Analytical determination of strain energy for the studies of coal mine bumps

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ANALYTICAL DETERMINATION OF STRAIN ENERGY FOR THE STUDIES OF COAL MINE BUMPS

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Thesis Submitted to the
College of Engineering and Mineral Resources
at West Virginia University
in Partial Fulfillment of the Requirements for
for the degree of

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in
Mining Engineering

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Coal mine bumps occur in most countries where coal is mined by underground methods. Coal bumps can be characterized as unstable releases of strain energy associated with energy changes that take place with progressive mining. This research is conducted to study the strain energy effect on coal bump problems associated with underground coal mining. The roofs are modeled as elastic beams on continuous elastic foundations subject to exponentially distributed abutment stress. Elastic beam theory is applied to develop analytical solutions for deflection of single-layer roof models. Methods for analyzing double layer roof and double layer foundation models are also discussed. Formulae for assessing critical spans of the roof beds and strain energy storage in the roof and foundation are developed. Based on a data bank of rock mechanics properties for coal measure strata from the results of 2813, 1102 and 126 tests for compression, tension and shear tests, respectively, from 50 coal seams in 90 coal mines by 63 coal companies in all the coalfields of the United States, the factors affecting roof cavability and energy accumulation are identified and analyzed. A parametric analysis reveals that mechanical characteristics of roof beds, foundation properties, and roof configurations may interact to influence roof cavability and energy storage.
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CHAPTER 1 INTRODUCTION

1.1 Research background

Coal bumps are sudden and violent bursts of coal from a pillar or pillars or even a block of coal, resulting in a section, the whole pillars, or the solid of coal being cast into an open entry with shattered coal stacking up to the roof line (Peng, 2008). Scenes of coal bumps are depicted in Figure 1.1.

![Figure 1.1 Scenes of coal bumps. Note that the roof in both cases are not damaged and oftentimes there is a gap between the roof and the top of the broken pillars (Peng, 2008)](image)

These events may lead to adverse effects such as fatalities and injuries, damage to mine facilities, economical losses from loss of production and premature abandonment of large reserves and environmental concerns. Many factors, such as abnormal geological conditions, improper mine design, physical and mechanical properties of roof strata and the like, may act together or separately to trigger bump events.

The presence of strong and massive roofs immediately overlying the mined-out areas has been long recognized as a substantial factor that contributes to mine tremors associated with pressure bumps and shock bumps. This factor has been observed and cited by numerous investigators (Avershin and Petukhov, 1964; Holland, 1958; Holland and Thomas, 1954; Jacobi, 1966; Lama, 1966; Rice, 1934). Typical competent roof strata in coal mines are sandstone, limestone, and sandyshale. Fine et al., (1964) proposed that the risk of bumps increases in proportion to the depth of the workings and described the bump effects shown in Figure 1.2.
Most U.S. bump-prone areas are located in the Southern Appalachian Basin of Kentucky, West Virginia, Virginia, and the Uinta and Piceance Creek Basins of Utah and Colorado (Iannacchione and DeMarco, 1992). Most reserves in these areas are deep with thick, strong and massive sandstone or sandyshale roofs close to the coal seam. Goode et al., (1984) documented that 20 coal bumps which occurred from 1964 to 1983 in these areas were associated with mining underneath strong sandstone or sandyshale roofs. Geological data compiled by Haramy et al., (1988) indicated that 35 Colorado and 38 Utah active and abandoned coal mines had strong sandstone roofs beds in the main roofs.

Laboratory tests showed that the average compressive strengths for the sandstone samples varying from 120 MPa (17,640 psi) to 230 MPa (33,810 psi) in the Southern Appalachian Coal Basin (Campoli, et al., 1993; Iannacchione and Mark, 1990; Khair, 1985), 70 MPa (10,290 psi) to 220 MPa (32,340 psi) in the Utah and Colorado coal mines, with a corresponding Young’s modulus ranging from 8 GPa (1.176 ×10^6 psi) to 50 GPa( 7.35×10^6 psi) ( Haramy, et al., 1988; Haramy and McDonnell, 1988).

Of the 172 bump events that are gathered in the USBM Coal Bump Database, lithologic descriptions of the mine roof are included for 95 bump sites. In 86 instances, reference is made to the presence of sandstone immediately above to within a few meters of the coalbed (Iannacchione and Zelanko, 1995).
It is believed that one of the most important factors favoring bump conditions is the sudden release of strain energy stored in the coal seam and the surrounding rock mass (Haramy and McDonnell, 1988). In longwall mining operations, the removal of coal will redistribute the overburden weight around the working faces. The strong roofs tend to bridge or cantilever over the adjoining gob area and transfer local stresses onto the working faces, the abutment pillars and the unmined panels. Meanwhile, massive strain energy is stored both in the coal seam and in the roof strata. The increased deflection of the roof beds with the increase of the unsupported span results in the superimposition of additional stresses to the already high front abutment stress concentration.

Iannacchione and Zelanko (1994) also proposed that the appearance of a dusting of “red coal” at the contact zone (Figure 1.3) is perhaps the most dramatic indicator of the imminence of a coal mine bump. This condition indicates the coalbed’s inability to resist shear forces generated by the tremendous confinement locally applied to the coal. The red zone in question probably represents coal that has been mechanically altered owing to the presence of excessive amount of shear strain.

Figure 1.3 Red dusts at the roof of the bump sites (Peng, 2008)

Pressure bumps may occur, if the local compressive stress concentration exceeds the local compressive strength of the coal, resulting in a violent release of stored strain energy in the form of elastic pulses radiating a considerable amount of seismic energy. Sudden catastrophic fracture of the roof strata may result in the rapid release of the stored strain energy and in the rapid stress transfer to the abutments, potentially bringing about shock bumps. Figure 1.4 illustrates the mechanics of pressure and shock bumps.
1.2 Research objectives

Although the problem of coal mine bumps has been extensively observed and investigated for many years and progress has been made in detecting bump-prone areas and the techniques in mitigating them, these events are still occurring. Very limited work has been done in terms of strain energy effect on coal bumps associated with sudden roof caving. The objective of this research is to achieve a better understanding of strain energy effect on coal mine bumps or rockbursts, as related to bump problems caused by their delayed caving. In order to accomplish the objective, the following goals should be achieved.

1. To develop models and their analytical solutions of the cantilevering and bridging roof strata based on elastic beam theory

2. To evaluate roof cavability in terms of determination of the critical spans of roof strata
③ To determine the amount of strain energy stored in the roof and the coal seam prior to the roof collapse

1.3 Research scope

In the context of longwall mining system, four configurations of roof types are identified according to the locations of strong roofs and the longwall weighting stages. Analytical solutions for single layer cantilevering and bridging models are developed using elastic beam theory. Following the analysis of single layer roof beam over single layer foundation (coal), analytical approaches for double layer roof beam and double layer foundation are discussed.
CHAPTER 2 LITERATURE REVIEW

2.1 Introduction

Although strong roof mitigates roof fall problems in panel entries, its incapacity to cave in timely may result in many ground control problems. Methods to control and reduce the possibility and severity of underground coal mine bumps usually deal with the behavior of roof overhanging over the mined-out areas. As far as underground coal mines are concerned, the common roof formations are stratified and horizontally bedded. This kind of mine roof can be assumed to be continuous over plan area. It is frequently treated as a “beam” or “plate” embedded at the edges by the overburden pressure.

To study ground control problems in longwall, roof cavability, strain energy storage and release, and assessment of bump-proneness potential associated with delayed collapse of the competent roof beds are the three particular important issues. A brief literature review pertaining to these issues is given in the following sections of this chapter.

2.2 Roof cavability

As early as 1905, it had been assumed that the roof is composed of many thin beams with beam supported by the underlying beam (Hackett, 1962). In recent years, theories of beam, plate, and Voussoir arch have been substantially improved by either more refined theoretical analysis or observations in the laboratory and in the field. Here, only beam theory is reviewed in detail which is mainly used later on in this research.

Beam theory

Horizontally bedded mine roofs bounded by bedding planes are usually treated as built-in or simply supported beams (Obert, et al., 1960; Caudle and Clark, 1955). Because of weak bond between bedding planes, the lower portions of the roof often detaches from the overlying rock, thus forming a layer loaded by its own weight. It is generally assumed that the beam is homogeneous, isotropic and elastic. Such kind of beam is free from any discontinuities. In addition to these assumptions, three geometric conditions should be taken into consideration specifically as follows:
① The span of the roof beam is at least twice of the beam thickness

② The length of the excavation is twice more than the roof span

③ The beam is of rectangular cross section

The applied load is uniform, and equal to the rock density multiplied by the overburden thickness. The stress and deflection are then assessed by simple beam theory. Rock being much stronger in compression than in tension, therefore, only the tensile stress is considered critical. The maximum span for a self-supporting built-in roof beam in tensile failure mode can be determined by the following expression (Adler and Sun, 1976):

$$L_{\text{max}} = \sqrt{\frac{2T_t}{\rho g}}$$  \hspace{1cm} (2.1)

where $L_{\text{max}}$ is the maximum roof span, m

$T_t$ is the tensile strength of rock, MPa

$\rho$ is the rock density, kg/m$^3$

$t$ is the roof thickness, m

Assuming that the overlying longwall roof beds are separated from each other along the bedding planes with minimum tensile resistance, the cantilevering length of the roof bed behind the working face can be expressed (Kidybinski, 1982) as follows:

$$L_{\text{max}} = \sqrt{\frac{T_f}{3\lambda}}$$  \hspace{1cm} (2.2)

where $L_{\text{max}}$ is span of cantilevering roof beam, m

$T_f$ is tensile strength of roof strata, MPa

$t$ is roof thickness, m

$\lambda$ is rock density, kg/m$^3$

Wilson (1986) assumed that the strong roofs acted as a fixed-end beam under a uniform overburden pressure and the failure modes were dominated by the overburden depth. A limit of the overburden depth is determined as follows:

$$H = \frac{C_o - T_t}{2kg}$$  \hspace{1cm} (2.3)

where $H$ is overburden depth, m

$C_o$ is compressive strength of rock, MPa

$T_t$ is tensile strength of rock, MPa

$k$ is ratio of horizontal stress to vertical stress
Above this limit, the tensile failure would occur on the upside of the rock beam, far from the mining. Below the limit, the severe compressive failure would take place on the underside of the rock beam, close to the mining, thus resulting in a bump. He also proposed that the in-situ horizontal stresses are of great importance when it comes to determining the cavability of strong roofs. The roof caving spans in tension and in compression can be determined by the following equations, respectively (Wilson, 1986):

\[ L_t = t \sqrt{\frac{2(T_o + kgH)}{gH}} \]  \hspace{1cm} (2.4)

\[ L_c = t \sqrt{\frac{2(C_o - kgH)}{gH}} \]  \hspace{1cm} (2.5)

where  
\( L_t \) is the roof span in tension, m  
\( L_c \) is the roof span in compression, m  
\( t \) is the roof thickness, m  
\( H \) is the overburden depth, m  
\( C_o \) is the compressive strength of rock, MPa  
\( T_o \) is the tensile strength of rock, MPa  
\( k \) is the ratio of horizontal stress to vertical stress.

As is known that the roof beds do not completely depend on rigid abutments, and that the elastic deformation of the abutments are supposed to affect the roof stability. Elastically supported beam theory, based on the differential equations of the elastic line, was proposed to provide an analytical basis for predicting the effects of elastic abutments on roof beam deflections (Hetenyi, 1946; Stephansson, 1971). Stephansson (1971) developed the mathematical solutions of deflection, bending moment, and longitudinal stresses for seven different roof configurations of single-, double- and multi-layer roofs on elastic abutments. In his analysis, the roof bed was assumed to act as a horizontal beam supported by the elastic, homogeneous, and isotropic abutments at both ends bearing a uniform loading.

### 2.3 Analysis of mining-induced energy changes

The initiation of underground mining induces transient stresses which may be greater than the final static stresses in the system. These transient influences on the stability of mine structures may be best studied through analysis of energy changes in the system. Coal bumps (in coal mining) or rockbursts (in hard rock mining) are caused by the violent release of kinetic, or seismic energy which is transformed from strain energy stored in stressed rock mass or coal, in the form of longitudinal and transverse elastic waves. Analysis of energy changes is the most effective method to study these violent events.
1. Mechanics of coal bumps

Crouch and Fairhurst (1972) best described the mechanics of coal bumps and Board and Fairhurst (1983) expanded the description. Although it attributes a lot to the earlier work on conventional rockburst studies summarized by Cook and Salamon (1983), the basic mechanics of coal bumps can be illustrated by the unloading deformation characteristics of a rock specimen under different stiff testing machines as shown in Figure 2.1 and Figure 2.2 (Brady and Brown, 1993).

Suppose that the specimen is at its peak strength and is further compressed by a small amount $\Delta S$. In order to accommodate this displacement, the load on the specimen must be reduced from $P_a$ to $P_b$, so that an amount of energy $\Delta W_s$, given by the area $abed$ in Figure 2.1 and Figure 2.2, is absorbed.

However, in displacing by $\Delta S$ from point $a$, the soft machine only unloads to $f$ and releases stored energy $\Delta W_m$, as given in Figure 2.1, the area $afed$. In this case, $\Delta W_m > \Delta W_s$, the energy released by the machine during unloading is greater than that which can be absorbed by the specimen in following the post-peak curve from $a$ to $b$. The excess of energy represented by the area $afb$ will be transformed into kinetic energy, causing catastrophic failure of the specimen. In the stiff machine case as shown in Figure 2.2, the post-peak failure of the specimen is stable because $\Delta W_m < \Delta W_s$, and energy in excess of that released by the machine as stored strain energy, represented by the area $abg$ in Figure 2.2, must be provided to deform the specimen along $abc$.

![Test machine unloading Specimen](image)

$\Delta W_s = abed, \Delta W_m = \Delta W_s + afb$

Figure 2.1 Post peak unloading for soft test machines (Brady and Brown, 1993)
Farmer (1985) described the bump conditions as the process of strain energy release in the form of kinetic energy.

① The rock being loaded must be subjected to a stress of sufficient magnitude over a sufficiently large volume to release a large amount of energy if it fractures.

② The loading conditions imposed by the surrounding strata must be such that their loading characteristic is less stiff than the fracturing rock.

These two factors are typical of the energy dissipation function of the stressed strata and the energy release rate of the rock mass. The interaction between these will determine the likelihood of coal bump. The energy release rate is the rate of energy released during initiation of underground mining, which is equal to the product of the mean force on the areal increment before mining and the mean convergence after mining. The energy dissipation function of the stressed strata has paramount to do with its ability to yield or fracture, absorbing accumulated strain energy in the stress concentration zone around the excavation.

2. Energy analysis due to mining

(1) Fundamental energy relationship during mining

The general concepts of fundamental energy storage and release process during mining were proposed by Blight (1984), Cook (1967b) and Salamon (1974). As summarized by Brady and Brown (1993), the energy redistribution caused by gradual creation of excavation follows the pattern as follows. As mining proceeds gradually, excavations in mines change in shape and grow in size with time, and the areas of induced stress are generated around the excavations. The previously stored strain energy in removing materials $W_r$ is gradually released. Partially released
energy $W_r$ is transformed into the surrounding induced stress zone, causing an increase of energy $\Delta W_r$. The remnant of the released energy is consumed in the form of rock fracture energy $W_f$ in the stress-induced zone. Energy conservation law requires that $W_r = \Delta W_r + W_f$. In the case of sudden creation of an excavation, the work that would have been done by the host rock, exterior to the excavation periphery, appears as excess energy $W_e$ at the excavation surface. This excess energy is subsequently released or propagated into the surrounding media in the form of kinetic energy. This process is similar to sudden loading applied to an elastic spring. Sudden loading produces imbalanced kinetic energy in the spring and radiates elastic waves. This extreme case gives an explanation of the possible source of kinetic or seismic energy.

(2) Energy changes for a thin tabular excavation

Since this kind of excavation is common when the coal seams are mined by longwall methods, energy changes or energy release associated with creating tabular excavations have been the subject of numerous researches.

Many of the original ideas associated with energy release evolved from studies of problems in deep mining in South African gold mines (Salamon, 1984). Using displacement-discontinuity techniques, special forms of the results for a single excavation were given by Salamon (1974, 1983, and 1984) and Walsh (1977). The important results from energy analysis for a tabular excavation include:

1. Sudden generation of a tabular excavation results in all strain energy stored in the removed materials being transformed into the surrounding stressed rock mass (Brady and Brown, 1993)

2. The amount of kinetic energy transferred from total energy released during excavating depends on how many mining steps were taken to reach the final step and size of the excavations as shown in Figure 2.3 (Salamon, 1983).

![Figure 2.3 Number of mining steps vs splitting of the total released energy (Salamon, 1983)](image-url)
(3) Sources of kinetic or seismic energy

A close examination of energy analysis and illustrated examples of circular cavity and thin tabular excavation discussed by Brady and Brown (1993) shows that the source of kinetic or seismic energy accompanied by a rockburst is sudden generation of excavation, and that the induced stress waves radiate from the periphery of the excavation. This explanation for kinetic energy source is valid on the assumption that the excavation is made in one step. When the excavation is factually expanded gradually, the estimation is grossly misleading. The error increases with the number of steps used to excavate the cavity. For instance, Salamon (1983) showed that 50 percent of the released energy can be transformed into kinetic energy if a circular cavity was made in one step. But if mining was done in 64 equal steps, the kinetic energy would be only 3.4 percent of the released energy $w_i$ (Figure 2.3). This suggests that the enlargement of mining excavations in small steps, which is the normal course of mining in most cases, does not result in the release of kinetic energy into the rock mass. Therefore, it cannot be the source of seismic energy. Another explanation of the kinetic energy source was given by Salamon (1983). He suggested that source of kinetic, or seismic energy comes from strain energy stored in a stress-concentrated zone surrounding the excavations. A seismic event would occur if the following conditions preexist (Salamon, 1983):

1. Substantial amount of energy must be stored in the rock around the instability to provide the source of kinetic energy. The origin of this energy is work done by: (a) gravitational forces and/or (b) tectonic forces and/or (c) stress induced by mining,

2. A region in the rock mass must be on the brink of unstable equilibrium,

3. Some induced stresses must affect the region in question, and however small, they must be sufficiently large to trigger the instability,

4. Sudden stress change of sizable amplitude must take place at the locus of instability to initiate the propagation of seismic waves.

(4) Strain energy stored in the coal and roof

Holland and Thomas (1954) and Phillips (1944) observed that the accumulation of strain energy in the coal and the adjacent strata is oftentimes the driving force behind coal bumps. They proposed that since coal is a relatively compressible material, it can store high amounts of strain energy even at the fairly low stress levels, and further reasoned that an overlying bed of massive sandstone contributes to both the accumulation and the release of this energy.

Holland (1955) analysed the strain energy stored in the roofs and the coal. The roof beds were modeled as either cantilever or fixed-end beams. The amounts of stored strain energy in the cantilever beam, the fixed-end beam, and the coal are given by Equations 2.6, 2.7, and 2.8, respectively (Holland, 1955). Figure 2.4 shows Poisson’s number for various coal measure strata (Holland, 1955).
\[ W_c = \frac{Q^2L}{40EI} \]  
\[ W_f = \frac{Q^2L}{1440EI} \]  
\[ W = \frac{1}{2E_c}\left[P_1^2 + 2\left(\frac{P_1}{m-1}\right)^2 - \frac{2}{m}\left(\frac{2P_1^2}{m-1}\right) + \left(\frac{P_1}{m-1}\right)^2\right] \]

where \( W_c \) is strain energy stored in the cantilever roof beam per unit volume, J  
\( W_f \) is strain energy stored in the fixed-end roof beam per unit volume, J  
\( W \) is strain energy stored in the coal per unit volume, J  
\( Q \) is load per unit length, MPa  
\( P_1 \) is principal stress, MPa  
\( L \) is length of the roof beam, m  
\( E \) is Young’s modulus of roof, MPa  
\( E_c \) is Young’s modulus of coal, MPa  
\( m \) is Poisson’s number of coal, the inverse of Poisson’s ratio, and  
\( I \) is moment of inertia, kg·m²

Figure 2.4 Poisson’s number for various coal measure strata (Holland, 1955)

Haramy et al., (1988) proposed a general analysis of strain energy accumulation associated with longwall mining by simulating the strong roof as an elastic cantilever beam over elastic foundation under uniformly distributed overburden load. On the assumption of a constant applied load, the effects of elastic modulus of the roof strata, roof thickness, and roof overhanging length on the strain energy accumulated in the roof and the coal were studied.
However, the model completely ignored the concentrated abutment pressure ahead of some longwall faces. This limitation prevents an accurate evaluation of the total amount of strain energy stored in the roof and the coal.

Wu and Karfakis (1993, 1994 and 1995) analysed ground control problems associated with longwall mining under strong roofs. The solutions for strain energy stored in the roof and the coal were thus developed (Wu and Karfakis, 1994a).
CHAPTER 3 MODEL FORMULATION AND SOLUTIONS

3.1 Introduction

The full-size structure considered in this research is the horizontally bedded roofs that are prevalent in most coal beds of the world. This kind of structure is frequently regarded as either cantilevers or fixed-end beams over rigid abutments loaded by uniformly distributed overburden pressure. The induced stresses and deflections by bending of the beam are then evaluated based on the classic simple beam theory. As is known to us, the roof beds do not rest completely on rigid abutments and the elastic behavior of the foundation should influence the stability of roof beam. Besides, the applied load on the roof beam is no longer uniform due to the stress redistributions caused by mining activities. Therefore, in order to reach a reasonable solution, the influence of foundations and non-uniformly applied loading conditions must be taken into considerations on the locations of strong roofs and the mining stages in longwall extractions. The abutment stress concentration is approximated as an exponentially decaying form. According to the elastic beam theory, analytical solutions of the deflection line for each roof model are developed. The influence of difference in the elastic moduli of rock materials under tension and compression on the flexural rigidity of the roof beam is investigated.

3.2 Model formulation

To form the models, assumptions must be made to simplify the problem in order to reach a reasonable solution. For the application of elastic beam theory to the problem, the following conditions are assumed:

① Two types of foundations are considered. One is coal seam, and the other consists of two layers, the coal seam and the overlying weak rock stratum, or weak floor stratum. Each foundation layer is assumed to be elastic, homogeneous and isotropic. The foundations are supposed to rest on the underlying floor strata.

② The strong roof beds are composed of elastic, isotropic and homogeneous rock materials and are void of discontinuities. Under consideration are competent roofs found in most bump-prone coal beds, in both US and other coal-producing countries with high compressive strengths.
③ The applied load is distributed in an exponentially decaying along the supported segment of the beam. For the unsupported part of the beam, the load is assumed to be uniformly distributed. The deflection of the roof beam does not appreciably change the load conditions.

④ Each layer is assumed to be a horizontally bedded formation with a rectangular cross-section and partially supported by elastic foundations, such as weak rock beds or coal pillars. The behavior of each layer conforms to the elastic beam principle. The length of each layer is twice of its width. The thickness of each individual layer is less than one fifth of the roof span. Roof configurations are categorized into four models according to the locations of overlying strong beds and the extraction stages in longwall mining.

(1) Model 1 Single layer cantilevering roofs

A single strong bed exists either in the immediate roof or in the main roof at the periodic weighting phase in longwall extraction or at pillar retreating phase in room-and-pillar extraction. This type of roof bed acts as cantilever resting on an elastic foundation (coal or weak roof) as illustrated in Figure 3.1. If the strong roof bed appears in the main roof, the weak immediate roof bed is regarded as part of a double layer foundation.

Figure 3.1 The single layer cantilevering beam
(2) Model 2 Single layer bridging roofs

The locations of strong roof beds are the same as in Model 1. This roof model represents the initial caving phase in longwall mining (Figure 3.2). For this type, both ends of the roof are supported by elastic foundations (coal or weak roof).

![Figure 3.2 The single layer bridging beam](image)

(3) Model 3 Double layer cantilevering roofs

For this roof model, both the immediate and the main roofs consist of competent roof beds with the ground movement occurring at the periodic weighting phase for longwall mining and at pillar retreating phase for room-and-pillar mining (Figure 3.3).

![Figure 3.3 The double layer cantilevering beam](image)
(4) Double-layer bridging roofs

This roof model is associated with the first weighting phase for a longwall system. Double-layer strong roofs beds are bridged on elastic foundation as shown in Figure 3.4.

![Figure 3.4 The double layer bridging beam](image)

As far as the following sections are concerned, the fundamentals of the elastic beam theory are discussed and then the analytical solutions to the deflection lines for each roof model are studied.

### 3.3 Fundamentals of elastic beam theory

#### 1. Differential equation of the bending beam

Suppose that a finite straight beam supported along its length by an elastic foundation and subjected to an arbitrarily distributed load \( p(x) \) as illustrated in Figure 3.5. Owing to the elastic assumption for the foundation, its reaction \( q(x) \) is proportional to the deflection \( y \) of the beam which is defined by the well-known fourth order differential equation:

\[
D \frac{d^4y}{dx^4} = p(x) - c(y) \tag{3.1}
\]

where
- \( D \) is flexural rigidity of the beam, \( \text{Pa} \cdot \text{m}^4 \)
- \( p(x) \) is arbitrarily distributed load on the beam, \( \text{MPa} \)
- \( c \) is modulus of the foundation, \( \text{MPa} \)
- \( y \) is deflection of the neutral axis of the beam.
Equation 3.1 is valid for deflection which is small compared to the thickness of the beam.

![Figure 3.5 A finite straight beam on an elastic beam (Wu, 1994)](image)

2. Characteristics of foundation

The foundation modulus, \( c \) in Equation 3.1 is a characteristic of the elastic foundation and is given as (Stephanson, 1971):

\[
c = \frac{E_c}{h_f(1-\nu^c_f)}
\]  

(3.2)

where \( c \) is modulus of foundation, MPa
\( E_c \) is Young’s modulus of the foundation, MPa
\( h_f \) is height of the foundation, \( m \) and
\( \nu_f \) is Poisson’s ratio of foundation.

Taking the notations in Figure 3.6, the moduli of a double-layer foundation \( c' \) can be determined by the following equations (Stephanson, 1971):

\[
c' = \frac{E_c}{(h_{t1} + h_{t2})(1-\nu^c_f)}
\]  

(3.3)

\[
E_c = \frac{(h_{t1} + h_{t2})E_{t1}E_{t2}}{h_{t1}E_{t2} + h_{t2}E_{t1}}
\]  

(3.4)

\[
\nu_f = \frac{(h_{t1} + h_{t2})E_{t1}E_{t2}(\nu_{t1}E_{t2} + \nu_{t2}E_{t1})}{(h_{t1}E_{t2} + h_{t2}E_{t1})(h_{t1}E_{t1} + h_{t2}E_{t2})}
\]  

(3.5)
where $c$ is equivalent modulus of the double-layer foundation, MPa
$E_{c1}$, and $E_{c2}$ are Young’s moduli of lower and upper layers for the double-layer foundation, respectively, MPa
$E_{c}$ is equivalent Young’s modulus of the double-layer foundation, MPa
$h_{c1}$ and $h_{c2}$ are thickness of the lower and upper layers for the double-layer foundation, respectively, m
$\nu_{c1}$ and $\nu_{c2}$ are Poisson’s ratios of the lower and upper layers for the double-layer foundation, respectively, and
$\nu_{c}$ is equivalent Poisson’s ratio of the double-layer foundation.

---

**Figure 3.6 Foundation comprised of two different layers**

### 3. Bending moment and shear force of the bending beam

Based on the elastic beam theory, the bending moment $M$, the shear force $V$, and the deflection $y$ of the beam have the following relationship:

\[
M = -D \frac{d^2y}{dx^2} \quad (3.6)
\]

\[
V = \frac{dM}{dx} = -D \frac{d^3y}{dx^3} \quad (3.7)
\]

where $M$ is bending moment of the beam, N·m
$V$ is shear force of the beam, MPa
$D$ is flexural rigidity of the beam, Pa·m$^4$.

### 4. Determining the equivalent elastic constants for combined beams

The modulus of elasticity in compression, $E_{c}$, for rock materials, is generally greater than the modulus of elasticity $E_{t}$ in tension. It is reported that for sandstone, $E_{c} = (1.5 — 4.0) E_{t}$ (Nasik and
Rzhevsky, 1971). The lower values of the modulus in tension will result in a shift of the neutral axis from the center line to the concave side of the beam as shown in Figure 3.7.

For a rectangular beam with a width of $b$ and a thickness of $h$, using the notations in Figure 3.7, the new position of the neutral axis is now defined by the following equations (Timoshenko, 1983):

$$h_1 = \frac{h \sqrt{E_t}}{\sqrt{E_t} + \sqrt{E_c}} \quad (3.8)$$

$$h_2 = \frac{h \sqrt{E_c}}{\sqrt{E_t} + \sqrt{E_c}} \quad (3.9)$$

where $h$ is thickness of the beam, m

$h_1$ is distance between the neutral axis and the underside of the beam, m

$h_2$ is distance between the neutral axis and the upside of the beam, m

$E_t$ is Young’s modulus in tension of the beam, MPa and

$E_c$ is Young’s modulus in compression of the beam, MPa.

![Figure 3.7](image)

**Figure 3.7** (a) Neutral axis shift $E_c > E_t$  
(b) Compressive and tensile stress-strain relationship, respectively

Because of the shift of the neutral axis, the equivalent modulus of elasticity $E_{eq}$ and the equivalent moment of inertia $I_{eq}$, of the beam are reduced and can be evaluated as follows (Jaeger, 1979):

$$E_{eq} = \frac{4E_tE_c}{(\sqrt{E_t} + \sqrt{E_c})^2} \quad (3.10)$$

$$I_{eq} = \frac{bh^2(\sqrt{E_t} + \sqrt{E_c})}{3\sqrt{E_c}} \quad (3.11)$$

where $E_{eq}$ is equivalent modulus of elasticity of the beam, MPa
\( I_{EV} \) is equivalent moment of inertia of the beam, kg·m\(^2\)
\( b \) is width of the beam, m
\( h \) is distance between the neutral axis and the underside of the beam, m
\( E_t \) is Young’s modulus in tension of the beam, MPa and
\( E_c \) is Young’s modulus in compression of the beam, MPa.

Therefore, the equivalent flexural rigidity of the bending beam \( D_{EV} \) becomes:

\[
D_{EV} = E_{EV} I_{EV}
\]

(3.12)

where \( D_{EV} \) is equivalent flexural rigidity of the beam, Pa·m\(^4\)
\( E_{EV} \) is equivalent Young’s modulus of the beam, MPa and
\( I_{EV} \) is equivalent moment of inertia of the beam, MPa.

Figure 3.8 illustrates the relationship of \( D_{EV} / D \) vs \( E_c / E_t \), which indicates that the actual flexural rigidity of the rock beam at \( E_c / E_t = 4 \), is over 20 percent lower than the corresponding value, when using \( E_t \) as the general modulus of elasticity for the entire beam.

![Graph illustrating the relationship of \( D_{EV} / D \) vs \( E_c / E_t \)](image)

Figure 3.8 Effect of \( E_c / E_t \) on the flexural rigidity of the beam (Wu, 1995)

5. The maximum tensile and compressive stresses in the bending beam

The maximum tensile stress, \( \sigma_{t,max} \) and the maximum compressive stress, \( \sigma_{c,max} \) at the external fibers of the beam under the bending moment, \( M \), can be determined by:
where \( \sigma_{\text{max}} \) is the maximum tensile stress in the bending beam, MPa
\( \sigma_{\text{max}} \) is the maximum compressive stress in the bending beam, MPa
\( h_1 \) is distance between the neutral axis and the underside of the beam, m
\( h_2 \) is distance between the neutral axis and the upside of the beam, m
\( M \) is bending moment of the beam, N·m
\( I_{\text{EV}} \) is equivalent moment of inertia of the beam, MPa.

### 3.4 Analytic solutions to the deflection line

Based on the roof configurations and the beam theories discussed in the previous sections, the solutions to deflection lines for each roof model are developed.

#### 1. Model 1 Single layer cantilevering roof beam

This model represents a single layer roof, partially supported by an elastic foundation (coal or weak roof) as shown in Figure 3.1. The applied load distributions are assumed to be:

\[
p(x) = \begin{cases} 
  p_0 e^{f_1} & 0 \leq x \leq l \\
  p & x > l, \text{and} -L < x < 0 
\end{cases}
\]  

(3.15)

where \( p \) is overburden pressure, MPa
\( p_0 \) is peak abutment pressure, MPa
\( f \) is characteristic of the abutment load distribution,
\( l \) is length of the stress concentration, m and
\( L \) is length of the unsupported part of the beam, m.

The parameter, \( f \) in Equation 3.15, denotes the abutment load distribution characteristics. It is defined by \( p, p_0 \) and \( l \).

\[
f = \frac{\ln\left(\frac{p}{p_0}\right)}{l}
\]  

(3.16)

By replacing \( D \) with \( D_{\text{EV}} \) in the differential Equation 3.1 and substituting \( p(x) \) in Equation 3.15, the differential equation of the deflection line for the supported part of the beam can be written as follows:
If a weak roof exists between the roof bed and the coal pillar, the foundation modulus, \( c \), in Equation 3.17, should be replaced by \( c' \) as defined in Equation 3.3.

The specific solution to Equation 3.17 takes the form:

\[ y_1(x) = \alpha e^{\alpha} \]  

(3.18)

where \( \alpha \) is constant.

Substituting \( y \) in Equation 3.17 with \( y_1(x) \) defined in Equation 3.18 and solving for \( \alpha \),

\[ \alpha = \frac{P_0}{D_{EV} f^4 + c} \]  

(3.19)

Hence, the specific solution can be taken as follows:

\[ y_1(x) = \frac{P_0}{D_{EV} f^4 + c} e^{\alpha} \]  

(3.20)

With regard to the general solution, \( y_2(x) \), of the deflection line of the beam which is defined in Equation 3.5, for points infinitely distant from the origin, the second term in Equation 3.5 must vanish. This condition can be satisfied only if the integration constant \( A_1 \) and \( A_2 \) in the equation are taken to equal zero. Hence, \( y_2(x) \) will take the form:

\[ y_2(x) = e^{-\beta x}(A_1 \sin \beta x + A_2 \cos \beta x) \]  

(3.21)

Combining Equations 3.20 and 3.21, we get the solutions for the deflection line of the beam:

\[ y = \frac{P_0}{D_{EV} f^4 + c} e^{\alpha} + e^{-\beta x}(A_1 \sin \beta x + A_2 \cos \beta x) \]  

(3.22)

The remaining integration constants \( A_1 \) and \( A_2 \) are determined as follows:

Denoting and substituting the following notations:

\[ y_0 = y|_{x=0} \]  

(3.23)

\[ y_0 = \frac{dy}{dx}|_{x=0} \]  

(3.24)

where \( y_0 \) is deflection of the beam at \( x=0 \) and

\( y_0' \) is the slope of the deflection line at \( x=0 \) .
Together with Equation 3.19, into Equation 3.25, we get:

\[
y_0 = \alpha + A_1 \tag{3.25}
\]

\[
y_0' = \alpha f + \beta(A_1 - A_2) \tag{3.26}
\]

From Equations 3.22 and 3.23, \( A_1 \) and \( A_2 \) can be expressed as follows:

\[
A_1 = \frac{y_0' - \alpha f}{\beta} + y_0 - \alpha \tag{3.27}
\]

\[
A_2 = y_0 - \alpha \tag{3.28}
\]

Now the deflection line (Equation 3.22) can be rewritten as:

\[
y = \alpha e^\alpha + e^{\beta x} \left[ \frac{y_0' - \alpha f}{\beta} + y_0 - \alpha \right] \sin \beta x + (y_0 - \alpha) \cos \beta x \tag{3.29}
\]

The second derivative of Equation 3.29 with respect to \( x \) together with Equation 3.6 gives the bending moment equation:

\[
M = -D_{kV} \left[ \alpha f^2 e^{\beta x} + 2\beta^2 e^{\beta x} \right] \left( y_0 - \alpha \right) \sin \beta x - \left( \frac{y_0' - \alpha f}{\beta} + y_0 - \alpha \right) \cos \beta x \tag{3.30}
\]

The bending moment at the point \( x = 0 \) can then be evaluated as follows:

\[
M_0 = D_{kV} \left[ 2\beta^2 \left( \frac{y_0' - \alpha f}{\beta} + y_0 - \alpha \right) - \alpha f^2 \right] \tag{3.31}
\]

The third derivative of Equation 3.29 with respect to \( x \) combining the known relation for shearing force in the bending beam (Equation 3.7) gives:

\[
V = -D_{kV} \left[ \alpha f^3 e^{\beta x} + 2\beta^3 e^{\beta x} \right] \left( \frac{y_0' - \alpha f}{\beta} \right) \sin \beta x - \left( \frac{y_0' - \alpha f}{\beta} + y_0 - \alpha \right) \cos \beta x \tag{3.32}
\]

for \( x = 0 \), we obtain,

\[
V_0 = -D_{kV} \left[ \alpha f^3 + 2\beta^3 \left( \frac{y_0' - \alpha f}{\beta} + 2y_0 - 2\alpha \right) \right] \tag{3.33}
\]

The magnitudes of \( M_0 \) and \( V_0 \) can be evaluated by the following known boundary conditions of the unsupported part of the beam:

\[
M_0 = \frac{pL^2}{2} \tag{3.34}
\]
From Equations 3.32 and 3.33, we can express $y_0$ and $y_0'$ in terms of $M_0$ and $V_0$ as follows:

$$y_0 = a - \frac{V_0 + \beta M_0 + \alpha D_{ev} f^2 (f + \beta)}{2D_{ev} \beta}$$  \hspace{1cm} (3.36)$$

$$y_0' = a f + \frac{V_0 + 2 \beta M_0 + \alpha D_{ev} f^2 (f + 2 \beta)}{2D_{ev} \beta^2}$$  \hspace{1cm} (3.37)$$

2. Model 2: Single layer bridging roof beam

A single-layer roof bed is bridged on the elastic foundation (coal or weak roof) at both ends as shown in Figure 3.2. Due to the constraints at both ends of the beam, the boundary conditions defined in Equations 3.34 and 3.35 for the cantilevering beam are no longer valid. In order to find the solution to the deflection line, let us first examine the unsupported part of the beam.

The differential equation for the unsupported part of the bending beam is:

$$D_{ev} \frac{d^4 y}{dx^4} = p$$  \hspace{1cm} (3.38)$$

with the following solution:

$$y = \frac{px^4}{24D_{ev}} + Ax^3 + Bx^2 + Cx + D$$ \hspace{1cm} (3.39)$$

where $A, B, C$ and $D$ are integration constants.

Inserting $y = y_0$ at $x = 0$, we obtain the constant $D$:

$$D = y_0$$ \hspace{1cm} (3.40)$$

Successive differentiation of Equation 3.39 with respect to $x$ and using notation for slope of the deflection line (Equation 3.24) along with moment and shear force using Equations 3.6 and 3.7 the following integration constants are obtained:

$$C = y_0'$$ \hspace{1cm} (3.41)$$

$$B = -\frac{M_0}{2D_{ev}}$$ \hspace{1cm} (3.42)$$

$$A = -\frac{T_0}{6D_{ev}}$$ \hspace{1cm} (3.43)$$
Substituting $M_c$ in Equation 3.31 and $V_c$ in Equation 3.33 into Equation 3.42 and 3.43, constants $A$ and $B$ can be expressed in terms of $y_0$ and $y_0'$ as follows:

$$B = \frac{1}{2} \alpha f^2 - \beta^2 \left( \frac{\alpha f}{\beta} \right) + y_0 - \alpha \quad (3.44)$$

$$A = \frac{1}{6} \alpha f^3 + \frac{1}{3} \beta^3 \left( \frac{\alpha f}{\beta} \right) + 2y_0 - \alpha \quad (3.45)$$

Yet, the deflection $y$ and the slope of the deflection curve $y_0$ at the point $x = 0$ remain. These can be evaluated by applying the following boundary conditions:

$$V|_{x=0} = -pL \quad (3.46)$$

$$\frac{dy}{dx} \bigg|_{x=L/2} = 0 \quad (3.47)$$

By combining Equations 3.33 and 3.46, we get:

$$D_{Lp} \left[ \alpha f^3 + 2\beta^3 \left( \frac{\alpha f}{\beta} + 2y_0 - 2\alpha \right) \right] = pL \quad (3.48)$$

Evaluating the first derivative of Equation 3.39 at the point $a = 0.5L$ and combining Equations 3.41, 3.44, 3.45 and 3.47, we obtain:

$$y_0 = \frac{pL}{48D_{Lp}} \left[ 3 \left( \frac{\alpha f}{6} \right) + \beta^3 \left( \frac{\alpha f}{\beta} + 2y_0 - 2\alpha \right) \right] - L \left[ \frac{\alpha f^3}{2} - \beta^3 \left( \frac{\alpha f}{\beta} + y_0 - \alpha \right) \right] = 0 \quad (3.49)$$

From Equations 3.48 and 3.49, we can find $y_0$ and $y_0'$:

$$y_0 = \frac{4a\beta^3 + 2f\alpha f^2 - \alpha f^3 - 2\beta^2 y_0}{4\beta^3} + \frac{pL}{4D_{Lp} \beta^3} \quad (3.50)$$

$$y_0 = \frac{(5p\beta L^2 + 12pL - 24D_{Lp} \alpha f \beta^2 - 12D_{Lp} \alpha f^3 - 24D_{Lp} \alpha \beta f^2) L}{24\beta D_{Lp} (L\beta + 2)} \quad (3.51)$$

For the supported part of the bending beam, the differential equation and deflection line equation are the same as the cantilevering beam which are defined by Equation 3.17 and Equation 3.29. However, due to different boundary conditions, $y_0$ and $y_0'$ are defined in Equations 3.50 and 3.51, respectively.

3. Model 3 and Model 4: Double layer roof beam
These models represent a double-layer roof with welded contacts between layers of different thickness and Young’s moduli resting on an elastic foundation (Figures 3.3 and 3.4).

The composite beam theory can be used to construct an equivalent layer of the same materials as the lower (or the upper) layer as shown in Figure 3.9 for \( E_{EY1} < E_{EY2} \). Following the notations in Figure 3.9, the relationship between the moduli and sections is:

\[
\frac{b_1}{b_2} = \frac{E_{EY1}}{E_{EY2}}
\]  

(3.52)

where  
\( b_1 \) is width of the upper layer, m  
\( b_2 \) is width of the lower layer, m  
\( E_{EY1} \) is equivalent modulus of elasticity of the upper layer, MPa and  
\( E_{EY2} \) is equivalent modulus of elasticity of the lower layer, MPa

![Figure 3.9 Double layer roof when \( E_{EY1} < E_{EY2} \) (a) Cross-section; (b) Cross-section transformed](image)

The moment of inertia about the neutral axis for the equivalent layer becomes:

\[
I_{EY} = \frac{1}{3} \left[ (h_1 + h_2 - d_2)^3 - \left( 1 - \frac{E_{EY1}}{E_{EY2}} \right) d_1^3 + \frac{E_{EY1}}{E_{EY2}} d_1^3 \right]
\]

(3.53)

where  
\( I_{EY} \) is equivalent moment of inertia for the double-layer roof beam, kg·m²  
\( h_1 \) is thickness of the upper-layer roof, m  
\( h_2 \) is thickness of the lower-layer roof, m  
\( d_2 \) is distance from the neutral axis to the upper fiber of the beam, m  
\( d_1 \) is distance from the neutral axis to the interface of two layers, \( d = h_1 - d_1 \) m.

The distance between the neutral axis and the upper fiber is given by:
\[
d_2 = \frac{E_{EV1} h_1^3 + \left(1 - \frac{E_{EV1}}{E_{EV2}}\right) h_2^3}{2 \left[ \frac{E_{EV1}}{E_{EV2}} h_1 + \left(1 - \frac{E_{EV1}}{E_{EV2}}\right) h_2 \right]} \quad (3.54)
\]

where \( h \) is total thickness of the double-layer beam, \( h = h_1 + h_2 \), m.

The flexural rigidity now becomes:

\[
D_{CEV} = E_{EV2} I_{CEV} \quad (3.55)
\]

The maximum tensile and compressive stresses can then be determined by the following equations:

\[
\sigma_{t,\text{max}} = \frac{M (h_1 - d_1)}{I_{CEV}} \quad (3.56)
\]

\[
\sigma_{c,\text{max}} = \frac{Md_1}{I_{CEV}} \quad (3.57)
\]

If \( E_{EV2} > E_{EV1} \), the new equivalent layer will have a cross-section as shown in Figure 3.10. The equivalent moment of inertia is the same as defined in Equation 3.53 by applying the notations in Figure 3.10.

\[\text{Figure 3.10 Double layer roof when } E_{EV1} > E_{EV2}\]

(a) Cross-section; (b) Cross-section transformed

The equivalent flexural rigidity of the composite beam can now be expressed as:

\[
D_{CEV} = E_{EV1} I_{CEV} \quad (3.58)
\]

However, the equations for the maximum tensile and compressive stresses will now become:

\[
\sigma_{t,\text{max}} = \frac{Md_1}{I_{CEV}} \quad (3.59)
\]

\[
\sigma_{c,\text{max}} = \frac{M (h_1 - d_1)}{I_{CEV}} \quad (3.60)
\]
where $d_2$ is the same as that in Equation 3.54.
CHAPTER 4 DETERMINATION OF CRITICAL SPANS AND ANALYSIS OF ROOF CAVABILITY

4.1 Introduction

In this chapter, equations for the critical spans are developed based on analytical solutions presented in Chapter 3. Because the tensile strength of rock materials is much less than its compressive strength, the failure mode is thus expected to be tensile.

Therefore, only tensile failure is taken into consideration in the following discussion. The critical spans are the roof spans for which the maximum tensile stress developed in the beam equates the tensile strength of the rock materials. The influence of parameters on the roof cavability is analyzed in order to assess the behavior of the major dependent variables. A set of design curves are developed for typical mechanical characteristics of the competent roof, foundation, and overburden.

4.2 Determining critical spans of roof beds

1. Critical spans for a single layer cantilevering roof

When the maximum tensile stresses in the bending beam equals to the tensile strength of rock material, the cantilevering length of the roof beam reaches its critical magnitudes. The following relations illustrate this condition:

\[ \sigma_{\text{max}} = -T_0 \]  \hspace{1cm} (4.1)
\[ M_{\text{max}} = -\frac{pL^2}{2} \]  \hspace{1cm} (4.2)

where \( \sigma_{\text{max}} \) is the maximum tensile stress, (negative), MPa
\( M_{\text{max}} \) is the maximum bending moment, N·m and
\( L \) is critical span of the beam, m

Applying the stress-moment relation defined in Equation 3.13, we obtain:
where \( h_1 \) is the distance between the neutral axis and the underside of the beam, m and 
\( I_{ev} \) is equivalent moment of inertia of the beam, kg\( \cdot \)m\(^2\).

Substituting \( h_1 \) into Equation 3.8 and \( I_{ev} \) in Equation 3.11 into Equation 4.3 results in:

\[
L_0 = 0.816h \sqrt{\frac{T_0}{p \sqrt{E'}}} \quad (4.4)
\]

For a given overburden pressure \( p \) and tensile strength \( T_0 \), Equation 4.4 indicates that, the critical spans of a single-layer cantilevering roof are dependent on the beam thickness \( h \) and the ratio of \( E, I/E' \) of rock materials.

2. Critical spans for a single layer bridging roof

For this kind of roof configuration, both ends of the beam are rested on foundation (coal or weak roof). Prior to the roof failure, the applied load \( p(x) \) acting on the beam should be continuous and evenly distributed as illustrated in Figure 4.1. The deflection line is defined by the following equation for the unsupported part of the bending beam (Stephanson, 1971):

\[
y = \frac{p}{c} \left[ \beta x \left( \frac{x^2}{6} - \frac{x^3}{3} \frac{Lx}{\beta} - \frac{L}{\beta^2} \right) + L\beta'k(1+ \beta x)^2 + 1 \right]
\]

(4.5)

where the parameter \( k \) is given by (Stephanson, 1971):

\[
k = \frac{\beta^2L + 66L + 6}{66(2 + L\beta)}
\]

(4.6)

The bending moment, \( M \) in the beam is given as:

\[
M = -\frac{px}{2} + \frac{pL}{2} \left( x + 1 \beta - k\beta \right)
\]

(4.7)

The maximum moment, \( M_{max} \) occurs at \( x = 0.5L \), and has a value:

\[
M_{max} = \frac{pL^2}{8} + \frac{pL}{2} \left( \frac{1}{\beta} - k\beta \right)
\]

(4.8)

By combining Equations 4.6 and 4.8 together, we obtain:
Assuming the in situ horizontal stress is zero and using Equations 3.16 and 4.9, the critical span, $L_c$, for tensile failure can be determined by the following equation:

$$p\beta^3 L_c^3 + 6p\beta L_c^2 \left[ 12p - \frac{8\beta^3 T_h \sqrt{E_h h^2}}{\sqrt{E_r} + \sqrt{E_i}} \right] L_c - \frac{16\beta T_h \sqrt{E_h h^2}}{\sqrt{E_r} + \sqrt{E_i}} = 0$$  \hspace{1cm} (4.10)

If the in situ horizontal stress is non-zero and has a magnitude of $\sigma_h$, Equation 4.10 becomes:

$$p\beta^3 L_c^3 + 6p\beta L_c^2 \left[ 12p - \frac{8\beta^3 (T_h + \sigma_h) \sqrt{E_h h^2}}{\sqrt{E_r} + \sqrt{E_i}} \right] L_c - \frac{16\beta (T_h + \sigma_h) \sqrt{E_h h^2}}{\sqrt{E_r} + \sqrt{E_i}} = 0$$  \hspace{1cm} (4.11)

From Equation 4.11, the critical spans of the bridging roof are not only dependent on overburden pressure, and mechanical properties of the roof beds, but also on the in-situ horizontal stress, and roof foundation system characteristics, $\beta$.

3. Critical spans for a double layer cantilevering roof

The critical span for a single-layer cantilevering roof, together with the stress-moment relationships defined in Equations 3.56 and 3.59 can be obtained.

When the upper layer has a higher magnitude of modulus of elasticity,
$$L_i = \frac{2T_i I_{CEV}}{h_i - d_z}$$  \hspace{1cm} (4.12)$$

When the lower layer has a higher magnitude of modulus elasticity,

$$L_i = \frac{2T_i I_{CEV}}{d_z}$$  \hspace{1cm} (4.13)$$

$I_{CEV}$ and $d_z$ are defined in Equations 3.53 and 3.54, respectively.

4. Critical spans for a double layer bridging roof

By equating the tensile stress $\sigma_t$ in Equations 3.56 and 3.59 to the tensile strength $T_0$ and substituting for $M_{max}$ in Equation 4.9, the critical spans under tensile failure can be determined as follows:

When the upper layer has a higher value of modulus of elasticity,

$$p\beta^2 L_i^3 + 6p\beta L_i^2 + \left[12p - \frac{24\beta^2 T_0 I_{CEV}}{h_z - d_z}\right] L_i - \frac{48\beta T_0 I_{CEV}}{h_z - d_z} = 0$$  \hspace{1cm} (4.14)$$

When the lower layer has a higher value of modulus of elasticity

$$p\beta^2 L_i^3 + 6p\beta L_i^2 + \left[12p - \frac{24\beta^2 T_0 I_{CEV}}{d_z}\right] L_i - \frac{48\beta T_0 I_{CEV}}{d_z} = 0$$  \hspace{1cm} (4.15)$$

$I_{CEV}$ and $d_z$ are defined in Equations 3.53 and 3.54, respectively.

4.3 Parametric analysis of roof cavability

Based on the critical span equations derived in the preceding sections, many selected variables that affect roof cavability are further examined in this section. The parameters analyzed are composed of overburden depth, $E_r/E$, ratio of rock materials, tensile strength, and foundation (coal) height. Since the double-layer models can be transformed into corresponding single-layer models using the composite beam principles, the single-layer cantilevering and bridging roofs are mainly analyzed as follows.
1. Overburden depth

Figure 4.2 shows the critical span variations for the cantilevering and bridging roof models for a range of roof thicknesses under different overburden depths. For both roof models, the spans increase with increasing roof thickness and decreasing overburden depth. However, the bridging spans are always greater than the cantilevering spans under the same conditions. As the overburden depth increases, this difference in spans between two roof models decreases.

\[ E_c = E_t = 30 \text{ GPa}, \quad T_0 = 4 \text{ MPa} \]

2. \( E_c/E_t \) ratio of roof beds

As illustrated in Chapter 3, for brittle rock materials, like sandstone, the modulus of elasticity in compression \( E_c \) is generally greater than the modulus of elasticity in tension, \( E_t \). This difference will affect the equivalent flexural rigidity of the roof beam, caving spans, and the capacity of energy storage. Figure 4.3 shows the effect of \( E_c/E_t \) ratio on the roof spans. The spans tend to increase with an increase of \( E_c/E_t \) ratio for both roof models. The bridging roofs are, however, more susceptible to variations in \( E_c/E_t \).
3. **Tensile strength**

The tensile strength of rock materials is the most important factor for affecting the roof cavability compared with other parameters (Figure 4.4). For the same roof thickness and tensile strength, the spans for the bridging beam are much greater than the spans for the cantilevering beam. As the roof thickness increases, the difference in spans between two models becomes even larger.

![Figure 4.3 Effect of $E_c / E_t$ on roof span](image)
4. Foundation height

The cantilevering spans are not affected by the foundation height as indicated in Figure 4.5. Whereas, the bridging spans increase as the foundation (coal) heights decrease as illustrated in Figure 4.5.
CHAPTER 5 STRAIN ENERGY ANALYSIS RELATED TO COAL BUMPS

5.1 Introduction

It is recognized that coal bumps may result from the violent release of seismic energy which is transformed from strain energy accumulated gradually in the coal seam and the surrounding rock mass. As mining operations continue, the virgin stress field is disturbed, resulting in stress concentrations around the working faces. In the meantime, great amount of strain energy, which is proportional to the square of the stresses, is accumulated both in the roofs and in the coal seam. Either roof breakage or foundation (coal or weak roof) failure may bring about rapid release of the stored strain energy. Therefore, an understanding of the strain energy accumulation behavior around the working faces and factors contributing to the energy accumulation, are of great importance to a safe and productive mining operation. Such understanding is able to provide a sound basis for the assessment of bump likelihood and its severity.

In this chapter, an attempt is made to analytically assess the strain energy accumulation caused by the cantilevering or bridging of competent roofs over the working faces. Strain energy formulae for cantilevering and bridging roof models are developed using elastic beam theory. Parameters probably affecting the energy storage are identified and studied. The parameters analyzed include roof geometry, mechanical properties of the roof beds, foundation (coal) characteristics and the overburden loading.

5.2 Strain energy stored in the roof and the foundation

1. Equations for basic energy

According to Hooke’s law (Buchanan, 1988), for elastic materials, the strain energy per unit volume, or strain energy density, can be expressed in terms of stress and strain components as follows:

\[ w = \frac{1}{2} \left( \sigma_\epsilon \epsilon + \sigma_\gamma \gamma + \sigma_\tau \tau + \tau_\sigma \sigma + \tau_\epsilon \epsilon + \tau_\gamma \gamma + \tau_\tau \tau \right) \]  \hspace{1cm} (5.1)

where \( w \) is strain energy density,
\( \sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{xz}, \tau_{yz} \) are stress tensors, and
\( \varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{xz}, \gamma_{yz} \) are strain tensors.

If the entire volume of the structural member is considered, the total stored strain energy is the integral of Equation 5.1 over the volume of the member, namely:

\[
W = \frac{1}{2} \int_{\text{volume}} (\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_z \varepsilon_z + \tau_{xy} \gamma_{xy} + \tau_{xz} \gamma_{xz} + \tau_{yz} \gamma_{yz}) \, dV
\]

(5.2)

where \( W \) is total stored strain energy in the structural member with a volume of \( V \), J and \( V \) is volume of the structural member, m\(^3\).

For the bending beam, the nonzero stress components are the flexural stress, \( \sigma_x \) and shear stress, \( \tau_{xy} \). Therefore, the strain energy due to bending, neglecting shear strain energy, is:

\[
W_b = \frac{1}{2} \int_{\text{volume}} \sigma_x \varepsilon_x \, dV = \frac{1}{2} \int_{\text{volume}} \frac{\sigma_x^2}{E} \, dV
\]

(5.3)

where \( W_b \) is total strain energy stored in the bending beam, J
\( \sigma_x, \epsilon_x \) are flexural stress and strain, respectively, MPa and \( E \) is Young’s modulus, MPa.

For the bending beam, flexural stress and strain can be evaluated by the following equations:

\[
\sigma_x = \frac{M y}{I}
\]

(5.4)

\[
\epsilon_x = \frac{\sigma_x}{E}
\]

(5.5)

\[
I = \int y^2 \, dA
\]

(5.6)

where \( M \) is bending moment of the beam, N\cdot m
\( y \) is distance from the neutral axis, m
\( A \) is cross-section area of the beam, m\(^2\) and
\( I \) is moment of inertia of the beam, kg\cdot m\(^2\).

Substituting above expressions into Equation 5.3, we obtain:

\[
W_b = \frac{1}{2} \int_{\text{area}} \frac{M^2 y^2}{EI} \, dA = \frac{1}{2} \int_{0}^{L} \frac{M^2}{D} \, dx
\]

(5.7)

where \( W_b \) is total strain energy stored in the bending beam, J
\( L \) is length of the beam, m
\( M \) is bending moment of the beam, N\cdot m
\( y \) is distance from the neutral axis, m
\( A \) is cross-section area of the beam, m\(^2\)
\( I \) is moment of inertia of the beam, kg·m\(^2\) and
\( D \) is flexural rigidity of the beam, Pa·m\(^4\) or N·m\(^2\).

Applying Equations 5.1 to 5.7, the stored strain energy in the roof beds and the coal can be evaluated.

2. Strain energy for the cantilevering roof model

The strain energy stored in this roof configuration is composed of three components, namely, energy stored in the unsupported roof portion, \( W_{1r} \), energy stored in the supported roof portion, \( W_{2r} \), and energy stored in the foundation (coal), \( W_c \). Based on Equations 5.1 to 5.7, each energy component can be assessed.

(1) Strain energy stored in the unsupported roof portion

The bending moment, \( M \), for the unsupported roof portion, is given by:

\[
M = \frac{1}{2} p x^2 \quad (5.8)
\]

Combining Equations 5.7 and 5.8, we obtain:

\[
W_{1r} = \frac{p^2 L^3}{40 D_{kv}} \quad (5.9)
\]

where \( W_{1r} \) is strain energy per unit width stored in the unsupported roof portion.

(2) Strain energy stored in the supported roof portion

The second derivative of deflection line (Equation 3.25) with respect to \( x \) together with Equation 3.9 gives the bending moment expression for the supported roof segment:

\[
M = D_{kv}\left[2\beta^2 e^{-\beta_1} (A_1 \cos \beta x - A_3 \sin \beta x) - \alpha f^2 e^{\beta_2} \right] \quad (5.10)
\]

Substituting Equation 5.10 into Equation 5.7, we obtain,

\[
W_{2r} = \frac{D_{kv} \alpha^2 f^2}{4} e^{2 \beta_1} + \frac{D_{kv} \beta^3}{4} e^{-2 \beta_1} (A_1^2 - A_3^2 + 2A_1A_3) \cos 2 \beta x - \frac{D_{kv} \beta^3}{2} (A_1^2 + A_3^2) e^{-2 \beta_1} (A_1^2 - A_3^2 + 2A_1A_3) \sin 2 \beta x +
\]

\[
\frac{2D_{kv} \alpha^2 f^2 \beta^3 e^{\beta_2} \beta_1}{(f - \beta)^2 + \beta^2} \left[ (A_1 f - A_1 \beta - A_3 \beta) \sin \beta x + (A_1 \beta - A_3 f - A_3 \beta) \cos \beta x \right] \quad (5.11)
\]

where \( W_{2r} \) is strain energy per unit width stored in the supported roof part.
(3) Strain energy stored in the foundation (coal)

According to the Hooke’s law, the stress and strain components are correlated as follows:

\[ \varepsilon_x = \frac{1}{E} \left[ \sigma_x - \nu(\sigma_y + \sigma_z) \right] \quad (5.12) \]
\[ \varepsilon_y = \frac{1}{E} \left[ \sigma_y - \nu(\sigma_x + \sigma_z) \right] \quad (5.13) \]
\[ \sigma_z = \frac{1}{E} \left[ \sigma_z - \nu(\sigma_x + \sigma_y) \right] \quad (5.14) \]
\[ \gamma_{xy} = \frac{\tau_{xy}}{G} \quad (5.15) \]
\[ \gamma_{yz} = \frac{\tau_{yz}}{G} \quad (5.16) \]
\[ \gamma_{zx} = \frac{\tau_{zx}}{G} \quad (5.17) \]

where \( E \) is modulus of elasticity, MPa
\( G \) is shear modulus, MPa and
\( \nu \) is Poisson’s ratio.

By substituting Equations 5.12 to 5.17 into Equation 5.1, the strain energy density can be expressed as:

\[ w_c = \frac{1}{2E} (\sigma_x^2 + \sigma_y^2 + \sigma_z^2) - \frac{\nu}{E_c} (\sigma_x \sigma_y + \sigma_y \sigma_z + \sigma_z \sigma_x) + \frac{1}{2G_c} (\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2) \quad (5.18) \]

where \( w_c \) is strain energy density in the coal,
\( E_c \) is Young’s modulus of the coal, MPa
\( G_c \) is shear modulus of the coal, MPa and
\( \nu_c \) is Poisson’s ratio of the coal.

The stress conditions in the coal are assumed to be biaxial, namely:

\[ \sigma_x = c_y \quad (5.19) \]
\[ \sigma_y = \sigma_z = \frac{\nu}{1-\nu_c} \sigma_x = \frac{\nu}{1-\nu_c} c_y \quad (5.20) \]
\[ \tau_{xy} = \tau_{yz} = \tau_{zx} = 0 \quad (5.21) \]

By combining Equations 5.18 to 5.21 and integrating, we obtain the strain energy per unit width in the coal seam:

\[ W_c = \frac{h c^2}{2E_c} \left[ \frac{(1+\nu)(1-2\nu)}{1-\nu} \right] \int y^2 dx \quad (5.22) \]
Substituting the deflection $y$, given in Equation 3.25, into Equation 5.22, and integrating, we get:

$$W_e = \frac{b_h c^2}{2E_v} \left[ \frac{\alpha^2 e^{2\beta x} - \alpha_x^2 e^{2\beta x} - 2e^{2\beta x} \cos 2\beta x - 2\beta e^{2\beta x}(\cos 2\beta x - \sin 2\beta x) + \frac{\alpha_x^2 e^{2\beta x} - \alpha^2 e^{2\beta x}}{8\beta} + 2\alpha e^{2\beta x}\{f - \beta'(f - \beta') + \beta^2[f(\alpha_x f - \alpha_f) - \beta(\alpha_f f - \beta)\cos \beta x}\} + \frac{\alpha \alpha_x}{4\beta} e^{-2\beta x}(\cos 2\beta x + \sin 2\beta x) \right]_0$$

(5.23)

where $w_e$ is strain energy per unit width stored in the coal.

Therefore, the total strain energy stored in the roof and the coal is given by:

$$W_{total} = W_{1r} + W_{2r} + W_{e}$$

(5.24)

where $W_{total}$ is total strain energy per unit width for the cantilevering roof model.

3. Strain energy stored in the bridging roof model

The formulae for assessing the strain energy stored in the supported part of the bending beam, $w_{1r}$, and the strain energy stored in the coal, $w_r$, are the same as those for the cantilevering beam, albeit with different initial magnitudes $y_0$ and $y_0'$ which are defined in Equations 3.53 and 3.54, respectively.

The moment equation for the unsupported equation can be established by second derivative of deflection line expression given in Equation 3.42 and substituting into Equation 3.9.

$$M = -(\frac{pL^3}{2} + 6B_1D_{cv} + 2B_2D_{cv})$$

(5.25)

Combining Equation 5.7 and 5.25 gives:

$$W_{1r} = \frac{p^2L^5}{40D_{cv}} - \frac{3}{4} pB_1L^4 + \frac{1}{6}(2pB_2 + 36B_1^2D_{cv})L^4 - 6B_1B_2D_{cv}L^4 + 2B_2D_{cv}L$$

(5.26)

where $w_{1r}$ is strain energy per unit width stored in the unsupported roof portion.

The total strain energy stored in the roof and the coal for the bridging model is thus obtained by:

$$W_{total} = W_{1r} + 2W_{2r} + 2W_{e}$$

(5.27)
where \( w_{\text{total}} \) is total strain energy per unit width for the bridging roof model.

### 5.3 Rock mechanics property data bank for coal measure strata

Sun and Peng (1993) developed a data bank of rock mechanics properties for coal measure strata. The data bank consists of data for more than 4,000 samples from 50 coal seams in 90 mines covering all the coalfields in the U.S. The unique features of coal measure rock strengths are that, for every type of rock, including coal, they differ enormously not only from mine to mine, but also from seam to seam (Hirt and Shakoor, 1992). Besides, the range of strengths overlaps each other, except that limestone is clearly the strongest and fireclay is the weakest.

From Figure 5.1, the maximum and the minimum tensile strength are 8.0MPa (1,180psi) and 3.1MPa (450psi), respectively. As Figure 5.2 shows, the maximum and the minimum tensile strength are 8.7MPa (1,280psi) and 2.2MPa (320psi), respectively. Figure 5.3 illustrates that the maximum and the minimum tensile strength are 6.8MPa (1,000psi) and 0.8MPa (120psi), respectively. The maximum and the minimum tensile strength in Figure 5.4 are 10.2MPa (1,500psi) and 3.4MPa (500psi), respectively. The magnitudes of the maximum and the minimum tensile strength in Figure 5.5 are 8.5MPa (1,250psi) and 2.1MPa (300psi), respectively. According to Figure 5.6, the maximum tensile strength is 6.8MPa (1,000psi) and the minimum tensile strength is 0.8MPa (120psi).

In summary, from Figure 5.1 to Figure 5.6, the magnitudes of the average maximum and the average minimum tensile strength are 8.2MPa (1205.4psi) and 2.1MPa (308.7psi), respectively. Because the tensile strength of rock materials is much less than its compressive strength, the failure mode is thus expected to be tensile. Therefore, only tensile failure is taken into account in the following discussions.

Based on the equations described in Chapters 4 and 5, parametric analysis and discussion are specifically illustrated in the following section 5.4.
Figure 5.1 Comparisons of compressive, tensile and shear strengths of sandstone by coal mines (Sun and Peng, 1993)

Figure 5.2 Comparisons of compressive, tensile and shear strength of shale by coal mines (Sun and Peng, 1993)
Figure 5.3 Comparisons of compressive, tensile and shear strengths of coal by mines (Sun and Peng, 1993)

Figure 5.4 Comparisons of compressive, tensile and shear strengths of sandstone by coal seams (Sun and Peng, 1993)
Figure 5.5 Comparisons of compressive, tensile and shear strengths of shale by coal seams
(Sun and Peng, 1993)

Figure 5.6 Comparisons of compressive, tensile and shear strengths of coal by seams
(Sun and Peng, 1993)
5.4 Parametric analysis and discussions

The parameters, such as overburden depth, tensile strength, foundation modulus, foundation height, which affect the strain energy accumulation and its distributions in the roof and foundations, can be analyzed to monitor the behavior of the major contributory variables, based on the strain energy expressions developed in the previous section.

1. Depth of overburden

Figure 5.7 illustrates the effects of overburden depth on the strain energy accumulation for a range of roof thickness. For both roof models, total stored energy increases with increasing overburden depth and roof thickness. However, energy increase rate of the bridging model is faster and higher than that of the cantilevering model. With the same depth of overburden and roof thickness, the bridging roof accumulates more energy than the cantilevering roof.

![Figure 5.7 Effect of the overburden depth on total strain energy](image)

Figure 5.8 shows energy distributions for the cantilevering roof model and reveals that even though both the roof energy and the foundation energy increase with overburden depth and the roof thickness, almost all energy is stored in the coal. The relative values of the roof energy to the total energy as plotted in Figure 5.9, \( \frac{W_r}{W_{total}} \) are very small.
Figure 5.10 shows that energy distribution for the bridging model is quite different from the cantilevering model. In general, both the roof energy and the foundation energy increase as the overburden depth and the roof thickness increase. However, the increase of the roof energy is at a relatively higher rate compared to the foundation energy. The energy distributions in the roof and the coal depend upon the overburden depth and the roof thickness (Figure 5.11). For a thin roof bed under a deep overburden, most of the energy is stored in the coal. Increasing the roof thickness or decreasing the overburden give a rise in roof energy and a decrease in foundation energy. The roof energy and the foundation energy arrive at the same value at an overburden depth of 300m and a roof thickness of 20m.
Figure 5.9 Energy percentages stored in roof and foundation (coal) for cantilevering roof

Figure 5.10 Strain energy distributions for bridging roof
Figure 5.11 Energy percentage stored in roof and foundation (coal) for bridging roof

2. Tensile strength

In both roof models, as shown in Figure 5.12, the stored strain energy increases with the increase of the tensile strength of the roof materials. The bridging roofs accumulate much more energy than the cantilevering roofs. The difference in the energy storage between two roof models increases as the roof thickness increases. Figure 5.13 illustrates that energy distributions in the cantilevering roof model indicates that most of energy is stored in the coal foundation. The roof carries only about 15% of total energy for tensile strength varying from 5MPa to 15MPa (Figure 5.14). Figure 5.15 shows the energy distributions for the bridging roof model. With the increase of tensile strength and roof thickness, the energy stored in the roof increases more rapidly than the energy stored in the coal. For a stiffer and thicker roof, the roof energy will exceed the foundation energy as shown in Figure 5.16.
Figure 5.12 Effect of tensile strength on total strain energy

Figure 5.13 Strain energy distributions for cantilevering roof
Figure 5.14 Energy percentage stored in roof and foundation (coal) for cantilevering roof

Figure 5.15 Strain energy distributions for bridging roof
3. Foundation modulus

Effect of the foundation modulus on the total strain energy accumulation is illustrated in Figure 5.17. For both models, increasing foundation modulus and decreasing the ratio of roof modulus to the coal modulus, $E_r/E_c$, results in decreasing the amount of stored energy. Under the same conditions, the bridging roof tends to store more energy than the cantilevering roof. Therefore, in order to minimize the energy buildup, roof modulus can be reduced by minimizing the ratio of $E_r/E_c$.

Figure 5.16 Energy percentage stored in roof and foundation (coal) for bridging roof

Figure 5.17 Effect of foundation (coal) modulus on total strain energy
Although variations in coal modulus have effects on the energy distributions as shown in Figure 5.18, the majority of energy is stored in the coal for the cantilevering model. In the bridging roof model, the energy stored in the roof and the foundation is greatly affected by the roof thickness and the foundation modulus as shown in Figure 5.19. With the increase of the roof thickness and the coal modulus, the energy stored in the roof will also increase, in contrast to the rapid decrease of energy in the coal.

Figure 5.18 Energy percentage stored in roof and foundation (coal) for cantilevering roof

Figure 5.19 Energy percentage stored in roof and foundation (coal) for bridging roof
4. Height of foundation

Variations in total stored energy for a range of foundation heights are illustrated in Figure 5.20. The total energy increases with increasing foundation height for both roof models. When the foundation height is the same, the energy stored in the bridging model is always greater than the energy in the cantilevering model.

![Figure 5.20 Effect of foundation (coal) height on total strain energy](image)

As far as the cantilevering roof is concerned, most of energy is stored in the coal as shown in Figure 5.21. Decreasing foundation height results in an increase in roof energy and a decrease in coal energy. However, as the foundation height decreases, for the bridging model, the foundation energy decreases at a rapid rate, corresponding to a sharp increase in roof energy as illustrated in Figure 5.22.
$E_c = E_t = 30 \text{ GPa}, \quad T_0 = 4 \text{ MPa}$

$E_0 = 3 \text{ GPa}$

Figure 5.21 Energy percentage accumulated and stored in roof and foundation (coal) for cantilevering roof

Figure 5.22 Energy percentage stored in roof and foundation (coal) for bridging roof
CHAPTER 6 CONCLUSIONS AND RECOMMENDATIONS

This research is conducted to study the strain energy effect on coal mine bumps or rockbursts directly overhanging longwall working faces. The major conclusions from this research work are summarized below.

Compared with the conventional method that treats the roof bed as simple or fix-end beams on a rigid foundation under a uniform load, the strong roof beds are modeled as either cantilevering or bridging beams on elastic foundations under exponentially decaying abutment stresses. These models combine the elastic deformation of the foundation, as well as the high stress concentrations around the working faces. Analyses show the importance of foundation compression and abutment stress concentration in resulting in roof cavability and strain energy storage.

Based on the elastic beam theory, mathematical solutions of deflection lines have been developed for the single-layer elastically supported cantilevering and bridging roof beams. These solutions provide an analytical basis for determining critical spans of roof beds and evaluating the strain energy stored in the roof and the foundation. The analyses show that the difference in the moduli of elasticity in tension and compression for rock materials alters the position of the neutral axis, decreases the actual flexural rigidity, and results in asymmetrical stress redistributions in the rock beam.

Formulae have been developed to determine the critical spans of cantilevering and bridging strong roof beds under tension failure. A set of design curves are plotted to provide a tool for determining critical spans. Study on the roof cavability shows as follows:

1. For a given roof information under the same overburden loading, the bridging roof has greater spans than the cantilevering roof, indicating that the initial caving intervals are greater than the periodic caving intervals during the longwall mining;

2. The critical spans for both cantilevering and bridging increase with increasing tensile strength of rock materials, ratio of the elastic modulus in tension, and decreasing overburden;

3. As far as the bridging roof is concerned, the cavability is also affected by the foundation characteristics. The critical spans increase with decreasing foundation height or increasing foundation modulus.

By applying the developed strain energy formulae, a comprehensive parametric analysis is used to examine the factors that determine the energy buildup and distributions caused by the
cantilevering and bridging of the strong roofs. The parametric analysis identifies that the strain energy accumulation in the roof and the foundation is affected by the following factors:

1. Roof configurations,
2. Foundation (coal) properties, namely, modulus of elasticity and height,
3. Mechanical properties of the strong roofs, such as, tensile strength, ratio of elastic modulus in compression to modulus in tension,
4. Applied stress characteristics, including overburden depth and stress concentration.

More facts regarding energy accumulations are revealed from the analysis:

1. Everything being equal, the energy storage associated with the bridging roof model is always greater than that for the cantilevering roof model, which indicates a higher bump is likely for the first weighting stage compared with the periodic weighting stage during longwall mining,
2. The stored strain energy increases with increasing roof thickness, overburden depth, $E_r/E_t$ ratio, tensile strength, stress concentration factor, foundation height, and decreasing foundation modulus.

As far as the energy analysis is concerned, for the cantilevering models, the roof carries only small percentage of the total energy. Therefore, measures and efforts to mitigate bump hazards should be concentrated on controlling of the stored energy in the coal. The stored energy in a thicker and massive bridging roof bed with a higher tensile strength may exceed the stored energy in the coal. Therefore, both the roof and the foundation energy accumulation must be concurrently taken into consideration.
REFERENCES


