turbulent fluxes of depleted angular momentum. The Reynolds number is assumed sufficiently large that laminar viscous stress terms may be neglected. Note that $\langle w \Gamma_d \rangle$ is not equal to zero at the surface but, rather, equal to a subgrid turbulent flux transported to the surface, which is parameterized in terms of an effective surface roughness, z_0 , in our model. Given (2) we can define a depleted angular momentum streamfunction in analogy with the usual streamfunction for a turbulent flow that is steady and axisymmetric in the mean, such that

$$\langle w \Gamma_d \rangle = \frac{1}{r} \frac{\partial \psi_{\delta\Gamma}}{\partial r}, \quad \langle u \Gamma_d \rangle = -\frac{1}{r} \frac{\partial \psi_{\delta\Gamma}}{\partial z}, \quad (3)$$

with

$$\begin{aligned} \psi_{\delta\Gamma}(r, z) - \psi_{\delta\Gamma}(r_0, z_0) \\ = \int r' (\langle w \Gamma_d \rangle dr' - \langle u \Gamma_d \rangle dz'). \end{aligned} \quad (4)$$

The line integral in (4) can be taken along any curve in the axial plane joining the point at (r, z) with the reference point at (r_0, z_0) . In plotting $\psi_{\delta\Gamma}$ in Figs. 1f and 3c the reference point was chosen on the axis so that $\psi_{\delta\Gamma}(r_0, z_0) = 0$. In the figures the depleted Γ flux streamlines are concentrated within the surface-corner-core flow regions, as advertised; were a larger radial domain shown, one could see some of the streamlines emanating from the surface, representing the loss of angular momentum there. In general, there are two primary “sources” of depleted Γ flux: the frictional loss of angular momentum from the outer swirling flow to the surface, and low Γ fluid from large radius outside or initially below the outer swirling flow. In addition, low Γ fluid from above can be drawn down the vortex core, and angular momentum can be turbulently transported radially out of the upper core, increasing the depleted Γ flux in the upper core.

We now define Y to be the total depleted Γ flux flowing through the corner flow region. Because there is very little angular momentum lost to the wall within the corner region itself (relative to the total), the flux into the corner equals the flux out of the corner to a good approximation; that is,

$$\begin{aligned} Y &\approx 2\pi \int_0^{z_1} -\langle u(r_1, z) \Gamma_d(r_1, z) \rangle r_1 dz \\ &\approx 2\pi \int_0^{r_2} \langle w(r, z_2) \Gamma_d(r, z_2) \rangle r dr, \end{aligned} \quad (5)$$

where r_1 is a radius close to, but outside of, the corner flow, z_1 is a height safely above the surface layer, z_2 is a height just above the corner flow, and r_2 is a radius safely outside of the upper-core region. Thus $Y = 2\pi\psi_{\delta r}$ evaluated immediately outside of the corner flow region. In our simulations, we have confirmed the equality of the last two terms in (5) and their insensitivity to the precise choices of r_1 , z_1 , r_2 , and z_2 , generally to a level of a few percent. Note that Y can be computed even when the time-averaged flow is not exactly axisymmetric, and, in fact, Eqs. (2)–(4) remain valid in this case.²

We can now define an $\mathcal{M}^*$ meeting our requirements if we take it to scale with Y/Γ_∞ . The specific form of the local corner flow swirl ratio (1) that we will use to analyze our simulations is then

$$S_c = r_c \Gamma_\infty^2 / Y. \quad (6)$$

Other possible implementations of (1) may differ from this one in practice to the extent that changes in the shape of the Γ and flux profiles in the core and surface layer are important, a level of secondary detail that we do not attempt to capture with a single parameter. For the high-swirl vortex of Fig. 1, $S_c = 6.9$; for the low-swirl case of Fig. 3, $S_c = 1.2$; and for the very low swirl case of Fig. 6, $S_c = 0.75$. We attribute the change in corner flow behavior in the three cases primarily to the change in depleted Γ flux in the surface layer (cf. Figs. 1f and 3c). For reference, the nontranslating medium-swirl simulation presented in LLS97 has $S_c = 2.6$.

c. Dependence of S_c on selected physical parameters

The S_c in (6) can be interpreted as the ratio of a characteristic swirl velocity in the surface–core flow ($\Gamma_\infty/r_c = V_c$) to a characteristic flow-through velocity ($Y/\Gamma_\infty r_c^2$), justifying its consideration as a swirl ratio. Unlike the swirl ratios commonly used in laboratory studies of vortex breakdown in vortex tubes or of vortex structure in tornado vortex chambers, however, it is by construction specific to the surface–core flow and does not explicitly involve the larger-scale geometry, for ex-

ample, the radius of the vortex chamber. This is useful since the analog of the latter scale is not easily identified in an actual thunderstorm system. On the other hand, while the ingredients of (6) can be measured, r_c and Y , in particular, cannot easily be set directly in any laboratory or numerical experiment. Nor are they completely independent of each other: the impact on S_c of an increase in Y is generally partly offset by the increase in r_c associated with carrying the increased depleted Γ flux up the core.

In section 3 we will examine to what extent the dependence of the corner flow on the velocity fields immediately outside of the corner flow is parameterized by S_c alone. Here we examine first, at least qualitatively, how S_c varies with some selected physical parameters affecting the larger-scale boundary conditions, assuming that each parameter is varied in isolation. First consider the dependence on the swirl ratio of the larger-scale converging swirling plume in which the core flow is embedded. We can define this, following LLS97, as

$$S_{\text{outer}} = \Gamma_\infty / (a_c r_0 h_{\text{inf}}), \quad (7)$$

where the inflow layer has a height h_{inf} , begins at a radius r_0 , has angular momentum Γ_∞ , and average horizontal convergence a_c . Increasing r_0 allows the flow more opportunity to lose angular momentum to the surface, thereby increasing Y and decreasing S_c . Increasing either a_c or h_{inf} will generally decrease the upper-core radius, again reducing S_c . With other conditions fixed, S_c rises or falls with S_{outer} , as one would expect.

On the other hand it is possible to vary S_c over a large swirl range with S_{outer} held fixed. For example, it has been noted by many studies that an increase in surface roughness leads to a “lower-swirl” corner flow behavior (e.g., Leslie 1977). This trend is incorporated into S_c , which provides a quantitative measure for it: as the surface roughness is increased, the angular momentum loss to the surface is increased, with a corresponding increase in Y and drop in S_c . Any source of low angular momentum fluid into the boundary layer flow at large radius also serves to reduce S_c by increasing Y (as seen, e.g., in the case of Fig. 6). Adding a translation velocity to the vortex can have this same effect; we consider such an example in section 4b.

In general, anything that increases the upper-core radius without affecting the surface layer flow outside of the corner flow region will increase S_c . Examples include decreasing the ratio of the inflow height to the domain height where the upper boundary conditions are imposed, adding an upper-level central downdraft, the addition of horizontal divergence in the upper flow, or the imposition of nonswirling flow through the lateral boundaries above some height.

We could, of course, include an arbitrary multiplicative constant in the definition of S_c in (6) but have chosen not to do so. The domain swirl ratio used in laboratory investigations of corner flow (e.g., Church et al. 1979) tends to have smaller values than S_c for

² The conservation of angular momentum is a consequence of the underlying rotational invariance of the Navier–Stokes equations themselves and requires no further assumptions about the symmetry of a particular flow solution. The simple form of Eq. (2) arises from the assumption of a steady mean flow, high Reynolds number, and the fact that torques exerted by pressure gradients and transport by the swirl velocity component both cancel out after performing an azimuthal integration.

similar corner flow behavior, but there is not a one-to-one correspondence between the two quantities in general. They measure different properties of the flow, and S_c is affected by other factors in addition to the domain swirl ratio.

3. Summary of dynamical relationships in the corner flow

In general, varying any physical parameter in the tornado flow will affect not only S_c , but the structure of the velocity profiles in the surface layer and upper core as well. In this section, we explore to what extent the effect on the corner flow of any such change is captured solely through its effect on S_c . In the limited space available in this paper it is not possible to show velocity distributions from all of the simulations performed for this study. Instead, we present here some dimensionless ratios of vortex flow parameters for a relatively large, representative set of simulations. In this dataset we tried to cover a range of different, but realistic, flow conditions occurring immediately outside of the corner flow region. We did not attempt to impose boundary conditions there directly, however. Instead, this was achieved indirectly by varying our far-field boundary conditions well outside of the corner flow region. In essence we used a large part of our simulation domain to provide more realistic turbulent boundary conditions for the corner flow, starting from more idealized steady flow conditions imposed on the outer tornado scale. The range of idealized conditions chosen was intended only as rough approximations to some of the wide variety of conditions expected to exist in real tornadoes on this larger-scale domain. In general, we imposed a region of inflow on the outside lateral boundaries of our simulation with a constant Γ and constant horizontal convergence, a_c . We varied the ratio of the inflow height to the full domain height, as well as the domain size, Γ and a_c . The boundary condition at our domain top was taken as zero slope on v and u with either a uniform outflow velocity across the full domain, or through a disk of fixed size, sometimes including an imposed central downflow (with no swirl) in a disk of given radius. In many of our runs the imposed constant Γ and a_c inflow region extended to the ground, so that the surface layer of depleted Γ flow developed on its own due to surface friction as the flow proceeded radially inward. In others, we imposed an analytic boundary condition on the surface flow, as described in LLS97, and in still others we included a layer of imposed inflow over the surface of fixed height with no swirl at all. The dataset includes an order of magnitude variation in the effective surface roughness and some simulations with a realistic translation velocity. In addition, we have included a simulation with horizontally diverging flow imposed in the upper half of the domain, and another in which the imposed lateral inflow has zero swirl over the top three-quarters of the domain. Obviously we have not inde-

FIG. 8. Summary of the surface intensification of the vortex for a set of 50 simulations as measured by the ratios $V_{\max}/V_c$ (*) and $\sqrt{P_{\min}/P_c}$ (O) vs S_c .

pendently covered the full parameter space in any of these variations; nonetheless, the dataset is large and we can only summarize some results in this work. It is our intent to present some of our more specialized results from this dataset in future work.

Each simulation was performed, and the data analyzed, as described in LLS97. We ran each simulation until quasi-steady conditions were achieved, so that the initial conditions were not critical, and continued to run for long enough to collect a good statistical sample for time averages. We employed a stretched grid spacing in all three dimensions so that we could achieve a fine grid spacing within the corner flow region of immediate interest while still taking our domain boundaries (where we set the inflow and outflow conditions described above) far from the corner flow. For computational efficiency we allowed the flow to spin up on a series of grids, progressing from coarse to fine. In each case, we refined the grid enough so that the average statistics (as well as the qualitative instantaneous flow structures) agreed for the finest and next-to-finest grid simulations. In some cases the finer grid simulations were performed in a smaller computational domain (but still much larger than the corner flow region) with boundary conditions inherited from the coarser simulation. The results presented here are only from the better resolved of our simulations, though the next-best-resolved cases gave results entirely consistent with these. Unless otherwise noted, the averages referred to involve both an axisymmetric and a time average in order to improve the statistics.

In Fig. 8 we show two variables as a function of swirl that provide measures of the surface intensification of the tornado relative to the core flow above. One is the ratio of the maximum average swirl velocity, $V_{\max}$, to the maximum average swirl velocity in the quasi-cylindrical region above the surface boundary layer, V_c . The

FIG. 9. Summary of the two ratios $V_c/\sqrt{\Delta p}$ (*) and $-U_{\min}/V_{\max}$ (○) vs S_c from sample simulations.

FIG. 10. Summary plot of the ratios $W_{\min}/V_{\max}$ (*) and $W_{\max}/V_{\max}$ (○) vs S_c from sample simulations.

other is the ratio of the minimum average pressure, $p_{\min}$, to that in the quasi-cylindrical region above the surface boundary layer, p_c . We show the square root of the pressure ratio to make it more similar to the velocity ratio. There is noticeable scatter, but both ratios show a distinct peaking of the intensification for S_c around 1.2. The peak intensification occurs for low-swirl corner flows such as in Fig. 4, where a sharp vortex breakdown caps a strong central jet just above the surface. For increasing values of S_c , the core size near the surface progressively increases to approach that of the core above as the radial overshoot in the surface layer weakens. For decreasing values of S_c below about 1, the vortex breakdown occurs at greater heights with decreasing changes across it, eventually weakening sufficiently that the term “breakdown” is no longer appropriate. A similar behavior has been observed in laboratory observations (e.g., Snow 1982). The most interesting feature of Fig. 8 is the sharpness of the intensification peak, though there is some inherent uncertainty in both its width and position due to the breadth of variations in boundary conditions we employed in order to vary S_c . Less scatter in the curve would be expected if we were to cover the swirl range by varying only a single physical parameter such as the outer-scale swirl ratio, or the surface roughness. The apparent gaps in the values of S_c represented are purely a selection effect: in generating a range of S_c values indirectly by varying different physical parameters we have not evenly populated the space.

Figure 9 shows the ratio of V_c to the square root of the pressure drop between the inflow and the center of the quasi-cylindrical region, and the ratio of the minimum average radial velocity, $U_{\min}$, to $V_{\max}$. Both of these ratios are remarkably insensitive to the swirl ratio. They show some scatter across the range of variations, but not as much as might be expected considering the range of flow profiles in the plots exhibited in sections 2 and

4. The values of $V_c/\sqrt{\Delta p}$ are scattered about a central value of 0.7, a little above the value of 0.6 associated with the cyclostrophic pressure drop of a Burger–Rott swirl velocity profile (Rott 1958). The near constancy of the ratio $U_{\min}/V_{\max}$ (with value ≈ -0.6) is a reflection of the direct dependence of the swirl overshoot on the overshooting low-level radial inflow. The degree of constancy is, however, surprising. The peak inflow occurs quite close to the surface a little outside the radius of the maximum swirl velocity; in general, the two peaks lie on different streamlines.

The velocity components in the corner flow do not all scale the same way with swirl ratio as Fig. 9 might suggest. Figure 10 shows the ratio of the maximum and minimum average vertical velocities to the maximum average swirl velocity. Not surprisingly, $W_{\max}/V_{\max}$ decreases with increasing swirl. It is not clear how significant the near constancy of this ratio is for $S_c < 1$. It is consistent with what is known about vortex breakdown that all of the low-swirl cases where a breakdown is observed have $W_{\max} > V_{\max}$. The values $W_{\max}/V_{\max} \approx 1.4$ and $V_{\max}/V_c \approx 2.5$ for the low-swirl case are not far from the values of 2 and 1.7 deduced in the model of Fiedler and Rotunno (1986) and the corresponding values of 1.6 and 1.7 from the axisymmetric simulations of Fiedler (1993). The maximum downward velocity in Fig. 10 occurs on the high-swirl side of the peak intensification in Fig. 8 where the bowl or conical nature of the dividing streamline between the high-swirl flow and the recirculating flow is more gradual than the very abrupt divergence in the breakdown shown in Figs. 3–5.

Figures 8–10 demonstrate that much of the influence of the outer-scale variables studied here is captured by our low-level swirl ratio. Figure 11, giving some information on the fluctuations within the flow, suggests that this is not the complete story. Rather than show the rms values of the fluctuations, we show the average of the peak fluctuations at any time: mean peak negative

FIG. 11. Measures of the turbulent intensity as given by the average of instantaneous peak quantities from sample simulations: $p_{\max}^{\text{inst}}/p_{\min}$ (*), $(W_{\max}^{\text{inst}} - W_{\max})/V_{\max}$ (O), $(W_{\min}^{\text{inst}} - W_{\min})/V_{\max}$ (+).

pressure normalized by $p_{\min}$, mean peak positive vertical velocity minus $W_{\max}$ normalized by $V_{\max}$, and mean peak negative vertical velocity minus $W_{\min}$ normalized by $V_{\max}$. None of these combinations shows a clear trend with S_c . They do show that the peak instantaneous values can have significantly greater magnitudes than the mean values, and that the scatter in these values for a given value of S_c is greater than it is for the average variables, a topic we turn to in the next section.

4. Some tornado vortex features not characterized by swirl ratio alone

The behavior of the tornadic corner flow is, of course, governed by the velocity and pressure fields immediately outside of the corner flow region. A single quantity such as S_c , based on integral invariants of these fields, cannot, on its own, completely parameterize this dependence, although it seems to do a good job on the basic behavior of the mean flow. We expect, for example, that the distributions of the mass and angular momentum fluxes (and not just their integrated totals) should also have some effect on the corner flow behavior. We present here a sample of some of these effects.

a. Secondary vortex structure

Figure 12 shows the normalized swirl velocity, angular momentum, and depleted angular momentum streamfunction for a simulation with $S_c = 6.6$, that is, effectively the same value as for the simulation of Fig. 1. The two simulations were performed with the same Γ_{∞} and achieve effectively the same depleted Γ flux in the surface layer and upper-core size as measured by r_c . The flows outside of the corner flow region differ chiefly in the angular momentum gradients in the upper-

core region. In the case of Fig. 12 a weaker horizontal convergence was imposed on the side boundaries than for the case of Fig. 1, but a uniform vertical outflow was imposed on the top boundary instead of a weak central down-flow as for Fig. 1. The result is an upper core with nearly the same r_c , but with much sharper radial Γ gradients in the case with imposed down-flow and stronger convergence. The effect on the mean profiles in the corner region is largely to shift the location of the peak swirl velocity relative to r_c (with the vortex without a central downdraft having the peak at much smaller radius), but without significantly affecting the magnitude of the velocity overshoot (cf. Fig. 8). The effect on the instantaneous flow structure in the corner is more dramatic (as suggested by the pressure results in Fig. 11). Figures 2 and 13 show the instantaneous pressure and vertical velocity contours for the two cases on a horizontal slice through the corner flow. The former case, with the stronger Γ gradients, displays strong secondary vortices as noted earlier. The number of secondaries is not constant in time, and their evolution is often chaotic, but there are always a number of strong secondaries in this case, typically with perturbation pressure deficits of twice that in the upper core of the main vortex. In contrast, Fig. 13 is dominated by the central vortex supported by a lower pressure aloft (in units of V_c^2), with no strong secondary vortices in evidence at this (or other) times. A high-swirl corner flow is necessary to obtain multiple strong secondary vortices but clearly not sufficient. High S_c assures an eruption of flow from the surface layer at relatively large radius, which is a necessary ingredient for multiple secondary vortices, but does not guarantee strong radial gradients in the swirl or vertical velocities, which are also required to produce strong secondaries. Note that the local pressure intensification apparent in the secondary vortices of Fig. 2 is washed out upon taking an axisymmetric average and so does not appear in the measure used in Fig. 8.

b. Translation effects

Superposing a translation velocity (relative to the ground) with an imposed axisymmetric, converging, swirling outer tornado flow has the effect of breaking the mean axisymmetry within the surface layer flow. We perform our simulations in the frame of reference moving at constant velocity with the translating tornado vortex, so that the surface plane is prescribed to move in the opposite direction. The added surface shear stress due to the translation of the surface provides a torque that tends to enhance the angular momentum in the surface layer (defined about the center of the vortex well off the ground) on the side of the vortex where the swirl velocity and surface motion are aligned, and to reduce the angular momentum on the opposite side where they are opposed. This effect is purely due to the surface friction; in the limit of a smooth surface and inviscid

FIG. 12. Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for a simulation with the same S_c as Fig. 1, but without the imposed central downdraft at the domain top and with a lower horizontal convergence. (a) Swirl velocity (solid contours) together with highest levels of total velocity variance (dashed, contour level is 0.1), (b) angular momentum, (c) depleted angular momentum streamfunction.

flow, the uniform translation would have no effect. The surface interaction is most important at larger radii where the relative magnitude of the translation velocity compared to the swirl velocity is greater, as is the lever arm for the torque and the area over which it acts. This asymmetry far from the vortex center is reduced as the

flow spirals in close to the center; near the center the mean flow of a persistent vortex will always tend toward axisymmetry. This is partly achieved in trivial fashion by a shift in the vortex center near the ground as well as by the mixing of the different fluid populations. More importantly, at large radii where the flow is more asymmetric, it is the fluid with lower angular momentum that is preferentially drawn toward the vortex center. Consequently, the depleted Γ flux entering into the corner flow is enhanced over what it would be without the surface translation, driving the flow toward lower corner flow swirl ratio. For example, we have found that the addition of translation to a medium-swirl vortex can drive it to the low-swirl configuration characterized by dramatic low-level intensification and a strong surface jet below a (now asymmetric) vortex breakdown.

The shift of the mean vortex center near the surface by the translation complicates the quantitative application of the formulas of section 2. For simplicity, we consider the high swirl flow of Fig. 1, with an imposed translation velocity of 40% of V_c (cf. LLS97 for a comparison of a medium-swirl vortex with and without translation). In addition to implementing the translation into our surface boundary condition, we incorporated it into the lateral domain boundary conditions by superposing a logarithmic boundary layer aligned with the translation (with depth of 10% of the domain radius) onto the existing axisymmetric outer boundary condition. When, as in this case, the shift in the vortex center induced by the translation is small compared to the upper-core radius, we can safely ignore the shift to lowest order in analyzing the flow. In this particular case, as advertised, the addition of the translation has the effect

FIG. 13. Instantaneous horizontal cross section of simulated tornado from Fig. 12 at a height of $0.2 r_c$, showing normalized perturbation pressure (contours) and vertical velocity (grayscale). The dashed contour indicates $W/V_c = -0.2$.

FIG. 14. Instantaneous horizontal cross section at a height of $0.2 r_c$ of a simulation like that of Fig. 2, but with the vortex translating relative to the ground to the right. Normalized perturbation pressure (contours) and vertical velocity (grayscale). The dashed contour indicates $W/V_c = -0.2$.

of lowering S_c , here from 6.9 to 4.9, due almost entirely to an increase in Y . In addition, the translation adds an asymmetry and increased unsteadiness to the instantaneous flow structures in the corner flow. Figure 14 shows the pressure and w contours on a horizontal slice as in Fig. 2, but for the translating case. While the particular features change in time, the general character does not. In the translating case there are generally fewer secondary vortices lying closer to the center of the main vortex than in the nontranslating case, consistent with the modestly lower swirl ratio. The secondaries are no longer on equal footing with one another: typically one or two are considerably stronger than the rest. In addition, the preferential inflow of lower angular momentum fluid from one side of the large-scale vortex leads to the appearance of inwardly spiraling rolls in the surface layer (just apparent on the right-hand side of Fig. 14).

c. More complex corner flows

The magnitude of the local velocity overshoot in the corner flow is a function of (among other things) the local Γ and radial mass flow. Equation (6) relates S_c to certain averages of these quantities without any detailed knowledge of how the mass flow is distributed among different Γ levels, although this could clearly influence the structure of the corner flow. In particular, this affects the Γ value giving the peak overshooting swirl velocity and allows nested corner flow behaviors on different radial scales as demonstrated by our next example. Fig-

FIG. 15. Normalized, axisymmetric, time-averaged contours in the radial-vertical plane for the simulation discussed in section 4c. (a) Swirl velocity (solid contours) together with highest levels of total velocity variance (dashed, contour level is 0.5), (b) magnitude of the velocity vector in the r - z plane, (c) perturbation pressure (dashed) and angular momentum (solid).

ure 15 shows the normalized axisymmetric, time-averaged velocity, pressure, and Γ fields for a simulation with boundary conditions as in the case of Fig. 6, but with the zero swirl inflow layer at the outer boundary now only half as thick. In the case of Fig. 6 we found the corner flow overshoot to be small and located well off the surface as is characterized by a very low swirl ratio. In the present case, by reducing the thickness of the zero swirl inflow layer at large radius, Y has been decreased and S_c correspondingly increased (to 1.4), so that the low-level overshoot is increased toward its maximal value (cf. Fig. 8). This occurs in an interesting fashion, as shown in Fig. 15. There are now two low-level maxima in the swirl velocity within the corner flow—in effect, nested low-swirl corner flows on different length scales. The larger, outer corner flow has its peak swirl velocity occurring where Γ is about 80% of Γ_∞ ; the inner one, at about 10%. The flat-bottomed vortex breakdown of Fig. 4 is modified here into a conical breakdown. The flow in the breakdown region is highly turbulent (as indicated by the peak turbulent kinetic energy contours in Fig. 15), but is often dominated by a single strong secondary vortex spiraling about the breakdown cone where the velocity gradients are largest.

5. Summary and comments

The qualitative features distinguishing different tornadic corner flows presented here are similar to those exhibited previously in laboratory flows and axisymmetric model simulations by varying the swirl ratio of the large-scale flow. We have added to this qualitative understanding in several ways. First, we have sampled a broader range of boundary conditions for the corner flow, including variations in the surface inflow and upper-core structures, and demonstrated the strong impact they can have on the corner flow structure. Second, we have defined a local corner flow swirl ratio, which incorporates much of the influence of a number of other variables, as well as the outer swirl ratio, on the surface intensification of the tornado. Third, we were able to make the influence of this swirl ratio on the dynamical structure of the flow more quantitative than was previously possible. And finally, we have exhibited the character of the relatively coherent turbulence for different corner flows, which often contributes directly to the greatest damage to structures on the surface. We have not addressed the important question of tornado-genesis on the larger scale, that is, what processes give rise to the converging swirling plume on the few-kilometer scale that is required for a tornado's existence?

The local corner flow swirl ratio, S_c , highlights the role of low angular momentum fluid in the near-surface layer which, upon exiting the boundary layer, forms much of the vertical core flow. It is the radial inertial overshoot of this fluid in the corner flow that provides the chief source of the intensification of swirl velocity and pressure deficit in the mean tornado vortex near the

surface. In defining S_c we have utilized only physical properties of the flow field outside of the corner flow that can be robustly measured, for example, whose values are relatively insensitive to the precise radial or vertical position at which they are measured. This new swirl parameter varies directly with the more familiar outer swirl ratio but depends on other parameters as well. It is reduced by anything that increases the surface layer inflow of low angular momentum fluid, such as increases in the effective surface roughness, or the tornado translation speed, or the presence of low-swirl fluid initially below or outside of the larger-scale circulation. It is generally increased by anything that increases the upper-core radius without changing the surface layer inflow, such as the presence of an upper-level central downdraft, or the addition of horizontal divergence in the upper flow. We have found that the effects of such changes on the mean flow structure of the tornadic corner flow are largely quantitatively parameterized by how they change the value of S_c . On the other hand, the magnitude of the outer swirl ratio, S_{outer} , does not by itself categorize the structure of the corner flow if other physical parameters are varied. Flows with the same outer swirl ratio can exhibit quite different corner flow structure.

It should be emphasized that S_c and S_{outer} measure different physical properties of the tornado flow; the former neither replaces nor redefines the latter. While each may be interpreted as the ratio of a characteristic swirl velocity to a characteristic flow through velocity, they are defined on different scales and have different utility. The ratio S_{outer} is a property of the swirling, converging tornado vortex as a whole. As such it represents one of the ingredients (but not the only one) determining the surface layer and upper-core properties of the flow and hence the corner flow structure. The definition of S_{outer} in an open flow necessarily depends on the extent to which the radius and height of the swirling, converging flow are unambiguously defined. The ratio S_c , on the other hand, is a property of the surface layer-core flow embedded within the larger-scale vortex. As such, it can more completely determine the corner flow structure than S_{outer} does but is dependent on, rather than partially determining, the upper-core and surface layer properties outside the corner flow. The S_c is well defined for an unbounded flow as long as there is a strong vortex interacting with a surface.

In section 3, some potentially useful relationships between important tornado dynamical structure parameters were presented as a function of S_c . As S_c decreases, the mean low-level vortex intensity rises to a maximal level; further decrease forces a transition from a very intense low-level tornado vortex to a lower swirl situation with little or no vortex intensification near the surface. Thus the enhancement of low-level, low angular momentum flow may either increase or decrease the tornado intensity depending upon whether it forces the flow toward or below the low-swirl peak intensification point. Mean

swirl velocities close to the surface reach more than 2.5 times the maximum mean swirl velocities aloft under conditions of peak intensification. At very low values of S_c , what little vortex intensification that does occur, occurs well off the surface. This raises the possibility that differences in the near-surface layer inflow may be one of the critical factors determining when or whether an existing kilometer-scale circulation can complete the spinup to a tornado. For a fixed circulation, the intensity of near-surface velocities may not reach levels recognizable as a tornado on the ground if the local corner flow swirl ratio is either too large or too small.

The nature of the coherent, but turbulent, eddy structure is quite different in the various regimes of S_c . Under conditions of peak intensification, there is an unsteadiness in the form of wandering of the narrow central stem of the vortex and in the height of the abrupt breakdown, but most of the large-scale random turbulence remains downstream of the breakdown. At high swirl, the eddies tend to be in the form of secondary vortices rotating around the main vortex in the region of the maximum radial gradients of the vertical and swirl velocities. These eddies are intensified by factors, such as the presence of an upper-level central downdraft, which tend to increase the maximum radial gradients of vertical and/or swirl velocity. In such cases the local intensification within these secondary vortices can be quite large, so that the question of whether low- or high-swirl-type corner flows are the most damaging does not have a simple answer. While S_c provides a useful first-order classification of corner flows, it must be emphasized that, at the level of quantitatively examining the turbulent structure, there is more than just a one-parameter family of possible corner flow behaviors.

To some lowest-order level of approximation, we have reduced the question of the dependence of the corner flow structure on a given physical variable to the dependence of S_c on that variable. We have quantitatively studied the dependence of the corner flow on S_c and qualitatively described the dependence of S_c on a variety of physical variables. A quantitative parameterization of this latter step remains to be presented and is a good candidate for future work.

In this work we have concentrated on the tornado-scale corner flow. It may prove fruitful to apply these results, particularly the definition of S_c , to mesocyclone-scale corner flows in cases where the two corner flows are distinct, for example, when the tornado behaves like a secondary vortex of a high swirl mesocyclone. In recent work, Wakimoto and Liu (1998) have examined the traditional outer-scale swirl ratio for the Garden City, Kansas, mesocyclone in an attempt to understand tornadogenesis in that storm. We expect that the near-surface layer flow will be important in determining the structure of the mesocyclone-scale corner flow, so S_c might prove to be a more useful measure. While it is problematic to obtain radar measurements of the surface layer flow, what we consider the property of the surface

layer flow of greatest importance in determining the corner flow structure—namely, the total flux of low angular momentum fluid—can, as we have seen, be measured outside of the surface layer, above the corner flow region.

We should note again that all of the flow patterns exhibited here involve quasi-steady distributions for the particular set of boundary conditions. Accordingly, the applicability of our conditions to real tornadoes is subject to the qualification that the relatively small scale phenomena in the corner flow must remain essentially determined by quasi-steady dynamics as the unsteady outer flow develops. The extent to which the interaction of the tornado with the surface varies, how the local corner flow swirl ratio evolves in time, and whether temporal overshoots are important on some scale, we leave for future work.

Acknowledgments. This research was supported by National Science Foundation Grant 9317599 with S. P. Nelson as technical monitor. We also wish to acknowledge support from the Army Research Office (Grant DAAH04-96-1-0196) in developing the subgrid-scale parameterization incorporating rotational damping given in the appendix, and useful input from the anonymous reviewers.

APPENDIX

Including Some Rotational Damping Effects in an LES Subgrid Model

In a large eddy simulation (LES) one explicitly simulates turbulent eddies large enough to be resolved on the computational grid, and models the effects of eddies that are smaller. If, in a given problem, one can resolve the scales of eddies most important for turbulent transport, then the most important role of the subgrid model is simply to provide an energy sink so that energy does not pile up on the grid scale. In such cases the simulation results should be insensitive to the details of the subgrid model. Sometimes, however, there are regions in a simulation where the larger-scale turbulence is suppressed for some physical reason, and the subgrid contribution becomes relatively more important. Two familiar examples are near a wall, where the scale of the most important eddies is limited by the distance to the wall, and in a stable temperature gradient where the turbulence is damped. One must ensure in these cases that the subgrid model is not overly dissipative, that is, that any physics suppressing the resolved turbulence also suppresses the turbulence represented by the subgrid model. In this appendix we sketch how to include the most important effects of rotational damping within our subgrid model, in analogy with how we have treated the damping in a stratified flow in our LES studies of the planetary boundary layer.

In our subgrid model we carry the subgrid turbulent

kinetic energy as a dynamical variable with evolution equation ($q^2 = 2 \times \text{TKE}$):

$$dq^2/dt + d(u_i q^2)/dx_i = -2\tau_{ij} du_i/dx_j + d(Kdq^2/dx_i)/dx_i - q^3/(4\Lambda), \quad (\text{A1})$$

where

$$\tau_{ij} = -\nu(du_i/dx_j + du_j/dx_i) + \delta_{ij}q^2/3$$

$$K = q\Lambda/3, \quad \nu = q\Lambda/4 \quad \text{and}$$

$$\Lambda = \min(c_1 \max(\Delta x, \Delta y, \Delta z), 0.65z, 0.5q/N, 0.5q/\xi). \quad (\text{A2})$$

A sum over repeated indices is assumed. The diagnostic variable Λ should be thought of (up to a constant factor) as the characteristic size of the eddies comprising the subgrid-scale turbulence. In general, we assume that the subgrid turbulence cascades down from the resolved turbulence so that its scale is related to the grid spacing, giving the first term in (A2) (where we typically take $c_1 = 0.25$). The next term in (A2) represents the reduction in eddy length scale as the distance z to the wall becomes small. Note that in this level of approximation the change in scale has been imposed isotropically (there is only a single scale Λ), even though it is only the vertical scale that is directly suppressed due to the proximity of the wall. This is an appropriate treatment within an LES: it is in directions where the resolved scale turbulence is suppressed that the subgrid diffusion might become competitive and hence should be modeled most accurately. In directions where the resolved turbulence is not suppressed, the resolved turbulent transport should dominate the subgrid, so underestimating the latter is of less concern.

The last two terms in (A2) limit the subgrid turbulence length scale in the presence of strong stratification or rotation, respectively. We have patterned the treatment of the latter after the former so we consider the simpler stratified case first. In a region of uniform increasing vertical temperature gradient, dT/dz , a parcel released initially from its equilibrium position with velocity q can (assuming no mixing and the Boussinesq approximation) travel to at most a height $\lambda = q/N$, where $N = \sqrt{(g/T_o)(dT/dz)}$ is the Brunt–Väisälä frequency, before its kinetic energy is all converted into potential energy. An eddy of a given energy in a given temperature gradient is thus limited in vertical extent. This fixes the form of the stratification limit in (A2). The coefficient depends on the “shape” of the eddy, its alignment with the gradient, etc., as well as the constant of proportionality between this scale λ and the subgrid turbulence length scale, Λ , as it appears in (A1). We choose the coefficient to match the theoretical critical Richardson number of 0.25 in stratified shear flow.

In a rotating fluid the angular momentum Γ of a small fluid parcel is conserved as it is displaced radially (again assuming no mixing), so that the displacement changes the rotational potential energy in analogy to the change

in gravitational potential energy of the vertically displaced parcel in the stratified flow. Given a center of rotation about $r = 0$, a parcel released initially from its equilibrium position with small velocity q directed radially outward in a region of stable rotation gradient, $\Gamma' \equiv d\Gamma/dr > 0$, can travel at most a distance $\lambda = q/\zeta$, where

$$\zeta = \sqrt{2\Gamma'/r^3}. \quad (\text{A3})$$

Assuming the same relation between λ and Λ as in the stratified case, this would give the last limit on Λ in (A2) with $\xi = \zeta$. The rotational damping constraint on the scale is not so easy to implement, however, because in general the center and orientation of the axis of rotation in the flow are unknown a priori; indeed there can be many local centers of rotation that can change position in time (cf. Fig. 2). For ξ , then, we need a local combination of fields that is coordinate invariant, is frame independent (under rotations and Galilean transformations), reproduces (A3) for a purely rotating flow, and does not lead to a strong constraint on Λ for flows (such as uniform shear) without strong streamline curvature. The following choice (written here in Cartesian coordinates for ease of application) fits these conditions:

$$\xi = 2\nabla p \cdot \frac{d\mathbf{V}}{dx_i} \left(\nabla p \cdot \frac{d\mathbf{V}}{dx_i} - \nabla p \cdot \nabla \mathbf{V}_i \right) / (\nabla p \cdot \nabla p), \quad (\text{A4})$$

where $\mathbf{V}$ is the velocity vector, p the pressure, and repeated indices are summed. The combination given in (A4) is not unique in fulfilling the required conditions but among the simplest and most local to implement in a finite difference scheme. We note in passing that the stabilizing effects of strong radial temperature gradients within a rotating flow can be included in the same level of approximation by adding the term $-\nabla T \cdot \nabla p/T_o$ to the right-hand side of (A4).

In this implementation of damping effects through the subgrid length scale there is an important feedback mechanism resulting in the subgrid turbulence level being dynamically adjusted to a large degree. In a region of stable rotation gradient, Λ is reduced according to (A2). This causes a reduction of q^2 as given in (A1) that, in turn, further reduces Λ via the q^2 dependence in (A2). Because of this feedback, the effects of employing this subgrid model within our LES has proved to be relatively insensitive to the value of the coefficient in front of the q/ξ term in (A2); when the rotational damping is strong the subgrid diffusion is reduced to where its precise level is unimportant.

We have attempted only to include the first-order effects of rotational damping on the subgrid model in this treatment. We could, for example, also include modifications to the q^2 production terms in (A1) due to the rotation (as we have included subgrid buoyancy production in the treatment of stratification in our boundary layer model). Nonetheless, as implemented it proves beneficial in our tornado simulations: the subgrid tur-

bulence length scale is reduced where it should be, in the main tornado core, and in the cores of the rotating secondary vortices. The corresponding reduction in subgrid diffusion helps to preserve the integrity of the latter, but its biggest practical benefit within our simulations is in reducing the dependence of the upper-core size on the grid spacing there. The turbulent transport of angular momentum radially out of the upper core is a contributing factor in determining the upper-core size. Without the rotational term in (A2), Λ was set by the grid spacing in the upper core and the subgrid transport was unphysically large for coarser grid spacing.

REFERENCES

- Burggraf, O. R., and M. R. Foster, 1977: Continuation of breakdown in tornado-like vortices. *J. Fluid Mech.*, **80**, 685–703.
- , K. Stewartson, and R. Belcher, 1971: Boundary layer induced by a potential vortex. *Phys. Fluids*, **14**, 1821–1833.
- Church, C. R., J. T. Snow, G. L. Baker, and E. M. Agee, 1979: Characteristics of tornado-like vortices as a function of swirl ratio: A laboratory investigation. *J. Atmos. Sci.*, **36**, 1755–1776.
- Davies-Jones, R. P., 1973: The dependence of core radius on swirl ratio in a tornado simulator. *J. Atmos. Sci.*, **30**, 1427–1430.
- , 1986: Tornado dynamics. *Thunderstorms: A Social and Technological Documentary*, E. Kessler, Ed., Vol. II, University of Oklahoma, 197–236.
- Fiedler, B. H., 1993: Numerical simulation of axisymmetric tornadogenesis in forced convection. *The Tornado: Its Structure, Dynamics, Prediction, and Hazards*, C. Church et al., Eds., Amer. Geophys. Union, 41–48.
- , and R. Rotunno, 1986: A theory for the maximum windspeed in tornado-like vortices. *J. Atmos. Sci.*, **43**, 2328–2340.
- Hall, M. G., 1972: Vortex breakdown. *Annu. Rev. Fluid Mech.*, **4**, 195–218.
- Leslie, F. W., 1977: Surface roughness effects on suction vortex formation: A laboratory simulation. *J. Atmos. Sci.*, **34**, 1022–1027.
- Lewellen, W. S., 1962: A solution for three-dimensional vortex flows with strong circulation. *J. Fluid Mech.*, **14**, 420–432.
- , 1993: Tornado vortex theory. *The Tornado: Its Structure, Dynamics, Prediction, and Hazards*, C. Church et al., Eds., Amer. Geophys. Union, 19–40.
- , and Y. P. Sheng, 1980: Modeling tornado dynamics. Tech. Rep. NTIS NUREG/CR-2585, U.S. Nuclear Regulatory Commission, 227 pp. [Available from NTIS, Sills Building, 5285 Port Royal Rd., Springfield, VA 22161.]
- , D. C. Lewellen, and R. I. Sykes, 1997: Large-eddy simulation of a tornado's interaction with the surface. *J. Atmos. Sci.*, **54**, 581–605.
- Long, R. R., 1958: Vortex motion in a viscous fluid. *J. Meteor.*, **15**, 108–112.
- Rott, N., 1958: On the viscous core of a line vortex. *Z. Math. Phys.*, **9**, 543–553.
- Shtern, V., and F. Hussain, 1993: Hysteresis in a swirling jet as a model tornado. *Phys. Fluids A*, **5**, 2183–2195.
- Snow, J. T., 1982: A review of recent advances in tornado vortex dynamics. *Rev. Geophys. Space Phys.*, **20**, 953–964.
- Wakimoto, R. M., and C. Liu, 1998: The Garden City, Kansas, storm during VORTEX 95. Part II: The wall cloud and tornado. *Mon. Wea. Rev.*, **126**, 393–408.
- Ward, N. B., 1972: The exploration of certain features of tornado dynamics using a laboratory model. *J. Atmos. Sci.*, **29**, 1149–1204.
- Wilson, T., and R. Rotunno, 1986: Numerical simulation of a laminar end-wall vortex and boundary layer. *Phys. Fluids*, **29**, 3993–4005.

Appendix B

Nonaxisymmetric, Unsteady Tornadic Corner Flows

D.C. Lewellen, W. S. Lewellen, and J. Xia

Proceedings of the AMS 20th Conference on Severe Local Storms
Orlando, FL, 11-15 September 2000, pp. 265-268

D. C. Lewellen*, W. S. Lewellen, and J. Xia
West Virginia University, Morgantown, WV.

1. INTRODUCTION

The corner flow is that region of a swirling flow where the surface layer inflow turns up into the vortex core. For tornadoes this region is of critical interest for at least two reasons, one simple, the other more profound: first, since we live just above the surface the damage potential in this region is of greatest interest, and second, due to the interaction of the vortex with the surface, the corner flow is generally the site of the highest velocities, shears, and pressure drops in the entire flow. In previous work, Lewellen *et al.* (2000) (referred to as LLX hereafter), we used large-eddy simulations to study the corner flow for quasi-steady conditions, generally with axisymmetric boundary conditions. Here we present some preliminary results of ongoing work aimed in part to extend that study to nonaxisymmetric, time dependent boundary conditions, as well as larger scale (mesocyclone) corner flows, in order to address tornadogenesis and variability. After a brief review of basic corner flow dynamics in section 2, we present examples of time transient behavior in section 3 (leading at times to dramatic increases in tornado intensification near the surface) and some effects of translation in section 4 (leading to more modest increases in intensification). The simulations are fully 3D and unsteady, and employ a stretched grid to simultaneously allow fine grid resolution in the corner flow region while imposing boundary conditions far from the corner flow. Details of the numerical model used and the simulation procedures employed can be found in LLX.

2. BASIC CORNER FLOW BEHAVIOR

Outside of the core in the surface layer underneath a swirling flow, the magnitude of the pressure is to a good approximation inherited from that in the flow above since normal pressure gradients tend to be small near a surface. This implies a strong radial pressure gradient since this is required to balance the strongly swirling flow. On the other hand, the swirl velocity near the surface necessarily drops to zero at least in

a thin layer due to surface friction and perhaps in a deeper layer depending on the time development of the vortex. The resulting deviation from cyclostrophic balance in the surface layer drives a strong radial inflow which can overshoot its equilibrium point in the corner flow before turning up into the core flow. This radial overshoot brings angular momentum levels to smaller radii than elsewhere in the flow leading to larger swirl velocities and lower pressures.

Figure 1: Summary of the near surface intensification of the vortex for a set of 50 simulations as a function of corner flow swirl ratio S_c .

The structure of the corner flow, in particular the degree of overshoot, is critically dependent on the magnitude of the flux of low swirl fluid in the surface layer, what we called the “depleted angular momentum flux” in LLX, as well as on the upper core size, r_c , and the angular momentum level in the outer flow, Γ_∞ . In LLX we formed a corner flow swirl ratio from these quantities (S_c , which can be interpreted as the ratio of a characteristic swirl velocity to a characteristic flow-through velocity for the surface-layer/corner/core flow) which successfully parameterizes the most basic corner flow variations induced by changing a variety of physical variables. Figure 1, from LLX, shows two measures of near surface intensification: the ratio of peak time

* Corresponding author address: D. C. Lewellen, MAE Dept.
PO Box 6106, WVU, Morgantown, WV, 26506-6106; e-mail:
dlewelle@wvu.edu.

averaged swirl velocity to maximum upper core velocity (well off the surface), and the square root of the analogous ratio for pressure drop. The largest intensification is for a critical value of corner flow swirl ratio, $S_c^* \approx 1.2$, for which the radial overshoot reaches the axis, turning into an intense vertical velocity jet off the surface capped by a vortex breakdown. Below S_c^* the unfavorable pressure gradient induced by the stagnating inflow forces the surface flow to turn upwards even outside of the upper core radius; above S_c^* the radial overshoot is only a fraction of the core radius, the centrifugal barrier preventing further radial penetration.

3. TEMPORAL OVERSHOOTS

One of the most important features of fig. 1 is the narrowness of the low swirl intensification peak. It would seem that conditions must be rather finely tuned to sit on this peak in a quasi-steady state. On the other hand, if the boundary conditions on the vortex change in time, then it is easy to sweep through a range of corner flow behaviors including the low swirl peak. The possibility of temporal overshoots (non-steady state behavior) also arises. Figure 2 shows relative intensification results as a function of time for two such cases. For simplicity and to increase the possibilities for temporal overshoots we begin in each case with a quasi-steady vortex, abruptly make a change in the boundary conditions, and then follow the simulations until they reach their new quasi-steady states. In case 1 we start with a very low swirl corner flow ($S_c=.8$)[†] achieved by including lower swirl inflow through the side boundaries in a layer above the surface. This layer of inflow was abruptly shut off in case 1, producing conditions which eventually lead to a high swirl quasi-steady state ($S_c=12$). This end point was used as the starting point for case 2, where the low swirl inflow layer was now abruptly reinstated. The two cases are thus in some sense time reversals of each other, but the differences between them are dramatic. Case 1 includes a period of near surface intensification far greater than anything found for the quasi-steady state cases of fig. 1 (even accounting for the time averaging in fig. 1), while case 2 does not. It should be noted that the square root of the pressure drop near the domain top varies less than 20% from the initial value ($\sqrt{p^*}$ used for normalization in the figure) for case 1 for $t < 1.4t^*$, before rising by a factor of 1.7 for the high swirl end condition.

[†]This is computed using the r_c found at mid-levels in the simulation domain. If we consider the corner flow on a larger scale, using r_c near the domain top (above an apparent upper level vortex breakdown) then we find $S_c=1.6$, a low swirl corner flow consistent with the intensification factors relative to the upper domain seen initially for case 1 in fig. 2.

Figure 2: Relative intensification as a function of time for two cases described in the text. p_{min} is the minimum pressure drop within the domain; p^* the average minimum pressure drop near the top of the domain for the low swirl quasi-steady state; and $t^* = r_d^2/\Gamma_\infty$ represents a characteristic time scale for the low swirl surface layer flow to be exhausted after the lateral boundary condition at radius r_d is changed.

We have performed related simulations restoring swirl to the low level inflow rather than shutting off the low swirl inflow, or varying the low-level inflow profile, top boundary conditions, or grid resolution, and have even switched the boundary conditions sequentially (beginning at one corner of the domain, working its way around in time). The depth of the intensification spike in fig. 2 and details of its shape vary with these changes, but the qualitative behavior encountered appears to be robust. At the very low swirl starting point the lowest pressures and highest velocities are well above the surface. This remains the case until the low swirl fluid near the surface has been mostly removed by the strong convergence, whereupon the lowest pressure region (as well as the vortex breakdown lying above it) descends. The peak intensification (fig. 3) occurs when the (now conical) breakdown lies just above the surface capped by a narrow central downdraft. This point is followed rapidly by this downdraft reaching the surface, opening the core somewhat (to produce a “medium swirl” corner flow configuration). The intensification level has dropped sharply at this point but still greatly exceeds the levels of fig. 1. In the final stages a much wider downdraft descends to the surface, generally pushing the surface level vor-

Figure 3: Vertical cross section of azimuthally averaged fields in the corner flow region for case 1 at time of peak pressure drop. Solid line: angular momentum fraction (relative to Γ_∞); dashed: pressure normalized by minimum average pressure drop near domain top at the given time.

text to the side rather than opening up its core. What was the primary vortex on the surface now appears as a secondary vortex rotating about a much larger core. It weakens in the process, leaving finally a high swirl corner flow with the updraft in a large annulus above the surface and with very weak multiple secondary vortices in evidence. The final stages of the once intense low level vortex are at least qualitatively suggestive of the “roping out” stage of tornado evolution.

A full range of corner flow configurations are also identifiable in the “reverse” simulation, case 2, again with the peak intensification point occurring for a low swirl configuration with breakdown just above the surface (fig. 4). There is a crucial difference however. In case 1 we have low swirl fluid preceding the higher swirl fluid through the corner flow, pulling the latter in to smaller radii in its wake to increase intensification (a temporal overshoot). In case 2 high swirl fluid precedes lower swirl fluid into the corner, impeding the radial overshoot of the latter.

4. TRANSLATION EFFECTS

Figures 5 and 6 illustrate some of the changes induced by adding a large (30ms^{-1}) surface translation to a high swirl vortex (S_c initially 6.9). The case is the

Figure 4: As in fig. 3 but for case 2. Note the order of magnitude change in pressure contour interval.

same as that described in LLX except a more modest 15ms^{-1} translation was applied there. The translation has three principle effects: shifting the center of the vortex at the surface relative to that above, reducing the corner flow swirl ratio by effectively increasing the surface depleted angular momentum flux into the core, and breaking the axisymmetry seen by the secondary vortices revolving about the main vortex. The latter two effects lead to fewer secondary vortices, with some decidedly stronger than others; for the strong translation illustrated there is effectively only one on the surface. Note how far from axisymmetric the near surface vortex becomes; this possibility should be kept in mind when trying to interpret near surface pressure traces or Doppler wind measurements in terms of idealized axisymmetric profiles. In related simulations we have added sufficient translation to a “medium swirl” ($S_c = 2.6$) vortex to effectively drive it to S_c^* , giving a large surface intensification and a vortex with a sharp bend between the vortex on the surface and that above.

Perhaps more surprising is that the addition of surface translation to a very low swirl vortex (S_c below S_c^*) can also lead to intensification (both in the steady state and through temporal overshoots). In contrast to the high swirl case, the translation now effectively *adds* angular momentum in the near surface layer (about the now shifted vortex center on the surface), raising S_c , and increasing the near surface vortex intensification. That the introduction of surface translation tends to drive S_c towards S_c^* from either the high or low swirl side, suggests that maybe the conditions necessary to

Figure 5: Cross section (400m×400m) at 26m height of high swirl corner flow showing perturbation pressure contours (500 Pa intervals) and horizontal velocity arrows (interpolated to a uniform 10m grid for clarity, with maximum length representing 74ms^{-1}).

be near S_c^* in a quasi-steady state do not require as much fine tuning as it would at first seem.

5. CONCLUDING REMARKS

In an ongoing study of nonaxisymmetric and/or time varying corner flows we have found different mechanisms for enhancing the near surface tornado intensification relative to the vortex strength far from the surface. The introduction of a large surface translation speed can lead to an increase by shifting the corner flow swirl ratio towards S_c^* , that value which gives the greatest intensification for quasi-steady state conditions. More significantly, a change in the near surface inflow conditions which allows the flow to evolve from a very low swirl corner flow ($S_c < S_c^*$) to a high swirl corner flow gives rise to a temporal overshoot producing dramatically larger near surface intensification factors. Both results reinforce one of our main conclusions in LLX: the critical role that the near surface inflow plays in tornado vortex intensification.

These results on corner flow structure should be relevant for both the tornado scale and mesocyclone scale corner flows, impacting tornado structure evolution in the former case and tornadogenesis in the latter. The changes in near surface flow conditions could arise from changes in surface topography, roughness, translation speed, or, probably most importantly, from the impact of other flow features such as a rear

Figure 6: As in fig. 5 but with the addition of a 30ms^{-1} surface translation (max arrow length = 82ms^{-1}).

flank downdraft wrapping around and cutting off the low swirl inflow near the surface. This latter scenario presents an attractive possibility for one type of tornadogenesis. A high swirl (referring to the vortex as a whole) mesocyclone which does not extend to the surface early in its evolution, overlies a layer of low swirl fluid, thus giving rise to a very low swirl mesocyclone corner flow. This gives little or no recognizable surface vortex but, crucially, provides strong low level central convergence. If a downdraft wrapping around this system either cuts off much of the low swirl inflow, or introduces angular momentum to the surface layer itself, then an intense, though transient, vortex on the surface can naturally arise. For example dimensioning the simulation of case 1 for a reasonable mesocyclone scale ($\Gamma_\infty = 4 \cdot 10^4 \text{m}^2 \text{s}^{-1}$, $r_d = 5\text{km}$), produces near surface swirl velocities exceeding 60ms^{-1} (9min after switching the boundary conditions), which last for 6min and reach a peak of 100ms^{-1} .

6. ACKNOWLEDGEMENTS

This research was supported by the National Science Foundation with S.P. Nelson as technical monitor.

References

- Lewellen, D. C., W. S. Lewellen, and J. Xia, 2000: The influence of a local swirl ratio on tornado intensification near the surface. *J. Atmos. Sci.*, **57**, 527–544.

JIANJUN XIA

- Education**
- Ph.D.** in Mechanical Engineering, December 2001
West Virginia University — Morgantown, WV, USA
Dissertation: “*Large-Eddy Simulation of a Three-Dimensional Compressible Tornado Vortex*”
Advisors: Dr. William Steve Lewellen, and Dr. David C. Lewellen
- M.E.** in Mechanical Engineering, March 1996
Zhejiang University — Hangzhou, P. R. China
Thesis: “*Numerical Simulation and Model Test on the Combustion Process in a Large-Scale Power Plant Furnace*”
Advisor: Professor Luo-Chan Shen
- B.E.** in Mechanical Engineering, July 1993
Zhejiang University — Hangzhou, P. R. China
- Experience**
- Fluent Incorporate** — Lebanon, NH, USA
Lead Test Engineer (January 2001 to present)
- West Virginia University** — Morgantown, WV, USA
Graduate Research Assistant (May 1997 to January 2001)
- Bao Steel Power Plant** — Shanghai, P. R. China
Engineer (March 1996 to November 1996)
- Zhejiang University** — Hangzhou, P. R. China
Graduate Research Assistant (September 1993 to March 1996)
Tutor (September 1993 to July 1995)
- Publications**
- Lewellen, D. C., W. S. Lewellen, and J. Xia (2000): “Nonaxisymmetric, unsteady tornadic corner flows”, in *Proceedings of the AMS 20th Conference on Severe Local Storms*, Orlando, FL, 11-15 September, 265-268.
- Lewellen, D. C., W. S. Lewellen, and J. Xia (2000): “The influence of a local swirl ratio on tornado intensification near the surface”, *Journal of the Atmospheric Sciences*, 57 (4), 527-44.
- Chi, Z. H., L. C. Shen, J. Xia, H. Zhou, and X. Jiang (1997): “Test and numerical simulation of multiphase dynamics in utility boiler with concentric firing system”, *Journal of Combustion Science and Technology*, 3(2): 206-214.
- Shen, L. C., J. Xia, Q. Yao, K. F. Cen, and Z. H. Chi (1996): “Zone method for numerical heat-transfer in combustion chamber of large power plant”, *Journal of Zhejiang University*, 30(1): 7-17.

- Zhu, Z. J., D. L. Zhang, J. Xia, and D. W. Yang (1996): "Research into characteristics of gas-solid multiphase flow in furnace with anti-tangential secondary air", *Journal of Shanghai Institute of Electric Power*, 12 (3): 16-21.
- Chi, Z. H., J. Xia, Z. J. Zhu, W. G. Pan, Z. L. Zhu, L. C. Shen, and K. F. Cen (1995): "Numerical simulation and aerodynamic experiment in large-scale tangential fired boiler with anti-tangential secondary air", in *Proceedings of the 9th Annual Conference of the Chinese Society of Thermal-Physics Engineering*, Yichang, P. R. China, October, No. 954105, V68-75.
- Xia, J., L. C. Shen, Z. H. Chi, Q. A. Yao, Z. J. Zhu, and K. F. Cen (1995): "Projection method for numerical simulation in the furnace of 600 MW utility boiler", in *Proceedings of the 3rd International Symposium on Coal Combustion*, Beijing, P. R. China, September, 329-335.
- Shen, L. C., and J. Xia (1994): "Applying Monte Carlo methods to simulate 3D unsteady conduction equation", in *Proceedings of the 5th Annual Conference of the Chinese College Society of Thermal-Physics Engineering*, Dalian, P. R. China, October, 599-602.