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Forced vibration and hygrothermal analysis of composite laminated beams under the action of moving loads

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**Forced Vibration and Hygrothermal Analysis of Composite Laminated
Beams under the Action of Moving Loads**

Praveen Kumar Kavipurapu

**Thesis submitted to the
College of Engineering and Mineral Resources
at West Virginia University
in partial fulfillment of the requirements
for the degree of**

**Master of Science
in
Mechanical Engineering**

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**Morgantown, West Virginia
2005**

**Keywords: Composite Beams, Hygrothermal, Moving Loads, Finite Element
Method.**

ABSTRACT

Forced Vibration and Hygrothermal Analysis of Composite Laminated Beams under the Action of Moving Loads

Praveen Kumar Kavipurapu

The dynamic response of simply supported glass/epoxy composite beams subjected to moving loads in a hygrothermal environment is carried out using the general purpose finite element program *ANSYS*. The moving load model and the hygrothermal incorporation are validated separately by reproducing results available in the literature. The hygrothermal effect is introduced by using empirical relations for degrading the material stiffness properties of the matrix. The corresponding ply properties are calculated using the rule of mixtures. A parametric study is conducted by varying the fiber volume fraction and the fiber orientation of the angle plies in the laminate. The results indicate that the dynamic magnification factors generally increase due to hygrothermal effects but all the laminates considered do not behave in a similar manner. Thus it is possible to minimize the environmental effect by judiciously selecting the laminate configuration.

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TABLE OF CONTENTS

ABSTRACT	ii
ACKNOWLEDGMENTS	iii
LIST OF SYMBOLS	vi
LIST OF TABLES	vii
LIST OF FIGURES	ix
1 INTRODUCTION	1
1.1 Problem statement	1
1.2 Laminated composites	1
1.3 Literature review	2
1.4 Need for present study	8
1.5 Objectives	9
1.6 Thesis overview	9
2 THEORETICAL FORMULATION	10
2.1 Introduction	10
2.2 Composite Lay-up Configuration	10
2.2.1 Force and Moment Resultants	11
2.2.2 Composite Plate Theories	12
2.3 Hygrothermal Analysis	13
2.4 Governing Equations for Cylindrical Bending of Plates	16
3 FINITE ELEMENT MODELING	19
3.1 Introduction	19
3.2 Overview of ANSYS	19
3.2.1 Modeling in ANSYS of present problem	20
3.3 Boundary Conditions and Meshing	23
3.3.1 Meshing	23
3.4 Modal Analysis	24
3.4.1 Subspace method of mode extraction	25
3.5 Transient Dynamic Analysis	25
4 RESULTS AND DISCUSSION	27
4.1 Introduction	27
4.2 Isotropic Beam Comparison	27
4.2.1 Static Central Deflection of Simply Supported Isotropic Beam	28
4.2.2 Moving Force on a Simply Supported Isotropic Beam	28
4.3 Composite Beam subjected to Moving Load	33
4.4 Hygrothermal Analysis on Composite Laminated Beams	39

4.4.1 Dry Conditions	40
4.4.1.1 Transverse Deflections of Pinned-Pinned Laminated Composite Beam under Uniform Transverse Load	40
4.4.1.2 Transverse Deflections of Clamped-Clamped Laminated Composite Beam under Uniform Transverse Load	43
4.4.1.3 Transverse Deflections of Pinned-Pinned Laminated Composite Beam under In-Plane Loads	44
4.4.2 Hygrothermal Conditions	46
4.5 Comparison of Hygrothermal Deflections with that of Dry Conditions for Laminate under Cylindrical Bending	50
4.6 Composite Beam subjected to Moving Loads under Hygrothermal Environment	53
4.6.1 Static Analysis	54
4.6.2 Dynamic Analysis	56
4.6.3 Dynamic Magnification Factors	57
5 CONCLUSIONS AND RECOMMENDATIONS	61
5.1 Contributions	61
5.2 Conclusions	61
5.3 Recommendations	62
REFERENCES	63

LIST OF SYMBOLS

a	Half span of laminate (in.)
$[A]$	Axial stiffness matrix (lb/in.)
$[B]$	Bending-extension stiffness matrix (lb)
$[D]$	Bending stiffness matrix (lb-in.)
E	Modulus of elasticity (lb/in. ²)
$\{F^e\}$	External element force vector
h	Height of laminate (in.)
I	Moment of inertia (in. ⁴)
$[K]$	Global stiffness matrix (lb/in.)
$\{K^e\}$	Element stiffness matrix (lb/in.)
L	Length of beam (in.)
M_x, M_y, M_{xy}	In plane moment resultants in xy plane (lb)
m	Moisture content (%)
n	Number of layers
N_x, N_y, N_{xy}	In plane force resultants in xy plane (lb/in.)
P	Vertical static load (lb)
$\{q^e\}$	Nodal values of generalized displacement (in.)
q	Uniform transverse load (lb/in. ²)
t	Time (s)
t_k	Thickness of k^{th} layer (in.)
u	Displacement in the x direction (in.)
V_f	Fiber volume fraction
w	Transverse deflection (in.)
x, y	In-plane coordinates
z	Thickness coordinate
z_k	Distance to the top of the k^{th} layer from the mid-plane of a laminate (in.)
$\bar{z}_k$	Distance to mid surface of the k^{th} layer from the mid-plane of a laminate (in.)
δ_s	Static deflection of the beam (in.)
δ_d	Dynamic deflection of the beam (in.)
$\emptyset$	Relative humidity
ρ	Mass density

LIST OF TABLES

4.1	Static mid-span deflection of a simply supported beam	28
4.2	Moving load velocities and time steps	29
4.3	Maximum centre deflection for various moving load velocities	30
4.4	Dynamic magnification factors for moving force on isotropic beam	30
4.5	Geometric and material properties of composite beam	33
4.6	Dynamic magnification factors for moving load on composite beam	35
4.7	Constituent properties of S glass fiber and Epoxy matrix	40
4.8	Parameters in hygrothermal analysis	40
4.9	Transverse deflections for pinned-pinned laminate due to transverse load in dry condition	41
4.10	Transverse deflections for clamped-clamped laminate due to uniform transverse load in dry condition	43
4.11	Transverse deflections for pinned-pinned laminate due to in-plane load in dry condition	44
4.12	Transverse deflections for pinned-pinned laminate due to transverse load for hygrothermal condition	46
4.13	Transverse deflections for clamped-clamped laminate due to uniform transverse load in hygrothermal condition	47
4.14	Transverse deflections for pinned-pinned laminate due to in-plane load in hygrothermal condition	49
4.15	Dry and wet condition mid-span deflections of pinned laminate under transverse load	51
4.16	Dry and wet condition mid-span deflections of clamped laminate under transverse load	52
4.17	Dry and wet condition mid-span deflections of pinned laminate under in-plane load	53

4.18	Material properties for fiber volume fraction of 0.52	54
4.19	Material properties for fiber volume fraction of 0.6	55
4.20	Material properties for fiber volume fraction of 0.7	55
4.21	Static mid-span deflection of beams made of various laminate configurations	56
4.22	Dynamic mid-span deflections of beams made of various laminate configurations	57
4.23	Dynamic magnification factors for different lay-ups and V_f	58

LIST OF FIGURES

2.1	Simply supported beam subjected to a moving load	10
2.2	Composite lay-up configuration	11
2.3	In-Plane forces and moment resultants	12
2.4	Deformation of transverse normal for CLPT, FSDT, HSDT	13
2.5	Laminate subjected to in-plane load, N_x	17
2.6	Laminate subjected to uniform pressure load, q	17
3.1	Simply supported beam subjected to moving load	20
3.2	Beam 3, 2-D Elastic Beam	21
3.3	Solid 191 Element	22
3.4	Shell 99 Element	22
4.1	Dynamic magnification factor of an isotropic beam	31
4.2	Time history mid span deflection for velocity $V = 51.16$ ft/s	31
4.3	Time history mid span deflection for velocity $V = 102.33$ ft/s	32
4.4	Time history mid span deflection for velocity $V = 204.66$ ft/s	32
4.5	Time history mid span deflection for velocity $V = 820.16$ ft/s	33
4.6	Dynamic magnification factor for composite laminated beam	35
4.7	Time history mid span deflection for velocity $V = 51.16$ ft/s	36
4.8	Time history mid span deflection for velocity $V = 102.33$ ft/s	36
4.9	Time history mid span deflection for velocity $V = 204.66$ ft/s	37
4.10	Time history mid span deflection for velocity $V = 307.08$ ft/s	37
4.11	Time history mid span deflection for velocity $V = 409.06$ ft/s	38
4.12	Time history mid span deflection for velocity $V = 511.75$ ft/s	38
4.13	Time history mid span deflection for velocity $V = 820.16$ ft/s	39
4.14	Pinned-pinned boundary conditions on composite laminate	41
4.15	Deflections for pinned-pinned laminate for uniform transverse load in dry condition	42
4.16	Transverse deflection distribution for 0.0348 psi	42

4.17	Deflections of clamped-clamped laminate for uniform transverse load in dry condition	43
4.18	Transverse deflection distribution for 0.0363 psi	44
4.19	Transverse deflections of pinned-pinned laminate under different tensile normal in-plane load in dry condition	45
4.20	Transverse deflection distribution for 3.426 lb/in.	45
4.21	Transverse deflections of pinned-pinned laminate for different uniform transverse loads in hygrothermal condition	46
4.22	Transverse deflection distribution for 0.0348 psi	47
4.23	Transverse deflections of clamped-clamped laminate due to uniform transverse loads in hygrothermal condition	48
4.24	Transverse deflection distribution for 0.0363 psi	48
4.25	Transverse deflections for pinned-pinned laminates for different in-plane loads in hygrothermal condition	49
4.26	Transverse deflection distribution for 3.426 lb/in.	50
4.27	Dry and wet condition mid span deflections of pinned-pinned laminate under transverse loads	51
4.28	Dry and wet condition mid span deflections of clamped-clamped laminate under transverse loads	52
4.29	Dry and wet condition mid span deflections of pinned-pinned laminate under in-plane loads	53
4.30	Dynamic magnification factor for 0/90/-15/15 laminate	58
4.31	Dynamic magnification factor for 0/90/-30/30 laminate	59
4.32	Dynamic magnification factor for 0/90/-45/45 laminate	59

1 INTRODUCTION

1.1 Problem Statement

The use of composite materials in engineering applications is increasing rapidly. Composite materials have higher strength-to-weight and stiffness-to-weight ratios than metals and find many applications such as composite bridge decks. Therefore, it is very important to understand the response of composite bridges to vehicle-induced vibrations. The strength, stiffness and eventually the life of composite bridges are affected more than conventional materials by the presence of moisture and temperature. Thus it is necessary to analyze the response of composite bridge decks under a hygrothermal environment.

1.2 Laminated Composites

A composite lamina consists of two or more distinct materials, combined at the macroscopic level, to attain desired properties that could not be achieved by either of the constituent materials alone. Composite materials can be divided into fibrous composites, particulate composites and laminated composites.

A laminate is a stack of lamina, with different fiber orientations, bonded together to attain desired properties. Laminates can be classified as symmetric, unsymmetric, balanced and unbalanced composites. In a symmetric laminate, the material and orientation of layers above the mid plane are identical to those below.

In a symmetric laminate the bending-stretching coupling is absent. That means, due to application of in-plane loads, bending and twisting curvatures will not be generated. This is not true for unsymmetric laminates.

The analysis of composite structures is more complex and several theories have been proposed for the analysis of laminated composites. Composite laminates have larger planar dimensions, so they can be treated as plate elements. Therefore, plate theories can be applied in the analysis of laminated composites. Classical Laminate theory (CLT) and First Order Shear Deformation Theory (FSDT) are the two most commonly used theories for the analysis [Reddy (1997)]. The plate theories can be systematically reduced to analyze beams.

1.3 Literature Review

The dynamic response of the beams subjected to moving loads has been of interest from the early 1900's. The goal of these studies has always been to design and build safe, economical, and durable bridges. In general the stresses and deflections produced due to moving loads are higher than that of stationary loads. Therefore, proper estimate of the dynamic effect of moving loads is very important to evaluate the load-carrying capacity of a beam. In the modern era, isotropic beams are replaced by light-weight and high-strength composite materials.

The dynamic response of bridge structures depends on many factors, which include vehicle configuration, pavement roughness and bridge parameters [(Cantieni, (1983); Chan and O' Connor, (1990)]. Much research has been done to find the influence of various factors on the response of bridges. Various computer models have been developed in simulating vehicle-bridge interaction [(Chang and Lee, (1994) and Fafard et al., (1998)]. Bridge models in these studies range from simple to very complex finite element simulations. The vehicle models encompass the spectrum of a single moving

force to complex ones with multidegree of freedom systems with suspensions, dashpots, and a concentrated mass representing the body of the vehicle.

Gonzalez and Brien (2001) have studied the dynamics of weight in motion considering the bridge and truck dynamics. In their work, they have investigated the parameters that affect the response of the bridge when a truck passes over it. The various parameters considered in this study are the road roughness, suspension type, and vehicle speed. In their simple case of a one-dimensional model, the bridge is a simply supported beam and the truck is composed of a body, suspension system and tires. The body of the vehicle is represented by a concentrated mass, while a mass element and a linear spring represent the cushioning effect of the tire.

In the same study, a moving load of constant magnitude acting on a beam has been considered. In this case, the mass of the load is neglected whereas the distributed mass of the beam is taken into account. The deflection of the beam is expressed in terms of modal coordinates and natural modes. LaGrange's equations are used to derive the equations of motion for each modal coordinate. It was found from this study that the only parameter that affects the bridge response is the vehicle speed.

Pesterev et al. (2001) have considered the vibration of a beam subjected to a constant moving force. They formulate simple tools to calculate the maximum deflection of the beam for any given velocity of the moving force. It is shown that there exists a unique response-velocity dependence function, which satisfies a particular boundary function.

A unique amplitude-velocity dependence function is formulated for simply supported and clamped-clamped beams. These unique functions are used to calculate the

maximum beam response without complex computations. The response of the beam is approximated by means of the first natural mode. The response is also calculated by including higher modes. These responses are compared with each other and the error range is less than one percent. Therefore, it is concluded from this study that the first fundamental mode alone is sufficient for finding the maximum deflection of a beam when subjected to a moving force.

O'Connor and Chan (1988) have defined some methods to predict the structural dynamic response of bridges subjected to moving loads. These methods are often referred to as predictive analysis problems. In this type of analysis two beam models are used to predict the response of a structure: one is a discrete beam model and other a continuous beam model. In the discrete beam model, the bridge is modeled as an assembly of lumped masses connected by massless beam elements. From the dynamic response equations, the nodal displacements and bending moments are calculated. In the continuous beam model, an Euler-Bernoulli beam is taken into consideration. In this beam model, the effects of shear deformation and rotary inertia are neglected. The beam has a constant cross section and mass per unit length with a linear viscous damping. The load is assumed to move from one end to the other with constant velocity. By using modal superposition, the beam deflection is found at any instant of time and distance.

Chan et al. (1998) have proposed an interpretive method to identify dynamic moving loads based on the response of the bridge due to these loads. Basically, this method is the inverse of the predictive analysis problem stated by O'Connor and Chan (1988). Euler's equations are used to identify the moving force. Bending moments and displacements of the bridge are calculated using strain gauges and transducers. These

responses are then used to identify constant and time varying moving forces. It is concluded that the identification using bending moments gives accurate results compared with that using displacements.

Saadeghvaziri (1993) has illustrated how a finite element package can be used to evaluate the dynamic transient effect of a moving load traversing a highway bridge. This is accomplished by using the load arrival time option in *ADINA*. The structure is assumed to be made of isotropic beam elements. The fundamental period of the bridge and the slowest velocity are taken into account to determine the number of elements. The weight of the moving load is neglected compared to that of the bridge and the velocity is assumed to be constant. The ratio of the time to travel the length of one element to the period of the highest mode is taken to be very small. This helps in avoiding unrealistic dynamic amplifications. The beam is assumed to be simply supported and subjected to a moving load of constant magnitude which travels from one end of the beam to the other. By using the load arrival time option in *ADINA*, the mid-span deflection and bending moments are determined. These results are compared and verified with exact analytical solutions.

A report submitted to the Virginia Transportation Research Council by Martin et al. (2000) describes the effect of design parameters on the dynamic response of a bridge. The commercial code *ANSYS* is used to simulate the moving load effect on the bridge structure. The effect of the damping is neglected in this analysis. The bridge is modeled using three dimensional isotropic beam elements. This beam model has the same dimensions and properties as the analytical model. The static and dynamic analysis results are compared with the analytical solution. This comparison validates the model

and the method of load representation in *ANSYS*. The stiffness and mass of the beam, loading frequency, and vehicle speed are varied to determine the dynamic response. These results are compared with the corresponding static load cases to gauge the relative effects of these parameters on dynamic response.

Taheri (1987) has studied the dynamic response of plates to moving loads. In his work, he develops numerical procedures for the study of dynamic interactions between moving vehicles and supporting guide way systems. Two computer programs PLATEFEM and PLATEIMP have been developed to analyze the responses of a simply supported isotropic beam and plate due to a moving vehicle. The accuracy of the code is verified by finding the dynamic magnification factors at the centre of the beam/plate and comparing with analytical solutions. A parametric study is conducted to find the effects of different parameters on the dynamic response of plates. These parameters include damping of the foundation, velocity of the moving load, and thickness of the plate. The following conclusions are drawn from this study. The shape of the deflection curve of a plate produced by a moving vehicle is quite different from that due to a static load. This difference is more significant for a moving mass solution.

Kadivar and Mohebpour (1997) have studied the dynamic response of an unsymmetric orthotropic laminated beam when subjected to moving loads. A one-dimensional element with 24 degrees of freedom is considered for the analysis. The effects of shear deformation and rotary inertia are also included. Orthotropic cross-ply and unsymmetric angle ply laminated beams are analyzed and the results are compared with those of an isotropic beam. Classical lamination theory, first order shear deformation theory, and higher-order shear deformation theory are used in the analysis of these

laminated beams. It is concluded that shear deformation and inertia effects can be neglected in the analysis of isotropic beams; but, for composite beams these effects play a significant role in the dynamic analysis.

Tita, Carvalho, and Lirani (2003) have experimentally studied the dynamic analysis of fiber reinforced composite beams. Beams with different lay ups have been manufactured using hand lay up and molding processes. Dynamic tests have been conducted to analyze the influence of fiber orientation on natural frequencies and modal damping. The experimental results are compared with that from a finite element analysis.

Pipes et al. (1976) have studied the hygrothermal response of laminated composite systems. A plate element is used to analyze the laminae stresses resulting from hygrothermal and mechanical loading. A six-ply graphite/epoxy laminate is considered for the analysis of hygrothermal effects. The effective moisture bending and in plane force resultants are developed and combined with thermal loads.

Shen and Springer (1976) have studied the effects of moisture and temperature on ultimate strength of Thorne 300/Fiberite 1034 epoxy composite. Eight-ply T300/1034 specimens have been placed in environmental chambers in which temperature and relative humidity have been controlled and kept constant. The ultimate tensile strength has been measured with material temperatures and moisture contents ranging from 200°K to 422°K and 0% to 1.5%, respectively. It is found that the ultimate strengths of 0° and 45° laminates have negligible change due to variation of temperature and moisture. But, in the case of 90° laminates, the reduction in strength is as high as 60 to 90%.

Upadhyay and Lyons (2000) have studied the effect of hygrothermal conditions on PMC laminates. The hygrothermal conditions are incorporated by changing the

stiffness coefficients of the laminate. Empirical relations between the moisture content and temperature level are given by Chamis (1983). Large deflection theory is adopted in calculating the deflections and the results are compared to that of a linear theory. Pinned-Pinned, clamped-clamped laminates are considered in this study. Two types of loads, namely, in-plane and uniform transverse load are applied on these laminates. It is found that the presence of moisture and temperature in a laminate significantly affects the deflections caused by the loads. It is also observed that the linear theory yields high differences between dry and wet results.

1.4 Need for Present Research

The use of composite materials in the modern engineering applications has been increasing rapidly. Bridges, aerospace structures are few examples of their application. Steel bridges are replaced by composite materials due to their superior qualities like higher strength-to-weight ratio. Bridge structures are constantly being exposed to various types of loads. The major loads that influence the life of a bridge are dynamic moving loads, hygrothermal loads and wind loads.

The study of stiffness degradation of composite structures due to hygrothermal environment is crucial in evaluating the service life of the structure. In reality, bridges are under constant influence of moving and hygrothermal loads. From the survey of the related literature, it is evident that the feasibility of using a commercial code such as *ANSYS* for modeling the effect of moving loads on composites has not been demonstrated. Further the combined effects of moving loads and hygrothermal exposure has not been studied. The present research attempts to fill these voids.

1.5 Objectives

- To simulate the effect of a moving load on simply supported isotropic and composite beams on *ANSYS* and to calculate dynamic deflection factors of the beam under various velocities of the moving load.
- To conduct a hygrothermal analysis of beams made of PMC laminates under the action of moving loads; to compare the transverse deflections of these beams under various boundary conditions for dry and wet environments.
- To conduct parametric studies involving the moving load analysis on laminated beams in dry and hygrothermal conditions.

1.6 Thesis Overview

Chapter Two explains the theory behind the present study. It deals with composite lay-up configurations and various theories that govern the composite behaviors. The formulation of governing equations and the equations that *ANSYS* solves in the analysis are discussed.

In chapter Three the finite element modeling of the present problem in *ANSYS* is detailed. The types of analysis used are outlined.

Chapter Four presents the results obtained in the present study. The graphs obtained from Ansys program are attached here.

Chapter Five has the conclusions of the present study and recommendations for the future work.

2 THEORETICAL FORMULATION

2.1 Introduction

This chapter explains the theoretical formulation that governs the dynamic response of a composite beam subjected to a moving load. The governing equations for the hygrothermal effect on the laminated composites are also explained here. The main theories in the modeling of composite laminates are also described in detail. In modern industry, heavy traditional materials are being substituted with light and strong composite materials. Bridges and railroads made of composite laminates are considerably affected by moving loads, temperature, and moisture. Therefore, it is very important to analyze these effects which would find practical applications in engineering designs.

Fig 2.1 Simply supported beam subjected to a moving load

2.2 Composite Lay-up Configuration

Figure 2.2 shows the lay-up configuration and the normal conventions that are used to describe a composite laminate. It consists of N layers numbered 1 to N from the bottom to the top of laminate. The total thickness of the laminate is represented by h . The

lateral coordinate z is measured from a reference plane located at the mid-surface of the laminate. The quantity z_k represents the distance of the top of the k^{th} layer from the reference plane. The quantity $\bar{z}_k$ represents the z coordinate of the middle surface of the k^{th} layer.

Figure 2.2 Composite lay-up configuration

2.2.1 Force and Moment Resultants

Figure 2.3 shows the positive directions of force and moment resultants acting on a plate. The N_x , N_y , N_{xy} are the in plane force resultants and M_x , M_y , M_{xy} are the moment resultants, and V_x , V_y , the transverse shear force resultants.

Figure 2.3 In-plane forces and moment resultants (Barbero, 1998)

2.2.2 Composite Plate Theories

There are several theories currently in use for the composite laminated plates [Reddy (1997)]. The simplest one, the classical laminated plate theory (CLPT) is based on following assumptions.

- 1) The transverse normals before deformation remain straight after deformation also.
- 2) The transverse normals do not experience any elongation ($\epsilon_{zz} = 0$).
- 3) The transverse normals rotate such that they remain perpendicular to the mid-surface after deformation. ($\epsilon_{xz} = 0$ and $\epsilon_{yz} = 0$).
- 4) The laminates are bonded perfectly.
- 5) The top and bottom surfaces of the plate do not experience any transverse shear stresses.

Composites have a very low transverse shear modulus compared to their in-plane moduli. Therefore, the CLPT may not be sufficient for the dynamic analysis of composite plates. To account for the shear deformation, the first order shear deformation theory (FSDT) can be adopted for the analysis. The FSDT assumes that the transverse normals

are straight but not perpendicular to the mid surface after deformation. Thus transverse shear strains are constant through the cross section. Since the actual shear stress is not constant, FSDT uses a shear correction factor. For plates and beams of rectangular cross section, the shear correction factor is 5/6. The third order theory is also based on the same assumptions as that of FSDT, except that the assumption on the straightness of the transverse normal after deformation is relaxed. The transverse normal is no longer inextensible, making the deformations as a function of the thickness coordinate. Figure 2.4 shows the deformation of transverse normals for CLPT, FSDT and HSDT.

Figure 2.4 Deformation of transverse normal for CLPT, FSDT and HSDT [Reddy (1997)]

2.3 Hygrothermal Analysis

The use of fiber reinforced matrix composites in engineering applications has been tremendously increasing. Therefore, it is very important to study the response of

these materials to environmental conditions like temperature and moisture. The performance of these materials degrades as a result of hygrothermal exposure. Due to moisture and temperature exposure, properties such as the stiffness, strength, life, modulus and conductivities of a composite material decrease considerably. Therefore, experimental and analytical methods have to be carried out to study their responses.

Shen and Springer (1976) have defined maximum moisture content (M_m) as the moisture level reached in a material when exposed to long periods of a constant temperature and moisture environment. The maximum moisture content of a material exposed to humid air can be approximated by the equation given by Springer, $M_m = a\phi^b$, where ϕ is relative humidity, and a and b are constants. In reality, the moisture content greatly depends on temperature. The glass transition temperature (T_g) is defined as the temperature at which the glassy form of a polymer changes to rubbery form.

Prolonged exposure to a hygrothermal environment has two major effects on a polymer matrix composite structure. The first one is of a residual nature. The coefficient of thermal expansion is usually higher for the matrix. Polymers absorb moisture readily whereas fibers are hardly affected by a wet environment. Further the absorption of moisture by the matrix is more pronounced at higher temperatures. An increase in the ambient temperature and/or humidity level causes the matrix to expand; contrarily, reduced temperature and/or moisture level of the surrounding environment results in a contraction of the matrix. Since the fibers are relatively immune to changes in temperature and moisture content, the differential expansion/contraction leads to residual stresses in a PMC lamina. A comparable phenomenon takes place at the laminate level due to the anisotropic nature of the laminate coefficient of thermal expansion and

coefficient of hygroscopic expansion. These residual stresses may starkly reduce the load carrying capacity of a structure made of a polymer matrix composite. Upadhyay and Lyons (2000 a) consider the effect of hygrothermal residual stresses on the large deflection of plates.

The second significant effect of an increased hygrothermal condition is the degradation of the mechanical properties of a PMC ply. Again it is the matrix that bears the brunt of this degradation while the fiber properties remain nearly unchanged. The degradation occurs in both the stiffness and strength values of the matrix and consequently in the corresponding values of the lamina. In the present research only this degradation effect is considered while ignoring the residual stresses.

It is observed from various studies conducted by Shen and Springer (1976, 1977) that the exposure to high moisture and temperature do not significantly affect the fiber related properties, but, the matrix properties get affected considerably. Therefore, the loss in performance can be related to temperature and moisture content of the matrix material. In this study, the relations given by Chamis (1983) are used to calculate the degraded properties of composite due to hygrothermal conditions. It is assumed that only matrix properties are affected by the raise in temperature and moisture content. According to Chamis, the relationship between the wet resin and dry resin mechanical properties can be stated as

$$\frac{P_{HTM}}{P_0} = \left[\frac{T_{gwr} - T}{T_{gdr} - T_0} \right]^{0.5} \quad (2.1)$$

where P is the property to be measured, HTM stands for hygrothermal mechanical, T_{gwr} is glass transition temperature of wet resin and T_{gdr} is the glass transition temperature of

dry resin. T is the temperature at which property to be measured and T_0 is the room temperature. This is a steady state equation that relates the properties of the matrix for dry and wet conditions at particular values of moisture content and temperature.

The relation between T_{gwr} and T_{gdr} in terms of moisture content is given by Chamis and is stated as

$$T_{gwr} = (0.005 m^2 - 0.1m + 1.0) T_{gdr} \quad (2.2)$$

where m is the moisture content expressed in weight percent ($m \leq 10\%$).

By using the rule of mixtures and changed properties of the matrix, elastic constants of the composite are calculated. According to the rule of mixtures, the values of longitudinal modulus, E_1 , transverse modulus, E_2 , and in plane shear modulus, G_{12} , are calculated as follows:

$$E_1 = E_f V_f + E_m V_m \quad (2.3)$$

$$E_2 = \frac{E_m E_f}{E_f V_f + E_m V_m} \quad (2.4)$$

$$G_{12} = \frac{G_m}{V_m + G_m V_f / G_f} \quad (2.5)$$

where E_f , E_m , V_f , V_m , G_f , and G_m are modulus of elasticity of fiber, modulus of elasticity of matrix, fiber volume fraction, matrix volume fraction, fiber in plane modulus and matrix in plane modulus, respectively.

2.4 Governing Equations for Cylindrical Bending of Plates

Cylindrical bending of plates is considered to validate the hygrothermal approach. Figures 2.5 and 2.6 shows a laminate of length L , thickness h subjected to in-plane tensile load N_x and uniform transverse load q , respectively. The equations 2.6, 2.7 and 2.8 are

referred from Upadhyay and Lyons (2000) and are formulated for transverse and in-plane loadings for pinned-pinned and clamped-clamped boundary conditions.

Figure 2.5 Laminate subjected to tensile in-plane load, N_x

Figure 2.6 Laminate subjected to uniform transverse load, q

For an asymmetric cross-plyed laminate subjected to only in-plane loading ($q=0$) and with simply supported boundary conditions along its edges ($x=\pm a$). According to classical linear theory, the equation for transverse deflection $w(x)$ is written as,

$$w(x) = \frac{1}{2} \frac{B_{11} N_x}{(D_{11} A_{11} - B_{11}^2)} (x^2 - a^2) \quad (2.6)$$

where A_{11} , B_{11} , and D_{11} are in-plane stiffness, extension-bending coupling coefficient and bending stiffness of the laminate, respectively.

Consider a laminate with coordinate axis at the mid-plane and z axis normal to the mid-plane. The laminate is simply supported at the edges $x = \pm a$. The bending moment and the in-plane displacement are zero at $x = \pm a$. For a uniform transverse loading, the classical linear theory yields the transverse deflection as,

$$w(x) = \frac{q_0}{24} (x^4 - a^4) - \left\{ \frac{3D_{11}A_{11} - 2B_{11}^2}{12D_{11}A_{11}} \right\} q_0 a^2 (x^2 - a^2) \quad (2.7)$$

Similarly, the transverse deflection of a clamped laminate subjected to transverse load can be stated as,

$$w(x) = \frac{q_0 a^2}{24} (a^2 - 2x^2) + \frac{q_0 x^4}{24} \quad (2.8)$$

3 FINITE ELEMENT MODELING

3.1 Introduction

The finite element analysis is a computer assisted numerical technique useful in solving for the response of a structure subjected to loading. In this technique, a mathematical/computer model of the structure is created and analyzed for deformations and stresses. The model is divided into small blocks called elements. These elements are connected by nodes at which the finite element boundary conditions are applied. A set of algebraic equations are created for each element and combined simultaneous equations are solved. A post processor to the finite element program may be helpful in displaying the stresses and deflections in the form of contours for easy comprehension. A number of finite element packages are available for the analysis of complex structures under various types of loads. A finite element package is chosen depending on the complexity of the model and the results needed.

3.2 Overview of ANSYS

The *ANSYS* is a versatile finite element package which can be used to analyze structural, thermal, electrical, and contact problems. The *ANSYS* graphical user interface (GUI) is divided into three modules, namely, preprocessor (/PREP7), solution (/SOLU) and post processor (/POST1). The preprocessor step consists of selecting the proper elements, providing real constants, material properties, and meshing the model; loads and displacement boundary conditions can also be applied in this module. Alternatively, the boundary conditions may be applied in the solution module, whose primary role is to solve for the desired quantities. The results of the solution are displayed using the general

post processor. Contour plots of stresses, deformations, nodal solution, and element solution can be carried out in this module.

The *ANSYS* solution module provides for a number of options such as static, transient, harmonic, modal, buckling and sub structuring analyses. In this thesis, only static, modal and transient analyses are carried out.

3.2.1 Modeling in *ANSYS* of the present problem

As noted earlier, the preprocessor is used to define suitable elements, real constants for those elements, material properties and to model the geometry. In the present analysis, Beam3, a two-dimensional beam element is used to model the isotropic beam. At each node, this beam has three degrees of freedom, namely, UX, UY and ROTZ. This is a uniaxial element which can account for axial and bending deformations. The beam is defined by two nodes, I and J, area of cross section, moment of inertia and beam height. As shown in figure 3.1, in the present study, the beam is modeled with twenty elements, simply supported at the two ends. At one end, the beam is constrained in the global X and Y directions and at the other end it is restricted only in the global Y direction. The material properties of the beam are entered in the form of modulus of rigidity, Poisson's ratio and density. Figure 3.2 describes the nodal coordinates of the BEAM 3 element.

Figure 3.1 Simply Supported beam subjected to a moving load

Figure 3.2 Beam3, 2-D Elastic Beam (ANSYS 7.0 Documentation)

Modeling of a beam made of composite materials is a more difficult task. Special attention has to be paid in defining the material properties, orientations of the layers and the element coordinate systems. For the transient analysis of a composite beam under a moving load, the Solid 191 element is used in this study. The Shell99 element is used for the analysis of asymmetric composite laminates under dry and wet conditions. The Solid191 element is a twenty-node layered structural solid element. This element has three degrees of freedom at each node, namely, translation in the X , Y , Z directions. This element can take a maximum of 100 layers and a user input option is available if more than 100 layers have to be modeled. An alternative type of element, the Shell element can be used to model composite beams. Unlike shell elements, solid elements can account for interlaminar shear stresses. Figure 3.3 shows description of a Solid 191 element.

x_0 = Element x-axis if ESYS is not supplied.

x = Element x-axis if ESYS is supplied.

Figure 3.3 Solid 191 Element (ANSYS 7.0 Documentation)

For, side-to-thickness ratio of ten or greater, the Shell99 element can be used. The Shell99 is an eight noded 3- D shell element that can take maximum of 250 equal thickness layers. This element has six degrees of freedom at each node.

x_{IJ} = Element x-axis if ESYS is not supplied.

x = Element x-axis if ESYS is supplied.

LN = Layer Number

NL = Total Number of Layers

Figure 3.4 Shell 99 Element (ANSYS 7.0 Documentation)

Figure 3.3 illustrates the typical configuration of the Shell99 element. The global and element coordinate systems are explained in this figure. Mid side nodes cannot be removed from this element. A triangular element can be formed by defining the same node numbers for three nodes as shown in the same figure. In this thesis work, Beam 3 element is used to model isotropic beam, whereas, Solid191 and Shell 99 elements are used to model the composite materials.

3.3 Boundary Conditions and Meshing

Boundary Conditions are the constraints and loads that can simulate the effect of the environment surrounding a body. Loads are applied in the form of forces, moments, pressures, and temperatures. The displacement constraints are applied by restricting the degrees of freedom at the corresponding nodes of a model. The proper specification of the boundary conditions is critical in a finite element analysis. Since, improper application of boundary conditions can create problems such as increased stiffness, rigid body motion, and high local stresses. In the present research, simply supported and clamped-clamped beams are used for analysis.

3.3.1 Meshing

Meshing is the process of dividing the model into elements, which are connected by nodes at the element boundaries. Meshing is one of the important steps in successfully performing a finite element analysis. The type and density of the mesh determines the economy and accuracy of the solution. Therefore, an analyst should use valid assumptions in determining the exact meshing parameters. A coarse mesh can lead

to inaccurate solutions and a very fine mesh may increase the time of analysis. In order to define a suitable mesh, an analyst should have an idea of the parameter distributions within the model. Also a convergence study needs to be carried out by increasing the number of elements.

Meshing can be done in two ways, free and map meshing. In free meshing, the elements that are created are of no definite shape and pattern. In contrast to this, a mapped mesh produces a definite element shape and follows a particular pattern.

3.4. Modal Analysis

The main objective of any modal analysis is to make sure the structure is not subject to resonant frequency under the range of operation. Natural frequency is the frequency at which the structure vibrates if the forcing function is identically zero. The lowest natural frequency is often referred to as the fundamental frequency, which is the most important parameter for design engineers as many of the systems are designed to operate below it.

The *ANSYS* has several mode extraction methods such as subspace, block Lanczos, power dynamics, reduced, unsymmetric, and damped. Each method has its own advantages and disadvantages. The method that is used in the present work is the subspace method of mode extraction. Modal analysis is the preliminary step of a dynamic transient analysis. The important thing about a modal analysis is that it ignores any nonlinearity present in the model.

3.4.1 Subspace method of mode extraction

This method of modal analysis is used when the number of modes that are to be extracted is less than 40. This method works better when the models have a definite shape of solid and shell elements. The modal analysis is nothing but the eigenvalue problem:

$$[K]\{\varphi_i\} = \omega_i^2[M]\{\varphi_i\} \quad (3.1)$$

where $[K]$ is stiffness matrix, $\{\varphi_i\}$ is mode shape vector of mode i , ω_i is the natural circular frequency, ω_i^2 is the eigenvalue and $[M]$ is the mass matrix.

The Subspace method is an iterative technique that makes use of Jacobi iterative algorithm. This is considered as a highly accurate method since it uses the full $[K]$ and $[M]$ matrices.

In the present work, a maximum of five mode shapes are extracted, but the point of interest is only the fundamental period of the beams. The fundamental period of the beam is the inverse of the first natural frequency. In the present research, modal analysis is the preliminary step in a transient dynamic analysis.

3.5 Transient Dynamic Analysis

This type of analysis is also known as time-history analysis. This method is generally used to determine response due to time varying loads. Through this analysis, we can find the time varying stresses, strains, and deflections produced when a system responds to the transient loads. A transient analysis is more complex and time consuming method compared to a static analysis, as it requires more engineering input data and better understanding of the system response. An analyst must have a good insight of the problem involved in the analysis.

There are three methods present in *ANSYS* to solve dynamic problems: full, reduced, and mode superposition. In this present work, the full transient analysis is used to solve the moving load problem. The *ANSYS* uses the Newmark method for time integration. The transient analysis can be treated as static analysis solved at discrete time steps.

4 RESULTS AND DISCUSSIONS

4.1 Introduction

This chapter describes the results obtained from the finite element analysis using *ANSYS*. Model validation is achieved by comparisons with analytical results and results available in literature. Results from three broad categories of analysis are presented. The first one deals with dynamic deflection factors of isotropic and laminated composite beams under different moving load velocities. The aim of this part of the analysis is to validate the moving load modeling. The second is that of in plane and transverse deflections of laminated composite plates under cylindrical bending and subjected to hygrothermal conditions. The third set consists of a parametric study on laminated composite beams subjected to moving loads in a hygrothermal environment, wherein new results are presented. This second part validates the modeling of hygrothermal effects.

4.2 Isotropic Beam Comparison

The dynamic deflection factor of an isotropic simply-supported steel beam under a moving load is evaluated using *ANSYS*. The dynamic magnification factor is the ratio of the maximum magnitude of the dynamic deflection at the mid-span of beam to the static deflection at the same location. First, a static analysis of the beam on *ANSYS* is done by applying a load at the center to get the maximum static deflection of the beam. The geometric and material properties of the beam used by Kadivar and Mohebpour (1997) are taken to simulate the model. The length, width, and thickness of the beam are 4 in., 0.25 in., and 0.25 in., respectively. The beam has a modulus of elasticity, $E = 30 \times 10^6$ psi, and with a fundamental period, $T_f = 8.149 \times 10^{-4}$ s. A moving force of magnitude 1 lb is

applied on this simply supported beam. Initially the load is applied at the left end of the beam and then the load moves toward the right end with a velocity, V .

4.2.1 Static Central Deflection of Simply Supported Isotropic Beam

The static deflection at the mid span of a simply supported beam can be calculated analytically as

$$\delta_s = \frac{PL^3}{48EI} \quad (4.1)$$

where P is the static vertical load applied at the center of the beam, L is the length of the beam, E the modulus of elasticity and I the moment of inertia.

The finite element model of the beam is modeled in *ANSYS* using 2D beam elements, namely Beam3. The value of P is taken to be 1 lb. The mid-span deflection of the finite element model in *ANSYS* is compared with the analytical result from Eq. (4.1) in Table 4.1 and they are in excellent agreement.

Table 4.1 Static mid span deflection of a simply supported beam

Analytical Result	<i>ANSYS</i> Result
1.36614×10^{-4} in	1.36601×10^{-4} in

4.2.2 Moving Force on a Simply Supported Isotropic Beam

A moving force of magnitude 1 lb is applied to a simply supported beam with the same properties as the previous example. As with the static case the beam is modeled in *ANSYS* using Beam3 elements. A transient dynamic analysis is carried out in *ANSYS* to move the load from one node to the next node. Twenty time steps are used to simulate the moving load from the left end to the right end of the beam. The time taken by the moving

load to travel from one end to the other is represented by T . The fundamental period of the beam is represented by T_f . Let V_f be the velocity of a reference load that would take a time of T_f (fundamental period) to traverse the beam of length L . Then the velocity ratio V/V_f can be expressed as

$$\frac{V}{V_f} = \frac{V}{L} \times \frac{L}{V_f} = \frac{T_f}{T} \quad (4.2)$$

Several velocity values for the moving load are considered. The moving load velocities and the corresponding velocity ratios and time steps are listed in Table 4.2.

Table 4.2 Moving load velocities and time steps

Moving load velocity		Velocity ratio T_f/T	Δt (s)
ft/s	mph		
51.16	34.88	0.125	3.257×10^{-4}
102.33	69.77	0.25	1.628×10^{-4}
204.66	139.54	0.5	0.814×10^{-4}
409.41	279.15	1	0.407×10^{-4}
820.16	559.20	2	0.203×10^{-4}

A convergence study is performed to decide the number of elements in the model. The beam is divided into 20 elements with the help of convergence study. The left end is constrained in the x and z directions whereas the right end is constrained only in the z direction, thus simulating simply supported boundary conditions. By using the transient analysis option in *ANSYS*, the point load is moved from one end to the other end. At a time step, the load moves from one node to the next. The maximum central deflection of the beam is found for various moving load velocities and listed in Table 4.3. The dynamic deflection magnification factors from the present analysis are compared with that of Taheri (1987) in Table 4.4 and are in excellent agreement.

Table 4.3 Maximum center deflection for various moving load velocities

T_f/T	Velocity (mph)	Velocity (ft/s)	Dynamic Deflection (in.)
0.125	34.88	51.16	1.422×10^{-4}
0.25	69.77	102.33	1.481×10^{-4}
0.5	139.54	204.66	1.742×10^{-4}
1.0	209.37	307.08	2.941×10^{-4}
2.0	559.20	820.16	2.054×10^{-4}

Table 4.4 Dynamic magnification factors for moving force on isotropic beam

			Dynamic magnification factor	
T_f/T	Velocity (mph)	Velocity (ft/s)	Present Analysis	Taheri (1987)
0.125	34.88	51.16	1.042	1.042
0.25	69.77	102.33	1.082	1.083
0.5	139.54	204.66	1.271	1.266
1.0	209.37	307.08	1.671	1.662
2.0	559.20	820.16	1.514	1.518

Figure 4.1 shows the dynamic magnification factor of the isotropic beam with respect to the velocity ratio. This plot consists of an undercritical region and an overcritical region. As seen from the graph, in the undercritical region the dynamic deflection of the beam increases with increasing velocity of the load. In the overcritical region, the dynamic deflection decreases with increasing velocity of the load. Figures 4.2-4.5 show the dynamic deflection of the center of the beam for moving load velocities of 51.16, 102.33, 204.66, and 820.16 ft/s, respectively. These graphs are obtained from the time-history processor of *ANSYS*. It can be observed that the deflection at the mid-span actually becomes positive (upward) toward the end of the traverse of the moving load for the lower velocity cases.

Figure 4.1 Dynamic magnification factor of an isotropic beam

Figure 4.2 Time history of mid-span deflection for $V = 51.16$ ft/s

Figure 4.3 Time history of mid-span deflection for $V = 102.33$ ft/s

Figure 4.4 Time history of mid-span deflection for $V = 204.66$ ft/s

Figure 4.5 Time history of mid-span deflection for $V = 820.16$ ft/s

4.3 Composite Beam subjected to Moving Load

A symmetric cross-ply laminated beam, $[0/90]_s$, made of graphite/epoxy is analyzed next. The geometric and material properties of this model are listed in Table 4.5.

Table 4.5 Geometric and material properties of composite beam

Property	Value
Beam length, L	4 in.
Beam width, b	0.25 in.
Beam height, h	0.293 in.
Mass density, ρ	0.05 lb/in. ³
Longitudinal Modulus, E_1	21 x 10 ⁶ psi
Transverse Modulus, E_2	1.4 x 10 ⁶ psi
Longitudinal Shear Modulus, G_{12}	0.6 x 10 ⁶ psi
Inter laminar Shear Modulus, G_{23}	0.5 x 10 ⁶ psi
Transverse Shear Modulus, G_{13}	0.6 x 10 ⁶ psi
Major in-plane Poisson's ratio, ν_{12}	0.25
Out of plane Poisson's ratio, ν_{23}	0.25
Out of plane Poisson's ratio, ν_{13}	0.25

The beam is meshed using solid 191 elements; there are 20 element divisions along the length of the beam, 4 divisions along the width and 1 division through the thickness. Thus the mesh consists of 80 elements and 621 nodes. A moving load of magnitude 1 lb is applied starting at the left end and traversing the span. The velocities and time steps considered are the same as that for the isotropic case (Table 4.2).

The beam has a fundamental period of $T_f = 3.187 \times 10^{-4}$ s. First a static analysis is carried out with a central load of 1 lb and mid-span deflection is found to be 0.000258 in. The dynamic magnification factors for various velocities of the moving load are calculated and presented in Table 4.6. The results (FSDT) from Kadivar and Mohebpour (1997) are also included in this table and the comparison indicates excellent agreement.

The dynamic magnification factor as a function of the moving load velocity is plotted in Figure 4.6. Similar to the isotropic beam, this plot contains an undercritical region and a super critical one. The maximum dynamic magnification factor occurs at the velocity ratio of 1.25, contrary to what one might intuitively expect this to occur at a velocity ratio of 1. The velocity ratio of 1.25 translates into a velocity of 511.8 ft/s and this is known as the critical velocity.

The time-history plots of the mid-span deflection for moving load velocities of 51.6, 102.33, 204.66, 307.08, 409.41, 511.75, 820.16 ft/s are shown in Figures 4.7- 4.13. It is seen, as with the isotropic beam, that at higher moving load velocities, the beam has an upward deflection toward the end of the load traverse. This phenomenon is attributed to wave-like motion of the beam in response to the moving load.

Table 4.6 Dynamic magnification factors for moving load on composite beam

			Dynamic Magnification Factor	
T_f/T	Velocity (ft/s)	Velocity (mph)	Present Analysis	Kadivar and Mohebpour (1997)
0.125	51.16	34.88	1.063	1.063
0.25	102.33	69.77	1.149	1.151
0.5	204.66	139.54	1.280	1.281
0.75	307.08	209.37	1.586	1.586
1.0	409.41	279.15	1.710	1.704
1.25	511.75	348.92	1.726	1.727
2.0	820.16	559.20	1.542	1.542

Figure 4.6 Dynamic magnification factors for composite laminated beam

Figure 4.7 Time history of mid-span deflection for $V=51.16$ ft/s

Figure 4.8 Time history of mid-span deflection for $V=102.33$ ft/s

Figure 4.9 Time history of mid-span deflection for $V=204.66$ ft/s

Figure 4.10 Time history of mid-span deflection for $V=307.08$ ft/s

Figure 4.11 Time history of mid-span deflection for $V=409.41$ ft/s

Figure 4.12 Time history of mid-span deflection for $V=511.75$ ft/s

Figure 4.13 Time history of mid-span deflection for $V=820.16$ ft/s

4.4 Hygrothermal Analysis on Laminated Composite Beams

In this section, the response of laminated composite beams to hygrothermal conditions is studied to validate the hygrothermal aspect of the model. In this study, the stiffness properties of the constituents are degraded by using the empirical relations (see section 2.3) proposed by Chamis (1983). It is assumed that only matrix properties are affected by temperature change and moisture content. By using the rule of mixtures and changed properties of the matrix, updated elastic constants of the lamina and the laminate are calculated. An asymmetric cross-ply laminate of $(0_4/90_4)$ under cylindrical bending configuration is considered in this study. The length, width and thickness of laminate (see Figure 2.5) are taken as 9 in., 1.5 in., and 0.04 in., respectively. These dimensions are similar to the ones taken by Sun and Chin (1988). S-glass fiber and epoxy composite properties used here are similar to those taken by Cairns and Adams (1984). The results of dry conditions are verified before proceeding to the hygrothermal analysis. The results of various loading and boundary conditions are discussed in the later sections. The

properties and values of various parameters of the composite are listed in Tables 4.7 and 4.8.

Table 4.7 Constituent properties of S glass fiber and Epoxy matrix

Property	Owens-Corning S2 Glass Fiber	Hercules 3501-6 Epoxy Matrix (Dry)
Modulus of elasticity, E	12.5×10^6 psi	0.62×10^6 psi
Shear Modulus, G	5.12×10^6 psi	0.23×10^6 psi
Poisson's ratio, ν	0.22	0.34

Table 4.8 Parameters in hygrothermal analysis

Property	Value
Moisture Content, m	5 %
Glass transition temperature of dry resin, T_{gdr}	342° F
Temperature at which property to be measured, T	156.2° F
Temperature at which initial parameter is measured, T_0	70° F
Glass transition temperature of wet resin, T_{gwr}	213.75° F
Fiber Volume Fraction, V_f	0.52

4.4.1 Dry Conditions

4.4.1.1 Transverse Deflections of Pinned-Pinned Laminate under Uniform Pressure Load

Pinned-Pinned boundary conditions are applied on the two edges of the composite laminate as shown in Figure 4.14. The laminate is subjected to a uniform transverse load under dry conditions. The temperature and moisture content at dry conditions are taken as 70° F and 0 % respectively. Linear static analysis is performed and the transverse deflections are calculated. The results are included in Table 4.9 and Figure 4.15 and show good agreement with the ones obtained by classical linear theory. The deflection along the length of the laminate for the transverse load value of 0.0348 psi is shown in Figure 4.16. It can be observed that the increase in load causes the deflection to increase linearly.

Figure 4.14 Pinned-pinned boundary conditions on composite laminate

Table 4.9 Deflections for pinned-pinned laminate due to uniform transverse load in dry condition

Transverse Load (psi)	Transverse deflection (in.) Present Study	Transverse Deflection (in.) Linear Theory
0.0058	0.0249	0.025
0.0116	0.0481	0.048
0.0174	0.0720	0.072
0.0231	0.0963	0.096
0.0292	0.0124	0.120
0.0348	0.1444	0.140
0.0406	0.1680	0.160
0.0464	0.9260	0.190
0.0522	0.2160	0.216
0.0581	0.2407	0.241

Figure 4.15 Deflections for pinned-pinned laminate for uniform transverse load in dry condition

Figure 4.16 Transverse deflection distribution for 0.0348 psi

4.4.1.2 Transverse Deflections of Clamped-Clamped Laminate under Uniform Transverse Load

In this section, the boundary conditions of the laminate are changed to clamped-clamped. The outer edges of the laminate are clamped and a uniform load is applied on the beam. Linear static analysis is carried out and the transverse deflections are calculated. The results are included in Table 4.10 and Figure 4.17. The deflection along the length of the laminate for the transverse load value of 0.0363 psi is shown in Figure 4.18.

Table 4.10 Transverse deflections for clamped-clamped laminate due to uniform transverse load in dry condition

Pressure Load (psi)	Transverse Deflection (in.) Present Study	Transverse Deflection (in.) Linear Theory
0.0072	0.00828	0.0083
0.0145	0.01767	0.0177
0.0217	0.02651	0.0265
0.0291	0.03536	0.0353
0.0363	0.04421	0.0442
0.0507	0.06188	0.0621

Figure 4.17 Deflections of clamped-clamped laminate for uniform transverse load in dry condition

Figure 4.18 Transverse deflection distribution for 0.0363 psi

4.4.1.3 Transverse Deflections of Pinned-Pinned Laminated Composites Beam under In-Plane Loads

Tensile normal in-plane loads of varying intensities are applied on outer edges of the laminate. The results are included in Table 4.11 and Figure 4.19, and show good agreement with the ones obtained by linear theory. The deflection along the length of the laminate for the inplane load value of 3.426 lb/in. is shown in Figure 4.20.

Table 4.11 Deflections for pinned-pinned laminate due to in-plane load in dry condition

In-Plane load (lb/in.)	Transverse Deflection (in.) Present study	Transverse Deflection (in.) Linear Theory
0.285	0.00145	0.00146
0.571	0.00291	0.00292
1.713	0.00875	0.00875
3.426	0.01752	0.01751
5.139	0.02630	0.02592
6.852	0.03506	0.03456
8.565	0.046	0.04601

Figure 4.19 Transverse deflection of pinned-pinned laminate under different tensile normal in-plane loads in dry conditions

Figure 4.20 Transverse deflection distribution for 3.426 lb/in.

4.4.2 Hygrothermal Conditions

The wet conditions represent a moisture content of 5% (by weight) and at a temperature of 156.2⁰ F. These values represent the severe atmospheric conditions. The analysis is carried out on pinned and clamped laminates and the maximum centre deflection of the laminates is found. The results are included in Tables 4.12-4.14 and Figures 4.21-4.26 show good agreement with the ones obtained by Upadhyay and Lyons (2000 b).

Table 4.12 Transverse deflections for pinned-pinned laminate due to transverse load for hygrothermal condition

Transverse Load (psi)	Transverse Deflection (in.) Present Study	Transverse Deflection (in.) Upadhyay and Lyons (2000)
0.0058	0.0280	0.028
0.0116	0.0561	0.056
0.0174	0.0842	0.084
0.0232	0.1122	0.112
0.0348	0.1683	0.168
0.0406	0.1964	0.196
0.0581	0.2805	0.28
0.1	0.4933	0.49

Figure 4.21 Transverse deflections of pinned-pinned laminate for different uniform transverse loads in hygrothermal condition

Figure 4.22 Transverse deflection distribution for 0.0348 psi

Table 4.13 Transverse deflections for clamped-clamped laminate due to transverse load in hygrothermal condition

Transverse load (psi)	Transverse deflection (in.) Present Study	Transverse deflection (in.) Upadhyay and Lyons (2000)
0.0072	0.01259	0.012
0.0145	0.02608	0.026
0.0217	0.03913	0.039
0.0291	0.05217	0.052
0.0363	0.06523	0.065
0.0435	0.07826	0.078
0.0507	0.09132	0.091
0.0581	0.10431	0.104

Figure 4.23 Deflections of clamped-clamped laminate due to transverse loads in hygrothermal condition

Figure 4.24 Transverse deflection distribution for 0.03626 psi

Table 4.14 Transverse deflections for pinned-pinned laminate due to in-plane loads in hygrothermal condition

In-plane load (lb/in.)	Transverse Deflection, (in.) Present study	Transverse Deflection (in.) Upadhyay and Lyons (2000)
0.2855	0.0025	0.0026
0.571	0.0051	0.0051
1.713	0.0154	0.0151
3.426	0.0308	0.0311
5.139	0.0463	0.0461
6.852	0.0617	0.0621
8.565	0.0763	0.0762

Figure 4.25 Transverse deflections for pinned-pinned laminates for different in-plane loads in hygrothermal conditions

Figure 4.26 Transverse deflection distribution for 3.426 lb/in.

4.5 Comparison of Hygrothermal Deflections with that of Dry conditions for Laminate under Cylindrical Bending

In this section, the wet and dry results for the laminate under cylindrical bending are compared. The first set of results (Table 4.15 and Figure 4.27) corresponds to the case of a pinned-pinned laminate subjected to a uniform transverse load. As expected, the dry condition plot of the transverse load versus mid-span deflection is linear whereas its wet counterpart exhibits slightly nonlinear behavior. The effect of the hygrothermal environment is an increase in the mid-span deflection that ranges from 16.58 % to 16.94%.

The corresponding results for the case of clamped-clamped laminate subjected to transverse loads are presented in Table 4.16 and Figure 4.28. The trends are similar to the pinned-pinned case except that the percentage differences are more than twice that of the previous case.

The comparison for a pinned-pinned laminate subjected to an in-plane load is shown in Table 4.17 and Figure 4.29. The deviation due to hygrothermal conditions is much higher in this case than the previous two cases. The percentage increase in mid-span deflection is almost constant for different values of N_x at about 76 %.

Table 4.15 Dry and wet condition mid-span deflections of pinned laminate under transverse load

Transverse Load (psi)	Dry Conditions (in.)	Hygrothermal Conditions (in.)	Percentage Difference
0.0058	0.024	0.028	16.66
0.0116	0.048	0.056	16.87
0.0174	0.072	0.084	16.94
0.0232	0.096	0.112	16.87
0.029	0.124	0.145	16.93
0.0348	0.144	0.168	16.66
0.0406	0.168	0.196	16.66
0.0464	0.193	0.225	16.58
0.0522	0.216	0.252	16.66

Figure 4.27 Dry and wet condition mid-span deflections of pinned laminate under transverse load

Table 4.16 Dry and wet condition mid-span deflections of clamped laminate under transverse load

Transverse load (psi)	Dry conditions (in.)	Hygrothermal Conditions (in.)	Percentage Difference
0.0145	0.01767	0.02608	47.59
0.0217	0.02651	0.03913	47.61
0.0291	0.03536	0.05217	47.54
0.0363	0.04420	0.06523	47.57
0.0435	0.05303	0.07826	47.57
0.0507	0.06188	0.09132	47.57
0.0581	0.07072	0.10431	47.49

Figure 4.28 Dry and wet condition mid-span deflections of clamped laminate uniform transverse load

Table 4.17 Dry and wet condition mid-span deflections of pinned laminate under inplane load

In-Plane load (lb/in.)	Dry conditions (in.)	Hygrothermal conditions (in.)	Percentage Difference
0.285	0.00145	0.00257	76.33
0.571	0.00291	0.00514	76.33
1.713	0.00874	0.01542	76.27
3.426	0.01749	0.03085	76.21
5.139	0.02624	0.04627	75.96
6.852	0.03499	0.06170	75.95
8.565	0.04374	0.07713	76.33

Figure 4.29 Dry and wet condition mid-span deflections of pinned-pinned laminate under uniform in-plane loads

4.6 Composite Beam subjected to Moving Loads under Hygrothermal Environment

Having validated the moving load model and the hygrothermal model, the two effects are combined in this phase of the analysis. A simply supported composite laminated beam has a moving load of constant magnitude traveling from the left end to

the right end with a constant velocity and the beam is subjected to a hygrothermal environment. The dynamic magnification factors of the mid-span deflection are calculated. A parametric study is carried out by varying the fiber volume fraction of the lamina and by considering different laminates. First a static analysis under dry and wet conditions is carried out to get the baseline mid-span deflections.

For wet conditions, the matrix properties are degraded using Eq.(2.1), and then the lamina properties are calculated using the rule of mixtures, Eq.(2.3,2.4, and 2.5).

4.6.1 STATIC ANALYSIS

The temperature and moisture content in dry conditions are taken as 72⁰ F and 0 % respectively. For hygrothermal conditions, the temperature and moisture content are taken as 154.2⁰ F and 5% respectively. The beam geometric properties are as follows:

Length = 4 in.

Width = 0.25 in.

Thickness = 0.293 in.

Tables (4.18, 4.19, and 4.20) provide the material properties for fiber volume fractions of 0.52, 0.6 and 0.7, respectively under dry and hygrothermal conditions.

Table 4.18 Material properties for fiber volume fraction of 0.52

Property	Dry Condition	Hygrothermal Condition
Longitudinal modulus, E_1	6.82 x 10 ⁶ psi	6.64 x10 ⁶ psi
Transverse modulus, E_2	1.23 x 10 ⁶ psi	0.59 x10 ⁶ psi
Longitudinal Shear Modulus, G_{12}	0.46 x 10 ⁶ psi	0.42 x10 ⁶ psi
Transverse Shear Modulus, G_{13}	0.46 x 10 ⁶ psi	0.42 x10 ⁶ psi
Inter laminar Shear Modulus, G_{23}	0.58 x 10 ⁶ psi	0.56 x10 ⁶ psi
Major in-plane Poisson's ratio, ν_{12}	0.25	0.25
Out of plane Poisson's ratio, ν_{23}	0.25	0.25
Out of plane Poisson's ratio, ν_{13}	0.25	0.25

Table 4.19 Material properties for fiber volume fraction of 0.6

Property	Dry Condition	Hygrothermal Condition
Longitudinal modulus, E_1	7.75×10^6 psi	7.62×10^6
Transverse modulus, E_2	1.44×10^6	0.70×10^6
Longitudinal Shear Modulus, G_{12}	0.54×10^6	0.32×10^6
Transverse Shear Modulus, G_{13}	0.54×10^6	0.37×10^6
Inter laminar Shear Modulus, G_{23}	0.70×10^6	0.62×10^6
Major in-plane Poisson's ratio, ν_{12}	0.25	0.25
Out of plane Poisson's ratio, ν_{23}	0.25	0.25
Out of plane Poisson's ratio, ν_{13}	0.25	0.25

Table 4.20 Material properties for fiber volume fraction of 0.7

Property	Dry Condition	Hygrothermal Condition
Longitudinal modulus, E_1	8.94×10^6 psi	8.84×10^6 psi
Transverse modulus, E_2	1.85×10^6 psi	0.92×10^6 psi
Longitudinal Shear Modulus, G_{12}	0.69×10^6 psi	0.32×10^6 psi
Transverse Shear Modulus, G_{13}	0.69×10^6 psi	0.32×10^6 psi
Inter laminar Shear Modulus, G_{23}	0.93×10^6 psi	0.90×10^6 psi
Major in-plane Poisson's ratio, ν_{12}	0.25	0.25
Out of plane Poisson's ratio, ν_{23}	0.25	0.25
Out of plane Poisson's ratio, ν_{13}	0.25	0.25

The laminate configurations that are taken in this analysis are

- 1) 0/90/-15/-15
- 2) 0/90/-30/30
- 3) 0/90/-45/45

The beam is simply supported and a load of 1 lb is applied at the center of the beam.

Table (4.21) gives the mid-span static deflection under dry and hygrothermal conditions, for the three fiber volume fractions and the three laminates considered.

Table 4.21 Static mid-span deflection of beams made of various laminate configurations

Fiber volume fraction, 0.52	Lay-up	Dry Condition (in.)	Hygrothermal Condition (in.)
	0/90/-15/15	1×10^{-3}	1.22×10^{-3}
	0/90/-30/30	0.989×10^{-3}	1.15×10^{-3}
	0/90/-45/45	0.897×10^{-3}	0.986×10^{-3}
Fiber volume fraction, 0.6	0/90/-15/15	0.868×10^{-3}	1.06×10^{-3}
	0/90/-30/30	0.858×10^{-3}	1.03×10^{-3}
	0/90/-45/45	0.779×10^{-3}	0.901×10^{-3}
Fiber volume fraction, 0.7	0/90/-15/15	0.722×10^{-3}	0.912×10^{-3}
	0/90/-30/30	0.716×10^{-3}	0.888×10^{-3}
	0/90/-45/45	0.651×10^{-3}	0.803×10^{-3}

4.6.2 DYNAMIC ANALYSIS

Dynamic transient analysis is performed on the above stated laminates. A force of magnitude 1 lb moving with a velocity of 102.33 ft/s (69.77) is applied on these simply supported beams. The maximum dynamic mid-span deflections in inches are presented in Table (4.22).

Table 4.22 Dynamic mid-span deflections of beams made of various laminate configurations

Fiber volume fraction, 0.52	Lay-up	Dry Condition (in.)	Hygrothermal Condition (in.)
	0/90/-15/15	1.15×10^{-3}	1.43×10^{-3}
	0/90/-30/30	1.14×10^{-3}	1.32×10^{-3}
	0/90/-45/45	0.98×10^{-3}	1.13×10^{-3}
Fiber volume fraction, 0.6	0/90/-15/15	0.93×10^{-3}	1.23×10^{-3}
	0/90/-30/30	0.92×10^{-3}	1.19×10^{-3}
	0/90/-45/45	0.91×10^{-3}	0.99×10^{-3}
Fiber volume fraction, 0.7	0/90/-15/15	0.76×10^{-3}	1.01×10^{-3}
	0/90/-30/30	0.73×10^{-3}	0.98×10^{-3}
	0/90/-45/45	0.69×10^{-3}	0.88×10^{-3}

4.6.3 Dynamic Magnification Factors (DMF)

The dynamic magnification factor (DMF) is the ratio of the maximum dynamic mid-span deflection to the static deflection at the mid-span. The magnification factors for the spectrum of the analysis are listed in Table (4.23). The DMF are also plotted in Figures 4.30, 4.31 and 4.32.

Table 4.23 Dynamic magnification factors for different lay-ups and V_f

Fiber volume fraction, 0.52	Lay-up	Dry Condition	Hygrothermal Condition
	0/90/-15/15	1.150	1.172
	0/90/-30/30	1.152	1.145
	0/90/-45/45	1.224	1.146
Fiber volume fraction, 0.6	0/90/-15/15	1.077	1.162
	0/90/-30/30	1.069	1.156
	0/90/-45/45	1.155	1.101
Fiber volume fraction, 0.7	0/90/-15/15	1.055	1.109
	0/90/-30/30	1.025	1.111
	0/90/-45/45	1.061	1.1

It can be observed that as the fiber volume fraction of the laminate increases, the deflection decreases for the same laminate configuration. This is due to increase in stiffness of the laminate.

Figure 4.30 Dynamic magnification factor for 0/90/-15/15 laminate

Figure 4.31 Dynamic magnification factor for 0/90/-30/30 laminate

Figure 4.32 Dynamic magnification factor for 0/90/-45/45 laminate

The ply orientation influences the dynamic behavior of a beam subjected to moving load. The dynamic magnification factors in dry and hygrothermal conditions for the laminates considered generally follow similar pattern. As the fiber volume fraction increases, the dynamic magnification factor generally decreases; the one exception being the 0/90/-30/30 laminate, wherein the dynamic magnification factor slightly increases at the fiber volume fraction of 0.6. In dry conditions, the 0/90/-45/45 laminate has the

highest DMF when compared to 0/90-15/15 and 0/90/-30/30 laminates. In hygrothermal conditions, 0/90/-15/15 laminate has the highest DMF when compared to 0/90/-30/30 and 0/90/-45/45 laminates. Based on these results a designer can choose the right ply orientations to control the dynamic behavior of laminated beams.

5 CONCLUSIONS AND RECOMMENDATIONS

5.1 CONTRIBUTIONS

1. A moving load on a composite laminated beam is modeled using *ANSYS*.
2. Hygrothermal analysis is carried out on asymmetric composite laminated beams subjected to a moving load.
3. A parametric study of the moving load hygrothermal problem is carried out by varying the fiber volume fraction and ply lay-ups.

5.2 CONCLUSIONS

1. Forced vibration analysis of laminated composite beams under the effect of moving loads using finite element method is successfully carried out with the help of a general finite element program like *ANSYS*.
2. The dynamic magnification factors for isotropic and composite beams are compared. The graph of dynamic magnification factor as a function of moving load velocity for isotropic and composite beams follows the same pattern. The maximum dynamic magnification factor occurs at the velocity ratio of 1.25, contrary to what one might expect this to occur at a velocity ratio of 1.
3. In the case of cylindrical bending problems subjected to transverse loads, the effect of the hygrothermal environment is to increase the maximum deflection by 16 % for pinned-pinned case while it is 47 % for the clamped-clamped case.
4. The ply orientation influences the dynamic behavior of a beam subjected to moving load. The dynamic magnification factors in dry and hygrothermal

conditions for the laminates considered generally follow similar pattern. As the fiber volume fraction increases, the dynamic magnification factor generally decreases; the one exception being the 0/90/-30/30 laminate, wherein the dynamic magnification factor slightly increases at the fiber volume fraction of 0.6.

5. In dry conditions, the 0/90/-45/45 laminate has the highest DMF when compared to 0/90-15/15 and 0/90/-30/30 laminates.
6. In hygrothermal conditions, 0/90/-15/15 laminate has the highest DMF when compared to 0/90/-30/30 and 0/90/-45/45 laminates.
7. Based on these results a designer can choose the right ply orientations to control the dynamic behavior of laminated beams.

5.3 RECOMMENDATIONS

1. Moving mass analysis may be performed on the beams. The results can be compared with the present study.
2. Multiple moving loads can be considered and the effects may be compared to that of a single moving load.
3. Nonlinear analysis based on large deflection theory may be adopted.
4. The effects of different moisture contents on the dynamic deflections of laminates may also be calculated.

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