Essays in international asset pricing

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Essays in International Asset Pricing

Hong Wu

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College of Business and Economics
at West Virginia University
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for the degree of

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in
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ABSTRACT

Essays in International Asset Pricing

Hong Wu

This dissertation attempts to assess the ability of various international asset pricing models in explaining asset returns across countries. The Introduction discusses the links between the chapters, the contributions I make and states what I have learned from the dissertation. Chapter 1 of the dissertation derives (in a discrete time framework to facilitate the transition from theory to empirical analysis) a consumption based capital asset pricing model with the added feature of a “keeping up with the Joneses” utility function and applies it in an international context to explain return differences across countries. Relying on the Lucas (1978) asset pricing framework, relating aggregate consumption to aggregate production, the model suggests that the exposure of assets to world-wide aggregate production risk as well as to U.S. aggregate production risk are the key factors in explaining cross-sectional differences in international returns. Chapter 2 of the dissertation examines and compares the standard international asset pricing models in the existing literature, including the international CAPM without exchange rate risks, the international CAPM with exchange rate risks (Adler and Dumas, 1983) and the international version of Fama-French three-factor model, (Fama and French, 1998). Results obtained from a Fama-MacBeth (1973) two-stage regression approach (with Shanken correction for standard errors) show that neither my two-factor CCAPM derived in Chapter 1 nor the existing models considered in Chapter 2 with constant risk premium and time-varying betas seem to be able to capture the return variations across countries. However, I do find some evidence that my two-factor CCAPM may be the one that is most consistent with the data. The international CAPM with exchange rate risks outperforms the international CAPM without exchange rate risks and the Fama-French three-factor model in explaining index returns across international markets. Chapter 3 of the dissertation investigates the role of the exchange rate risks in international asset pricing by looking at the effects of changes in exchange rate regime on the variability of stock returns, based on Hong Kong’s experience. Two propositions regarding the variability of stock returns as well as the variability of other key macroeconomic fundamentals influenced by the shift of exchange rate regime are derived theoretically from a combination of a stochastic dynamic Mundell-Fleming framework and the Lucas Asset Pricing Model. My results support the importance of exchange rate risks from a different perspective.
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INTRODUCTION
Introduction

How are international assets priced? Have researchers found the very underlying systematic risks that are able to capture the cross-sectional variation in expected returns? In an attempt to answer these questions, in this dissertation, I derive theoretically a consumption based capital asset pricing model (CCAPM) with the added feature of a “keeping up with the Joneses” utility function and apply it in an international context (Chapter 1). I also evaluate empirically the standard international asset pricing models in the existing literature (Chapter 2). As the evidence (from this dissertation as well as from the literature) shows, foreign exchange rate risks play an important role in understanding international asset pricing. I investigate this role indirectly by examining whether a shift in exchange rate regime has significant effects on asset returns (Chapter 3).

The consumption-based capital asset pricing model (CCAPM) of Breeden (1979) is on theoretical grounds the model of choice in a non-international context. Based on time separable utility functions, this traditional CCAPM does not perform well empirically. Meanwhile, habit formation and catching up with the Joneses formulations have been the focus of quite some research in the asset pricing literature. Habit formation is modeled by assuming that a consumer’s current utility is determined by current consumption relative to a reference level of consumption. Catching up with the Joneses implies that if others consume more today, a consumer will experience higher marginal utility from an additional unit of consumption in the future. Details provided in Chapter 1
show that there is substantial evidence against time separable utility function vis-à-vis habit formation and catching up with the Joneses formulations.

The preceding considerations motivate deriving a consumption based asset pricing model by incorporating time non-separable preferences. I use different countries to delineate different representative individuals to open the way to apply the CCAPM internationally. The “keeping up with the Joneses” preference formulation captures the fact that consumers/investors are always trying to “keep up with” at least the economy’s average standard of living because they care about what they consume today as well as about what others consume today. Relying on the Lucas (1978) asset pricing framework, relating aggregate consumption to aggregate production, the model suggests that the exposure of assets to world-wide aggregate production risk as well as to U.S. aggregate production risk are the key factors in explaining cross-sectional differences in international returns.

Employing the Fama-MacBeth (1973) two-stage regression method with time-varying betas and constant risk premia, my conditional two-factor CCAPM seems not to be supported significantly. However, adding the U.S. industrial production factor improves the standard CCAPM to some extent. And I also find that the two-factor CCAPM may be the one that is mostly consistent with the data compared to other benchmark models, namely the international CAPM and the international version of the Fama-French three-factor model.
As introduced above, Chapter 1 mainly focuses on a new derived model that can be used in international asset pricing and looks empirically at country market index returns only. In addition, only the international CAPM without foreign exchange rate risks is considered as a benchmark model. To further examine the question of how international assets are priced, in Chapter 2, I carefully survey the existing literature, putting my effort in assessing and comparing the two standard models that have been considered as the ones that perform relatively well in explaining the cross-sectional return spreads in an international context. The two models I refer to here are the international version of the Fama-French three-factor model and the international CAPM with foreign exchange rate risks.

Several recent papers come very close to what I do in Chapter 2, for instance, Dahlquist and Sallstrom (2001), Zhang (2001) and Griffin (2001). I explain in detail how my work can be distinguished from theirs in the chapter. Briefly, we do not have the same model specifications (regarding the property of betas and risk premia); we employ different evaluation methods and the base assets we use are different.

Griffin (2001) finds that the Fama and French factors are local rather than global. My results also find no support for the value factor. However, unlike Dahlquist and Sallstrom (2001) and Zhang (2001), I only find support for the international CAPM with foreign exchange rate risks when explaining the cross-sectional variation of country market expected returns. All specifications considered in chapter 2 are unable to capture the cross-section of returns on the country value portfolios and country value-minus-
growth portfolios. The performance of the international asset pricing models seems to be sensitive to the empirical method that is employed to evaluate those specifications.

This dissertation poses special challenges for the existing international asset pricing models. More serious consideration of the underlying risks that truly capture the asset return variations across countries is needed in future research.

Chapter 3 of the dissertation may look less integrated with the former two chapters at first glance. However, this is not so. Consider the transition from a flexible exchange rate regime to a fixed or linked exchange rate regime. What would this imply in terms of exchange rate risks? If less volatile exchange rates indicate less variability of the underlying risks, then one would expect that stock returns should also be less variable if the underlying risks are priced. Thus investigating the effects of changes in exchange rate regime on stock returns offers an interesting way of testing the role of exchange rate risks in asset pricing.

On October 17, 1983, Hong Kong began to link its currency with the U.S. dollar after having applied a clear flexible exchange rate policy in the prior period. Hong Kong’s experience provides me with a nice empirical example. To reach the goal, I first examine theoretically the implications of alternative exchange rate regimes on stock returns as well as on other key macroeconomic variables in a modern stochastic dynamic version of the Mundell-Fleming framework, following Obstfeld, 1985 and Mark, 2001. As specified clearly in Chapter 3, the sticky-price adjustment rule, the forward-looking
Taylor rule monetary reaction function and the integration of the Lucas Asset Pricing formula are the major theoretical contributions relative to Obstfeld and Mark.

Two propositions derived suggest how alternative exchange rate regimes would affect key macroeconomic variables depending on the characteristics of the economy; for instance, flexible prices or fixed prices. Although it is difficult to make unambiguous predictions, overall, stock return variability decreases under the linked exchange rate regime, both theoretically and empirically, supporting the model.
CHAPTER 1

AN INTERNATIONAL CCAPM WITH “KEEPING UP WITH THE JONESES” PREFERENCES
Chapter 1

An International CCAPM with
“Keeping up with the Joneses Preferences”

1.1 Introduction

The two standard models that attempt to explain the cross-sectional variations in international asset returns are the international capital asset pricing model (Adler and Dumas, 1983) and the Fama-French two-factor model (Fama and French, 1998). Basically, the international capital asset pricing model (ICAPM) argues that when purchasing power parity (PPP) does not hold, additional exchange rate risks should be considered. Fama and French (1998) provide evidence that their two-factor (global market beta and the value factor) model captures the value premium in international returns. However, the international capital asset pricing model does not have much empirical support and the Fama-French two-factor model is not well grounded theoretically.

The consumption-based capital asset pricing model (CCAPM) of Breeden (1979) is on theoretical grounds the model of choice in a non-international context. See for instance, Grossman and Laroque (1990), Marshall and Parekh (1999), Lettau and Ludvigson (2001) and Valckx (2001). Little has been done to estimate the CCAPM with international portfolios. One relevant paper is Evans and Hasan (1998). They modify the consumption-based capital asset pricing model to allow for the possibility that households have finite horizons and find that
introducing finite horizons into the CCAPM does not enhance its ability to account for real-world data.

This chapter derives a consumption based capital asset pricing model with the added feature of a “keeping up with the Joneses” utility function and applies it in an international context to explain return differences across countries and further compares it with the aforementioned two standard international asset pricing models as well as the standard CCAPM to see which one provides an empirically more useful framework for understanding international cross-sectional stock returns.

Time separable utility functions have been the mainstay in much of the literature on asset pricing, optimal consumption, and portfolio choice. The traditional consumption based asset pricing model is based on this kind of utility function too. But the CCAPM does not perform well empirically.¹ A search for factors other than aggregate consumption that can explain asset returns is ongoing.

“Habit formation” and “catching up with the Joneses” formulations now play an important role in explaining many economic puzzles; several papers show how the theory or empirical results improve after these two issues are taken into account.

Habit formation has a long history in the study of consumption. Marshall (1920) discusses the notion that tastes can be cultivated and that they are affected by past consumption.

¹ Mankiw and Shapiro (1986) calculate both a market beta and a consumption beta for each of 464 stocks from 1959 to 1982. When both are included in a cross-sectional regression, the market beta clearly outperforms the consumption beta.

Abel (1990) introduces a utility function that nests three classes of utility functions: 1) time-separable utility functions; 2) “catching up with the Joneses” utility functions that depend on the consumer’s level of consumption relative to the lagged cross-sectional average level of consumption; and 3) utility functions that display habit formation.

At time $t$, each consumer chooses the level of consumption, $c_t$, to maximize $E_t(U_t)$ where $E_t()$ is the conditional expectation operator at time $t$ and the utility function is given by

$$U_t = \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, v_{t+j})$$

where $v_{t+j}$ is a preference parameter. Suppose that the preference parameter $v_t$ is specified as

$$v_t = [c_{t-1}^{D} C_{t-1}^{1-D}]^\gamma, \quad \gamma \geq 0 \text{ and } D \geq 0,$$
where \( c_{t-1} \) is a consumer’s own consumption in period \( t-1 \) and \( C_{t-1} \) is aggregate consumption per capita in period \( t-1 \). If \( \gamma = 0 \), then \( v_t = 1 \) and the utility function in (1) is time separable. If \( \gamma > 0 \) and \( D = 0 \), the parameter \( v_t \) depends only on the lagged level of aggregate consumption per capita. This formation is the relative consumption model or “catching up with the Joneses”. Finally, if \( \gamma > 0 \) and \( D = 1 \), the parameter \( v_t \) depends only on the consumer’s own past consumption. This formulation is the habit formation model. Abel’s utility specification has been widely used in research on habit formation or on catching up with the Joneses.

More directly, habit formation is modeled by assuming that a consumer’s current utility is determined by current consumption relative to a reference level of consumption. Habit formation can explain why a consumer’s reported sense of well being often seems more related to recent changes in consumption than to the absolute level of consumption. Catching up with the Joneses implies that if others consume more today, a consumer will experience a higher marginal utility from an additional unit of consumption in the future. In some ways the idea of catching up with the Joneses is a variation on the theme of habit formation. When habit formation is external, an individual’s habit level depends on the history of aggregate consumption rather than on the individual’s own past consumption.

This chapter is organized as follows: Section 1.2 presents the achievements of habit formation and catching up with the Joneses formulation in the existing literature. Section 1.3 describes the motivation and derives the model. Section 1.4 focuses on the testable
propositions, the methodology and the data. Section 1.5 presents the empirical results. Section 1.6 concludes.

1.2 More on Habit Formation and Catching up with the Joneses

Papers in various areas have developed considerable theoretical and empirical support for habit formation.

1.2.1 Aggregate Consumption:

Past studies of time-non-separable preferences based on aggregate consumption data yield mixed conclusions about the strength of habit formation.

Dunn and Singleton (1986), Eichenbaum (1988) and Heaton (1993) find very little evidence of habit formation in U.S. aggregate monthly consumption data, and Muellbauer (1988) produces similar results with U.S. quarterly consumption data. Dynan (2000) tests the presence of habit formation by using household data. His estimation results yield no evidence of habit formation at the annual frequency. This finding is robust to a number of changes in the specification, and it holds for several proxies for non-durables and services consumption.

In contrast, Ferson and Constantinides (1991) find large and statistically significant amounts of habit formation in monthly, quarterly, annual U.S. consumption data, and Braun (1993) finds some evidence of habit formation in aggregate Japanese consumption. Several
recent empirical papers in the microeconomic consumption literatures have argued that habits may play an important role in determining consumption. Contributions include Carroll and Weil (1994), Deaton and Paxson (1994), and van de Stadt et al. (1985). Two very recent papers make the case that habit formation may be essential in understanding the high-frequency dynamics of aggregate consumption data in the United States (Fuhrer, 2000) and OECD countries (Fuhrer and Klein, 1998).

1.2.2 The Growth Literature:

Much of the recent growth literature has attempted to explain the finding that growth Granger-causes saving (Carroll and Weil, 1994). This finding constitutes a serious violation of the Permanent Income Hypothesis (PIH), because a PIH consumer would save less today in the face of strong growth that augments life-time resources.

Carroll, Overland and Weil (1995) suggest that habit implies a sluggish response of consumption to income shocks. Thus, after an income shock, growth in income could temporarily exceed consumption growth, raising savings while consumers gradually respond to the increase in income.

Carroll, Overland & Weil (2000) show that if utility depends partly on how consumption compares to a “habit stock” determined by past consumption, an otherwise growth model can imply that increases in growth can cause increased saving. The argument that habit formation is important seems even stronger when dynamic evidence is considered.
1.2.3 Monetary Policy and the Excess Smoothness Puzzle:

Fuhrer (2000) explores a monetary-policy model with habit formation for consumers. The empirical tests developed in the paper show that one can reject the hypothesis of no habit formation with tremendous confidence. The paper then embeds the habit-consumption specification in a monetary-policy model and finds that the responses of both spending and inflation to monetary-policy actions are significantly improved by this modification. Augmenting the model in this way allows the model to replicate the key dynamic correlation among consumption, output, interest rates and inflation to a degree that standard models cannot. The other contribution this paper makes is an empirically successful solution to the excess smoothness puzzle (Consumption does not respond immediately to news, consumption exhibits “excess smoothness” that can not be reconciled with the PIH model (Campbell & Deaton, 1989)).

1.2.4 Tax Policy and Catching up with the Joneses:

Ljungqvist and Uhlig (2000) examine a catching up with the Joneses utility function, a near cousin to habit formation models, in a productivity shock driven model. They demonstrate that optimal tax policy in such a model can be pro-cyclical. Such a “Keynesian” tax policy is optimal because it dampens booms that arise sub-optimally due to the failure of the individuals to take account of the external effect of their own consumption on the consumption of others.
1.2.5 Asset Pricing and the Equity Premium Puzzle:

Habit formation and catching up with the Joneses has been the focus of quite some research in the asset pricing literature as well.

Bergman (1985) shows that the assumption of a separable time-additive preference structure is crucial for the derivation of a single consumption beta CAPM. When more general preferences are used, like intertemporal preferences for consumption streams, which are more realistic, Merton’s (1973) Multi-beta intertemporal CAPM still holds but it can no longer be collapsed to Breeden’s (1979) single consumption beta model.

Sundaresan (1989) constructs a model in which a consumer's utility depends on the consumption history. The sample paths of consumption generated from his model imply lower variability in consumption growth rates compared to those generated by models with separable utility functions. He then presents a partial equilibrium model based on optimal consumption and portfolio rules to accommodate non-separability in preferences. Asset pricing implications of his framework are briefly explored. When the opportunity set is stochastic, the asset pricing structure has a multi-β representation similar to Merton (1973), while the single consumption β representation obtained by Breeden (1979) no longer obtains in this setting. The result also suggests that factors other than aggregate consumption may be relevant in describing the cross sectional variations in security returns.

Abel (1990) derives the price of risky capital with habit formation purely theoretically and he calculates the unconditional expected returns for stocks, bills and consols using three
kinds of preferences respectively, which are time-separable preferences, relative consumption and habit formation. He confirms the equity premium puzzle for time-separable preferences. For the relative consumption and habit formation models, the equity premium puzzle could in some sense be solved but the solution is sensitive to the choice of parameters and utility functional forms.

Constantinides (1990) shows that the equity premium puzzle is resolved in a rational expectations model, once one relaxes the time separability of preferences and allows for habit persistence.

Detemple and Zapatero (1991) analyze asset prices in an exchange economy in which the preferences of the representative agent exhibit habit formation. Their analysis demonstrates that consumption smoothness may obtain even when the interest rate is stochastic. The interest rate depends on the growth in the standard of living. Historical consumption as well as possible future consumption paths are determinants of the current interest rate. The analysis provides new testable restrictions on the behavior of financial assets.

Gali (1994) formalizes the notion of investment in financial assets as a social activity by introducing a particular type of consumption externalities in standard portfolio and asset pricing models. He examines the effect of consumption externalities on portfolio decisions and equilibrium asset prices. In the CAPM model, the presence of consumption externalities makes the optimal risky share either larger or smaller than in the standard model, depending on the sign of externalities. The introduction of consumption externalities in a multi-period asset pricing
model yields, on his assumptions, a basic equivalence result: Equilibrium asset prices and returns in an economy with externalities are identical to those of an externality-free economy with a properly adjusted degree of risk aversion.

Campbell and Cochrane (1999) show that many of the puzzles in asset pricing can be understood with a simple modification of the standard representative agent consumption-based model. The central ingredient is a slow-moving habit, or time-varying subsistence level, added to the basic power utility function. They document a broad variety of empirical successes for their consumption-based model with external habit formation (catching up with the Joneses). The model resolves many puzzles that face the standard power utility consumption-based model, including the equity premium and risk-free rate puzzles and the low unconditional correlation of consumption growth with stock returns. The model is consistent with an even sharper long-run equity premium puzzle that results from mean reversion in stock prices, together with low long-run consumption volatility.

Boldrin, Christiano and Fisher (2001) introduce two modifications, habit preferences and a two-sector technology into the standard real business-cycle model and show that the model is consistent with the observed mean risk-free rate, equity premium, and Sharp ratio on equity. In addition, its business-cycle implications represent a substantial improvement over the standard model.

As explained above, habit-forming consumers dislike large and rapid cuts in consumption. As a result the premium that they require to hold risky assets that might force a
rapid cut in consumption will be large relative to that implied by the time-separable utility model. While habit formation may not explain all asset pricing anomalies, it is now widely agreed, to “fit the data” better than time-separable utility asset pricing models.

1.3 The Motivation and the Model

The derivation of a standard CCAPM is based on a time-separable utility function. From section 1.2, there is substantial evidence against this type of utility function vis-à-vis habit formation and catching up with the Joneses formulations. Why does the CCAPM perform so poorly empirically? Is it because the whole idea is wrong or only because the utility function used was not the relevant one?

The purpose of this chapter is to use a “keeping up with the Joneses” utility function in the framework of the CCAPM to derive asset returns. I use the term “keeping up with the Joneses” instead of “catching up with the Joneses” because in my model, utility at time \( t \) is a function of the individual’s time \( t \) consumption level as well as the average aggregate consumption at same period \( t \), not \( t-1 \). Investors are always trying to “keep up with” at least the economy’s average standard of living because they care about what they consume today as well as about what others consume today.

The theoretical model is based on Merton’s (1973) Intertemporal CAPM and Breeden’s (1979) CCAPM. These models are derived in a continuous time stochastic dynamic
programming framework. To facilitate the transition from theory to empirical analysis, I will derive instead a discrete Intertemporal CAPM and CCAPM under a normality assumption.

An individual investor maximizes his expected life-time utility. Because individual consumption data is hard to obtain, I use different countries to delineate different individuals. This opens the way to apply the CCAPM internationally. By doing so, a representative consumer/investor must exist in each country. It has been shown that “Given time-additive preferences that are not state dependent, homogeneous expectations, and complete markets, a representative investor always exists. Assets may be priced based on the preferences of this investor only”.\(^2\)


Under the assumption of a representative consumer/investor, each country’s aggregate consumption level can be considered as the country-individual’s own consumption. The representative country-individual cares about his own consumption as well as the consumption

\(^2\) See Constantinides (1982). Time-additive preferences are not state dependent within each individual country so a representative investor exists at a country level. Although “keeping up with the Joneses” formulation does imply state dependent preferences in the world, a world-representative investor is not needed in the derivation of the model. It also has been shown that “If all investors have HARA (Hyperbolic Absolute Risk Aversion) preferences with discount factor and exponent equal across investors but with potentially different constants, then a representative investor exists”. See Cass and Stiglitz (1970). Note that HARA preferences are quite general and most utility functions may be reasonably well approximated by some member of the HARA class.
level of the wealthiest country-individual, namely, the US. Then the US consumption level is what he wants to “keep up with”.  

Choice variables are the portfolio shares $s_{ik}^k$ and the consumption level $c_k^k$ in each period and the constraint is life-time wealth.

Maximize:

$$ E_0 \beta^t u(c_t^k, usc_t), $$(1)

where $0 < \beta < 1$, Subject to:

$$ W_{t+1}^k = R_{t+1}^k (W_t^k - c_t^k), $$

$$ R_{t+1}^k = R_t^f + \sum_{i=1}^n s_{i,t}^k (R_{t+1}^f - R_t^f), $$

$$ R_{t+1}^f = F(R_t^f, e_{t+1}), $$

where $\beta$ is a subjective discount factor, $c_t^k$ is aggregate consumption in period $t$ for country $k$, $usc_t$ is the United States aggregate consumption in period $t$, $W$ denotes wealth, $R$ denotes gross return, individual assets are indicated by superscript $i$ and with superscript $f$ indicating the risk

---

3 The world average consumption level is also a reasonable variable for country-individuals to “keep up with”. Since I use 18 developed countries’ data in this chapter, the wealthiest country-individual’s consumption seems to be the appropriate one that influences other country-individual’s utility functions.
free asset. Portfolio shares $s^t_i$ add to one in each period. The risk free rate changes stochastically over time.\footnote{Changes in investment opportunities assumed here are not necessary in deriving my model. By assuming a constant investment opportunity set, the model can still be derived in the same way and the same outcome will be obtained. I consider here a more general situation.} Returns on all other securities are assumed to be uncorrelated over time and are multi-variate normally distributed conditional on the risk free rate for the upcoming period. This is a simplifying assumption. Assuming more than one state variable capturing shifts in investment opportunities does not affect the key results.

Note that from equation (1), the U.S. consumer really only cares about one factor. Still, in general equilibrium, two factors will matter.

The decision problem in equations (1) and (2) can be reformulated using the dynamic programming approach.

\[
V^k(W^t_k, R^t_f) = \max_{\{s^t_i\}^n_{i=1}} \left\{ u^k(c^t_k, usc^t_k) + \beta_k E^t V^k(W^t_{k+1}, R^t_{f+1}) \right\}, \quad (3)
\]

First order conditions for this decision problem are:

with respect to $c^t_k$,

\[
u^t_k(c^t_k, usc^t_k) = \beta_k E^t [R^t_{f+1} V^t_k(W^t_{k+1}, R^t_{f+1})], \quad (4)
\]

with respect to $s^t_i$,
\[ E_t \left[ (R_{t+1}^i - R_t^i) V_{W_t}^k (W_{t+1}^k, R_{t+1}^f) \right] = 0, \text{ for all } i, \quad (5) \]

Using the definition of covariance and converting to net returns (represented by \( r \)), rewriting equation (5):

\[ E_t \left[ V_{W_t}^k (W_{t+1}^k, r_{t+1}^f) \right] (\mu_{t+1}^i - r_{t+1}^f) = -\text{Cov}_i \left[ V_{W_t}^k (W_{t+1}^k, r_{t+1}^f), r_{t+1}^i \right], \quad (6) \]

where \( E_t(r_{t+1}^i) = \mu_{t+1}^i, \quad E_t(r_{t+1}^f) = r_{t+1}^f \). \( \mu^i \) represents the expected return on asset \( i \).

The envelope condition is:

\[ V_{W_t}^k (W_{t}^k, R_{t}^f) = u_{c_t}^k (c_{t}^k, \text{usc}_t) \], \quad (7) \]

Substituting equation (7) into equation (6) yields:

\[ E_t[u_{c_t}^k (c_{t+1}^k, \text{usc}_{t+1})](\mu_{t+1}^i - r_{t+1}^f) = -\text{Cov}_i \left[ u_{c_t}^k (c_{t+1}^k, \text{usc}_{t+1}), r_{t+1}^i \right], \quad (8) \]

Assuming that consumption of each country is normally distributed, applying Stein’s generalized Lemma\(^5\) to equation (8) produces:

\(^5\) See the Appendix of Balvers (2001).
\[- E_t [u^k_c (c^k_{t+1}, \text{usc}_{t+1})](\mu^f_{t+1} - r^f_i) = E_t[u^k_c (c^k_{t+1}, \text{usc}_{t+1})]\text{Cov}_t(c^k_{t+1}, r^f_{t+1})
\]
\[+ E_t [u^k_{usc} (c^k_{t+1}, \text{usc}_{t+1})]\text{Cov}_t (\text{usc}_{t+1}, r^f_{t+1}), \quad (9)\]

Dividing by the term in front of the first covariance gives:

\[
(\mu^f_{t+1} - r^f_i) \frac{- E_t [u^k_c (c^k_{t+1}, \text{usc}_{t+1})]}{E_t[u^k_c (c^k_{t+1}, \text{usc}_{t+1})]} = \text{Cov}_t (c^k_{t+1}, r^f_{t+1})
\]
\[+ \frac{E_t [u^k_{usc} (c^k_{t+1}, \text{usc}_{t+1})]}{E_t[u^k_c (c^k_{t+1}, \text{usc}_{t+1})]} \text{Cov}_t (\text{usc}_{t+1}, r^f_{t+1}), \quad (10)\]

The expression includes various terms that are specific to individual country \(k\). The next step thus is to consider market equilibrium by aggregating over all individual countries:

\[
\sum_{k=1}^{K} - E_t [u^k_c (c^k_{t+1}, \text{usc}_{t+1})] \frac{1}{E_t[u^k_c (c^k_{t+1}, \text{usc}_{t+1})]} (\mu^f_{t+1} - r^f_i) = \text{Cov}_t \left(\sum_{k=1}^{K} c^k_{t+1}, r^f_{t+1}\right)
\]
\[+ \sum_{k=1}^{K} \frac{E_t [u^k_{usc} (c^k_{t+1}, \text{usc}_{t+1})]}{E_t[u^k_c (c^k_{t+1}, \text{usc}_{t+1})]} \text{Cov}_t (\text{usc}_{t+1}, r^f_{t+1}), \quad (11)\]
Note that \( \sum_{k=1}^{K} c_{t+1}^k = c_{t+1} \), the world-wide consumption level which is the consumption level of all individual countries added together. Then we can rewrite equation (11) as:

\[
A_1 ( \mu_{t+1}^i - r_{t}^i ) = \text{Cov}_t (c_{t+1}^i , r_{t+1}^i ) + A_2 \text{Cov}_t (usc_{t+1}^i , r_{t+1}^i ),
\]

(12)

where

\[
A_1 = \sum_{k=1}^{K} \frac{E_t [u_k^i (c_{t+1}^k , usc_{t+1}^k )]}{E_t [u_{usc}^i (c_{t+1}^k , usc_{t+1}^k )]}, \quad A_2 = \sum_{k=1}^{K} \frac{E_t [u_k^i (c_{t+1}^k , usc_{t+1}^k )]}{E_t [u_{usc}^i (c_{t+1}^k , usc_{t+1}^k )]}.
\]

### 1.3.1 An asset exists that is perfectly correlated with consumption

If there exists an asset whose return is conditionally perfectly correlated with world consumption and U.S. consumption such that \( c_{t+1}^i = g_t + h_t r_{t+1}^i \), \( usc_{t+1}^i = p_t + q_t r_{t+1}^{usc} \), then

\[
A_1 ( \mu_{t+1}^i - r_{t}^i ) = h_t \text{Cov}_t (r_{t+1}^i , r_{t+1}^i ) + A_2 q_t \text{Cov}_t (r_{t+1}^{usc}, r_{t+1}^i ),
\]

(13)

\[
( \mu_{t+1}^i - r_{t}^i ) = (h_t/A_1) \text{Cov}_t (r_{t+1}^i , r_{t+1}^i ) + (A_2q_t/A_1) \text{Cov}_t (r_{t+1}^{usc}, r_{t+1}^i ),
\]

(14)

\[
( \mu_{t+1}^i - r_{t}^i ) = B_1 \text{Cov}_t (r_{t+1}^i , r_{t+1}^i ) + B_2 \text{Cov}_t (r_{t+1}^{usc}, r_{t+1}^i ),
\]

(15)
where \( B_1 = h/A_1 \), \( B_2 = A_2q/A_1 \).

Apply equation (15) to the asset with return perfectly correlated with world wide aggregate consumption (\( i=c \)) and the United States aggregate consumption (\( i=usc \)):

\[
(\mu_{t+1}^i - r_t^f) = B_1 \text{Var}_t(r_{t+1}^c) + B_2 \text{Cov}_t(r_{t+1}^{usc}, r_{t+1}^c),
\]

(16)

\[
(\mu_{t+1}^{usc} - r_t^f) = B_1 \text{Cov}_t(r_{t+1}^c, r_{t+1}^{usc}) + B_2 \text{Var}_t(r_{t+1}^{usc}),
\]

(17)

Use equation (16) and (17) to solve \( B_1 \) and \( B_2 \); substitute the solutions into equation (15) to obtain:

\[
\mu_{t+1}^i - r_t^f = \beta_{ic} (\mu_{t+1}^c - r_t^f) + \beta_{usc} (\mu_{t+1}^{usc} - r_t^f),
\]

(18)

with \( \beta_{ic} = \frac{\sigma_{c^2} \sigma_{ic}^2 - \sigma_{c^2} \sigma_{usc}^2}{\sigma_{usc}^2 \sigma_{c^2} - \sigma_{c^2} \sigma_{usc}^2} \), \( \beta_{usc} = \frac{\sigma_{usc^2} \sigma_{ic}^2 - \sigma_{usc}^2 \sigma_{c^2}}{\sigma_{usc}^2 \sigma_{c^2} - \sigma_{c^2} \sigma_{usc}^2} \)

where \( \beta_{ic} \) measures the exposure of an asset \( i \) to the world aggregate consumption risk; \( \beta_{usc} \) measures the exposure of an asset \( i \) to the U.S. aggregate consumption risk.

Equation (18) is the key equation. I find a two-beta CCAPM model using the same procedure as can be used to derive the regular CCAPM. The only difference is that there are
two factors which affect an individual country’s representative consumer/investor’s utility, one is
the world’s aggregate consumption level, the other is the world’s wealthiest country’s
consumption level that the individual country investors would like to keep up with. World-wide
aggregate consumption by itself can no longer explain asset returns, one should consider the
covariance risk between a particular asset and the asset which is perfectly correlated with
world-wide consumption as well as the covariance risk between the same individual asset and
the asset which is perfectly correlated with United States consumption.

In asset pricing, only systematic risk can be rewarded. Why are investors concerned
with aggregate consumption risk? When future aggregate consumption is high, the growth rate of
consumption is high. The return on the asset that is perfectly correlated with aggregate
consumption (indicated by the growth rate of aggregated consumption) is high. Thus the
individual asset’s return is high. It pays out when aggregate consumption is high. Since high
aggregate consumption means low marginal utility, investors need to have a higher mean return
to compensate this risk. Conversely, low aggregate consumption indicates high marginal utility
and low required stock returns. Investors gain less when they need it most. They are paid at the
wrong time. If the individual beta is negative, this kind of risk can be hedged.

1.3.2 No asset exists that is perfectly correlated with consumption

If no asset exists that is perfectly correlated with consumption, equations (13), (16) and
(17) cannot be used. Instead, define $r_{i+1}^{C_i} = c_{i+1}/c_i$ as the growth rate of world-wide
consumption, $r_{t+1}^{usc} = usc_{t+1}/usc_t$ as the growth rate of the US consumption. Then equation (13) becomes

\[
(\mu_{t+1}^i - r_f^i) = c_t/A_1 \text{Cov}_t (r_{gsc_t}^i, r_{i+1}^i) + (usc_t A_2/A_1) \text{Cov}_t (r_{gusc_t}^i, r_{i+1}^i),
\]  

(19)

Consider now an asset that is highly correlated with consumption. For instance, the market:

\[
(\mu_{t+1}^m - r_f^m) = c_t/A_1 \text{Cov}_t (r_{gsc_t}^m, r_{m+1}^m) + (usc_t A_2/A_1) \text{Cov}_t (r_{gusc_t}^m, r_{m+1}^m),
\]  

(20)

\[
(\mu_{t+1}^{usm} - r_f^{usm}) = c_t/A_1 \text{Cov}_t (r_{gsc_t}^{usm}, r_{usm+1}^{usm}) + (usc_t A_2/A_1) \text{Cov}_t (r_{gusc_t}^{usm}, r_{usm+1}^{usm}),
\]  

(21)

where m represents the world market and usm represents the U.S. market.

Use equations (20) and (21) to solve $c_t/A_1$ and $usc_t A_2/A_1$; substitute the solutions into equation (19) to obtain:

\[
\mu_{t+1}^i - r_f^i = \beta_{igm} (\mu_{t+1}^m - r_f^m) + \beta_{igusm} (\mu_{t+1}^{usm} - r_f^{usm}),
\]  

(22)

where $\beta_{igm}$ and $\beta_{igusm}$ are both functions of $\text{Cov}_t (r_{gsc_t}^i, r_{i+1}^i)$, $\text{Cov}_t (r_{gsc_t}^m, r_{m+1}^m)$, $\text{Cov}_t (r_{gsc_t}^{usm}, r_{usm+1}^{usm})$, $\text{Cov}_t (r_{gusc_t}^i, r_{i+1}^i)$, $\text{Cov}_t (r_{gusc_t}^m, r_{m+1}^m)$, $\text{Cov}_t (r_{gusc_t}^{usm}, r_{usm+1}^{usm})$. 

27
Empirically, this version of the CCAPM is estimated using an Instrumental Variable (IV) technique: first regress the growth rate of consumption on the market return; then use the predicted value of the growth rate of consumption based on the market return to regress against the time series of returns of each asset. This yields the appropriate beta.

My result is consistent with what other authors find (see section 1.2.5, Bergman (1985) and Sundaresan (1989)). Sundaresan (1989) applies a nonseparable utility function to explain the observed consumption smoothing, not focusing on asset pricing directly although he does provide some implications regarding asset pricing. He argues that the single consumption beta representation obtained by Breeden (1979) no longer holds when utility functions are not time separable and suggests that factors other than aggregate consumption may be relevant in describing cross-sectional variation in security returns without further examining what these “other factors” are.

1.4 Evaluating the Model Empirically

A consumption-oriented capital asset pricing model with keeping up with the Joneses utility function is estimated in this section. If we find that the two-beta model (equation (18)) well explains the cross-sectional variations in stock returns, then the poor performance of the standard CCAPM may be due to its not very realistic time-separable utility function.
1.4.1 Methodology

The methodology in estimating the CAPM used in this chapter is the Fama and Macbeth (1973) two-pass regression method (FM), one of the regression based tests of linear factor models (see Cochrane, 2001). I choose the FM method because it allows changing betas, which other method such as the generalized method of moments (GMM) cannot easily handle.

The basic idea of this method is: each month the cross-section of returns on stocks is regressed on variables (factor loadings) hypothesized to explain expected returns. The time-series means of the monthly regression slopes then provide standard tests of whether different explanatory variables are on average priced.

Equation (23) provides the empirically testable form for the solution (equation (18)) of the model.

\[
r_i - r_f = a_0 + a_1 \beta_{FMC} + a_2 \beta_{USC} + \varepsilon_i ,
\]

(23)

If the betas for each stock were directly observable, we could run (23) on a cross section of stocks. The betas however are not observable. So in the “first pass”, I regress
equation (24) by using 5 years of monthly time series data\(^6\) of \(r^i_t, r^f_t, r^c_t\) and \(r^{usc}_t\) to get the estimated factor loadings \(\hat{\beta}_c (\beta_c)\) and \(\hat{\beta}_{usc} (\beta_{usc})\) for each selected portfolio \(i\).

\[
r^i_t - r^f_t = b_0 + b_1 (r^c_t - r^f_t) + b_2 (r^{usc}_t - r^f_t) + e^i_t,
\]

(24)

Where \(r^c_t\) and \(r^{usc}_t\) represent returns on assets that are perfectly correlated with the world and the U.S. aggregate consumption.

In more detail: I obtain the first set of betas for each portfolio (country market index) by running equation (24) using time series data from July 1975 to June 1980 (60 months). Then I obtain the second set of betas for each country by using time series data from August 1975 to July 1980 (one month forward 60 months). Then the third set of betas, the fourth set and so forth until I reach the last month, October 2000. By doing so, I assume that individual factor loadings vary over time.

In the “second pass”, I regress the July 1980 cross-section of stock returns (the 61\(^{st}\) cross-sectional observation among the 304 months) on the first set of betas: \(\beta_c\) and \(\beta_{usc}\) (see equation (23)) obtained from the first 60 months time series data for each country. And regress the August 1980 cross-section stock returns (the 62\(^{nd}\) cross-section observation among the 304

\(^6\) There is a trade off between more accurate beta estimation by using a longer time period and misestimation due to drift in beta over time. Five years of monthly data (60 observations) is the norm in estimating betas (see Alexander and Chervany, 1980). Although some have argued that one could use the full sample to estimate beta if the process by which beta changes over time is stationary.
months) on the second estimated set of betas. Continuing this process month by month I end up with a series of “second pass” intercepts and regression slopes for each hypothesized explanatory variable.

The estimated series of $\hat{a}_0$, $\hat{a}_1$, and $\hat{a}_2$ (from equation (23)) are then averaged over all time periods used in the second pass (244 months) to provide the evaluation criterion of the CAPM. The model is rejected if the intercept $\hat{a}_0$ deviates significantly from zero; or if $\hat{a}_1$, $\hat{a}_2$ deviate significantly from their corresponding actual mean risk premiums $\mu_c - r_f$ and $\mu_{usc} - r_f$ predicted by the model. Significance here is based on the t-statistic obtained by dividing each mean estimated coefficient by its standard error.

The Fama-MacBeth standard errors do not correct for the fact that betas are generated regressors. To use this approach, I calculate the Shanken (1992) correction factors and report t-statistic adjusted for the errors-in-variables (EIV) problem. Shanken derives the asymptotic variance of the second-pass estimator. He finds that the Fama-MacBeth procedure for computing standard errors fails to reflect measurement error in the betas and overstates the precision of the risk factor estimates. He then argues to adjust for EIV by a scalar $c$ (Shanken 1992, p12). The adjustment reflects the fact that the variance of the beta estimates is directly related to residual variance and inversely related to factor variability. To calculate $c$, second-stage estimated risk factors and the variance-covariance matrix of the factors are needed.
1.4.2 Data and Summery Statistics

I collect the Morgan Stanley Capital International (MSCI) monthly country index returns (with dividends reinvested) for 18 developed countries as $r_t^i$. The U.S. one-month T-bill rate\(^7\) is the risk free asset return $r_t^f$. Since there are measurement problems\(^8\) using actual consumption data to represent the theoretical concept of consumption (see Breeden, Gibbons and Litzenberger, 1989), I replace it with industrial production data as is appropriate in the world of the Lucas production based CAPM (Lucas, 1978). See for instance Balvers, Cosimano and McDonald (1990) for a similar approach. Then $r_t^c$ and $r_t^{usc}$ can be considered as the returns on assets that are perfectly correlated with world industrial production and U.S. industrial production.

The problem is $r_t^c$ and $r_t^{usc}$ are not observable. To mimic theses returns, I use the growth rate of world industrial production $\Delta y$ and the growth rate of U.S. industrial production $\Delta usy$ ($y$ and $usy$ are in log terms), where

\[
\begin{align*}
    r_t^c &= \gamma + \delta \Delta y_t, \\
    r_t^{usc} &= \lambda + \Theta \Delta usy_t. 
\end{align*}
\]  

\(^7\) Compared with other assets, for instance, long-term government bonds, the rate on one-month T-bill is more appropriate to be considered as risk free because of its short maturity. A longer maturity is risky if there is a chance that liquidity is needed since selling a long-term bond before maturity may involve a substantial capital loss. One-month or three-month T-bill rate has been widely used as risk free rate in empirical work on asset pricing.

\(^8\) It is difficult for instance to measure the stream of durable good consumption services during one period.
Monthly data on the U.S. inflation rate (to convert to real returns) and industrial production (seasonally adjusted) and the world industrial production (seasonally adjusted) are obtained from International Financial Statistics (IFS). The data ranges from July 1975 to Oct. 2000 (304 observations). All excess returns are in real terms denominated in U.S. dollars.²³

I also want to compare my two-factor CCAPM with some benchmark models, such as the standard CCAPM, the international CAPM and the international version of the Fama-French three-factor model.⁴ The world market index return can be obtained from MSCI. The US one-month T-bill rates and the High-Minus-Low (H-L, or value minus growth) portfolio returns can be obtained directly from Kenneth French’s homepage.⁵

Table 1.1 provides statistics for the 18 country market indices excess returns that are to be explained, \( r_i - r_f \). All excess returns are positive; however only a few are significantly so.

Table 1.2 provides statistics for the hypothesized explanatory variables from the four international asset pricing models considered in this chapter. Note that the growth rates of both world and US industrial production are significantly positive and that the rate for the US is

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²³ When global Purchasing Power Parity holds, the real exchange rate is equal to one for any bilateral exchange rate. Real returns would be same no matter in what currency. This chapter assumes a world in which capital markets are integrated and investors are unconcerned with deviations from purchasing power parity as in Fama and French (1998). Thus it is appropriate to look primarily at U.S. dollar-denominated returns. Although I also do conduct my empirical work by using German Mark denominated returns for robustness in section 1.5.2.

⁴ I use here a two-factor (market beta and the value factor) instead of a three-factor model because my data for 18 developed countries and the world from MSCI include only large firms. A set of indexes consisting of large firms only does not allow meaningful inclusion of a size factor.

⁵ http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
higher. The real risk free rate is significantly positive and about .17% monthly. There exist a significantly positive global market excess return and a value premium.

Table 1.3 provides a correlation matrix for 21 variables including 18 country market index excess returns (dependent variables in my two-factor CCAPM), the growth rate of world industrial production, the growth rate of U.S. industrial production and the real risk free rate (explanatory variables in my two-factor CCAPM). Note that excess returns on the market indices are all positively correlated across countries and all (with few exceptions) negatively correlated with the hypothesized risk factors, but the correlation is low. However, the second factor that I add to the standard CCAPM, which is the growth rate of the U.S. industrial production, is positively correlated with the world industrial production. Adding this additional factor thus will affect the results.
Table 1.1
Summary Statistics for 18 Developed Countries

Excess dollar returns here refer to the dollar-denominated returns on 18 country market indices in excess of the U.S. 1-month T-bill rate. Mean is the average monthly excess return over 304 months in our sample; Std. represents the standard deviation of monthly excess returns; t is the ratio of the average return to its standard error. The time period is from July 1975 to Oct. 2000. The individual countries are (in order): Australia, Austria, Belgium, Canada, Denmark, France, Germany, Hong Kong, Italy, Japan, Netherlands, Norway, Singapore, Spain, Sweden, Switzerland, the United Kingdom and the United States.

<table>
<thead>
<tr>
<th>Excess Return</th>
<th>Mean</th>
<th>Std.</th>
<th>t value</th>
</tr>
</thead>
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<tr>
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<td>7.23</td>
<td>.79</td>
</tr>
<tr>
<td>AUT</td>
<td>.02</td>
<td>6.34</td>
<td>.05</td>
</tr>
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<td>5.68</td>
<td>1.11</td>
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<td>5.21</td>
<td>1.71</td>
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<td>6.52</td>
<td>1.59</td>
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Table 1.2
Statistics for the Hypothesized Explanatory Variables

$\Delta y$ is the growth rate of world industrial production. $\Delta usy$ is the growth rate of US industrial production. Both $\Delta y$ and $\Delta usy$ are from IFS. $r_m$ is the global market return from MSCI. $r_f$ is the one-month U.S. Treasury bill rate. H-L is an international version of HML, the distress factor in the three-factor model for U.S. stock returns in Fama and French (1993). H-L represents the difference between the returns on global value (high book-to-market: H) portfolios and global growth (low book-to-market: L) portfolios. $r_f$ and H-L are from Kenneth French’s homepage (see footnote 9). Mean is the average monthly factor value over the 304 months in my sample; Std. is the standard deviation of each point estimate; t is the ratio of the average factor value to its standard error. All dollar returns are monthly. The time period is from July 1975 to October 2000.

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Table 1.3

Correlation Matrix for 18 Dependent Variables and 3 Explanatory Variables

The numbers listed in the table are the correlation coefficients between every two variables. Dependent variables are 18 countries’ market indices excess returns. Explanatory variables are growth rate of world industrial production $\Delta y$, the growth rate of U.S. industrial production $\Delta u$sy and the real risk free rate $r_f$ in my two-factor CCAPM. Both $\Delta y$ and $\Delta u$sy are from IFS. $r_f$ is the one-month U.S. Treasury bill rate from Kenneth French’s homepage (see footnote 9). All dollar returns are monthly. The time period is from July 1975 to October 2000.
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1.4.3 Testable Propositions

Since $r_t^c$ and $r_t^{usc}$ are not observable, the FM two-stage method cannot be used based on equations (23) and (24). The growth rates of industrial production are not returns. Substitute equation (25) and (26) into equation (24) to give

$$r_t^i - r_t^f = d_0 + d_1 \Delta y_t + d_2 \Delta usy_t + d_3 (r_t^{\text{nominal}} f - \text{infl}_t) + v_t^i,$$  \hspace{1cm} (27)

where $\text{infl}$ represents the inflation rate in the U.S; $d_0 = b_0 + b_1 \gamma + b_2 \lambda$, $d_1 = b_1 \delta$, $d_2 = b_2 \theta$, $d_3 = -(b_1 + b_2)$.

Apply the FM method that is explained in detail in section 1.4.1 to regress equation (27) by time series data in the first stage. Use the estimated $\hat{d}_1$, $\hat{d}_2$ and $\hat{d}_3$ from the first stage as the regressors in the second stage, regress equation (28) cross-sectionally month by month.

$$r_t^i - r_f = h_0 + h_1 \hat{d}_1 + h_2 \hat{d}_2 + h_3 \hat{d}_3 + \eta,$$  \hspace{1cm} (28)

The model can be evaluated by checking whether or not the coefficients $h_1$, $h_2$ and $h_3$ are significant. More directly check whether $h_1$ significantly deviates from the mean of $\Delta y$; $h_2$ significantly deviates from the mean of $\Delta usy$ and $h_3$ significantly deviates from the mean of $r_f -$ infl. Note that $h_0$ is not restricted because the empirical factors are not
returns. Significance here is based on the t-statistic obtained by dividing the mean point estimate by its standard error with calculated Shanken corrections.

1.5 Empirical Results

1.5.1 Basic Empirical Results:

Table 1.4 shows four international asset-pricing models’ two-stage regression results for U.S. dollar denominated real returns. The dependent variables are the 18 country market portfolio excess returns. The explanatory variables include the growth rate of world industrial production $\Delta y$, the growth rate of US industrial production $\Delta usy$, the real risk free rate (US one-month T-bill rate), the excess return on the global market portfolio $r_m - r_f$, and the difference between the global high and low book-to-market returns (H-L).

Not encouragingly, the two-factor CCAPM with added feature of “keeping up with the Joneses” preference does not perform well empirically. The growth rate of world industrial production, the US industrial production and the real risk free rate are unable to capture the cross-sectional variations in national market indices returns. They are not statistically significant. Economically, the estimated factors all have the right sign (positive) but still are not close to the actual averages predicted by the model (see Table 1.2). The adjusted R square is notably low, about 8% on average.
However, the other three-benchmark models perform equally badly or slightly worse than my two-factor CCAPM. Consider the standard CCAPM first. The growth rate of world industrial production is insignificant and has the wrong sign, negative. The real risk free rate is also insignificant. The adjusted R square is lower (6% on average) compared to the one in my two-factor CCAPM (8% on average). Although the hypothesized factors are not significant after adding the US industrial production, they also do not significantly deviate from what the model predicts.

The international CAPM and the Fama-French two-factor model are also rejected. Both the global market excess return and the value factor are insignificant and negative (which is contrary to the actual mean value predicted by the model, see Table 1.2). Since in these two models, the risk factors are true returns, the intercept has its economic meaning, which is the return after risk adjustment and should be insignificantly different from zero. Note that from Table 1.4, the intercepts in both models are significant statistically at the 10% level and also significant economically, about 10% annually. The adjusted R squares are low in both models.

Although all models considered in this chapter do not perform well, the above observations suggest that the two-factor CCAPM is more consistent with the data than the other three.
Table 1.4

Two-Pass Regression Results (for dollar denominated returns)

The dependent variables are the 18 country market portfolio excess returns. The explanatory variables are the growth rate of the world industrial production $\Delta y$, the growth rate of the US industrial production $\Delta usy$, the real risk free rate (US one-month T-bill rate), the return on the global market portfolio in excess of the one-month U.S. Treasury bill return $r_m-r_f$, and the difference between the global high and low book-to-market returns (H-L). Mean is the time series average of the intercepts, slopes and adj. R$^2$ from the month-by-month Fama-MacBeth (FM) regressions over the 244 months in my sample; Std. is the standard deviation of the coefficient estimates; t is the ratio of the average value to its Shanken corrected standard error. All dollar returns are monthly. The time period is from July 1975 to October 2000.

<table>
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<tr>
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<th>The International CAPM</th>
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1.5.2 Robustness

To check robustness, I apply the FM two-stage regression method in three different ways: 1) fewer dependent variables: 15 market indices excess returns (leave out three countries/areas in Asia including Japan, Singapore and Hong Kong); 2) use German Mark denominated returns and the German real risk free rate; 3) divide the entire time period (July 1975 to October 2000, 304 observations) into two equal length sub periods, from July 1975 to February 1988 and from March 1988 to October 2000 (152 observations each) and do the empirical work separately. Table 1.5 (for 15 dependent variables), Table 1.6 (for German Mark denominated returns), Table 1.7 (for the first subperiod) and Table 1.8. (for the second subperiod) show the results.

Overall, the basic empirical results explained in section 1.5.1 still hold. All models perform poorly empirically. The hypothesized factors are not statistically significant. Economically, the estimated factors may not have the right signs or may not be close to the actual averages as predicted by the model. The adjusted R squares are all notably low. The intercepts in the international CAPM and the Fama-French two-factor model are either significantly different from zero at the 10% level or are high economically as indicators of pricing errors.

To further check which model is more consistent with the data --which model provides an empirically more useful framework for understanding international asset returns-- I find similar answers to these questions compared to what I find from the basic empirical work. The Fama-French two-factor model always performs better than the
international CAPM in terms of the adjusted R squares. However it is often rejected significantly due to its high pricing errors. My two-factor CCAPM seems to be the one that is most consistent with the data. Although the hypothesized factors are not significant, they also do not significantly deviate from what the model predicts.

It is worth noting that for the time period of March 1988 to October 2000 (see Table 1.8), all models perform better compared to the performances in both the whole time period and the first subperiod. In particular, the two-factor CCAPM clearly outperforms the standard CCAPM. The t-values are a lot higher and the risk free rate is statistically significant at the 10% level. The predicted values for the hypothesized risk factors are much closer to the true means. Higher adjusted R squares are also observed. These findings provide some support for the two-factor CCAPM.
Table 1.5
Two-Pass Regression Results (for 15 dependent variables)

The dependent variables are the 15 country market portfolio excess returns (leaving out Hong Kong, Japan and Singapore). The explanatory variables are the growth rate of the world industrial production $\Delta y$, the growth rate of the US industrial production $\Delta usy$, the real risk free rate (US one-month T-bill rate), the return on the global market portfolio in excess of the one-month U.S. Treasury bill $r_{m}-r_{f}$, and the difference between the global high and low book-to-market returns (H-L). Mean is the time series average of the intercepts, slopes and adj. $R^2$ from the month-by-month Fama-MacBeth (FM) regressions over the 244 months in my sample; Std. is the standard deviation of the coefficient estimates; $t$ is the ratio of the average value to its Shanken corrected standard error. All dollar returns are monthly. The time period is from July 1975 to October 2000.

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Table 1.6
Two-Pass Regression Results (for German Mark denominated real returns)

The dependent variables are the 18 country market portfolio excess returns. The explanatory variables are the growth rate of the world industrial production $\Delta y$, the growth rate of the US industrial production $\Delta usy$, the real risk free rate (German T-bill rate), the return on the global market portfolio in excess of the German Treasury bill $r_m-r_f$ and the difference between the global high and low book-to-market returns (H-L). Mean is the time series average of the intercepts, slopes and adj. $R^2$ from the month-by-month Fama-MacBeth (FM) regressions over the 244 months in my sample; Std. is the standard deviation of the coefficient estimates; t is the ratio of the average value to its Shanken corrected standard error. All German Mark returns are monthly. The time period is from July 1975 to October 2000.

<table>
<thead>
<tr>
<th>The Standard CCAPM</th>
<th>The Two-Factor CCAPM</th>
<th>The International CAPM</th>
<th>The FF Two-Factor Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>std.</td>
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<td>mean</td>
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<tr>
<td>intercept</td>
<td>1.01</td>
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<td>3.12</td>
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<td>$\Delta y$</td>
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<tr>
<td>real $r_f$</td>
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<tr>
<td>$\Delta usy$</td>
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</tr>
<tr>
<td>real $r_f$</td>
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<td></td>
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<tr>
<td>adj. $R^2$</td>
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Table 1.7
Two-Pass Regression Results (July 1975-February 1988)

The dependent variables are the 18 country market portfolio excess returns. The explanatory variables are the growth rate of the world industrial production \( \Delta y \), the growth rate of the US industrial production \( \Delta usy \), the real risk free rate (US one-month T-bill rate), the return on the global market portfolio in excess of the one-month U.S. Treasury bill return \( r_m-r_f \), and the difference between the global high and low book-to-market returns (H-L). Mean is the time series average of the intercepts, slopes and adj. \( R^2 \) from the month-by-month Fama-MacBeth (FM) regressions over the 92 months in my sample; Std. is the standard deviation of the coefficient estimates; t is the ratio of the average value to its Shanken corrected standard error. All dollar returns are monthly. The time period is from July 1975 to February 1988.

<table>
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<tr>
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<td>real ( r_f )</td>
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<td>0</td>
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<td>adj. ( R^2 )</td>
<td>.04</td>
<td>.14</td>
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Table 1.8
Two-Pass Regression Results (March 1988-October 2000)

The dependent variables are the 18 country market portfolio excess returns. The explanatory variables are the growth rate of the world industrial production $\Delta y$, the growth rate of the US industrial production $\Delta usy$, the real risk free rate (US one-month T-bill rate), the return on the global market portfolio in excess of the one-month U.S. Treasury bill return $r_m - r_f$, and the difference between the global high and low book-to-market returns (H-L). Mean is the time series average of the intercepts, slopes and adj. $R^2$ from the month-by-month Fama-MacBeth (FM) regressions over the 92 months in my sample; Std. is the standard deviation of the coefficient estimates; t is the ratio of the average value to its Shanken corrected standard error. All dollar returns are monthly. The time period is from March 1988 to October 2000.

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<td>.61</td>
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1.5.3 **Instrumental Variable Approach**

If markets are not complete and no asset perfectly correlated with aggregate consumption exists, an instrumental variable approach should be used. First regress the growth rate of consumption on the market return; then use the predicted value of the growth rate of consumption based on the market return to regress against the time series of returns of each asset. This yields the appropriate beta.

Table 1.9 provides the regression results. All models perform equally (not well) in terms of the significance and the estimation accuracy of the factors. The adjusted R squares for the two-factor CCAPM and the Fama-French two-factor model are the same and higher than the other two models.

To what extent this conclusion is supported depends mainly on how good the instrumental variable is. But due to the extremely low R square that I observe in the instrumental regression of estimating the mimicking returns based on the market return, the results presented in Table 1.9 can hardly be trusted.
Table 1.9
Two-Pass Regression Results (Instrumental Variable Approach)

The dependent variables are the 18 country market portfolio excess returns. The explanatory variables are the predicted growth rate of the world aggregate consumption and the predicted growth rate of the U.S. aggregate consumption based on the world market return and the U.S. market return respectively in excess of the one-month U.S. Treasury bill rate, $r_{c}-r_{f}$ and $r_{usc}-r_{f}$; the return on the global market portfolio in excess of the one-month U.S. Treasury bill rate $r_{m}-r_{f}$, and the difference between the global high and low book-to-market returns (H-L). Mean is the time series average of the intercepts, slopes and adj. $R^2$ from the month-by-month Fama-MacBeth (FM) regressions over the 244 months in my sample; Std. is the standard deviation of the coefficient estimates; t is the ratio of the average value to its Shanken corrected standard error. All dollar returns are monthly. The time period is from July 1975 to October 2000.

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<td>intercept</td>
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50
1.6 Conclusions

Time separable utility functions have been the mainstay in much of the literature on asset pricing, optimal consumption, and portfolio choice. The traditional consumption based asset pricing model is also based on this kind of utility function. But the standard CCAPM does not perform well empirically. This chapter tries to search theoretically for factors other than aggregate consumption that can explain asset returns.

Papers in various areas have developed considerable theoretical and empirical support for habit formation and catching up with the Joneses formulations. A representative consumer will experience a higher marginal utility from an additional unit of consumption in the future if others consume more today. Once one considers this “keeping up with the Joneses” featured utility function in the derivation of a consumption based capital asset pricing model, aggregate consumption itself is no longer sufficient to explain the cross-sectional variations in stock returns.

In an international context, this chapter derives a discrete Intertemporal CAPM and CCAPM under a normality assumption and shows that modifying the standard CCAPM with added feature of “keeping up with the Joneses preference” implies one more key factor, the aggregate consumption in the wealthiest country (U.S.) which is the consumption level that representative consumers in other countries want to “keep up with”. I replace aggregate consumption with industrial production as motivated by the Lucas production based CAPM (Lucas, 1978).
Applying the Fama and Macbeth (1973) two-pass regression method I have shown that all international asset pricing models (including three benchmark models, the standard CCAPM, the international CAPM and the Fama-French three-factor model as well as my two-factor CCAPM) considered in this chapter do not perform well empirically.

The Fama-French two-factor model always performs better than the international CAPM in terms of the adjusted R squares. However it is often rejected significantly due to its high pricing errors. My two-factor CCAPM seems to be the one that is most consistent with the data. Although the hypothesized factors are not significant, they also do not significantly deviate from what the model predicts.

The aforementioned findings are robust to various changes in the empirical specification.

The current study can be extended by solving the model for the more general case of $s$ state variables affecting investment opportunities. It is also worth examining that how the aforementioned models perform when other method like generalized method of moments, maximum likelihood estimation is employed as the evaluation metric. It may well be possible that the strict application of the FM procedure, requiring adequate out-of-sample performance, provides too high a hurdle for the current generation of asset pricing models.
CHAPTER 2

EXCHANGE RISK VERSUS THE VALUE FACTOR
IN INTERNATIONAL ASSET PRICING
2.1 Introduction

In an international context, if purchasing power parity (PPP) does not hold continuously, investing in foreign markets entails exposure to exchange rate risks. Any investment in a foreign asset is a combination of an investment in the performance of the foreign asset and an investment in the performance of the domestic currency relative to the foreign currency. The presence of this additional source of risk should be considered in the asset pricing model. Several theoretical models of international asset pricing incorporate this additional source of risk (see Solnik, 1974; Stulz, 1981; Adler and Dumas, 1983; for a survey see Stulz, 1992). Empirically, Dumas and Solnik (1995) support the existence of a foreign exchange risk premium by using a conditional approach that allows for time variation in the rewards for exchange rate risk. De Santis and Gerard (1998) estimate the conditional version of an International Capital Asset Pricing Model. Their findings strongly support a model that includes both market and foreign exchange risk.

Outside the direct context of the CAPM, the Fama and French (1992, 1993, 1996) three-factor model has been used in the international context. Fama and French (1998) argue that the CAPM cannot explain returns in a cross-section of national value portfolios
but their two-factor (global market β and value factor) model captures the value premium in international returns. They use here a two-factor instead of a three-factor model because their data for 12 EAFE (Europe, Australia, and the Far East) countries and the U.S. from MSCI include only large firms. A set of indexes consisting of large firms only does not allow meaningful inclusion of a size effect. See Banz (1981) for US returns and Heston, Rouwenhorst, and Wessels (1995) for international returns.

In order to explain the cross-sectional average returns of the international portfolios it is natural to consider the International CAPM model. However the Fama and French (1998) two-factor model avoids the conditions leading to the International CAPM by “assuming a world in which capital markets are integrated and investors are unconcerned with deviations from purchasing power parity”. Thus, they “ignore other risk factors that might affect expected returns”. But what if the assumption of purchasing power parity is dropped? In addition, Fama and French (1998) do not employ the more exacting Fama-MacBeth methodology as they did in their 1992 paper.

The purpose of this chapter is to assess the performance of the following three models: 1. The Standard CAPM, 2. The International CAPM and 3. The Fama and French two-factor model, in explaining the average returns of international portfolios by

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12 From MSCI Methodology & Index Policy (1997), the MSCI selection criteria, include among others: capture 60% of the market capitalization of the country across all industry groups; select the most liquid securities within each industry; apply full market capitalization weights.

13 According to the “Roll (1977) Critique”, the CAPM is not testable in part because the proxy used for the market return is imprecise. Any apparent rejection of the CAPM could be defended by saying that results are biased due to measurement error related to the improper measure of the market return. Roll’s true critique is more extensive. I am not testing the model; I simply want to evaluate the performance of asset pricing models in explaining the cross-sectional expected returns. Seeing how well the model fits the data and how useful the model is in out-of-sample forecasting.
employing the Fama and MacBeth (1973) two-stage regression approach (FM). There are alternative approaches, for instance, the generalized method of moments (GMM) of Hansen (1982). The advantage of the FM method is that it allows time-varying risk exposures\textsuperscript{14} and provides clear implications on a model’s out-of-sample forecasting power. The problem of generated regressors in the second stage can be resolved by using Shanken (1992) corrections.

Two closely related papers are Dahlquist and Sallstrom (2001) and Zhang (2001).

Dahlquist and Sallstrom (2001) examine whether an International CAPM performs better or worse than an international version of the empirical three-factor model (with size effect) in explaining the cross-sectional variation of returns. They estimate and evaluate the models for the national market portfolios, the global portfolios sorted according to earnings-price ratios and market values, and the global industry portfolios by using the generalized method of moments (GMM). They find that all the models considered seem to capture national market returns fairly well. An International CAPM without foreign exchange risk cannot explain the variation in average returns of the characteristic-sorted industry portfolios, even when allowing for time-varying expected returns. While an asset pricing model, which includes foreign exchange risk, is able to explain nearly 60% of the variation in average returns. Their empirical work suggests that the International CAPM with exchange risk has at least the same explanatory ability as the international three-factor model.

\textsuperscript{14} So I assume the model holds conditionally in terms of rolling betas. However, in this study, time-varying risk premia are not considered.
Zhang (2001) evaluates the cross-sectional pricing performance of several international asset pricing models (the International CAPM without exchange risk, the International CAPM with exchange risk, and the international version of the Fama-French three-factor model with size effect). The comparison metric is the Hansen-Jagannathan (1997) distance, and the base assets are size and book-to-market portfolios from the US, the UK and Japan. By allowing time-varying betas and risk premiums, most of the conditional models can capture the cross-sectional return spreads and can pass the test. The Fama-French factors are redundant in conditional models. Exchange risk exposures contribute significantly to the international asset returns, and the conditional International CAPM performs the best.

Although we all intend to assess the ability of the same set of international asset pricing models in explaining cross-sectional stock return variation, this chapter still differs from those of Dahlquist and Sallstrom (2001) and Zhang (2001) by the model specification\(^{15}\), the evaluation method\(^{16}\) and by the test assets used\(^{17}\).

It is interesting to see whether my results reinforce their conclusions or not.

\(^{15}\)Dahlquist and Sallstrom (2001) estimate conditional models in the sense of time-varying risk premia and assume constant betas; Zhang (2001) considers conditional models including variations in both risk premia and betas. Here, in this chapter, I examine conditional models by allowing risk exposures to vary over time but assume that risk premia do not vary over time.

\(^{16}\)Dahlquist and Sallstrom (2001) use the generalized method of moments (GMM), which does not allow time-varying risk exposure; Zhang (2001) uses the Hansen-Jagannathan distance metric and uses GMM for robustness. I employ the FM two-stage regression method with Shanken \(t\) corrections.

\(^{17}\)Griffin (2001) and Zhang (2001) use a large number of individual firms in three or four major countries, and they primarily focus on the characteristics-sorted portfolios on a country-by-country basis. Griffin (2001) finds that the Fama and French factors are local rather than global. My goal, however, is to study a large cross-section of international portfolios as is in Dahlquist and Sallstrom (2001), rather than portfolios sorted on a country-by-country basis. In addition, my test assets are not identical to those in Dahlquist and Sallstrom (2001), although we both include national market portfolios.
The chapter is organized as follows. Section 2.2 presents the model, the method employed and the evaluation criteria. Section 2.3 describes data and preliminary statistics. Section 2.4 provides the empirical results. Section 2.5 concludes.

2.2 Methodology and Evaluation Criteria

2.2.1 The Standard CAPM

Suppose the relevant model is the Standard CAPM. Then the global market portfolio is mean-variance efficient, and the dollar expected return on any security or portfolio, $R$, is fully explained by its loading on the dollar global market return, $M$. Employing the U.S. one-month Treasury bill as the risk free asset (with return $F$), equation (1) provides the form of the Standard CAPM model:

\[ R-F = \alpha + \beta [M-F] + \epsilon, \quad (1) \]

2.2.2 The Two-Factor Model (Fama and French, 1998)

Applying the Fama and French three-factor model in an international context and ignoring the size effect as explained before, equation (2) reflects the two-factor model Fama and French favor instead of the standard CAPM. The global market return and the difference between the returns on global value (high book-to-market: H) portfolios and
global growth (low book-to-market: L) portfolios can be used as the mimicking factors to explain the expected returns.

\[ R-F = \alpha + \beta_1 [M-F] + \beta_2 [H-L] + \epsilon, \]

(2)

H-L is the second explanatory return. Fama and French argue that the two-factor model in equation (2) provides a better description of country value portfolios. The GRS F-test\(^{18}\) clearly rejects equation (1), the standard CAPM when exchange rate risk is not considered.

### 2.2.3 The International CAPM

“When PPP does not hold, the heterogeneity of portfolio-choice behaviors limits the aggregation of individual demands into a CAPM” (Adler and Dumas 1983, p. 928). Equation (3) is obtained based on equation (16) from Adler and Dumas (1983), except that I only include three key exchange risks here.

\[ R-F = \alpha + \beta_1 [M-F] + \beta_2 [Mark-F] + \beta_3 [Yen-F] + \beta_4 [Pound-F] + \epsilon, \]

(3)

The extension of the CAPM to a multi-country case leads to a multi-factor solution for the pricing of assets. The new factors are the excess returns on assets that are

\(^{18}\) Gibbons, Ross and Shanken (1989) analyze a test for the ex ante efficiency of a given portfolio of assets. The null hypothesis of joint significance of the estimated intercepts across a set of equations is tested based on a central F distribution.
perfectly correlated with the exchange rate appreciations for each currency but the benchmark country. For efficiency purpose, Dumas and Solnik (1995), De Santis and Gerard (1998), Dahlquist and Sallstrom (2001) and Zhang (2001) use three important currencies to test the existence of an exchange risk premium. This chapter considers the same three currencies: the German Mark, the Japanese Yen and the British Pound (the United States is the reference country). Mark, Yen, and Pound in equation (3) represent the return of the assets that are perfectly correlated with the exchange rate appreciations for these three currencies.

To find these returns, practically, we select one existing zero-net-investment choice: borrowing in the reference country and investing in a foreign country’s risk free asset, for example, German Treasury bills. Then the actual Dollar return for this asset would be the return on the German Treasury bill minus the appreciation of the Dollar over the Mark (or plus the appreciation of the Mark over the Dollar).

In more detail, suppose one investor borrows a dollar at the end of January 1975, exchanges this one dollar to the German Mark at the end of January at the nominal exchange rate $s_1$ (German Mark/US dollar) and invests $s_1$ German Marks in the German Treasury bill. Suppose the February German Treasury bill rate is $i_2$. Then at the end of February, the investor ends up with $s_1(1+i_2)$ German Marks. To convert to a dollar return, divide this amount $s_1(1+i_2)$ by the end-of-February nominal exchange rate $s_2$. So the February dollar return is given by $\frac{s_1(1+i_2)}{s_2}$. Take the natural logarithm to obtain the continuous net return. Note that when $\ln(s_1/s_2)$ is positive (the US dollar depreciated...
during the period), the investor gains more than the risk free rate from investing in the foreign asset. When \( \ln(s_1/s_2) \) is negative (the US dollar appreciated during the period), the actual return is lower than the risk free rate because the payoff is worth less in terms of the dollar. The return on the zero-net-investment portfolio then subtracts the U.S. risk free rate.

If we consider the risk free rate to be relatively constant, the return on this investment mostly reflects the exchange rate risk. Similarly I find the factor returns for the Japanese Yen and the British Pound.

Why are these extra factors affecting expected returns? From a foreign consumer’s perspective, utility varies not just because of variation in wealth but also because of variation in the purchasing power of the wealth. For given returns denominated in the foreign currency their purchasing power would be less when the domestic currency appreciates. Investors in the foreign market may hedge against this kind of risk by holding their own currency.

### 2.2.4 Methodology

The question is how to evaluate these models and compare them empirically. Fama and French use the GRS F-test to judge the models and conclude that their two-factor model performs better by looking at the intercepts of a set of univariate regressions. However when they first considered the three-factor model (Fama and French, 1992) their asset pricing tests were the cross-sectional approach of Fama and
MacBeth (1973). Each month the cross-section of returns on stocks is regressed on variables hypothesized to explain expected returns. The time-series means of the monthly regression slopes then provide standard tests of whether different explanatory variables are on average priced.

Because my focus is on already well-specified international portfolios with considerable return variation, there is no need for portfolio formation. Consider the example of the International CAPM model. In the “first pass”, I regress equation (3) by using time series data of R, F, M, Mark, Yen and Pound to get the estimated factor loadings $\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3$ and $\hat{\beta}_4$ for each selected portfolio. Fama and French (1992) estimate betas in the first pass based on the full sample time period (from July 1963 to December 1990, 330 months). By doing so they assume the true betas of the portfolios do not vary over time, or are, at least, stationary. Here I discard this assumption and allow betas to vary over time. So in my first pass, I estimate betas based on only 5 years of monthly data.\footnote{There is a trade off between more accurate beta estimation by using a longer time period and misestimation due to drift in beta over time. Five years of monthly data (60 observations) is the norm in estimating betas (see Alexander and Chervany, 1980).} In more detail: I obtain the first set of betas for each portfolio by running equation (3) using time series data from July 1975 to June 1980 (60 months). Then I obtain the second set of betas for each portfolio by using time series data from August 1975 to July 1980 (60 months). Then the third set of betas, the fourth set and so forth until I reach the last month, December 2000.
In the “second pass”, Fama and French (1992) regress the cross-section of stock returns on the unique set of betas they estimate in the first pass month-by-month then calculate the average slopes that provide the standard FM tests. In my second pass I do the same except that my betas vary over time. To assess the usefulness of the model in out-of-sample forecasting, lagged estimated betas are used to explain future cross-sectional stock returns. I regress the July 1980 cross-section stock returns (the 61\textsuperscript{st} month of the full time period, or one month ahead right after the first time series sample period used to estimate betas) on the first set of betas ($\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3$ and $\hat{\beta}_4$), see equation (4). And I regress the August 1980 cross-section of stock returns (the 62\textsuperscript{nd} month among the whole time period, or one month ahead right after the second time series sample period used to estimate betas) on the second estimated set of betas. Continuing this process month by month I end up with a series of “second pass” intercepts and regression slopes for each hypothesized explanatory variable.

$$R - F = a_0 + a_1 \hat{\beta}_1 + a_2 \hat{\beta}_2 + a_3 \hat{\beta}_3 + a_4 \hat{\beta}_4 + e,$$ (4)

Where $\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3$ and $\hat{\beta}_4$ are the coefficient estimates from the first pass based on equation (3).

The estimated series of $\hat{a}_0, \hat{a}_1, \hat{a}_2, \hat{a}_3$ and $\hat{a}_4$ are then averaged over all time periods used in the second pass to provide the criteria for evaluating the model. A non-zero
constant indicates that a model cannot price the assets on average. The model is assessed by seeing if the intercept $\hat{a}_0$ deviates significantly from zero; if $\hat{a}_1$, $\hat{a}_2$, $\hat{a}_3$ and $\hat{a}_4$ deviate significantly from their corresponding actual mean risk premiums $\bar{M} - \bar{F}$, $\bar{Mark} - \bar{F}$, $\bar{Yen} - \bar{F}$ and $\bar{Pound} - \bar{F}$ and whether the model specification is able to capture the cross-sectional variation in average returns as measured by a cross-sectional R-square. Significance here is based on the t-statistic obtained by dividing each average estimated coefficient by its Shanken corrected standard error.

A similar procedure and testable propositions apply to the standard CAPM and the two-factor model. The model’s usefulness is challenged if the second stage time series average intercept deviates significantly from zero; or if the second stage time series average slopes deviate significantly from the model-predicted value, namely the mean excess returns for the hypothesized risk factors; or if the cross-sectional R-square is notably low.

The out-of-sample feature of the FM method discussed above motivates the choice of this method in this chapter in estimating the by far most common linear factor models together with its ability of allowing time-varying betas. The GMM approach is a natural fit for a discount factor formulation of asset pricing theories since it uses sample moments in the place of population moments. GMM does not allow risk exposures (betas) to vary over time. Another competing approach is maximum likelihood estimation, which is the most efficient asymptotically. Hansen and Jagannathan (1997) develop alternative ways to compare asset-pricing models when it is understood that their implied stochastic
discount factors do not price all portfolios correctly. They focus on model misspecification by measuring maximum pricing errors. However, the FM method is the one that allows evaluating various asset-pricing models based on their out-of-sample predictability power, which other methods cannot.

2.3 Data and Preliminary Statistics

Dumas and Solnik (1995) explain the cross-sectional differences in the returns on eight assets in addition to U.S. dollar deposits: the equity index of each country (Germany, the United Kingdom, Japan and the United States), deutsche mark deposits, pound sterling deposits, yen deposits, and the world index of equities. De Santis and Gerard (1998) explain monthly returns on stock indexes for the same four countries plus a value-weighted world index. Dahlquist and Sallstrom (2001) examine the national market portfolios, the global portfolios sorted according to earnings-price ratios and market values and the global industry portfolios. The base assets used in Zhang (2001) are size and book-to-market portfolios from the US, the UK and Japan.

In this chapter I study returns on the market, value, and growth portfolios for the United States and twelve major EAFE countries (Europe, Australia, and the Far East) as in Fama and French (1998). The global market portfolio returns come from MSCI (Morgan Stanley’s Capital International). All the other portfolio returns (including H-L)
and the US one-month T-bill rates are directly from Kenneth French’s homepage.\footnote{http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html} I use the monthly end-of-period exchange rate information for the Mark, Yen, and Pound (Dollar is the base currency) from International Financial Statistics (IFS). I further use the appropriate IFS Treasury-bill rates as risk free returns for Germany and the United Kingdom. For Japan I use IFS “money market rates” as the risk free asset returns. All dollar returns are monthly. The time period is from July 1975 to December 2000 (the longest available time period common to all my variables).

Table 2.1 shows that there exist a significant positive global market excess return and a value premium during the period from July 1975 to December 2000. No significant excess returns are observed for the assets that are perfectly correlated with country exchange rates.

Table 2.2 shows the correlation matrix for the five explanatory variables. Note that all three currencies, Mark, Yen and Pound are positively correlated with M-F. Including these extra factors in the model thus may affect the results for the market factor. All three currencies are positively correlated with each other.

Table 2.3 provides the statistics for 13 country international portfolio excess returns including the country value, country growth and country market portfolios and the global market, value, and growth excess returns.
Table 2.1
Statistics for the Hypothesized Explanatory Variables

M is the global market return from MSCI. F is the one-month U.S. Treasury bill rate. H-L is an international version of HML, the distress factor in the three-factor model for U.S. stock returns in Fama and French (1993). H-L represents the difference between the returns on global value (high book-to-market: H) portfolios and global growth (low book-to-market: L) portfolios. F and H-L are from Kenneth French’s homepage (see footnote 6). Mark, Yen, and Pound represent the return on investing in the German, Japanese, and United Kingdom risk free assets respectively in dollar terms. The exchange rate information and the risk free asset return for Germany, Japan and the United Kingdom are from IFS. Mean is the average monthly excess return over the 306 months in our sample; Std. is the standard deviation of each excess return; t is the ratio of the average return to its standard error. All dollar returns are monthly. The time period is from July 1975 to December 2000.

<table>
<thead>
<tr>
<th>Excess Return</th>
<th>Mean</th>
<th>Std.</th>
<th>t value</th>
</tr>
</thead>
<tbody>
<tr>
<td>M-F</td>
<td>.50</td>
<td>4.03</td>
<td>2.19</td>
</tr>
<tr>
<td>H-L</td>
<td>.49</td>
<td>2.74</td>
<td>3.16</td>
</tr>
<tr>
<td>Mark-F</td>
<td>-0.02</td>
<td>3.30</td>
<td>-0.10</td>
</tr>
<tr>
<td>Yen-F</td>
<td>.18</td>
<td>3.50</td>
<td>0.92</td>
</tr>
<tr>
<td>Pound-F</td>
<td>.12</td>
<td>3.20</td>
<td>0.64</td>
</tr>
</tbody>
</table>
Table 2.2

Correlation Matrix for the Five Explanatory Variables

The numbers listed in the table are the correlation coefficients between every two variables. M is the global market return from MSCI. F is the one-month U.S. Treasury bill rate. H-L is an international version of HML, the distress factor in the three-factor model for U.S. stock returns in Fama and French (1993). H-L represents the difference between the returns on global value (high book-to-market: H) portfolios and global growth (low book-to-market: L) portfolios. F and H-L are from Kenneth French’s homepage (see footnote 6). Mark, Yen, and Pound represent the return on investing in the German, Japanese, and United Kingdom risk free assets respectively in dollar terms. The exchange rate information and the risk free asset return for Germany, Japan and the United Kingdom are from IFS. All dollar returns are monthly. The time period is from July 1975 to December 2000.

<table>
<thead>
<tr>
<th></th>
<th>M-F</th>
<th>H-L</th>
<th>Mark-F</th>
<th>Yen-F</th>
<th>Pound-F</th>
</tr>
</thead>
<tbody>
<tr>
<td>M-F</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H-L</td>
<td>-0.12</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mark-F</td>
<td>0.21</td>
<td>-0.09</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yen-F</td>
<td>0.32</td>
<td>-0.12</td>
<td>0.55</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Pound-F</td>
<td>0.28</td>
<td>-0.11</td>
<td>0.66</td>
<td>0.44</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 2.3
Statistics for Country Portfolio Excess Returns

Excess returns here refer to the excess returns on market, high book-to-market (value), and low book-to-market (growth) portfolios for individual countries and the world. Mean is the average monthly excess return over 306 months in our sample; Std. represents the standard deviation; t is the ratio of the average return to its standard error. These portfolio excess returns are denoted by $R - F$ in equations (1), (2), (3) and (4). The time period is from July 1975 to December 2000. The individual countries are (in order): Australia, Belgium, France, Germany, Hong Kong, Italy, Japan, Netherlands, Singapore, Sweden, Switzerland, the United Kingdom and the United States.

<table>
<thead>
<tr>
<th>Country</th>
<th>Market Portfolios</th>
<th>Value Portfolios</th>
<th>Growth Portfolios</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean  Std. t</td>
<td>Mean  Std. t</td>
<td>Mean  Std. t</td>
</tr>
<tr>
<td>AUS</td>
<td>0.57  6.83 1.47</td>
<td>1.02  6.86 2.59</td>
<td>0.34  7.68 0.78</td>
</tr>
<tr>
<td>BEL</td>
<td>0.77  5.46 2.48</td>
<td>0.91  6.41 2.50</td>
<td>0.61  5.64 1.89</td>
</tr>
<tr>
<td>FRA</td>
<td>0.78  6.55 2.08</td>
<td>1.11  7.44 2.61</td>
<td>0.64  6.60 1.71</td>
</tr>
<tr>
<td>GER</td>
<td>0.64  5.69 1.95</td>
<td>1.06  6.02 3.08</td>
<td>0.60  6.14 1.71</td>
</tr>
<tr>
<td>HKG</td>
<td>1.26  9.34 2.37</td>
<td>1.38 11.37 2.12</td>
<td>1.05  8.63 2.12</td>
</tr>
<tr>
<td>ITA</td>
<td>0.63  7.78 1.42</td>
<td>0.62  8.64 1.25</td>
<td>0.70  7.74 1.58</td>
</tr>
<tr>
<td>JPN</td>
<td>0.55  6.69 1.44</td>
<td>1.00  7.17 2.44</td>
<td>0.24  7.12 0.58</td>
</tr>
<tr>
<td>NLD</td>
<td>0.93  4.98 3.26</td>
<td>0.98  6.70 2.57</td>
<td>0.81  5.29 2.69</td>
</tr>
<tr>
<td>SIG</td>
<td>0.67  7.71 1.52</td>
<td>1.14 10.21 1.95</td>
<td>0.57  7.69 1.29</td>
</tr>
<tr>
<td>SWE</td>
<td>0.93  6.58 2.47</td>
<td>1.15  7.96 2.54</td>
<td>0.94  6.75 2.44</td>
</tr>
<tr>
<td>SWT</td>
<td>0.69  5.22 2.30</td>
<td>1.07  5.80 3.23</td>
<td>0.59  5.32 1.94</td>
</tr>
<tr>
<td>UKM</td>
<td>0.86  5.81 2.59</td>
<td>1.06  6.32 2.94</td>
<td>0.76  6.15 2.15</td>
</tr>
<tr>
<td>USA</td>
<td>0.70  4.41 2.76</td>
<td>0.95  4.14 4.01</td>
<td>0.64  4.88 2.30</td>
</tr>
<tr>
<td>World</td>
<td>0.50  4.03 2.19</td>
<td>0.91  4.91 3.24</td>
<td>0.41  5.00 1.45</td>
</tr>
</tbody>
</table>
2.4 Main Results

2.4.1 Country Market Portfolios:

Overall, the results in the literature suggest that an international asset-pricing model with or without foreign exchange risk captures national market returns fairly well.\(^{21}\)

Table 2.4 lists the empirical results obtained via the methodology\(^{22}\) explained in section 2.2.4. The value factor which Fama and French strongly supported is not significant and negative, (t=-0.45, while the inferred value premium should be positive, see Table 2.1). The intercept represents the return after risk adjustment and should be insignificantly different from zero. Note that from Table 2.4, the intercept of the standard CAPM model is 1.12 with a t-value of 1.77, significantly different from zero both statistically (at the 10% level) and economically (14.3\% annualized); the two-factor model gives us an intercept of 1.14 with a t-value of 1.61, significantly different from zero both statistically (at the 10% level) and economically (14.57\% annualized) while the intercept from the International CAPM is only 0.34 with a t-value of 0.49 (statistically insignificant). In addition, the estimated average M-F is positive and equal to 0.34, which is much closer to the actual average M-F, 0.50 (see Table 2.1) than for the other two


\(^{22}\) Appendix 1 presents the “in-sample” FM method and the associated empirical results. Instead of applying estimated betas from the first pass to explain future cross-section stock returns, in the second pass I regress the cross-section stock returns on the betas obtained from the same time period.
models. Also it is obvious that the International CAPM has a dramatically higher adj. $R^2$ of 18% compare to 5% for the standard CAPM and 10% for the two-factor model.

Unlike what had been found in the literature on national market portfolios, these observations provide no support for the standard CAPM and the Fama-French two-factor model. Only the international CAPM with foreign exchange risks is able to explain reasonably well the cross-sectional variations in country market returns when the FM two-stage rolling beta method is employed.

One may argue that the exchange rate factors here are not significant. But although these factors are not significant by themselves they do not differ significantly from what the model predicts. Including these factors affects the standard CAPM beta since they are correlated with the market risk premium (see Table 2.2)

2.4.2 Country Value Portfolios and Value-Minus-Growth Portfolios:

Dahlquist and Sallstrom (2001) demonstrate that it is difficult to evaluate the ability of the asset pricing models to explain the cross-section of expected returns using national market portfolios. The reason for this is the low dispersion in the returns. To deal with this issue, country value portfolios and country value-minus-growth portfolios are also used as test assets in this chapter. Results in Table 2.5 and 2.6 indicate that

\footnote{Returns on these portfolios are obtained by subtracting each country’s growth portfolio return from the same country’s value portfolio return. Instead of using growth portfolios, value-minus-growth portfolios may have more return dispersion and therefore be good base assets to assess the ability of various asset pricing models.}
estimating international asset pricing models by using these portfolios poses a special challenge.

All three specifications considered here are unable to capture reasonably the cross-section of returns on the country value portfolios and country value-minus-growth portfolios. The null hypothesis of zero pricing errors is rejected (all intercepts are significantly different from zero). None of the hypothesized risk factors are significant. All market risk premia are negative. The adjusted Rsquare is somewhat higher for the international CAPM but still notably low.

These findings differ from the results in Fama and French (1998), Dahlquist and Sallstrom (2001) and Zhang (2001). Fama and French’s (1998) analysis is in the time series and applies the GRS F-test to unconditional models; Dahlquist and Sallstrom (2001) use GMM to simultaneously estimate time-series and cross-sectional parameters with time-varying risk premiums but without allowing time-varying betas; while here the FM two-stage regression method with the featured advantage of time-varying risk exposures contributes to assess the model. Zhang (2001) incorporates time-variation in both betas and risk premiums and draws conclusions based on the Hansen-Jagannathan distance (and examines portfolios on a country-by-country basis). The performance of the international asset pricing models seems to be sensitive to the empirical method that is employed to evaluate these specifications.
2.5 Conclusions

This chapter assesses the ability of the standard CAPM, the international CAPM and the international version of the FF two-factor model to explain the cross-sectional variation in expected returns. When applied to the cross-section of national market returns, the International CAPM model performs best. However, my empirical work suggests that the requirement of zero pricing errors can be rejected for any specification when the country value portfolios and country value-minus-growth portfolios are taken to be the test assets. To reach this conclusion I use the most exacting methodology, which Fama developed with MacBeth and which Fama and French (1992) use to support their three-factor model (with Shanken corrected standard errors).

The failure of all three models may result from the fact that the constant risk premiums implied in this study misspecify the behavior of the underlying risks.\(^{24}\) In addition, this chapter only examines an international version of the Fama-French three-factor model without incorporating the size effect. Using DataStream data, which cover many small firms, allows investigating the global size effect. This provides several directions in which the current study can be extended. The inconsistent results regarding the explanatory power of the international asset pricing models stemming from various empirical evaluation methods is also worth exploring in the future. It may well be

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possible that the strict application of the FM procedure, requiring adequate out-of-sample performance, provides too high a hurdle for the current generation of asset pricing models.
Table 2.4  
Two-Pass Regression Results for the Country Market Portfolios

The dependent variables are the 13 country market portfolio excess returns. The explanatory variables are the return on the global market portfolio in excess of the one-month U.S. Treasury bill return (M-F), the difference between the global high and low book-to-market returns (H-L), and the excess returns obtained by borrowing dollars and investing in German, Japanese, and U.K. risk free assets (Mark-F, Yen-F, Pound-F). Mean is the time series average of the intercepts, slopes and adj. $R^2$ from the month-by-month Fama-MacBeth (FM) regressions over the 246 months in my sample; Std. is the standard deviation of the coefficient estimates; $t$ is the ratio of the average value to its Shanken corrected standard error. All dollar returns are monthly. The time period is from July 1975 to December 2000.

<table>
<thead>
<tr>
<th></th>
<th>The Standard CAPM</th>
<th>The Two-Factor Model</th>
<th>The International CAPM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std.</td>
<td>$t$</td>
</tr>
<tr>
<td>Intercept</td>
<td>1.12</td>
<td>9.94</td>
<td>1.77</td>
</tr>
<tr>
<td>M-F</td>
<td>-0.37</td>
<td>9.82</td>
<td>-0.59</td>
</tr>
<tr>
<td>H-L</td>
<td>-0.17</td>
<td>5.9</td>
<td>-.45</td>
</tr>
<tr>
<td>Pound-F</td>
<td>0.25</td>
<td>5.74</td>
<td>0.68</td>
</tr>
</tbody>
</table>
Table 2.5
Two-Pass Regression Results for the Country Value Portfolios

The dependent variables are the 13 country-value portfolio excess returns. The explanatory variables are the return on the global market portfolio in excess of the one-month U.S. Treasury bill return (M-F), the difference between the global high and low book-to-market returns (H-L), and the excess returns obtained by borrowing dollars and investing in German, Japanese, and U.K. risk free assets (Mark-F, Yen-F, Pound-F). Mean is the time series average of the intercepts, slopes and adj. $R^2$ from the month-by-month FM regressions over the 246 months in my sample; Std. is the standard deviation of the coefficient estimate; $t$ is the ratio of the average value to its Shanken corrected standard error. All dollar returns are monthly. The time period is from July 1975 to December 2000.

<table>
<thead>
<tr>
<th>The Standard CAPM</th>
<th>The Two-Factor Model</th>
<th>The International CAPM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>Std.</td>
<td>$t$</td>
</tr>
<tr>
<td>Intercept</td>
<td>1.16</td>
<td>9.73</td>
</tr>
<tr>
<td>M-F</td>
<td>-0.09</td>
<td>10.1</td>
</tr>
<tr>
<td>H-L</td>
<td>0.29</td>
<td>5.82</td>
</tr>
<tr>
<td>Pound-F</td>
<td>0.17</td>
<td>6.87</td>
</tr>
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</table>
Table 2.6  
Two-Pass Regression Results for the Country Value-Minus-Growth Portfolios

The dependent variables are the 13 country value-minus-growth portfolio returns. The explanatory variables are the return on the global market portfolio in excess of the one-month U.S. Treasury bill return (M-F), the difference between the global high and low book-to-market returns (H-L), and the excess returns obtained by borrowing dollars and investing in German, Japanese, and U.K. risk free assets (Mark-F, Yen-F, Pound-F). Mean is the time series average of the intercepts, slopes and adj. $R^2$ from the month-by-month Fama-MacBeth (FM) regressions over the 246 months in my sample; Std. is the standard deviation of the coefficient estimates; $t$ is the ratio of the average value to its Shanken corrected standard error. All dollar returns are monthly. The time period is from July 1975 to December 2000.

<table>
<thead>
<tr>
<th></th>
<th>The Standard CAPM</th>
<th>The Two-Factor Model</th>
<th>The International CAPM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>Mean 0.33</td>
<td>Std. 2.19</td>
<td>$t$ 2.36</td>
</tr>
<tr>
<td>M-F</td>
<td>-0.66</td>
<td>8.51</td>
<td>$-1.22$</td>
</tr>
<tr>
<td>H-L</td>
<td>0.1</td>
<td>3.26</td>
<td>$0.48$</td>
</tr>
<tr>
<td>Pound-F</td>
<td>0.77</td>
<td>9.97</td>
<td>1.21</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.01</td>
<td>0.12</td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER 3

VARIABILITY OF STOCK RETURNS AND MACROECONOMIC FUNDAMENTALS UNDER DIFFERENT EXCHANGE RATE REGIMES: THEORY AND APPLICATION TO HONG KONG DATA
Chapter 3

Variability of Stock Returns and Macroeconomic Fundamentals Under Different Exchange Rate Regimes: Theory and Application to Hong Kong Data

3.1 Introduction

Many arguments have been made both for and against flexible exchange rate systems. Monetary models of the nominal exchange rate imply that stabilization of the exchange rate is achieved at the cost of a more volatile money supply and resulting volatility in the real exchange rate. The major advantages of a flexible exchange rate are the increased independence of government policies to be effective in stabilizing macroeconomic fundamentals and the less painful adjustment mechanism to trade imbalances. But the Mundell-Fleming model (Mundell, 1963 and Fleming, 1962) argues that fixed exchange rates are superior to floating rates because the fixed rates can reduce trade and investment uncertainties.

Standard classical or international real business cycle models imply that a shift in exchange rate regime would have no real effects. The International Capital Asset Pricing Model of Adler and Dumas (1983) on the other hand implies that the exchange rates are important determinants of cross-sectional differences in stock returns so a decrease in exchange rate volatility should decrease stock price volatility and expected returns based on “exchange rate betas”. Currently, most countries in the world manage their exchange
rates in some way. Is it true that a fixed exchange rate reduces uncertainty? An interesting way of testing these models and of answering these questions is to consider the effect of a change in exchange rate regime. In fact, much of the existing literature shows great interest on the same issue from different perspectives.

Frenkel and Mussa (1980) argue that for a given set of random shocks, fixing the exchange rate induces higher volatility of interest rates, money supplies, prices and output. Flood and Hodrick (1986) shows that the degree of credibility of the peg matters: an imperfectly credible exchange rate may result in higher volatility of domestic interest rates and asset prices than what would be the case in a permanently fixed and credible regime.

Baxter and Stockman (1989) are unable to find systematic differences in the volatility of real macroeconomic aggregates when they compare the pre-1973 and post-1973 periods for a sample of OECD and non-OECD countries. Hutchison and Walsh (1992) find that in Japan output appears to be more stable after it moved to the floating exchange rate system. Roll (1992) considers returns of national stock indices as determined by industry factors and exchange rates. Exchange rates explain a significant portion of common currency denominated national index returns, although the amount explained by exchange rates is less than the amount explained by industrial structure for most countries.

Regarding the EMS (European Monetary System) experience, Fratianni and von Hagen (1990) document that the EMS had a stabilizing impact on nominal and real
exchange rates, inflation and output. Artis and Taylor (1994) confirm this reduction in the conditional variances of nominal and real exchange rates over the 1979-1992 periods and show that there is no simultaneous increase in the conditional variance of interest rates.

Flood and Rose (1995) examine the typical OECD countries and conclude that conditional exchange rate volatility is substantially higher in floating rate regimes than it is during regimes of fixed rates. But the volatility of macroeconomic variables such as money, output, stock return and inflation does not change very much across exchange rate regimes. This suggests that there is no clear tradeoff between reduced exchange rate volatility and macroeconomic stability.

Bodart and Reding (1999) inspect the behavior of domestic daily returns on bond and stock markets of the ERM (Exchange Rate Mechanism) and non-ERM countries and provide substantial evidence that a credible peg is associated with a decline in bond market volatility. The essentially empirical analysis also shows that an increase in exchange rate volatility is accompanied by a decline in international correlations between bond and, to a lesser extent, stock markets.

Ran (2002) shows, contrary to the dominating empirical literature for the G-7 countries, that the volatilities of more than half of the nominal and real aggregate variables for Hong Kong under the float are significantly higher than those under the link.

A large body of the aforementioned studies is empirical and focuses on developed economies such as G-7 countries and the EMS. The purpose of this chapter is to seek theoretical support from a combination of a stochastic dynamic Mundell-Fleming
framework and the Lucas Asset Pricing Model to predict the influence of the shift in exchange rate regime on key macroeconomic fundamentals, especially on asset prices and test the predictions based on Hong Kong’s experience, with the link to the U.S. dollar.

Hong Kong has gone through different exchange rate regimes in its recent history. The Hong Kong dollar had been linked to the pound sterling from 1946 to 1972. On October 17, 1983, Hong Kong began to link its currency with the U.S. dollar after having applied a clear flexible exchange rate policy in the prior period. The Asian Financial Crisis of 1997 to 1998 hit Hong Kong very hard and its government has been active in adopting various policies in meeting this challenge. Because trade is very important to Hong Kong the influence of exchange rate policies is likely to be easily detectable.

This chapter is organized as follows. Section 3.2 lays out the model. Section 3.3 derives the testable propositions. Explanations are also provided in the same section. Section 3.4 describes the data and the method to be used empirically. Section 3.5 presents the results. Section 3.6 concludes.

### 3.2 The Model

The Mundell-Fleming model is one of the classic models in international finance. Mundell (2001) provides a history of the Mundell-Fleming model. Obstfeld (2001) presents a broad overview of postwar analytical thinking on international macroeconomics beyond the Mundell-Fleming model. Although the model is rather old
and ad hoc, the basic framework continues to be used in policy-related research (see Williamson, 1994; Hinkle and Montiel, 1999; MacDonald and Stein, 1999).

In this chapter I examine the implications of different exchange rate regimes in a modern stochastic dynamic version of the Mundell-Fleming framework, following Obstfeld, 1985 and Mark, 2001. The hallmark of the Mundell-Fleming framework is that goods prices adjust slowly, whereas asset markets—including the foreign exchange market—are continuously in equilibrium. The actions of policy-makers play a major role in these models, because the presence of nominal rigidities opens the way for nominal shocks to have real effects.

3.2.1 The (main body of the) Stochastic Dynamic Mundell-Fleming Model

Let \( y_t^d \) be aggregate demand, \( y_t^p \) potential production, \( s_t \) the nominal exchange rate (domestic currency per unit of foreign currency), \( p_t \) the domestic price level, \( i_t \) the domestic nominal interest rate, \( m_t \) the nominal money stock, and \( E_t(X) \) the mathematical expectation of the random variable \( X \), conditioned on date-\( t \) information. All variables except the interest rates are in natural logarithms. Variables with a superscript * represent foreign variables.

The IS curve in the stochastic dynamic Mundell-Fleming model is

\[
y_t^d = \eta (s_t - p_t) - \sigma [i_t - (E_t p_{t+1} - p_t)] + d_t,
\]

(1)
where $d_t$ is an aggregate demand shock and $i_t - (E_{t}p_{t+1} - p_t)$ is the *ax ante* real interest rate. Demand for home goods depends positively on the real exchange rate $s_t + p_t^* - p_t$ (I study a small open economy, so that for simplicity, the world price level $p_t^*$ can be set to be 0). Domestic goods demand depends negatively on the real interest rate through the investment-saving channel. The parameters $\eta$ and $\sigma$ (as are all other parameters) are defined to be positive.

The LM curve is

$$m_t - p_t = y_t^d - \lambda i_t,$$

(2)

where the income elasticity of money demand is assumed to be 1 and the interest rate elasticity equals $\lambda > 0$.

Capital market equilibrium is given by uncovered interest rate parity:

$$i_t = i_t^* + E_t s_{t+1} - s_t,$$

(3)

The long-run or the steady state is not conveniently characterized in a stochastic environment, because the economy is constantly being hit by shocks to the nonstationary exogenous state variables. Instead of a long-run equilibrium, I work with an equilibrium concept given by the solution formed under hypothetically fully flexible prices following Obstfeld (1985) and Mark (2001). Then as long as there is some degree of price-level
stickiness that prevents complete instantaneous adjustment, the disequilibrium can be characterized by the gap between sticky-price solution and the shadow flexible-price equilibrium.

Let the shadow price associated with the flexible-price equilibrium be denoted with a “tilde.” The sticky-price adjustment rule used in the standard stochastic dynamic Mundell-Fleming framework (Obstfeld, 1985 and Mark 2001) is

\[ p_t = \theta E_{t-1} \tilde{p}_t + (1-\theta) \tilde{p}_t, \quad (4)' \]

According to equation (4)', goods prices display rigidity for at most one period.

In this chapter, I use a different sticky-price adjustment rule following Rotemberg (1982). The stickiness is due to costs of adjusting prices.

\[ p_t = \theta p_{t-1} + (1-\theta) \tilde{p}_t, \quad (4) \]

At date \( t \), part of the firms set their price level exactly equal to last period’s actual price \( p_{t-1} \) while the rest of the firms set the prices such that the markets are in equilibrium, namely they allow prices to be flexible, \( p_t = \tilde{p}_t \). Prices are perfectly flexible if \( \theta = 0 \), and they are completely fixed if \( \theta = 1 \). Intermediate degrees of price rigidity are characterized by \( 0 < \theta < 1 \).
Rotemberg and Giovannini (1988) estimate the Deutsche mark/dollar exchange rate dynamics with sticky prices formed this way. The main results support the specified sticky-price adjustment rule. Rotemberg (1996) shows that the same sticky-price adjustment rule is consistent with a variety of facts concerning the correlation of prices, hours, and output. The sticky-price adjustment rule stated in equation (4) is one of the features that distinguish my work from the other work.

To derive $\hat{p}_t$, under fully flexible prices, $p_t = \hat{p}_t (\theta = 0)$, the goods market is continuously in equilibrium: $y_t^d = y_t$. Using equation (1) and the fact that potential production $y_t$ is exogenous gives:

$$y_t = \eta (s_t - \hat{p}_t) - \sigma [i_t - (E_t \hat{p}_{t+1} - \hat{p}_t)] + d_t,$$

(5)

The exogenous state variables are potential production $y_t$, the demand shock $d_t$, and the foreign interest rate $i_t^*$, and they are governed by unit-root processes.

$$y_t = y_{t-1} + z_t,$$

(6)

$$d_t = d_{t-1} + u_t,$$

(7)

$$i_t^* = i_{t-1}^* + v_t,$$

(8)
where $z_t \sim \text{i.i.d. } N(0, \sigma_z^2)$, $u_t \sim \text{i.i.d. } N(0, \sigma_u^2)$ and $v_t \sim \text{i.i.d. } N(0, \sigma_v^2)$. Cross correlations among $z_t$, $u_t$ and $v_t$ are all zero.

A second contribution in this chapter is to apply the Lucas Asset Pricing Model (1978) in my stochastic dynamic Mundell-Fleming framework to predict stock returns.

The Lucas production based asset pricing formula (see Appendix 4 for derivation) can be described by equation (9): $R_t^i$ is the stock return at time $t$ for asset $i$, $i_t$ is the risk free interest rate, $\beta_i$ measures the exposure to aggregate production risk for asset $i$, $\Delta$ denotes the change of a variable so $\Delta y_t^d$ represents the growth rate of production. The return on an asset that is perfectly correlated with aggregate production is measured by $a + b \Delta y_t^d$ where $a$ and $b$ are parameters that transform the growth rates of aggregate production to returns.

$$R_t^i = i_t + \beta_i [(a + b\Delta y_t^d) - i_t] = a \beta_i + (1 - \beta_i) i_t + b \beta_i \Delta y_t^d,$$  \hspace{1cm} (9)

### 3.2.2 The Stochastic Dynamic Mundell-Fleming Model under a Fixed Exchange Rate Regime

When the Hong Kong dollar is linked to the U.S. dollar, Hong Kong monetary authorities must adjust its nominal interest rate to track adjustments to the U.S. nominal interest rate conducted by the U.S. monetary authorities. Otherwise, with perfect capital mobility, the discrepancy between the local interest rate $i_t$ and the foreign interest rate $i_t^*$
would cause capitals flow in and out which could break the link. So we can say that Hong Kong’s monetary authorities are committed to fixing the exchange rate with the U.S. Consider the United States as the foreign environment, Hong Kong has a fixed exchange rate regime so that \( s_{t+1} = s_t = E_t s_{t+1} \). Substituting this into equation (3), the uncovered interest rate parity condition, I end up with equation \((10)_L\). The subscript \( L \) refers to the linked exchange rate regime.

\[ i_t = i_t^* \quad (10)_L \]

In order to maintain a fixed exchange rate, the domestic nominal interest rate is set to be equal to the nominal interest rate in a foreign country to which the bilateral exchange rate is linked. Monetary authorities in the home country lose the ability to control the economy via monetary policies.

Equations (1) through \((10)_L\) provide the complete stochastic dynamic Mundell-Fleming model under a linked exchange rate regime. By incorporating a lag operator \( L \) (for example, \( L \ p_t = p_{t-1} \)), the system can be solved (see Appendix 2). Let \( \pi_t \) (\( p_t - p_{t-1} \)) be the inflation rate and let \( q_t \) be the real exchange rate \( (s_t - p_t) \), then the solutions are:

\[ \Delta i_t = v_t \quad (11)_L \]
\[ \Delta y_t^d = \frac{1-\theta}{1-\theta L} \left( \sigma v_t + z_t - u_t \right) + u_t - \sigma v_t - \theta(1-\theta) \frac{\sigma^2}{\eta} (v_{t-1} - v_t) + \frac{\sigma}{\eta} [z_t - z_{t-1} - (u_t - u_{t-1})] \] 

(12)_L

\[ \Delta q_t = \frac{1-\theta}{1-\theta L} \left( \frac{\sigma v_t + z_t - u_t}{\eta} \right) \] 

(13)_L

\[ \pi_t = -\frac{1-\theta}{1-\theta L} \left( \frac{\sigma v_t + z_t - u_t}{\eta} \right) \] 

(14)_L

\[ \Delta m_t = \pi_t + \Delta y_t^d - \lambda \Delta i_t \] 

(15)_L

\[ R_t^i = a\beta_i + (1 - \beta_i) i_t^* + b\beta_i \Delta y_t^d \] 

(16)_L

Under the linked exchange rate regime, it is not surprising to find that the change in the domestic nominal interest rate at date \( t \) is determined solely by date \( t \) shocks to the foreign interest rate, because \( i_t \) is set to be equal to \( i_t^* \) and \( i_t^* \) is assumed to follow a random walk process.

Slow adjustment in goods prices enables past shocks to play a role in the current economy. Under the linked exchange rate regime, monetary authorities are unable to control these shocks by letting the interest rate and the exchange rate vary over time. Note that, from equations (12)_L through (16)_L, output growth \( \Delta y_t^d \), the growth rate of the real exchange rate \( \Delta q_t \), inflation \( \pi_t \), the growth rate of money \( \Delta m_t \) and stock returns \( R_t^i \)
depend on the domestic supply and demand shocks and foreign interest rate shocks at time $t$ as well as those shocks previous to time $t$.

### 3.2.3 The Stochastic Dynamic Mundell-Fleming Model under a Flexible Exchange Rate Regime

When the authorities do not intervene in the foreign exchange market, the nominal exchange rate is free to vary over time and domestic authorities regain control over monetary policy. Many authors have considered the estimation of monetary policy rules that relate the interest rate to inflation and the output gap, known as Taylor rules, both for the United States and for other countries (see Taylor (1993), Judd and Rudebusch (1998), Clarida, Gali and Gertler (1998) and Faust, Rogers and Wright (2001)).

As in Faust, Rogers and Wright (2001), I use a forward-looking Taylor rule (relating the monetary policy to expected future economic conditions, equation (10)$_F$) in this chapter as the monetary reaction function for the domestic authorities. The subscript $F$ refers to the flexible exchange rate regime.

$$i_t = \bar{i} + \alpha (E_t p_{t+1} - p_t) + \gamma (y^d_t - y_t), \quad (10)_F$$

where $\bar{i}$ is a constant (containing the targeted interest and inflation rate information), $E_t p_{t+1} - p_t$ is the expected inflation rate and $y^d_t - y_t$ is the output gap. This kind of monetary
reaction function indicates that the central bank is targeting inflation (or forecasted inflation) but that it is doing so with at least some separate attention paid to the output gap, and it is using the short-term interest rate as the instrument of monetary policy.

Most central banks may not characterize their behavior in this way. However, the reason for the popularity of this kind of monetary policy rule is that it provides a good simple empirical representation of central bank behavior under a wide variety of monetary policy regimes.

If the parameter $\alpha$ is greater than one, an increase in inflation causes the real interest rate to rise. If $\alpha$ is less than one, the system is not stable and self-fulfilling bursts of inflation rate are possible (Henderson and McKibbin, 1993). Clarida, Gali and Gertler (2000) show, based on the U.S. data, that the estimate of $\alpha$, the coefficient associated with expected inflation is significantly greater than one for the Volcker-Greenspan period (79:3-96:4). On the other hand, the estimate of $\gamma$, the coefficient measuring the sensitivity to the cyclical variable, is marginally significantly positive but less than one for the Volcker-Greenspan era. Faust, Rogers and Wright (2001) study the monetary policy for the ECB (European Central Bank), and compare it with a simple empirical representation of the monetary policy of the Bundesbank prior to 1999. By using monthly German data (85:01-98:12), they find that $\alpha$ is somewhat greater than one, $\gamma$ is small but positive.

This model fits the literature with one more contribution of adding a forward-looking Taylor rule as monetary authorities’ reaction function into the stochastic dynamic
Mundell-Fleming framework. Equations (1) through (10) provide a complete Mundell-Fleming system under a flexible exchange rate regime.

In order to solve the system (and to require the system to be stable), the following condition must hold\(^{25}\) (see Appendix 3):

\[
\rho = \theta - \alpha \theta + \alpha + \theta \gamma \eta > 0, \quad \text{(17)}
\]

The solutions then are as follows:

\[
\Delta i_t = \frac{\rho + 1 - \theta \rho}{\rho} \nu_t, \quad \text{(11)}_F
\]

\[
\Delta y_t^d = \frac{\theta \eta}{\rho} \nu_t + z_t, \quad \text{(12)}_F
\]

\[
\Delta q_t = \frac{\theta \eta + \sigma \rho}{\rho \eta} \nu_t + \frac{z_t - u_t}{\eta}, \quad \text{(13)}_F
\]

\[
\pi_t = \frac{1 - \theta}{\rho} (i_t^* - i_t), \quad \text{(14)}_F
\]

\(^{25}\) Note that from Appendix 3, equation (A3.13) is a stochastic difference equation in \(p_t - p_{t-1}\). Iterating forward on (A3.13), the solution for \(p_t - p_{t-1}\) can be found by the present-value formula if the parameter, \(\frac{1 - \alpha (1 - \theta) + \gamma \sigma \theta}{\gamma \theta (\sigma + \eta) + \theta}\), on the term of \(E_p p_{t-1} - p_t\), is less than one. This results in the precondition, equation (17).
\[ \Delta m_t = \pi_t + \Delta y_t^d - \lambda \Delta i_t, \quad (15) \]

\[ R_t^i = a\beta_i + (1 - \beta_i) i_t^* + \frac{(1 - \beta_i)(1 - \theta)}{\rho} (i_t^* - \bar{i}) + b\beta_i \Delta y_t, \quad (16) \]

Note that under a flexible exchange rate regime, all the macroeconomic fundamentals depend only on domestic supply and demand shocks as well as foreign exchange rate shocks at time \( t \). Shocks previous to time \( t \) do not play a role like they do under the linked exchange rate regime. Independent monetary policies (as stated in the Taylor rule) are effective in stabilizing the macroeconomic fundamentals from the view of eliminating past shocks to the economy.

### 3.3 Testable Propositions under Two Extreme Cases for the Degree of Price Rigidity

The influence of different exchange rate regimes on macroeconomic fundamentals can be studied now by looking at the solutions from the classical point of view when goods prices are perfectly flexible and also from the Keynesian point of view when goods prices are fixed. In other words, I analyze the solutions suggested by the model for two extreme degrees of price rigidity, \( \theta=0 \) (flexible prices) and \( \theta=1 \) (fixed prices).
3.3.1 The Classical Case (θ=0)

When \( \theta = 0 \), the precondition (equation (17)) to solve the above flexible Mundell-Fleming model is: \( \alpha > 1 \). This is consistent with the results obtained by Henderson and McKibbin (1993), Clarida, Gali and Gertler (2000) and Faust, Rogers and Wright (2001) mentioned in section II, part C. If the parameter \( \alpha \) is greater than one, an increase in inflation causes the real interest rate to rise. If \( \alpha \) is less than one, the system is not stable and self-fulfilling bursts of inflation rate are possible (Henderson and McKibbin, 1993).

Substitute \( \theta = 0 \) into equations (11)\(_L\) through (16)\(_L\), and equations (11)\(_F\) through (16)\(_F\), when goods prices are perfectly flexible, I end up with six pairs of comparisons for key macroeconomic fundamentals under different exchange rate regimes.

\[
\begin{align*}
\left\{ \begin{array}{l}
(\Delta y^d_t)_F = z_t \\
(\Delta y^d_t)_L = z_t 
\end{array} \right.,
\left\{ \begin{array}{l}
(\Delta i_t)_F = \frac{\alpha}{\alpha-1} v_t \\
(\Delta i_t)_L = v_t,
\end{array} \right.
\left\{ \begin{array}{l}
(\Delta q_t)_F = \frac{\sigma}{\eta} v_t + \frac{z_t - u_t}{\eta} \\
(\Delta q_t)_L = \frac{\sigma}{\eta} v_t + \frac{z_t - u_t}{\eta},
\end{array} \right.
\end{align*}
\]

\[
\begin{align*}
\left\{ \begin{array}{l}
(\pi_t)_F = \frac{1}{\alpha-1} (i^*_t - \bar{i}) \\
(\pi_t)_L = - \left( \frac{\sigma}{\eta} v_t + \frac{z_t - u_t}{\eta} \right)
\end{array} \right.,
\left\{ \begin{array}{l}
(\Delta m_t)_F = \frac{1}{\alpha-1} (i^*_t - \bar{i}) + z_t - \frac{\lambda \alpha}{\alpha-1} v_t \\
(\Delta m_t)_L = - \left( \frac{\sigma}{\eta} v_t + \frac{z_t - u_t}{\eta} \right) + z_t - \lambda v_t,
\end{array} \right.
\end{align*}
\]

\[
\begin{align*}
\left\{ \begin{array}{l}
(R_t^i)^F = a \beta_i + (1 - \beta_i) i_t^* + \frac{(1 - \beta_i)}{\alpha-1} (i^*_t - \bar{i}) + \beta_i z_t \\
(R_t^i)^L = a \beta_i + (1 - \beta_i) i_t^* + \beta_i z_t.
\end{array} \right.
\end{align*}
\]

These findings are explained as follows:
If prices are free to change, the goods market is always in equilibrium \((y_t^d = y_t)\), actual production is determined purely by the economy’s potential capacity to produce, the choice of the exchange rate regime does not have real effect on \(\Delta y_t^d\) so variability of the growth rate of the output stays the same under both exchange regimes.

The nominal interest rate is more stable under the linked exchange rate regime. Based on the uncovered interest rate parity, equation (3), the domestic nominal interest rate \(i_t\) is equal to the foreign nominal interest rate \(i_t^*\) plus the expected change in the nominal exchange rate, \(E_t s_{t+1} - s_t\). When the exchange rate is fixed, \(E_t s_{t+1} - s_t\) is zero, \(i_t\) is determined solely by \(i_t^*\). However, in the flexible case, \(i_t\) is affected by the variations both in \(i_t^*\) and in \(s_t\). The nominal exchange rate \(s_t\) varies over time as the domestic authorities try to stabilize the economy following a Taylor rule reaction function.

More volatile stock returns under the flexible exchange rate system are due to the risk free interest rate being more variable given that the variability of output growth stays the same under both regimes.

Regarding the real exchange rate, there is no difference in the volatility among two alternative regimes suggested by the solutions. Since \(q_t = s_t - p_t\), and the nominal exchange rate \(s_t\) is obviously more volatile under the flexible exchange rate regime, so in order to satisfy an equal variability in the real exchange rate \(q_t\), it must be true that under the flexible exchange rate regime, the inflation rate \(\pi_t\) is less volatile. Actually we would expect that this is true intuitively given the model set up. Under the floating exchange rate system, domestic authorities are able to stabilize the inflation rate by their own
independent monetary policies while they cannot do so under a linked exchange rate regime when they have to use the monetary instruments to maintain a fixed exchange rate. Meanwhile, the fact that the real exchange rate exhibits the same variability under both regimes can also be explained. Although the nominal exchange rate is more volatile when it is free to vary, low volatility of the inflation rate enables the possibility that the exchange rate regime has no effect on the variability of the real exchange rate.

From equation (15), we know that variability of the money stock depends on the variability of output growth, nominal interest rates and inflation. We have concluded that output growth variability stays the same under both exchange rate regimes but that inflation is less volatile and the nominal interest rate is more volatile under the flexible exchange rate environment, so it is hard to tell whether the money stock should be more or less stable without further assumptions on the parameters and precise mathematic derivations. The overall effect is ambiguous. The ambiguity comes from the opposite influences of the choice of the exchange rate regime on the variability of inflation and nominal interest rates.

Proposition I summarizes these findings and provides us with interesting testable implications.
**Proposition I:** when goods prices are perfectly flexible, when the forward-looking Taylor rule is used as the monetary authority’s reaction function under a flexible exchange rate regime, a stochastic dynamic Mundell-Fleming framework combined with the Lucas Asset Pricing Model indicates:

\[(I\ a)\ \text{Variance (}\Delta y^d)_{F} = \text{Variance (}\Delta y^d)_{L};\]

\[(I\ b)\ \text{Variance (}\Delta q)_{F} = \text{Variance (}\Delta q)_{L};\]

\[(I\ c)\ \text{Variance (}\Delta i)_{F} > \text{Variance (}\Delta i)_{L};\]

\[(I\ d)\ \text{Variance (}\pi_{r})_{F} > \text{Variance (}\pi_{r})_{L};\]

\[(I\ e)\ \text{Variance (}\pi_{y})_{F} < \text{Variance (}\pi_{y})_{L}.\]

### 3.3.2 The Keynesian Case (\(\theta = 1\))

When \(\theta = 1\), the precondition (equation (17)) to solve the above flexible Mundell-Fleming model is: \(\gamma \eta > 0\). \(\eta\) is defined to be positive. A positive \(\gamma\) is consistent with the results obtained by Clarida, Gali and Gertler (2000) and Faust, Rogers and Wright (2001) mentioned in section II, part C. Note that from equation (10)\(_F\), the Taylor rule monetary reaction function, \(\gamma\) is the coefficient measuring the sensitivity of nominal interest rate to the cyclical variable (the output gap). When the output gap is positive (demand is above potential production), monetary authorities need to increase nominal interest rates to cool
down the economy and therefore eliminate the gap. Lower interest rates indicated by a negative $\gamma$ will deepen the gap and make the economy unstable.

Substituting $\theta=1$ into equations (11)$_L$ through (16)$_L$, and equations (11)$_F$ through (16)$_F$, when goods prices are fixed, I again end up with six pairs of comparisons for key macroeconomic fundamentals under different exchange rate regimes.

\[
\begin{align*}
(\Delta y^d_t) F &= z_t + \frac{1}{\gamma} v_t, \\
(\Delta y^d_t) L &= u_t - \sigma v_t, \\
(\Delta i_t) F &= v_t, \\
(\Delta i_t) L &= v_t, \\
(\Delta q_t) F &= \frac{1}{\gamma \eta} v_t + \frac{z_t - u_t}{\eta}, \\
(\Delta q_t) L &= 0
\end{align*}
\]

\[
\begin{align*}
(\Delta m_t) F &= z_t + \left(\frac{1}{\gamma} - \lambda\right) v_t, \\
(\Delta m_t) L &= u_t - (\sigma + \lambda) v_t, \\
(\Delta R_t) F &= \alpha \beta_i + (1 - \beta_i) i_t^* + \beta_i (z_t + \frac{1}{\gamma} v_t), \\
(\Delta R_t) L &= \alpha \beta_i + (1 - \beta_i) i_t^* + \beta_i (u_t - \sigma v_t).
\end{align*}
\]

It is clear that when prices are fixed, the inflation rate is stable and does not depend on the choice of the exchange rate regime. Obviously the nominal exchange rate is more volatile under the flexible system. With the same (zero) variability of inflation under both regimes, the real exchange rate $q_t$ must be more variable under the flexible environment since $q_t = s_t - p_t$.

It is interesting to find that the variability of the nominal interest rate stays the same under both regimes when prices are fixed and is fully determined by shocks on the foreign interest rate. Based on the uncovered interest rate parity condition, equation (3), the domestic nominal interest rate $i_t$ is equal to the foreign nominal interest rate $i_t^*$ plus the expected change in the nominal exchange rate $E_t s_{t+1} - s_t$. It can be shown that (see
Appendix 3, equation A3.17), under the flexible exchange rate regime, \( E_t s_{t+1} - s_t = p_t - p_{t-1} \).

Expected variations in the nominal exchange rate next period are determined by the actual inflation rate this period. Given that prices are fixed, the zero inflation rate implies zero changes in the expected nominal exchange rate. The term \( E_t s_{t+1} - s_t \) disappears in the uncovered interest rate parity, making \( i_t \) equals \( i_t^* \), like the case in which the exchange rate is fixed.

Without further knowledge about the parameters’ magnitudes (\( \gamma \) and \( \sigma \)) and more precise assumptions on the shocks, it is unclear what sign should be assigned to the comparison of \( (\Delta y_t^d)_F \) with \( (\Delta y_t^d)_L \). But note that once we can conclude under which regime \( \Delta y_t^d \) is more stable, the same question for \( \Delta m_t \) and \( R_t^i \) can be easily determined because they both contain \( \Delta y_t^d \) in the expression and it is the only different part over the two alternative regimes. Proposition II summarizes the above findings.

**Proposition II:** when goods prices are fixed, when the forward-looking Taylor rule is used as the monetary authority’s reaction function under the flexible exchange rate regime, a stochastic dynamic Mundell-Fleming framework combined with the Lucas Asset Pricing Model indicates either:
(II a1) \( \text{Variance} (\pi_t)_F = \text{Variance} (\pi_t)_L \);

(II b1) \( \text{Variance} (\Delta q_t)_F > \text{Variance} (\Delta q_t)_L \);

(II c1) \( \text{Variance} (\Delta i_t)_F = \text{Variance} (\Delta i_t)_L \);

(II d1) \( \text{Variance} (\Delta y_{td}^d)_F > \text{Variance} (\Delta y_{td}^d)_L \);

(II e1) \( \text{Variance} (R_t^i)_F > \text{Variance} (R_t^i)_L \);

(II f1) \( \text{Variance} (\Delta m_t)_F > \text{Variance} (\Delta m_t)_L \);

Or

(II a2) \( \text{Variance} (\pi_t)_F = \text{Variance} (\pi_t)_L \);

(II b2) \( \text{Variance} (\Delta q_t)_F > \text{Variance} (\Delta q_t)_L \);

(II c2) \( \text{Variance} (\Delta i_t)_F = \text{Variance} (\Delta i_t)_L \);

(II d2) \( \text{Variance} (\Delta y_{td}^d)_F < \text{Variance} (\Delta y_{td}^d)_L \);

(II e2) \( \text{Variance} (R_t^i)_F < \text{Variance} (R_t^i)_L \);

(II f2) \( \text{Variance} (\Delta m_t)_F < \text{Variance} (\Delta m_t)_L \).

My results are much more subtle than those in previous studies that have looked at the influence of changes in exchange rate regime on variability of key macroeconomic variables. Instead of guessing the effects intuitively or testing purely empirically, my
predictions are built on rigorous theoretical derivations. Moving from a flexible exchange rate regime to a fixed exchange rate regime does not simply indicate that macroeconomic fundamentals are more volatile or less volatile as argued by other studies.

3.4 **Conditional Variance and the Data**

Note that from the expressions of the key macroeconomic variables that we are interested in, variability of these variables can be determined by variances of $u_t$, $v_t$ and $z_t$ in either the Classical case or the Keynesian case. By assumption, $z_t \sim \text{i.i.d. } N(0, \sigma_z^2)$, $u_t \sim \text{i.i.d. } N(0, \sigma_u^2)$ and $v_t \sim \text{i.i.d. } N(0, \sigma_v^2)$, see equation (6), (7) and (8). Then the conditional and unconditional variances of each variable $u_t$, $v_t$, and $z_t$ would be same and equal to $\sigma_u^2$, $\sigma_v^2$, and $\sigma_z^2$ respectively. $i_t^*$ is governed by unit-root process as specified in equation (8), $i_t^* = i_{t-1}^* + v_t$. While the unconditional variance of $i_t^*$ is infinity, the conditional mean of $i_t^*$, $E_t(i_t^*) = E_t(i_{t-1}^*) + E_t(v_t) = i_{t-1}^*$. The conditional variance of $i_t^*$ then could be calculated theoretically by $E_t((i_t^* - i_{t-1}^*)^2) = E_t(v_t^2) = \sigma_v^2$, which is a constant. By looking at the conditional variances, volatility of key macroeconomic variables in terms of $u_t$, $v_t$, $z_t$ and $i_t^*$ can be compared over the alternative exchange rate regimes.

This chapter uses standard deviation as a measure of conditional variance. Standard deviation for each key macroeconomic variable (the growth rate of Gross Domestic Production, the growth rate of nominal money supply, inflation, the change in nominal interest rate, the growth rate of real exchange rate, and the stock return on Hong
Kong market index) is calculated before and after the link, and a standard F-test is used to test the hypothesis of no significant differences in variance.

Quarterly data (1975Q1-1999Q2) for GDP, interest rate, money supply, and the Consumer Price Index are obtained from Hong Kong’s Research Center. The nominal exchange rates (end of period Hong Kong dollar per U.S. dollar) and the U.S. CPI are collected from International Financial Statistics (IFS). Hong Kong market index returns R_t can be obtained directly from Kenneth French’s homepage.

3.5 Major Results

Table 3.1 lists the standard deviations for each key macroeconomic variable under alternative exchange rate regimes and the associated variance-ratio F-test.

The data tell us that after the link of the Hong Kong dollar to the US dollar, Hong Kong market index stock returns, the nominal interest rate, the nominal money supply and the real exchange rate become less volatile. For output growth and the inflation rate, no significant changes in variability are observed.

Note that from Table 3.1, neither Proposition I nor Proposition II is able to predict the facts implied by the data perfectly well. It is relatively easy to see that compared to the second part of Proposition II, the first part is more consistent with the data. However,

\footnote{I am grateful to Professor Jimmy Ran (Lingnan University, Hong Kong) for kindly providing these valuable data to me.}

\footnote{http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html}
the choice between Proposition I and the first part of Proposition II to characterize Hong Kong’s economy is hard.

If prices are perfectly flexible in Hong Kong, Proposition I works well for stock returns, the change in nominal interest rates and output growth. It also predicts that under the linked exchange rate regime, the inflation rate should be more variable and the variability of the real exchange rate is unchanged. Individual F-test weakly supports my prediction on the inflation rate but clearly rejects my prediction on the real exchange rate. The link to the US dollar dramatically decreased the volatility of the real exchange rate in Hong Kong. The probability that Proposition I holds for all variables is almost zero.

If prices are perfectly fixed in Hong Kong, Proposition II (the first part) works well for stock returns, the growth rate of the nominal money supply, the inflation rate and the real exchange rate. It also predicts that under the linked exchange rate regime, output growth should be less variable and that the variability of the nominal interest rate be unchanged. Individual F-tests weakly support my prediction on output growth but clearly rejects my prediction on the nominal interest rate.

In this case, a joint test for the two Propositions is preferred. However, it is very difficult to construct such a test. The null hypotheses for the two propositions are very complex. And the number of possible alternative hypotheses is large. The test statistic based on a joint distribution of these F-test ratios is hard to derive.

---

28 For Proposition I, H₀: \( \sigma_1^2(R) > \sigma_1^2(R), \sigma_1^2(\Delta y) = \sigma_1^2(\Delta y), \sigma_1^2(\Delta i) > \sigma_1^2(\Delta i), \sigma_1^2(\pi) > \sigma_1^2(\pi), \sigma_1^2(\Delta q) = \sigma_1^2(\Delta q) \). For Proposition II, first part, H₀: \( \sigma_1^2(R) > \sigma_1^2(R), \sigma_1^2(\Delta y) > \sigma_1^2(\Delta y), \sigma_1^2(\Delta i) > \sigma_1^2(\Delta i), \sigma_1^2(\Delta m) > \sigma_1^2(\Delta m), \sigma_1^2(\pi) = \sigma_1^2(\pi), \sigma_1^2(\Delta q) > \sigma_1^2(\Delta q) \).
Table 3.1  
**Standard Deviations for Hong Kong’s Key Macroeconomic Variables**

Std\(_F\) represents standard deviation under flexible exchange rate regime (1975Q1-1983Q3, 35 observations), Std\(_L\) represents standard deviation under linked exchange rate regime (1983Q4-1999Q2, 63 observations). R represents stock returns on the market index, Δy represents the growth rate of nominal GDP, Δi represents the change in the nominal interest rate, Δm represents the growth rate of nominal money supply, π represents inflation and Δq represents the growth rate of real Hong Kong dollar-US dollar exchange rate. F refers to the variance-ratio F-test. In the row of “Conclusion”, “↓” indicates that from flexible to linked exchange rate regime, variability of the corresponding variables significantly decreased at 1% level and “nd” indicates that no significant differences are observed. Two propositions are also listed in the same table. “↓” indicates that from flexible to linked exchange rate regime, the predicted variability of the corresponding variables decrease, “↑” indicates that from flexible to linked exchange rate regime, the predicted variability of the corresponding variables increase and “nd” indicates that no significant differences should be observed.

<table>
<thead>
<tr>
<th>Fundamentals</th>
<th>R</th>
<th>Δy</th>
<th>Δi</th>
<th>Δm</th>
<th>π</th>
<th>Δq</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std(_F)</td>
<td>10.25</td>
<td>5.21</td>
<td>2.01</td>
<td>7.58</td>
<td>1.36</td>
<td>3.56</td>
</tr>
<tr>
<td>Std(_L)</td>
<td>7.05</td>
<td>6.51</td>
<td>1.21</td>
<td>5.02</td>
<td>1.18</td>
<td>1.35</td>
</tr>
<tr>
<td>F</td>
<td>2.11</td>
<td>1.56</td>
<td>2.74</td>
<td>2.28</td>
<td>1.35</td>
<td>6.95</td>
</tr>
<tr>
<td>Conclusion</td>
<td>↓</td>
<td>nd</td>
<td>↓</td>
<td>↓</td>
<td>nd</td>
<td>↓</td>
</tr>
</tbody>
</table>

**Proposition I**

|                | Down | nd  | Down | no prediction | Up  | nd   |

**Proposition II**

(1) | Down | Down | nd   | Down | nd   | Down |
(2) | Up   | Up   | nd   | Up   | nd   | Down |
Although the individual variance ratio is unable to clearly support one Proposition and reject the other, it seems that the first part of Proposition II outperforms Proposition I. The decrease in the variability of the real exchange rate in Hong Kong after the link was so remarkable that any theory that fails to capture this decrease can not be true. It may be more appropriate to characterize Hong Kong’s economy by a relatively fixed price system.

My propositions stem from two extreme assumptions regarding the price adjustment rule (see equation (4)), either perfect flexible (θ = 0) or perfect fixed (θ = 1). The results obtained in this chapter suggest a value of θ that is close to 1 for the Hong Kong’s economy. It is also worth noting that both Proposition I and the first part of the Proposition II (between which a choice is hard to make) predict that after the link, stock market returns should be more stable as is verified by the data. This finding supports the exchange rate risk theory in international asset pricing. When moving from a flexible exchange rate regime to a linked regime, the variability of the underlying exchange risks decreases, resulting in the more stable stock returns.

3.6 Conclusions

On October 17, 1983, Hong Kong began to link its currency with the U.S. dollar after having applied a clear flexible exchange rate policy in the prior period. How would this change in the exchange rate regime affect the Hong Kong’s economy? Will the link
associate with more volatile key macroeconomic variables as argued by the monetary models? Or does the link reduce the trade and investment uncertainties as implied by the Mundell-Fleming framework and therefore benefit Hong Kong’s economy? Or would the link have no real effects according to standard classical or international real business cycle models?

This chapter seeks theoretical support from a combination of a stochastic dynamic Mundell-Fleming framework and the Lucas Asset Pricing Model to predict the influence of the shift in exchange rate regime on key macroeconomic fundamentals, especially on asset prices and test the predictions based on Hong Kong’s experience with the link to the U.S. dollar.

Rotemberg’s (1982) sticky-price adjustment rule, a forward-looking Taylor rule monetary reaction function and the Lucas Asset Pricing formula (1978) employed in the stochastic dynamic Mundell-Fleming framework are the major contributions.

The solutions of the model are studied both from the classical point of view when goods prices are perfectly flexible and also from the Keynesian point of view when goods prices are fixed. The results obtained in this chapter suggest that prices in Hong Kong are relatively fixed; stock market returns are more stable after the link, which supports the exchange rate risk theory in international asset pricing. When moving from a flexible exchange rate regime to a linked regime, the variability of the underlying exchange risks decreases, resulting in the more stable stock returns.
The major conclusion in this chapter is: there is no certain direction for the changes in the variability of key macroeconomic fundamentals when the exchange rate regime shifts. The interaction between the variables under alternative market structures provides mixed predictions.

Future research can be focused on solving the model for the case where foreign interest rates follow a white noise process rather than a random walk; controlling for the external effects on macroeconomic indicators. In more detail, the changes in the variability of the macroeconomic fundamentals may be caused by the changes in the general global environment rather than by the shift of the exchange rate regime. Since other countries would be subject to the same external influences, their macroeconomic variables can serve as effective instruments to control for such influences as long as their exchange rate regimes did not change during the study period.

Furthermore, I study a small open economy in this chapter and the foreign price level \( p_t^* \) is set to be 0 for simplicity. To apply the model more generally, the effect of changes in foreign prices on the system should also be carefully considered and examined.
APPENDICES
Appendix 1
In Sample FM Two-Pass Method and Results

This appendix presents the work I have done related to Chapter 2. It is left out of the main body of the chapter because I use here an adjusted Fama-Macbeth methodology. Everything else is unchanged.

A.1.1 Methodology

If one is not concerned with the model's out-of-sample forecasting power and cares more about whether the data support the model, the FM two-pass regression methodology, with some differences suggested by the context, is also used. Instead of applying estimated betas from the first pass to explain future cross-section stock returns according to the exact FM method, in the second pass I regress the cross-section stock returns on the betas obtained from the same time period. When the FM approach is used in sample it is more similar to the Black, Jensen, and Scholes (1972) approach.

In more detail: I obtain the first set of betas for each portfolio by running equation (3) using time series data from July 1975 to May 1980 (59 months). I use 59 months instead of 60 months because it is easier for the second-pass to find a middle observation, which is the return that can be explained best by the betas obtained from the 59 months period. Then I obtain the second set of betas for each portfolio by using time series data from August 1975 to June 1980 (59 months). Then the third set of betas, the fourth set and so forth until reach the last month, December 2000.
In the “second pass”, Fama and French (1992) regress the cross-section of stock returns on the unique set of betas they estimate in the first pass month-by-month then calculate the average slopes that provide the standard FM tests. In my second pass I do the same except that my betas vary over time. I regress the December 1977 cross-section stock returns (the 30th month among the first 59 months, or the middle observation in the first time series sample period) on the first set of betas ($\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3$ and $\hat{\beta}_4$). And I regress the January 1978 cross-section stock returns (again, the middle observation in the second time series sample period) on the second estimated set of betas. Continuing this process month by month I end up with a series of the “second pass” intercepts and regression slopes for each hypothesized explanatory variable.

A.1.2 In Support of the International CAPM:

Fama and French (1998) found strong evidence that for the country value portfolios the standard CAPM was dead (the GRS F-test clearly rejects it) while their two-factor model does much better in explaining the value premium. Table A.1.1 lists the empirical results obtained via the methodology explained in section A.1.1.

I use the same portfolios as Fama and French (1998), 13 country value portfolios, but a relatively longer time period29 to estimate the standard CAPM, the two-factor model and the International CAPM. The value factor which Fama and French strongly supported is not significant and negative, ($t=-0.87$, while the inferred value premium

---

29 Mine is from July 1975 to December 2000 while Fama and French investigated the period from January 1975 to December 1995.
should be positive, see Table 2.1). The intercept represents the return after risk adjustment and should be insignificantly different from zero. Note that from Table A.1.1, the intercept of the standard CAPM model is 0.95 with a t value of 1.79 (significantly from zero at 10% level); the two-factor model gives us an intercept of 0.96 with a t-value of 1.88, significantly different from zero both statistically (at 10% level) and economically (12.15% annualized) while the intercept from the International CAPM is only 0.03 with a t-value of 0.05. Also Table A.1.1 shows that market beta has no explanatory power in the standard CAPM (t=0.35) and the two-factor model (t=0.62) while it is significantly different from zero at the 10% level in the international CAPM model (t=1.82). And it is obvious that the International CAPM has a dramatically higher adj. $R^2$ of 20% compare to 5% for the standard CAPM and 8% for the two-factor model. This evidence supports the International CAPM model.

So, in an international context, the International CAPM model provides the best explanation of the cross-sectional stock returns, at least compared with the standard CAPM and the Fama-French two-factor model. This is true not only for country value portfolios. For robustness, I apply the standard CAPM, the International CAPM and the two-factor model to two other sets of portfolios: 13 country market portfolios and 26 country value and growth portfolios (13 value and 13 growth portfolios).

Country Market Portfolios (Table A.1.2): The standard CAPM is rejected because it has a statistically significant (t=1.64, 10% level) positive intercept and the value is high economically for a return after risk adjustment, 0.90% monthly. For the same reason the
two-factor model is rejected. The intercept is 1.02% monthly with a t of 1.76. Besides, the value factor H-L is significantly (t=-2.06) negative (-0.76) which is contrary to the value of 0.49 the model predicts (See Table 2.1). However the International CAPM survives. First the intercept is only 0.48 and insignificant (t=0.86); second the estimated average M-F is 0.37 which is relatively close to the actual average M-F, 0.50 (see Table 2.1); last, the 25% average adj. $R^2$ is higher than either 5% for the standard CAPM or 11% for the two-factor model. None of the factors significantly differs from the model predicted value.

Country Value and Growth Portfolios (Table A.1.3): The standard CAPM is rejected because it has a statistically significant (t=2.51) positive intercept and the value is quite high economically for a return after risk adjustment, 1.15% monthly. For the same reason the two-factor model is rejected. The intercept is 1.05% monthly with a t of 2.36. Besides, the value factor H-L is negative and insignificant. However the International CAPM again survives. First the intercept is only 0.49 and insignificant (t=1.18); second the estimated average M-F is 0.49 which is much closer to the actual average M-F, 0.50 (see Table 2.1) than for the other two models; last, the 27% average adj. $R^2$ is higher than either 6% for the standard CAPM or 10% for the two-factor model. None of the factors significantly differs from the model predicted value.

Other regressions, not shown in this appendix, indicate that a value factor added to the International CAPM is insignificant and does not affect the other results.
The dependent variables are the 13 country-value portfolio excess returns. The explanatory variables are the return on the global market portfolio in excess of the one-month U.S. Treasury bill return (M-F), the difference between the global high and low book-to-market returns (H-L), and the excess returns obtained by borrowing dollars and investing in German, Japanese, and U.K. risk free assets (Mark-F, Yen-F, Pound-F). Mean is the time series average of the intercepts, slopes and adj. $R^2$ from the month-by-month FM regressions over the 248 months in my sample; Std. is the standard deviation of the coefficient estimate; $t$ is the ratio of the average value to its standard error. All dollar returns are monthly. The time period is from July 1975 to December 2000.

### Table A.1.1  
**In Sample Two-Pass Regression Results for the Country Value Portfolios**

<table>
<thead>
<tr>
<th>The Standard CAPM</th>
<th>The Two-Factor Model</th>
<th>The International CAPM</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td><strong>Std.</strong></td>
<td><strong>t</strong></td>
</tr>
<tr>
<td>Intercept</td>
<td>0.95</td>
<td>8.38</td>
</tr>
<tr>
<td>M-F</td>
<td>0.21</td>
<td>9.77</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.05</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table A.1.2
In Sample Two-Pass Regression Results for the Country Market Portfolios

The dependent variables are the 13 country market portfolio excess returns. The explanatory variables are the return on the global market portfolio in excess of the one-month U.S. Treasury bill return (M-F), the difference between the global high and low book-to-market returns (H-L), and the excess returns obtained by borrowing dollars and investing in German, Japanese, and U.K. risk free assets (Mark-F, Yen-F, Pound-F). Mean is the time series average of the intercepts, slopes and adj. $R^2$ from the month-by-month Fama-MacBeth (FM) regressions over the 248 months in my sample; Std. is the standard deviation of the coefficient estimates; $t$ is the ratio of the average value to its standard error. All dollar returns are monthly. The time period is from July 1975 to December 2000.

<table>
<thead>
<tr>
<th></th>
<th>The Standard CAPM</th>
<th>The Two-Factor Model</th>
<th>The International CAPM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std.</td>
<td>$t$</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.90</td>
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<td>1.64</td>
</tr>
<tr>
<td>M-F</td>
<td>-0.04</td>
<td>9.17</td>
<td>-0.06</td>
</tr>
<tr>
<td>H-L</td>
<td>-0.76</td>
<td>5.82</td>
<td>-2.06</td>
</tr>
<tr>
<td>Pound-F</td>
<td>-0.08</td>
<td>6.01</td>
<td>-0.20</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.05</td>
<td>0.17</td>
<td></td>
</tr>
</tbody>
</table>
Table A.1.3
In Sample Two-Pass Regression Results for the Country Value Growth Portfolios

The dependent variables are the 26 country value and growth portfolio excess returns. The explanatory variables are the return on the global market portfolio in excess of the one-month U.S. Treasury bill return (M-F), the difference between the global high and low book-to-market returns (H-L), and the excess returns obtained by borrowing dollars and investing in German, Japanese, and U.K. risk free assets (Mark-F, Yen-F, Pound-F). Mean is the time series average of the intercepts, slopes and adj. $R^2$ from the month-by-month Fama-MacBeth (FM) regressions over the 248 months in my sample; Std. is the standard deviation of the coefficient estimates; t is the ratio of the average value to its standard error. All dollar returns are monthly. The time period is from July 1975 to December 2000.

<table>
<thead>
<tr>
<th>The Standard CAPM</th>
<th>The Two-Factor Model</th>
<th>The International CAPM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std.</td>
</tr>
<tr>
<td>Intercept</td>
<td>1.15</td>
<td>7.24</td>
</tr>
<tr>
<td>M-F</td>
<td>-0.21</td>
<td>7.83</td>
</tr>
<tr>
<td>H-L</td>
<td>-0.09</td>
<td>3.76</td>
</tr>
<tr>
<td>Pound-F</td>
<td>-0.13</td>
<td>5.67</td>
</tr>
<tr>
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<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix 2

Solving a Stochastic Dynamic Mundell-Fleming Model Combined with the Lucas Asset Pricing Formula under a Fixed Exchange Rate Regime

The model:

\[ y_t^d = \eta (s_t - p_t) - \sigma [i_t - (E_t p_{t+1} - p_t)] + d_t, \quad (1) \]

\[ m_t - p_t = y_t^d - \lambda i_t, \quad (2) \]

\[ i_t = i_t^* + E_t s_{t+1} - s_t, \quad (3) \]

\[ p_t = \theta p_{t-1} + (1-\theta) \bar{p}_t, \quad (4) \]

\[ y_t = \eta (s_t - \bar{p}_t) - \sigma [i_t - (E_t \bar{p}_{t+1} - \bar{p}_t)] + d_t, \quad (5) \]

\[ y_t = y_{t-1} + z_t, \quad (6) \]

\[ d_t = d_{t-1} + u_t, \quad (7) \]

\[ i_t^* = i_{t-1}^* + v_t, \quad (8) \]

\[ R_t^i = i_t + \beta_i (\Delta y_t^d - i_t) = (1 - \beta_i) i_t + \beta_i \Delta y_t^d, \quad (9) \]

\[ i_t = i_t^*, \text{ or } s_{t+1} = s_t = E_t s_{t+1}, \quad (10) \]

To solve the model:

Define \( q_t = s_t - \bar{p}_t \), \( q_t = s_t - p_t \). \( \text{(A2.1)} \)

A2 indicates Appendix 2.
Substitute (3) into (5) and rearrange to give

$$\tilde{q}_t = \frac{y_t - d_t}{\sigma + \eta} + \sigma \left( i_t^* + E_t q_{t+1} \right),$$

(A2.2)

(A2.2) is a stochastic difference equation in $\tilde{q}$. The solution is given by the present-value formula by iterating forward on (A2.2) since $\frac{\sigma}{\sigma + \eta} < 1$.

$$\tilde{q}_t = \frac{\sigma i_t^* + y_t - d_t}{\eta},$$

(A2.3)

By using a lag operator $L$ in (4),

$$p_t = \frac{1 - \theta}{1 - \theta L} \tilde{p}_t,$$

(A2.4)

where $L p_t = p_{t-1}$, $L$ is equal to 1 when multiplied by a constant.

Rearrange (A2.1) to give

$$\tilde{p}_t = s_t - \tilde{q}_t,$$

(A2.5)

Substitute (A2.3), (A2.5) into (A2.4),

$$p_t = s_t - \frac{1 - \theta}{1 - \theta L} \frac{\sigma i_t^* + y_t - d_t}{\eta},$$

(A2.6)

Based on (A2.6), (6), (7), (8) and (10)$_L$ to find

$$E_t p_{t+1} - p_t = - \frac{\theta (1 - \theta)}{1 - \theta L} \frac{\sigma v_t + z_t - u_t}{\eta},$$

(A2.7)

Substitute (A2.6), (10)$_L$ and (A2.7) into (1),

$$y_t^d = \frac{1 - \theta}{1 - \theta L} \left( \sigma i_t^* + y_t - d_t \right) + d_t - \sigma i_t^* - \frac{\theta (1 - \theta)}{1 - \theta L} \frac{\sigma^2 v_t + \sigma (z_t - u_t)}{\eta},$$

(A2.8)
Based on (A2.1) and (A2.6),

\[ q_t = \frac{1-\theta}{1-\theta L} \frac{\sigma i_t^* + y_t - d_t}{\eta}, \quad (A2.9) \]

Now it is straightforward to find the solutions:

\[ \Delta i_t = v_t \quad (11)_L \]

\[ \Delta y_t^d = \frac{1-\theta}{1-\theta L} (\sigma v_t + z_t - u_t) + \sigma v_t - \theta(1-\theta) \frac{\sigma^2}{\eta} (v_t - v_{t-1}) + \frac{\sigma}{\eta} [z_t - z_{t-1} - (u_t - u_{t-1})] \]

\[ (12)_L \]

\[ \Delta q_t = \frac{1-\theta}{1-\theta L} \left( \frac{\sigma v_t + z_t - u_t}{\eta} \right) \quad (13)_L \]

\[ \pi_t = p_t - p_{t-1} = -\frac{1-\theta}{1-\theta L} \left( \frac{\sigma v_t + z_t - u_t}{\eta} \right) \quad (14)_L \]

\[ \Delta m_t = \pi_t + \Delta y_t^d - \lambda \Delta i_t \quad (15)_L \]

\[ R^i_t = (1 - \beta_i) i_t^* + \beta_i \Delta y_t^d \quad (16)_L \]
Appendix 3

Solving a Stochastic Dynamic Mundell-Fleming Model
Combined with the Lucas Asset Pricing Formula
under
a Flexible Exchange Rate Regime

The model:

\[ y_t^d = \eta (s_t - p_t) - \sigma \left[ i_t - (E_t p_{t+1} - p_t) \right] + d_t, \quad (1) \]

\[ m_t - p_t = y_t^d - \lambda i_t, \quad (2) \]

\[ i_t = i_t^* + E_t s_{t+1} - s_t, \quad (3) \]

\[ p_t = \theta p_{t-1} + (1-\theta) \tilde{p}_t, \quad (4) \]

\[ y_t = \eta (s_t - \tilde{p}_t) - \sigma \left[ i_t - (E_t \tilde{p}_{t+1} - \tilde{p}_t) \right] + d_t, \quad (5) \]

\[ y_t = y_{t-1} + z_t, \quad (6) \]

\[ d_t = d_{t-1} + u_t, \quad (7) \]

\[ i_t^* = i_{t-1}^* + v_t, \quad (8) \]

\[ R_t = i_t + \beta (\Delta y_t^d - i_t) = (1 - \beta_i) i_t + \beta_i \Delta y_t^d, \quad (9) \]

\[ i_t = \bar{i} + \alpha (E_t p_{t+1} - p_t) + \gamma (y_t^d - y_t), \quad (10) \]

To solve the model:

Define \( \tilde{q}_t = s_t - \tilde{p}_t, \ q_t = s_t - p_t \), \quad (A3.1)

A3 indicates Appendix 3.
Substitute (3) into (5) and rearrange to give

\[ \tilde{q}_t = \frac{y_t - d_t}{\sigma + \eta} + \frac{\sigma}{\sigma + \eta} (i_t^* + E_t \tilde{q}_{t+1}) , \]

(A3.2)

(A3.2) is a stochastic difference equation in \( \tilde{q} \). The solution is given by the present-value formula by iterating forward on (A3.2) since \( \frac{\sigma}{\sigma + \eta} < 1 \).

\[ \tilde{q}_t = \frac{\sigma i_t^* + y_t - d_t}{\eta} , \]

(A3.3)

By using a lag operator \( L \) in (4),

\[ \tilde{p}_t = \frac{1 - \theta L}{1 - \theta} p_t , \]

(A3.4)

where \( L p_t = p_{t-1} \), \( L \) is equal to 1 when multiplied by a constant.

From (A3.4),

\[ \tilde{p}_t - p_t = \frac{\theta}{1 - \theta} (p_t - p_{t-1}) , \]

(A3.5)

\[ E_t \tilde{p}_{t+1} - \tilde{p}_t = \frac{1 - \theta L}{1 - \theta} (E_t p_{t+1} - p_t) , \]

(A3.6)

Rearrange (A3.1) to give

\[ s_t = \tilde{p}_t + \tilde{q}_t , \]

(A3.7)

Substitute (A3.3) into (A3.5),

\[ s_t = \tilde{p}_t + \frac{\sigma i_t^* + y_t - d_t}{\eta} , \]

(A3.8)

Based on (A3.6), (6), (7) and (8) find

\[ E_t s_{t+1} - s_t = E_t \tilde{p}_{t+1} - \tilde{p}_t , \]

(A3.9)
Substitute (A3.6) into (A3.9),

\[ E_t s_{t+1} - s_t = \frac{1-\theta}{1-\theta} (E_t p_{t+1} - p_t), \]  
\[ (A3.10) \]

Subtract (5) from (1),

\[ y_t^d - y_t = \eta (\tilde{p}_t - p_t) + \sigma [E_t p_{t+1} - p_t - (E_t \tilde{p}_{t+1} - \tilde{p}_t)], \]  
\[ (A3.11) \]

Substitute (A3.5) and (A3.6) into (A3.11),

\[ y_t^d - y_t = \frac{(\sigma+\eta)\theta}{1-\theta} (p_t - p_{t-1}) - \frac{\sigma\theta}{1-\theta} (E_t p_{t+1} - p_t), \]  
\[ (A3.12) \]

Substitute (A3.10) into (3), substitute (A3.12) into (10)\(_F\), and rearrange to give

\[ (p_t - p_{t-1}) = \frac{1-\theta}{\gamma \theta (\sigma+\eta) + \theta} (i_t^* - \bar{i}) + \frac{1-\alpha(1-\theta) + \gamma \sigma \theta}{\gamma \theta (\sigma+\eta) + \theta} (E_t p_{t+1} - p_t), \]  
\[ (A3.13) \]

(A3.13) is a stochastic difference equation in \(p_t - p_{t-1}\). The solution is given by the present-value formula by iterating forward on (A3.13) as long as

\[ \frac{1-\alpha(1-\theta) + \gamma \sigma \theta}{\gamma \theta (\sigma+\eta) + \theta} < 1, \]  
\[ (A3.14) \]

From (A3.14), the precondition to solve the model is

\[ \rho = \theta - \alpha \theta + \alpha - 1 + \theta \gamma \eta > 0, \]  
\[ (17) \]

Given that (17) is true, solve (A3.13),

\[ p_t - p_{t-1} = \frac{1-\theta}{\rho} (i_t^* - \bar{i}), \]  
\[ (A3.15) \]

Based on (A3.15) and (6), (7), (8),

\[ E_t p_{t+1} - p_t = p_t - p_{t-1}, \]  
\[ (A3.16) \]

Substitute (A3.16) into (A3.10),

\[ E_t s_{t+1} - s_t = p_t - p_{t-1}, \]  
\[ (A3.17) \]
From (A3.8) and (A3.5),

\[ q_t = s_t - p_t = \frac{\theta}{1-\theta} (p_t - p_{t-1}) + \frac{\sigma_i^* + y_t - d_t}{\eta}, \]  

(A3.18)

Everything has been solved in terms of \( p_t - p_{t-1} \), (A3.15) provides the solution for \( p_t - p_{t-1} \).

Now it is straightforward to find the solutions for the model as follows:

\[ \Delta i_t = \frac{\rho + 1 - \theta}{\rho} v_t, \]  

(11)_F

\[ \Delta y_t^d = \frac{\theta_1}{\rho} v_t + z_t, \]  

(12)_F

\[ \Delta q_t = \frac{\theta_1 + \sigma \rho}{\rho \eta} v_t + \frac{z_t - u_t}{\eta}, \]  

(13)_F

\[ \pi_t = \frac{1 - \theta}{\rho} (i_t^* - \bar{i}), \]  

(14)_F

\[ \Delta m_t = \pi_t + \Delta y_t^d - \lambda \Delta i_t, \]  

(15)_F

\[ R_t^i = (1 - \beta_i) i_t^* + \frac{(1 - \beta_i)(1-\theta)}{\rho} (i_t^* - \bar{i}) + \beta_i \Delta y_t^d, \]  

(16)_F
Appendix 4
Lucas Production-Based Asset Pricing Formula

The Lucas (1978) model is based on a simple endowment economy. A representative investor maximizes expected lifetime utility

\[ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \quad 0 < \beta < 1, \]  

subject to a wealth constraint:

\[ \sum_{i=1}^{n} p_i^t s_{t+1}^i + q_t b_{t+1} = \sum_{i=1}^{n} (p_i^t + d_i^t) s_{t+1}^i + b_t - c_t, \]  

Here \( p_i^t \) indicates the price of a share in asset \( i \), \( s_i^t \) indicates the number of shares held at the beginning of period \( t \), \( q_t \) the price of a discount bond, and \( b_t \) the number of discount bonds held.

Defining returns as \( R_{t+1}^i = (d_{t+1}^i + p_{t+1}^i)/p_t^i \), the first order conditions for the risky assets and the riskless asset can be shown to yield:

\[ E_t[(R_{t+1}^i - R_{t+1}^f) u(c_{t+1})] = 0, \quad \text{for all } i. \]  

(A4.3)
\[ u_i(c_i) = \beta E[R_{i,t+1}^i u_i(c_{i,t+1})], \text{ for all } i. \quad (A4.4) \]

Focus first on equation (4) for an asset representing the market portfolio. This asset should be worth the sum of the prices of all assets and generate a dividend that equals the dividend for the whole market.

In equilibrium, the demands for all shares should equal their supplies. But, given the interpretation that share \( \gamma \) represents ownership to a \( \gamma \) share of the dividends, the supply of shares for any asset must equal one. If there is no riskless “fruit” technology, then the riskless asset must be in zero net supply under the presumption that, in principle, borrowing and lending may occur at the same rate. Thus, in equilibrium:

\[ s_t^i = 1, \quad b_t = 0, \quad \text{for all } i \text{ and } t. \quad (A4.5) \]

Substituting the equilibrium conditions into equation (2) produces:

\[ c_t = \sum_{i=1}^n d_t^i = y_t. \quad (A4.6) \]

The second equality follows since dividends are the only form of production and the only source of income in this endowment economy. Equation (6), of course, must hold given
equation (5) due to Walras’ Law—the goods market must clear automatically once all other markets clear.

It is straightforward, based on equations (3), (4) and (6), and assuming normality, to derive an asset pricing equation for each asset analogously to the derivation of the CCAPM:

\[ \mu_{t+1}^i - r_t^f = \beta_{iy}(\mu_{t+1}^y - r_t^f), \]  
\[ \text{with } \beta_{iy} = \frac{\text{Cov}_t(r_{t+1}^y, r_{t+1}^i)}{\text{Var}_t(r_{t+1}^y)}. \]  

(A4.7)

Here \( r_{t+1}^y \) may represent either the return on an asset perfectly correlated with aggregate production or the growth rate of aggregate production itself.
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