2005

Methodology to determine post yield material mechanical properties from spherical indentation

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Methodology to Determine Post Yield Material Mechanical Properties from Spherical Indentation

Bharath Konda

Thesis submitted to the
College of Engineering and Mineral Resources
at West Virginia University
in partial fulfillment of the requirements
for the degree of

Master of Science
In
Mechanical Engineering

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Morgantown, West Virginia
2005

Keywords: Spherical indentation, Finite element method, mechanical properties
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Abstract

Methodology to Determine Post Yield Material Mechanical Properties from Spherical Indentation

Bharath Konda

Conventional material testing methods, such as tensile tests require the preparation of specimens and is not applicable for small-size coupons or on-site, in-situ testing. In this research, a method of determining material stress-strain curve based on spherical indentation is studied. Spherical-indentation process is analyzed by finite element (FE) method and a systematic analysis of relationships between indentation parameters and true stress/plastic-strain (σ_t - ϵ_p) curve is performed for a range of material properties. A detailed methodology based on Tabor's empirical formula and Meyer's law is developed to determine the material strain hardening properties, Young's modulus and yield strength. It is noticed that Tabor's empirical formula is valid in the plastic region (deep indentation) with a small percentage error in the calculated values of yield strength. Based on the FE results, strain values have been predicted corresponding to which indentation gives satisfactory output. Steel and aluminum alloys, with same Young's modulus and yield strength but with different strain hardening coefficients, were selected for the simulation. FE simulation of 20 material models was performed. It is observed that the Tabor's constant is not equal to the value of 2.8 but takes different values for different materials. From the results it is concluded that the value of Tabor's constant decreases as the strain hardening value increases.

Acknowledgements

I would like to thank Dr.Bruce Kang for providing me with an opportunity to work with him, and for his guidance and technical advice. His support and patience during the course of my research work deserve a special mention. I would also like to thank Dr.Jacky C.Prucz, and Dr.Charles F.Stanley, for their technical guidance and advice.

I wish to express my sincere thanks to Ziheng Yao. Without his help it would have been very difficult to accomplish the amount of work that I have been able to in the last few years. Ziheng Yao had provided me invaluable analysis knowledge and for this I thank him.

Finally, I would like to thank my parents and brother for their moral support and encouragement throughout my graduate studies. I dedicate my research thesis to them.

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List of Symbols

F	Load
d	Contact diameter of spherical indenter
D	Diameter of spherical indenter
E	Young's modulus
HV	Vickers hardness
P	Force applied on the indenter
h	contact depth
S	indentation stiffness
E_r	reduced modulus
ν	Poisson's ratio
p_m	Meyer hardness
ε	Strain
ε_i	Indentation strain (Tabor's relation)
ε_p	plastic strain
σ	Stress
σ_i	Indentation stress (Tabor's relation)
n	Strain hardening coefficient
m	Linear strain hardening coefficient
μ	Micro
S_y	Yield stress

Chapter 1

Introduction

1.1 Indentation hardness

In general hardness implies the resistance to deformation. Hardness has conventionally been defined as the resistance of a material to permanent penetration by another harder material with measurement being made after the applied force has been removed, such that elastic deformation is ignored. Today the indentation hardness test is used in practically every metal working plant as a means of checking the quality and uniformity of metals and metal parts. The test serves as a control or designation of the heat-treating or processing of the metals, or is used in estimating the tensile strength. Because of the simplicity of the indentation hardness test as carried out by modern hardness testers, and relatively small cost, it is probably the most commonly employed test in industry. Hardness measurements usually fall into three main categories: scratch hardness, static indentation hardness, and rebound or dynamic hardness. With the exception of the Brinell tester, most modern testers use diamond penetrators or indenters. Diamond is the hardest of the all known materials. The indentation hardness is calculated from the test force, F , divided by the projected area of the indenter in contact with the test piece at maximum load.

1.2 Static indentation hardness

The methods most widely used in determining the hardness of the metals are static indentation methods. Primarily used in engineering and metallurgy, indentation

hardness seeks to characterise a material's resistance to permanent, and in particular plastic deformation. It is usually measured by loading an indenter of specified geometry onto the material and measuring the dimensions of the resulting indentation. The most commonly used methods are:

Brinell hardness test,

Vickers hardness test and

Rockwell hardness test.

1.2.1 Brinell hardness test

The first widely accepted and standardized indentation hardness test was proposed by J. A. Brinell in 1900. The Brinell hardness test consists of indenting the metal surface with a 10-mm-diameter steel ball at a load of 3,000 kg mass (29400 N). For soft metals the load is reduced to 500 kg to avoid deep impression, and for very hard metals a tungsten carbide ball is used to minimize distortion of the indenter. The load is applied for a standard time, usually 30 s, and the diameter of the indentation is measured with a low-power microscope after removal of the load. The Brinell hardness number is calculated by dividing the load applied by the surface area of the indentation.

Figure 1.1 Schematic diagram of brinell hardness test [23]

Where,

BHN = the Brinell hardness number

F = the imposed load in kg

D = the diameter of the spherical indenter in mm

D_i = diameter of the resulting indenter impression in mm

1.2.2 Meyer Hardness

Meyer suggested that a more rational definition of hardness than that proposed by Brinell would be one based on the projected area of the impression rather than the surface area. The mean pressure between the surface of the indenter and the indentation is equal to the load divided by the projected area of the indentation. Meyer proposed that this mean pressure should be taken as the measure of hardness. It is referred to as the Meyer hardness.

1.2.3 Vickers hardness

The Vickers hardness test method consists of indenting the test material with a diamond indenter, in the form of a right pyramid with a square base and an angle of 136 degrees between opposite faces subjected to a load of 1 to 100 kgf. The full load is normally applied for 10 to 15 seconds. The two diagonals of the indentation left in the surface of the material after removal of the load are measured using a microscope and their average calculated. The area of the sloping surface of the indentation is calculated. The Vickers hardness is the quotient obtained by dividing the kgf load by the square mm area of indentation.

Figure 1.2 Schematic diagram of Vickers hardness test [24]

$$HV = \frac{2F \sin \frac{136^\circ}{2}}{d^2} \quad HV = 1.854 \frac{F}{d^2} \text{ approximately} \quad (1.1)$$

Where,

F= Load in kgf

d = Arithmetic mean of the two diagonals, d1 and d2 in mm

HV = Vickers hardness

1.2.4 Rockwell hardness test

Stanley P. Rockwell invented the Rockwell hardness test. The most widely used hardness test is the Rockwell hardness test. Its general acceptance is due to its speed, freedom from personal error, ability to distinguish small hardness differences in hardened steel, and the small size of the indentation, so that finished heat-treated parts can be tested without damage. This test utilizes the depth of indentation, under constant load, as a measure of hardness. A minor load of 10 kg is first applied to seat the specimen. This

minimizes the amount of surface preparation needed and reduces the tendency for ridging or sinking in by the indenter. The major load is then applied, and the depth of indentation is automatically recorded on a dial gage in terms of arbitrary hardness numbers.

Figure 1.3 Schematic diagram of Rockwell hardness test [25]

The resulting Rockwell number represents the difference in depth from the zero reference position as a result of the application of the major load.

1.3 Research Objective

The objective of this research is to develop a methodology to determine material mechanical properties like Young's modulus, yield stress and strain hardening coefficient based on Meyer's law and Tabor's empirical equation using indentation technique. Numerical finite element simulations of spherical indentation will be carried out on various metallic alloys and a detailed methodology will be proposed based on the results obtained.

Indentation technique is very promising and convenient for evaluating mechanical properties of materials at micro or nano scale level, for thin surface layers, coatings, ion implanted layer, corroded surface layer, irradiated materials etc. Empirical equation relating the applied load, the radius of the sphere and the radius of the indentation, known as Meyer's law is available. Tabor's empirical equation provides the necessary relationships between stress, strain and the load-induced contact radius. Focus will be placed on determining the material mechanical properties based on the data obtained from two outputs: the applied load (N) and contact diameter (d). Special attention will be placed to determine the range of strain corresponding to which the results obtained are in good agreement with the input values. Recent studies at different research institutes and laboratories have proved that the Tabor's constant is not a fixed value and varies with materials. Here, we will work to determine the correct Tabor's constant for the present set of materials in order to obtain the yield stress with less error percentage.

Chapter 2

Literature Review

2.1 Instrumented Indentation for Hardness and Material Properties

Instrumented indentation, also known as depth-sensing indentation is increasingly being used to probe the mechanical response of materials from metals and ceramics to polymeric and biological materials. It is used to determine the properties of materials by putting an indentation in the surface of a material sample. Instrumented indentation hardness provides the ability to measure the indenter penetration (h) under the applied force (F) throughout the testing cycle and is therefore capable of measuring both the plastic and elastic deformation of the material under test.

Like a hardness test, an indenter with a well-defined geometry is indented onto the sample surface. Unlike a hardness test, where only the indent size or depth is measured after the total force has been removed, instrumented indentation testing utilizes high-precision instrumentation to continuously monitor and control the displacement of the indenter as it penetrates and then is withdrawn from the sample. The data points generated during the indentation process are stored for post data analysis. A visual load/unload curve as shown in Figure 2.2, is normally generated from the data. Below Figure 2.1 shows the assembly of an instrumented indentation system.

Figure 2.1 Instrumented indentations assembly

2.2 Analysis Techniques

The analysis of force-displacement curves produced by instrumented indentation systems is based on work by Doerner and Nix [1] and Oliver and Pharr [2]. Their analyses were in turn based upon relationships developed by Sneddon [3] for the penetration of a flat elastic half space by different probes with particular axisymmetric shapes (e.g., a flat-ended cylindrical punch, a paraboloid of revolution, and a cone). These elasticity-based analyses are normally applied to the unloading data of an indentation measurement, assuming the unloading behavior of the material is characterized by elastic recovery only. In general, the relationships between penetration depth, h , and force, P , during unloading is represented in the form

$$P = \alpha (h - h_f)^m \quad (2.1)$$

The parameter α contains geometric constants, the sample elastic modulus, E , the sample Poisson's ratio, ν , the indenter elastic modulus, E_i , and the indenter Poisson's ratio, ν_i ; the parameter h_f is the final unloading depth; and m is a power law exponent that is related to the geometry of the indenter. A nonlinear power law fit to the unloading data, where α , h_f and m are fitting parameters, yields a good estimate of the data. Once an appropriate fit is obtained, a derivative, dP/dh , applied at the maximum loading point ($h_{\max}$, $P_{\max}$) yields information about the state of contact at that point. This derivative is termed the contact stiffness, S^* , and is given analytically by

$$S^* = 2\alpha E_r = \frac{2}{\sqrt{\pi}} E_r \sqrt{A} \quad (2.2)$$

The reduced modulus, E_r , accounts for elastic deformation of both the indenter and the sample and is given by

$$\frac{1}{E_r} = \frac{(1-\nu^2)}{E} + \frac{(1-\nu_i^2)}{E_i} \quad (2.3)$$

An indentation force-displacement curve is illustrated in Figure 2.2 along with several important parameters used in the Oliver and Pharr analysis. The measured stiffness, S^* , is the slope of the tangent line to the unloading curve at the maximum loading point ($P_{\max}$) and is given by

$$S^* = \left[\frac{dP}{dH} \right]_{P_{\max}} = am(h_{\max} - h_f)^{m-1} \quad (2.4)$$

Figure 2.2 Indentation force-displacement curve [26]

2.3 Use of instrumented indentation testing

Instrumented indentation testing was developed to determine the properties of materials that, due to their configuration, could not be tested using other conventional methods. After approximately 20 years of development, instrumented indentation testing has become the primary technique to examine the properties of thin films and coatings as well as surfaces that have been specially treated such as laser heat-treated or ion implanted. Ceramics and other hard coatings and materials are also frequently tested using instrumented indentation testing. Hardness values can be determined from instrumented indentation testing test data, however, a major objective for doing instrumented indentation testing is to determine additional mechanical properties of the subject materials.

The most universally accepted property determined from instrumented indentation testing is elastic modulus E and Poisson's ratio ν . The material's modulus can be easily determined from the slope of the unload curve (see Figure 2.2). Almost any property that can be measured by uniaxial tension or compression testing can be measured or estimated using instrumented indentation testing. In addition, coating separation can be detected by observing discontinuities in the load curve, and fracture toughness may be estimated by optically measuring the cracks created by the indenter.

2.4 Previous FEM analyses

Giannakopoulos, *et al.* [4] investigated in detail the influence of mechanical properties, Young's modulus, yield stress, and hardening coefficient, on the deformation of materials under a Vickers indenter by using FEM analyses. The same method was applied to creep materials by Bower *et al.* and Storakers *et al.*, using results from Hill. In all these cases, the indentation problem for the entire loading history is reduced to the solution of a single non-linear boundary value problem. This greatly facilitates the numerical solution of the reduced problem, since the finite element mesh can be chosen appropriately to the extent of the contact area and discretization errors as the contact area expands over the mesh are eliminated.

Field and Swain [5] focused on the behavior of pilling up and sinking in to characterize mechanical properties. Based on the FEM results, Cheng, *et al.* [6] addressed the limitation for the determination of the stress-strain relationship from the load-depth curve measured using the depth sensing indentation technique with conical and pyramidal indenters.

Spherical indentation overcomes many of the limitations associated with pyramidal indenters and allows mechanical properties to be evaluated by following the transition from elastic to plastic behavior. This also enables the yield stress to be calculated. Indentation behavior depends on the ratio between the actual strain and the yield strain of the material; low ratios produce elastic behavior whereas high ratios produce plastic behavior. Here, Spherical-indentation technique is analyzed by finite element (FE) method using ABAQUSTM.

2.5 Meyer's Law

Professor Eugene Meyer of the material testing laboratory at the imperial school of technology, Charlottenburg, made an intensive study of the Brinell ball indentation hardness test and published his results in 1908. Meyer's work showed that resistance to penetration by a ball penetrator varies with the degree of penetration of the ball and follows the relation

$$P = ad^n \quad (2.5)$$

Where,

P = load

d = diameter of indentation

a and n are constants of the material under test.

The above relation is known as Meyer's law.

The mean pressure between the surface of the indenter and the indentation is equal to the ratio of the load to the projected area of the indentation. This quantity, as a measure of the hardness, was first proposed by Meyer in 1908 and is referred to as the Meyer hardness.

$$p_m = \text{Meyer hardness} = \frac{4P}{\pi d^2} \quad (2.6)$$

Meyer's work, carried out using a ball indenter, has been instrumental in giving a much clearer picture of the mechanics of the ball indentation test, especially the role of work-hardening capacity or strain-hardening ability of a metal.

Direct relation between the hardness number, the amount of deformation or strain, and the yield stress of any metal was given by Tabor (1948). In Tabor's empirical work, it was pointed out that for spherical indentation; the Meyer's hardness has a close agreement with the uniaxial stress-strain curve. He came up with the following stress and strain relations:

$$\epsilon_i = 0.2 \frac{d}{D} \quad (2.7)$$

$$\sigma_i = \frac{p_m}{2.8} \quad (2.8)$$

Where, d is the diameter of indentation, D is the ball indenter diameter and p_m is mean contact pressure.

Chapter 3

Methodology

3.1 Methodology proposed in the current research

We aim to determine the material properties using the indentation technique. We will discuss the methodology to determine these using two available outputs from the Finite element analysis. Applied load which is calculated from the reaction forces at the bottom of the specimen and the contact diameter of the indenter are used in this process. This data is extracted from multiple frames available in the same step of the analysis. Then, we will be using the Meyer's and Tabors empirical equations to obtain the complete stress strain curve in order determine the material properties.

The relation between contact radius (a), indenter diameter (D) and the applied load (P) was proposed by Meyer, which governs spherical indentation, namely:

$$P = K \frac{a^{n+2}}{D^n} \quad (3.1)$$

Where n and K being material constants. The work of O'Neill and Tabor [27] showed that the value of n from indentation was equal to the strain hardening exponent of the material.

3.1.1 Determining strain hardening exponent

Graph is plotted between the applied load (P) and contact radius (a) of the indenter. Then the points obtained in the plane are curve fitted based on Equation (3.1) whose exponent is of the form $n + 2$, where n is material's strain hardening. Thus, the

value of the power minus 2 gives the strain hardening coefficient of the material being analyzed. Let the curve-fitted equation obtained is of the form:

$$p = ya^n \quad (3.2)$$

Where, y is some constant.

This is followed by the use of Tabor's empirical equations of stress and strain:

$$\varepsilon_i = 0.2 \frac{d}{D} \quad (3.3)$$

$$\sigma_i = \frac{P_m}{2.8} \quad (3.4)$$

Where,

d = Contact diameter

D = Diameter of the ball indenter and

$$P_m = \frac{4p}{\pi d^2}$$

p = Curve represented by Equation (3.2)

3.1.2 Determining yield stress from stress-strain curve

Graph is plotted between stress and strain. The resultant points obtained are the post yielding region of the stress strain curve. The points obtained are power curve fitted.

The resultant equation is of the form:

$$\sigma_i = \delta \cdot \varepsilon_i^n \quad (3.5)$$

Where, δ is a constant and n is strain hardening coefficient.

The material follows Hook's law before yielding i.e. it behaves linearly before the yield point.

$$\sigma = E \cdot \varepsilon \quad (3.6)$$

In order to obtain the yield stress, the point of intersection of the linear and non linear part of the stress strain curve is to be found. Solving the above Equations (3.5) and (3.6) we obtain

$$S_y = \left(\frac{\delta}{E^n} \right)^{\frac{1}{1-n}} \quad (3.7)$$

Where,

S_y = Yield Stress

δ = Constant obtained from Equation (3.5)

E = Young's Modulus

n = Strain hardening coefficient.

The value obtained by using Equation (3.7) is the material yield stress. Thus by employing the above methodology, the material mechanical properties: yield stress and strain hardening exponent are found from the finite element simulation of the indentation technique.

Chapter 4

Finite Element Modeling

4.1 ABAQUS™ software

ABAQUS™ was founded in 1978 by David Hibbitt, Bengt Karlsson, and Paul Sorensen. Commercial FEM simulation software ABAQUS™ 6.4 is used to carry out the indentation simulation. The ABAQUS™ suite of software for finite element analysis (FEA) is known for its high performance, quality and ability to solve challenging simulations ranging from simple linear to complex nonlinear simulations.

The ABAQUS™ suite consists of three core products - ABAQUS™ /Standard, ABAQUS™ /Explicit and ABAQUS™ /CAE.

ABAQUS™ /Standard is a general-purpose, finite-element program designed to simulate large scale, complex linear problems and highly nonlinear problems.

ABAQUS™ /Explicit provides ABAQUS™ analysis technology focused on transient dynamics and quasi-static analyses using an explicit approach appropriate in many applications such as drop test, crushing and many manufacturing processes.

ABAQUS™ /CAE provides a complete modeling and visualization environment for ABAQUS analysis products. With direct access to CAD models, advanced meshing and visualization, and with an exclusive view towards ABAQUS™ analysis products, ABAQUS™ /CAE is the modeling environment of choice for many ABAQUS™ users.

In our present simulation ABAQUS™ /Standard and ABAQUS™ /CAE has been used.

ABAQUSTM /CAE is used as the preprocessor for the creation of the part, inputting the property of the materials, assembly of the model, for defining the interaction, applying the load and meshing the model. It is also used as the postprocessor for viewing the results.

4.2 Solving the problem using ABAQUSTM

The present analysis is a nonlinear simulation. There are basically two types of nonlinearity involved in this simulation: material nonlinearity and the other being geometrical in nature.

Many metals display material nonlinearity at high strain values. At high strains, the materials yield and thus stress/strain relation is no longer linear. The plastic behavior of a material is described by its yield point and yield hardening. In general, the yield stress of the material is 0.05 to 0.1 % of the materials yield's modulus. Many metals show work hardening. Work hardening is when a metal is strained beyond the yield point. An increasing stress is required to produce additional plastic deformation and the metal apparently becomes stronger and more difficult to deform. Indentation process is a highly localized phenomenon. There is high deformation in the area just beneath the indenter and the specimen, thus giving rise to geometrical nonlinearity. Geometric nonlinearity needs to be accounted for when the structure deforms to such an extent that the original geometry and/or position and direction of the loads significantly affect the structural behavior.

4.2.1 Solver used

ABAQUSTM uses Newton-Raphson method to obtain solution for nonlinear simulations. In a nonlinear analysis the solution cannot be calculated by solving a single system of linear equations. Instead, the solution is found by specifying the loading as a function of time and incrementing time to obtain the nonlinear response. Therefore, ABAQUSTM /Standard breaks the simulation into a number of time increments and finds the approximate equilibrium configuration at the end of each time increment.

An increment is part of a step. In nonlinear analyses each step is broken into increments so that the nonlinear solution path can be followed. The user suggests the size of the first increment, and ABAQUSTM /Standard automatically chooses the size of the subsequent increments. At the end of each increment the structure is in (approximate) equilibrium and results are available for writing to the restart, data, results, or output database files. Iteration is an attempt at finding an equilibrium solution in an increment. If the model is not in equilibrium at the end of the iteration, ABAQUSTM /Standard tries another iteration.

4.3 Indentation modeling

It is generally known that an axisymmetric two- dimensional finite element model can be used to capture the result of a full three-dimensional model as long as the projected area/depth of the two models are equivalent. This is done to take calculative efficiency into account. Computations were performed using the general purpose finite element package ABAQUSTM. The complete set in the indentation model consists of a deformable Spherical indenter and specimen. The spherical indenter in the analysis is

made up of tungsten material and the specimens are steel and aluminum alloys. Since, most of the practical indenters are made up of Tungsten, here the indenter material is chosen as tungsten. The radius of the indenter being $80\text{ }\mu\text{m}$ and the length and height of the specimen are $1000\text{ }\mu\text{m}$ each. Thus in reality, it being a spherical indenter of diameter $160\text{ }\mu\text{m}$ and a cylindrical specimen of dimensions $1000\text{ }\mu\text{m}$ radius and $1000\text{ }\mu\text{m}$ in height.

4.4 Element

4-node linear axisymmetric quadrilateral, reduced integration, hourglass control 'CAX4R' element was used. Reduced integration signifies that there is only one Gauss integration point in the element as shown by x1 below.

Figure 4.1 CAX4R element with one gaussian integration point

4.5 Indenter modeling

Deformable indenter of Tungsten material and radius of $800\text{ }\mu\text{m}$ is modeled. The indenter model consists of 1600 elements and 1671 nodes in total. The indenter is bias meshed with high mesh density at the bottom (point of contact). The element size is lowest at the bottom with a measure of $3.91\text{ }\mu\text{m}$ and highest at the top with measure of $19.63\text{ }\mu\text{m}$. Indentation being a highly localized phenomena, very high stresses and high

deformation are produced at the point of contact. Therefore, our main focus is at the area around the point of contact. Elastic property of Tungsten with Young's modulus $E=750$ GPa and Poisson's ratio $\nu=0.33$ is considered. The Young's modulus of indenter is 3 to 5 times the Young's modulus of the specimen.

Figure 4.2 Indenter with biased meshed

4.6 Specimen modeling

Elastic power hardening plastic material property is assumed for the deformable specimen. Steel and Aluminum alloy with different yield stress and strain hardening coefficient are chosen. In total, analysis on 20 such different materials has been performed. The materials in a given set have same Young's modulus and yield strength but with different stain hardening coefficients. The list of materials chosen is listed below.

Aluminum alloys ($E=69$ GPa, $\nu=0.33$)							
$S_y = 275MPa$ (6061-T6)				$S_y = 500MPa$ (7075-T651)			
n=0.09	n=0.18	n=0.27	n=0.36	n=0.09	n=0.18	n=0.27	n=0.36

Steel alloys (E=200 GPa, v=0.3)											
$S_y = 500MPa$				$S_y = 750MPa$				$S_y = 1000MPa$			
n=0.1	n=0.2	n=0.3	n=0.4	n=0.1	n=0.2	n=0.3	n=0.4	n=0.1	n=0.2	n=0.3	n=0.4

Table 4.1 List of materials being analyzed

For Plastic property, the uniaxial true stress (σ) true strain (ϵ_p) data is required to input. There are several math models which describe the true stress strain curve. Some among them are:

a) Ramberg –Osgood model

$$\epsilon = \begin{cases} = \sigma/E; & \sigma < \sigma_y \\ = \epsilon_e + \epsilon_p = \frac{\sigma}{E} + a\left(\frac{\sigma}{b}\right)^n & otherwise \end{cases} \quad (4.1)$$

b) Linear elastic – linear strain hardening plastic model

$$\epsilon = \begin{cases} \sigma/E; & \sigma < \sigma_y \\ (\sigma - \sigma_y)/m + \sigma_y/E & otherwise \end{cases} \quad (4.2)$$

Where, m is the linear strain hardening coefficient.

c) Some other available form is

$$\sigma = \begin{cases} E\epsilon & \sigma < \sigma_y \\ \sigma_y + K\left(\epsilon - \frac{\sigma_y}{E}\right)^n & otherwise \end{cases} \quad (4.3)$$

d) piecewise linear elastic – power-law plastic model

$$\varepsilon = \begin{cases} \sigma/E; & \sigma < \sigma_Y \\ \frac{\sigma_Y}{E} \left(\frac{\sigma}{\sigma_Y} \right)^{1/n} & \text{otherwise} \end{cases} \quad (\text{or}) \quad (4.4)$$

$$\sigma = \begin{cases} E\varepsilon & \sigma < \sigma_Y \\ \sigma_Y \left(\frac{E}{\sigma_Y} \right)^n \varepsilon^n & \text{otherwise} \end{cases} \quad (4.5)$$

Most widely used math model's are linear elastic – power-law plastic, Ramberg – Osgood and power law hardening model.

In the present work, piecewise linear elastic – power-law plastic model represented by Equation (4.4) is used. Few simulations have been carried out for Ramberg –Osgood model represented by Equation (4.1) and power law hardening model.

Figure 4.3 Specimen with bias meshed.

The specimen is also bias meshed in a similar way as the indenter. This approach is applied to reduce the computational time. Very fine mesh with a least size of 10μm is

modeled at the top surface of the specimen and the size of the mesh gets coarser as we move away. The maximum size of the mesh used in the specimen is $640\mu\text{m}$. It is essential to have such a highly refined mesh in the contact area in order to calculate the contact radius accurately. It has been observed that a small error in the calculation of the contact radius results in high error in calculation of yield stress.

The trapezoidal shaped element is used in the transition area to connect the fine mesh with the coarse. Figure 4.4 below shows the transition area where trapezoidal elements are used to connect a finer with a coarser region. This process is employed through out the model giving rise to a stair case mesh model.

Figure 4.4 Trapezoidal element used in transition region

In total, 21573 elements and 21930 nodes are in the specimen model. Convergence of the present mesh was compared with the simulation of still finer mesh. Results showed no significant changes. Thus, the fineness of the present mesh is validated.

ABAQUSTM has no designated unit system I.e. the unit system is arbitrary as long as there is a consistency in the entire unit being used in the analysis. Here, we will be inputting the force in Newton (N) and the length is micrometer (μm).

4.7 Boundary condition and load

The specimen has fixed bottom and the side along the axis of symmetry is constrained in X-direction, as it is an axis-symmetry model. The specimen is free to move in Y-direction. The other two sides are left free. The specimen size is chosen large enough so that the results obtained are independent of the boundary conditions setup. The Figure 4.5 below shows the constrained specimen.

Figure 4.5 Boundary conditions applied on the specimen

The indenter is also constrained in X-direction along the axis of symmetry. The moment of the indenter is controlled along the Y-direction.

The load can be applied on the indenter in two methods: load controlled and displacement controlled. In the load controlled method, the desired load is applied at the top surface of the indenter. In displacement control, the moment of the indenter is controlled by inputting the desired displacement to the top surface nodes. Here we have used the displacement controlled method in order to control the moment of the indenter. Thus, all the nodes at the top surface are given a constant displacement value. It has been noticed

that no matter which method is employed to input the load, the results obtained are the same. Arrows at the top surface, in the below Figure 4.6 shows the direction of the displacement of the indenter.

Figure 4.6 Boundary conditions applied on the deformable indenter

In this work, load applied on the indenter is calculated in an indirect method. For a given displacement of the indenter, the load applied is calculated by summing up the total reaction forces at the bottom of the specimen. The results are extracted from multiple frames for a given step. Thus, the Contact radius and the total force applied for a particular displacement of the indenter are extracted from all the frames.

4.8 Interaction between indenter and specimen

ABAQUSTM defines contact between two bodies in terms of two surfaces that may interact: these surfaces are called ‘contact pair’. Contact interaction occurs between two surfaces. However, in strict master-slave contact formulation in ABAQUSTM /Standard, only the master surface is used as a surface. The only data required by the

ABAQUS™ from the slave surface are the location of its nodes and the surface area associated with each node.

4.8.1 Choosing the master and slave surfaces of a contact pair

ABAQUS™ defines that, slave surfaces must always be attached to deformable bodies. Rigid bodies must always be the master surfaces in the contact pair. When both the bodies are deformable, the stiffer body is chosen as the master surface.

In the present simulation, the interaction between the indenter and the specimen is modeled as contact pair. The indenter surface is defined as ‘master’ surface and the specimen surface as ‘slave’ surface because the indenter is stiffer than the specimen.

Figure 4.7 Demonstrating the master and the slave surface

Earlier work by Nix [22] was carried out under the assumption of frictionless conditions between the indenter and the specimen. He claimed no considerable change in load displacement response even with a friction coefficient of 1. But this has been challenged by the subsequent work of Mesarovic and Fleck claiming that friction could

decrease the contact area by 8%. In the present work, we will be assuming the friction coefficient to be 0.1 in all the analysis models.

4.9 Simulation of the Indentation process

The finite element model is prepared by following the procedure discussed above. Maximum depth of the indentation is 320 μm in all the models. Pile-up is observed for materials with strain hardening exponent less than 0.2 and sink-in for material with strain hardening exponent greater than 0.2. Figures 4.8 and 4.9 below demonstrate the pile up and sink during the indentation process.

Figure 4.8 Pileup of the material
with $n=0.18$

Figure 4.9 Sink in of the material
with $n=0.36$

Chapter 5

Results

5.1 Materials analyzed

Finite element simulation has been performed on wide variety of steel and aluminum alloys. The table below gives the list of material matrix.

Aluminum alloys (E=69 GPa, $\nu=0.33$)											
$S_y = 275MPa$ (6061-T6)						$S_y = 500MPa$ (7075-T651)					
n=0.09	n=0.18	n=0.27	n=0.36	n=0.09	n=0.18	n=0.27	n=0.36	n=0.09	n=0.18	n=0.27	n=0.36
Steel alloys (E=200 GPa, $\nu=0.3$)											
$S_y = 500MPa$				$S_y = 750MPa$				$S_y = 1000MPa$			
n=0.1	n=0.2	n=0.3	n=0.4	n=0.1	n=0.2	n=0.3	n=0.4	n=0.1	n=0.2	n=0.3	n=0.4

Table 5.1 List of material properties

As a result of various heat treatment processes, the materials in a given set of alloys may have same Young's modulus but different yield stress and strain hardening coefficient. Thus, we have chosen materials with same Young's modulus and yield stress but with different strain hardening coefficient to perform the indentation simulation.

Material of the indenter is tungsten with Young's modulus of 650 GPa and Poisson's ratio 0.2. The indenter has much higher hardness value when compared with the materials being analyses. In practice, many commercially available indenters are made up of tungsten material.

5.2 Application of the methodology

The indenter is indented to a depth of 320 μm . It has been applied in multiple steps. The purpose of this is to obtain sufficient number of frames within a step, so that the required output can be extracted from the frames. It is then followed by calculating the force applied and the contact diameter of the indenter for a given displacement of the indenter. The force is calculated by summing up the total reaction force at the bottom of the specimen and at the same time noting the contact diameter. These two are the only required outputs in order to obtain the complete stress-strain curve of a material by applying the present proposed methodology. This is then followed by applying the Tabor's stress strain Equation (5.1).

$$\varepsilon_i = 0.2 \frac{d}{D} \text{ and } \sigma_i = \frac{P_m}{2.8} \quad (5.1)$$

Where,

d = Contact diameter

D = Diameter of the ball indenter and

$$P_m = \frac{4p}{\pi d^2}$$

p= Applied load

Now, the methodology explained in Chapter 5 is employed. The applied load versus the contact radius and stress versus strain graphs are plotted and the required material properties are calculated. The following results were obtained.

Aluminum, $E=69$ GPa, $S_y=275$ MPa

Figure 5.1 Load Versus contact radius for aluminum

$E=69$ GPa, $S_y=275$ MPa, $n=0.09$

Figure 5.2 Stress Versus strain for aluminum

$E=69$ GPa, $S_y=275$ MPa, $n=0.09$

Figure 5.3 Load Versus contact radius for aluminum
 $E=69 \text{ GPa}$, $S_y=275 \text{ MPa}$, $n=0.18$

Figure 5.4 Stress Versus strain for aluminum
 $E=69 \text{ GPa}$, $S_y=275 \text{ MPa}$, $n=0.18$

Figure 5.5 Load Versus contact radius for aluminum
 $E=69 \text{ GPa}$, $S_y=275 \text{ MPa}$, $n=0.27$

Figure 5.6 Stress Versus strain for aluminum, $E=69 \text{ GPa}$
 $S_y=275 \text{ MPa}$, $n=0.27$

Figure 5.7 Load Versus contact radius for aluminum
 $E=69 \text{ GPa}$, $S_y=275 \text{ MPa}$, $n=0.36$

Figure 5.8 Stress Versus strain for aluminum, $E=69 \text{ GPa}$,
 $S_y=275 \text{ MPa}$, $n=0.36$

From load versus contact radius graph, the strain hardening exponent of the material is obtained by subtracting 2 from the exponent of the curve obtained. And then using the Tabor's empirical stress and strain equation, we obtain the stress versus strain graph.

Numerical results with the theoretical values are tabulated below.

Input n	n obtained from Load Versus Contact Radius graph	Input yield (Mpa)	Yield stress from Graph (Mpa)	Error (%)
0.09	0.094	275	280.7	2.09 %
0.18	0.182	275	284.2	3.37 %
0.27	0.28	275	270.39	1.67 %
0.36	0.368	275	240.13	12.67 %

Table 5.2 Comparison of the results obtained for aluminum
 $E=69 \text{ GPa}$, $S_y=275 \text{ MPa}$

From the above Table 5.2 we observe that values of the strain hardening coefficient, n , obtained from the finite element simulations using the proposed methodology are in good agreement with the input values. Yield stresses are also in good agreement with error percentage ranging from 2 to 12 %. Yield stress values calculated for the material with n values of 0.9, 0.18, and 0.27 are in good agreement, however, the error percentage is high for material with $n = 0.36$. This error can be attributed to the Tabor's constant. This issue will be discussed in detail in the later part of this chapter.

Aluminum, $E = 69 \text{ GPa}$, $S_y = 500 \text{ MPa}$

Figure 5.9 Load Versus contact radius for aluminum

$E=69 \text{ GPa}$, $S_y=500 \text{ MPa}$, $n=0.09$

Figure 5.10 Stress Versus strain for aluminum, $E=69 \text{ GPa}$

$S_y=500 \text{ MPa}$, $n=0.09$

Figure 5.11 Load Versus contact radius for aluminum
 $E=69 \text{ GPa}$, $S_y=500 \text{ MPa}$, $n=0.18$

Figure 5.12 Stress Versus strain for aluminum, $E=69 \text{ GPa}$
 $S_y=500 \text{ MPa}$, $n=0.18$

Figure 5.13 Load Versus contact radius for aluminum
 $E=69 \text{ GPa}$, $S_y=500 \text{ MPa}$, $n=0.27$

Figure 5.14 Stress Versus strain for aluminum, $E=69 \text{ GPa}$
 $S_y=500 \text{ MPa}$, $n=0.27$

Figure 5.15 Load Versus contact radius for aluminum
 $E=69 \text{ GPa}$, $S_y=500 \text{ MPa}$, $n=0.36$

Figure 5.16 Stress Versus strain for aluminum, $E=69 \text{ GPa}$
 $S_y=500 \text{ MPa}$, $n=0.36$

Numerical results with the theoretical values are tabulated below.

Input n	n obtained from Load Versus Contact Radius graph	Input yield (MPa)	Yield stress from Graph (MPa)	Error (%)
0.09	0.096	500	503.9	0.79 %
0.18	0.18	500	501.2	0.24 %
0.27	0.275	500	484.5	3.08 %
0.36	0.376	500	413.1	17.36 %

Table 5.3 Comparison of the results obtained for aluminum

$$E=69 \text{ GPa}, S_y=500 \text{ MPa}$$

From the above Table 5.3, values of the strain hardening coefficient, n , obtained from the finite element simulations using the proposed methodology are in good agreement with the input values. Yield stress is also in good agreement for strain hardening materials of 0.09, 0.18, and 0.27. However, the error is 17% for material with $n = 0.36$. This error in yield stress calculation can be attributed to the Tabor's constant. This issue will be discussed in detail in the later part of this chapter.

STEEL ALLOY

$E = 200 \text{ GPa}$, $S_y = 500 \text{ MPa}$

Figure 5.17 Load Versus contact radius for steel, $E=200 \text{ GPa}$
 $S_y=500 \text{ MPa}$, $n=0.1$

Figure 5.18 Stress Versus strain for steel, $E=200 \text{ GPa}$
 $S_y=500 \text{ MPa}$, $n=0.1$

Figure 5.19 Load Versus contact radius for steel, $E=200$ GPa
 $S_y=500$ MPa, $n=0.2$

Figure 5.20 Stress Versus strain for steel, $E=200$ GPa
 $S_y=500$ MPa, $n=0.2$

Figure 5.21 Load Versus contact radius for steel, E=200 GPa

$S_y = 500$ MPa, $n=0.3$

Figure 5.22 Stress Versus strain for steel, E=200 GPa

$S_y = 500$ MPa, $n=0.3$

Figure 5.23 Load Versus contact radius for steel, E=200 GPa

$$S_y = 500 \text{ MPa}, n=0.4$$

Figure 5.24 Stress Versus strain for steel, E=200 GPa

$$S_y = 500 \text{ MPa}, n=0.4$$

Numerical results with the theoretical values are tabulated below.

Input n	n obtained from Load Versus Contact Radius graph	Input yield (MPa)	Yield stress from Graph (MPa)	Error (%)
0.1	0.1012	500	518.5	3.71 %
0.2	0.2059	500	514.3	2.87 %
0.3	0.3072	500	477.5	4.49 %
0.4	0.4036	500	430.04	14.01%

Table 5.4 Comparison of the results obtained for steel

$$E=200 \text{ GPa}, S_y=500 \text{ MPa}$$

From the above Table 5.4, the strain hardening coefficient values obtained from the finite element simulations are in good agreement with the input values. Yield stress is also in good agreement, with error being 2 to 5 % for material with n values less than 0.3. High error percentage is observed for material with $n = 0.4$.

E=200 GPa, $S_y=750$ MPa

Figure 5.25 Load Versus contact radius for steel, E=200 GPa
 $S_y=750$ MPa, $n=0.1$

Figure 5.26 Stress Versus strain for steel, E=200 GPa
 $S_y=750$ MPa, $n=0.1$

Figure 5.27 Load Versus contact radius for steel, $E=200$ GPa
 $S_y=750$ MPa, $n=0.2$

Figure 5.28 Stress Versus strain for steel, $E=200$ GPa
 $S_y=750$ MPa, $n=0.2$

Figure 5.29 Load Versus contact radius for steel, $E=200$ GPa
 $S_y=750$ MPa, $n=0.3$

Figure 5.30 Stress Versus strain for steel, $E=200$ GPa
 $S_y=750$ MPa, $n=0.3$

Figure 5.31 Load Versus contact radius for steel, $E=200$ GPa
 $S_y=750$ MPa, $n=0.4$

Figure 5.32 Stress Versus strain for steel, $E=200$ GPa
 $S_y=750$ MPa, $n=0.4$

Numerical results with the theoretical values are tabulated below:

Input n	n obtained from Load Versus Contact Radius graph	Input yield (MPa)	Yield stress from Graph (MPa)	Error (%)
0.1	0.1019	750	782.36	4.31 %
0.2	0.2015	750	769.02	2.53 %
0.3	0.3077	750	724.67	3.37 %
0.4	0.4098	750	650.77	15.24%

Table 5.5 CoMparison of the results obtained for Steel

$E=200 \text{ GPa}$, $S_y=750 \text{ MPa}$

Values of the strain hardening coefficient, n , obtained from the finite element simulations are in good agreement with the input values. Yield stresses are also in good agreement with errors ranging from 2 to 4 % for strain hardening exponent less than 0.3. However, the error is around 14 % for material with strain hardening coefficient 0.4.

Steel, $E=200$ GPa, $S_y=1000$ MPa

Figure 5.33 Load Versus contact radius for steel $E=200$ GPa
 $S_y=1000$ MPa, $n=0.1$

Figure 5.34 Stress Versus strain for steel, $E=200$ GPa
 $S_y=1000$ MPa, $n=0.1$

Figure 5.35 Load Versus contact radius for steel, $E=200$ GPa
 $S_y=1000$ MPa, $n=0.2$

Figure 5.36 Stress Versus strain for steel, $E=200$ GPa
 $S_y=1000$ MPa, $n=0.2$

Figure 5.37 Load Versus contact radius for steel, $E=200$ GPa
 $S_y=1000$ MPa, $n=0.3$

Figure 5.38 Stress Versus strain for steel, $E=200$ GPa
 $S_y=1000$ MPa, $n=0.3$

Figure 5.39 Load Versus contact radius for steel, $E=200$ GPa
 $S_y=1000$ MPa, $n=0.4$

Figure 5.40 Stress Versus strain for steel, $E=200$ GPa
 $S_y=1000$ MPa, $n=0.4$

Numerical results with the theoretical values are tabulated below.

Input n	n obtained from Load Versus Contact Radius graph	Input yield (MPa)	Yield stress from Graph (MPa)	Error (%)
0.1	0.1008	1000	1017.62	1.76%
0.2	0.204	1000	1039.75	3.97%
0.3	0.3028	1000	989.59	1.04%
0.4	0.4074	1000	876.05	12.39%

Table 5.6 Comparison of the results obtained for steel

$$E=200 \text{ GPa}, S_y=1000 \text{ MPa}$$

It is observed that the values of the strain hardening coefficient, obtained from the finite element simulations are in good agreement with the input values. Yield stresses are also in good agreement, with errors ranging from 1 to 4 % for n less than 0.3. Error is around 13% for material with n 0.4. Error in the yield stress calculation can be attributed to the Tabor's constant. This issue will be discussed in detail in the later part of this chapter.

5.3 Variation in error percentage with increase of strain hardening coefficient

Figure 5.41 Percentage error variations with strain hardening coefficient in aluminum alloys

From the above Figure 5.41, it is clear that percentage error in the calculation of yield stress is high for material with n over 0.3. It is observed that the percentage error increases with the increase of material strain hardening coefficient.

Figure 5.42 Percentage error variations with strain hardening coefficient of steel alloys

From the above discussions, we can see that in spite of good agreement in determining the strain hardening coefficient, there is error in the calculation of the yield stress of the material for n values larger than 0.3.

5.4 Results from Modified Tabor's Constant

It is necessary to study the effect of Tabor's Constant on the behavior of stress strain curve. It is noticed that by varying the Tabor's constant, nature of the stress strain curve remains same but the curve shifts either up or down depending on the constants value.

Figure 5.43 Effect of Tabor's constant on stress strain curve

From the above Figure 5.43, we notice that the curve shifts down when the constant has been changed from 2.8 to 2.92. This behavior has to be observed carefully as this effects the calculation of the yield stress. As the yield stress is calculated by the intersection of post yielding stress-strain curve with the linear part of the stress strain

curve, the location of the former is important for the correct calculation of the yield stress.

It can be concluded that error in the calculation of yield stress is an effect of the Tabor's constant. One can minimize the error by varying the Tabor's constant for a given materials with different strain hardening. This understanding has been employed to see how the Tabor's constant varies with the increase of strain hardening exponent. In this process, we will try to either increase or decrease the value of the Tabor's constant so that the yield stress value obtained from the stress strain curve is close to the input values. Then we will study how the value of Tabor's constant changes with the change in strain hardening coefficient. The above discussed method has been employed to all the 20 material models being analyzed in this research.

Curve blue in color corresponds to the Tabor's constant 2.8 and the curve pink in color corresponds to the modified value of Tabor's constant which is different for different materials and depends on the value of strain hardening exponent. The following results were obtained.

Aluminum, $E = 69 \text{ GPa}$, $S_y = 275 \text{ MPa}$

Figure 5.44 Comparison of stress Versus strain curves for aluminum, $E=69 \text{ GPa}$, $S_y=275 \text{ MPa}$, $n=0.09$

Figure 5.45 Comparison of stress Versus strain curves for aluminum, $E=69 \text{ GPa}$, $S_y=275 \text{ MPa}$, $n=0.18$

Figure 5.46 Comparison of stress Versus strain curves for aluminum, $E=69 \text{ GPa}$, $S_y=275 \text{ MPa}$, $n=0.27$

Figure 5.47 Comparison of stress Versus strain curves for aluminum, $E=69 \text{ GPa}$, $S_y=275 \text{ MPa}$, $n=0.36$

5.4.1 Comparison of the yield stress values obtained from Tabor's Constant 2.8 and modified Tabor's Constant for Aluminum $E=69$ GPa, $S_y=275$ MPa.

Input n	Input S_y (MPa)	Tabor's constant	S_y (MPa) obtained	Error (%)	Modified Tabor's constant	S_y from modified constant	Error (%)
0.09	275	2.8	280.7	2.09%	2.85	275.3	0.11%
0.18	275	2.8	284.2	3.37%	2.87	275.82	0.30%
0.27	275	2.8	270.3	1.67%	2.76	275.84	0.30%
0.36	275	2.8	240.13	12.67%	2.57	275.04	0.05%

Table 5.7 Comparison of the results obtained before and after the change in Tabor's constant for aluminum, $E=69$ GPa, $S_y=275$ MPa

From the above table we can see that the percentage error in the calculation of yield stress with modified Tabor's constant is low, i.e. less than 1%. Figure 5.48 below shows the variation of the modified Tabor's constant with the strain hardening exponent. We can see that the constant value decreases as the n increases.

Figure 5.48 Strain hardening exponent Versus modified constant for aluminum, $E=69$ GPa, $S_y=275$ MPa

Aluminum, $E=69$ GPa, $S_y=500$ MPa

Figure 5.49 Comparison of stress Versus strain curves for aluminum, $E=69$ GPa, $S_y=500$ MPa, $n=0.09$

Figure 5.50 Comparison of stress Versus strain curves for aluminum, $E=69$ GPa, $S_y=500$ MPa, $n=0.18$

Figure 5.51 Comparison of stress Versus strain curves for aluminum, $E=69$ GPa, $S_y=500$ MPa, $n=0.27$

Figure 5.52 Comparison of stress Versus strain curves for aluminum, $E=69$ GPa, $S_y=500$ MPa, $n=0.36$

5.4.2 Comparison of the yield stress values obtained from Tabor's Constant 2.8 and modified Tabor's Constant for Aluminum $E=69$ GPa, $S_y=500$ MPa

Input n	Input S_y (MPa)	Tabor's constant	S_y (MPa) obtained	Error (%)	Modified Tabor's constant	S_y from modified constant	Error (%)
0.09	500	2.8	503.9	0.79%	2.82	500.03	0.06%
0.18	500	2.8	501.2	0.24%	2.81	499.04	0.19%
0.27	500	2.8	484.5	3.08%	2.73	501.8	0.36%
0.36	500	2.8	413.1	17.36%	2.48	501.92	0.38%

Table 5.8 Comparison of the results obtained before and after the change in Tabor's constant for aluminum, $E=69$ GPa, $S_y=500$ MPa

Table 5.8 above shows the percentage error in the calculation of yield stress with modified Tabor's constant is less than 0.5% when compared with 17% with Tabor's constant set to 2.8. Figure 5.53 below shows the variation of the modified Tabor's constant with the strain hardening exponent. We can see that the constant value decreases as the value of n increases. It decreases very rapidly for material with n over 0.3.

Figure 5.53 Strain hardening exponent Versus modified constant for aluminum, $E=69$ GPa, $S_y=500$ MPa

Steel, E=200 GPa, S_y=500 MPa

Figure 5.54 Comparison of stress Versus strain curves for steel, E=200 GPa, S_y=500 MPa, n=0.1

Figure 5.55 Comparison of stress Versus strain curves for steel, E=200 GPa, S_y=500 MPa, n=0.2

Figure 5.56 Comparison of stress Versus strain curves for steel, E=200 GPa, S_y=500 MPa, n=0.3

Figure 5.57 Comparison of stress Versus strain curves for steel, E=200 GPa, S_y=500 MPa, n=0.4

5.4.3 Comparison of the yield stress values obtained from Tabor's Constant 2.8 and modified Tabor's constant for steel $E=200$ GPa, $S_y=500$ MPa

Input n	Input S_y (MPa)	Tabor's constant	S_y (MPa) obtained	Error (%)	Modified Tabor's constant	S_y from modified constant	Error (%)
0.1	500	2.8	518.55	3.71%	2.89	500.61	0.12%
0.2	500	2.8	514.39	2.87%	2.86	500.82	0.16%
0.3	500	2.8	477.50	4.49%	2.71	500.55	0.11%
0.4	500	2.8	430.04	14.01%	2.6	501.62	0.32%

Table 5.9 Comparison of the results obtained before and after the change in Tabor's constant for steel, $E=200$ GPa, $S_y=500$ MPa

Yield stress values obtained from the methodology proposed after varying the Tabor's constant are in good agreement with the input values. The constant value drops rapidly for materials with n greater than 0.3 as shown below.

Figure 5.58 Strain hardening exponent Versus modified constant for steel, $E=200$ GPa, $S_y=500$ MPa

Steel, E=200 GPa, $S_y=750$ MPa

Figure 5.59 Comparison of stress Versus strain curves for steel, E=200 GPa, $S_y=750$ MPa, n=0.1

Figure 5.60 Comparison of stress Versus strain curves for steel, E=200 GPa, $S_y=750$ MPa, n=0.2

Figure 5.61 Comparison of stress Versus strain curves for steel, E=200 GPa, $S_y=750$ MPa, n=0.3

Figure 5.62 Comparison of stress Versus strain curves for steel, E=200 GPa, $S_y=750$ MPa, n=0.4

5.4.4 Comparison of the yield stress values obtained from Tabor's constant 2.8 and modified Tabor's constant for steel $E=200$ GPa, $S_y=750$ MPa

Input n	Input S_y (MPa)	Tabor's constant	S_y (MPa) obtained	Error (%)	Modified Tabor's constant	S_y from modified constant	Error (%)
0.1	750	2.8	782.36	4.31%	2.905	750.97	0.12%
0.2	750	2.8	769.02	2.53%	2.857	749.83	0.02%
0.3	750	2.8	724.67	3.37%	2.734	750.07	0.05%
0.4	750	2.8	650.77	15.24%	2.57	750.07	0.03%

Table 5.10 Comparison of the results obtained before and after the change in Tabor's constant for steel, $E=200$ GPa, $S_y=750$ MPa

The behavior in the variation of the Tabor constant with n is the same as in the previous case. There is a steep decrease in the value of the modified constant for the material with n greater than 0.3.

Figure 5.63 Strain hardening exponent Versus modified constant for steel, $E=200$ GPa, $S_y=750$ MPa

Steel, $E=200$ GPa, $S_y=1000$ MPa

Figure 5.64 Comparison of stress Versus strain curves for steel, $E=200$ GPa, $S_y=1000$ MPa, $n=0.1$

Figure 5.65 Comparison of stress Versus strain curves for steel, $E=200$ GPa, $S_y=1000$ MPa, $n=0.2$

Figure 5.66 Comparison of stress Versus strain curves for steel, $E=200$ GPa, $S_y=1000$ MPa, $n=0.3$

Figure 5.67 Comparison of stress Versus strain curves for steel, $E=200$ GPa, $S_y=1000$ MPa, $n=0.4$

5.4.5 Comparison of the yield stress values obtained from Tabor's Constant 2.8 and modified Tabor's constant for steel $E=200$ GPa, $S_y=1000$ MPa

Input n	Input S_y (MPa)	Tabor's constant	S_y (MPa) obtained	Error (%)	Modified Tabor's constant	S_y from modified constant	Error (%)
0.1	1000	2.8	1017.6	1.76%	2.84	1001.6	0.36%
0.2	1000	2.8	1039.7	3.97%	2.88	1003.8	0.13%
0.3	1000	2.8	989.5	1.04%	2.62	995.1	0.07%
0.4	1000	2.8	876.05	12.39%	2.55	999.2	0.01%

Table 5.11 Comparison of the results obtained before and after the change in Tabor's constant for steel, $E=200$ GPa, $S_y=1000$ MPa

Good results are obtained with the modified Tabor's constant. The error is small when compared to the results obtained by taking the Tabor's constant as 2.8. Variation of the constant with n is shown in the Figure 5.68 below.

Figure 5.68 Strain hardening exponent Versus modified constant for steel, $E=200$ GPa, $S_y=1000$ MPa

Figure 5.69 below shows the variation of Tabors constant with strain hardening coefficient in all the materials.

Figure 5.69 Strain hardening exponent Versus modified Tabor constant

Figure 5.70 Results from the work of B. Taljat [12]

Variation of the Tabors constant observed in the present research has been compared with the results obtained by B.Taljat, *et al.*[12] for the material A533-B Steel, with material properties: Young's modulus 210 GPa and Yield stress 400 MPa. Results have also been compared with the work of Hagag [28].

5.5 Simulation results for Ramberg-Osgood model

Finite element simulations were also carried out on steel alloys whose stress-strain curve is defined by Ramberg-Osgood model with $E=208$ GPa, $\nu=0.333$, $S_y=256$

MPa, yield offset $\alpha=17.176$, exponent $N=3.585$ where $N=\frac{1}{n}$ and $n=0.2889$.

Figure 5.72 Stress Versus strain obtained for Ramberg-Osgood model

The value of the strain hardening exponent obtained from the above is in good agreement with the input n value defined by Ramberg-osgood model.

5.6 Simulation results for Power Law hardening model

One of the most commonly used models to define the stress-strain curve is power law hardening. Curve defined by this model has no definite elastic region. This model typically describes soft ductile materials such as lead. Equation below describes the power law hardening model.

$$\sigma = A \varepsilon^n \quad (5.2)$$

For the FE simulation, the stress-strain curve defined by equation $\sigma = 1000 \varepsilon^{0.3968}$ is used as an input. The following results were obtained.

Figure 5.73 Input Stress Versus strain curve describing power law model

Figure 5.74 Stress Versus strain curve obtained from simulation for power law model

Value of strain hardening exponent obtained for the power law hardening model is in good agreement with the input data.

Based on the results obtained in this research, validity of the proposed methodology for metallic alloys is established. It can be successfully employed to determine stress-strain curve and thus material mechanical properties. Strain hardening exponent obtained using the Meyer's empirical equations are in good agreement with the input data. Large percentage error in the calculation of yield stress is an effect of Tabors constant. Analysis of the results shows that the Tabors constant is not a fixed value and varies with material strain hardening. Error in the calculation of yield stress can be minimized by picking the appropriate Tabors constant value.

Chapter 6

Conclusions and Discussions

Finite element spherical indentation simulations of 20 materials (aluminum and steel alloys) were carried out. A methodology is proposed to determine the material mechanical properties such as yield stress and strain hardening parameter, from the indentation data using Meyer's and Tabor's empirical equations.

It has been found that results obtained from the proposed methodology and input material properties are in good agreement. Strain hardening exponent calculated from the Meyer's equation is in close agreement to the input value. It is observed that there is high error percentage in the calculation of yield stress using Tabor's empirical equation for materials with strain hardening exponent over 0.3. Based on the series of results obtained, this error in the calculation of yield stress has been attributed to the Tabor's constant. It is concluded that Tabor's constant (2.8) is not a universal value for all the materials. It changes from material to material and is a function of strain hardening coefficient. Results have been analyzed by modifying the Tabor's constant. Error percentage is low when Tabor's empirical equation is used with modified Tabor's constant. Variation of the modified Tabor's constant with strain hardening exponent has been studied. It is observed that the constant takes value ranging from 2.5 to 2.9 depending on the material strain hardening coefficient. Variation of the constant for materials with n less than 0.25 is small but high for n over 0.25.

Another finding is that data of indentation load and contact diameter corresponding to 3 to 12 % of strain will give good curve fitting and results of strain hardening exponent and yield stress. The above observation is helpful to guide

indentation test as one can concentrate on the data of load and contact diameter corresponding to the proposed strain range. And once the strain hardening coefficient is obtained from the load versus contact diameter curve, the appropriate Tabor's constant can be picked from the strain hardening exponent versus modified Tabor's constant curve (Figure 5.69). Use of this constant in Tabor's equation reduces the percentage error in the calculation the yield stress of the material.

6.1 Limitations

From the finite element results, it is clear that the Tabors constant is not fixed, thus arising the question, how does one pick the right Tabor constant when the indentation of an unknown material is carried out experimentally? How can one determine the right stress-strain model that the material follows? These are few questions that have to be looked upon in future work.

More importantly, the above discussed indentation procedure is limited only to ductile materials. Thus, the research has been done for the application of this procedure on ductile metals. Very little is known about the validity in employment of this approach to brittle materials. Development of cracks in early stages at relatively small loads limits the use of indentation procedure on brittle materials.

Future work should focus on producing the relation between strain hardening coefficient and the correct Tabors constant so that the yield stress can be calculated experimentally with less percentage error. Future research work should also try to develop a methodology to determine the correct stress-strain model from the indentation data. Effort should be placed to extend the indentation technique to determine the mechanical properties of brittle materials.

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DATA SHEETS

Aluminum, $E=69 \text{ GPa}$, $S_y = 275 \text{ MPa}$, $n=0.09$

Aluminum		
E=69 GPa, $S_y=275 \text{ MPa}$, $n=0.09$		
Time Increment	Load (N)	a, Contact Radius (μm)
0.0299609	6.50424	49.9904
0.0662982	17.9222	80.1135
0.135376	42.996	120.39
0.168861	54.8339	140.744
0.202186	69.9999	150.865
0.242179	83.9379	171.274
0.277838	100.619	181.442
0.313283	117.122	191.732
0.350278	132.795	202.058
0.388998	148.794	212.45
0.440436	172.109	233.112
0.47996	189.795	243.549
0.514273	203.223	254.071
0.560952	227.523	264.564
0.600212	244.111	275.167
0.646821	264.547	285.759
0.703065	295.143	296.553
0.751876	317.998	307.381
0.79929	340.051	318.124
0.868789	370.518	329.418
0.920914	395.681	340.369
0.97202	420.651	351.287
1	428.409	352.158
1.03781	450.542	362.666
1.10505	480.9	374.039
1.16679	510.186	385.303
1.23858	542.64	396.892
1.3013	572.917	408.133
1.36695	603.289	419.464
1.4867	651.743	432.838
1.55577	684.562	444.454
1.67906	733.88	466.512
1.74456	766.492	469.477
1.8551	812.226	482.575
2	879.921	505.289

Aluminum, $E=69$ GPa, $S_y = 275$ MPa, $n=0.18$

Aluminum		
$E=69$ GPa, $S_y=275$ MPa, $n=0.18$		
Time Increment	Load (N)	a, Contact Radius (μm)
0.08	23.8864	90.0413
0.1525	52.9732	130.374
0.190215	71.4869	140.451
0.244554	96.7641	160.586
0.303683	126.181	180.918
0.395386	171.925	211.47
0.458462	203.63	232.023
0.513786	236.15	242.309
0.553927	258.203	252.721
0.641833	303.82	273.339
0.714077	346.209	283.983
0.786818	383.245	304.767
0.836578	411.981	315.313
0.900369	449.782	325.895
0.994103	505.804	337.026
1	507.99	337.149
1.085	557.355	357.911
1.175	613.092	369.137
1.27249	664.692	380.724
1.35114	709.293	401.296
1.4375	762.844	412.428
1.52289	815.493	423.66
1.60683	865.223	434.899
1.69352	915.392	446.197
1.8185	986.531	458.586
1.90628	1037.89	470.008
2	1091.87	481.485

Aluminum, $E=69$ GPa, $S_y = 275$ MPa, $n=0.27$

Aluminum		
$E=69$ GPa, $S_y=275$ MPa, $n=0.27$		
Time Increment	Load (N)	a, Contact Radius (μm)
0.11	38.3793	99.9484
0.259062	115.374	160.184
0.404609	199.446	200.586
0.492412	254.545	231.134
0.537342	284.479	241.325
0.643775	356.078	261.698
0.731914	414.49	282.25
0.798565	462.659	292.558
0.901058	532.916	313.184
0.969949	583.057	323.657
1	600.426	333.986
1.06445	646.464	334.515
1.19438	740.806	355.409
1.38659	884.701	386.755
1.49547	961.604	398.084
1.55777	1011.48	408.48
1.69849	1115.16	429.412
1.80384	1196.16	440.685
1.89286	1262.33	451.46
2	1341.58	462.651

Aluminum, $E=69$ GPa, $S_y = 275$ MPa, $n=0.36$

Aluminum		
$E=69$ GPa, $S_y=275$ MPa, $n=0.36$		
Time Increment	Load (N)	a, Contact Radius (μm)
0.0875	30.9241	89.8789
0.2675	133.222	159.828
0.444687	256.464	210.045
0.531172	321.998	230.215
0.645078	411.335	250.459
0.777969	518.892	280.829
0.948828	662.769	311.369
1	707.209	321.603
1.1025	801.567	331.958
1.2825	958.428	362.663
1.485	1149.3	383.743
1.6875	1334.32	414.567
1.86469	1503.64	435.58
1.97121	1604.77	446.16

Aluminum, $E=69$ GPa, $S_y = 500$ MPa, $n=0.09$

Aluminum		
$E=69$ GPa, $S_y=500$ MPa, $n=0.09$		
Time Increment	Load (N)	a, Contact Radius (μm)
0.105625	42.6889	99.9896
0.195625	93.2377	140.29
0.250007	127.262	160.555
0.302614	158.412	180.766
0.439455	247.533	221.55
0.511618	297.766	242.109
0.563936	336.188	252.435
0.628248	384.623	262.85
0.674806	418.398	273.293
0.749578	471.27	294.134
0.803423	511.053	304.663
0.852904	546.754	315.258
0.909454	589.203	325.878
0.981951	647.229	336.686
1	656.619	337.018
1.05719	700.421	347.838
1.13108	752.298	358.906
1.19215	799.244	369.82
1.27915	859	390.772
1.3417	907.532	401.935
1.42677	970.793	413.145
1.50383	1025.14	424.41
1.58065	1081.91	435.808
1.64648	1135.73	437.785
1.72963	1197.32	449.369
1.80904	1257.92	460.826
1.88374	1315.49	472.222
2	1395.01	484.828

Aluminum, $E=69$ GPa, $S_y = 500$ MPa, $n=0.18$

Aluminum		
E=69 GPa, $S_y=500$ MPa, $n=0.18$		
Time Increment	Load (N)	a, Contact Radius (μm)
0.1225	53.3268	109.999
0.251875	135.637	160.192
0.40375	246.95	200.534
0.541387	350.912	241.301
0.601306	404.643	251.535
0.648422	443.564	261.775
0.770059	542.179	282.377
0.857723	620.455	303
0.920565	674.994	313.428
1	739.979	334.23
1.07906	806.862	344.721
1.15263	876.945	355.339
1.23639	954.141	366.086
1.33522	1036.69	377.196
1.42449	1112.23	397.835
1.50753	1189.39	408.672
1.59786	1275.13	419.642
1.68546	1355.5	430.639
1.77163	1433.35	441.656
1.86161	1512.21	452.699
1.97035	1614.2	464.197

Aluminum, $E=69$ GPa, $S_y = 500$ MPa, $n=0.27$

Aluminum		
$E=69$ GPa, $S_y = 500$ MPa, $n=0.27$		
Time Increment	Load (N)	a, Contact Radius (μm)
0.1025	44.5212	89.7962
0.2375	135.652	149.846
0.342266	218.7	179.973
0.456172	315.248	210.159
0.589063	434.303	240.384
0.759922	594.068	270.87
0.930781	763.363	311.744
1	831.938	321.991
1.1025	943.093	332.413
1.2375	1084.7	353.119
1.3725	1228.44	373.88
1.44984	1312.96	384.375
1.53824	1408.01	395.002
1.6824	1560.96	415.788
1.77551	1664.35	426.465
1.86631	1765.25	437.161
2	1910.54	448.466

Aluminum, $E=69$ GPa, $S_y = 500$ MPa, $n=0.36$

Aluminum		
$E=69$ GPa, $S_y = 500$ MPa, $n=0.36$		
Time Increment	Load (N)	a, Contact Radius (μm)
0.1025	46.9819	89.7298
0.276875	176.84	159.558
0.442988	329.925	199.426
0.579142	468.554	229.498
0.723304	624.713	259.716
0.891494	814.881	289.957
1	942.28	310.287
1.10125	1064.71	320.611
1.30375	1315.56	351.155
1.50625	1569.29	381.687
1.72563	1857.8	402.769
2	2222.14	433.854

Steel, $E=200$ GPa, $S_y = 500$ MPa, $n=0.1$

Steel		
$E=200$ GPa, $S_y = 500$ MPa, $n=0.1$		
Time Increment	Load (N)	a, Contact Radius (μm)
0.044375	21.9989	70.1006
0.0715	41.0664	90.2563
0.106665	67.946	110.415
0.143835	96.0989	130.702
0.165024	113.091	140.882
0.186716	129.54	151.078
0.20767	145.862	161.271
0.234611	167.841	171.519
0.264536	194.296	181.801
0.290477	215.84	192.098
0.32341	244.871	202.448
0.350497	269.672	202.666
0.380723	298.256	213.07
0.405028	324.995	223.359
0.430141	348.889	233.697
0.466955	381.203	244.189
0.501725	409.082	254.754
0.542866	443.454	265.324
0.588708	492.391	275.979
0.629533	525.913	286.69
0.680959	582.598	297.482
0.736512	634.823	308.469
0.78075	677.464	319.25
0.826283	710.717	330.129
0.876479	752.354	341.067
0.928236	802.663	352.145
0.968698	847.883	353.294
1	880.139	363.502
1.1225	980.852	385.924
1.15454	1021.38	387.142
1.26066	1111.59	409.056
1.37103	1219.1	422.235
1.46289	1298.39	434.772
1.57514	1389.69	456.631
1.74834	1543.54	472.662
1.83649	1622.61	484.936
1.96461	1726.89	506.812

Steel, $E=200$ GPa, $S_y = 500$ MPa, $n=0.2$

Steel		
E=200 GPa, $S_y = 500$ MPa, $n=0.2$		
Time Increment	Load (N)	a, Contact Radius (μm)
0.06625	41.6859	80.0184
0.111825	79.8989	110.229
0.141005	107.417	120.28
0.174902	136.732	140.516
0.20976	175.287	150.611
0.260963	227.794	170.874
0.298872	268.244	181.078
0.350438	320.394	201.484
0.38655	361.658	211.724
0.419983	396.368	222.004
0.490443	477.555	232.334
0.525374	521.162	242.689
0.562305	567.274	252.992
0.613494	625.387	263.511
0.655608	677.715	273.851
0.697186	730.395	284.245
0.73855	782.109	294.65
0.817256	870.198	305.572
0.864164	931.577	316.048
0.902176	983.308	326.366
0.974088	1069.97	337.186
1	1091.14	337.745
1.04797	1157.56	348.313
1.14129	1269.27	369.095
1.19676	1336.11	370.357
1.26118	1419.76	381.169
1.35441	1536.11	401.824
1.43141	1638.23	412.888
1.50795	1722.41	423.943
1.58302	1817.59	435.079
1.66914	1925.28	446.31
1.76526	2042.27	457.698
1.82947	2133.2	459.682
1.95299	2274.69	480.67
2	2350.32	482.086

Steel, $E=200$ GPa, $S_y = 500$ MPa, $n=0.3$

Steel		
$E=200$ GPa, $S_y = 500$ MPa, $n=0.3$		
Time Increment	Load (N)	a, Contact Radius (μm)
0.06	39.1234	69.9173
0.125	103.058	109.986
0.215	203.444	150.172
0.305	315.593	180.415
0.374375	409.017	200.626
0.421836	480.225	210.724
0.460398	535.917	220.905
0.539612	646.139	241.179
0.581663	707.353	251.466
0.630047	780.624	261.667
0.683989	868.099	271.88
0.782241	1013.12	292.419
0.827281	1085.79	302.744
0.88437	1177.03	313.071
0.944256	1273.43	323.404
0.999727	1366.12	323.827
1	1366.45	323.831
1.07594	1485.73	344.192
1.14169	1594.59	354.651
1.21743	1720.18	365.107
1.2463	1782.57	365.408
1.29963	1869.23	375.694
1.36852	1982.17	386.212
1.45467	2132.33	396.83
1.53725	2276.01	407.5
1.62057	2417.19	418.187
1.69763	2539.58	428.825
1.78152	2678.69	439.526
1.86379	2821.98	441.174
1.93081	2947.95	451.448

Steel, $E=200$ GPa, $S_y = 500$ MPa, $n=0.4$

Steel		
$E=200$ GPa, $S_y = 500$ MPa, $n=0.4$		
Time Increment	Load (N)	a, Contact Radius (μm)
0.354531	450.133	179.795
0.468437	646.866	209.86
0.582344	855.637	240.065
0.734219	1146.15	270.328
0.905078	1491.35	300.723
1	1695.35	311.067
1.075	1853.4	321.397
1.2325	2190.65	351.719
1.38648	2534.11	372.198
1.46758	2727	382.569

Steel, $E=200$ GPa, $S_y = 750$ MPa, $n=0.1$

Steel		
E=200 GPa, $S_y = 750$ MPa, $n=0.1$		
Time Increment	Load (N)	a, Contact Radius (μm)
0.035938	20.9589	60.0106
0.076213	54.5578	90.1478
0.110047	86.0401	110.288
0.148835	125.017	130.485
0.190909	172.055	140.566
0.227008	212.747	160.896
0.264918	258.4	171.075
0.291173	290.07	181.264
0.342612	343.346	201.783
0.377476	386.987	212.1
0.411789	428.693	222.425
0.451129	477.774	232.751
0.527225	568.992	253.602
0.56635	616.118	264.097
0.614928	686.255	274.648
0.65732	733.38	285.219
0.701967	789.19	295.821
0.744318	847.867	296.563
0.796323	914.345	317.2
0.83783	974.816	317.937
0.888331	1042.8	328.713
0.955332	1124.19	339.955
1	1190.41	350.618
1.05531	1265.33	361.545
1.12374	1351.9	372.879
1.18096	1430.22	383.857
1.25544	1523.78	395.421
1.33346	1622.35	407.106
1.41623	1724.45	419.033
1.50365	1832.25	431.045
1.5774	1928.94	442.684
1.64221	2017.86	453.8
1.8019	2204.57	476.971
1.88896	2298.7	489.002
1.97786	2411.99	501.399

Steel, $E=200$ GPa, $S_y = 750$ MPa, $n=0.2$

Steel		
$E=200$ GPa, $S_y = 750$ MPa, $n=0.2$		
Time Increment	Load (N)	a, Contact Radius (μm)
0.07	52.7568	79.9325
0.16	150.474	130.22
0.192842	195.669	140.267
0.234556	245.367	160.482
0.263592	283.906	170.593
0.298538	330.98	180.728
0.325937	369.843	190.861
0.364039	421.364	201.03
0.42364	514.473	211.124
0.475301	584.842	231.613
0.51508	645.693	241.855
0.535767	686.418	241.828
0.571218	739.139	252.145
0.617747	811.503	262.437
0.659233	872.51	272.778
0.706414	942.047	283.134
0.777578	1067.57	293.512
0.863187	1190.78	314.386
0.90025	1262.96	314.707
0.981511	1386.63	335.411
1	1434.59	335.503
1.0525	1516.1	346.076
1.10907	1602.25	356.68
1.16313	1691.02	367.287
1.2167	1788.79	368.054
1.2586	1868.03	378.195
1.32969	1986.15	389.01
1.40338	2109.01	399.825
1.48364	2246.55	410.814
1.56233	2376.97	421.755
1.64804	2523.93	432.86
1.77106	2720.91	444.969
1.83368	2837.25	455.454
1.92914	2996.5	466.806

Steel, $E=200$ GPa, $S_y = 750$ MPa, $n=0.3$

Steel		
$E=200$ GPa, $S_y = 750$ MPa, $n=0.3$		
Time Increment	Load (N)	a, Contact Radius (μm)
0.08	67.2827	79.8865
0.17	180.996	129.962
0.26	316.603	159.994
0.365	487.546	190.098
0.5	722.816	220.322
0.635	976.096	250.77
0.75875	1215.79	281.364
0.888125	1482.61	301.862
0.971182	1647.54	322.407
1	1712.63	322.382
1.05203	1817.86	332.788
1.11446	1945.64	343.1
1.18242	2087.31	353.471
1.25819	2257.43	363.889
1.35206	2467.88	374.471
1.45545	2685.22	385.364
1.54729	2881.07	405.81
1.65118	3108.76	416.359
1.75533	3353.99	427.216
1.84654	3558.21	437.946
1.94293	3771.24	448.678

Steel, $E=200$ GPa, $S_y = 750$ MPa, $n=0.4$

Steel		
$E=200$ GPa, $S_y = 750$ MPa, $n=0.4$		
Time Increment	Load (N)	a, Contact Radius (μm)
0.1475	165.559	109.721
0.539053	916.379	219.49
0.875432	1722.79	290.032
1.045	2152.46	320.425
1.28125	2788.57	350.891

Steel, $E=200$ GPa, $S_y = 1000$ MPa, $n=0.1$

Steel		
$E=200$ GPa, $S_y = 1000$ MPa, $n=0.1$		
Time Increment	Load (N)	a, Contact Radius (μm)
0.499334	645.253	232.232
0.546072	712.977	252.853
0.585617	777.065	253.064
0.646168	885.647	273.674
0.715984	989.26	284.431
0.783944	1091.7	305.261
0.838903	1187.72	315.935
0.920283	1323.18	327.007
1	1437.39	348.079
1.0592	1535.43	358.957
1.13202	1663.99	370.029
1.19897	1780.08	381.117
1.27009	1900	392.311
1.34926	2034.54	403.724
1.4432	2180.11	415.733
1.56649	2365.21	437.643
1.65704	2521.56	449.448
1.78274	2721.46	462.69

Steel, $E=200$ GPa, $S_y = 1000$ MPa, $n=0.2$

Steel		
$E=200$ GPa, $S_y = 1000$ MPa, $n=0.2$		
Time Increment	Load (N)	a, Contact Radius (μm)
0.881567	1458.13	303.402
1	1692.72	324.294
1.28562	2278.24	376.608
1.35075	2411.14	387.353
1.4439	2616.94	398.254
1.54387	2819.16	409.536
1.64975	3024.44	421.042
1.76083	3264.4	441.698
1.87363	3506.73	453.224
2	3756.31	465.26

Steel, $E=200$ GPa, $S_y = 1000$ MPa, $n=0.3$

Steel		
E=200 GPa, $S_y = 1000$ MPa, $n=0.3$		
Time Increment	Load (N)	a, Contact Radius (μm)
0.1025	103.404	89.7681
0.2825	396.239	159.757
0.485	793.324	220.099
0.636875	1131.02	250.381
0.733906		270.704
0.885781	1699.38	301.268

Steel, $E=200$ GPa, $S_y = 1000$ MPa, $n=0.4$

Steel		
E=200 GPa, $S_y = 1000$ MPa, $n=0.4$		
Time Increment	Load (N)	a, Contact Radius (μm)
0.645005	1304.34	239.222
0.773149	1651.76	269.399
0.901294		289.505
1	2301.77	299.712
1.0575	2479.64	309.832
1.20938	2936.72	340.064
1.28215	3165.33	350.242
1.45301	3708.07	370.553
1.62387	4268.1	390.989

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