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Determining low permeability formation properties from absolute open flow potential

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Determining Low Permeability Formation Properties from Absolute

Open Flow Potential

Fatemeh Belyadi

**Thesis submitted to the
College of Engineering and Mineral Resources
at West Virginia University
in Partial Fulfillment of the Requirements
for the Degree of**

**Master of Science
In
Petroleum and Natural Gas Engineering**

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Department of Petroleum and Natural Gas Engineering

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2006**

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ABSTRACT

DETERMINING LOW PERMEABILITY FORMATION PROPERTIES FROM ABSOLUTE OPEN FLOW POTENTIAL

Fatemeh Belyadi

Absolute Open Flow Potential (AOFP) is often the only productivity information available from low permeability gas formations particularly in the Appalachian Basin. Absolute Open Flow Potential (AOFP) is commonly evaluated through a single point test due to its cost effectiveness. In low permeability reservoirs, the time required for obtaining stabilized flow rate is often very long. Therefore, stabilized AOFP may not be achieved in short tests. Furthermore, most low permeability gas wells are hydraulically fractured to improve their productivity. Therefore, the post-fracture measured AOFP could be significantly influenced by fracture properties.

The objective of this was to develop a methodology to estimate formation properties from AOFP. Conventional methods for pressure transient analysis or deliverability testing analysis cannot be used since they require multiple pressure and rate data. To achieve the objective, the impact of formation properties on AOFP was studied. Several different solutions were considered to determine the relationship among AOFP, permeability, and skin factor. They included pseudo-steady state radial flow, transient radial flow, and bi-linear flow. The non-Darcy flow effects were also included in all these cases. The results of this study are summarized by a series of correlations for estimation of formation properties particularly the permeability from the AOFP.

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DEDICATION

I thank God for giving me the opportunity to finish this segment of my education. This work is dedicated to my extraordinary parents. I thank them for being supportive, patient, and my best friend. I also dedicate my work to my brother who passed away 10 years ago. Ali, I love you, and I am saddened that you are not here today.

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Nomenclature:

q Flow rate (MSCF/D)

S = Skin factor

S^* = Apparent skin factor

k = permeability (md)

t = Time (Hrs)

ϕ = Porosity (%)

μ_i = Initial gas Viscosity (cp)

c_{gi} = Initial Compressibility (psia⁻¹)

$m(\bar{p}_R)$ = Reservoir pseudo-pressure (psia²/ cp)

$m(P_i)$ = Initial Reservoir pseudo-pressure (psia²/ cp)

r_w = Wellbore radius (ft)

h = Net pay thickness (ft)

C_D = dimensionless wellbore storage coefficient

C_R = Fracture Conductivity (md-ft)

C_{fD} = dimensionless fracture conductivity

c_t = total compressibility (psi⁻¹)

v = velocity (ft/hr)

h = formation thickness (ft)

k = permeability (md)

M = molecular weight of the gas (lbm/lbmole)

p = pressure (psi)

q = production rate (Mscf/d)

q_{sf} = sandface production rate (Mscf/d)

q_{DND} = dimensionless flow rate constant

r_e = reservoir radius(ft)

S_{ND} = non-Darcy flow skin

t = time (hr)

T = reservoir temperature (R)

w_f = fracture width, in (ft)

L_f = fracture half-length (ft)

Z = gas compressibility factor

a = constant

b = constant

Greek symbols

λ = non-Darcy flow coefficient (ft⁻¹)

ϕ = porosity

γ_g = specific gravity of gas

μ = viscosity (cp)

Subscripts and superscripts

D = dimensionless

f = fracture

i = initial

w = wellbore

w_f = flowing wellbore

CHAPTER I

INTRODUCTION

The absolute open flow potential (AOFP) represent the maximum flow rate that a well could sustain against atmospheric backpressure. Different definitions of stabilized AOFP of a gas well can be found in the literature. The definition of AOFP is often confused; it does add confusion to a discussion about stabilized AOFP determination. Since a natural gas well will not exhibit a flowing sandface pressure of less than atmospheric pressure for normal production operations, we shall use the definition of stabilized AOFP as the theoretical stabilized rate at which the well would produce at a stabilized flowing sandface backpressure of atmospheric pressure. While this definition of stabilized AOFP has the limitation of variable atmospheric pressure values, the limitation is negligible since in most areas, the standard atmospheric pressure is regarded to be about 14.7 psia (Lee et al 1982).

The stabilized deliverability equation can be used to predict the AOFP of a gas well. To develop the stabilized deliverability equation; four-point tests such as flow-after-flow, isochronal, and modified isochronal tests are usually conducted (Rawlins et al 1936). Four-point tests generally are not used in the Appalachian Basin due to long stabilization time in low permeability formations. Most of the operators however utilize single point test to determine AOFP of a gas well.

It is desirable to develop a methodology for estimating formation properties from AOFP since it is often the only productivity information available from low permeability gas formations particularly in the Appalachian Basin. In low permeability reservoirs, the time required for obtaining stabilized flow rate is often very long. Therefore, stabilized AOFP may not be achieved in short tests. Furthermore, most low permeability gas wells are hydraulically fractured to improve their productivity. Therefore, the post-fracture measured AOFP could be significantly influenced by fracture properties.

CHAPTER II

LITERATURE REVIEW

2.1 Introduction

There are a number of available solutions to general partial differential equation that describe the gas flow through porous media. Two aspect of gas must be included in the solutions for gas flow through porous media. They are the dependency of gas properties, i.e. viscosity and density, on the pressure and additional pressure drop caused by high gas velocity (non-Darcy flow effects).

2.2 Pseudo-pressure

The pressure dependency of gas properties is accounted by introducing pseudo-pressure.

Al-Hussainy and Ramey (1966) presented pseudo-pressure as follows:

$$m(p) = \frac{2p}{\mu z} dp \quad (1)$$

2.3 Non-Darcy Flow

Darcy's (1856) law describes the flow of fluid in a porous medium at low velocities.

Darcy's law is mathematically described by the following equation:

$$\frac{p}{L} = \frac{\mu}{K} \quad (2)$$

Forchheimer (1901) added the second term to Darcy's law equation by discovering that pressure gradient required to maintain a certain flow rate through porous media was higher than that predicted by Darcy's law :

$$\frac{p}{L} = \frac{\mu}{K} + \quad (3)$$

2.4 Unsteady-State Radial Gas Flow Equation

Unsteady state constant rate solution, which is usually used for drawdown analysis, is defined as follows (Houpeurt, 1959):

$$m(p_i) - m(p_{wf}) = \frac{1637qT}{kh} \left[\log t + \log \frac{k}{\mu_i c_{ti} r_w^2} - 3.23 + 0.869S' \right] \quad (4)$$

Muskut (1969), Katz (1959), and Wattenbarger and Ramy (1968) all describe non-Darcy flow close to the wellbore as a rate dependent skin:

$$S' = S + Dq \quad (5)$$

As Equation 5 indicates, the rate dependent or total skin factor (S') consist of a constant component (S), and a rate dependent part (Dq). Skin factor (S) represents formation alteration (damage or stimulation), whereas rate dependent part represents non-Darcy flow pressure drop. The flow rate coefficient (D) is defined by following equation (Katz et al 1959):

$$D = \frac{2.22 \cdot 10^{-15} \frac{g}{k}}{\mu h r_w} \quad (6)$$

2.5 Pseudo-Steady State Gas Flow Equation

Pseudo-steady state constant rate solution, which is usually used for the deliverability test analysis, is defined as follows (Houpeurt, 1959):

$$m(\overline{P_R}) - m(p_{wf}) = \frac{1422Tq_{sc}}{kh} \left[\ln\left(\frac{0.472r_e}{r_w}\right) + S + Dq_{sc} \right] \quad (7)$$

2.6 Non-Darcy Flow Coefficient

The critical factor for evaluating the non-Darcy effect is to obtain an estimate of non-Darcy coefficient, β . Janicek and Katz (1955) were the first to develop a correlation for non-Darcy coefficient using sandstone, limestone, and dolomite cores in their laboratory experiment. They proposed non-Darcy coefficient for natural porous media which was defined as following:

$$\beta = 1.82 \times 10^8 k^{-5/4} \mu^{-3/4} \quad (8)$$

Where:

k = Permeability (md)

μ = non-Darcy coefficient (1/cm)

They found that there was no difference among the non-Darcy coefficient for various rock types. Tek et al. (1962) developed a correlation for non-Darcy coefficient, by analyzing Janicek and Katz data. Their correlation is as follows:

$$\beta = \frac{2.33 \times 10^{10}}{k^{1.2}} \quad (9)$$

Where:

k = Permeability (md)

β = non-Darcy coefficient (1/ft)

Cooke (1973) studied non-Darcy coefficient for several fluids in propped fracture at various temperatures and introduced the following equation:

$$\beta = b k^{-a} \quad (10)$$

Where k is fracture permeability (md), λ is measured in ft^{-1} , a and b are depend on proppant type. Table 2.1, summarizes the constant values for Cooke's equation.

Table 2.1: Constant a and b in Cooke's equation

sand size (mesh)	a'	b'
8 to 12	1.24	17423.61
10 to 20	1.34	27539.48
20 to 40	1.54	1104070.39
40 to 60	1.6	69405.31

Geertsma (1974) analyzed the data obtained from unconsolidated sandstones, consolidated sandstones, limestones, and dolomites from his experiments, and defined the following equation:

$$\lambda = \frac{0.005}{k^{0.55}} \quad (11)$$

Where:

K = Permeability (cm^2)

λ = Non-Darcy coefficient (1/cm)

Pascal et al. (1980) proposed a mathematical model to estimate the fracture length and the non-Darcy coefficient, using variable rate tests for low permeability hydraulically fractured wells. Based on their analysis, the following correlation for non-Darcy coefficient was defined:

$$\lambda = \frac{4.8 \times 10^{12}}{k^{1.176}} \quad (12)$$

Where:

k= Permeability (md)

= Non-Darcy coefficient (1/cm)

Jones (1987) carried out his experiment on 355 sandstone and 29 limestone cores, with different core types such as fine grain sandstone, vuggy limestone, and crystalline limestone. Based on his analysis the following correlation for non-Darcy coefficient was developed:

$$= \frac{6.15 \times 10^{10}}{k^{1.55}} \quad (13)$$

Where:

k= Permeability (md)

= Non-Darcy coefficient (1/ft)

Li et al. (1995) studied the effect of non-Darcy using a reservoir simulator. They also conducted a series of experiments by injecting the Nitrogen at different flow rates, in several different directions in to a wafer-shaped Berea sandstone core. Then, the pressure drops from simulations and experiments were compared and the following correlation for non-Darcy coefficient was obtained:

$$= \frac{11500}{k} \quad (14)$$

Where:

k= Permeability (d)

=Non-Darcy coefficient (1/cm)

Coles and Hartman (1998) proposed a correlation for non-Darcy coefficient by conducting their experiment on limestone and sandstone samples,

$$= \frac{1.07 \times 10^{12} \times 0.449}{k^{1.88}} \quad (15)$$

Where:

k= Permeability (md)

= non-Darcy coefficient (1/ft)

According to Thauvin and Monhaty's (1998) analysis, the comparison of Erugun's flow equation with Forchheimer equation leads to following equation:

$$= ab^{-1/2} (10^{-8} k)^{-1/2} \quad -3/2 \quad (16)$$

Where:

a= 1.75

b= 150

k= Permeability (d)

=Non-Darcy coefficient (1/cm)

2.7 Flow Regimes in Hydraulically Fractured Formations

Presence of a hydraulic fracture will alter the pressure response of the well since the flow geometry in the fracture is linear. In well test analysis three types of fracture behaviors are usually observed (Gringarten et al 1974)

(a) Finite conductivity fractures (low conductivity)

(b) Infinite conductivity fractures (high conductivity)

(c) Uniform flux fractures.

Cinco, Samaniego and Dominguez (1978) and Cinco and Samaniego (1981) considered the finite conductivity fractures. In the finite conductivity fractures, different flow regimes are observed at different time. At early times, two linear flows occur one in the fracture and one in the formation as illustrated in Figure 2.1. These two simultaneous linear flows give rise to the bilinear flow period. This flow regime is recognized as a

straight line with the slope of $1/4$ in both pressure and the pressure derivative against time on a log-log paper (diagnostic plot). Following the bilinear flow period, linear flow period will take place. This flow regime is recognized as a straight line with the slope of $1/2$ on the diagnostic plot. Upon conclusion of the linear flow regime, a transitional flow regime will initiate and eventually, radial flow will take place (See Figure 2.1 c).

Figure 2.1: Different flow regimes for finite conductivity fracture (Cinco et al, 1978)

2.8 Effect of non-Darcy on Fractured Well

The effect of the non-Darcy flow on hydraulically fractured well has been discussed in several papers. Millheim and Cichowicz (1968) were first to observe the effect of non-Darcy flow on vertically fractured well. Holditch and Morse (1976) discussed the effect of the non-Darcy flow in both reservoir and fracture system by using numerical models. They concluded that non-Darcy flow in the fracture system reduced the apparent fracture conductivity. Guppy et al. (1982) developed a semi-analytical model for non-Darcy flow in wells with finite conductivity fractures. This model was able to account for the fracture and reservoir interaction under non-Darcy flow conditions in the fracture. They presented

the changes in flux distribution in the fracture under non-Darcy flow and concluded reduction in apparent conductivity of the fracture.

Cinco and Sameniego (1977) have shown that during the bilinear flow the dimensionless pseudo-pressure responses of finite-conductivity fracture under Darcy flow condition may be expressed by following form:

$$m_{wD}(\text{Darcy}) = \frac{t_D^{1/4}}{(5/4)\sqrt{2C_{fD}}} \quad (17)$$

Guppy et al. (1982) have shown that during the bilinear flow the dimensionless pseudo-pressure responses of finite-conductivity fracture under non-Darcy flow condition may be expressed by following form:

$$m_{wD}(\text{non - Darcy}) = \frac{t_D^{1/4}}{(5/4)\sqrt{2C_{fD,app}}} \quad (18)$$

According to Camacho (1984), during the linear flow the dimensionless pseudo-pressure responses of finite-conductivity fractures under Darcy and non-Darcy flow can be defined respectively by following equations:

$$m_{wD}(\text{Darcy}) = \sqrt{t_D} + \frac{a}{C_{fD}} \quad (19)$$

$$m_{wD}(\text{non - Darcy}) = \sqrt{t_D} + \frac{a}{C_{fD,app}} \quad (20)$$

Where:

$$a = \beta \text{ for } C_{fD} \quad 25$$

$a=0.944$ for $C_{fd} = 10$

$a=0.902$ for $C_{fd} = 5$

Dimensionless pseudo-pressure and dimensionless time are defined as following (Guppy et al 1982):

$$m_D(t_D) = \frac{kh}{1,422qT} [m(p_i) - m(p_{wf})] \quad (21)$$

$$t_D = \frac{2.637 * 10^{-4} k}{(\mu c_t)_i x_f^2} t \quad (22)$$

Transient pressure responses of finite conductivity fractures with Non-Darcy flow are governed by dimensionless fracture conductivity and dimensionless flow rate (Guppy et al 1982):

$$C_{fd} = \frac{k_f w_f}{k x_f} \quad (23)$$

$$q_{DND} = \frac{4.64 * 10^{-16} k_f M q}{w_f h_i} \quad (24)$$

To take the non-Darcy flow in to the consideration the apparent fracture conductivity ($C_{fd,app}$) is defined by following equation (Guppy et al 1982):

$$C_{fd,app} = \frac{C_{fd}}{(1 + 0.31 q_{DND})} \quad (25)$$

2.9 Dimensionless Fracture Conductivity and Skin Factor

Cinco-Ley, Heber, and Samaniego (1981), defined skin factor as a function of dimensionless fracture conductivity and fracture half length. For bilinear flow regime (in

finite conductivity fracture) ($c_{fd} < 100$), $\frac{r_w e^s}{x_f} > 0.5$, skin factor can be calculated from

estimation of $\frac{r_w e^s}{x_f}$ value from the figure 2.2 for each value of dimensionless fracture conductivity.

Figure 2.2: Skin factor as a function of fracture length and conductivity (Cinco-Ley, Heber, and Samaniego, 1981)

CHAPTER III

OBJECTIVE AND METHODOLOGY

The goal of this study was to develop a correlation between permeability, skin factor and AOFP in low permeability formation. To achieve the objective, a methodology consisting of the following 5 steps was employed:

1. The impact of skin factor and permeability on absolute open flow potential was evaluated using unsteady state transient radial flow solution.
2. The impact of skin factor and permeability on absolute open flow potential was evaluated using pseudo-steady state radial flow solution.
3. The impact of hydraulic fracture on absolute open flow potential was evaluated using bilinear flow solution (Darcy and non-Darcy).
4. The impact of hydraulic fracture on absolute open flow potential was evaluated using linear flow solution (Darcy and non-Darcy).
5. The impact of test duration (time) on absolute open flow potential was evaluated using linear, bilinear and radial flow solutions.

Table 3.1 and 3.2 shows the parameters and ranges that were used through out this study.

Table 3.1: Parameters used, throughout this study

Formation porosity (%)	0.2
Gas Density (lbm/ft ³)	13.84
k _f (md)	50000
w _f (ft)	1.00E-02
x _f (ft)	5.00E+02
T (°R)	5.40E+02
r _w (ft)	3.30E-01
(ft ⁻¹)	1.79E+05

Table 3.2: Ranges used, throughout this study

Parameter	Range
Pressure (psia)	1000, 1500 and 2000
Permeability	0.1 to 1
Skin Factor	-3 to 3
Dimensionless fracture conductivity	10 to 100

3.1 Impact of Skin factor and Permeability on AOFP (Transient Radial Flow)

The first step of this study was to evaluate the impact of permeability and skin factor on absolute open flow potential using unsteady state transient radial flow equation. Gas viscosity and z factor were calculated at the initial reservoir pressure of 1000, 1500, and 2000 psia respectively. In addition, pseudo-pressure was calculated by numerical integration using equation 1 (Refer to Figure A.1 and A.2 in the Appendix for interface of simulation program and relation between pressure and pseudo-pressure respectively). AOFP value was calculated using unsteady state transient radial flow solution. Furthermore, unsteady state transient radial flow equation was divided into three parts as follows:

$$a = \frac{1637T}{kh}[0.869D] = \frac{1422TD}{kh}$$
$$b = \frac{1637T}{kh}[\log t + \log \frac{k}{\mu_i c_u r_w^2} - 3.23 + 0.869S]$$
$$c = m(p_i) - m(p_{wf})$$

To calculate AOFP, quadratic formula in the excel program was used.

$$AOFP = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In order to develop a correlation between permeability, skin factor and AOFP the following steps were considered:

AOFP values were calculated for permeability data range from 0.1 to 1 md, keeping skin factor, initial reservoir pressure and time constant, then skin factor values were varied from -3 to 3 (Refer to Table B.1 in the Appendix B for tabulated result).

The permeability was plotted against AOFP for various skin factors at specified initial reservoir pressure and test duration as illustrated in Figure 3.1.

Figure 3.1: Permeability vs. AOFP for $P_i=1000$ psia, $t=1$ hr, and $S= -3$ to 3

Permeability was plotted against AOFP, for different test duration as specified initial reservoir pressure and skin factor. Figure 4.2 (in the result section) illustrates the impact of time on AOFP for a particular range of permeability.

Permeability was plotted against AOFP, for various initial reservoir pressures, as specified skin factor and test duration as illustrated in Figure 3.2.

Figure 3.2: Permeability vs. AOFP for $P_i=1000, 1500,$ and 2000 psia

After showing the effect of skin factor, time and initial reservoir pressure on AOFP, using $\text{Log}k + 0.87S$, a combined range (Refer to Table B-2 in the Appendix for range) was calculated. Then AOFP values were calculated for various initial reservoir pressures and non-Darcy flow coefficient respectively. Through out this study, Tek et al. non-Darcy coefficient (equation 9) was used in unsteady state transient radial flow equation to study the behavior of low permeability formation properties in Appalachian basin on AOFP. Tek et al. correlation is usually used for low permeability formation.

$\text{Log}k + 0.87S$ was plotted against AOFP, for various initial reservoir pressures, as specified test duration as illustrated in Figure 4.3.

$\log k + 0.87S$ was plotted against AOFP, for various non-Darcy coefficient, as specified test duration (Refer to Figure 4.4 in result section and Table B.2 in the Appendix B for tabulated result).

3.2 Impact of Skin Factor and Permeability on AOFP (Pseudo-Steady State Radial Flow)

The second step of this study was to evaluate the impact of permeability and skin factor on AOFP using pseudo-steady state radial flow solution. To attain this goal, AOFP value was calculated using pseudo-steady state radial flow solution (See Equation 7). In order to find AOFP, the quadratic formula was used. To find all parameters of the quadratic formula, equation 7 was divided into three parts as following:

$$a = D \times C_1$$

$$b = C_1 \times \ln\left(\frac{0.472r_e}{r_w}\right) + S$$

$$c = -m(\overline{p_r}) - m(\overline{p_{wf}})$$

Where

$$C_1 = \frac{1422T}{kh}$$

In order to develop a correlation between permeability, skin factor and AOFP the following steps were utilized:

AOFP values were calculated for permeability data range from 0.1 to 1, keeping skin factor and reservoir pressure constant, then skin factor values were varied from -3 to 3 (Refer to Table B.1 in the Appendix B for tabulated result).

Permeability was plotted against AOFP for various skin factors at specified reservoir pressure as illustrated in figure 4.5 (Refer to result).

The skin factor was plotted against AOFP for various permeability values at specified reservoir pressure as illustrated in Figure 3.3.

Figure 3.3: Skin Factor vs. AOFP for Various Permeability Values at $P_R=1500$ psia.

Permeability was plotted against AOFP for various reservoir pressures at specified skin factor and time duration as illustrated in figure 3.4.

Figure 3.4: AOFP vs. Permeability for zero skin factor, $P_R=1000, 1500,$ and 2000 psia

3.3 The Impact of Hydraulic Fracture on Absolute Open Flow Potential (Bilinear Flow, Darcy and Non-Darcy)

The third step of this study was to determine the AOFP based on analytical expression of bilinear flow regimes in finite conductivity fracture for Darcy and non-Darcy flow conditions. To attain this goal, following procedure was performed:

The dimensionless variable ($m_D(t_D)$, t_d , and C_{fd}) from equations 21-23, were substituted in equation 17. By rearranging the terms in equation 17, the equation for AOFP in finite conductivity fracture under Darcy flow conditions is shown below (Refer to Appendix D, for derivation of AOFP):

$$\text{AOFP} = \frac{kh[m(p_i) - m(p_{wf})] (5/4)(2C_{fd})^{\frac{1}{2}}}{(t_d^{1/4})(1422t)} \quad (26)$$

The dimensionless variable ($m_D(t_D)$, t_D , $C_{fd,app}$ and q_{DND}) from equations 21-25, were substituted in equation 18. Rearranging the terms in equation 18 gives the following equation (Refer to Appendix D, for derivation of AOFP):

$$-(1422T)^2 - 0.31 - 4.64 \cdot 10^{-16} \left(\frac{1}{t_D^4} \right)^2 k_f M q^3 - (1422T)^2 w_f h \mu_i \left(\frac{1}{t_D^4} \right)^2 q^2 + [(1.25)]^2 [kh \{ m(p_i) - m(p_{wf}) \}]^2 (2C_{fd}) w_f h \mu_i = 0 \quad (27)$$

In order to find AOFP, cubic solution was used (see Appendix C). To find all parameters for cubic solution, equation 27 was divided into four parts:

$$a = (1422T)^2 - 0.31 - 4.64 \cdot 10^{-16} \left(\frac{1}{t_d^4} \right)^2 k_f B M q^3$$

$$b = (1422T)^2 w_f h \mu_i \left(\frac{1}{t_d^4} \right)^2 q^2$$

$$c = 0$$

$$d = []^2 [kh \{ m(p_i) - m(p_{wf}) \}]^2 (2C_{fd}) w_f h \mu_i$$

In order to develop a correlation between permeability, dimensionless fracture conductivity and AOFP the following steps were utilized:

AOFP values were calculated using equation 27 for dimensionless fracture conductivity data range from 10 to 40, keeping initial reservoir pressure and time constant, then permeability values were varied from 0.1 to 0.5 md (Refer to Table D.1 in the Appendix D for tabulated result).

The dimensionless fracture conductivity was plotted against AOFP for various permeability values at specified initial reservoir pressure as illustrated in figure 3.5.

Figure 3.5: Dimensionless Fracture Conductivity vs. AOFP (for $P_i=1000$ psia, $t=1$ hr, and $k=0.1$ to 0.5 md)

The initial reservoir pressure was plotted against AOFP for various permeability values at specified test duration.

3.4 The Impact of Hydraulic Fracture on AOFP (Linear Flow in Finite Conductivity Fracture Darcy and Non-Darcy)

The 4th step of this study was to determine the AOFP based on analytical expression of linear flow regimes in finite conductivity fracture for Darcy and non-Darcy flow conditions. To attain this goal, following procedure was performed:

For finite conductivity fracture under Darcy flow conditions, equation 19 was used to find the AOFP. The dimensionless variable ($m_D(t_D)$, t_d , and C_{fd}) from

equations 21-23, were substituted in equation 19. By rearranging the terms in equation 19, the equation for AOFP in finite conductivity fracture under Darcy flow conditions is shown below (Refer to Appendix D, for derivation of AOFP):

$$\text{AOFP} = \frac{C_{fd} kh [m(p_i) - m(p_{wf})]}{1422TC_{fd} (t_D)^{\frac{1}{2}} + 1422T a} \quad (28)$$

The dimensionless variable ($m_D(t_D)$), t_d , C_{fd} and q_{DND} from equations 21-25, were substituted in equation 20. Rearranging the terms in equation 20 gives the following equation (Refer to Appendix D, for derivation of AOFP).

$$[1422T \cdot 0.31 \cdot 4.64 \cdot 10^{-16} k_f \cdot Ma] q^2 - [1422T a w_f h_i - 1422T (t_D)^{\frac{1}{2}} C_{fd} w_f h_i] q - kh[m(p_i) - m(p_{wf})] C_{fd} w_f h_i = 0 \quad (29)$$

AOFP value was calculated using quadratic formula. To find all parameter for the quadratic formula, equation 29 was divided in to three parts:

$$a = -[1422 \times T \times 0.31 \times 4.64 \times 10^{-16} k_f \times M \times a]$$

$$b = [-1422 \times T \times a \times w_f \times h \times \mu_i - 1422 \times T \times (t_D)^{\frac{1}{2}} \times C_{fd} \times w_f \times h \times \mu_i]$$

$$c = k \times h [m(p_i) - m(p_{wf})] \times C_{fd} \times w_f \times h \times \mu_i$$

In order to develop a correlation between permeability, dimensionless fracture conductivity and AOFP the following steps were utilized:

AOFP values were calculated using equation 29 for dimensionless fracture conductivity data range from 10 to 100, keeping initial reservoir pressure and time constant, then

permeability values were varied from 0.1 to 0.5 md (Refer to Table D.2 in the Appendix D for tabulated result).

Dimensionless fracture conductivity was plotted against AOFP for various permeability values at specified initial reservoir pressure.

Initial reservoir pressure was plotted against AOFP for various permeability values at specified test duration.

3.5 Impact of Time on AOFP for Linear, Bilinear and Radial Flow

The 5th step of this study was to determine a correlation between AOFP and time for linear, bilinear and radial flow solutions with identical skin factor. To attain this goal, following procedure was performed:

C_{fd} values were converted to skin factor in linear and bilinear flow regime, for comparison purposes with radial flow regime, using Figure 2.2.

AOFP was plotted against test duration at specified initial reservoir pressure, permeability and skin factor, for bilinear (Darcy and non-Darcy) and radial flow (non-Darcy) comparison.

AOFP was plotted against test duration at specified initial reservoir pressure, permeability and skin factor, for linear (Darcy and non-Darcy) and radial flow (non-Darcy) comparison (For tabulated result refer to Appendix D).

CHAPTER IV

RESULTS AND DISCUSSION

The main objective of this study was to determine the formation properties of low permeability formation from AOF. To achieve this objective, several different solutions were considered to determine the relationship between AOF, permeability, and skin factor.

The first step of analysis is the impact of skin factor, permeability, and initial reservoir pressure, on AOF using unsteady state transient radial flow equation. Figure 4.1 illustrates the impact of skin factor on relationship between permeability and AOF.

Figure 4.1: Permeability vs. AOF for ($P_i=1500 \text{ psi}$, $t=1 \text{ hr}$, and $S = -3 \text{ to } 3$)

Figure 4.1, illustrates that as skin factor decreases within the ranges of -3 to 3, the curve shift to the right. For negative skin factor the permeability does not have significant

impact on AOF (non-Darcy). However for positive skin factor lower permeability values have some impact on AOF (non-Darcy).

From Figure 4.1, it is possible to predict permeability values for skin factor range of -3 to 3 and AOF range of 0 to 25000 MSCF/D (Refer to Figure B-1 in Appendix for pressure 2000 psia).

Figure 4.2 shows the correlation between permeability and AOF, for skin factor of 0, time range of 1 to 3 hours, and initial reservoir pressure of 1500 psia.

Figure 4.2: Permeability vs. AOF for $P_i=1500$ psia and $t = 1, 2, 3$ hrs

Figure 4.2, illustrates that, time does not have significant impact on AOF (non-Darcy) using unsteady state transient radial flow solution for low permeability range of 0.1 to 1 md.

Figure 4.3: $\text{Log } K+0.87S$ vs. AOF for $P_i=1000, 1500$, and 2000 psia

Figure 4.3, a graph of $\text{Log } K+0.87S$ vs. AOF shows the relationship between permeability, skin factor, and AOF. From Figure 4.3, Permeability can be predicted, assuming skin factor is known for initial reservoir pressure of 1000, 1500, and 2000 psia respectively.

Figure 4.4, shows the different non-Darcy coefficient () correlation, and how they impact the AOF. Janicek and Katz correlation is typically used for sandstone formation. Similar to Jones non-Darcy coefficient () correlation, Coles, and Hartman correlation is commonly used for different core types such as limestone and fine grain sandstone (high permeability) formation. Pascal et al. non-Darcy coefficient () correlation is used for low permeability fractured formation.

Figure 4.4: $\log K + 0.87S$ vs. AOF for (different correlation)

Figure 4.4, shows that for negative skin factor ($\log K + 0.87S < 0$), the AOF (non-Darcy) differs for various non-Darcy coefficient () correlation. However for positive skin factor ($\log K + 0.87S > 0$), the AOF (non-Darcy) acts similarly for various non-Darcy coefficients () correlation.

From figure 4.4, it is also possible to read permeability value for different non-Darcy coefficient () correlation based on the formation of interest, having the value of skin factor.

The next step was to evaluate the impact of skin factor and permeability on AOF using pseudo-steady state equation. Figure 4.5 illustrates that there is linear relationship

between permeability and AOFP for various skin factors. Furthermore unlike unsteady state solution permeability does not have significant impact on AOFP (non-Darcy).

Skin effect can be negative due to stimulation, which means more production. Due to damage the skin effect could be positive which means less production. As the fluid gets close to the wellbore, there is a decrease in the permeability and positive skin factor.

Permeability can be estimated from using pseudo-steady state equation assuming the known value of the skin factor for AOFP (Refer to Table C-1 in the Appendix C for tabulated result).

Figure 4.5: Permeability vs. AOFP for $P_R=1500$ Psia.

Since most low permeability gas wells are hydraulically fractured to improve their productivity. The impact of hydraulic fracture on absolute open flow potential was evaluated using bilinear flow solution in finite conductivity fracture (non-Darcy).

Figure 4.6 illustrates the impact of permeability on relationship between dimensionless fracture conductivity and AOFP.

Figure 4.6: Dimensionless Fracture Conductivity vs. AOFP
($P_i=1500$ psia, $t=1$ hr, and $k=0.1$ to 0.5 md)

From Figure 4.6, it is possible to predict dimensionless fracture conductivity values for permeability range of 0.1 to 0.5 md and AOFP range of 0 to 25000 MSCF/D.

Figure 4.7 illustrates the impact of permeability on relationship between initial reservoir pressure and AOFP using bilinear flow solution in finite conductivity fracture.

Figure 4.7: Initial Reservoir Pressure vs. AOFP for $C_{fd}=10$ and $t=1 \text{ hr}$

From Figure 4.7, it is possible to predict initial reservoir pressure values for permeability range of 0.1 to 0.5 md , AOFP range of 4000 to 21000 MSCF/D.

The next step was to evaluate the impact of hydraulic fracture on absolute open flow potential using linear flow solution in finite conductivity fracture (non-Darcy).

Figure 4.8 illustrates the impact of permeability on relationship between dimensionless fracture conductivity and AOFP.

Figure 4.8: Dimensionless Fracture Conductivity vs. AOFP
($P_i=1500$ psia and $t=1$ hr)

In the final step of this study, C_{fd} values were converted to skin factor in linear and bilinear flow regime. Furthermore, skin factor for linear and bilinear flow were adjusted for comparison purpose with radial flow regime.

Figure 4.9 illustrates a correlation between AOFP and time for bilinear (Darcy and non-Darcy) and radial flow (non-Darcy) comparison using adjusted skin factor ($p_i=1500$, $t=1, 2, 4$ hrs, $S=-4.24$).

Figure 4.9: AOFP vs. Time for Bilinear Flow
($P_i=1500$ psia, $K=0.5$ md, and $S=-4.24$)

Figure 4.10 illustrates a correlation between AOFP and time for bilinear (Darcy and non-Darcy) and radial flow (non-Darcy) comparison using adjusted skin factor ($p_i=2000$, $t=1, 2, 4$ hrs, $S=-4.24$).

Figure 4.10: AOFP vs. Time for Bilinear Flow
($P_i=2000$ psia, $K=0.5$ md, and $S=-4.24$).

Figure 4.10 shows that for pressure of 2000 psia, radial flow curve (non-Darcy) is close to bilinear flow (non-Darcy) which indicates that using unsteady state radial flow equation for adjusted skin factor, leads to reasonable estimate of AOF (non-Darcy). Also, linearity of these two curves (bilinear and radial flow) illustrates that time does not have significant impact on non-Darcy AOF.

Figures 4.9 and 4.10 show that as pressure increase, the difference between radial and bilinear flow for (non-Darcy) condition decrease.

Figure 4.11 shows a correlation between AOF and time for linear (Darcy and non-Darcy) and radial flow (non-Darcy) comparison using adjusted skin factor ($P = 1500$, $K=0.5$ md, $t = 1, 2, 4$ hrs, and $S = -5.94$).

Figure 4.11: AOF vs. Time for Linear Flow
($P = 1500$, $K = 0.5$ md and $S = - 5.94$).

Figure 4.11 shows that time does not have significant impact on AOFP for both linear and radial (non-Darcy) flow.

Figure 4.12 shows a correlation between AOFP and time for linear (Darcy and non-Darcy) and radial flow (non-Darcy) comparison using adjusted skin factor ($P = 1500$, $K=0.1$ md, $t = 1, 2, 4$ hrs, and $S = -5.94$).

Figure 4.12: AOFP vs. Time for Linear Flow

($P = 1500$, $K = 0.1$ md and $S = -5.94$).

Figures 4.11 and 4.12 illustrate that as permeability increase, the difference between radial and bilinear flow for (non-Darcy) condition decrease.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

After studying and analyzing the impact of skin factor and permeability on AOF, the following conclusions have been reached in this study:

1. Correlation between permeability and AOF have been developed using unsteady state and pseudo-steady state solutions.
2. The plot of AOF vs. time illustrates that time does not have significant impact on bilinear, linear and radial flow solutions.
3. Transient radial flow equation can give a reasonable estimate of AOF in fractured formation as compared to radial flow solution. As pressure increase, the difference between radial and bilinear flow for (non-Darcy) condition decrease. Furthermore, as permeability increase, the difference between radial and linear flow for (non-Darcy) condition decrease.

Non-Darcy coefficient is an important parameter that has significant impact in result of this study. It is recommended to study this parameter in detail in developing correlation between AOF, permeability, and skin factor.

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APPENDIXES

APPENDIX A

PSEUDO-PRESSURE CALCULATION

Steps for pseudo-pressure calculation:

Pseudo-pressure is simply described by Eq.1. Pseudo-pressure can be computed using numerical integration. In order to determine the relationship among pressure and pseudo-pressure, the following steps was performed (See Eq.30-32) (Alvarez et al 2000).

$$\frac{2P}{\mu z} \quad (30)$$

$$\frac{\overline{2P}}{z} \quad (31)$$

$$\frac{\overline{2p}}{z} \quad (32)$$

Dr. Aminian Program is written using the following equations:

Initially, Z factor is calculated by performing the following series of calculations (See Equations 33-36) (Alvarez et al 2000).

$$P_{pc} = 709.6 - 58.7 * g \quad (33)$$

$$T_{pc} = 170.5 + 307.3 * g \quad (34)$$

$$Tr = T / T_{pc} \quad (35)$$

$$Pr = P / P_{pc} \quad (36)$$

The reduced density was calculated using Newton's method. This method also used to estimate the z factor with the procedure described by Dranchuk, Purvis and Robinson.

The p_{k+1} estimation is determined from the P_k estimate using the Equation 37

$$P_{k+1} = P_k - \frac{f(P_k)}{f'(P_k)} \quad (37)$$

Where:

$$f(p) = ap^6 + bp^3 + cp^2 + dp + ep^3(1 - fp^2)\exp[-fp^2] - g$$

$$f'(p) = 6ap^5 + 3bp^2 + 2cp + d + ep^2(3 - fp^2)\exp(-fp^2)$$

Where:

$$a = 0.06423$$

$$b = 0.5353 T_r - 0.6123$$

$$c = 0.3151 T_r - 1.0467 - \frac{0.5783}{T_r^2}$$

$$d = T_r$$

$$e = \frac{0.6816}{T_r^2}$$

$$f = 0.6845$$

$$g = 0.27 p_r$$

$$p_o = 0.27 \frac{p_r}{T_r}$$

$$z = \frac{0.27 p_r}{p T_r}$$

The next step is the calculation of gas viscosity, μ_g . This is done in two steps, first the Carr, Kobayashi and Burrows gas viscosity μ_{g1} was determined, and next the Dempsey method (1965) was used for final calculation:

$$\mu_{g1} = (1.709 \times 10^{-5} + 2.062 \times 10^{-6} p_r) T_F^{0.8} (8.188 \times 10^{-3} + 6.15 \times 10^{-3} \log p_r) \quad (38)$$

$$y_{N_2} (8.48 \times 10^{-3} \log p_r + 9.59 \times 10^{-3})$$

$$y_{CO_2} (9.08 \times 10^{-3} \log p_r + 6.24 \times 10^{-3})$$

$$y_{H_2S} (8.49 \times 10^{-3} \log p_r + 3.73 \times 10^{-3})$$

$$\ln \mu_r = \frac{a_0}{T_r} + a_1 p_r + a_2 p_r^2 + a_3 p_r^3 + T_r (a_4 + a_5 p_r + a_6 p_r^2 + a_7 p_r^7) \quad (39)$$

$$T_r^2 (a_8 + a_9 p_r + a_{10} p_r^2 + a_{11} p_r^3) + T_r^3 (a_{12} + a_{13} p_r + a_{14} p_r^2 + a_{15} p_r^3)$$

Constant's values are listed below:

$$a_0 = -2.46211820$$

$$a_1 = 2.97054714$$

$$a_2 = -2.86264054 \times 10^{-1}$$

$$a_3 = 8.05420522 \times 10^{-3}$$

$$a_4 = 2.80860949$$

$$a_5 = -3.49803305$$

$$a_6 = 3.60373020 \times 10^{-1}$$

$$a_7 = -1.04432413 \times 10^{-2}$$

$$a_8 = -7.93385684 \times 10^{-1}$$

$$a_9 = 1.39643306$$

$$a_{10} = -1.49144925 \times 10^{-1}$$

$$a_{11} = 4.41015512 \times 10^{-3}$$

$$a_{12} = 8.39387176 \times 10^{-2}$$

$$a_{13} = -1.86408848 \times 10^{-1}$$

$$a_{14} = 2.03367881 \times 10^{-2}$$

$$a_{15} = -6.09579263 \times 10^{-4}$$

Table A-1: Z factor, viscosity, and compressibility results

P, psia	z	μ, cp
100	0.9888498	1.15E-02
200	0.9778506	1.16E-02
300	0.9670271	1.17E-02
400	0.9564064	1.19E-02
500	0.9460179	1.20E-02
600	0.9358935	1.22E-02
700	0.9260672	1.23E-02
800	0.9165752	1.25E-02
900	0.9074554	1.27E-02
1000	0.8987471	1.29E-02
1100	0.8904899	1.32E-02
1200	0.8827233	1.34E-02
1300	0.8754857	0.0136592
1400	0.8688135	1.39E-02
1500	0.8627393	1.42E-02
1600	0.8572918	1.44E-02
1700	0.8524943	0.0147049
1800	0.848364	0.0149649
1900	0.8449116	1.52E-02
2000	0.8421413	1.55E-02
2100	0.8400503	1.58E-02
2200	0.83863	1.61E-02
2300	0.8378659	0.0164531
2400	0.8377392	1.68E-02
2500	0.8382268	1.71E-02
2600	0.839303	1.74E-02
2700	0.8409398	1.78E-02

2800	0.8431082	1.81E-02
2900	0.8457785	1.85E-02
3000	0.8489212	1.88E-02

Table A-2: Pseudo-pressure calculation using the z factor, viscosity, compressibility values from stimulator

2P/ μ z	avg(2P/ μ z)	dP	avg(2P/ μ z) X dP	m(p), psia/cp ²
12292.83149	6146.415745	100	614641.5745	614641.5745
24376.28826	18334.55987	100	1833455.987	2448097.562
3.63E+04	30320.60355	100	3032060.355	5480157.917
47970.58322	42117.75104	100	4211775.104	9691933.021
59500.95612	53735.76967	100	5373576.967	15065509.99
105175.9764	82338.46626	100	8233846.626	23299356.61
123050.7353	114113.3558	100	11411335.58	34710692.2
139592.4511	131321.5932	100	13132159.32	47842851.52
156084.2471	147838.3491	100	14783834.91	62626686.42
172167.9061	164126.0766	100	16412607.66	79039294.08
187788.9995	179978.4528	100	17997845.28	97037139.36
203015.5071	195402.2533	100	19540225.33	116577364.7
217419.7676	210217.6373	100	21021763.73	137599128.4
231440.1819	224429.9747	100	22442997.47	160042125.9
245140.5117	238290.3468	100	23829034.68	183871160.6
258408.3056	251774.4086	100	25177440.86	209048601.4
271222.277	264815.2913	100	26481529.13	235530130.6
283560.9682	277391.6226	100	27739162.26	263269292.8
295405.2167	289483.0924	100	28948309.24	292217602.1
306737.5189	301071.3678	100	30107136.78	322324738.8
316493.8367	311615.6778	100	31161567.78	353486306.6
325371.1615	320932.4991	100	32093249.91	385579556.5
333684.1505	329527.656	100	32952765.6	418532322.1
341438.5016	337561.3261	100	33756132.61	452288454.8
348644.2979	345041.3998	100	34504139.98	486792594.7
355313.7821	351979.04	100	35197904	521990498.7
361453.2641	358383.5231	100	35838352.31	557828851
366724.1437	364088.7039	100	36408870.39	594237721.4
371515.2057	369119.6747	100	36911967.47	631149688.9
375848.3187	373681.7622	100	37368176.22	668517865.1

Figure A.1: Illustrates the nonlinearity of pseudo-pressure as a function of pressure

Figure A.2: z factor, viscosity, and compressibility calculation program (Aminian, 2000)

APPENDIX B

AOFD DETERMINATION USING UNSTEADY STATE TRANSIENT RADIAL FLOW SOLUTION

In order to find AOFD, unsteady state radial flow equation was used. Furthermore, Equation 4 was divided into three parts as follows:

$$\begin{aligned}
 a &= \frac{1637T}{kh} [0.869D] - \frac{1422TD}{kh} \\
 b &= \frac{1637T}{kh} \left[\log t - \log \frac{k}{c_{ti} r_w^2} - 3.23 - 0.869 S \right] \\
 c &= m(p_i) - m(p_{wf})
 \end{aligned}$$

To calculate AOFD, quadratic formula in the excel program was used.

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Table B-1 shows, the steps in calculation of AOF for a skin factor of 3, using unsteady state transient radial flow equation.

Table B-1: The calculation steps for predicting AOF (P =1500 psi, t =1 hr, and S = 3)

K(md)	D,(MSCF/D) ⁻¹	a	b	c	AOF, MSCF/D
0.1	0.00010523	8.080418	382148.2	-183781637	476.1238
0.2	9.16082E-05	3.517206	204379.3	-183781637	885.7177
0.3	8.44727E-05	2.162163	141441.6	-183781637	1274.515
0.4	7.97496E-05	1.530953	108842.3	-183781637	1650.209
0.5	7.62687E-05	1.171305	88787.16	-183781637	2016.281
0.6	7.35377E-05	0.941136	75155.87	-183781637	2374.722
0.7	7.13051E-05	0.782197	65264.74	-183781637	2726.825
0.8	6.94261E-05	0.666386	57747.45	-183781637	3073.499
0.9	6.78097E-05	0.578553	51833.48	-183781637	3415.414
1	6.63958E-05	0.50984	47054.62	-183781637	3753.089

Table B-2: The calculation steps for predicting AOF, using different correlation (Janecicek, Katz)

K, md		D, (MSCF/D) ⁻¹	S	a	b	c	AOF, MSCF/D	Log k+0.87S
0.1	3.30E+11	9.39E-05	-5	7.746223	-278190.6	-183781636.6	36561.97907	-5.4
0.2	1.39E+11	7.90E-05	-4	3.256885	-83550.50	-183781636.6	27691.27333	-4.2
0.3	8.36E+10	7.14E-05	-3	1.961952	-22624.59	-183781636.6	17031.61245	-3.1
0.4	5.83E+10	6.64E-05	-2	1.369351	6624.182	-183781636.6	9416.00505	-2.1
0.5	4.41E+10	6.28E-05	-1	1.036042	23641.20	-183781636.6	6128.068178	-1.2
0.6	3.51E+10	6.00E-05	0	0.824899	34705.33	-183781636.6	4757.508747	-0.2
0.7	2.90E+10	5.77E-05	1	0.68032	42442.34	-183781636.6	4065.242196	0.7
0.8	2.45E+10	5.58E-05	2	0.575741	48138.82	-183781636.6	3657.729329	1.6
0.9	2.12E+10	5.42E-05	3	0.496920	52497.26	-183781636.6	3391.88367	2.6
1	1.86E+10	5.28E-05	4	0.435602	55932.79	-183781636.6	3205.723733	3.5

Figure B-1 illustrates the impact of skin on relationship between permeability and AOFP for ($P=2000$ psia, $t=1$ hr, and $S = -3$ to 3).

Figure B-1: Permeability vs. AOFP
($P=2000$ psia, $t=1$ hr, and $S = -3$ to 3)

APPENDIX C

AOFD DETERMINATION USING PSEUDO-STEADY STATE RADIAL FLOW SOLUTION

In order to find AOFD, the quadratic formula was used. To find all parameters of the quadratic formula, equation 7 was divided into three parts as following:

$$a = D C_1$$

$$b = C_1 \ln\left(\frac{0.472r_e}{r_w}\right) S$$

$$c = m(\overline{p_r}) - m(\overline{p_{wf}})$$

Where

$$C_1 = \frac{1422T}{kh}$$

The following table shows the tabulated results of AOFD for constant permeability (k=0.5) and skin factor ranges of -3 to 5.

Table C-1: Steps for estimation AOFD using pseudo-steady state equation

D, (MSCF/D)⁻¹	S	C1	a	b	c	AOFD, MSCF/D
0.000126467						
0.000126467	-3.0	15357.6	1.942229389	59128.69761	-183960684.6	2845.271965
0.000126467	-2.75	15357.6	1.942229389	62968.09761	-183960684.6	2697.113279
0.000126467	-2.5	15357.6	1.942229389	66807.49761	-183960684.6	2562.669417
0.000126467	-2.25	15357.6	1.942229389	70646.89761	-183960684.6	2440.237097
0.000126467	-2.0	15357.6	1.942229389	74486.29761	-183960684.6	2328.365239
0.000126467	-1.75	15357.6	1.942229389	78325.69761	-183960684.6	2225.813589

0.000126467	-1.5	15357.6	1.942229389	82165.09761	-183960684.6	2131.51839
0.000126467	-1.25	15357.6	1.942229389	86004.49761	-183960684.6	2044.563959
0.000126467	-1	15357.6	1.942229389	89843.89761	-183960684.6	1964.159182
0.000126467	-0.75	15357.6	1.942229389	93683.29761	-183960684.6	1889.618057
0.000126467	-0.5	15357.6	1.942229389	97522.69761	-183960684.6	1820.343563
0.000126467	0	15357.6	1.942229389	105201.4976	-183960684.6	1695.573167
0.000126467	0.5	15357.6	1.942229389	112880.2976	-183960684.6	1586.395294
0.000126467	0.75	15357.6	1.942229389	116719.6976	-183960684.6	1536.790095
0.000126467	1.0	15357.6	1.942229389	120559.0976	-183960684.6	1490.124147
0.000126467	1.25	15357.6	1.942229389	124398.4976	-183960684.6	1446.149355
0.000126467	1.5	15357.6	1.942229389	128237.8976	-183960684.6	1404.644183
0.000126467	1.75	15357.6	1.942229389	132077.2976	-183960684.6	1365.41027
0.000126467	2.0	15357.6	1.942229389	135916.6976	-183960684.6	1328.269539
0.000126467	2.25	15357.6	1.942229389	139756.0976	-183960684.6	1293.061722
0.000126467	2.5	15357.6	1.942229389	143595.4976	-183960684.6	1259.642225
0.000126467	2.75	15357.6	1.942229389	147434.8976	-183960684.6	1227.880289
0.000126467	3.0	15357.6	1.942229389	151274.2976	-183960684.6	1197.657408
0.000126467	3.25	15357.6	1.942229389	155113.6976	-183960684.6	1168.865942
0.000126467	3.75	15357.6	1.942229389	162792.4976	-183960684.6	1115.193997
0.000126467	4.0	15357.6	1.942229389	166631.8976	-183960684.6	1090.142529
0.000126467	4.25	15357.6	1.942229389	170471.2976	-183960684.6	1066.17878
0.000126467	4.5	15357.6	1.942229389	174310.6976	-183960684.6	1043.234212
0.000126467	4.75	15357.6	1.942229389	178150.0976	-183960684.6	1021.245861
0.000126467	5	15357.6	1.942229389	181989.4976	-183960684.6	1000.155791

APPENDIX D

AOFD DETERMINATION USING BILINEAR, LINEAR, RADIAL FLOW REGIME

AOFD derivation steps based on analytical expression of bilinear flow regimes in finite conductivity fracture under Darcy flow conditions are defined as follows:

$$\frac{kh}{(5/4)\sqrt{2C_{fD}}} t_D^{1/4} \frac{[m(p_i) - m(p_{wf})]}{1,422qT}$$

$$q = \frac{kh[m(p_i) - m(p_{wf})] * (5/4)(2C_{fD})^{\frac{1}{2}}}{(t_D^{1/4})(1422t)}$$

AOFD derivation steps based on analytical expression of bilinear flow regimes in finite conductivity fracture under non-Darcy flow conditions are defined as follows:

$$\frac{kh}{(5/4)\sqrt{2C_{fD,app}}} t_D^{1/4} \frac{[m(p_i) - m(p_{wf})]}{1,422qT}$$

$$\frac{t_D^{\frac{1}{4}}}{\left(\frac{5}{4}\right) \left(\frac{2C_{fD}}{0.31 * 4.64 * 10^{-16} k_f Mq} \right)^{\frac{1}{2}}}$$

$$\frac{kh[m(p_i) - m(p_{wf})]}{1422 qT}$$

$$\left[\left(\frac{kh[m(p_i) - m(p_{wf})]}{1422Tq} \right) \left(\frac{2C_{fD}}{0.31 * 4.64 * 10^{-16} k_f Mq} \right)^{\frac{1}{2}} \right]^2 \left[\frac{t_D^{\frac{1}{4}}}{\left(\frac{5}{4}\right)} \right]^2$$

$$\begin{aligned}
& \frac{1}{\frac{2C_{fD}}{0.31 * 4.64 * 10^{-16} k_f Mq} * \frac{[kh\{m(p_i) - m(p_{wf})\}]^2}{(1422 qT)^2} \left(-\frac{t_D^{\frac{1}{4}}}{5}\right)^2} w_f h_i \\
& \frac{[kh\{m(p_i) - m(p_{wf})\}]^2}{(1422 qT)^2} * \frac{2C_{fD}}{w_f h_i \frac{0.31 * 4.64 * 10^{-16} k_f Mq}{w_f h_i}} \left(-\frac{t_D^{\frac{1}{4}}}{5}\right)^2 \\
& \frac{[kh\{m(p_i) - m(p_{wf})\}]^2 * 2C_{fD}}{(1422 T)^2 w_f h_i q^2 - (1422 T)^2 * 0.31 * 4.64 * 10^{-16} k_f Mq^3} \left(-\frac{t_D^{\frac{1}{4}}}{5}\right)^2 \\
& \frac{[kh\{m(p_i) - m(p_{wf})\}]^2 * 2C_{fD} * w_f h_i}{(1422 T)^2 w_f h_i q^2 - (1422 T)^2 * 0.31 * 4.64 * 10^{-16} k_f Mq^3} \left(-\frac{t_D^{\frac{1}{4}}}{5}\right)^2
\end{aligned}$$

The procedure for AOFPP calculation, for bilinear flow (non-Darcy) is as follows:

The first step of the procedure was to break down the cubic equation 31, in to four parts as follow:

$$A = (1422 T)^2 * 0.31 * 4.64 * 10^{-16} * \left(t_d^{\frac{1}{4}}\right)^2 k_f B M q^3$$

$$B = (1422 T)^2 w_f h_i \left(t_d^{\frac{1}{4}}\right)^2 q^2$$

$$C = 0$$

$$D = []^2 [kh\{m(p_i) - m(p_{wf})\}]^2 (2C_{fD}) w_f h_i$$

Cubic equations have to be solved in several steps. First we define a variable 'f' (see equation 40):

$$f = \frac{\frac{3c}{a} - \frac{b^2}{a^2}}{3} \quad (40)$$

Next g value was defined:

$$g = \frac{\frac{2b^3}{a^3} - \frac{9bc}{a^2} + \frac{27d}{a}}{27} \quad (41)$$

Then "h" value was found:

$$h = \left(\frac{g^2}{4}\right) - \left(\frac{f^3}{27}\right) \quad (42)$$

For the special case where f=0, g=0, and h=0, all 3 roots are real and equal.

When h ≤ 0, as is the case here, all 3 roots are real and is solved by another method.

When only 1 root is real (h > 0), the R, S, T, and U values are defined by following equation (see equation 43-12):

$$R = \left(\frac{g}{2}\right) - \left(\frac{f^3}{27}\right) \quad (43)$$

$$S = (R)^{1/3} \quad (44)$$

$$T = \left(\frac{g}{2}\right) + (h)^{1/2} \quad (45)$$

$$U = (T)^{1/3} \quad (46)$$

$$X1 = (S - U) - \left(\frac{b}{3a}\right) \quad (47)$$

$$X2 = \frac{(S - U)}{2} - \frac{b}{3a} - \frac{i(S + U)3^{1/2}}{2} \quad (48)$$

$$X3 = \frac{(S - U)}{2} - \frac{b}{3a} + \frac{i(S + U)3^{1/2}}{2} \quad (49)$$

AOFP derivation steps based on analytical expression of linear flow regimes in finite conductivity fracture under Darcy flow conditions are defined as follows:

$$\begin{aligned}
& \frac{kh}{1,422qT} [m(p_i) - m(p_{wf})] \sqrt{t_D} - \frac{a}{C_{fD}} \\
& \frac{\sqrt{t_D}}{1} - \frac{a}{C_{fD}} - \frac{kh}{1,422qT} [m(p_i) - m(p_{wf})] \\
& \frac{a - C_{fD} \sqrt{t_D}}{C_{fD}} - \frac{kh}{1,422qT} [m(p_i) - m(p_{wf})] \\
& 1,422qTC_{fD} \sqrt{t_D} - 1,422qTa - C_{fD}kh[m(p_i) - m(p_{wf})] \\
& q^* [1,422TC_{fD} \sqrt{t_D} - 1,422Ta] - C_{fD}kh[m(p_i) - m(p_{wf})]
\end{aligned}$$

AOFP derivation steps based on analytical expression of linear flow regimes in finite conductivity fracture under non-Darcy flow conditions are defined as follows:

$$\begin{aligned}
& \frac{kh}{1,422qT} [m(p_i) - m(p_{wf})] \sqrt{t_D} - \frac{a}{C_{fD,app}} \\
& \frac{kh}{1,422qT} [m(p_i) - m(p_{wf})] \sqrt{t_D} - \frac{a}{C_{fD}} \\
& \left(1 - \frac{0.31 - 4.64 \cdot 10^{-16} k_f Mq}{w_f h_i} \right)
\end{aligned}$$

$$\sqrt{t_D} \frac{kh}{1,422qT} [m(p_i) - m(p_{wf})] = \frac{\frac{a}{C_{fD}}}{(1 - \frac{0.31 \cdot 4.64 \cdot 10^{-16} k_f Mq}{w_f h_i})}$$

Next, make the left side of the equation the same denominator.

$$\frac{\sqrt{t_D} \cdot 1422qT \cdot kh[m(p_i) - m(p_{wf})]}{1,422qT} = \frac{\frac{a}{C_{fD}}}{(1 - \frac{0.31 \cdot 4.64 \cdot 10^{-16} k_f Mq}{w_f h_i})}$$

$$\frac{\sqrt{t_D} \cdot 1422qTC_{fD} \cdot kh[m(p_i) - m(p_{wf})]C_{fD}}{1422Tw_f h_i q \cdot 1422T \cdot 0.31 \cdot 4.64 \cdot 10^{-16} k_f Mq^2} = \frac{a}{1}$$

$$\frac{(\sqrt{t_D})^{\frac{1}{2}} w_f h_i \cdot 1422qTC_{fD} \cdot kh[m(p_i) - m(p_{wf})]C_{fD} w_f h_i}{1422Tw_f h_i q \cdot 1422T \cdot 0.31 \cdot 4.64 \cdot 10^{-16} k_f Mq^2} = \frac{a}{1}$$

To attain the AOF, the quadratic formula was used. To find all parameter for the quadratic formula, equation 32 was divided in to three parts. The following equations were used:

$$a = [1422T \cdot 0.31 \cdot 4.64 \cdot 10^{-16} k_f Mq]$$

$$b = [1422Taw_f h_i - 1422T(\sqrt{t_D})^{\frac{1}{2}} C_{fD} w_f h_i]$$

$$c = kh[m(p_i) - m(p_{wf})]C_{fD} w_f h_i$$

By substituting all the formula in to the quadratic equation, the AOF was found.

Table D-1 and D-2 shows the results for steps of AOFD calculation using bilinear and linear Non-Darcy flow regime for C_{fd} ranges (10-100).

Table D-1: AOFD calculation using bilinear non-Darcy flow regime (p=1500 psia and K=0.5)

C_{fd}	h	g	d	R	S	T	U	AOFD, MSCF/D
1.00E+01	1.39E+25	-7.46E+12	-7.87E+19	7.46E+12	1.95E+04	0	0	1.95E+04
2.00E+01	5.57E+25	-1.49E+13	-1.57E+20	1.49E+13	2.46E+04	0	0	2.46E+04
3.00E+01	1.25E+26	-2.24E+13	-2.36E+20	2.24E+13	2.82E+04	0	0	2.81E+04
4.00E+01	2.23E+26	-2.99E+13	-3.15E+20	2.99E+13	3.10E+04	0	0	3.10E+04
5.00E+01	3.48E+26	-3.73E+13	-3.94E+20	3.73E+13	3.34E+04	0	0	3.34E+04
6.00E+01	5.01E+26	-4.48E+13	-4.72E+20	4.48E+13	3.55E+04	0	0	3.55E+04
7.00E+01	6.82E+26	-5.22E+13	-5.51E+20	5.22E+13	3.74E+04	0	0	3.73E+04
8.00E+01	8.91E+26	-5.97E+13	-6.30E+20	5.97E+13	3.91E+04	0	0	3.90E+04
9.00E+01	1.13E+27	-6.72E+13	-7.09E+20	6.72E+13	4.07E+04	0	0	4.06E+04
1.00E+02	1.39E+27	-7.46E+13	-7.87E+20	7.46E+13	4.21E+04	0	0	4.21E+04

Table D-2: AOFD calculation using linear non-Darcy flow regime (p=1500 psia and K=0.5)

C_{fd}	a	b	b	b	c	AOFD, MSCF/D
10	1.04E+01	257058.49	3.65E+04	2.94E+05	-3.26E+10	4.37E+04
20	1.04E+01	257058.49	7.29E+04	3.30E+05	-6.52E+10	6.50E+04
30	1.15E+01	285923.10	1.09E+05	3.95E+05	-9.78E+10	7.65E+04
40	1.15E+01	285923.10	1.46E+05	4.32E+05	-1.30E+11	8.93E+04
50	1.15E+01	285923.10	1.82E+05	4.68E+05	-1.63E+11	1.00E+05
60	1.15E+01	285923.10	2.19E+05	5.05E+05	-1.96E+11	1.10E+05
70	1.15E+01	285923.10	2.55E+05	5.41E+05	-2.28E+11	1.19E+05
80	1.15E+01	285923.10	2.92E+05	5.78E+05	-2.61E+11	1.27E+05
90	1.15E+01	285923.10	3.28E+05	6.14E+05	-2.93E+11	1.35E+05
100	1.15E+01	285923.10	3.65E+05	6.50E+05	-3.26E+11	1.42E+05

Table D-3 and D-4 shows the results for steps of AOFD calculation using bilinear Non-Darcy and Darcy flow regime for C_{fd} ranges (10-100).

Table D-3: Converting C_{fd} values to skin factor and AOF calculation using bilinear non-Darcy flow regime, (p=1500 psia and K=0.5, t=1hr)*

C_{fd}	S	Xf, ft	t_D	a	b	c
10	-4.892852	1.00E+02	6.34E-04	4.91E+06	2.08E+09	0
20	-4.244157	5.00E+01	2.54E-03	9.83E+06	4.15E+09	
30	-3.860198	3.33E+01	5.71E-03	1.47E+07	6.23E+09	
40	-3.593569	2.50E+01	1.01E-02	1.97E+07	8.31E+09	
50	-3.380789	2.00E+01	1.59E-02	2.46E+07	1.04E+10	
60	-3.208723	1.67E+01	2.28E-02	2.95E+07	1.25E+10	
70	-3.064725	1.43E+01	3.11E-02	3.44E+07	1.45E+10	
80	-2.933212	1.25E+01	4.06E-02	3.93E+07	1.66E+10	
90	-2.821459	1.11E+01	5.14E-02	4.42E+07	1.87E+10	
100	-2.718101	1.00E+01	6.34E-02	4.91E+07	2.08E+10	

d	f	h	g	d	R	S
0.00E+00	-5.95E+04	4.01E+24	-4.01E+12	-1.97E+19	4.01E+12	1.59E+04
0.00E+00	-5.95E+04	4.01E+24	-4.01E+12	-3.94E+19	4.01E+12	1.59E+04
0.00E+00	-5.95E+04	4.01E+24	-4.01E+12	-5.90E+19	4.01E+12	1.59E+04
0.00E+00	-5.95E+04	4.01E+24	-4.01E+12	-7.87E+19	4.01E+12	1.59E+04
0.00E+00	-5.95E+04	4.01E+24	-4.01E+12	-9.84E+19	4.01E+12	1.59E+04
0.00E+00	-5.95E+04	4.01E+24	-4.01E+12	-1.18E+20	4.01E+12	1.59E+04
0.00E+00	-5.95E+04	4.01E+24	-4.01E+12	-1.38E+20	4.01E+12	1.59E+04
0.00E+00	-5.95E+04	4.01E+24	-4.01E+12	-1.57E+20	4.01E+12	1.59E+04
0.00E+00	-5.95E+04	4.01E+24	-4.01E+12	-1.77E+20	4.01E+12	1.59E+04
0.00E+00	-5.95E+04	4.01E+24	-4.01E+12	-1.97E+20	4.01E+12	1.59E+04

T	U	AOF, MSCF
1.950928	1.249253	1.57E+04
1.950928	1.249253	1.57E+04
1.950928	1.249253	1.57E+04
1.950928	1.249253	1.57E+04
1.950928	1.249253	1.57E+04
1.950928	1.249253	1.57E+04
1.950928	1.249253	1.57E+04
1.950928	1.249253	1.57E+04
1.950928	1.249253	1.57E+04
1.950928	1.249253	1.57E+04

Table D-4: Converting C_{fd} values to skin factor and AOFB calculation using bilinear Darcy flow regime ($p=1500$ psia and $K=0.5$, $t=1hr$)*

C_{fD}	data for skin	S	X_f , ft	t_D	AOFP, MSCF/D
10.00	2.27	-4.89	100.00	0.00	97354.07
20.00	2.17	-4.24	50.00	0.00	97354.07
30.00	2.13	-3.86	33.33	0.01	97354.07
40.00	2.08	-3.59	25.00	0.01	97354.07
50.00	2.06	-3.38	20.00	0.02	97354.07
60.00	2.04	-3.21	16.67	0.02	97354.07
70.00	2.02	-3.06	14.29	0.03	97354.07
80.00	2.02	-2.93	12.50	0.04	97354.07
90.00	2.00	-2.82	11.11	0.05	97354.07
100.00	2.00	-2.72	10.00	0.06	97354.07

Table D-5 shows the results for steps of AOFB calculation using transient equation and adjusted skin factor* ($p=1500$ psia, $K=0.5$ md, $S=-4.24$)

Table D-5: AOF calculation using pressure transient equation for radial flow regime, ($p=1500$ psia and $K=0.5$)*

t(hrs)	D	s	a	b	c	AOFP, MSCF/D
0	0.000108186					
0.09	0.000108186	-4.24415684	1.661484987	-46319.636	-183781636.6	31401.04241
0.19	0.000108186	-4.24415684	1.661484987	-40582.41	-183781636.6	28329.84893
0.29	0.000108186	-4.24415684	1.661484987	-37335.651	-183781636.6	26625.62976
0.39	0.000108186	-4.24415684	1.661484987	-35060.877	-183781636.6	25448.64592
0.49	0.000108186	-4.24415684	1.661484987	-33308.272	-183781636.6	24552.45578
1	0.000108186	-4.24415684	1.661484987	-27831.062	-183781636.6	21820.04056
2	0.000108186	-4.24415684	1.661484987	-22508.972	-183781636.6	19283.61093
4	0.000108186	-4.24415684	1.661484987	-17186.883	-183781636.6	16892.38228
4.99	0.000108186	-4.24415684	1.661484987	-15488.924	-183781636.6	16165.05391
5.99	0.000108186	-4.24415684	1.661484987	-14086.467	-183781636.6	15578.56459
6.99	0.000108186	-4.24415684	1.661484987	-12901.044	-183781636.6	15093.35039
9.99	0.000108186	-4.24415684	1.661484987	-10159.144	-183781636.6	14009.85738
11.99	0.000108186	-4.24415684	1.661484987	-8757.9707	-183781636.6	13478.05676
13.99	0.000108186	-4.24415684	1.661484987	-7573.4635	-183781636.6	13040.50434
15.99	0.000108186	-4.24415684	1.661484987	-6547.503	-183781636.6	12670.62277
17.99	0.000108186	-4.24415684	1.661484987	-5642.6134	-183781636.6	12351.5274

19.99	0.000108186	-4.24415684	1.661484987	-4833.2125	-183781636.6	12071.84922
21.99	0.000108186	-4.24415684	1.661484987	-4101.0572	-183781636.6	11823.58353
23.99	0.000108186	-4.24415684	1.661484987	-3432.6796	-183781636.6	11600.89134
25.99	0.000108186	-4.24415684	1.661484987	-2817.8532	-183781636.6	11399.38895
27.99	0.000108186	-4.24415684	1.661484987	-2248.6299	-183781636.6	11215.70564
29.99	0.000108186	-4.24415684	1.661484987	-1718.7092	-183781636.6	11047.19686
31.99	0.000108186	-4.24415684	1.661484987	-1223.0126	-183781636.6	10891.75165
33.99	0.000108186	-4.24415684	1.661484987	-757.38629	-183781636.6	10747.65967
35.99	0.000108186	-4.24415684	1.661484987	-318.38976	-183781636.6	10613.51698
37.99	0.000108186	-4.24415684	1.661484987	96.858906	-183781636.6	10488.15782
47.99	0.000108186	-4.24415684	1.661484987	1891.0105	-183781636.6	9963.577879
48.99	0.000108186	-4.24415684	1.661484987	2049.3611	-183781636.6	9918.606657
49.99	0.000108186	-4.24415684	1.661484987	2204.512	-183781636.6	9874.75253

*Note: Adjusted skin factor was bolded for table D-3, D-4, and D-5