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Solution techniques for a crane sequencing problem

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Abstract: In the areas of power plant maintenance, shipyard and warehouse management, resources (items) assigned to locations need to be relocated. It is essential to develop efficient techniques for relocating items to new locations using a crane such that the sum of the cost of moving the items and the cost of loading/unloading the items is minimised. This problem is defined as the crane sequencing problem (CSP). Since the CSP determines the routes for a crane to relocate items, it is closely related to some variants of the travelling salesman problem. However, the CSP considers the capacities of locations and intermediate drops (i.e. preemptions) during a multiple period planning horizon. In this article, a mathematical model and hybrid ant systems are developed for the CSP. Computational experiments were conducted to evaluate the performances of the proposed techniques, and results show that the proposed heuristics are effective.

Keywords: crane sequencing problem; hybrid ant systems; integer programming; meta-heuristics; simulated annealing.

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1 Introduction

The crane sequencing problem (CSP) is the task of determining the sequences in which a single overhead crane moves items (or resources) to their new locations with respect to minimising the total travel and loading/unloading costs. More specifically, when items are rearranged (reassigned to new locations), the crane is used to relocate the items. First, the crane arrives at the location of an item to be moved. Second, the item is loaded onto the crane. Third, the item is moved from its original position to its new position if the position is available. Otherwise, the item is moved to a temporary location. Nevertheless, the item is unloaded from the crane. This process is repeated for each item being rearranged. An item assigned to a temporary location is moved to its assigned location when the location is available and either the crane is at its temporary location or at the end of the sequence of moves. Hence, the CSP determines the order in which the items should be moved such that the total cost is minimised. The total cost consists of two major components. First, loading/unloading cost is the cost of loading the items onto the crane plus the cost of unloading the items. Second, the crane travel cost is the cost of moving the items from their original locations to their destination locations using the crane, which is the sum of the product of the distances the crane travels and the cost of moving items per distance unit.

The assumptions for the CSP are as follows.

- 1 The location of each item in each period, the distances between locations, the capacities of the locations, initial location of the crane, cost of moving items per distance unit and the costs of loading/unloading items are known.
- 2 All items may not be relocated at each period.
- 3 The capacity for a single overhead crane is one unit of an item.
- 4 After the second period, the initial position of the crane is the last position of the crane in the previous period.
- 5 Preemption can occur, and any available locations can be used as temporary storage locations.
- 6 The resources are fixed at their locations if they are not scheduled to be moved at a specific time period.

- 7 The objective of the CSP is to find the sequences in which items are to be moved by the crane such that the sum of the loading/unloading and crane travel costs is minimised.

The CSP is a problem which considers multiple time periods (i.e. items may be reassigned to locations in different periods). As a result, the CSP determines the crane route for each period items are rearranged. It is important to note that all items may not be rearranged from one period to the next as stated in Assumption (2). Without loss of generality, the initial location of the crane at the beginning of each period, not including the initial period, is the last position of the crane in the previous period as defined in Assumption (4). However, if the initial position of the crane at the beginning of each period is known, the problem can be solved as a series of static CSPs, which is much easier than the CSP defined in this article. Furthermore, it is easy to modify the mathematical model and heuristics below to consider this case. In Assumption (5), temporary storage locations can be used to store items temporarily when:

- 1 The destination location of an item currently being moved is at full capacity. In this case, either the item has to be moved to another available location (called a temporary storage space) or the item is moved at a later time when the destination location is available.
- 2 Routes of the crane can be improved (i.e. the sum of total travel and loading/unloading costs for the crane is reduced) if temporary storage space is used.

Both of these cases are illustrated below. Nevertheless, once an item is moved to a temporary storage space, it is stored there until its assigned location is available and either the crane is at its temporary location or after all the other items has been moved to their assigned locations as stated previously. Also, available locations (temporary locations) are defined as storage locations not at full capacities and workspaces that are currently available (i.e. no activities are currently being performed in the workspaces). In the literature (e.g. Charikar, Khuller and Raghavachari, 2002), when an item can be put into an intermediate location before it is delivered to its final destination location, it is called ‘preemption’ or ‘intermediate drop’. There are different variations of the CSP where locations can be workspaces (i.e. locations use to perform activities which require resources) and/or storage locations (locations used to store resources not to perform activities). For instance, during planned outages at electric power plants, the locations can be defined as workspaces where outage activities are performed and resources (items) are required to perform the activities. Also, within the same application, locations can be either workspaces, as defined above, or storage spaces where idle resources are stored until they are needed to perform an activity. On the other hand, locations can be defined as storage locations only where products are stored until they are needed (e.g. as in storing products in a warehouse). Nevertheless, the techniques presented in this article can be applied to all the above scenarios.

The CSP was first defined by McKendall et al. (2006) in which items, during outages at electric power plants, are to be moved using a single overhead crane such that the total distance travelled by the crane is minimised. The authors presented a simulated annealing heuristic for the CSP. McKendall and Shang (2008) presented several tabu search heuristics for the CSP, as defined in this article (i.e. items are to be moved using a single overhead crane such that the sum of the crane travel cost and loading/unloading costs is minimised). The first tabu search heuristic is a simple tabu search heuristic, the second is

a probabilistic tabu search heuristic, and the third adds diversification and intensification strategies to the first. These are the only papers, known to the authors, which consider the CSP defined in this article.

Besides determining the order in which items are relocated during outages at electric power plants, the CSP has many potential applications in the real world. Another application of the CSP occurs in the context of warehouse rearrangement. Since the demand of products is dynamic and some products may become obsolete, warehouse managers may rearrange the layout of their products in warehouses. For example, products with high demand usually are located close to the input/output locations. As a result, the locations of products may change. Therefore, the CSP can be used to determine the sequence in which the crane (or vehicle) moves the products reassigned to new locations in a warehouse. This problem is called the warehouse rearrangement problem and was presented in Christofides and Colloff (1973). Also, the CSP can be used to determine the order in which a crane removes containers from ships or vice versa (i.e. determine the order in which a crane loads containers onto ships).

In this article, a binary integer linear program for the CSP is developed to obtain optimal solutions for small-size problems. Since only small-size problems can be solved optimally in reasonable computational time, hybrid ant systems (HASs) are developed to solve large-size CSPs. The article is organised as follows. Section 2 presents a mathematical model for the CSP. Also, a small CSP instance is presented. Section 3 describes the proposed HASs to solve the CSP, and the computational results are summarised in Section 4. Section 5 concludes the article.

2 Mathematical formulation

In this section, a binary integer linear program is presented for the CSP. First the notation is given below. The indices are as follows:

$i, j, h = 1, \dots, L$, where L is the number of locations

$r = 1, \dots, R+1$, where R is the number of items (or resources) and $R+1$ represents a dummy item (empty move)

$k, k_2 = 1, \dots, K$, where K is an upper bound on the number of moves

$t = 1, \dots, T$, where T is the total number of periods.

The parameters are given below:

A_t = set of items to be moved at period t (not including $R+1$)

$B_t = A_t \cup \{R+1\}$, which is a set of items to be moved at period t including $R+1$

Q_t = set of locations which cannot be used as temporary storage locations because activities are being performed in those locations at period t

UB_t = upper bound for the number of moves in period t , where $UB_t = 3|A_t|$ and $K = \max\{UB_t\}$

d_{ij} = distance between locations i and j

l_{rt} = the location of item r at period t

n_{jt} = the number of items in location j at period t

p = the initial location of the crane at the beginning of period 2 (assume location 1)

C_j = capacity of location j

M = a larger number

W_1 = travel cost per distance unit

W_2 = loading/unloading cost per item moved.

The decision variables are defined as

$$x_{t,k,r,i,j} = \begin{cases} 1, & \text{If at the end of period } t, \text{ item } r \text{ is moved from location } i \text{ to } j \text{ at } k\text{th crane moved;} \\ 0, & \text{Otherwise.} \end{cases}$$

The binary integer linear program for the CSP is as follows.

$$\begin{aligned} \text{Minimise total cost} = & W_1 \sum_t \sum_{k \leq UB_t} \sum_{r \in B_t} \sum_i \sum_j d_{ij} x_{t,k,r,i,j} \\ & + W_2 \sum_t \sum_{k \leq UB_t} \sum_{r \in A_t} \sum_i \sum_j x_{t,k,r,i,j} \end{aligned} \quad (0)$$

subject to:

$$\sum_{k \leq k2} \sum_i \sum_j x_{t,k,r,i,j} \leq M \sum_{k \leq k2} \sum_j x_{t,k,r,i=l_{rt},j} \quad \forall t, k2, r \in A_t \quad (1)$$

$$\sum_k \sum_j x_{t,k,r,i=l_{rt},j} = 1 \quad \forall t, r \in A_t \quad (2)$$

$$M \left(1 - \sum_{k \leq k2} \sum_i x_{t,k,r,i,j=l_{rt+1}} \right) \geq \sum_{k \geq k2+1} \sum_i \sum_j x_{t,k,r,i,j} \quad \forall t, k2, r \in A_t \quad (3)$$

$$\sum_k \sum_i x_{t,k,r,i,j=l_{rt+1}} = 1 \quad \forall t, r \in A_t \quad (4)$$

$$n_{jt} + \sum_{k \leq k2} \sum_{r \in A_t} \sum_i x_{t,k,t,i,j} - \sum_{k \leq k2-1} \sum_{r \in A_t} \sum_i x_{t,k,t,j,i} \leq C_j \quad \forall t, k2, j \quad (5)$$

$$\sum_{r \in B_t} \sum_i \sum_j x_{t,k,r,i,j} \leq 1 \quad \forall t, k \quad (6)$$

$$\sum_{r \in B_t} \sum_j x_{t=1,k=1,r,i=p,j} = 1 \quad (7)$$

$$\sum_{r \in B_t} \sum_i x_{t,k,r,i,j=h} \leq \sum_{r \in B_t} \sum_j x_{t,k+1,r,i=h,j} + 1 - \sum_{r \in B_t} \sum_i \sum_j x_{t,k+1,r,i,j} \quad \forall t, h, k < K \quad (8)$$

$$\begin{aligned}
\sum_{r \in B_t} \sum_i x_{t-1, k=k2, r, i, j=h} - \sum_{r \in B_t} \sum_i \sum_j x_{t-1, k=k2+1, r, i, j} \\
\leq \sum_{r \in B_t} \sum_j x_{t, k=1, r, i=h, j} \quad \forall t > 2, h, k2
\end{aligned} \tag{9}$$

$$\sum_i x_{t, k=k2, r, i, j=h} \leq \sum_{k > k2} \sum_j x_{t, k, r, i=h, j} \quad \forall t, r \in A_t, h \neq l_{t+1}, k2 \tag{10}$$

$$\sum_i \sum_{k < k2} x_{t, k, r, i, j=h} \geq \sum_j x_{t, k=k2, r, i=h, j} \quad \forall t, r \in A_t, h \neq l_t, k2 \tag{11}$$

$$\sum_r \sum_i \sum_j x_{t, k, r, i, j} \geq \sum_r \sum_i \sum_j x_{t, k=k+1, r, i, j} \quad \forall t, k < K \tag{12}$$

$$\sum_k \sum_{r \in A_t} \sum_i x_{t, k, r, i, j=Q_i} = 0 \quad \forall t, Q_i \tag{13}$$

$$x_{t, k, r, i, j} = 0 \text{ or } 1 \quad \forall t, k, r, i, j. \tag{14}$$

The objective function (0) minimises the sum of the total travel and loading/unloading costs, where the first term considers total crane travel cost and the second term considers total loading/unloading cost. Constraint (1) ensures that no item (or resource) can be moved before it is moved from its initial position, and constraint (2) ensures that each item being rearranged is always moved from its initial location. Similarly, constraint (3) ensures that no item can be moved after it is moved to its destination location, and constraint (4) guarantees that each item being rearranged is always moved to its destination location. Constraint (5) restricts an item from being moved to a location at full capacity. Constraint (6) ensures that for each possible crane move, the crane can move only between two locations and can carry at most one item. Constraint (7) makes sure that the crane starts from its initial location p at the initial period. Constraint (8) guarantees that each crane move destination is the starting position of the next crane move. For instance, if a crane moves from locations 1 to 5, then the starting position of the next crane move is location 5. Constraint (9) ensures that the last crane move destination in period $t-1$ is the initial position of the first crane move in period t . For example, if the last move in period 3 terminates at location 4, then the first move in period 4 starts at location 4. Constraints (10) and (11) are used to temporarily store items in temporary storage locations. Constraint (10) ensures that an item not at its destination location (i.e. either at its initial location or a temporary location) at the current move will be moved to its destination location later. Constraint (11) allows an item to be moved to a temporary storage space. Constraint (12) ensures that once a crane starts to rearrange items in a period, it does not stop until all items have been rearranged. Constraint (13) restricts workspaces, currently used to perform activities, from being used as temporary storage spaces. Last, the restrictions on the decision variables are given in constraint (14).

It is important to note that the condition which requires an item stored in a temporary storage space to be moved to its assigned location when the location is available and either the crane is at its temporary location or after all the other items has been moved to their assigned location is relaxed for the mathematical model. In other words, an item stored in a temporary storage space is moved to its assigned location when the location is

available and either the crane is at or close to its temporary location or after all the other items has been moved to their assigned locations. This increases the solution space tremendously. Since the CSP is computationally intractable, only small-size CSPs can be solved in reasonable time using the mathematical model. Therefore, in Section 3, HASs are presented for the CSP, but first the solution representation for the HASs is defined, followed by a presentation of a small CSP instance.

In this article, π_t is used to represent an ordered list of items (i.e. a sequence of items) to be moved by the crane at the beginning of period t such that $t = 2, \dots, T$. In other words, π_t is a permutation of items needed to be moved at the beginning of period t and is represented as follows:

$$\pi_t = (\pi_{1t}, \pi_{2t}, \dots, \pi_{n_t}), \text{ for } t=2, \dots, T$$

where π_{it} is the i th item moved by the crane in period t , and π_{n_t} is the last item to be moved by the crane in period t . Hence, the entire solution can be represented as

$$\pi = \{\pi_2, \dots, \pi_T\} = \{(\pi_{12}, \pi_{22}, \dots, \pi_{n_2}), (\pi_{13}, \pi_{23}, \dots, \pi_{n_3}), \dots, (\pi_{1T}, \pi_{2T}, \dots, \pi_{n_T})\}.$$

Note in some cases, where temporary locations are used, the permutation may represent different possible solutions (different crane routes) with different cost unless a specific algorithm is used that leads to a unique crane route. Therefore, the serial method presented by McKendall et al. (2006) is used to obtain the actual crane routes.

To illustrate the CSP, its assumptions, and the above concepts, consider the CSP instance where the layout configuration and distance matrix (d_{ij}) are shown in Figures 1 and 2, respectively. Also, the assignment of activities (and their required resources) to workspaces and the assignment of the idle resources to storage spaces are given in Figure 3. This CSP instance has 3 time periods ($T=3$), 9 resources ($R=9$), 6 locations ($L=6$) and 4 activities. Also, each location has a capacity of 3 items ($C_j=3\forall j$). In period 1, activities 1 and 2 are performed in locations 1 and 2 (or workspaces 1 and 2), respectively. Since resources (4) and (2, 3) are used to perform activities 1 and 2, they are also assigned to locations 1 and 2, respectively (i.e. $l_{41}=1, l_{21}=l_{31}=2$). In addition, idle resources (1, 8, 9), (5, 6) and (7) are assigned to locations 4, 5 and 6, respectively (i.e. $l_{11}=l_{81}=l_{91}=4, l_{51}=l_{61}=5, l_{71}=6$). Therefore, the number of items in location 4 in period 1 is 3 (i.e. $n_{41}=3$). Notice activity 2 is also performed in both periods 1 and 2 at location 2, and activity 3 is performed at location 1 in period 2. Therefore, at the beginning of period 2, location 2 is not available as a temporary location (i.e. $Q_2=\{2\}$), since activity 2 is still being performed there, but location 1 is available. All the items in bold font represent the items which are to be relocated at the beginning of periods 2 and 3. More specifically, items 1 and 4 are reassigned to locations 1 and 4 at the beginning of period 2 (i.e. $A_2=\{1, 4\}, B_2=\{1, 4, 10\}, l_{12}=1, l_{42}=4$). In addition, items (1), (2, 3) and (5, 6) are reassigned to locations 6, 5 and 2, respectively, at the beginning of period 3 (i.e. $A_3=\{1, 2, 3, 5, 6\}, B_3=\{1, 2, 3, 5, 6, 10\}, l_{13}=6, l_{23}=l_{33}=5, l_{53}=l_{63}=2$). As a result, the upper bound for the number of moves at period 3 is the product of 3 and the cardinality of the set A_3 , which is K =the maximum upper bound (i.e. $K=UB_3=3|A_3|=3(5)=15$). Therefore, the CSP determines the order in which the crane moves the items for each of the periods (periods 2 and 3) such that the sum of the travel and loading/unloading costs is minimised.

Figure 1 Layout configuration for the crane sequencing problem instance

Work space 1 (Location 1)	Work space 2 (Location 2)	Work space 3 (Location 3)
Storage space 1 (Location 4)	Storage space 2 (Location 5)	Storage space 3 (Location 6)

Figure 2 Distance matrix d_{ij} for the crane sequencing problem instance

i/j	1	2	3	4	5	6
1	0	1	2	1	2	3
2	1	0	1	2	1	2
3	2	1	0	3	2	1
4	1	2	3	0	1	2
5	2	1	2	1	0	1
6	3	2	1	2	1	0

Figure 3 Item assignments for the crane sequencing problem instance

$t = 1$	4 ($A1$)	2,3 ($A2$)	
	1,8,9	5,6	7
$t = 2$	1 ($A3$)	2,3 ($A2$)	
	4,8,9	5,6	7
$t = 3$		5,6 ($A4$)	
	4,8,9	2,3	1,7

As mentioned previously, there are multiple periods (i.e. $T = 3$) considered in this CSP instance. As a result, the CSP will be used to generate $T - 1$ crane routes (i.e. crane routes for periods 2 and 3). Also, the capacity of the crane is one unit of an item. Assume that the cost per distance unit is \$5 (i.e. $W_1 = 5$), and the cost of loading and unloading each item is \$3 (i.e. $W_2 = 3$). In addition, the initial location of the crane at the beginning of period 2 is location 1 (i.e. $p = 1$) for this specific instance. At the beginning of period 2, the crane needs to move items 1 and 4 to locations 1 and 4, respectively. Hence, there are two (2!) possible sequences for the crane to relocate items 1 and 4 as shown in Table 1. If the sequence of items to be moved is 1–4 (i.e. $\pi_2 = (1, 4)$), the crane first needs to perform an empty move from its initial location (i.e. location 1) to the location of item 1 (i.e. location 4) which gives a travel cost of 1 distance unit. Once the crane arrives at location 4, item 1 is loaded and prepared for moving. Next, the crane moves item 1 from locations 4 to 1 which gives a travel cost of 1 distance unit. Also, item 1 is unloaded.

Therefore, loading/unloading cost for item 1 is \$3. Next, item 4 is considered to be moved, since it is the next item in the sequence. Since the crane is currently at location 1 where item 4 is located, item 4 is loaded and prepared for moving. Then, the crane moves item 4 from locations 1 to 4, which gives a travel cost of 1 distance unit, and it is unloaded. As a result, loading/unloading cost of item 4 is \$3. Therefore, for this sequence, the total loading/unloading cost is \$6, and the total crane travel cost is \$15. Hence, the total cost of the sequence 1–4 (i.e. $\pi_2 = (1, 4)$) is \$21. On the other hand, if the sequence of items to be moved is 4–1 (i.e. $\pi_2 = (4, 1)$), the number of item moves is 3 and the crane travels 7 distance units. Therefore, the total cost of this sequence is \$44. More specifically, for this sequence, the crane starts at location 1 where item 4 is loaded and moved from location 1 to a temporary storage location 5, since its destination location 4 is at full capacity. This is case (1) of temporary storage space usage. Initially, locations 1, 3, 5 and 6 can be used as temporary storage locations, since either no activity is currently being performed at these locations (unlike workspace 2) or the locations are not at full capacities (unlike location 4). However, since location 5 is closest to item 4's destination location (location 4), location 5 is selected as the temporary storage location. Ties are broken arbitrarily. This specific move gives a loading/unloading cost of \$3 and a crane travel cost of \$10. Then the crane moves from locations 5 to 4 (crane travel cost is \$5), item 1 is loaded and moved from locations 4 to 1, and unloaded (crane travel cost = \$5 and loading/unloading cost = \$3). Now crane moves from locations 1 to 5 (crane travel cost = \$10), item 4 is loaded and moved to location 4, and unloaded (crane travel cost = \$5 and loading/unloading cost = \$3). Thus, the sequence 4–1 (i.e. $\pi_2 = (4, 1)$) gives the crane route 1–5–4–1–5–4 indicating the sequence in which the locations are visited by the crane. Nevertheless, sequence 1–4 (i.e. $\pi_2 = (1, 4)$), which cost \$21, is much better than sequence 4–1 (i.e. $\pi_2 = (4, 1)$), which cost \$44.

Table 1 The crane sequencing problem instance solutions at period $t = 2$

Period	Sequence of items moved by the crane	Actual moves of the crane	Distance units the crane travels	Number of item moves	Cost
$t = 2$	1–4	1–4–1–4	3	2	\$21
$t = 2$	4–1	1–5–4–1–5–4	7	3	\$44

Table 2 The crane sequencing problem instance solutions at period $t = 3$

Period	Sequence of items moved by the crane	Actual moves of the crane	Distance units the crane travels	Number of item moves	Cost
$t = 3$	1–2–5–3–6	4–1–6–2–5–2–5–6	10	5	\$65
$t = 3$	1–2–5–3–6	4–1–2–5–2–5–2–6	8	6	\$58

Table 3 Optimal solution for the crane sequencing problem instance

Period	Sequence of items moved by the crane	Actual moves of the crane	Distance units the crane travels	Number of item moves	Cost
$t = 2$	1–4	1–4–1–4	3	2	\$21
$t = 3$	1–2–5–3–6	4–1–2–5–2–5–2–6	8	6	\$58
					\$79

For period 3, the initial location of the crane is the location of the last position of the crane in period 2. As a result, the crane is located at location 4. Also, the number of possible crane sequences is $5! = 120$. In Table 2, one of the crane sequences is considered to illustrate temporary storage location issues, specifically, how different crane routes are formed from the same sequence. Consider crane sequence 1–2–5–3–6 (i.e. $\pi_3 = (1, 2, 5, 3, 6)$). First, the crane moves empty from locations 4 to 1 (crane travel cost = \$5). Second, item 1 is loaded and moved from locations 1 to 2 (crane travel cost = \$5 and loading/unloading cost = \$3), since using location 2 as temporary storage space reduces total cost, which considers case (2) above for temporary storage space usage. Next, items 2, 5, 3 and 6 are loaded and moved in order from locations 2, 4, 2 and 4 to their assigned locations 4, 2, 4 and 2, respectively (crane travel cost = \$20 and loading/unloading cost = \$12). Because all other items had been moved to their destination locations, item 1 is loaded and moved from its temporary location (location 2) to its required location (location 6) and is unloaded (crane travel cost = \$10 and loading/unloading cost = \$3). Therefore, the cost of sequence 1–2–5–3–6 (i.e. $\pi_3 = (1, 2, 5, 3, 6)$) is \$58. However, the sequence does not indicate when an item is moved to/from a temporary storage location, and more than one crane route may be obtained for a specific sequence. More specifically, the sequence 1–2–5–3–6 (i.e. $\pi_3 = (1, 2, 5, 3, 6)$) yields a crane route 4–1–2–5–2–5–2–6 with total cost of \$58. For the same sequence, a crane route of 4–1–6–2–5–2–5–6 may be obtained with a total cost of \$65. Notice in this crane route, item 1 is moved directly to its destination location. Therefore, using temporary storage location can reduce total cost. Recall, the serial method presented by McKendall et al. (2006) is used to obtain an efficient crane route given a sequence.

In Table 3, the optimal solution for this CSP instance is given as the following sequences for periods 2 and 3: 1–4 and 1–2–5–3–6, respectively (i.e. $\pi = \{\pi_2, \pi_3\} = \{(1, 4), (1, 2, 5, 3, 6)\}$). The total cost of this solution is $f(\pi) = \$79 = \$21 + \$58$. The optimal solution was obtained using the binary integer linear program presented above for the CSP and the CPLEX solver, Version 6.6. It took 0.1 minutes on a Pentium IV 2.8 GHz PC. However, for another small CSP with 9 items and 4 periods, it required 8.1 hours of computation time. Therefore, only small-size CSPs are solved in reasonable computation time using the mathematical model. As a result, HASs are presented next to solve large-size CSPs in reasonable time.

3 Methodology

A local neighbourhood search technique starts with an initial solution π , as the current solution, and randomly selects a neighbouring solution π' in the neighbourhood of π , $\pi' \in N(\pi)$. A neighbouring solution π' is obtained by randomly selecting a period t ($1 < t \leq T$) and randomly exchanging the locations of two items in π_t . If $f(\pi') < f(\pi)$, then π' becomes the current solution (set $\pi = \pi'$). Otherwise, the current solution π does not change. This process is repeated until a predefined stopping criterion has been met. This technique is known as the random descent heuristic. Since only improving solutions are accepted during the search process, this local search technique as well as others (e.g. steepest descent, first improvement), converge to the local optimum of the initial solution. Often times, this may result in a low-quality solution. Hence, this method does not guarantee a global optimum, but it is used within many meta-heuristics such as the simulated annealing and HASs which are used to improve the performances of simple

local neighbourhood search techniques. More specifically, the random descent heuristic is used to search the neighbourhoods of solutions, and other components of the proposed HASs will be used to accept non-improving solutions so that the global optimum may be obtained. Next, the proposed HASs are presented for the CSP.

Gambardella, Taillard and Dorigo (1999) presented an ant system optimisation technique, called HAS, for the quadratic assignment problem (HAS-QAP). It is used to improve a set of randomly generated solutions. HAS is a modification of the ant colony optimisation (ACO) heuristics presented in Coloni, Dorigo and Maniezzo (1991) and Dorigo and Gambardella (1997). In both papers, the ACO heuristics were used to construct solutions for the travelling salesman problem (TSP). Dorigo et al., (1996) discuss the applications of ant systems to construct solutions for some well-known problems such as the TSP and vehicle routing problem. Several authors presented ant systems for the quadratic assignment problem (QAP) such as Gambardella, Taillard and Dorigo (1999), Maniezzo and Coloni (1999) and Stutzle and Dorigo (1999). The ACO and HAS heuristics use the idea of how ants search for food and leave a chemical substance called pheromone so that other ants can find the food source to solve combinatorial optimisation problems. The pheromone trails in these ant systems serve as distributed, numerical information which the ants use to probabilistically construct (as in ACO) or improve (as in HAS) solutions to combinatorial optimisation problems.

The artificial ants in the HAS represent solutions which are stochastically modified based on the pheromone trail matrix (i.e. entries in matrix represent the desirability of moving item r in the j th position in period t). The HAS also provides mechanisms to either intensify or diversify the search. McKendall and Shang (2006) modified the HAS-QAP heuristic and presented three HASs for the dynamic facility layout problem (multi-period QAP). The first heuristic is a direct modification of HAS-QAP for the dynamic facility layout problem. The second heuristic uses a simulated annealing heuristic, instead of a random descent heuristic as in HAS-QAP, to improve the solutions obtained after performing pheromone trail swaps. The third heuristic is exactly like the first, except that the random descent heuristic has a look-ahead/look-back strategy. In this article, two HASs are presented for the CSP. The first is similar to the HAS-QAP (use random descent heuristic as a local search technique), and the second uses simulated annealing instead of a random descent heuristic. The first HAS, called HAS I, is briefly summarised below.

Step 1. Set parameters and counters:

M = number of ants (solutions);

Q = parameter used to determine the initial values of the pheromone trail matrix P ;

E = number of pheromone trail exchanges used to modify each CSP solution;

q = the probability that pheromone trail exchange policy 1 will be selected to modify a CSP solution. As a result, policy 2 is chosen with a probability of $1 - q$; the policies are discussed below.

α_1, α_2 = parameters used to weaken and enforce pheromone trails, respectively;

S = number of consecutive iterations, without improving best solution π^* , before implementing diversification strategy;

Set *intense* = 1 (intensification strategy is active);

run_time = run time of heuristic;

Set trail swap counter $e = 0$;

Set iteration counter $it = 0$;

Set iteration without improvement counter $itw = 0$;

Step 2. Randomly generate a set of initial CSP solutions, called Ω . That is, list items to be moved in random order in each period for each ant, π^m , where $m = 1, 2, \dots, M$ and $\pi^m \in \Omega$.

Step 3. Use the random descent method presented above to improve all M solutions $\pi^m \in \Omega$. The set of improved solutions are defined as $\hat{\Omega}$ (i.e. $\hat{\pi}^m \in \hat{\Omega}$).

Set $f(\pi^*) = \min\{f(\hat{\pi}^m) \mid \forall m\}$.

Step 4. Initialise pheromone trail matrix P :

Set entries of P , $p_{rjt} = Q / f(\pi^*) \quad \forall r \in A_t, j, t$, where the entries p_{rjt} measures the desirability of assigning item $r = \hat{\pi}_{it}^m$ (item r in i th position in period t) to be moved to the j th position in $\hat{\pi}_t^m$ of $\hat{\pi}^m$ and A_t is a set of items to be moved.

Step 5. Modify all $\hat{\pi}^m \in \hat{\Omega}$ by using the pheromone trail matrix and one of two policies to obtain another set of solutions $\bar{\Omega}$ (i.e. $\bar{\pi}^m \in \bar{\Omega}$). More specifically, for each $\hat{\pi}^m$:

Set counter $e = 1$;

Step 5a):

If $e < E$,

Randomly select a period t and an item $x \in \hat{\pi}_t^m$, say $\hat{\pi}_{ut}^m$;

Randomly generate a number w between 0 and 1;

If $w \leq q$,

Then use policy 1. That is, select a new position $v \neq u$ for item x such that $p_{xvt} + p_{yut}$ is maximised where item y is currently in position v in period t (i.e. $y = \hat{\pi}_{vt}^m$);

Else,

Use policy 2. That is, $v \neq u$ is chosen with a probability of $p_{xvt} + p_{yut} / \sum_{v \neq u} (p_{xvt} + p_{yut})$ where y is currently in position v in period t (i.e. $y = \hat{\pi}_{vt}^m$);

Exchange the locations of items x and y in $\hat{\pi}_t^m$;

Set $e = e + 1$, and go to the beginning of Step 5a);

Else, E trail swaps have been performed;

After all solutions $\hat{\pi}^m \in \hat{\Omega}$ have been modified such that the new set of solutions $\bar{\Omega}$ is obtained, go to Step 6.

Step 6. Use the random descent heuristic to improve solutions $\bar{\pi}^m$ in the set $\bar{\Omega}$ and obtain the set of solutions $\tilde{\Omega}$ ($\tilde{\pi}^m \in \tilde{\Omega}$).

If $\exists \tilde{\pi}^m \in \tilde{\Omega}$ such that $f(\tilde{\pi}^m) < f(\pi^*)$, then

set $\pi^* = \tilde{\pi}^m$ where $f(\pi^*) = \min\{f(\tilde{\pi}^m) \mid \tilde{\pi}^m \in \tilde{\Omega}\}$;

set $itw = 0$;
 Else, $itw = itw + 1$;

Step 7. If $itw = S$, set $itw = 0$, and randomly generate a set of $M - 1$ solutions. The $M - 1$ solutions and π^* make up a diverse set of solutions define as Ω . Then go to Step 3;

Step 8. If $intense = 1$,
 Set $temp = 0$;
 For each m , if $f(\hat{\pi}^m) > f(\tilde{\pi}^m)$,
 set $\hat{\pi}^m = \tilde{\pi}^m$ and $temp = 1$;
 Set $intense = temp$;
 Else, set $\hat{\Omega} = \tilde{\Omega}$.
 If $itw = 0$, then set $intense = 1$;

Step 9. Update pheromone trail matrix as discussed below.

Step 10. If heuristic run time $> run_time$, then terminate the heuristic.
 Else, go to Step 5.

In Step 3, the randomly generated solutions obtained in Step 2 are improved using the random descent heuristic. Based on the value of the best solution obtained in Step 3, the pheromone trail matrix is initialised in Step 4. In Step 5, a period t and an item x in position u are randomly selected. The order in which the crane will move this item is exchanged with another item y in position v which is selected based on one of two policies. The first policy uses the pheromone trail matrix to select item y in location v based on the most desirable exchange between item x in position u and all other items. The second policy selects an item y in location v randomly, but there is a higher probability that item y with the largest $p_{xvt} + p_{yut}$ will be accepted. After E exchanges using the pheromone trail matrix, the solutions are improved using the random descent heuristic in Step 6. If S iterations are performed without improving the best found solution, the diversification strategy is implemented in Step 7. This strategy is used to explore diverse areas of the solution space by taking the best solution found π^* , randomly generating $M - 1$ solutions to obtain a different set of solutions (Ω), and continuing the heuristic at Step 3. Otherwise, the intensification strategy is invoked in Step 8, if it is active (i.e. $intense = 1$). If $intense = 0$, then the intensification strategy is not active. Intensification is used to explore promising areas of the solution space. In other words, the set of solutions $\hat{\Omega}$ for the next iteration is obtained based on the intensification strategy. More specifically, initially the intensification strategy is activated, which means that the better solution between $\hat{\pi}^m$ and $\tilde{\pi}^m$ is used as the starting solution at the next iteration ($\hat{\pi}^m$) for each solution m . This strategy remains active if at least one solution m is improved during an iteration. Otherwise, the intensification strategy is inactive, and the solution at the next iteration for each ant is $\tilde{\pi}^m$ (i.e. $\hat{\pi}^m = \tilde{\pi}^m \forall m$). If the strategy is inactive, then it becomes active when the best solution π^* is improved. In step 9, the pheromone trail matrix P is updated such that only the best solution found so far is permitted to deposit pheromone. The pheromone trail matrix is updated according to Dorigo and Gambardella (1997). Set

$$p_{rjt} = (1 - \alpha_1)p_{rjt} + \alpha_2\delta(\pi_{rjt}) \quad \forall r \in A_t, j, t$$

where

$$\delta(\pi_{rjt}) = \begin{cases} 1/f(\pi^*) & \text{if } \pi_{rjt} \in \text{best solution } \pi^* \\ 0 & \text{otherwise} \end{cases}.$$

The first term in the expression, $(1 - \alpha_1)p_{rjt}$, is used to weaken the pheromone trails where $0 < \alpha_1 < 1$ is a parameter used to control the evaporation rate of the trails. A value close to zero indicates the trails will remain active for a longer time, and a value close to one indicates a high degree of evaporation and a shorter memory of the system. However, the expression $\alpha_2\delta(\pi_{rjt})$ is used to strengthen the pheromone trails where α_2 is a parameter used to control the reinforcement of the pheromone trails corresponding to the best solution π^* . Using only the best solution π^* speeds up the convergence of the heuristic (Gambardella, Taillard and Dorigo, 1999). In step 10, the heuristic is terminated, if heuristic run time is at least *run_time* minutes.

The second HAS, called HAS II, uses a simulated annealing heuristic instead of a random descent heuristic, to improve solutions initially and after performing E pheromone trail exchanges. McKendall and Shang (2006) presented a similar HAS for the dynamic facility layout problem, which out-performed the HAS with a random descent heuristic. Therefore, it is applied to the CSP presented in this article. More specifically, the random descent heuristic presented in Steps 3 and 6 is replaced by the following simulated annealing heuristic.

Step 1. Set parameters and counters:

T_0 = initial temperature;

ϵ = cooling ratio ($\epsilon = 0.95$);

H_0 = number of iterations performed at initial temperature epoch length, ($H_0 = T$ where T = number of periods);

γ = parameter used to increase epoch length ($\gamma = 0.04$);

T_f = final temperature ($T_f = 0.1$);

π^m = current solution;

π^* = best solution found;

Set current temperature $Temp = T_0$;

Set current epoch length $H = H_0$;

Set iteration counter at each temperature $ic = 0$;

Set $\pi^* = \pi^m$;

Step 2. Set $ic = ic + 1$;

Generate a neighbouring solution π' in the neighbourhood of current solution π^m by random pairwise exchange (i.e. $\pi' \in N(\pi^m)$).

Step 3. If $f(\pi') \leq f(\pi^*)$, then set $\pi^* = \pi'$, $\pi^m = \pi'$, and go to step 4;

Else,

if $f(\pi') \leq f(\pi^m)$, then set $\pi^m = \pi'$;

Else, if $\text{rand}(0, 1) < \exp(-[f(\pi') - f(\pi^m)]/Temp)$, set $\pi^m = \pi'$;

Step 4. If $ic < H$, then go to step 2;
 Else, set $Temp = \epsilon Temp$ and $H = H(1 + \gamma)$;
 If $Temp > T_f$, set $ic = 0$ and go to step 2;
 Else, terminate heuristic.

This simulated annealing heuristic is repeated for each ant. First, the temperature $Temp$ has been set to the initial value T_0 at the beginning. The initial temperature T_0 was determined by using the formula $\exp(-[f(\pi') - f(\pi^m)]/T_0) = 0.25$ and $f(\pi') - f(\pi^m) = 0.1 * f(\pi^m)$, where 0.10 and 0.25 were obtained experimentally. In other words, $T_0 = -0.10 * f(\pi^m) / \ln(0.25)$ and the initial probability of accepting a non-improving solution, which is 10% worse than solution π^m , is 25%. Then, $Temp$ is decreased by $\epsilon Temp$ (i.e. $Temp = \epsilon Temp$) at every temperature reduction. In other words, after performing H number of random pairwise exchanges, the temperature is reduced by $\epsilon Temp$. The epoch length H is the number of iterations (random pairwise exchanges) performed at each temperature. At the initial temperature T_0 , the number of random pairwise exchanges is the number of periods (i.e. $H_0 = T$). Otherwise, the number of random pairwise exchanges is $H(1 + \gamma)$, as in the simulated annealing heuristic presented by Bouleimen and Lecocq (2003) for the resource constrained project scheduling problem. The process is continued until a predefined stopping temperature has been met (i.e. $Temp \leq T_f$). Initially, the probability of allowing a non-improved move, $\exp(-[f(\pi') - f(\pi^m)]/Temp)$ will be relatively large. However, as $Temp$ decreases, this probability also decreases. Furthermore, ϵ , γ and H_0 were set experimentally.

4 Computational experiments and analysis

Two test data sets are presented in this article. Data set 1 is randomly generated based on the following factors: the total number of items, R ; the number of periods, T and the number of locations, L . Hence, there are $L/2$ workspace locations and $L/2$ storage space locations and distance between any two locations i and j are defined in a distance matrix D . The number of items to be moved (NOIM) is uniformly distributed in $[1, R]$; the number of destination locations (NODL) is uniformly distributed in $[1, NOIM]$, and the number of items in destination location is uniformly distributed in $[1, Cap]$. For instance, if $NOIM = 3$ for a specific period t , then NODL is uniformly distributed in $[1, 3]$, say 2. Therefore, 3 items in period t is moved and assigned to two locations. First, randomly generate 2 locations uniformly distributed in $[1, L]$. Next, randomly assign the 3 items to two locations. This is done for each period t . In the first data set, the number of items R is 9, the number of locations L is 6 and the number of periods T is from 2 to 7 as shown in Table 4.

Data set 2 was obtained by solving the dynamic space allocation problem (DSAP) instances presented in McKendall et al., (2005). The DSAP assigns activities and their required items (resources) to workspaces and idle items to storage spaces with respect to minimising the sum of the distances the items travel. Therefore, the output of the DSAP can be used as input data for the CSP. This data set has 96 test problems. The sizes of the instances in this data set are shown in Table 5.

For HAS I and II heuristics, the following parameters setting are used. The numbers of ants (M) are set to 5, 10, 12 and 15 for group 1 (i.e. data set 1 and all problems with $R = 9$ in data set 2), 2 (i.e. all problems with $R = 18$ in data set 2), 3 (i.e. all problems with $R = 30$ in data set 2) and 4 (i.e. all problems with $R = 48$ in data set 2), respectively. $E = R * T$, $\alpha_1 = 0.1$ and $\alpha_2 = Q = T * 10^3$, $q = 0.7$, $S = R * T$. Furthermore, the settings for the simulated annealing heuristic embedded within HAS II are $T_0 = -0.10 * f(\pi^m) / \ln(0.25)$, as well as ϵ , γ , T_f and H_0 were set to 0.95, 0.04, 0.1 and number of periods (T), respectively.

In all experiments, a Pentium IV 2.4GHz PC was used to solve the CSP instances from data sets I and II using the proposed heuristics. The heuristics were coded using the C++ programming language under Windows XP operating system. The proposed HAS heuristics were compared with the simulated annealing heuristic presented in McKendall et al. (2006) except that the objective function includes both crane travel cost and loading/unloading cost. In order to make fair comparisons, the proposed HAS and simulated annealing heuristics ran for the same amount of computational time for each CSP problem instance.

Table 4 Problem sizes for data set 1

<i>Pr. number</i>	<i>R</i>	<i>L</i>	<i>T</i>
P01–P04	9	6	2
P05–P08	9	6	3
P09–P12	9	6	4
P13–P16	9	6	5
P17–P20	9	6	6
P21–P24	9	6	7

Table 5 Problem sizes for data set 2

<i>Pr. number</i>	<i>R</i>	<i>L</i>	<i>T</i>
P01–P08	9	6	10
P09–P16	9	6	15
P17–P24	9	6	20
P25–P32	18	12	10
P33–P40	18	12	15
P41–P48	18	12	20
P49–P56	30	20	10
P57–P64	30	20	15
P65–P72	30	20	20
P73–P80	48	32	10
P81–P88	48	32	15
P89–P96	48	32	20

Table 6 Computational results for generated data set 1

<i>Pr. number</i>	<i>Optimal solution</i>		<i>Simulated annealing</i>	<i>HASI/II</i>	<i>Heuristic time</i>
	<i>Final</i>	<i>Time</i>			
P01	57	0.01	57	57	0.00
P02	83	0.05	83	83	0.00
P03	62	0.03	62	62	0.00
P04	88	0.11	88	88	0.01
P05	65	0.08	65	65	0.01
P06	119	7.60	119	119	0.01
P07	72	0.02	72	72	0.01
P08	91	0.23	91	91	0.02
P09	121	0.33	121	121	0.02
P10	150	19.42	150	150	0.02
P11	118	2.02	118	118	0.02
P12	150	19.53	150	150	0.01
P13	104	0.10	104	104	0.01
P14	189	1527.36	189	189	0.02
P15	114	0.12	114	114	0.02
P16	229	8.48	229	229	0.02
P17	145	0.12	145	145	0.02
P18	278	1947.32	278	278	0.02
P19	157	60.97	157	157	0.02
P20	246	1014.73	246	246	0.03
P21	169	2.93	169	169	0.03
P22	310	2915.87	310	310	0.04
P23	197	318.12	197	197	0.04
P24	347	2755.59	347	347	0.03

The optimal solutions in Table 6 were obtained for all of the test problems in data set 1 and were obtained using the mathematical model presented previously and the CPLEX solver, Version 6.6. The following CPLEX parameter settings were used: strong branching for variable selection, depth first for node selection, dual simplex for start algorithm and the rest were default settings. Also, their computational times are listed. The bold numbers represent the optimal objective function values (OFVs). All times in the tables are given in minutes. As a result, both HAS and simulated annealing heuristics obtained the optimal solutions for all 24 test problems. Notice the run times for the heuristics are much less than the run times for the mathematical model with CPLEX. The main reason for using this data set was to test the mathematical model.

The second data set is used to further verify the speed and robustness of the proposed heuristics. This data set has test problems with 9–48 items, 6–32 locations 10, 15 and 20 periods as shown in Table 5. Compared to data set 1, data set 2 is much larger and has much larger problems, which cannot be solved by using exact methods in reasonable time. simulated annealing, HAS I and II, ran 15 times each with the same randomly generated initial solutions. Data set 2 is divided into four groups, each of which has

24 test problems. From groups 1 to 4, the sizes of the test problems are increased. The test problems in group 1 are relative small, groups 2 and 3 have medium size test problems, and the test problems in group 4 are the largest.

For data set 2, Table 7 shows the number of best solutions obtained by each of the proposed heuristics, and Table 8 gives the average percent each heuristic is away from best found solutions. For the first group (i.e. test problems 01–24), the test problems have 9 resources, 6 locations and 3–5 periods, which are the smaller size problems. All heuristics performed well for this group of test problems. More specifically, HAS I obtained the best found solution 23 times, HAS II, 22 times, and simulated annealing 23 times. The detail results for the first group are given in Table 9. Note, the bolded OFVs are the best found OFVs.

However, the HAS and simulated annealing heuristics perform differently for the other three groups of test problems. More specifically, HAS I obtained the best found solution 12, 7 and 6 times, HAS II obtained best 12, 16 and 17 times, and simulated annealing obtained best 16, 3 and 5 times. See the results for each test problem in groups 2, 3 and 4 in Tables 10, 11 and 12, respectively.

Overall in Table 7, HAS II outperformed the other two heuristics since it obtained 67 best solutions of 96 test problems, which is the most. Moreover, HAS I performed slightly better than simulated annealing since HAS I obtained best found 48 times and simulated annealing obtained best for 47 of 96 test problems. Since this information may be misleading, the average percent each heuristic is away from the best found solution is given in Table 8. More specifically, the average percent the HAS II, HAS I and simulated annealing heuristics are above the best found solutions are 0.20, 0.28 and 0.74%, respectively. In other words, although simulated annealing and HAS I obtained about the same number of best found solutions, HAS I obtained solutions closer to the best found than simulated annealing, on average.

Tables 9–12 list detail results and computational times for each group of test problems in data set 2. The bold numbers are the best found OFVs, and times are given in minutes. Recall, all heuristics ran for the same amount of time with 15 runs. Although the HAS II heuristic is a combination of the HAS I with simulated annealing, it outperformed the HAS I and simulated annealing heuristics with the same run times. Although this integration of heuristics increases the computational complexity, the results show that HAS II improved HAS I. In other words, even though an iteration in the HAS II require more computation time, less iterations need to be performed to obtain superior solutions. Hence, HAS II is the preferred heuristic.

Table 7 Number of best solutions found obtained by the proposed heuristics

<i>No</i>	<i>HASI</i>	<i>HASII</i>	<i>Simulated annealing</i>
P01–P24	23	22	23
P25–P48	12	12	16
P49–P72	7	16	3
P73–P96	6	17	5
Total	48	67	47
Percent	50.0	69.8	49.0

Table 8 Average percent each heuristic is away from the best found solutions

<i>No</i>	<i>HASI %</i>	<i>HASII %</i>	<i>Simulated annealing %</i>
P01–P24	0.09	0.12	0.09
P25–P48	0.45	0.34	0.42
P49–P72	0.32	0.21	1.35
P73–P96	0.27	0.14	1.11
Average	0.28	0.20	0.74

Table 9 Computational results for data set 2 with $R = 9$ and $L = 6$

<i>No</i>	<i>Initial cost</i>	<i>HAS</i>			<i>Simulated annealing</i>	<i>Best of all</i>	<i>Time</i>
		<i>HASI</i>	<i>HASII</i>	<i>Best</i>			
1	205	205	205	205	205	205	0.03
2	359	339	339	339	339	339	0.05
3	243	233	233	233	233	233	0.03
4	324	314	314	314	314	314	0.05
5	213	213	213	213	213	213	0.03
6	385	375	375	375	375	375	0.05
7	274	254	254	254	254	254	0.05
8	369	346	346	346	346	346	0.03
9	410	390	390	390	390	390	0.07
10	655	645	645	645	645	645	0.07
11	462	442	442	442	442	442	0.07
12	720	637	647	637	637	637	0.07
13	403	373	373	373	373	373	0.05
14	595	585	585	585	585	585	0.05
15	566	460	460	460	460	460	0.07
16	733	670	670	670	670	670	0.07
17	467	467	467	467	467	467	0.08
18	856	806	806	806	806	806	0.08
19	651	621	621	621	621	621	0.10
20	1,031	938	918	918	938	918	0.08
21	711	615	615	615	615	615	0.08
22	926	853	853	853	853	853	0.08
23	848	692	702	692	692	692	0.10
24	981	786	786	786	786	786	0.08

Table 10 Computational results for data set 2 with $R = 18$ and $L = 12$

<i>No</i>	<i>Initial cost</i>	<i>HAS</i>			<i>Simulated annealing</i>	<i>Best of all</i>	<i>Time</i>
		<i>HASI</i>	<i>HASII</i>	<i>Best</i>			
25	610	490	490	490	490	490	0.45
26	813	632	622	622	622	622	0.43
27	982	513	513	513	513	513	0.50
28	1,038	730	730	730	730	730	0.47
29	544	441	441	441	441	441	0.43
30	783	660	660	660	660	660	0.45
31	791	557	537	537	537	537	0.48
32	1,167	827	837	827	824	824	0.48
33	1,107	794	794	794	794	794	0.70
34	1,434	1,065	1,065	1,065	1,065	1,065	0.70
35	1,249	856	880	856	896	856	0.77
36	1,670	1,214	1,212	1,212	1,193	1,193	0.73
37	895	687	682	682	692	682	0.68
38	1,312	1,085	1,082	1,082	1,079	1,079	0.70
39	1,276	819	817	817	827	817	0.75
40	1,793	1,241	1,238	1,238	1,229	1,229	0.75
41	1,587	1,159	1,160	1,159	1,171	1,159	0.93
42	1,844	1,408	1,409	1,408	1,409	1,408	0.95
43	1,765	1,275	1,273	1,273	1,272	1,272	1.05
44	2,566	1,864	1,858	1,858	1,860	1,858	1.03
45	1,326	1,016	1,014	1,014	1,013	1,013	0.90
46	1,798	1,457	1,457	1,457	1,455	1,455	0.93
47	1,958	1,291	1,296	1,291	1,313	1,291	1.05
48	2,562	1,828	1,835	1,828	1,830	1,828	1.02

Table 11 Computational results for data set 2 with $R = 30$ and $L = 20$

<i>No</i>	<i>Initial cost</i>	<i>HAS</i>			<i>Simulated annealing</i>	<i>Best of all</i>	<i>Time</i>
		<i>HASI</i>	<i>HASII</i>	<i>Best</i>			
49	1,085	816	802	802	812	802	2.82
50	1,210	936	943	936	951	936	2.87
51	1,162	799	798	798	809	798	3.12
52	1,892	1,317	1,315	1,315	1,328	1,315	3.05
53	1,164	791	796	791	781	781	2.78
54	1,351	943	940	940	932	932	2.83
55	1,404	851	847	847	885	847	3.15
56	2,002	1,304	1,297	1,297	1,305	1,297	3.12
57	1,576	1,107	1,106	1,106	1,117	1,106	4.28
58	2,047	1,530	1,534	1,530	1,525	1,525	4.30
59	2,166	1,335	1,330	1,330	1,363	1,330	4.88
60	2,166	1,334	1,336	1,334	1,356	1,334	4.82

Table 11 Computational results for data set 2 with $R = 30$ and $L = 20$ (continued)

No	Initial cost	HAS			Simulated annealing	Best of all	Time
		HASI	HASII	Best			
61	1,677	1,134	1,138	1,134	1,154	1,134	4.43
62	2,416	1,697	1,693	1,693	1,706	1,693	4.38
63	2,494	1,638	1,632	1,632	1,681	1,632	5.03
64	3,775	2,411	2,409	2,409	2,441	2,409	4.97
65	2,265	1,544	1,539	1,539	1,553	1,539	5.90
66	3,094	2,224	2,224	2,224	2,242	2,224	5.90
67	3,025	2,074	2,075	2,074	2,107	2,074	6.80
68	4,364	3,031	3,025	3,025	3,051	3,025	6.70
69	2,728	1,708	1,706	1,706	1,720	1,706	6.22
70	3,646	2,472	2,469	2,469	2,525	2,469	6.20
71	3,531	2,425	2,436	2,425	2,447	2,425	7.18
72	5,223	3,547	3,547	3,547	3,642	3,547	7.15

Table 12 Computational results for data set 2 with $R = 48$ and $L = 32$

No	Initial cost	HAS			Simulated annealing	Best of all	Time
		HASI	HASII	Best			
73	2,022	1,347	1,340	1,340	1,373	1,340	17.28
74	2,101	1,543	1,540	1,540	1,548	1,540	17.23
75	2,640	1,736	1,729	1,729	1,705	1,705	18.93
76	3,636	2,560	2,556	2,556	2,538	2,538	18.48
77	2,024	1,200	1,200	1,200	1,215	1,200	16.22
78	2,436	1,634	1,636	1,634	1,645	1,634	16.25
79	3,001	1,710	1,706	1,706	1,720	1,706	18.52
80	4,323	2,496	2,493	2,493	2,483	2,483	18.57
81	2,933	1,984	1,984	1,984	2,034	1,984	27.50
82	4,445	2,936	2,927	2,927	2,937	2,927	27.17
83	4,924	2,910	2,907	2,907	2,962	2,907	30.27
84	6,585	4,008	4,005	4,005	4,054	4,005	29.85
85	3,241	1,976	1,976	1,976	1,978	1,976	26.78
86	4,787	3,102	3,104	3,102	3,087	3,087	26.97
87	5,082	2,914	2,910	2,910	3,007	2,910	30.62
88	6,896	4,451	4,440	4,440	4,444	4,440	30.77
89	4,387	2,911	2,901	2,901	2,936	2,901	36.49
90	6,202	3,937	3,939	3,937	4,010	3,937	39.44
91	6,883	4,450	4,448	4,448	4,517	4,448	41.63
92	9,285	5,903	5,912	5,903	5,902	5,902	44.98
93	4,866	2,870	2,862	2,862	2,925	2,862	40.66
94	5,982	3,968	3,960	3,960	4,070	3,960	41.78
95	7,741	4,767	4,767	4,767	4,777	4,767	49.56
96	10,542	6,523	6,521	6,521	6,655	6,521	48.17

5 Conclusions

In this article, a mathematical model and two HASs were presented to solve the CSP. More specifically, a binary integer linear programming model was presented for the CSP. Since the CSP is computationally intractable and only small-size problems can be solved using the mathematical model, two HASs were presented for the CSP. The first HAS (HAS I) is a direct modification of HAS-QAP for the CSP. The second HAS (HAS II) uses a simulated annealing heuristic, instead of a random descent heuristic as in HAS I. The proposed heuristics performed well on two data sets presented in the literature.

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