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Experimental characterization of fuel cell gas turbine power system and determination of optimal trajectories of operation using a model predictive controller

Bernardo Restrepo
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**EXPERIMENTAL CHARACTERIZATION OF FUEL CELL GAS TURBINE POWER SYSTEM
AND DETERMINATION OF OPTIMAL TRAJECTORIES OF OPERATION USING A MODEL
PREDICTIVE CONTROLLER**

Bernardo Restrepo

**Dissertation Submitted to the
College of Engineering and Mineral Resources at
West Virginia University
in partial fulfillment of the
requirements for the degree of**

Doctor of Philosophy in Mechanical Engineering

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Morgantown, West Virginia

2011

Abstract

EXPERIMENTAL CHARACTERIZATION OF FUEL CELL GAS TURBINE POWER SYSTEM AND DETERMINATION OF OPTIMAL TRAJECTORIES OF OPERATION USING A MODEL PREDICTIVE CONTROLLER

Bernardo Restrepo

Hybrid Performance (HyPer) hardware simulation facility installed in the National Energy Technology Laboratory (NETL), U.S. Department of Energy is a hardware in the loop technology that couples a real recuperated gas turbine cycle with a Solid Oxide Fuel Cell (SOFC) Model. The system is composed of a gas turbine, two high-efficiency recuperators, three different bypasses, and several associated pressure vessels and pipes that represent the volumes and flow impedance of the fuel cell. The real-time fuel cell model is used to control a gas burner which replicates the thermal output of a SOFC. Control of thermal energy in and out of the fuel cell, especially during load transients, is fundamental to maintain safe fuel cell/gas turbine operation. This is achieved in the HyPer system by diverting air around the fuel cell system. Three bypass sub-systems are employed for this purpose. A full factorial experimental design and a replicated fractional factorial design are carried out in the HyPer system. The HyPer system has been experimentally tested mostly using a one factor at a time analysis.

The objectives of this work are first, to expand the envelope of operation by performing a full factorial experimental design and a replicated fractional factorial experimental design to enlarge characterization of the HyPer system. A 3^4 factorial design is selected to study the effect of four factors (input variables) and their interactions: cold air, hot air, bleed air bypass valves, and the electric load on different parameters such as cathode and turbine inlet temperatures, pressure and mass flow. The results obtained show the effects over the response variables of interaction and nonlinearities between the factors in the range of the operation selected in this experiment. This work describes the methodology, strategy, and some results of these experiments that enhance the understanding of the complex thermo-fluid characteristics of hybrid operation. Second, a Model Predictive Control strategy is used to design a controller that allows the system to regulate these factors and to control the different parameters of interaction between both sub-systems, based on models obtained by system identification techniques. Different off-design scenarios of operation have been tested to confirm the estimated implementation behavior of the plant-controller dynamics.

“If you can’t explain it simply, you don’t understand it well enough” – Albert Einstein.

To my inspirational and spiritual angels Javier Anibal and Daniel Dario Restrepo, to my unconditional and supportive sister Rubiela Restrepo, to my patient and beloved parents Adelfina Torres and Heriberto Restrepo, to my funny, joyful, and lovely wife Miriam Baez and to my beautiful and sweetie princess daughter Miriam Rubi Restrepo.

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Table of Contents

Abstract	ii
Acknowledgments	v
List of Figures	xi
List of Tables	xvii
Nomenclature	xviii
Chapter 1 Introduction	1
1.1 Introduction	1
1.2 The Hybrid Performance Research Facility	1
1.2.1 Hybrid Components	2
1.2.1.1 Solid Oxide Fuel Cells (SOFC)	2
1.2.1.2 Gas Turbine Cycle	3
1.2.1.3 HyPer Theoretical Configuration	6
1.2.1.4 HyPer Facility (Real Configuration)	7
1.3 Historical Development of Hybrid Systems	9
1.3.1 Hybrid Configuration Review	9
1.3.2 Dynamic Simulation	10
1.3.3 Empirical Characterization	13
1.3.4 Control Strategies	15
1.3.5 Model Predictive Control (MPC) Review	19
1.3.6 MPC Fuel Cell and Gas Turbine Applications	23
1.4 Research Needs	24
1.4.1 New HyPer Characterization	24
1.4.1.1 Motivation of the Design of Experiments on the HyPer Facility	25
1.4.2 Multi-Input Multi-Output Approach	32
1.5 Design of Experiments	32
1.6 Model Predictive Control Selection	33

1.6.1	MPC Motivation	34
1.6.1.1	Input Constraints Handling.....	34
1.6.1.2	Input-Output Models	34
1.6.1.3	The Receding Horizon Idea	35
1.6.1.4	Computing the Optimal Inputs	35
1.6.1.5	Multivariable Control Systems	36
1.6.1.6	The Capacity of Anticipation	36
1.6.1.7	The Reference Trajectory	37
1.7	Research Objectives	37
1.7.1	Specific Objectives	38
1.8	Contributions to State of the Art	39
Chapter 2	Control Strategy	40
2.1	Control Strategy	40
2.1.1	Dynamics of FC Load Increase.....	40
2.1.1.1	Control Action for FC Load Increase	41
2.1.2	Dynamics of FC Load Decrease	42
2.1.2.1	FC load Decrease Control Action.....	43
2.1.3	Anode-Cathode Pressure Differential	45
Chapter 3	Design of Experiments	47
3.1	Experimental Method and Results	47
3.1.1	Experimental Variables.....	48
3.1.1.1	Input Selection and Levels of the Treatment	48
3.1.1.2	Cold Air Valve.....	48
3.1.1.3	Hot Air Valve	50
3.1.1.4	Bleed Air Valve	50
3.1.1.5	Electric Load.....	51

3.1.1.6	Output Parameters	51
3.2	Test Procedure	55
3.2.1	Experimental Constraints	56
3.2.2	Data Processing	56
3.2.3	Summary of Bypass Testing	57
3.3	Results	58
3.3.1	Full Factorial Design Results	58
3.4	Analysis of the Design of Experiments	64
3.4.1.1	Statistical Fuel Flow Analysis (FT432)	64
3.4.1.2	Regression Equation for Fuel Flow (FT432)	68
3.4.1.3	Cathode Inlet Mass Flow Statistical Analysis	71
3.4.1.4	Regression Analysis of the Cathode Air Mass Flow (FT380)	73
3.4.2	Cathode and Turbine Inlet Temperature Statistics (TE326A and TE350, respectively)	75
3.4.2.1	Regression Analysis of the Cathode Inlet Temperature (TE326)	80
3.4.2.2	Regression Analysis of the Turbine Inlet Temperature (T350)	82
3.4.3	Fractional Factorial Replication	84
3.4.4	Replication Results	85
3.5	Operating Envelope	88
3.5.1	Fuel Flow Envelope	89
3.5.2	Cathode Air Mass Flow Envelope	92
3.5.3	Cathode Inlet Temperature Envelope	94
3.5.4	Turbine Inlet Temperature Envelope	96
Chapter 4	Model Predictive Control	99
4.1	Introduction	99
4.2	Description of MPC	100
4.3	MPC Logic and Algorithm Sequence	103

4.3.1	Plant Inputs and Outputs	105
4.3.1.1	Plant and Horizons.....	116
4.3.1.2	Anticipation Action	117
4.3.1.3	Weight Tuning.....	117
4.3.1.4	Blocking.....	117
4.3.2	Performance Index of the Optimization Problem with Constraints.....	118
4.3.3	Model Predictive Control Computation.....	120
4.4	System Identification and Control System Simulation	120
4.4.1	System Identification Plant Modeling	120
4.4.2	Data Quality Requirements.....	121
4.4.3	Limitations working with linear models	121
4.4.4	Selecting Black-Box Model Structure and Order	121
4.4.4.1	Order Selection, Fit Criterion and Time Delay	124
4.4.5	Polynomial (ARX) Model Identification and MPC.....	125
4.4.5.1	HyPer Auto Regressive Exogenous (ARX) Identification.	127
4.4.5.2	MPC Design Using On-line ARX Models	135
4.4.6	State-Space Identification and MPC.....	139
4.4.6.1	State-Space Identification: Example 1.	139
4.4.6.2	State-Space Identification: Example 2.	147
4.4.6.3	MPC Design Using On-line State-Space Models.....	152
4.4.7	Transfer Functions Model and MPC.....	158
4.4.7.1	MPC Design Using Transfer Functions Model	161
Chapter 5	Conclusions	166
	Design of Experiments.....	167
	Model Predictive Control.....	169
Chapter 6	Contribution Overview and Recommendations	172

Contribution Overview	172
6.1 Future Work and Recommendations.....	173
References.....	176
Appendix A ASME Fuel Cell Conference Paper	181
Appendix B Design of Experiments Test Plan Purpose	195
Appendix C Design of Experiments Results	197
Appendix D Fuel Flow Regression Analysis.....	202
Appendix E Fuel Flow Standard Deviation.....	204
Appendix F Cathode Air Flow Regression Analysis	205
Appendix G Cathode Inlet Temperature Regression Analysis	207
Appendix H Turbine Inlet Temperature Regression Analysis.....	209
Appendix I Replicate using a 1/9 fractional factorial experiment.....	211
Appendix J Matlab Program for State Space Identification	212
Appendix K Table of Equivalence Ratio	213
Appendix L Guidelines for Determining Model Parameters	214
Appendix M System Identification –advice”.....	217
Appendix N Simulink Representation of the MPC and transfer functions Model	219

List of Figures

Figure 1.1 Schematic of a solid oxide fuel cell (SOFC)	3
Figure 1.2 Schematic of a gas turbine cycle.	4
Figure 1.3 (a) T-s diagram for the ideal gas turbine cycle and (b) thermal efficiency as a function of the pressure ratio (Reproduced from [72]).....	5
Figure 1.4 Schematic of a recuperated gas turbine cycle.....	5
Figure 1.5 Theoretical layout of direct fired integrated FC/GT hybrid cycle.....	6
Figure 1.6 HyPer research facility overview (Hardware in the loop layout).....	7
Figure 1.7 Diagram of the pressurized 220 kW SOFC/GT hybrid system (Roberts et al., 2006 [12]).....	12
Figure 1.8 Bottoming SOFC/GT hybrid system with variable speed and supplementary oxidizer fuel (Roberts et al., 2006 [21]).....	16
Figure 1.9 Layout diagram for atmospheric SOFC/GT system (Junker et al., 2006 [24]).	18
Figure 1.10 Basic structure of MPC (Camacho et al. 2007 [43]).	19
Figure 1.11 Window of MPC design at each present time	20
Figure 1.12 MPC (DMCPlus) control structure (Goodhart et al. 2000 [33]).	22
Figure 1.13 Fuel mass flow (Natural Gas) as a function of the bypassed flow (OFAT) at 0, 25, and 50 kW generator load (Tucker et al. 2009 [4]).....	26
Figure 1.14 Cathode air mass flow as a function of the bypassed flow (OFAT) at 0, 25, and 50 kW generator load (Tucker et al. 2009 [4]).	26
Figure 1.15 Turbine inlet temperature as a function of the bypassed flow (OFAT) at 0, 25, and 50 kW generator load (Tucker et al. 2009 [4]).....	27
Figure 1.16 Pressure drop as a function of the bypassed flow (OFAT) at 0, 25, and 50 kW generator load (Tucker et al. 2009 [4]).	27
Figure 1.17 Bypass valve positions and compressor stall line at 50 kW generator load (Tucker et al. 2009 [4]).	28
Figure 1.18 Envelope of cathode inlet temperature as function of cathode inlet mass flow (Tucker et al. 2009 [4]).	29
Figure 1.19 Cathode inlet air flow as a function of CA valve position and electric load.	30

Figure 1.20 Envelope of the cathode air mass flow as a function of CA and HA valve position (BA=90%, and EL=50kW).	31
Figure 1.21 Cathode air mass flow contour plot as a function of CA and HA valve position (BA=90%, and EL=50kW).	31
Figure 1.22 Anticipation response of turbine speed for different setting of the control horizon (m samples) for a prediction horizon of 12 samples (obtained from [38]).	36
Figure 2.1 Dynamic effect of the hybrid integrated system after of a FC load increased.	41
Figure 2.2 Control action over the FC and GT to mitigate the adverse dynamic effects of the FC load (increased).	42
Figure 2.3 Dynamic effect of the hybrid integrated system after of a FC load decreased.	43
Figure 2.4 Control action over the FC and GT to mitigate the adverse dynamic effects of the FC load increased.	45
Figure 3.1 CA mass flow as a function of CA valve position at 50 kW electric load.	49
Figure 3.2 Turbine inlet temperature as a function of CA valve position at 50 kW electric load.	50
Figure 3.3 3-D output distribution as function of the different level factors combinations.	59
Figure 3.4 Cathode inlet mass flow as function of the different treatments.	60
Figure 3.5 Fuel flow as function of the different treatments.	61
Figure 3.6 Cathode inlet temperature as function of the different treatments	62
Figure 3.7 Turbine inlet temperature as function of the different treatments	63
Figure 3.8. Normal distribution of the residual in the fuel flow variable.	64
Figure 3.9. Computed F-value for the fuel valve, FT432	66
Figure 3.10. Interaction effect plots on fuel flow mean.	67
Figure 3.11. Residual vs fitted value of fuel flow for regression models 2 and 3.	69
Figure 3.12. Surface plot of fuel flow as a function of EL and CA valve.	70
Figure 3.13. Contour plot of fuel flow mean as a function of EL and CA valve.	70
Figure 3.14. Normal distribution of the residual in the cathode flow variable.	71
Figure 3.15. Computed F-value for the cathode inlet flow.	72
Figure 3.16. Interaction effect plots on cathode flow.	73
Figure 3.17 Surface plot of cathode inlet flow as a function of CA and HA valves.	74
Figure 3.18. Contour plot of fuel flow mean as a function of CA and HA valves.	75

Figure 3.19. Normal distribution of the residual in the cathode inlet temperature variable.	75
Figure 3.20 Normal distribution of the residual in the turbine inlet temperature variable.	76
Figure 3.21 Computed F-value for the cathode inlet temperature.	77
Figure 3.22 Computed F-value for the turbine inlet temperature.	78
Figure 3.23. Interaction effect plots on cathode inlet temperature.	79
Figure 3.24 Interaction effect plots on turbine inlet temperature.	79
Figure 3.25 Surface plot of cathode inlet temperature as a function of EL and BA valve.	81
Figure 3.26 Contour plot of cathode inlet temperature mean as a function of EL and BA valve.	81
Figure 3.27 Surface plot of turbine inlet temperature as a function of EL and BA valve.	83
Figure 3.28 Contour plot of turbine inlet temperature mean as a function of EL and BA valve.	83
Figure 3.29 3^{4-2} fractional factorial design selected to replicate set-points.	84
Figure 3.30 Replicate of compressor flow and the relative error between tests.	85
Figure 3.31 Replicate of cathode inlet flow and the relative error between tests.	86
Figure 3.32 Replicate of fuel flow and the relative error between tests.	86
Figure 3.33 Replicate of cathode inlet temperature and the relative error between tests.	87
Figure 3.34 Replicate of turbine inlet temperature and the relative error between tests.	87
Figure 3.35 Replicate of turbine exhaust temperature and the relative error between tests.	88
Figure 3.36 Fuel flow as a function of bypass air flow	90
Figure 3.37 Fuel flow as a function of CA bypass air flow (including the stall zone (brown quadrant)).	91
Figure 3.38 Fuel flow as a function of HA bypass air flow (including the stall zone (brown quadrant)).	92
Figure 3.39 Envelope of bypass air mass flow as function of compressor corrected flow.....	93
Figure 3.40 Envelope of CA bypass mass flow as function of compressor corrected flow (including the stall zone (brown quadrant)).	93
Figure 3.41 Envelope of HA bypass mass flow as function of compressor corrected flow (including the stall zone (brown quadrant)).	94
Figure 3.42 Cathode inlet temperature as a function of bypass air flow.	95

Figure 3.43 Cathode inlet temperature as a function of CA bypass air flow (including the stall zone (brown quadrant)).	95
Figure 3.44 Cathode inlet temperature as a function of HA bypass air flow (including the zone out of DoE (brown quadrant)).	96
Figure 3.45 Turbine inlet temperature as a function of bypass air flow.	97
Figure 3.46 Turbine inlet temperature as a function of CA bypass air flow (including the stall zone (brown quadrant)).	97
Figure 3.47 Turbine inlet temperature as a function of HA bypass air flow (including the zone out of DoE (brown quadrant)).	98
Figure 4.1 Flow Diagram of the Matlab MPC Toolbox.	100
Figure 4.2 Block diagram of HyPer MPC control application.	102
Figure 4.3 Past, present and future of the controller parameters.	104
Figure 4.4 Time response of step up/down change of CA valve.	107
Figure 4.5 Dead time and tracking of cathode airflow to CA valve.	107
Figure 4.6 Steady state time for cathode airflow after the 10 % step up of CA valve	108
Figure 4.7 Time response of step up/down change of HA valve.	109
Figure 4.8 Dead time and tracking of cathode airflow to HA valve.	110
Figure 4.9 Time response of step up/down change of BA valve.	111
Figure 4.10 Dead time of cathode airflow and turbine speed vs BA valve.	112
Figure 4.11 Steady state time for cathode airflow and turbine speed after the 2 % step opening of BA valve.	113
Figure 4.12 Time response of step up/down of electric load.	114
Figure 4.13 Dead time of cathode airflow and turbine speed vs electric load.	115
Figure 4.14 Comparison between model vs measured outputs and the input dynamics.	130
Figure 4.15 Comparison between model versus measured outputs and the input dynamics at $t = 60$ seconds.	131
Figure 4.16 Comparison between model versus measured outputs and the input dynamics at $t = 60.8$ seconds.	132
Figure 4.17 Comparison between model versus measured outputs and the input dynamics at $t = 61.6$ seconds.	133

Figure 4.18 Comparison between model versus measured outputs and the input dynamics at $t = 62.4$ seconds.	134
Figure 4.19 MPC input and output dynamics with some set-point variation using default values of Matlab weighted matrices (without anticipation) using the ARX identification model.	136
Figure 4.20 MPC input and output dynamics with some set-point variation after tuning the controller weighted matrices (without anticipation) using the ARX identification model.....	137
Figure 4.21 MPC input and output dynamics with some set-point variation after tuning the controller and using the MPC anticipation feature and the ARX identification model.....	138
Figure 4.22 HyPer Data measured for state space identification.	141
Figure 4.23 Comparison between on-line state-space identification model data and measured output (time 1 and 2).....	143
Figure 4.24 Comparison between on-line state-space identification model data and measured output (time 3 and 4).	144
Figure 4.25 Comparison between on-line state-space identification model data and measured output (time 5 and 6).....	145
Figure 4.26 Comparison between on-line state-space identification model data and measured output (time 7 and 8).....	146
Figure 4.27 HyPer Data measured for state space identification.	148
Figure 4.28 Comparison between on-line state-space identification model data and measured output (time 1 and 2).....	149
Figure 4.29 Comparison between on-line state-space identification model data and measured output (time 3 and 4).....	150
Figure 4.30 Comparison between on-line state-space identification model data and measured output (time 5 and 6).....	151
Figure 4.31 Variable type, description, units and nominal values.....	153
Figure 4.32 MPC input and output dynamics with some set-point variation using default values of Matlab weighted matrices (without anticipation) using the State-Space identification model.	155

Figure 4.33 MPC inputs and outputs dynamics with some set-point variation after tuning the controller weighted matrices (without anticipation) using the State-Space identification model.	156
Figure 4.34 MPC inputs and outputs dynamics with some set-point variation after tuning the controller and using the MPC anticipation feature and the State-Space identification model.	157
Figure 4.35 MPC structure, inputs and outputs type, units and nominal values (Matlab MPC Toolbox).....	160
Figure 4.36 MPC inputs and outputs constraints (Matlab MPC Toolbox).	161
Figure 4.37 MPC inputs and outputs dynamics with some set-point variation using default values of Matlab weighted matrices (without anticipation) using transfer functions model.....	163
Figure 4.38 MPC inputs and outputs dynamics with some set-point variation after tuning the controller weighted matrices (without anticipation) using the transfer functions model.....	164
Figure 4.39 MPC inputs and outputs dynamics with some set-point variation after tuning the controller and using the MPC anticipation feature and the transfer functions model.....	165
Figure 5.1 Comparison of turndown capability between standard gas turbine generator, extended turndown generator and FC/GT hybrid system.	168

List of Tables

Table 1.1 Summary of linear MPC applications by areas from different industrial vendors (Qin and Badgwell, 2002 [30])	21
Table 1.2 Summary of nonlinear MPC applications by areas from different industrial vendors (Qin and Badgwell, 2002 [30])	21
Table 3.1 Test levels (set-points) for valves and electric load.....	51
Table 3.2 Standard order of treatments designed for the experiments.	53
Table 3.3 Arrangements of “run order” and “standard order” for the test.....	54
Table 3.4 Parameter averages and standard deviations as a function of sample size for CA=40%, HA=20%, BA=90%, and EL=50 kW.	57
Table 3.5 Minimum and maximum average values for the different parameters.	58
Table 3.6 ANOVA analysis of the fuel flow (FT432).	66
Table 3.7 Fuel flow (FT432) regression equations.	68
Table 3.8 ANOVA analysis of the cathode air mass flow (FT380).....	72
Table 3.9 Cathode air flow (FT380) regression equations.	74
Table 3.10 ANOVA analysis of the cathode inlet temperature (TE326).....	77
Table 3.11 ANOVA analysis of the turbine inlet temperature (TE350).....	78
Table 3.12 Cathode inlet temperature (TE326) regression equations.....	80
Table 3.13 Turbine inlet temperature (TE326) regression equations.	82
Table 4.1 Upper/lower bounds of output/input variables and up/down rates of input variables	116
Table 4.2 Weight default matrices of the MPC performance index.	118

Nomenclature

Symbols

J	Cost Function
P	Pressure
Q	Heat or Output Weight Matrix
R	Input Rate Weight Matrix
T	Temperature
W	work
k	specific heat
d	Unmeasured disturbance.
r	Set-point (or reference).
u	Manipulated variable (or input).
v	Measured disturbance.
$\hat{y}$	Output (or controlled variable).
y	Measured output.
z	Measurement noise.

Acronyms

ANOVA	Analysis of Variance
APU	Auxiliary Power Unit
ARX	Autoregressive Exogenous Model
BA	Bleed Air
CA	Cold Air
CAF	Cathode Airflow
CIT	Cathode Inlet Temperature
CV	Controlled Variables
DMC	Dynamic Matrix Control
DoE	Design of Experiments
DOE	Department of Energy
EGT	Exhaust Gas Temperature
EL	Electric Load
ETFE	Empirical Transfer Function Estimates
FC	Fuel Cell
FC/GT	Fuel Cell/Gas Turbine hybrid system
FFD	Full Factorial Design
ffd	Fractional Factorial Design
FH	Fuzzy Hammerstein
FO	First Order
FUF	Fuel Utilization Factor
GPC	General Predictive Control
GT	Gas Turbine

HA	Hot Air
HAT	Humid Air Turbine
HIL	Hardware in the Loop
HyPer	Hybrid Performance
IDCOM	Identification and Command
IGFC	Integrated Gasifier Fuel Cell
LDM	Linear Dynamic Model
LQR	Linear Quadratic Control
MIMO	Multi-input Multi-output
MPC	Model Predictive Control
MPHC	Model Predictive Heuristic Control
MV	Manipulated Variables
NAARX	Nonlinear Additive Autoregressive Exogenous Model
NETL	National Energy Technology Laboratory
NLSM	Non-Linear Static Model
NMPC	Nonlinear Model Predictive Control
OFAT	One Factor at a Time
PEM	Predictive Error Minimization Method
PEN	Positive-electrolyte-Negative FC membrane Configuration
PI	Proportional + Integrated
PID	Proportional + Integrated + Derivative
RBFNN	Radial Basis Function Neural Network
RGA	Relative Gain Array
RPLDM	Real Time Piecewise Linear Dynamic Model
RSM	Response Surface Methodology
SIMO	Single Input Multiple Output
SISO	Single Input Single Output
SO	Second Order
SOFC	Solid Oxide Fuel Cell
SOFC/GT	SOFC and Gas Turbine Hybrid System
STD	Standard
TF	Transfer Function
TIT	Turbine Inlet Temperature
TPG	Thermochemical Power Group

Process Variables

FT110	Airflow measured at the compressor inlet
FT162	Airflow bypass by CA and BA valve
FT380	Airflow measured at the cathode inlet
FT432	Fuel Flow measured
PT151	Pressure measured at the compressor outlet
PT180	Pressure measured at the turbine inlet
PT305	Pressure measured at the cathode inlet
TE202	Temperature measured at the turbine outlet
TE326	Temperature measured at the cathode inlet
TE350	Temperature measured at the turbine inlet

Chapter 1

Introduction

1.1 Introduction

Fuel Cell/Gas Turbine hybrid system has emerged in recent years as a low emission, high efficiency source of power for the future. Around the world, research has been carried out to characterize the system, describe the behavior, and find the best ways to combine and control electrochemical devices with a turbomachine to improve system efficiency. In terms of efficiency and low emission alternative power, hybrid systems will play an important role in the next generation of electric production.

In this chapter, hybrid Solid Oxide Fuel Cell (SOFC) and Gas Turbine (GT) systems are introduced. A description of each component is presented and the theoretical idea of the pressurized configuration is illustrated. After that, the hybrid performance (HyPer) facility is depicted with all its components. Then the research progress of the hybrid system in the last ten years is reviewed. The review includes the main areas of hybrid development such as numerical and empirical models, static and dynamic characterization, and control strategies. Finally, the objectives and the contributions of this research are enumerated.

1.2 The Hybrid Performance Research Facility

The National Energy Technology Laboratory (NETL) of the U.S. Department of Energy is located in Morgantown WV, and has been investigating in recent years, ways to couple a high temperature SOFC with GT into a hybrid electric generation system. Researchers at NETL have designed and constructed one of the world's first laboratories for testing such systems, called the HyPer facility. The HyPer system runs an embedded real time fuel cell model coupled with the gas turbine cycle where the conditions of the cathode air flow and other variables are taken by the model to perform a real time fuel cell simulation. The output of the model: effluent heat, pressure, and temperature are matched with the real hardware through a fuel valve and plenum

volume installed in the facility. The HyPer system also contains three different manipulated bypass systems that allow the control of different and important variables in the functionality of the system: Bleed Air bypass (BA), Cold air bypass (CA), and Hot Air bypass (HA).

1.2.1 Hybrid Components

The hybrid system is formed by two basic power generation systems, the fuel cell and gas turbine power system. In the next section a brief description of both the SOFC and GT are given. The advantage of a recuperated GT cycle for hybrid applications is highlighted. Then, a theoretical hybrid configuration is presented. And finally, the HyPer facility is introduced and explained in some detail.

1.2.1.1 Solid Oxide Fuel Cells (SOFC)

Fuel cells, like a battery, transform chemical energy directly into electricity. FCs can provide continuous output power because the reactants are continuously supplied and the products are continuously removed. The main advantage of the FCs is its high theoretical efficiency compared with other thermal engines. In addition, FCs are very low pollutant emission devices, they scale very well to the megawatt range, have no moving parts, and operate quietly. Even though the electrochemical principles are the same, there exist several types of FCs differing mainly in their electrolyte. Each type of FC operates at different temperature regimes. SOFCs work at high temperatures (over 900 K) making them more suitable for stationary power generation. Heat is generated as result of the electrochemical reaction and a large amount of unoxidized fuel is typically found in the exhaust products.

Figure 1.1 shows a schematic of a standard SOFC. The basic electrochemical process occurs on the electrolyte interface. The hydrogen supplied in the anode side of the fuel cell reacts with the negatively charged oxygen producing water, electrons and heat according to:

The electrons are carried to the cathode through an external circuit producing electrical work. On the cathode side, oxygen continuously reacts with the electrons producing the negatively charged oxygen ions, according to:

The oxygen ions are transported again from the cathode to the anode through the electrolyte membrane, and the process is repeated continuously.

Figure 1.1 Schematic of a solid oxide fuel cell (SOFC)

1.2.1.2 Gas Turbine Cycle

The GT cycle or Brayton cycle is an open flow-through cycle. It receives a continuous supply of ambient air that becomes the working fluid. The air is exhausted directly back into the atmosphere, and not returned into the system. Figure 1.2 shows a typical GT cycle. In a simple gas turbine cycle, air is compressed at the inlet by a compressor. The compressor air then goes to a combustion chamber, where fuel is burned, raising the temperature. The air enters the power

turbine at high pressure and expands thorough the turbine, rotating the turbine, and generating work.

Figure 1.2 Schematic of a gas turbine cycle.

The thermal efficiency of an ideal Brayton cycle depends on the pressure ratio of the GT and the specific heat ratio of the working fluid, given by:

$$\eta_{th,Brayton} = 1 - \frac{1}{r_p^{\frac{(k-1)}{k}}} = 1 - \frac{T_4}{T_3}, \quad \text{where } r_p = \frac{P_2}{P_1} \quad (1.3)$$

Figure 1.3 shows (a) the temperature-entropy diagram for a gas turbine cycle and (b) the ideal thermal efficiency as a function of pressure ratio. It is clear in the figure, that in order to increase the efficiency the pressure ratio must be increased. In most common cases, the pressure ratio ranges between 11 to 16. This kind of high pressure ratio is not favorable to embed a FC because the fragile anode-electrolyte-cathode membrane could be destroyed easily. Another way to improve the efficiency of the cycle is by recovering heat from the exhaust gases using a heat exchanger called a recuperator. Figure 1.4 shows a schematic of a recuperated gas turbine cycle.

This configuration works at pressure ratios lower than the standard cycle which is advantageous for hybrid applications.

Figure 1.3 (a) T-s diagram for the ideal gas turbine cycle and (b) thermal efficiency as a function of the pressure ratio (Reproduced from [72])

Figure 1.4 Schematic of a recuperated gas turbine cycle.

1.2.1.3 HyPer Theoretical Configuration

The theoretical configuration of the HyPer facility is shown in Figure 1.5. The cathode side of a SOFC is embedded after the recuperator of a GT Brayton Cycle. The idea of the hybrid FC/GT system is to convert the heat generated in the FC and the unoxidized fuel in the anode exhaust into useful work at the gas turbine shaft. The extra power extracted in the gas turbine cycle would improve the efficiency of the fuel cell generation system. System efficiencies approaching 75% are theoretically possible using the hybrid configuration.

The difficulty with SOFC/GT hybrids is to match the requirements of both subsystems over a wide range of operating conditions. For instance, the heat effluent from the FC must match the heat required by the turbine in order to supply load and maintain constant speed. If the heat effluent and heat required do not match in operation, some mechanism or action must be taken in order to absorb or reject the difference. At the same time, the mass flow rate of air through the cathode of the FC must be managed to maintain thermal regulation of the FC while simultaneously operating in a stable region for the gas turbine compressor system.

Figure 1.5 Theoretical layout of direct fired integrated FC/GT hybrid cycle.

1.2.1.4 HyPer Facility (Real Configuration)

In the Laboratory, the HyPer facility uses a combination of hardware and real time dynamic models to simulate a solid oxide FC/GT hybrid system as shown in Figure 1.6. The facility is used to study the dynamic responses due to changes in parameters of the fuel cell, gas turbine, or peripheral equipment necessary to run the system. A real compressor, turbine, generator and other physical hardware are combined with models of the FC stack and fuel system component to create a “hardware in the loop” simulation. This arrangement avoids using a real FC in the loop because of its fragility to abrupt changes in operating pressure or to large temperature gradients. Therefore, the best option has been to combine hardware elements with numerical models in order to have a robust and flexible test bed facility for research.

Figure 1.6 HyPer research facility overview (Hardware in the loop layout)

The hardware components of the HyPer research facility comprise the recuperated GT cycle, the air bypass subsystems, and pressure vessels to simulate the volume and thermal capacitance of the FC stack. The gasifier and the fuel cell are real time models operating in conjunction with

the GT cycle using hardware in the loop technique (HIL) as described in detail by Smith [1]. The FC model captures the mass and thermal parameters coming from the HyPer facility to the cathode inlet: air mass flow (FT380), pressure (PT305), temperature (TE326). The mathematical FC model simulates the electrochemical process, calculating the heat effluent coming out of the fuel cell.

Subsequently, fuel burned in the combustor emulates and matches the numerically calculated heat effluent from the FC model. This match is done by a reverse fuel valve model that dynamically and continuously adjusts the fuel valve (FV432) position in order to reflect the model's thermal effluent prediction. The heated air system is sent downstream to the GT to generate electrical power and to drive the compressor.

The bypass subsystems are used to divert the air flow coming from the compressor to different specific points of the facility. The BA bypass is used to deliver air directly from the compressor to the atmosphere and the flow is regulated by the BA valve. This bypass valve has the benefit of mitigating stall in the compressor. In addition, the BA bypass has the ability to add load to the turbomachine and is therefore capable of helping to control the turbine speed. The drawback of this bypass valve is that it decreases the efficiency of the system [2], [3].

The Cold Air (CA) bypass is employed to deliver air from the compressor directly to the turbine, bypassing the heat exchanger and the fuel cell. The CA flow is regulated by the CA valve. The CA allows bypassing large amounts of air and is therefore useful in controlling the air mass flow to the cathode. This bypass has the advantage of lowering the turbine inlet temperature by delivering fresh air downstream of the combustor and also enhances the compressor stall margin by decreasing system pressure drop. The disadvantage is in the cost of the efficiency of the system, since the amount of energy recovered in the recuperator is decreased with increasing CA bypass.

The Hot Air (HA) bypass is used to route air from the recuperator around fuel cell. The flow is managed by the HA valve. It offers effective control of the mass flow rate of air through the FC cathode while still recovering energy from the turbine exhaust. In addition, the HA bypass enhances the stall margin somewhat. The drawback is its initial high investment because it must

be a high temperature valve, since the air passing through it has been heated by the recuperators. The components and subsystems of the HyPer facility have been well described in specific detail by Tucker et al. in [2], [3], and [4].

1.3 Historical Development of Hybrid Systems

The following paragraphs describe former work in the area of hybrid systems based on different areas of research. Progress in Hybrid systems has been associated with advances in the understanding of the different controlled variables (CV) and their relationship with the different manipulated variables (MV) in order to maintain the FC and GT under safe operation. For the FC, variables such as current, voltage, fuel composition, fuel utilization, cathode and anode mass flow, electrochemical losses, heat generated, degradation, temperature, and pressure of air and fuel must be controlled in order to minimize thermal and mechanical stresses in the anode-electrolyte-cathode membranes and maximize the efficiency of the system. The pressure difference between cathode and anode must be less than, 50 mbar (0.73 lb/in^2), to avoid damage by mechanical stress. Temperature variation across the FC must be less than 150°C , and the rate of temperature change less than 1°C/sec . For the GT, disturbances, air mass flow, stall margin, compressor surge, the speed, pressure and operational temperatures must be kept between the limits imposed by the turbine materials and the requirements of the generation system.

1.3.1 Hybrid Configuration Review

Comparison of efficiency and performance of different system configurations from the literature is difficult because the basis and assumptions of individual simulations are often dissimilar. Rao et al. 2003 [5] and [6] compared the thermal efficiencies, exergy destruction, and specific power produced by analytical models for three pressurized tubular SOFC hybrid configurations: an intercooled preheated SOFC recuperated GT cycle, a single SOFC humid air turbine (HAT) cycle, and a dual SOFC-HAT cycle that incorporates a second low pressure SOFC. Exergy calculations were used to define the thermal efficiency of each cycle, since Carnot efficiencies are not suitable for power cycles incorporating electrochemical reactions.

The integrated configuration (atmospheric and pressurized FC) has also been studied in the past by Veyo et al. 2003 [7], and the higher hybrid efficiency and lower emissions were found for the pressurized than for the atmospheric configurations. In this study, the atmospheric system achieved efficiency of 52%, and the pressurized system about 59% for 300kW plants. The pressurized hybrid systems also produced a significant reduction of NO_x and CO₂ emissions, a higher FC stack voltage, and a higher cell operating efficiency for a set stack current.

Tucker et al. 2006 [8] compared the effects of three different configurations for coal based SOFC/GT configurations. The comparison was made on the basis of efficiency, operability issues, and component integration for one high pressure cycle (pressure ratio 44) and two lower pressure cycles (pressure ratio 5). The recuperated cases showed high efficiency and exhibited excellent potential for effective control strategies. It was shown that recuperation provides a means to increase the power contribution of the turbine in a hybrid, maintaining a high efficiency (60.9%). Work in press compares these cycles to an atmospheric IGFC with a maximum efficiency of 48%. Thus, the HyPer facility has been implemented as a recuperated GT cycle.

1.3.2 Dynamic Simulation

Most of the work in the area of hybrid systems has been related to the development of dynamic models without experimental results to validate the models. In some cases, research models have been validated, but do not represent the total characteristics of the system described.

In 2004, Liese et al. [9] developed a study of the transient behavior of a SOFC using a simplified approach. All the components were modeled with a lumped volume approach using global balance equations and empirical equations for losses. These models were not compared to any empirical data.

Shelton et al. [10] in 2005 presented a lumped parameter model of the HyPer facility. Empirical constants were used to tune the model with experimental data. In this work, the cases studied were: 1.) Speed controlled load shedding and 2.) Speed set point changing. In the first case, the model underpredicted the fuel flow required to control the speed. In the second case the fuel flow was overpredicted by the model. Bypass valves were not included in this model.

In 2006 Ferrari et al. [11] presented a work describing the experimental validation of two different transient models of the hybrid FC/GT facility of the NETL at Morgantown. A real-time transient model developed by Liese and the detailed transient modeling activity using the TRANSEO program developed by the Thermochemical Power Group (TPG) of the University of Genoa were compared. Both models were successfully compared with the experimental data of two different load step decreases. But both models failed to describe the use of the valves, especially when they were operated simultaneously. The detailed model was more time-consuming than the real-time model. Ferrari's model was more accurate than Liese's model and the results were reported in [11]. In these models the Cold Air valve was maintained opened at 40%. While Liese's model was less accurate in transient response over a broad range of off-design operating conditions, Ferrari's model was more precise and more time consuming.

In 2006, R. A. Roberts et al. [12] reported in the literature a dynamic simulation of a pressurized 220 kW SOFC/GT, Figure 1.7. The results were compared with measured data from the system developed by Siemens-Westinghouse (S-W). The model was built based on dynamic principles and provided good insight into transient and steady state performance, but was not fully validated. Computational time was not reported. Also, the bypasses of the real physical system were different from the HyPer facility, because the S-W system does not have BA or CA bypass valves and includes a heat exchanger bypass. The S-W system does have a HA valve placed to bypass the cathode side of the FC.

Figure 1.7 Diagram of the pressurized 220 kW SOFC/GT hybrid system (Roberts et al., 2006 [12]).

In 2006, Wächter et al. [13] presented control strategies based on a dynamic simulation, a nonlinear model with bulk parameters and 19 dynamic states. This nonlinear model was also linearized and a state-space representation was derived. The two models were compared. These models have not been compared with measured data and no controller was developed. The detailed nonlinear model had a high computation time (about 21 hours of calculations) although the linear model reduced the computation time to 2 minutes.

In 2010 Restrepo et al. [14] presented a real time piecewise linear dynamic model (RPLDM) that consists of the combination of a nonlinear static and a linear dynamic model of the HyPer system based on experimental data. A linear model was identified based on experimental data. The linearized model representing the nonlinear plant over the total region of operation of CA valve employed linear dynamic models scheduled by parameters obtained from the properties of nonlinear steady state values. In this work, the RPLDM was modeled and validated, but was limited to control of the cathode air flow using the CA valve at different electric loads (Single input-multiple outputs (SIMO) model). The paper is presented in Appendix A.

1.3.3 Empirical Characterization

Other research has been focused on the characterization of the system based on models or in hardware simulations. Tucker et al. [2] presented in 2005 a characterization of air flow in the HyPer system to show how it is possible to manage thermal gradients in the fuel cell component during transient load periods. The bleed air bypass and cold air bypass were characterized quantitatively in terms of compressor inlet flow, process limits, system efficiency and system performance. In that paper, the characterization of air flow was performed with zero electric load imposed on the system generator. CA bypass was shown to be a very effective strategy to reduce system pressure drop, and hence improve compressor surge margin. CA bypass was also shown to have an insignificant specific energy requirement compared with bleed air bypass. BA bypass was also shown to be an effective means to increase the compressor surge margin. These experiments were performed by experimentally varying one factor at a time (OFAT), in other words while a bypass test was carried out; the other valves were kept closed.

Traverso et al. [15] published in 2005 a characterization of air flow through a pressurized SOFC integrated into a hybrid system using two different computer models. It was concluded that a variable speed microturbine is the best option for off-design operation. This is because during an abrupt transient such as load trip, the bypass bleed valve fails to provide adequate speed control. It was stated that the air management on the cathode side controlled by the turbine cycle is the best way to ensure safe SOFC operation. The system studied contains a BA and a CA bypass valve similar to the HyPer facility. But the strategies analyzed in this work were assessed separately to show specific features. The results showed that the BA is not able to adequately control the flow to the FC because it could cause overheating in the recuperator.

In 2006, Tucker et al. [3] presented a characterization of the cathode inlet air flow, this time including the hot air bypass with a 45 kW load on the turbine generator. Bleed air bypass was shown to have almost no effect on cathode inlet flow. This means that BA bypass operation can increase compressor surge margin without disturbing the fuel cell cathode cooling flow. HA bypass had a relatively large impact on cathode inlet flow but not on surge margin. CA bypass showed the largest effect on both the cathode flow and the compressor inlet flow. Compressor discharge pressure showed little change (3kPa) for any valve. The CA bypass showed the

greatest reduction of system pressure losses between the compressor outlet and turbine inlet over its full range of operation. The HA bypass over its full range of operation lowers the system pressure drop about 10%. System pressure loss between compressor and turbine was not affected using the BA valve because the flow and path between turbine and compressor does not change. BA bypass had a big negative impact on the efficiency of the system. Thus, its application can be only considered for control of transient events since the energy cost of the method is too high. HA bypass was shown to require no increase in fuel over its entire range of operation, resulting in the most efficient means of airflow management to the fuel cell cathode. CA bypass was shown to have high energy cost over its full range of operation, especially at higher bypass flows. These experiments were performed using the OFAT technique.

In 2009, Tucker et al [4] presented a paper determining the operating envelope of the HyPer facility based on the modulation of the three bypass valves independently. Turbine load was also varied and each point of operation was matched to the thermal, pressure and flow output of a corresponding fuel cell operating condition using a 1D fuel cell model. Parametric variation of CA, HA, BA valves was shown to provide a wide range of operating conditions in a fuel cell turbine hybrid system without variation in turbine speed. In this paper, fuel cell power output could be varied from a 30% increase and a 57% turndown from nominal conditions. Cathode airflow could be varied from 17% increase to a 39% decrease from nominal conditions. A system operating range of 228kW to 737kW was demonstrated by the simulations, representing a possible turndown of 69%. This paper is discussed in more detailed in section 1.4.1.1.

Factorial analysis has been used by Cali et al. [16], and [17]. They applied this approach to a fuel cell mathematical model and ran a simulated computer experimental test. A 2^k factorial analysis was performed using the ANOVA technique, and the effects of the main independent variables and their interactions were analyzed. In Cali's paper, air preheating was shown to have a significant effect on the stack electric power, which is one of the reasons to implement the bypass analysis in the hybrid system and to control the temperature and flow in the cathode side of the FC. The approach did not consider possible curvature (nonlinearities) in the model and it is established for the fuel cell only. Cali et al. also applied the response surface methodology to

the former design of experiments to investigate optimal operation points in terms of generated power and heat effluent.

Thus, no work has been published in the literature about the characterization of a SOFC/GT system by applying methodologies of design of experiments and its respective statistical analysis of the results.

1.3.4 Control Strategies

Important results have been found in the literature describing different ways to manage fundamental variables to couple the electrochemical FC and the thermomechanical GT system. For example, the air flow coming from the compressor to the FC must be managed and controlled in order to prevent high temperature gradients in the anode-electrolyte-cathode (positive-electrolyte-negative PEN configuration) of the FC when the electrochemical reaction is perturbed, for example by load changes or fuel composition variations. Also, the heat effluent of the FC must match the required heat of the turbine to allow controlling the speed and load of the GT. In addition, anode and cathode pressure difference must also maintain a small gap of operation to avoid rupture of the PEN structure by stress or delamination. This description is the base to elaborate effective control strategies of interaction of the hybrid systems.

In 2003, Jurado et al. [18] incorporated an adaptive minimum variance controller, identifying the parameters of the controller through their estimation. A linear-discrete plant disturbed in a stochastic manner was considered. The electric load was the stochastic part of the disturbance. This model did not report measured data or computational time.

In 2005, Stiller et al. [19] presented a multi-loop controller including a dynamic model for a SOFC/GT hybrid system. The control strategy was to keep constant the temperature in the FC under any condition. The controller is adaptable under malfunction or degradation of the system to the new characteristic. Calculation time for a steady state point was between 5 and 10 seconds and for load changes was about 5 minutes. Load profile simulations required about 20 hours in a 2.5 GHz Intel Pentium 4 Processor.

Ming Xu et al. [20] presented a novel power tracking control method to address the issue of power partitioning between two energy sources. A detailed system simulation was developed. The simulation results demonstrated that power tracking control can effectively control the load power distribution between these two sources of power. These results were not compared with any empirical data.

In 2006, Roberts et al. [21] studied the impact of the gas turbine rotational speed on the dynamics and controllability of a hybrid atmospheric FC/GT configuration. The investigation was focused on two operational strategies: fixed speed and variable speed operation. The configuration is shown in Figure 1.8. The results showed that the variable speed operation is superior for the FC/GT hybrid configuration studied, for two reasons. The variable speed operation is able to manipulate the mass flow through the system without using bypass valves or an auxiliary combustor, and the efficiencies were higher than using constant speed. Linear controllers (proportional plus integral, PI) were applied to different system configurations. In this study, for fixed speed GT, the air flow to the cathode was manipulated using a bypass valve, and in this way the FC temperature was controlled. It was demonstrated that the variable speed GT increased the efficiency and also the range of power operation of both a pressurized and an atmospheric system.

Figure 1.8 Bottoming SOFC/GT hybrid system with variable speed and supplementary oxidizer fuel (Roberts et al., 2006 [21]).

Mueller et al. [22] in 2005 conceptualized and designed a control approach for a bottoming SOFC/GT hybrid system with variable speed and supplemental oxidizer fuel. The schematic of this system is shown in Figure 1.8. This control approach was based on a dynamic simulation developed for the 275kW SOFC mentioned in [22]. The model was a nonlinear description of the facility. A relative gain array (RGA) analysis was also performed for several operating points to characterize the input/output relationship. One of the conclusions is that for plants that use voltage as a controlled variable, it is beneficial to control power by manipulating fuel cell current and to control fuel cell voltage by manipulating the anode fuel flowrate. Airflow was controlled by varying the gas turbine speed. Using voltage as a state variable was shown to be a potential solution to making the SOFC system robust. A decentralized and linearized multiloop control design has been suggested because of the different time scales of each control loop. The GT shaft speed was controlled as the fast inner loop. It receives its set point from the slow outer loop that controls the fuel cell (stack) temperatures.

The development by Mueller et al. in 2006 of a centralized linear quadratic regulator was presented [23] because a decentralized controller [22] had been showing coupling among variables at time scales greater than one second. The centralized controller manipulates the anode fuel and water flow, combustor fuel flow, the fuel cell current, and the gas turbine power. The optimal controller was designed under a linearized state space model and the results were implemented in the nonlinear model. This implementation was sluggish when compared to the linear model, but good.

Fuel Cell Energy and the Department of Energy worked to develop advanced and intelligent control strategies for hybrid systems [24], and [25]. The system was composed of a GT cycle coupled to an atmospheric FC. Figure 1.9 shows a layout of the hybrid system employed for analysis. A dynamic model was developed and used as a test bed for controller simulations. Components of the advanced control module included a neural network supervisor, robust feedback controllers, and predictive system models.

Figure 1.9 Layout diagram for atmospheric SOFC/GT system (Junker et al., 2006 [24]).

Tsai et al. [26], [27], [28], and [29] presented a multivariable robust control and a relative gain array (RGA) analysis of HyPer system. The multivariable empirical model was derived via frequency response. Transfer functions were derived after the modulation of the different bypasses valves, electric load, and fuel valve. An integrated transfer functions matrix served as a nominal plant to design a H-infinity robust control algorithm. The multivariable controller was designed to work within the boundaries of a nominal operational point under constraints imposed by the cathode airflow and the speed of the turbomachinery. The H-infinity (H_∞) study proved the functionality of a robust, stable, controller based on an empirical mathematical formulation. The RGA analysis demonstrated that the mass flow rate is best controlled with the HA valve, the TIT with CA valve, and the turbine speed with the fuel valve. The turbine inlet temperature is not a critical manipulated variable but rather a constraint of operation. However, cathode inlet temperature and pressure are critical variables but were not controlled by the system proposed by Tsai. No practical implementation of this controller has been done.

The strategy of control for the HyPer is very important in order to select the right action addressed by the inputs in a particular operational situation of the system. The HyPer research team continually searched for control methods that can manage both inputs and output constraints, incorporate a simple model of the facility, cope with nonlinearities, reject disturbances, and optimize operation.

1.3.5 Model Predictive Control (MPC) Review

As defined by Camacho et al. [43] the term MPC does not designate a specific control strategy but rather a range of control methods which make explicit use of a model of the process to obtain the control signal by minimizing an objective function. The MPC structure is presented in Figure 1.10. A model based on the past and current inputs and outputs, and the future control actions is used to predict the future outputs of the plant. This predicted output is compared with a reference trajectory, the error is obtained, and the constraints and the cost function are used to calculate new control actions.

Figure 1.10 Basic structure of MPC (Camacho et al. 2007 [43]).

The receding horizon principle proposed by Propoi [44] is one of the central ideas of MPC. The receding horizon principle consists of calculation in the present time of a window containing a trajectory of input and outputs to reach specific objectives. Then, after one or more control actions, the window is abandoned and a new window is created pointing toward the objectives of the controller. Figure 1.11 shows a window at the present time k of controller calculation. The future interval P is the time of the predicted response. In the figure, the target is the tracking set point of the system, and the interval M is the time selected by the designer to achieve the desired set point; the control loop trajectory is obtained after the optimized control inputs are determined

by minimizing a specific cost function. It is typical in MPC, that the first control action move is implemented and the algorithm repeats the same procedure to calculate the next control window. In spite of many successful applications in the petrochemical sector, the MPC method lacks a formal demonstration of stability and robustness.

Figure 1.11 Window of MPC design at each present time

The paper presented by Qin et al. [30] in 2002, is an excellent overview of the commercially available MPC technology. In the paper, a survey of industrial MPC technology is presented, along with a good condensed history of the MPC development. In the survey, some vendors offered information about their MPC technology products and their application. Table 1.1 and Table 1.2 show the different areas and the number of linear and nonlinear MPC applications, respectively. Although the data was collected in 1999, and the numbers must have changed very fast since then, the relatively high number of applications reflects the standing of the MPC in the state of the art of the industrial process.

In 1978 Richalet et al. [31] presented the first description of an MPC application. The approach was known as model predictive heuristic control (MPHC) and the software algorithm was named IDCOM (Identification and Command). The characteristic of this formulation were

the use of an impulse response model for the plant, the quadratic performance objective over a finite prediction horizon, input and output constraints, and the future output trajectory specified by reference.

Table 1.1 Summary of linear MPC applications by areas from different industrial vendors (Qin and Badgwell, 2002 [30])

Area	Aspen Technology	HoneyWell Hi-Spec	Adersa	Invensys	SGS	Total
Refining	1200	480	280	25		1985
Petrochemicals	450	80	...	20		550
Chemicals	100	20	3	21		144
Pulp and Paper	18	50	...	...		68
Air & Gas	...	10	...	...		10
Utility	...	10	...	4		14
Mining/Metallurgy	8	6	7	16		37
Food Processing	...	...	41	10		51
Polymer	17	...	...	...		17
Furnaces	...	...	42	3		45
Aerospace/Defense	...	...	13	...		13
Automotive	...	...	7	...		7
Unclassified	40	40	1045	26	450	1601
Total	1833	696	1438	125	450	4542
	DMC:1985	PCT:1984	IDCOM:1973			
First App.	IDCOM-M:1987	RMPCT:1991	HIECON:1986	1984	1985	
	OPC:1987					
Largest App.	603x283	225x85	...	31x12	...	

Table 1.2 Summary of nonlinear MPC applications by areas from different industrial vendors (Qin and Badgwell, 2002 [30])

Area	Adersa	Aspen Technology	Continental Controls	DOT Products	Pavilion Technologies	Total
Air & Gas			18			18
Chemicals	2		15		5	22
Food Processing					9	9
Polymers		1		5	15	21
Pulp & Paper					1	1
Refining					13	13
Utilities		5	2			7
Unclassified	1		1			2
Total	3	6	36	5	43	93

Cutler and Ramaker [32], engineers at Shell Oil, published the MPC methodology termed Dynamic Matrix Control (DMC). The dynamic process model is created using the step response of the plant. The future control actions are determined by minimizing the error between the output and the reference trajectory subject to operational constraints.

An article presented by Goodhart et al. [33] in 2000 illustrates the generic DMCplus™ advanced control software structure and is shown in Figure 1.12. In this software structure, a steady-state optimization based on the economics of the process is solved at each controller iteration using the current state of independent manipulated variables and predicted value of dependent controlled variables. The difference between DMCplus and DMC is that the first includes input constraints in its formulation and the second is a free constraint formulation.

Figure 1.12 MPC (DMCPlus) control structure (Goodhart et al. 2000 [33]).

In 1987 Clarke et al. [34], [35], [36], and [37] developed a Generalized Predictive Control (GPC). It is one of the most popular methods used and it has been chosen in this present work as one of the methods of design. The details of the method will be explained in Chapter 4.

1.3.6 MPC Fuel Cell and Gas Turbine Applications

In 2001 van Essen et al. [38] applied a successful real time nonlinear model predictive control (NMPC) to a real gas turbine laboratory installation. The sample time was 1.2 seconds limited by the velocity of the control valves' operation. It was recommended to decrease the sample time by using faster and more accurate control valves to obtain faster control. In spite of this, the advantages of MPC, i.e. constraint handling in inputs and outputs, optimal set point tracking, and anticipation of future step set-points changes, were fulfilled. The steady state and transient tracking were very good. The NMPC was shown to be a promising new control strategy for industrial turbomachinery configurations. This design was performed for a gas turbine, not for a hybrid system.

In 2007 Wang et al. [39] presented a predictive controller for a SOFC that was completely data based. The proposed MPC was data-driven since it only required a set of input-output open-loop data. This approach provided a good alternative to the SOFC control problem because the explicit numerical model of the SOFC for MPC is complex and time consuming. The design was investigated for a FC alone.

Jurado in 2006 [40] and Huo et al in 2008 [41] presented a NMPC of SOFC based on a Hammerstein model, in which the nonlinear static part was approximated by a fuzzy Hammerstein model and a radial basis function neural network (RBFNN) by Jurado and Huo, respectively. The linear dynamic parts were modeled by an auto regressive exogenous ARX model in both cases. The Jurado formulation was a (MIMO) configuration, meanwhile the Huo design was carried out in a SISO framework. In both case just the FC was used, not a FC/GT configuration.

In 2010, Bhattacharyya et al. [42] presented a NMPC of a SOFC. This work considered a SISO and a MIMO control. Both the ARX model and the nonlinear additive autoregressive with exogenous input (NAARX) model were used in the SISO identification. Both showed very good correlation with the data, and the NAARX model captured the nonlinearities of the FC dynamics very well. Different polynomial approximations of the NAARX model were used for the MIMO case. The identification process data was generated from a FC numerical model using uniform

random steps as the input signal for system excitation. In the SISO case, the anode flow is manipulated to supply the right power to the grid, and it was found that a PID control scheme can be sufficient to control the power output of the FC. In the MIMO case, power and fuel utilization factor (FUF) were the controlled variables, while anode flow and voltage were the manipulated variables. The PID controller was found to perform poorly for the MIMO case, since very large overshoot for the power and the FUF were observed using a PID controller. Meanwhile the NMPC could satisfy the step change in load without overshoot in power or FUF.

1.4 Research Needs

The research concerns and gaps outlined above must be addressed with a new approach for overcoming hybrid difficulties. The new strategies must specifically work to solve the main drawbacks and leverage the advantages to drive the research to a new level.

1.4.1 New HyPer Characterization

In 2005, Tucker et al. [2] characterized the air flow and thermal energy coming from the turbomachinery to the fuel cell cathode in terms of the BA and CA bypass. This characterization also quantified the compressor inlet flow, process limits, system efficiency, and system performance. In 2006, Tucker et al. [3] repeated the experiments using the former bypasses plus HA bypass and turbine load. In 2009, Tucker et al. [4] used the results of those tests to determine the operating envelope of the hybrid system. The method used in that work manipulated the different bypass valves one at a time. This experiment technique, called one factor at a time (OFAT) has the disadvantage that it does not permit discovery of the complete interacting behavior of the highly coupled, nonlinear system. In addition, OFAT experiments do not permit observation of the complete working envelope of the system. The factorial experiments performed in the present research provide new understanding, and reveal new relationships between the different manipulated variables and the plant response.

1.4.1.1 Motivation of the Design of Experiments on the HyPer Facility

Figure 1.13 shows the fuel mass flow that both simulates the heat effluent coming from the FC plus injects auxiliary fuel as necessary to keep turbine speed constant. It is illustrated that the CA and BA valves can bypass a considerable amount of compressor air with the disadvantage of increasing the amount of auxiliary fuel required, i.e. lowering the efficiency of the system. However, the hot air valve is able to provide similar FC bypass capacity while keeping constant efficiency. Figure 1.14 shows the cathode air flow when the bypass valves are employed. The CA and HA valves are very effective in management of the cathode air flow and therefore have the potential to control thermal stresses in the FC. Figure 1.15 shows the effects of the CA and HA valves on the turbine inlet temperature (TIT). Specifically, the CA valve bypass delivers air mass flow at the compressor outlet temperature to the turbine inlet. This fresh air decreases the TIT and exhaust gas temperature (EGT) keeping the turbine working in the safe range of operation. The TIT and EGT constitute an important constraint in the hybrid system operation. The pressure drop through the system as a function of the bypass valves is shown in Figure 1.16. In this figure it is illustrated that the BA valve has little effect on the compressor outlet pressure. Meanwhile, diverted flow in the system using CA and HA valves bring benefits in terms of the pressure drop between the compressor and turbine. The CA bypass is the most effective means to decrease the pressure drop through the system.

Figure 1.13 Fuel mass flow (Natural Gas) as a function of the bypassed flow (OFAT) at 0, 25, and 50 kW generator load (Tucker et al. 2009 [4]).

Figure 1.14 Cathode air mass flow as a function of the bypassed flow (OFAT) at 0, 25, and 50 kW generator load (Tucker et al. 2009 [4]).

Figure 1.15 Turbine inlet temperature as a function of the bypassed flow (OFAT) at 0, 25, and 50 kW generator load (Tucker et al. 2009 [4]).

Figure 1.16 Pressure drop as a function of the bypassed flow (OFAT) at 0, 25, and 50 kW generator load (Tucker et al. 2009 [4]).

Another important constraint in the system is shown in Figure 1.17. In this figure, the stall margin is improved when the bypass valves are operating compared with zero bypasses. The CA and HA valves were shown to be a good method to mitigate steady state stall. Figure 1.18 shows the maximum operating envelope of the cathode inlet temperature as a function of the cathode mass flow and electric load. During transient operation it is important to control the temperature of the fuel cell in order to avoid thermal stress in its components. An effective way to control the temperature gradients is to manage the air mass flow to the cathode and to supervise the overall temperature of the FC. A strategy for implementing cathode temperature control was presented by Banta et. al. [57]. A supervising system to control the speed in the turbomachine and the thermal gradients in the FC must be implemented in the HyPer facility based on the characteristics of each bypass subsystem. These characteristics mentioned formerly in [2-4] were investigated examining the range of FC and turbine operation using single bypass flow modulation. Therefore, the effects of the interaction of the different bypass valves and the electric load on the turbine have not ever been investigated.

The new experiments conducted here were successful in finding points of operation where the benefits of each valve could be synergistically added.

Figure 1.17 Bypass valve positions and compressor stall line at 50 kW generator load (Tucker et al. 2009 [4]).

Figure 1.18 Envelope of cathode inlet temperature as function of cathode inlet mass flow (Tucker et al. 2009 [4]).

The Hybrid system concept has shown a promising potential for low emission power generation, including high efficiency and the ability to use a variety of working fuels. However, nobody has shown experimental proficiency to control the equipment. Most of the research efforts in the experimental area have focused on mass flow management and steady state limited regions of the envelope using OFAT. Multivariable experiments can unveil more information that is relevant in the behavior of the system. For instance, Figure 1.19 compares the cathode inlet air flow with the CA valve varying from 40 to 80%, leaving the BA and HA valves completely closed (OFAT), and the same range for CA valve when the BA and HA valves are opened 14% and 80%, respectively. It is clear that the mass flow envelope is extended in this new setup. Figure 1.20 shows the surface of variability of cathode inlet mass flow when the CA and HA (BA=10% opened, and EL=50kW) are operated simultaneously. It was found that it is possible to bypass up to 86% of the mass flow coming from the compressor. Figure 1.21 shows a contour plot of the former graph. It illustrates how it is possible to follow trajectories of equal mass flow by manipulating the HA and CA simultaneously. As mentioned before, each valve has benefits and drawbacks. Therefore, operating the valves individually could improve one system parameter, but at the same time have a negative impact on other parameters. This study

examines this lack of synergetic operation and determines new benefits that hybrid systems can have during operation. Detailed description of the experimental design methodology, performance, and analysis will be presented in Chapter 3.

Throughout the process of designing and analyzing the experiments, several critical issues have been identified, such as: interaction of factors, operating constraints, linearity, variability, and repeatability. These design issues and their impact on system performance are described and analyzed in this dissertation. Because the underlying hybrid system has not been fully understood, an empirical model based on the operation of simultaneous variables is appropriate. Response surface methodology (RSM) and optimization techniques are good tools to find trajectories of operation which allow integrating these two systems and clarifying how to control the system. Also, it is possible to develop a controller following trajectories of the different parameters investigated here.

* The air mass flow at 0 kW was taken to 19 °C, and the other two were taken at 32 °C.

Figure 1.19 Cathode inlet air flow as a function of CA valve position and electric load.

Figure 1.20 Envelope of the cathode air mass flow as a function of CA and HA valve position (BA=90%, and EL=50kW).

Figure 1.21 Cathode air mass flow contour plot as a function of CA and HA valve position (BA=90%, and EL=50kW).

1.4.2 Multi-Input Multi-Output Approach

In 2010, Tsai et al. [26] demonstrated that it is not possible to control one state variable without affecting drastically another, under a realistic strategy of control and the current HyPer configuration. This work emphasizes that the three different bypasses must be used simultaneously as a part of a multivariable centralized input and output controller. His work was concentrated around the limits of one operating point and did not cover the wide range of operation of the system. The strong interaction between parameters, the nonlinearities and the complexities of the system have made it difficult to understand the non-intuitive behavior of the system in order to implement a MIMO controller.

1.5 Design of Experiments

Design of experiments (DoE) is a tool established and commonly applied in industry. DoE is based on statistical principles and is used to determine systematically the effect of manipulated variables (called factors in the DoE terminology) and their interactions over the controlled variables (response) of the experiments. Application of DoE has not been done to study a hybrid FC/GT system. Only the works of Cali et al. [16], and [17], 2005 and 2006 have been found in the literature. They applied DoE to a FC to understand the impact of FC input variables (fuel and air utilization factors, fuel and air preheating, and anode recycling rate) on the FC power and thermal recovery power. Their experiments were carried out using a computer model of the 100 kW SOFC installed in Torino, Italy and did not include a hybrid system.

The DoE work performed and presented here constitute the first time that factorial experiments have been applied to a hybrid SOFC/GT system using a real physical hardware configuration. This approach allows to understand the effect of each of the bypass valves working simultaneously on the different parameters of the turbomachinery and fuel cell system.

DoE permits to have a greater visualization of the effects of the variables and their interactions in different states of the system; to have a new operating envelope of the system; to develop strategies for control by defining optimal points of operation, and to determine trajectories of operation while the system moves from one state to another. For example,

optimizing fuel consumption or safely changing modes of operation will require this type of knowledge. It will permit great flexibility in the manipulation of all the valves to accomplish safe and efficient operation of the system. In addition, a replicate of the experiments was performed to ensure repeatability of the experiments. Former experiments in the HyPer facility have not included replication and no prior analysis of variance has been done.

1.6 Model Predictive Control Selection

In the last twenty years MPC has flourished as one of the top emerging control techniques in the process industries. The use of a simple model, the ability to incorporate an optimal framework, the ability to deal with multivariable linear and nonlinear systems and to handle system constraints gives to the method vast advantages over classical control. MPC has been widely used in the process industries such as chemical plants and oil refineries. MPC leads to an optimization problem which is solved on-line in real time at each sampling interval. MPC takes full advantage of the power available in today's control computer hardware. The components of a MPC controller are basically:

- Explicit use of a model to predict the process output at future time instants.
- Calculation of an online control sequence minimizing a cost function at each control execution.
- Calculation of a new horizon by applying the first control signal of the sequence calculated at each step.

The HyPer facility is a complex multivariable system whose dynamic behavior, (nonlinearities, combustion, turbomachinery and valves dynamics) is very difficult to express by models based on fundamental physics and chemistry principles. Any MPC explicit model formulation offers the opportunity to update the model at each instant of time. In this purpose, system identification plays an important role in MPC design, allowing quick estimation of empirical dynamic models from test data, reducing the cost of model development as described by [43], [48], and [55].

1.6.1 MPC Motivation

Many reasons arise to select MPC to control the fuel cell/gas turbine system. Next is presented the detailed description of the advantages offered by MPC as a controller for the HyPer system.

1.6.1.1 Input Constraints Handling

The real physical constraints found in the HyPer facility and in other hybrid configurations constitute one of the main advantages of MPC as the best strategy for control. Constraints such as limiting the range of valve opening, limiting the maximum electric load to avoid overload of the turbine or overheating of the system, and maintaining adequate stall margin are representative examples of the limitations on inputs and outputs in the cathode circuit of the hybrid system. The anode side presents other real constraints, such as fuel utilization factor, anode/cathode pressure difference, and average temperature of the FC.

It is known that when a valve or actuator achieves its upper or lower bound of operation, saturation of the actuator is achieved and in these limits the controllability of the plant could be lost. In some cases the closed-loop system may also become unstable or show an oscillating behavior. This undesired phenomenon is called the “*windup effect*”. The “wind up” situation is present in this system, and an anti-windup design must be included in the controller in order to maintain control of the plant (Tsai, [26]). The MPC is formulated based on an optimization mathematical framework which allows a very straight forward formulation of the input and output constraints (Arora [52], Venkataraman [53], and Elster [54]). Therefore, the wind up problem does not arise in MPC formulation.

1.6.1.2 Input-Output Models

The other problem in the HyPer control design is the lack of a model based on first principles to study the complex dynamics present among the different equipment and processes of this hybrid system. For instance, the compressor/turbine dynamic is not easy to characterize in a mathematical framework, the internal physical properties of the heat exchanger are not supplied by the manufacturer and are unknown. In addition, the valves are nonlinear in their response; the

chemical processes of the FC are complex, as are the fluid and thermal characteristic of the system. Fortunately, the availability of measured data in the HyPer system offers a good path to establish dynamical models based on experimentally measured input/output relationships. Typically, as described by Qin et al. [30], Camacho et al. [43], and Maciejowski [55], the MPC is mathematically formulated using system identification plant models.

1.6.1.3 The Receding Horizon Idea

In MPC control, a sequence of “optimal” control actions is computed to achieve a desired plant state at some point P time steps in the future. Although the entire sequence is computed, only one or perhaps a few of the control actions is initiated before re-computing the optimal sequence and starting over. This method is used to mitigate problems that could arise from model inaccuracy, external system upsets or any unexpected system change during the control interval that was not predicted by or included in the model (Seron [78]).

The receding horizon allows the controller to account for any changes in set-points or in the plant model and to make high quality decisions concerning control. The plant operates with this *constant* input until the controller obtains new measurements and *totally revises its plan*. This sequence is repeated indefinitely. Recalculation at each sampling instant is essential for good control. That is, the receding horizon optimization strategy is really a time-invariant state feedback control law while the system evolves.

1.6.1.4 Computing the Optimal Inputs

The MPC solves an optimization problem similar or identical to LQG optimal control. The main difference is that the optimization problem includes input/output constraints and also rate constraints. Thus, the inputs applied to the plant are considered optimal to accomplish an objective function and to maintain the input/output constraints inside some upper and lower bounds.

1.6.1.5 Multivariable Control Systems

One advantage of MPC design (relative to classical multi-loop control) is that it generalizes directly to plants having multiple inputs and outputs. Moreover, the plant can be *non-square*, i.e., having an unequal number of actuators and outputs. Industrial applications involving hundreds of actuators and controller outputs have been reported.

1.6.1.6 The Capacity of Anticipation

MPC has the ability to react before an actual set-point change is commanded. Without anticipation, the controller starts responding as soon as the set-point change is activated. The phenomenon of anticipation appears to be strongly related to the relative size of the control horizon with respect to the prediction horizon. Figure 1.22 (van Essen et al. [38]) shows an example of anticipating turbine speed response for different settings of the control horizon (m samples) and for a prediction horizon of 12 samples. For instance, the simulated control scenarios could help to design the system response and configure the controller to anticipate the cathode air flow.

Figure 1.22 Anticipation response of turbine speed for different setting of the control horizon (m samples) for a prediction horizon of 12 samples (obtained from [38])

1.6.1.7 The Reference Trajectory

MPC can also follow reference trajectories by different means, including overshooting or softening the tracking policies. The reference trajectory can be used with a smooth approximation from the current value of the system output towards the known reference by means of a first order system of the form, or other mathematical approximations. It is important to note that the shape of the reference trajectory determine the desired speed of approach to the set-point.

1.7 Research Objectives

This research describes the design and implementation of a full and factorial design of the experiments, performed in the HyPer Facility, to experimentally characterize points of operation, interaction between variables, and a behavior of the system when three bypass valves and the turbine-generator electric load are manipulated simultaneously. The HyPer facility, as any hybrid system, is highly nonlinear in much of its behavior and is difficult to model accurately from first principles. This research shows the results of a factorial 3^4 design of the experiments for different parameters in the turbomachinery and the SOFC as functions of the temperature, pressure, and air flow in the cathode inlet. It is the first time that the HyPer facility has been operated or experiments have been done while manipulating all three valves and the electric load at the same time. It is also important to mention that dynamic experimental data were recorded when the factors were step changed between treatments as part of the testing.

Also this research describes the methodology of design of a model predictive controller that satisfies a control strategy proposed in this work. The control strategy addresses the minimum requirements to integrate the FC/GT during dynamic excursions. The MPC design uses as input variables the same valves employed in the DoE experiments, plus the manipulation of electric load to control the turbine speed and cathode air flow as the main manipulated variables of operation.

The approach selected to accomplish the objectives proposed here is to design and perform a factorial design of experiments in order to determine the relationship between variables, record

the transient step changes in the experiment to create the dynamic model, and finally design a MPC algorithm to drive the plant in the desired optimal trajectories.

1.7.1 Specific Objectives

The specific objectives of this dissertation are to expand the knowledge of the HyPer facility performance, as follows:

- First, a multidimensional operating space was characterized based on the manipulation of the different bypass subsystems and the electric load imposed on the turbine. Characterization of operation plays an important role because it determines the impact of manipulated variables over the controlled parameters, and allows description of the related behavior, and detection of strong or weak coupling between them. In addition, replicates of the experiments allowed checking repeatability of the system.
- Second, an extended envelope of operation and the off-design FC/GT integration analysis has been obtained from the DoE data.
- Third, a steady state hybrid-parameter regression model was obtained to describe the relationship between controlled and manipulated variables.
- Fourth, a steady state analysis was performed which provides general guidelines about the optimal control policy for fuel and mass flow with active state constraints during operation. From the analysis, a set of control-design guidelines is presented to select the gains and parameters of the control scheme.
- And fifth, the development of a MPC control algorithm was undertaken to control the facility by manipulating simultaneously the three valves used during these experiments. The study was developed to predict the setting of controlled variables capable of guiding the system along the optimal trajectories under several constraints

imposed by the system itself. This is the first time that MPC control strategy has been applied to SOFC/GT hybrid system.

The outcomes of this work will be to increase knowledge of the process and to identify new potential methods to regulate airflow, manage thermal gradients, and mitigate compressor stall and surge during operational transients using a multi-input envelope based on the manipulation of the three bypasses simultaneously.

1.8 Contributions to State of the Art

From objectives 1, 2 and 3: The experimental plant behavior considering interaction of variables has never been defined completely. The plant behavior results in the mapping of the state space and the understanding of variable combinations to obtain steady state optimal points of operation.

From objective 4: In the past, only uncertain theoretical models and one factor at a time experiments were used to define steady state plant behavior as the basis of the control systems. This new approach enables us to extend the system envelope and to define optimum trajectories for moving between system operating states.

From objective 5: The experimental plant behavior and the experiments performed from Objective 1 were used to design the predictive control system of the HyPer facility.

The results of this research contribute to improvement of the present and future of hybrid systems, and in particular the HyPer System.

Chapter 2

Control Strategy

2.1 Control Strategy

The role of the hybrid control system is to monitor, control, and coordinate the components of the hybrid system in order to keep the FC and GT operating at safe, stable optimal conditions. During operation, many variables such as turbine speed, stack temperature, anode fuel flow, anode-cathode differential pressure, etc. need to be monitored and controlled. The main interest in this hybrid system is to try to control the turbine speed and the anode-electrolyte-cathode membrane temperature. Other hybrid phenomena must be addressed in a whole perspective of control of the integrated FC/GT system such as anode-cathode differential pressure, compressor stall, degradation, flooding, etc. However, at this point in the history of the research performed on hybrid system, the design of a controller suitable to control the turbine speed and the stack temperature would represent a tremendous advance in the hurdles of hybrid technology. In order to control these variables, two different cases have been identified. First case is when the FC load is increased from its nominal operation. Second case is when the load is suddenly decreased in the FC.

2.1.1 Dynamics of FC Load Increase

Figure 2.1 displays a sequential flow diagram of a step increase in the FC load and the transient reaction of the FC and GT variables. When the load is increased in the FC, the anode fuel is converted electrochemically to power output in the FC, and less unreacted fuel is allowed to come to the combustor, initially lowering the heat coming out from the FC. The electrochemical reaction takes place very rapidly; increasing the fuel utilization factor more quickly than the fuel can be replaced by opening the fuel valve to the anode. The total thermal power flowing to the turbine decreases initially. In addition, the FC average temperature will increase because the increasing activity of the FC, but this happens at a slower rate due to the large thermal mass of the FC stack. On the turbine side, this “low heat” coming from the FC

would act to lower the turbine speed, which is unacceptable for either synchronous generation systems or for variable speed systems, since it would lead to decreased cathode air flow at a time when increased waste heat is being generated by the fuel cell. Instead, the turbine speed must be controlled as is done in an actual GT cycle, by injecting auxiliary fuel into the system at the combustor.

Figure 2.1 Dynamic effect of the hybrid integrated system after of a FC load increased.

2.1.1.1 Control Action for FC Load Increase

In this situation, it is necessary to add additional thermal power to the turbine inlet to maintain constant turbine speed. From previous experiments, Tucker et al, [3]and [4], have shown that this can be achieved by closing the CA valve, and/or adding supplementary fuel to the combustor, as is done in a typical simple gas turbine cycle. Alternatively, the turbine load

can be decreased, either by decreasing the generator load or by closing the BA valve, if it is open. In addition, it is necessary to increase the cathode air flow to mitigate any increase in stack temperature. This can be done by closing the CA and HA valves from their nominal point of operation. Figure 2.2 displays the control action strategy necessary to mitigate the effect of lowering temperature in the FC and increasing speed in the turbine. In the meantime, the anode fuel flow must be increased to achieve steady state for the new operating point.

Figure 2.2 Control action over the FC and GT to mitigate the adverse dynamic effects of the FC load (increased).

2.1.2 Dynamics of FC Load Decrease

The second case is presented when the load is suddenly decreased in the FC. Figure 2.3 shows a flow diagram of a FC with the load decreased from its nominal operation and the transient reactions of the FC and GT variables. In this case, the electrochemical reaction of the FC is lowered very quickly, decreasing heat power generated (Q_g) inside the FC, and increasing

the unreacted fuel coming from the anode circuit. The unreacted fuel is burned in the combustor, so the total heat effluent power is increased, and the turbine speed will also increase. Control of heat effluent power in the FC has a large time constant compared to turbine speed. Thus, it is necessary to take quick action to control turbine speed. During this dynamic excursion the FC average temperature decreases, since the heat generated by the electrochemical process in the FC is reduced. At this instant, the cathode air flow must be decreased to compensate for the reduced average temperature in the FC.

Figure 2.3 Dynamic effect of the hybrid integrated system after of a FC load decreased.

2.1.2.1 FC load Decrease Control Action

One way to quickly control the turbine speed in this case is to increase the turbine-generator load, but this may not be feasible because the total electric load is decreasing. The other approach identified by Tucker et al. ([3] and [4]), to decrease speed is opening CA and BA valves. The CA valve has the faculty when opening to decrease speed, decrease the cathode inlet

flow, and at the same time decrease TIT. But the CA valve is limited in its range of operation and thus its ability to control turbine speed. The BA valve can also accomplish or help the CA valve in this assignment. The BA valve has the same effect on the turbine shaft as the generator load (Tucker et al. [3], and [4]). Thus, the proposed strategy is to open the CA and BA valves to absorb the “high” thermal power output of the FC and control the turbine speed. The BA valve should be used just during the transient excursions of the FC, while providing enough time for the anode fuel flow controller to reduce the anode fuel flow. In addition to controlling turbine speed, it is necessary to avoid decreasing the FC stack temperature. This is achieved by opening the CA valve to reduce the amount of air flow through the FC cathode. The HA valve can also be opened to assist with the objective of lowering cathode air flow. Figure 2.4 illustrates the sequence of control actions necessary to mitigate the effects of a sudden drop in the electric load on the FC.

These two different cases described in detail above clearly justify the use of a MIMO controller to control the two most important variables of FC/GT hybrid integration: turbine speed and cathode air flow (FC temperature).

Based on the transient data collected during the design of experiments presented and detailed in chapter 3, a linear dynamic model will be built using system identification techniques and a model predictive controller will be designed to control turbine speed and cathode air flow in chapter 4.

Figure 2.4 Control action over the FC and GT to mitigate the adverse dynamic effects of the FC load increased.

2.1.3 Anode-Cathode Pressure Differential

Even a modest difference between anode and cathode pressure can cause damage in the FC by physical stress or delamination on the electrolyte until it ruptures. No more than a few kilopascals are thought presently to be sustainable across the electrolyte during operation. At this instance, nothing has been mentioned in the control strategy about this important controlled variable. As shown by Tsai et al. [28], using an RGA analysis, an ill-conditioned situation is presented in the FC with the actuators and the control output variables. The cathode pressure, turbine speed and cathode air flow cannot all be controlled at the same time using only the bypass valve positions as manipulated variables. A control strategy is suggested here, but is not

included in this work, because it has to be designed on the anode flow side and it is an independent controller, as described next.

In order to control the anode-cathode difference, the cathode pressure can take on a small range of values depending on the positions of the valve actuators, the ambient temperature and pressure, and the turbine load. Cathode pressure is primarily dictated by compressor speed, but can vary over a range of 10 or more kPa with the other parameters. We propose to mitigate the occurrence of large pressure differentials across the electrolyte by manipulating the anode pressure to track cathode pressure. This suggestion is prompted by several circumstances:

- The anode will be largely isolated from ambient conditions due to its position downstream of the gasifier and fuel conditioning equipment,
- Mass flow rates through the anode are much smaller than those through the cathode,
- The compressor for the anode gases is not constrained to run at a synchronous speed with the grid,
- The anode circuit is much less complex and less prone to instability than the cathode circuit.

The control of the anode side pressure thus seems a reasonable candidate for FC pressure management. The anode controller must be designed to simultaneously control FC anode fuel flow and pressure. This can be accomplished using a variable speed compressor with a bypass valve. The set value for anode pressure must be read from the cathode side. Thus, a tracking pressure problem could be the solution to maintain minimum anode-cathode pressure difference.

Chapter 3

Design of Experiments

A full factorial experimental design and a replicated fractional factorial design were carried out using the HyPer project facility installed at the National Energy Technology Laboratory (NETL), U.S. Department of Energy (DOE) to simulate FC/GT hybrid power systems. The HyPer facility uses hardware in the loop technology that couples a modified recuperated GT cycle with hardware driven by a SOFC model. A 3^4 full factorial design (FFD) was selected to study the effects of four factors: cold-air, hot-air, bleed-air bypass valves, and the electric load on different parameters such as cathode and turbine inlet temperatures, cathode pressure and air mass flow. The results obtained, compared with former results where the experiments were made using one-factor-at-a-time, show that no strong interactions between the factors are present in the different parameters of the system. This work also presents a fractional factorial design (ffd) 3^{4-2} in order to analyze repeatability of the experiments. In addition, a new envelope is described based on the results of DoE, compared with OFAT experiments, and analyzed in an off-design integrated FC/GT framework. This work describes the methodology, strategy, and analysis of the results of these experiments that bring new knowledge concerning the operating state space for this kind of power generation system.

3.1 Experimental Method and Results

The approach addressed to accomplish the design of experiments objectives is to design, and perform a FFD of experiments in order to determine the relationship between variables, record the steady states parameters, analyze the data using statistical DoE tools, and finally present and discuss the significance of different DoE plots. As part of the methodology, a fractional factorial approach is designed to look at for repeatability of the data and to get more system performance reliability.

The purpose of this operational test was to map an envelope that represents a space of operation of the whole system. All the variables (inputs) were manipulated simultaneously under the limitations imposed by the environment and the equipment itself. In order to maintain complete independence of the treatments and minimize systematic noise, the experiments were carried out randomly. This randomized test sequence was necessary to prevent the effects of unknown nuisance variables, for example, the warm up effect of the equipment or environmental conditions. Ambient temperature, pressure, humidity, constant turbine speed, stall margin, and limited temperatures of operation were constraints of the experiments.

3.1.1 Experimental Variables

3.1.1.1 Input Selection and Levels of the Treatment.

Scoping tests were done prior to the experiments to determine the operation at limits due to possible surge or stall, overheating or insufficient air in the combustor caused by the extreme positions of the bypass valves and the high demands of the electrical load. The input variables to conduct the experiment are the Cold Air valve (CA), Hot Air valve (HA), Bleed Air valve (BA), and Electric Load (EL). The valve positions and the electric load setting values tested in the experiments were all command positions. A three level FFD of the experiments was selected to take into account the curvature in the output variables of the system and because of a priori knowledge [2], [3] and [4] of the nonlinearities of the system. The levels of each variable were selected based on the constraints imposed by the system and the experience in the use of the equipment, and are summarized as follows: All the factors were set to specific fixed levels and performed in a full 3^4 combinatorial factorial design.

3.1.1.2 Cold Air Valve

The three level positions selected for the CA valve are: 40, 60, and 80% opened (1, 2, and 3). The lower limit of 40% is selected for safety reasons because the compressor presented the onset of surge below about 30% CA valve opening. Also in preliminary tests the compressor stalled above 80% CA valve opening. The position 60% was selected in order to maintain symmetric levels in the experiment design. Valve position changes performed in former research studies [4] showed that the mass flow is nonlinear with respect to CA valve changes. At low opening of the

CA (0 – 15%) the air mass flow change through the compressor was very small; between (15 – 70%) has significant variation, but in practical terms looks linear; and for 70% and beyond the flow change was practically nothing. In Figure 3.1 it is shown that the same change percentage in the valve (30%) has different mass flow changes. The change in mass flow through the valve was 0.377 kg/s when the valve opens from 40 to 70%, and it was less than half that (0.151 kg/s), when the valve was opened from 70 to 100%.

The system parameters are also nonlinear with respect to the mass flow bypassed in the CA valve. For instance, Figure 3.2 shows the presence of nonlinearities in the turbine inlet temperature with respect to mass flow. In one case, a 10% change in the CA valve position (from 40% open to 50% open) produced an air flow change of 0.148 kg/s and a turbine inlet temperature drop of 24 K. However, when the valve was opened from 70 to 100% (30% change) approximately the same amount of flow change was experienced (0.151 kg/s), but the turbine inlet temperature changed just 1.5 K.

Figure 3.1 CA mass flow as a function of CA valve position at 50 kW electric load.

Figure 3.2 Turbine inlet temperature as a function of CA valve position at 50 kW electric load.

3.1.1.3 Hot Air Valve

The three level positions selected for the HA valve are: 20, 50, and 80% opened (1, 2, and 3). The 20% and 80% openings were selected because below 20% and above 80% the nonlinear HA valve showed in preliminary tests to have the same mass flow characteristic. The amount of air mass flow bypassed by the HA valve was very similar to the CA valve. The 50% value was selected to maintain symmetry in the design of the experiment.

3.1.1.4 Bleed Air Valve

The three level positions selected for the BA valve are: 90, 88, and 86% closed (1, 2, and 3). BA is used in the system as a safety valve, remaining closed in regular operation. However, plots shown in this work used the values 10, 12, and 14% open for better comparison and presentation of the results. The limit 90% closed was chosen because at this value the valve is in effect practically closed. The value 86% was selected due to two reasons: 1) high openings in the BA valve increase the skin temperature in the combustor, and could exceed the limit value of 1600 K, and 2) high openings of the BA valve combined with the high levels of CA and HA valves increase the equivalence ratio in the combustor to levels over the safe limit.

3.1.1.5 Electric Load

The three levels selected for the Electric Load bank were: 0, 25, and 50 kW (0, 1, and 2). Electric load above of 50 kW could lead to surge in the compressor.

The levels selected for the four factors based on a screening test where the equipment was run close to the limits of operation for different parameters such as temperature and stall margin are summarized in Table 3.1:

Table 3.1 Test levels (set-points) for valves and electric load.

Factor	Symbol	Low Level	Intermed. Level	High Level
		1	2	3
<i>CA Valve (%)</i>	CA	40	60	80
<i>HA Valve (%)</i>	HA	20	50	80
<i>BA Valve (%)</i>	BA	10	12	14
<i>Electric Load (kW)</i>	EL	0	25	50

3.1.1.6 Output Parameters.

The parameters (outputs) of interest measured in the facility and available for analysis are:

- Fuel Flow (FT432)
- Compressor Inlet Air Flow (FT110)
- Bleed and Cold Air Bypass Flow (FT162)
- Cathode Inlet Air Flow (FT380)
- System Total Pressure Drop (PDT158)
- Compressor Outlet Pressure (PT151)
- Turbine Inlet Pressure (PT180)
- Cathode Inlet Pressure (PT305)
- Turbine Exhaust Temperature (TE202)
- Cathode Inlet Temperature (TE326)
- Turbine Inlet Temperature (TE350)

In this work not all the above listed parameters are selected for analysis. The turbine speed was not analyzed as an output parameter because the test was performed under turbine speed feedback controlled by the fuel valve. The turbine speed Proportional + Integral control was

used to keep turbine/generator running safe during the abrupt step changes carried out during the test of the experiment. The 40,500 RPM turbine speed set-point is needed to run the generator at nominal current frequency and avoid failure of the generator by heating of its components. The parameters analyzed here are directly related to the integrated performance between FC and GT, and they are listed below:

- Fuel Flow (FT432)
- Cathode Inlet Air Flow (FT380)
- Cathode Inlet Temperature (TE326)
- Turbine Inlet Temperature (TE350)

A simple combination at specific levels of the factors constitute a treatment (i.e. CA= 40%, HA=80%, BA=14%, EL=25 kW). A *test number* was assigned to each treatment in the the experiments. This test number was based on an ordered arrangement of all level combinations of the factors. The number of the treatments based on the design order is “*Standard Order*” and they are listed in

Table 3.2.

The treatments were performed in a random order. Therefore, a *test sequence* is the actual order that the treatments were run in the facility after randomization. The number of the treatments in a test sequence is called *Run Order*” and they are listed in Table 3.3. For instance, the treatment (observation) CA=40%, HA=20%, BA=14%, EL=50kW labeled in the designed list with the *standard order* number 55 was performed first (1st) in the *Run order* list”. This randomized test sequence was necessary to prevent the effects of unknown nuisance variables, for example the warm up effect of the equipment and the environmental conditions.

Table 3.2 Standard order of treatments designed for the experiments.

Exp.	CA	HA	BA	EL		Exp.	CA	HA	BA	EL		Exp.	CA	HA	BA	EL
1	40	20	10	0		28	40	20	10	25		55	40	20	10	50
2	40	20	12	0		29	40	20	12	25		56	40	20	12	50
3	40	20	14	0		30	40	20	14	25		57	40	20	14	50
4	40	50	10	0		31	40	50	10	25		58	40	50	10	50
5	40	50	12	0		32	40	50	12	25		59	40	50	12	50
6	40	50	14	0		33	40	50	14	25		60	40	50	14	50
7	40	80	10	0		34	40	80	10	25		61	40	80	10	50
8	40	80	12	0		35	40	80	12	25		62	40	80	12	50
9	40	80	14	0		36	40	80	14	25		63	40	80	14	50
10	60	20	10	0		37	60	20	10	25		64	60	20	10	50
11	60	20	12	0		38	60	20	12	25		65	60	20	12	50
12	60	20	14	0		39	60	20	14	25		66	60	20	14	50
13	60	50	10	0		40	60	50	10	25		67	60	50	10	50
14	60	50	12	0		41	60	50	12	25		68	60	50	12	50
15	60	50	14	0		42	60	50	14	25		69	60	50	14	50
16	60	80	10	0		43	60	80	10	25		70	60	80	10	50
17	60	80	12	0		44	60	80	12	25		71	60	80	12	50
18	60	80	14	0		45	60	80	14	25		72	60	80	14	50
19	80	20	10	0		46	80	20	10	25		73	80	20	10	50
20	80	20	12	0		47	80	20	12	25		74	80	20	12	50
21	80	20	14	0		48	80	20	14	25		75	80	20	14	50
22	80	50	10	0		49	80	50	10	25		76	80	50	10	50
23	80	50	12	0		50	80	50	12	25		77	80	50	12	50
24	80	50	14	0		51	80	50	14	25		78	80	50	14	50
25	80	80	10	0		52	80	80	10	25		79	80	80	10	50
26	80	80	12	0		53	80	80	12	25		80	80	80	12	50
27	80	80	14	0		54	80	80	14	25		81	80	80	14	50

Table 3.3 Arrangements of “run order” and “standard order” for the test.

Run Order	Std Order		Run Order	Std Order		Run Order	Std Order
1	55		28	24		55	50
2	10		29	64		56	13
3	53		30	57		57	1
4	52		31	25		58	12
5	58		32	36		59	33
6	43		33	76		60	21
7	2		34	19		61	37
8	56		35	78		62	4
9	54		36	14		63	81
10	3		37	18		64	77
11	9		38	66		65	22
12	34		39	39		66	65
13	28		40	17		67	71
14	11		41	45		68	63
15	46		42	75		69	79
16	60		43	70		70	51
17	7		44	31		71	5
18	67		45	35		72	16
19	6		46	80		73	30
20	29		47	47		74	73
21	32		48	68		75	69
22	38		49	40		76	8
23	41		50	42		77	23
24	48		51	20		78	72
25	61		52	15		79	74
26	49		53	62		80	44
27	27		54	26		81	59

3.2 Test Procedure

The 3^4 full factorial design (81 treatments) was performed in the HyPer facility during approximately five days of continuous operation. Compressor inlet conditions between testing of separate valves varied with the normal seasonal ambient conditions in temperature and humidity. The temperature inside the test facility was maintained at a fairly consistent level, varying little during the day and between tests. The minimum and maximum ambient temperatures recorded during this test were 303 K and 313 K, respectively. And the compressor inlet varied between 306 and 316 K.

The tests were initiated by running the turbine up to an operational steady state nominal speed of 40,500 rpm, with a compressor bleed-air valve setting of 16% open, and a load of 50kW applied to the generator to provide an initial preheat of system equipment. The experiments were run varying the CA, HA, and BA valves, with the EL simultaneously. All of the HyPer variables were combined to develop a full factorial experimental design analysis.

The tests were carried out after preheating the system (T344 around 800K). The command for the three bypass valves and the electric load were set simultaneously in order to record the transient behavior between treatments. After the settings, the slowest (rate of change) temperature of the system, the post-combustor skin temperature (T344), was monitored for the steady state condition. This took between 40 to 75 minutes, depending on the electric load, valve positions and the size of the step change relative to the former set of values. The factor which most affected the waiting time was the electric load. Changes in electric load demanded changes in fuel burned in the combustor and in consequence, more time for the system to achieve thermal equilibrium itself and with the environment. Therefore, enough time had to be left to allow the system to cool or heat up completely. After that, a time period of about 10-15 minutes was left in order to average the steady state data. As abovementioned, the order of the treatments was decided randomly and after each treatment was set and achieved steady state, a new treatment was set in the facility. The steady state temperature of the system is defined as when the skin temperature of the post-combustor is changing less than 1°C. The steady state “maintained time” used in former experiments carried out in the facility was between 2 to 6 minutes before any further change in process conditions. In DoE experiments, when a steady state condition was

reached, the system was maintained at this condition for about 8 minutes. Appendix B details the test plan purpose and the main procedure concern advised for the experiments.

3.2.1 Experimental Constraints

A HyPer facility pre-test was performed before the experiments to screen the operation points and determine the maximum or minimum values of operation for the factors when they were manipulated simultaneously. The temperature of the plenum, temperature in the combustor, stall margin, equivalence ratio, and large change in load were the most important constraints during performance. Compressor inlet conditions during the 5 continuous days of the experiment varied with the normal seasonal ambient conditions in pressure and humidity.

Airflow management was evaluated at steady-state conditions and under this presumption, the FC was considered to operate at the steady conditions. In other words, the dynamic fuel cell model was not used in the real time simulations. Turbine speed was maintained at nominal conditions throughout by a standard PI fuel valve controller.

3.2.2 Data Processing

For the plots presented in this study, the processed data were collected at a 400ms sampling rate and averaged over a 120s (300 samples) time period upon reaching steady conditions. The steady state condition was verified by calculating standard deviation for different parameters in three separate periods of time, 240, 400, and 480s (600, 1000 and 1200 samples respectively). These standard deviations were all close to the values obtained in the 120s period. Table 2 shows the average and standard deviation of different output variables as functions of the number of samples.

After the 3⁴ design of experiments was tested and a preliminary statistical analysis of the data was done, some preliminary conclusions could be obtained. Table 3.4 shows the average and standard deviation values for different parameters using different time averages (sample sizes) for the treatment CA=40%, HA=20%, BA=90%, and EL=50 kW. The consistency between averages and standard deviations for different sample sizes shows that the definition of steady state value of this particular treatment is very accurate. Most of the parameters show a close and

repeatable average value for different sample sizes. The fuel flow showed the higher standard deviations for sample sizes of 600 and 300 samples. This is due to the flowmeter sensor, which has high variability (noise) in measuring the fuel flow.

Table 3.4 Parameter averages and standard deviations as a function of sample size for CA=40%, HA=20%, BA=90%, and EL=50 kW.

Parameters →	FT432		FT110		FT380		PDT158		PT151B	
	(g/min)		(kg/s)		(kg/s)		(kPa)		(kPa)	
Sample Size (time s)	Avg.	Std Dev	Avg.	Std Dev	Avg.	Std Dev	Avg.	Std Dev	Avg.	Std Dev
1200 (480)	852	2.34	2.03	0.019	0.925	0.007	23.1	0.071	351.7	0.514
1000 (400)	851	2.38	2.03	0.019	0.925	0.007	23.1	0.068	351.8	0.469
600 (240)	846	5.50	2.03	0.020	0.923	0.007	23.1	0.073	351.7	0.673
300 (120)	849	4.74	2.03	0.020	0.924	0.007	23.1	0.065	351.7	0.656
Parameters →	PT180A		PT305B		TE202A		TE326A		TE350A	
	(kPa)		(kPa)		(K)		(K)		(K)	
Sample Size (time s)	Avg.	Std Dev	Avg.	Std Dev	Avg.	Std Dev	Avg.	Std Dev	Avg.	Std Dev
1200 (480)	326.1	0.49	342.1	0.62	761.3	0.59	750.1	0.507	1020.2	1.147
1000 (400)	326.2	0.44	342.2	0.56	761.2	0.60	750.0	0.519	1020.3	1.136
600 (240)	326.1	0.68	342.1	0.73	761.6	1.06	752.3	2.195	1020.1	1.806
300 (120)	326.1	0.65	342.1	0.70	761.2	0.854	750.5	0.822	1019.7	1.838

3.2.3 Summary of Bypass Testing

Table 3.5 shows the minimum and maximum average values for different parameters and the corresponding setting of the factors where the extremum was achieved. Also, it is shown their respective standard deviation, variance, and coefficient of variation. The compressor air mass flow (FT110) has shown to be very consistent at providing between 2.0 and 2.2 kg/s of air to the system. The measurement of the air mass flow has a very low variability as observed by the values of variance in the Table 3.4 and Table 3.5. The fuel flow measurement has the highest variance compared to any other parameter in the table. The consistency in the pressure and temperature is also high, as shown by the low values of variance. The sample size used to build this table was 300 samples equivalent in time to 120 seconds. The coefficient of variation is a ratio that represents the relationship between noise and measurement, and it is calculated by:

$$\text{Coefficient of variation} = 100 \times \frac{\sigma(\text{std Deviation})}{\bar{x}(\text{Average})} \quad (3.1)$$

These values were very low for most of the parameters found in the Table 3.5.

Table 3.5 Minimum and maximum average values for the different parameters.

Parameters ↓		Treatment				Average	Standard Deviation	Variance σ^2	Coefficient of Variation (%)	Run Order
FT432 (g/min)	Min	40	80	10	0	653.2	2.199	4.84	0.337	17
	Max	80	20	14	50	986.3	6.775	45.90	0.687	42
FT110 (kg/s)	Min	40	20	10	50	2.03	0.020	0.000	1.010	1
	Max	80	20	12	0	2.21	0.024	0.001	1.090	51
FT162 (kg/s)	Min	40	80	10	50	0.42	0.014	0.000	3.380	25
	Max	80	20	14	50	1.07	0.011	0.000	1.001	42
FT380 (kg/s)	Min	80	80	14	50	0.33	0.006	0.000	1.955	63
	Max	40	20	12	0	0.96	0.007	0.000	0.739	7
PDT158 (kPa)	Min	80	80	10	25	12.3	0.077	0.006	0.625	4
	Max	40	20	12	50	23.4	0.209	0.044	0.896	8
PT151 (kPa)	Min	80	20	14	0	346.0	0.622	0.387	0.180	60
	Max	60	50	12	50	360.6	0.982	0.965	0.272	48
PT180 (kPa)	Min	40	20	12	0	322.8	0.523	0.274	0.162	7
	Max	80	80	14	50	343.0	1.120	1.255	0.327	63
PT305 (kPa)	Min	40	20	12	0	337.8	0.596	0.356	0.177	7
	Max	80	20	12	25	354.8	1.325	1.756	0.373	47
T202 (K)	Min	60	80	10	0	629.0	0.701	0.491	0.111	72
	Max	40	20	14	50	776.3	0.634	0.401	0.082	30
T326 (K)	Min	60	80	10	0	623.0	0.546	0.299	0.088	72
	Max	40	20	14	50	761.6	0.414	0.171	0.054	30
T350 (K)	Min	80	20	10	0	846.3	2.697	7.275	0.319	34
	Max	40	20	14	50	1038.5	1.903	3.620	0.183	30

3.3 Results

3.3.1 Full Factorial Design Results

The outputs of the DoE methodology described in the preceding section are presented below. Figure 3.3 shows a 3-D distribution for the different levels of the three different factors. The variation in the fourth factor is shown in two extra 3-D plots similar to this one. The electric load was selected as the fourth variable, so it is fixed for each cube shown.

Figure 3.3 3-D output distribution as function of the different level factors combinations.

As abovementioned, the outputs selected here for analysis are presented in this section. Results for cathode inlet mass flow (FT380), fuel flow (FT432), cathode inlet temperature (TE326), and turbine inlet temperature (TE350) are shown in Figure 3.4, Figure 3.5, Figure 3.6, and Figure 3.7, respectively. The rest of the output variables are shown in Appendix C. These figures do not show any intuitive tendency of variation and so it is hard to say in which direction an output variable is increasing or decreasing. Therefore, a statistical analysis is required to understand the authentic relationship between factors and outputs. An analysis of variance (ANOVA) analysis is the recommended statistical tool to understand the effect of the factors over the output variables.

Figure 3.4 Cathode inlet mass flow as function of the different treatments.

Figure 3.5 Fuel flow as function of the different treatments.

Figure 3.6 Cathode inlet temperature as function of the different treatments

Figure 3.7 Turbine inlet temperature as function of the different treatments

3.4 Analysis of the Design of Experiments

In this section, the FC heat effluent (fuel flow, FT432), the cathode mass flow (FT380), the cathode and turbine inlet temperature (TE326 and TE350, respectively) will be analyzed in terms of the different factors. The statistical software Minitab version 16 is used for this work.

3.4.1.1 Statistical Fuel Flow Analysis (FT432)

The heat effluent from the FC is simulated by burning natural gas in a combustor placed upstream of the turbine. This fuel flow represents the sum of the heat generated by the electrochemical anode-cathode reaction of the fuel-air in the FC, plus the complementary heat when unreacted anode and supplementary fuel is burned downstream to the FC to keep constant turbine speed. Figure 3.8 shows different subplots illustrating the normal nature of the residuals. Normal nature is a basic principle of DoE statistical formulation because it is indicative that the source of error is an *experimental error* and not attribute to a factor of the experiment.

Figure 3.8. Normal distribution of the residual in the fuel flow variable.

Table 3.6 shows an ANOVA analysis of the fuel flow and the F-values are plotted in the histogram of the Figure 3.9. Because the 81 experiments were performed without replication,

the error in the ANOVA analysis was obtained by aliasing the four interaction terms (CA*HA*BA*EL) with the error. It can be seen that the F-value shows how the CA valve and EL have a high impact on the fuel flow. Opening the CA valve increases the fuel consumption, because it reduces the amount of energy recovered by the recuperator. This means in an off-design FC/GT interaction that this valve could be used to absorb heat rejected by the FC while maintaining constant turbine speed. Variation of EL could be also a good way to control speed, depending on whether EL is an independent variable or a perturbation that it is out of control. So, it is better to think of the CA valve as a possible speed and mass flow actuator than EL. The HA valve shows no effect on the fuel flow. Thus, the HA valve can manage air mass flow without perturbing turbine speed significantly. This is the same result reported in [4].

The BA valve was shown in former papers as a possible heat sink for the FC. In this DoE, this effect was not as apparent because the range of the BA valve opening variation was small. This was a constraint imposed by high temperatures in the steady state system operation. However, in this small range, BA had an important effect compared with any one of its other interactions (CA*BA, HA*BA, and BA*EL). BA valve interactions show the lower significant values in the analysis. At this point, it is important to know that the effect of manipulating BA is reflected in the system as if it was operating completely independently. In other words, the BA has the same effect over FT432 acting alone than when the other valves are opening. So, the advantages of using the BA valve to control turbine speed are confirmed by this experiment and BA valve could be used at higher opening values if it is used as a transient mechanism to absorb heat. One possible application would be during the period immediately following a large FC electric load reduction. In this case, the FC quickly decreases the fuel utilization, and the unreacted fuel increases. Therefore surplus fuel and thermal energy are entering to the turbine causing it to overspeed. By opening the BA valve, some of this surplus thermal energy would be absorbed by the compressor work. After the transition finishes, the anode fuel flow is harmonized with the new electric load and the turbine speed, the BA valve must be completely closed in order to save energy. This is clearly a second choice option (after opening the CA valve), because the compressor energy is simply wasted by compressing and then dumping the air to ambient, but it is preferable to allowing the turbine to overspeed. In this work, the largest BA opening was 14%, compared with up to 20% in other OFAT experiments at 0 kW.

Table 3.6 ANOVA analysis of the fuel flow (FT432).

General Linear Model: FT432 versus CA, HA, BA, EL

Factor	Type	Levels	Values
CA	fixed	3	1, 2, 3
HA	fixed	3	1, 2, 3
BA	fixed	3	1, 2, 3
EL	fixed	3	1, 2, 3

Analysis of Variance for FT432, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
CA	2	107401	107401	53701	303.72	0.000
HA	2	6899	6899	3450	19.51	0.000
BA	2	11554	11554	5777	32.67	0.000
EL	2	505399	505399	252700	1429.20	0.000
CA*HA	4	782	782	196	1.11	0.388
CA*BA	4	261	261	65	0.37	0.827
CA*EL	4	3660	3660	915	5.17	0.007
HA*BA	4	469	469	117	0.66	0.627
HA*EL	4	514	514	128	0.73	0.587
BA*EL	4	374	374	94	0.53	0.716
CA*HA*BA	8	362	362	45	0.26	0.972
CA*HA*EL	8	1343	1343	168	0.95	0.505
CA*BA*EL	8	1440	1440	180	1.02	0.461
HA*BA*EL	8	1798	1798	225	1.27	0.324
Error	16	2829	2829	177		
Total	80	645086

Figure 3.9. Computed F-value for the fuel valve, FT432

Figure 3.10 shows the interaction plot. The lines are very nearly parallel in most of the cases, meaning that interactions are not present in these particular cases. Lines like this mean that the

factors behave in the same way if they act separately or together. Neither the interactions nor the BA and HA valve main effects have any significance in the fuel flow for the ranges selected here. Thus, CA and EL have the most important effect on the heat effluent management. The column 1 (CA column) in Figure 3.10 has a slight inclination. This is an indicative that the CA valve has some effect on the fuel consumption. Column 2 and Column 3 (HA column, and BA column, respectively) are practically horizontal. This means that they have little effect on the fuel flow response. Column 4 is the electric load. This factor shows the highest slope in the plot, indicative of that is the factor with highest effect over the fuel flow.

Figure 3.10. Interaction effect plots on fuel flow mean.

In the former analysis, it was seen that the main factor affecting the fuel flow is the EL. In the case of a sudden electric load decrease, the heat effluent from the FC will increase due to the burning of the unreacted fuel coming from the anode. The extra thermal energy arriving to the turbine would have to be absorbed in order to keep constant speed. A mechanism available to manage this transient situation is the BA valve as described before.

3.4.1.2 Regression Equation for Fuel Flow (FT432)

Three different regression models are shown in Table 3.7. First, the linear model (not considering any cross terms in the equation). Second, is a model that considers the individual factors, and all of the two and three cross terms of the equation. And third, is a reduced regression model that consider the most significant terms obtained of the ANOVA analysis of model 2.

Appendix D shows the ANOVA analysis of the coefficients and the R^2 correlation for the three models. The R^2 value shows that regression model 1 explains 97.5% of the variance in fuel flow, indicating that the model fits the data fairly well. Model 2 considers all the cross interaction terms and the improvement with respect the model 1 is less than 1%. Model 3 has practically the same R^2 value that model 2. Model 3 is selected to predict for specific values of fuel flow at steady state. The regression equations presented in Table 3.5 are functions of the level values 1, 2 and 3. In order to convert percent position values to levels values in the models the following expressions can be used:

$$\begin{cases} CA^* = 1 + \frac{CA-20}{30} \\ HA^* = 1 + \frac{HA-20}{30} \\ BA^* = 1 + \frac{BA-86}{2} \end{cases} \quad (3.2)$$

Table 3.7 Fuel flow (FT432) regression equations.

Model 1 ($R^2 = 97.5\%$)	FT432 = 519 + 44.3 CA - 11.2 HA + 14.2 BA + 96.7 EL
Model 2 ($R^2 = 98.2\%$)	FT432 = 548.588 + 29.267CA + 4.408HA + 1.178BA + 75.571EL - 8.629CA*HA + 2.948CA*BA + 12.305CA*EL + 1.048HA*BA - 4.310HA*EL + 7.805BA*EL + 1.693CA*HA*BA + 1.727CA*HA*EL - 2.968CA*BA*EL - 1.372HA*BA*EL
Model 3 ($R^2 = 98.1\%$)	FT432 = 547.275 + 28.254CA - 7.602HA + 15.94BA + 78.824EL - 1.789CA*HA + 9.823CA*EL - 0.875BA*EL

Figure 3.11 shows the residual values as functions of fitted values of the fuel flow regression models 2 and 3. Both models illustrate practically the same range of error in the residuals. The

fuel flow measurement as shown above has the highest variance value in the experiment. The maximum variance value was achieved for the treatment CA=80%, HA=80%, BA=90%, and EL=50% with a standard deviation of 15.94 g/min (see Appendix E). Therefore, in a confidence interval of 95% (corresponding to 3 standard deviations of probability), a prediction error around 40 g/min is inside of the range of deviation achieved in the experiments.

Figure 3.11. Residual vs fitted value of fuel flow for regression models 2 and 3.

Figure 3.12 and Figure 3.13 show the surface and contour plots of the fuel flow as a function of CA and EL. The surface shows some curvature corroborated by the smooth curve line in the contour plot. The range of response of fuel flow was between 660 and 950 g/min in this DoE.

Figure 3.12. Surface plot of fuel flow as a function of EL and CA valve.

Figure 3.13. Contour plot of fuel flow mean as a function of EL and CA valve.

3.4.1.3 Cathode Inlet Mass Flow Statistical Analysis

Figure 3.14 shows the normal residual behavior of the cathode mass flow. The residuals are very small compared with the measured values of the air mass flow.

Figure 3.14. Normal distribution of the residual in the cathode flow variable.

Table 3.8 and Figure 3.15 show the F-value for all of the factors and their interactions. It is shown that the HA valve has the most significant effect on the air mass flow to the cathode, follow by the CA valve. But remember, the range of operation for the HA valve in this experiment was higher (20-80%) than the CA (40-80%). In previous experiment [3] and [4], the CA and HA valves showed the same capacity of bypass when they were opened 100%. The CA and HA valves opening decrease the cathode flow. Meanwhile, the BA valve and the EL do not show major effects on the cathode flow. The CA and HA valves show a small interaction compared with the main effect. But, they act predominantly independent of each other. For control, the CA valve has the capacity to absorb FC heat, the HA valve is indifferent to FC heat, but HA valve is robust to control cathode air flow. In previous work the minimum flow to the cathode was found using the CA valve, and it was around 0.7 kg/s. Using simultaneous operation of the CA and HA valves will allow supply to the cathode of the FC a minimum

amount of around 0.3 kg/s. This is an important difference taking into account that this flow is used to control the temperature of the FC during a turn down in the electric demand, keeping the FC safe from temperature gradients.

Table 3.8 ANOVA analysis of the cathode air mass flow (FT380).

General Linear Model: FT380 versus CA, HA, BA, EL

Factor	Type	Levels	Values
CA	fixed	3	1, 2, 3
HA	fixed	3	1, 2, 3
BA	fixed	3	1, 2, 3
EL	fixed	3	1, 2, 3

Analysis of Variance for FT380, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
CA	2	0.595830	0.595830	0.297915	2767.57	0.000
HA	2	1.976297	1.976297	0.988148	9179.70	0.000
BA	2	0.000165	0.000165	0.000082	0.77	0.481
EL	2	0.011448	0.011448	0.005724	53.18	0.000
CA*HA	4	0.064677	0.064677	0.016169	150.21	0.000
CA*BA	4	0.000104	0.000104	0.000026	0.24	0.911
CA*EL	4	0.000271	0.000271	0.000068	0.63	0.649
HA*BA	4	0.000467	0.000467	0.000117	1.09	0.397
HA*EL	4	0.002010	0.002010	0.000502	4.67	0.011
BA*EL	4	0.000359	0.000359	0.000090	0.83	0.524
CA*HA*BA	8	0.000308	0.000308	0.000038	0.36	0.928
CA*HA*EL	8	0.000580	0.000580	0.000072	0.67	0.708
CA*BA*EL	8	0.000131	0.000131	0.000016	0.15	0.995
HA*BA*EL	8	0.000557	0.000557	0.000070	0.65	0.729
Error	16	0.001722	0.001722	0.000108		
Total	80	2.654925				

S = 0.0103752 R-Sq = 99.94% R-Sq(adj) = 99.68%

Figure 3.15. Computed F-value for the cathode inlet flow.

Figure 3.16 is the interactions plot. The lines are very nearly parallels in most of the case. This means that interactions are not present for the factors over cathode flow. The small interaction is noticed because the subplot HA versus CA shows lightly non-parallel lines. It is clearly noted that neither the BA valve, or EL, or their interactions show any effect on the cathode air mass flow.

Figure 3.16. Interaction effect plots on cathode flow.

3.4.1.4 Regression Analysis of the Cathode Air Mass Flow (FT380)

The three different regression models are shown in Table 3.9. Appendix F shows the ANOVA analysis of the coefficients and the R^2 correlation for the three models. The R^2 value shows that regression model 1 explains 91.9% of the variance in the cathode flow, indicating that the model fits the data fairly well. Model 2 considers all the cross interaction terms and the improvement with respect to the model 1 is 2.3%. Model 3 shows a R^2 value of 93.7%, very close to model 2 with just six terms instead of the 15 employed in model 2. Model 3 is selected to predict for specific values of fuel flow at steady state. The regression equations presented in Table 3.9 are functions of the level values 1, 2 and 3. The same conversion made for fuel flow using equation 3.2 must be used for cathode air flow regressions.

Table 3.9 Cathode air flow (FT380) regression equations.

Model 1 (R ² =91.9%)	FT380 = 1.168 - 0.105 CA - 0.184 HA - 0.0145 EL
Model 2 (R ² =94.2%)	FT380 = 1.366 - 0.1886CA - 0.278HA + 0.0031BA - 0.0305EL + 0.040CA*HA - 0.0023CA*BA + 0.00262CA*EL - 0.0021HA*BA + 0.0074HA*EL - 0.00251BA*EL + 0.00115CA*HA*BA - 0.00098CA*HA*EL + 0.00063CA*BA*EL + 0.00060HA*BA*EL
Model 3 (R ² =93.7%)	FT380 = 1.304 - 0.185CA - 0.265HA - 0.00119BA + 0.0404CA*HA

Figure 3.17 and Figure 3.18 show the surface and contour plots of the cathode air flow as a function of CA and HA valves. The surface plot shows that the cathode air mass flow for these experiments ranges between 0.30 and 0.93 kg/s as CA and HA valves ranges from high levels to low levels. It is illustrated from the contour plot that it is possible to have trajectories of constant air mass flow to the cathode for widely different combinations of HA and CA valves. The slope in the contour plot shows that the system is more sensitive to changes in the HA valve than to the CA in this range of operation.

Figure 3.17 Surface plot of cathode inlet flow as a function of CA and HA valves.

Figure 3.18. Contour plot of fuel flow mean as a function of CA and HA valves.

3.4.2 Cathode and Turbine Inlet Temperature Statistics (TE326A and TE350, respectively)

Figure 3.19 and Figure 3.20 show the normal residual behavior of the CIT and TIT.

Figure 3.19. Normal distribution of the residual in the cathode inlet temperature variable.

Figure 3.20 Normal distribution of the residual in the turbine inlet temperature variable.

The ANOVA analysis in Table 3.10 and the F-value histogram in Figure 3.21, show how EL has a huge impact on the cathode inlet temperatures followed by the BA valve with significant but less dramatic impact. Meanwhile the CA and HA valves do not show significant effect on this temperature.

The ANOVA analysis in Table 3.11 and the F-value histogram in Figure 3.22, show how EL has a huge impact in the TIT follow by BA valve. Meanwhile the CA valve shows a relative major impact in TIT than in the CIT. This is because the air bypassing by the CA valve is place directly in the post-combustor vessel. This vessel is placed just in front of the inlet of the turbine. Meanwhile the CIT is controlled by the recuperators. When air is bypassed by the CA valve the inlet and the exhaust temperature of the turbine are lowering. But the CIT is lightly affected because the flow to the cool side of the recuperators is also decreased and the valve does not show effect over this temperature.

The “low” CA valve effect does not agree with results of former experiments. The possible reason of the low effect showed here could be that the electric load and BA valve show such high

effects that the CA valve effect is hidden. It is also true that the range of CA valve adjustment was limited to between 40 – 80% open for the experiments.

Table 3.10 ANOVA analysis of the cathode inlet temperature (TE326)

General Linear Model: TE_326A versus CA, HA, BA, EL

Factor	Type	Levels	Values
CA	fixed	3	1, 2, 3
HA	fixed	3	1, 2, 3
BA	fixed	3	1, 2, 3
EL	fixed	3	1, 2, 3

Analysis of Variance for TE_326A, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
CA	2	325.6	325.6	162.8	2.26	0.137
HA	2	979.7	979.7	489.9	6.80	0.007
BA	2	3699.6	3699.6	1849.8	25.68	0.000
EL	2	120284.4	120284.4	60142.2	834.86	0.000
CA*HA	4	835.9	835.9	209.0	2.90	0.056
CA*BA	4	72.7	72.7	18.2	0.25	0.904
CA*EL	4	352.6	352.6	88.2	1.22	0.340
HA*BA	4	170.7	170.7	42.7	0.59	0.673
HA*EL	4	275.9	275.9	69.0	0.96	0.457
BA*EL	4	140.9	140.9	35.2	0.49	0.744
CA*HA*BA	8	522.6	522.6	65.3	0.91	0.534
CA*HA*EL	8	334.7	334.7	41.8	0.58	0.779
CA*BA*EL	8	99.5	99.5	12.4	0.17	0.992
HA*BA*EL	8	429.8	429.8	53.7	0.75	0.652
Error	16	1152.6	1152.6	72.0		
Total	80	129677.3				

S = 8.48756 R-Sq = 99.11% R-Sq(adj) = 95.56%

Figure 3.21 Computed F-value for the cathode inlet temperature.

Table 3.11 ANOVA analysis of the turbine inlet temperature (TE350)

General Linear Model: TE_350A versus CA, HA, BA, EL

Factor	Type	Levels	Values
CA	fixed	3	1, 2, 3
HA	fixed	3	1, 2, 3
BA	fixed	3	1, 2, 3
EL	fixed	3	1, 2, 3

Analysis of Variance for TE_350A, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
CA	2	2901.0	2901.0	1450.5	13.50	0.000
HA	2	447.6	447.6	223.8	2.08	0.157
BA	2	7629.3	7629.3	3814.6	35.51	0.000
EL	2	234384.2	234384.2	117192.1	1090.97	0.000
CA*HA	4	3583.1	3583.1	895.8	8.34	0.001
CA*BA	4	67.7	67.7	16.9	0.16	0.957
CA*EL	4	579.6	579.6	144.9	1.35	0.295
HA*BA	4	342.1	342.1	85.5	0.80	0.545
HA*EL	4	342.1	342.1	85.5	0.80	0.545
BA*EL	4	215.6	215.6	53.9	0.50	0.735
CA*HA*BA	8	766.2	766.2	95.8	0.89	0.545
CA*HA*EL	8	858.3	858.3	107.3	1.00	0.473
CA*BA*EL	8	136.6	136.6	17.1	0.16	0.994
HA*BA*EL	8	757.8	757.8	94.7	0.88	0.552
Error	16	1718.7	1718.7	107.4		
Total	80	254729.8				

S = 10.3644 R-Sq = 99.33% R-Sq(adj) = 96.63%

Figure 3.22 Computed F-value for the turbine inlet temperature.

Figure 3.23 and Figure 3.24 are the interaction plots for cathode and turbine inlet temperature, respectively. The lines are very nearly parallels in most of the case. This means that interactions are not present in these particular cases as was showed by the F-values. Tucker works has illustrated how BA valve has important effects in temperatures. Although in these

experiments the EL shows to be the most important factor; it is important take in mind that the BA valve opening also increases the temperatures, specially the TIT. Meaning in term of FC/GT integration that EL and BA valve have capacity to absorb heat from the FC.

Figure 3.23. Interaction effect plots on cathode inlet temperature.

Figure 3.24 Interaction effect plots on turbine inlet temperature.

3.4.2.1 Regression Analysis of the Cathode Inlet Temperature (TE326)

Table 3.12 shows three different regression models: The R^2 value in regression model 1 explains 96.25% of the variance in CIT, indicating that the model fits the data very well. Model 2 considers all the cross interaction terms and the improvement with respect to model 1 is less than 1%. Model 3 shows a R^2 value of 97.09%, very close to model 2 with just eleven terms instead of the 15 employed in model 2. Model 1 is selected to predict for specific values of cathode inlet temperature at steady state. Model 1 is purely linear and indicates that curvatures are minimum between factors and CIT. Appendix G shows the ANOVA analysis of the coefficients and the R^2 correlation for the three models. The regression equations presented in Table 3.12 are functions of the level values 1, 2 and 3. The same conversion made for fuel flow using equation 3.2 must be used for CIT regressions.

Table 3.12 Cathode inlet temperature (TE326) regression equations.

Model 1 ($R^2=96.25\%$)	$T_{326} = 585.996 - 2.225CA - 4.0018HA + 8.272BA + 47.139EL$
Model 2 ($R^2=97.22\%$)	$T_{326} = 599.594 - 12.9571 CA - 10.9647 HA - 0.691198 BA$ $+ 59.2023 EL + 7.287CA*HA + 4.245CA*BA - 3.403CA*EL$ $+ 3.115HA*BA - 6.253HA*EL - 0.596BA*EL$ $- 2.359CA*HA*BA + 0.892CA*HA*EL + 0.085CA*BA*EL$ $+ 1.133HA*BA*EL$
Model 3 ($R^2=97.09\%$)	$T_{326} = 609.12 - 16.867CA - 19.064HA - 1.882BA + 54.442EL$ $+ 9.072CA*HA + 4.415CA*BA - 1.448CA*EL$ $+ 5.380HA*BA - 2.203HA*EL - 2.359CA*HA*BA$

Figure 3.25 and Figure 3.26 show the surface and contour plots of the cathode inlet temperature as a function of EL and BA valve. The surface plot and contour plot corroborate that the relationship between the factors and the cathode inlet temperature is linear. Practically zero curvature is expressed in surface plot and linear trajectories are observed in the contour plot. The surface plot shows that the cathode inlet temperature for these experiments ranges between 610 and 770 K as the EL and the BA valve range from low levels to high levels. The slope in the contour plot shows that the system is more sensitive to changes in the EL than to the BA valve in this range of operation.

Figure 3.25 Surface plot of cathode inlet temperature as a function of EL and BA valve.

Figure 3.26 Contour plot of cathode inlet temperature mean as a function of EL and BA valve.

3.4.2.2 Regression Analysis of the Turbine Inlet Temperature (T350)

Table 3.13 shows three different regression models: The R^2 value in regression model 1 explains 95.8% of the variance in the TIT, indicating that the model fits the data fairly well. The model 2 considers all the cross interaction terms and the improvement respect to model 1 is 1.5%. Model 3 shows a R^2 value of 97.1%, very close to model 2 with just nine terms instead of the 15 employed in model 2. Model 3 is selected to predict for specific values of TIT at steady state. Model 1 is purely linear and indicates that curvatures are minimum between factors and TIT. But regression model 3 includes some cross terms that indicate there is presence of curvature in TIT response. Appendix H shows the ANOVA analysis of the coefficients and the R^2 correlation for the three models. The regression equations presented in Table 3.13 are functions of the level values 1, 2 and 3. The same conversion made for fuel flow using equation 4.2 must be used for TIT regressions.

Table 3.13 Turbine inlet temperature (TE326) regression equations.

Model 1 ($R^2=95.8\%$)	$T350 = 797 - 6.08 CA - 2.21 HA + 11.9 BA + 65.9 EL$
Model 2 ($R^2=97.3\%$)	$T350 = 827.78 - 24.338 CA - 15.820 HA - 1.841BA + 80.654EL$ $+ 10.398 CA*HA + 6.144CA*BA - 5.034CA*EL$ $+ 3.724 HA*BA - 8.445HA*EL + 0.558BA*EL$ $- 2.645CA*HA*BA + 1.901CA*HA*EL - 0.446CA*BA*EL$ $+ 1.308HA*BA*EL$
Model 3 ($R^2=97.1\%$)	$T350 = 815.389 - 19.575CA - 15.976HA + 11.957BA + 74.167 EL$ $+ 8.909CA*HA - 0.0392CA*BA - 2.124CA*EL$ $- 2.027 HA*EL$

Figure 3.27 and Figure 3.28 show the surface and contour plots of the TIT as a function of EL and BA valve. The contour plot shows some minor curvature between the factors and the TIT. The surface plot shows that the TIT for these experiments ranges between 840 and 1020 K as the EL and the BA valve range from low levels to high levels. The slope in the contour plot shows that the system is more sensitive to changes in the EL than to the BA valve in this range of operation.

Figure 3.27 Surface plot of turbine inlet temperature as a function of EL and BA valve.

Figure 3.28 Contour plot of turbine inlet temperature mean as a function of EL and BA valve.

3.4.3 Fractional Factorial Replication

The replication was a fractional factor design (ffd) based on the design of treatment aliases (see Montgomery [46]). The objective of the replication was to check for repeatability of the experiments. The fractional factorial design (9 treatments) was performed in the HyPer facility during approximately 10 hours of continuous operation. The different treatments designed under a 3^{4-2} fraction of the full factorial experiment, are shown in Appendix I. The test procedure described above for the full factorial design and all considerations were the same as those used in the fractional factorial design.

A fractional (3^{4-2}) replication of the experiments was a test run 30 days ahead of the experiments to assure consistency of the data (ability). The temperature inside the test facility was maintained at a fairly consistent level, varying by no more than 3K between tests. The test procedure described above for the full factorial design and all considerations were the same used in the fractional factorial design. Figure 3.29 illustrates the distribution of the nine replicate points designed in the fractional factorial design. All of these set-points are orthogonal by design and guarantee contrast between experiments.

Figure 3.29 3^{4-2} fractional factorial design selected to replicate set-points.

3.4.4 Replication Results

Following are the results comparing the same treatments in the FFD with those of the ffd, a relative percentage error calculated between these two values based on the formula below, was obtained:

$$\text{Percent Error (\%)} = \frac{|x_1 - x_2|}{x_1} \times 100 \quad (3.3)$$

Figure 3.30 to Figure 3.35 show the results obtained. It is important mention that the first experiment (x_1) is selected as the “true” value of calculation in equation 3.3.

Figure 3.30 Replicate of compressor flow and the relative error between tests.

Figure 3.31 Replicate of cathode inlet flow and the relative error between tests.

Figure 3.32 Replicate of fuel flow and the relative error between tests.

Figure 3.33 Replicate of cathode inlet temperature and the relative error between tests.

Figure 3.34 Replicate of turbine inlet temperature and the relative error between tests.

Figure 3.35 Replicate of turbine exhaust temperature and the relative error between tests.

3.5 Operating Envelope

Based on this design of experiments, a theoretical performance envelope for the HyPer system can be inferred. The low and high levels of each valve, plus the electric load, in the factorial experiments and in the OFAT experiments presented in [4] are used to depict envelopes of operation. The advantages and disadvantages of these envelopes are analyzed from a control point of view. The proposed control strategy is to determine a nominal operating point for the FC and analyze the possible manipulation of the valves to maintain safe FC operation and constant turbine speed.

The two different cases presented and study in Chapter 2 serve as supporting material to understand the significance of the results described in this section. The analysis is discussed including the results found in the OFAT and DoE experiments. The blue area in the next four figures represents the envelope of the CA valve between 40 and 80% opening, with both the HA and BA valves completely closed and EL at 0 and 50 kW. The green area is the HA valve envelope ranging from 20 to 80% open, with the CA and BA valves closed and setting EL at 0 and 50kW. The brown area is for the BA valve ranging from 10 to 14% keeping the other valves

completely closed, and the EL between 0 and 50kW. It is important to mention that in the BA valve experiment the maximum opening of the BA valve was 12% since high temperatures were achieved in the combustor. The red area is the DoE envelope based on simultaneous valve operations and the limits discussed above. It is worth mentioning that the OFAT experiments were run with the total range of operation (0 – 100%) of CA and HA valves. Unfortunately, in the DoE experiments the CA and HA valves could not be run in the full range because some stall problems were encountered in the compressor. The limits selected here were chosen based on this issue. The texture area is the OFAT envelope for CA valve below 40% and above 80% and also for HA valve below 20% and above 80%.

3.5.1 Fuel Flow Envelope

The fuel flow represents to some extent the amount of energy rejected by the FC. The energy is in two forms—direct heating of the cathode air from the electrochemical reaction, and unreacted fuel from the anode that is mixed with the cathode air and burnt in the combustor. The combustor in the HyPer system simulates these two energy sources, and also provides auxiliary heat to regulate turbine speed under varying load. In cases of rapid change in the electrical load on the FC, there will also be rapid changes in the thermal effluent from the stack and dramatic changes in the amount of unreacted fuel coming from the anode to the cathode circuit. It is of interest to determine how well the HyPer system can absorb these transients via manipulation of the available parameters of the three valve positions and the electric load on the generator.

Figure 3.36 shows the envelopes for each valve, using both OFAT (blue, green and brown quadrangle) and the simultaneous operation of valves (red triangle). The CA valve envelopes (blue quadrangle) cover the same range of fuel flow/energy absorption compared at simultaneous operation (red triangle) as measured on the vertical axis, while the fraction of bypass ranges are essentially non-overlapping.

It is fortuitous that the point of maximum energy absorption by the cathode circuit (i.e. maximum FT432 fuel flow) occurs at the point where the air flow through the cathode is minimum, i.e. maximum % bypass flow. The maximum thermal transient will occur with a sudden drop in the FC electric load, which will lead to a slow decline in waste heat via the

cathode air, but a rapid rise in the unreacted fuel from the anode. Under low FC electric load, it will be necessary to reduce cathode airflow to a minimum to avoid overcooling the fuel cell stack. Note that changing the HA valve (green quadrangle) from minimum to maximum has almost no effect on fuel flow but a large effect on the airflow path (i.e. bypass fraction). HA is thus the preferred way to control cathode air flow under steady FC electric load, but is not effective in absorbing thermal transients.

Conversely, the BA valve (brown quadrangle) has almost no impact on air bypass fraction but is very capable of absorbing energy from the FC stack, as witnessed by its tall, narrow shape. Under normal operating conditions the BA valve should be closed, since it simply dumps compressor work to the atmosphere. But Figure 3.36 implies that the BA valve could be an effective “rapid response” control to modulate turbine speed while the HA and CA valves coordinate with the anode fuel flow system to manage the slower thermal transition of the FC between operating points.

Figure 3.36 Fuel flow as a function of bypass air flow

Overlap could and would be achieved by extending the boundary conditions of the red triangle to allow HA and CA valves to be 0% and 100% open. Figure 3.37 shows the extended CA valve envelope (texture quadrangle) based on OFAT experiments. The texture is the actual stall compressor zone (not stall present in old experiments). Irrespective of that, the data shows

as expected that simultaneous operation of the three valves significantly expands the operating envelope of the HyPer system. It should be noted that the two sets of data were taken with different compressor impellers installed in the turbine, and if the blue data were retaken the quadrangle would likely shift to the left slightly.

Figure 3.37 Fuel flow as a function of CA bypass air flow (including the stall zone (brown quadrant)).

Figure 3.38 shows the extended HA valve envelope (textured quadrangle) based on OFAT experiments. The data shows that simultaneous operation of the three valves significantly expands the operating envelope of the HyPer system. It is important to note that the extended CA or HA valves envelopes show an important range of operable bypass flow (20 to 85%) at the same electric load (0 or 50 kW). This show high flexibility of heat absorption and control of cathode air flow, meanwhile the electric load is varied.

Figure 3.38 Fuel flow as a function of HA bypass air flow (including the stall zone (brown quadrant)).

3.5.2 Cathode Air Mass Flow Envelope

Figure 3.39 shows the percent bypass air as a function of total compressor air flow. In the OFAT experiments, the minimum measured flow to the cathode was around 0.589 kg/s, representing about 30% of the total compressor flow. Using the simultaneous manipulation of all input variables, it was possible to set the measured flow at 0.328 kg/s, representing around 15% of the total compressor flow. This is an important difference considering that the cathode air flow is used to control the average temperature of the FC, one of the most critical controlled variables used to protect the FC electrolyte. Figure 3.39 also illustrates that the compressor flow is higher with simultaneous valve operation. It means that simultaneous operation allows the compressor to mitigate the stall and work at higher stall margin.

Figure 3.40 shows the extended CA valve envelope (textured quadrangle) based on OFAT experiments. The data shows that simultaneous operation of the three valves significantly expands the operating envelope of the HyPer system. Figure 3.41 shows the extended HA valve envelope. It is important to note that the extended CA or HA envelopes show an important range of operable bypass flow (20 to 85%) at the same electric load (0 or 50 kW).

Figure 3.39 Envelope of bypass air mass flow as function of compressor corrected flow.

Figure 3.40 Envelope of CA bypass mass flow as function of compressor corrected flow (including the stall zone (brown quadrant)).

Figure 3.41 Envelope of HA bypass mass flow as function of compressor corrected flow (including the stall zone (brown quadrant)).

3.5.3 Cathode Inlet Temperature Envelope

Figure 3.42 shows CIT as a function of percent bypass airflow. Again, the DoE data (red quadrangle) significantly extends the operating envelope of the system to allow up to 85% bypass of cathode air without significantly affecting CIT (slope of the top edge is very small). On the other hand, judicious manipulation of the HA and CA valves allows for significant variation of the TIT at essentially constant bypass fraction (nearly vertical left edge of red quadrangle). Note that the vertical edges of the blue and green quadrangles correspond to significant changes in turbine generator load, and are thus more a function of auxiliary fuel firing rate than they are of HA or CA valves influence. The BA valve (brown quadrangle) again shows little impact on the bypass fraction, but significant impact on TIT. This is because opening the BA valve mechanically loads the turbine and the combustor burns auxiliary fuel to regulate the turbine speed.

Figure 3.43 and Figure 3.44 show similar results to the others extended bypass range and greater selectivity of control with the MIMO control. In all cases, cathode inlet temperature is below what will be needed for an SOFC, indicating that a heat source ahead of the cathode will be necessary. However, Simultaneous operation offers higher efficiency because the higher

envelope temperatures in the cathode side, mitigating in part the input energy necessary to increase this temperature.

Figure 3.42 Cathode inlet temperature as a function of bypass air flow.

Figure 3.43 Cathode inlet temperature as a function of CA bypass air flow (including the stall zone (brown quadrant)).

Figure 3.44 Cathode inlet temperature as a function of HA bypass air flow (including the zone out of DoE (brown quadrant)).

3.5.4 Turbine Inlet Temperature Envelope

Figure 3.45 shows the TIT as a function of percent bypass airflow. Again, the DoE data (red quadrangle) significantly extends the operating envelope of the system to allow up to 85% bypass of cathode air without significantly affecting TIT (slope of the top edge very small). On the other hand, judicious manipulation of the HA and CA valves allow for significant variation of the TIT at essentially constant bypass fraction (nearly vertical left edge of red quadrangle). Note that the vertical edges of the blue and green quadrangles correspond to significant changes in turbine generator load, and are thus more a function of auxiliary fuel firing rate than they are of HA or CA valve influence.

The BA valve (brown quadrangle) again shows little impact on the bypass fraction, but significant impact on TIT. This is because opening the BA mechanically loads the turbine and the combustor burns auxiliary fuel to regulate the turbine speed. In a MIMO control scheme

such as it is here presenting, some of the TIT excursion could be mitigated by opening the CA valve along with the BA valve to provide cooling air to the turbine inlet.

Figure 3.46 and Figure 3.47 show similar results to the others: extended bypass range and greater selectivity of control with the MIMO control.

Figure 3.45 Turbine inlet temperature as a function of bypass air flow.

Figure 3.46 Turbine inlet temperature as a function of CA bypass air flow (including the stall zone (brown quadrant)).

Figure 3.47 Turbine inlet temperature as a function of HA bypass air flow (including the zone out of DoE (brown quadrant)).

Chapter 4

Model Predictive Control

4.1 Introduction

Model Predictive Control (MPC) has been used for many applications in the petrochemical industry. In this section the description, logic, and algorithms for the design of a MPC for the HyPer facility are presented. The design will be developed based on the control strategy detailed in Chapter 3. The advantages that this method offers to control hybrid systems are summarized and analyzed. A specific model for control that fits the dynamics of the plant is necessary in MPC. In this chapter, two different approaches are used to model the plant for MPC formulation. These two modeling formulations are built on identification of the plant and used to design and simulate control scenarios for off-line tuning of the controller.

The first demonstration is accomplished in this work using system identification based on the data experimentally collected via the designed experiments described earlier. MPC and system identification are combined in an adaptive control technique as used in [84], [85], [88], and [89]. In this work, the model is parameter adjusted (adapted) as the plant is operating. The MPC controller designs will be presented combined with the Auto-Regressive Exogenous (ARX) and state-space MIMO identification models. This approach is carried out using the Matlab MPC toolbox using the MPC GUI.

The second case uses transfer functions (TFs) identified previously by Tsai ([26], [27], [28], and [29]). These TFs were identified to linearize the HyPer plant around an operating point, in order to design a multivariable control system. Tsai used experimental frequency tests and Bode plot analysis to obtain TFs which describe the dynamic interactions between inputs and outputs of the facility. This analysis has been detailed in Tsai's dissertation and in several papers already published and mentioned above.

4.2 Description of MPC

This section is devoted to describe the MPC structure and apply it to the HyPer system. Figure 4.1 shows a block diagram of the different possible variables encountered by the MPC controller. This diagram is based on the MPC documentation developed by MathWorks™ to run the Matlab MPC algorithm.

Figure 4.1 Flow Diagram of the Matlab MPC Toolbox.

The input variables of the MPC presented in Figure 4.1 are of three types:

Measured disturbances are variables that are not directly manipulated by the controller, but can be used for feedforward compensation. Examples of this type of variable in the hybrid system could be a sudden change in the composition of the anode fuel flow or in the ambient conditions.

Manipulated variables or actuators are the variables that the controller adjusts and regulates to keep the plant running according to the design objectives. The three bypass valves and the electric load used in the design of the experiments described in Chapter 3 are considered the manipulated variables of the HyPer system in this design.

Unmeasured disturbances are independent inputs of which the controller has no direct knowledge, and for which it must compensate. An example of this type of variable is the heat

generated by the FC and the heat coming out of the unreacted fuel burned in the combustor after a sudden change of the electric load demand.

The plant outputs are the dependent variables (outcomes) that the system wishes to control or monitor. As shown in Figure 4.1, there are two types:

Measured outputs are the variables actually measured and that the controller uses as feedback on the success of its adjustments. The output variables of interest in this work are the rotational turbine speed and the airflow to the cathode side of the FC.

Unmeasured outputs are the output variables which the system does not have access to measure. The controller must estimate these output variables based on available measurements and the plant model. The controller can also hold unmeasured outputs at set-points or within constraint boundaries. For now, no unmeasured outputs are included in this work.

Set-points are the reference values of the output variables which the controller desires to achieve via the manipulation of the inputs. The set-point for the rotational speed is certainly known because it is function of the synchronized frequency of the generator. For the HyPer system this value is 40500 rpm. The set-point for the airflow has not been studied yet. This set-point is a function of the FC temperature. The fuel cell electrolyte temperature must be maintained inside certain limits to avoid delamination or damage by overheating or excessive cooling of the material.

Figure 4.2 shows the block diagram of the HyPer MIMO MPC controller. The *manipulated variables* or *actuators*, u , in the facility are: CA, HA, BA valves and the EL. The signals of the *controller* adjust these actuators to achieve the desired values of the *outputs*. The *outputs* of the system, y , are the turbine speed and the cathode air mass flow. These variables must be held close to the *set-points* or control *reference* r of the MPC. These values are sometimes corrupted by noise z when they are measured by the sensors and transmitted to the controller.

A *measured disturbance*, v , in the HyPer system would be the total heat effluent coming out from the FC, and this is indirectly represented by the measured fuel flow to the combustor. Other *measured disturbances* would be the ambient conditions. The ambient conditions vary

very little in the bay of the HyPer facility and this was shown in Chapter 3 and by [14] that the variation does not have a remarkable effect on the *output* measured variables of the system. Moreover, the effluent heat is a huge disturbance compared with the ambient conditions. For this reason, the ambient conditions will not be included in the model as a *measured disturbance*. In an application where the hybrid system may be placed outdoors, this argument may not be valid. In that case, an ambient condition variation study should be performed and included in the controller design.

The unmeasured disturbances d for any plant are unmanipulated variables that have effects on the plant output. In MPC, the measured disturbances are compensated by the controller using a feedforward action to mitigate the effects on the outputs before they can be manifested. The controller must provide feedback to compensate for such disturbances. In this way, the MPC design always provides feedforward compensation for measured disturbances and feedback compensation for unmeasured disturbances.

Figure 4.2 Block diagram of HyPer MPC control application.

In MPC two important characteristics of the input values are: they are calculated to be in some way “optimal”, and they consider the effect of constraints on the adjustments. The constraints could be the upper and lower bounds of the actuators such as maximum and minimum opening of the valves, or limits imposed by the system such as maximum electric load supported by the facility. One important physical constraint is the stall margin in the compressor. In order to avoid compressor stall the CA valve could be set with a low limit value of 30% as an example of a safety decision on the bound of an actuator. Other kinds of limits are the rate or how rapidly the inputs can vary. Also, constraint limitations can be specified on output variables. Constraint handling in MPC is very important in order to avoid actuator saturation or other nonlinear effects that complicate the controller design.

4.3 MPC Logic and Algorithm Sequence

The Matlab MPC algorithm developed here is designed as a discrete-time controller. Figure 4.3 illustrates the logistics of MPC. At time k , the past output/input relationship (model) is used to predict the probabilistic future. The controller at this time k uses these previous measurements to drive the system to achieve certain goals. One of the goals is to minimize the error between the actual values of the outputs and their references. To guarantee this minimization, penalties are imposed on the controller performance in order to enforce the realization of the objectives.

To calculate the next move (command) u_k , the controller needs to predict the best intelligent (optimal) movement. The moves are the solution of a *constrained* (bounded) optimization problem. When the controller finishes the calculations, the controller sends move commands to the plant. Typically, in MPC design, just the first movement of the actuators is sent to the plant. The plant operates with these *constant* inputs until the next sampling instant. At the next sampling instant, Δt time units later, the controller obtains new measurements and totally revises its plan. This sequence is repeated indefinitely. Recalculation at each sampling instant is essential for good control. The predictions made during the optimization stage are changed by the output updates. Continuous measurement feedback allows the controller to account for the errors and for unexpected disturbances. The recalculations and control action are also functions

of the computation time of the MPC algorithm, the delays between inputs and outputs, the data sampling time and the time taken for the plant to manifest the effect of the inputs.

Figure 4.3 Past, present and future of the controller parameters.

Figure 4.3 shows the instant k representing the present time. The past and future times of the window are represented by i , and P , respectively. Values of set-points, measured disturbances, and constraints are specified over a finite *horizon* of future sampling instants, $k+1, k+2, \dots, k+P$, where P (a finite integer ≥ 1) is the *prediction horizon*. The controller computes M moves $u_k, u_{k+1}, \dots, u_{k+M-1}$, where M ($\geq 1, \leq P$) is the *control horizon*.

The terms used in the MPC framework design are:

1. Moving horizon window, $(k-i, k+P)$: the total time of the window presented in the Figure 4.3. Here, i is the number of historical data points used to predict the future outputs. In this work, $i = 200$ for the ARX model and 400 for the State Space model. The choice of i for a particular application is dependent on the system, the noise levels in the data and other factors, and must often be found by trial and error.

2. Prediction horizon, P : the time interval into the future for which the controller can predict the plant response.
3. Control horizon, M : the time period within which the control actions attempt to drive the plant to the desired state.
4. Receding horizon control: the practice of implementing only the first move u_k from an entire computed sequence and then neglecting (receding) the rest of the trajectory in favor of re-computing a new optimal control signal sequence.

The main challenge in MPC is to tune the controller to achieve multiple objectives. For example, if turbine speed is the main output to be controlled, it might be necessary to prioritize so that the controller provides accurate set-point tracking for the most important output, sacrificing cathode inlet air mass flow when necessary, e.g., when it encounters constraints. MPC supports such prioritization. Tuning also may be done in the inputs, such as to set the maximum rate of change or to penalize a particular input by definition of an optimal target point.

Specifically, the controller predicts how much the turbine speed and cathode inlet flow will deviate from their set-point values each sample time within the prediction horizon. It multiplies a weight value assigned to each output error, and computes the weighted sum of squared deviations. Selecting the weights is a critical step in MPC. Usually, it is necessary to tune the controller, varying the weights to achieve the desired behavior. Also, it is often of interest to maintain some variables inside certain limits rather than to adjust them to a reference value. This could be the case for the turbine exhaust temperature, whose value must be kept inside a minimum and maximum boundary, but does not need to be referenced to a particular set-point. This is also possible to do using MPC methodology.

4.3.1 Plant Inputs and Outputs

The plant inputs to control the turbine speed and the cathode inlet airflow are the CA, HA, BA valves and the EL. Some researchers consider the electric load to be a good way to control the turbine speed and others consider it as a mere disturbance in the system. Historically, the electric demand on the system has been considered an uncontrolled disturbance with the load split between the FC and the turbine more or less fixed. However, recent work by Banta and

Magee (not yet published) has defined a strategy under which the “split” between the FC and turbine generator can be manipulated and the turbine EL can be used for speed control of the turbine. The turbine speed change very quickly after a disturbance in the effluent heat coming out of the FC, and turbine electric load is attractive as an input because it can be manipulated very quickly. In this work, electric load will be considered an input variable that can be used to control turbine speed excursions.

Time is another important parameter in MPC applications. At the beginning, the implementation of MPC was restricted to slow systems because of the computation time necessary to solve the real time optimization problem. With the advent of faster computing machines, this drawback has been overcome for modest sized systems. In this case, it is important to know the actuator response times, which are more a limitation than the computational times. Knowing the response times of the inputs and outputs permits tuning the controller parameters to optimize performance.

In Figure 4.4 two subplots of CA valve position step changes are shown. The left subplot shows a step opening of the CA valve from 40% to 60% open. The measured signals are read each 0.4 second and the sampled value is stored with a zero order hold (ZOH) approximation. After the first sample time (0.4 seconds) the valve was around 43% open, the second sample interval the CA valve moved from 43% to 58%, representing a change of 15%, and it stayed in this value for another sample interval. Finally, in the next sample interval the valve opened the remaining 2% and the reading was around 60%. The total time spent by the CA valve to open 20% was 1.2 seconds. This opening took around 3 sample intervals to be performed. Based on these results, it is expected that a CA valve opening of 10% would last less than 2 sample intervals. The right-hand subplot shows the CA valve closing from 60% to 40%. Opening and closing appear to have similar behavior and the same elapsed time. These equal opening/closing times are very important in order to consider a linear model of the facility. As suggested by Ljung in [68], if the system shows hysteresis in actuator operation, sometimes this behavior requires a nonlinear model identification of the system.

Figure 4.4 Time response of step up/down change of CA valve.

Figure 4.5 shows the reaction and the delay times for cathode airflow when the CA valve is opened from 0% to 15%. This example was obtained with the electric load set at 50 kW and using a fuel valve-speed control feedback. The CA valve needs 0.4 seconds to achieve this 15% change. This is equivalent to 1 sample intervals. The effect on the cathode airflow of this CA valve movement was achieved after one sample interval (0.4 s). This value concurs with a time delay of 0.51 seconds, according to data obtained by Tsai [29] using transfer function identification. The time delay for the turbine speed with respect to the CA valve using the transfer function identification was 0.66 seconds [29].

Figure 4.5 Dead time and tracking of cathode airflow to CA valve.

Figure 4.6 shows the cathode airflow taking two sample intervals (0.8 seconds) to reach the steady state value. This is a relatively short time compared with the 0.8 seconds spent by the CA valve to open. The change in airflow was 0.13 kg/s, representing a 12% change. The cathode air flow tracks the CA valve opening in terms of the time spent to change. This tracking is very important because it means that the cathode air flow is modified at the same rate as the CA valve position.

Figure 4.6 Steady state time for cathode airflow after the 10 % step up of CA valve .

Taking account of the response time of the CA valve and the delay time with respect to the cathode airflow, the maximum slewing rate for the CA valve has been set at 10% in the MPC constraints. This will be a constraint imposed on the valve to obtain soft changes in the output parameters and also to have time for this change make an effect in the output variable. This means that the valve can move no more than 10% of its range in each optimal calculation. It is expected that the “real” time required by the valve to effect this move will be approximately one sample interval (see Figure 4.5). In addition, the time for the valve change to be reflected in the cathode air flow will be approximately one sample period. Therefore, at least a 3-sample period delay must be imposed on any changes in the input command.

In Figure 4.7 two subplots of HA valve position step changes are shown. The left subplot shows a step opening of HA valve from 50% to 80% open. The initial value was 50% and the final value was 80% opening. The measured signals are read each 0.4 second and the sampled value is stored with a zero order hold (ZOH) approximation. The total time spent by the HA

valve to open 30% was 2.4 seconds. This opening took 6 sample intervals to be performed. The right subplot shows the HA valve closing from 80% to 50%. The total time spends by the HA valve to closed 30% was also 3.0 seconds. Based on these results, it is expected that a HA valve opening of 10% would last less than 2 sample intervals. The right-hand subplot shows the HA valve closing from 80% to 50%. Opening and closing appear to have similar behavior and the same elapsed time.

Figure 4.7 Time response of step up/down change of HA valve.

Figure 4.8 shows the reaction and delay times for cathode airflow when the HA valve is closing from 60% to 50%. This example was obtained with the electric load set at 50 kW and using a fuel valve-speed control feedback. The HA valve needs 1.2 seconds to achieve this 10% change. The effect on the cathode airflow of this HA valve movement was achieved after one sample interval (0.4 s). This value concurs is slightly less than a time delay of 0.77 seconds, according to data obtained by Tsai [29] using transfer function identification. The time delay for the turbine speed with respect to the HA valve using the transfer function identification was 0.85 seconds [29].

Figure 4.8 shows that the cathode airflow tracks the HA valve closing after one sample time of delay. This tracking is very important because it means that the cathode air flow is modified at the same rate as the HA valve position. The HA valve and the cathode air flowmeter are placed very close each other. The change in airflow was 0.06 kg/s, representing a 9% change.

Figure 4.8 Dead time and tracking of cathode airflow to HA valve

Taking account of the response time of the HA valve and the delay time with respect to the cathode airflow and turbine speed, the maximum slewing rate for the HA valve is set at 10%. This will be a constraint imposed on the valve to obtain soft changes in the output parameters and also to prevent overshoot in the output variables. The HA valve is free to move from 0 to 100%. This range of operation allows the valve to manage a significant amount of air flow and to effectively control the cathode inlet flow. The maximum slewing rate is the same as was used for the CA valve.

Figure 4.9 shows two subplots of a BA valve position step change. Note that the readings for the BA valve are given in %closed rather than %open like the other valves. The left subplot shows a step opening of the BA valve position command of 8%, i.e. the BA valve was initially commanded to be 100% closed and the final value was commanded to be 92% closed. The measured signals are read from a position sensor on the valve stem and show the “actual” position of the valve to be changing from about 99.8% closed to about 93% closed. This example is taken from old data and it is unknown whether the error is in the valve position versus the commanded position, or in the sensor reading. The difference is not important—the information desired from this data is the approximate response time and that is available from this data. The data is taken each 0.4 second and the reading is maintained with the zero order hold (ZOH) approximation. The total time required by the BA valve to open was 2.0 seconds.

This opening took around 5 sample intervals to be performed. The right subplot shows the BA valve closing from 90% to 100%. The total time spends by the BA valve to closed was also 0.8 seconds. This closing time is not too bad if it is taking account that the normal operation of this actuator is being closed and its relevant application is while is being opening to control overspeed as it was explained in Chapter 2. The opening and closing cases showed to have different behavior and the elapsed times were totally different in both cases.

Figure 4.9 Time response of step up/down change of BA valve.

Figure 4.10 shows the delay time for cathode airflow when the BA valve is opening from 92% closed to 90% closed (i.e. 8% open to 10% open). This example was obtained with the electric load set at 50 kW and using a fuel valve-speed control feedback. The BA valve needs 2.0 seconds to achieve this 2% change. This is equivalent to 5 sample intervals. This time is not consistent with the time required to open 8% shown above. The effect on the cathode airflow of this BA valve movement is very low. In fact, the BA valve position does not by itself have a significant effect on the system flow patterns at all. Its effect is to add mechanical load to the turbine shaft and thus to momentarily reduce the turbine speed. The turbine speed reduction is the cause of mass flow decreasing. This result has already been shown by Tucker in former work [3] and [4], and confirmed in the DoE described in Chapter 3. The time delay obtained by Tsai using transfer function identification for this case was 2.29 seconds (around 6 time intervals). In the transfer function identification the time delay was 1.19 seconds (3 time intervals) between BA valve and turbine speed. This last time delay can be observed in this subplot.

Figure 4.10 Dead time of cathode airflow and turbine speed vs BA valve.

Figure 4.11 shows that the cathode airflow is only slightly affected by the BA valve opening, specifically because the BA valve has the effect of loading the turbine and lowering the turbine speed. It is observed in Figure 4.11 that when the turbine speed recovers its nominal speed, the cathode airflow recovers its initial value. Thus, the turbine speed is strongly affected by the BA valve. In this example, an opening of 2% in the BA valve changes the turbine speed around 300 RPM under speed feedback control. In open loop this turbine speed change could be higher. Figure 4.11 shows that the cathode air flow clearly tracks the turbine speed rather than the BA valve position.

Figure 4.11 Steady state time for cathode airflow and turbine speed after the 2 % step opening of BA valve.

The limits of this valve are set at 100% (completely closed) and 80% closed. This last limit can be softened because the valve will be opened only for short periods of time. Taking account of the former section, the time response of the BA valve and the delay time with respect to the cathode airflow and turbine speed, the maximum slewing rate for the BA valve is set at 5%. This will be a constraint imposed on the valve to allow flexibility in the control of turbine speed if needed.

The last manipulated variable is the electric load. This is a very fast actuator, with good ability to control turbine speed. The lower limit can be set to zero and the maximum value 50 kW to minimize the danger of stalling the compressor. The upper limit was chosen to be conservative for these experiments to avoid possible disruption caused by a stall in the a lengthy test sequence.

Figure 4.12 shows two subplots of electric load change. The left subplot shows a step increase of 25 kW EL. The measured signals are read each 0.4 second and the reading is maintained with the zero order hold (ZOH) approximation. The total time elapsed for the 25 kW electric load change was 1.2 seconds or about 3 sample intervals. The right subplot shows the 25

kW electric load being reduced from 50kW to 25 kW. The total time required for the load decrease was 0.8 seconds. The step up and step down cases have different behavior and the elapsed times were different in both cases. But this actuator has the ability to make relatively large load changes in short periods of time. The electric load is suggested to be used as an appropriate actuator to control turbine speed.

Figure 4.12 Time response of step up/down of electric load.

Figure 4.13 shows the delay time for cathode airflow and turbine speed when the electric load is changed from 0kW to 25kW. This example was obtained using fuel valve-speed control feedback. The electric load needed 1.2 seconds to make this change. This is equivalent to 3 sample intervals. The effect on the cathode airflow of this electric load movement is related to the variation of the turbine speed due to the electric load in the same way that the BA valve affects the turbine speed. The time delay obtained by Tsai using transfer function identification for cathode air flow versus electric load was 1.59 seconds so it disagrees with this data. The time delay between electric load change and the corresponding turbine speed reaction is zero. In the transfer function identification, the time delay was 0.153 seconds (less than one time interval). This last value is very near to the one illustrated in Figure 4.23. This quick variation in turbine speed can explain the quick variation in cathode airflow.

Figure 4.13 Dead time of cathode airflow and turbine speed vs electric load.

On the output side, the generator is synchronized to the electrical grid frequency. The turbine speed must match the synchronized speed with the generator. Transient speed excursions must be mitigated very quickly to avoid large phase displacement to the nominal generator frequency. Turbine speed must be hard constrained to avoid damage of the generator. The tracking of the speed set-point is strongly weighted in the performance index definition in order to maintain minimum error between the control output and the nominal generator frequency of operation. This turbine speed error minimization also avoids large transient changes in mass flow and pressure in the cathode side of the FC. The cathode air flow is also constrained to maintain low equivalence ratio in the combustor. The minimum amount of air in the combustor to keep the equivalence ratio below one is around 0.3 kg/s of cathode inlet air to insure that all of the fuel coming to the combustor will be burned. This limitation is imposed because any unburned fuel passing through the combustor could explode in the turbine or heat exchanger or stack if it encounters oxygen from other sources such as leakage between the compressor and turbine or leakage between the high- and low-pressure sides of the heat exchanger. This value

must be a hard constrained in the facility. Appendix K shows a table of equivalence ratio as a function of cathode air flow and fuel flow to support this limit.

In Table 4.1 the constraints of the manipulated variables and output variables are given. These constraints will be used for both State Space and ARX modeling approaches. A slightly different constraint on the BA valve is used in the Transfer Function model.

Table 4.1 Upper/lower bounds of output/input variables and up/down rates of input variables

Constraints on Manipulated Variables					
Name	Units	Minimum	Maximum	Max Down Rate	Max Up Rate
CA Valve	% Open	30	100	10	10
HA Valve	% Open	0	100	10	10
BA Valve	% Closed	80	100	5	5
Electric Load	kW	0	50	10	10

Constraints on Output Variables			
Name	Units	Minimum	Maximum
Turbine Speed	RPM	38000	44000
Cathode Airflow (CAF)	kg/s	0.3	1

4.3.1.1 Plant and Horizons

Now, the output/input horizons, the constraints, the performance index weighting matrices, and the output/input disturbances will be modified to create the controller test scenarios. First the plant was selected, along with the input/output horizon. The prediction horizon recommended in the literature by many authors for MPC should be between 20 and 30 sample times. The prediction horizon (P) must not be too long making the computation time an obstacle of implementation and not so short that overshoot and constraints will be violated. The control horizon also is a very important variable. Short control horizons are reflected in the controller by strong actions of the inputs. On the other hand, a long control horizon would lead to very

sluggish control action in the system. Different combinations of weighted matrices were tested and the best result is presented below for different case studies.

4.3.1.2 Anticipation Action

The anticipation feature of MPC as described in section 1.6.1.6 is also available in the MPC Toolbox. The “*look ahead*” check box enables an anticipative action on the corresponding signal. This option becomes available when you define reference signals. In this way, the controller accounts for known future reference variations in its predictions, which usually improve set-point tracking.

4.3.1.3 Weight Tuning

The appropriate matrices Q and R of the performance index must be adjusted. This is an important step in tuning the controller. For now, an off-line pre-tuning has to be done to test the controller and define how to find the control law. Eventually, this step should be done online during the implementation of the controller. In Table 4.2, the values of the weight matrices are equal for the inputs and the outputs. This means that a change in any of the inputs and the error between the outputs and the reference have the same penalty in the performance index. Once the controller is implemented, the relative values of the weights can be adjusted to place more or less importance on specific output variables. This will allow the customization of the system response to maintain tighter control on critical system parameters.

4.3.1.4 Blocking

Blocking is a special feature of the MPC toolbox. A block is a one or more successive sampling periods during which the manipulated variable remains constant. This blocking action can be done at the beginning, uniformly throughout, or at the end of the input horizon selection in the Toolbox. The *block durations* are the number of sampling periods in each block. This is very important because it permits control of the duration of the inputs’ actions. In fact, the HyPer valves take some period of time to complete some actions and there are also delay times before the control actions are manifested in the outputs. Thus, blocking helps to keep the actuators from taking action until the plant reacts to the new input values.

Table 4.2 Weight default matrices of the MPC performance index.

Input Weights				
Name	Description	Units	Weight (Q)	Rate Weight
CA	Cold Air Valve	% Open	0	1
HA	Hot Air Valve	% Open	0	1
BA	Bleed Air Valve	% Closed	0	1
EL	Turbine Electric Load	kW	0	1

Output Weights			
Name	Description	Units	Weight
Speed	Turbine Speed (Rotation)	RPM	1
CAF	FC Cathode Inlet Flow	kg/s	1

4.3.2 Performance Index of the Optimization Problem with Constraints

The MPC law is obtained after solving the MIMO optimization problem described by the following equations,

$$J = \left\{ \sum_{p=0}^{P-1} \left(\sum_{j=1}^{n_y} \left[q_{p+1,j} \left(y_j(k+p+1|k) - \bar{w}_j(k+p+1) \right) \right]^2 \right) \right\} + \sum_{j=1}^{n_u} \left[r_{p,j} \Delta u_j(k+p|k) \right]^2 + \sum_{p=1}^M \sum_{j=1}^{n_u} \{ s_j [\bar{u}_{j\text{ref}} - u_j(k+p-1)] + \rho \varepsilon^2 \} \quad (4.1)$$

where J is the cost function or performance index. P and M are the prediction and control horizons, respectively, given in number of samples. The index k is the present time instant and p is a future prediction time instant. And n_y and n_u account for the number of outputs and inputs. $y(k+p+1|k)$ is the predicted value of y at time k . The first term on the right side of equation 4.1 is the primary control objective in this work. It forces the plant to track the state variable set-points. The controller tries to minimize the difference between the set-points and the outputs. It multiplies the error by the output weight and computes the weighted sum of squared deviations. The second term in the equation is the movement penalty. This term is used to penalize big changes in the input variables. This term helps to preserve the equipment and leads to control system stability. The third term penalizes the deviations of an input from a defined nominal value. This could be very valuable for example with the electric load because it allows the turbine generator load to be used as a manipulated variable that tracks a load split value

synchronized with the FC electric load. q , r , and s are elements of the weight matrices $\mathbf{Q}$, $\mathbf{R}$ and $\mathbf{S}$. The smaller q , r , and s , the less important is the behavior of the corresponding variable to the overall performance index. They allow the controller to penalize any deviation of the outputs from the references, high input rates, and deviations of the inputs from input references, respectively. P and M are the prediction and control horizon, respectively. The cost function is subject to the following constraints:

$$\left\{ \begin{array}{l} u_{jmin}(i) - \varepsilon V_{jmin}^u(i) \leq u_j(k+i|k) \leq u_{jmax}(i) + \varepsilon V_{jmax}^u(i) \\ \Delta u_{jmin}(i) - \varepsilon V_{jmin}^{\Delta u}(i) \leq \Delta u_j(k+i|k) \leq \Delta u_{jmax}(i) + \varepsilon V_{jmax}^{\Delta u}(i) \\ y_{jmin}(i) - \varepsilon V_{jmin}^y(i) \leq y_j(k+i+1|k) \leq y_{jmax}(i) + \varepsilon V_{jmax}^y(i) \\ \Delta u(k+h|k) = 0, \quad h = m, \dots, p-1 \\ \varepsilon \geq 0 \end{array} \right. \quad (4.2)$$

$u_{j,min}$, $u_{j,max}$, $\Delta u_{j,min}$, $\Delta u_{j,max}$, $y_{j,min}$, $y_{j,max}$ are lower/upper bounds on inputs, inputs rates, and output variables, respectively. In equation 4.2, the constraints on u , Δu , and y are relaxed by introducing the slack variable $\varepsilon \geq 0$. The weight ρ on the equation 4.1 penalizes the violation of the constraints. The fourth term in the cost function ($\rho \varepsilon^2$) is the constraint violation penalty if soft constraints are indicated in the optimization. The larger ρ is with respect to input and output weights, the more the constraint violation is penalized. The Equal Concern for the Relaxation (ECR) vectors V_{min}^u , V_{max}^u , $V_{min}^{\Delta u}$, $V_{max}^{\Delta u}$, V_{min}^y , V_{max}^y have nonnegative entries which represent the concern for relaxing the corresponding constraint; the larger V , the *softer* the constraint. $V=0$ means that the constraint is a *hard* one that cannot be violated. For the valves, all the limit constraints are hard ($V_{min}^u = V_{max}^u = V_{min}^{\Delta u} = V_{max}^{\Delta u} = 0$) and for the outputs constraints are soft ($V_{min}^y = V_{max}^y = 1$).

The MPC Toolbox also computes a *state observer* to estimate the unmeasured states $x(k)$ of the plant model. The estimates are computed from the measured output $y_m(k)$ by the linear state observer. The observer is designed using Kalman filtering techniques on the extended model (Wang [56]).

4.3.3 Model Predictive Control Computation

The model predictive control optimization problem is solved at each time step k by using the matrices built at initialization. If the system is unconstrained, the optimal solution is computed analytically. On the other hand, if the system is constrained, the solution is obtained by solving the quadratic program described by equations 4.1 and 4.2. The MPC Toolbox use the *QP* solver coded in the *qp solver.mex* function using the Dantzig-Wolfe algorithm (Matlab command *qp dantz*).

4.4 System Identification and Control System Simulation

For the HyPer facility it has been very difficult to find a first-principles model that predicts with relative accuracy the behavior of the air flowing through the turbomachinery, heat exchangers and all the valves, pipes and tanks in the air stream. As was mentioned in the literature review, none of the models previously developed for the HyPer facility are able to simulate the valves and electric load operation simultaneously. Because a multidimensional controller is the goal for HyPer and since for MPC a model is necessary for prediction, HyPer models will be created from the measured data using system identification. To be used with MPC, the internal model must be capable of simulating the plant behavior faster than real time. The idea is to curve fit a plant model of the data up to the present time and use that model for prediction in MPC.

4.4.1 System Identification Plant Modeling

In the system identification approach, a linear plant model has been assumed. This is partly because in the design of experiments the system was shown to have a weak nonlinear behavior, but more importantly, it is because the system identification model is updated continuously and the plant is never far away from the last operating point. Thus, any change caused either by system wear or by its nonlinear character is updated in the model with the identification. In control design, this kind of information can be involved in the computation of control actions, running on-line.

4.4.2 Data Quality Requirements

MPC requires a model in order to calculate the input adjustments needed to predict the future outputs and track the reference set-points. Since a HyPer model is necessary to control the plant, this model is determined after the system identification ARX/State-Space algorithm is run. The dynamic data gathered in the design of experiments was used to identify the system as a back preprocessing step in order to design the optimal constraint model predictive control. System identification requires that the input/output data capture the essential dynamics of the system. In general, the experiment must (see Ljung, Chapter 14 [68], and Matlab documentation):

1. Use inputs that excite the system dynamics effectively. For example, a single step is seldom enough excitation. In the DoE experiments steps were used between all treatments and this transient data has been used for identification.
2. Measure data long enough to capture the important time constants.
3. Have good signal-to-noise ratio.
4. Measure data at appropriate sampling intervals or frequency resolution.

4.4.3 Limitations working with linear models

MPC is typically configured using linear models. The linear approximation is effective because the plant is updated continuously and the identification model obtained in this way should work very well in such a narrow region of operation of the plant. It is possible that even a non-linear plant generally admits a locally-linearized model when considering regulation about a particular operating point. In addition, in the controller, the inputs are computed to minimize the error between plant outputs and set-points. So at each time step when the plant is nearest to the reference value, it can be adequately represented by a linear model.

4.4.4 Selecting Black-Box Model Structure and Order

Black-box modeling is useful when the primary interest is in fitting the data without regard to a particular mathematical structure of the model. System identification provides several model structures, which have traditionally been useful for representing dynamic systems for MPC.

Typical linear models for MPC strategy include impulse and step models, ARX model structures, and State-Space models.

The impulse and step models are the most widely used in the industry because during the development of MPC, they were the typical approach selected by the operators for developing models of plant response. The drawback has been the number of parameters used by the models (sometimes over 60) after truncation of the high order parameters of these models. The selection of ARX and State-Space representation is chosen because they now have been introduced by the scientific community and accepted by many people in industry (see [30], [36],[43], [48], [55], and [56]). They are also the simplest linear black-box structures. They vary in complexity depending on the flexibility needed to account for the dynamics, disturbance and noise in the system.

Black-box modeling is usually a trial-and-error process, where the parameters are estimated using various structures and the results are compared. System Identification can choose one of these structures and compute its parameters to fit the measured response data. Typically, the identification starts for example with a simple linear model structure, low order system, without delay, etc., and then, progresses to more complex structures. A model structure might also be chosen because it looks similar to the assumed system first-principles structure or because it is necessary for a specific application. In this dissertation both structures, ARX and State-Space, will be tested to account for the advantage and drawback of each one.

It is possible to configure a model structure using the *model order*. The definition of model order varies depending on the type of model selected. For example, if the transfer function representation is chosen, the model order is related to the number of poles and zeros. For State-Space representation, the model order corresponds to the number of states. In some cases, such as for linear ARX and State-Space model structures, one can estimate the model order from the data. If the simple model structures do not produce good models, one can select more complex model structures.

It is usually recommended to try different model structures when using black box modeling. Each model structure has its own advantages and disadvantages. The ARX and State-Space

models will be used in this algorithm to account for the specific adaptive input-output relationships of the facility. These adaptive models are used to build online recursive models. The plant models created by recursive system identification will be used with the model predictive controller algorithm to illustrate optimized predicted plant behavior. A guideline to obtain suitable models from system identification is presented in Appendix L. The ARX and State-Space model identification methods will be explained in detail in section 4.4.5 and 4.4.6, respectively.

System identification requires measurement data. The data selected here has been taken from the DoE described in detail in Chapter 3. Although this experiment was not performed specifically to develop a system identification model and for control analysis, the step changes used to move from one treatment to another, contain enough dynamic information to be captured by the system identification methods. This static and transient data serves to establish the foundation which will be used for implementation of the MPC approach in the HyPer facility.

The DoE was run under feedback control of the turbine speed, since the HyPer system contains safety interlocks that will automatically shut the turbine down if turbine speed excursions exceed $\pm 5\%$ of nominal. The turbine speed was controlled by injecting fuel to the combustor system using a very accurate, fast and linear fuel valve. In the discussions to follow, the term “closed loop” means that the PID controller for turbine speed is active. “Open loop”, conversely, would be an experiment in which the turbine speed control was defeated in order to avoid confounding the response data.

During the DoE experiments, the direction of control commands (i.e. HA, CA, BA valves open and/or close and EL increase/decrease) was not formulated to control turbine speed or airflow in the system; the valves were just moved to achieve the randomized treatment points of the DoE. For example, in order to control an increase in turbine speed the BA valve must be opened and/or the turbine electric load must be increased. In DoE experiments, it is possible that the BA valve was opening while the electric load was decreased. Such a situation would have triggered the speed control system to reduce fuel flow to the combustor, making the interpretation of the data more difficult. Because a “good” model depends on how well the measured data reflects the behavior of the system for real control applications, it is recommended

for implementation of the controller that either the system identification model be tested in open loop or that special measures be taken to isolate the influence of the controller from the plant dynamics. Such testing will require careful design of the test procedure to avoid creating undesirable system conditions, but should be possible during the tuning stages of the controller implementation.

4.4.4.1 Order Selection, Fit Criterion and Time Delay

Both identification methods, ARX/State-Space, use the “*fit*” parameter to quantify the correlation between the data measurement and the model simulation. The precise definition of the fit is:

$$Fit = \left[1 - \frac{\|y - y_{hat}\|}{\|y - mean(y)\|} \right] * 100 \quad (4.3)$$

where y is the measured output and y_{hat} is the simulated model output, and the norm is the norm of a vector defined as:

$$\|x\| = \sqrt{x^T x}$$

The Akaike information criterion (AIC) is a statistical model fit measure defined by

$$AIC = \log \left\{ \det \left[\frac{1}{N} \sum_{i=1}^N \varepsilon(i, \theta) \varepsilon^T(i, \theta) \right] \right\} + \frac{l}{N} \quad (4.4)$$

where N is the number of measurement points, the θ vector contains the identified parameters, l is the number of the parameters to be identified, and ε is the error between measured and simulated values. In Matlab, this can be computed using the function “ $f = aic(G)$ ”, where G is an object calculated by the “ $arx()$ ” or “ pem ” function. The AIC smallest value is the best accurate model of the data. If any further increase of identified parameters does not produce an improvement of the AIC number, then the order used in equation (4.4) is large enough to adequately model the system. The “Akaike Final Prediction Error” is another parameter computed from:

$$FPE = V * (1 + \frac{2*l}{N}) \quad (4.5)$$

where l is the number of estimated parameters, N is the number of estimation data samples (assuming that $l \ll N$), and V is the “Loss Function” and is defined by the following equation:

$$V = \det \left[\frac{1}{N} \sum_{t=1}^N \varepsilon(t, \theta_N) (\varepsilon(t, \theta_N))^T \right] \quad (4.6)$$

where $\varepsilon(t, \theta_N)$ is the error between measured and estimated value, and θ_N represents the estimated parameters. According to Akaike's theory, the most accurate model has the smallest FPE.

In order to estimate the time delay the command “~~delayest~~” in Matlab is used for this purpose. The time delay in this way is estimated in sample times. The “~~delayest~~” command was used one input/output at a time and used in the MIMO ARX model equations.

4.4.5 Polynomial (ARX) Model Identification and MPC

The ARX model is the simplest model incorporating the stimulus signal. It is also the most efficient of the polynomial models as their solution is analytical and unique which is preferable when the model order is high. The ARX model uses the least squares method to minimize the error between measured and model outputs. The mathematical algorithm used for least squares is convex which means it is less mathematically complex ([84], [87], and [88]). The ARX model operates only with inputs and outputs; therefore it does not need any observers. The disadvantage is when modeling disturbance dynamics occur, because coupling between the plant and disturbance model can show unrealistic results. This disadvantage is reduced when the signal has a “good” signal-to-noise ratio. The “good” signal-to-noise ratio premise is assumed in this work as found in the DoE analysis. Another disadvantage of ARX methods is that they can experience many local minima in the performance function and thereby a lack of convergence to global minima. The user will need to specify complicated parameterizations of system orders and delays. ARX models may also suffer potential problems with numerical instability. For MIMO systems and large model order, computation time to execute the iterative numerical minimization can become “excessive”. A typical discrete-time SISO ARX model is given by:

$$y(t) + a_1y(t-1) + a_2y(t-2) + \dots + a_{n_a}y(t-n_a) = b_1u(t-d) + b_2u(t-d-1) + \dots + b_{n_b}u(t-d-n_b+1) + \varepsilon(t) \quad (4.7)$$

where u and y represent the input and output measured data, respectively. The a 's and b 's are the coefficient of the linear difference equation to be found by the minimization of the squared error ε , as detailed below. n_a represents the number of poles, n_b is equal to the number of zeros, and d is the pure time delay. So, if N samples of the data are taken to find the coefficients a and b , the N equations of the model are:

$$\begin{aligned} y(1) &= -a_1y(0) - \dots - a_{n_a}y(1-n_a) + b_1u(1-d) + \dots + b_{n_b}u(2-n_b-d) + \varepsilon(1) \\ y(2) &= -a_1y(1) - \dots - a_{n_a}y(2-n_a) + b_1u(2-d) + \dots + b_{n_b}u(3-n_b-d) + \varepsilon(2) \\ &\vdots \\ y(N) &= -a_1y(N-1) - \dots - a_{n_a}y(N-n_a) + b_1u(N-d) + \dots + b_{n_b}u(N+1-n_b-d) + \varepsilon(N) \end{aligned}$$

so, selecting $N \gg n_a + n_b$ allow to get a good differential equation description for the system.

The matrix form of the above equations can be written as:

$$\begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(N) \end{bmatrix} = \begin{bmatrix} y(0) & \dots & y(1-n_a) & u(1-d) & \dots & u(2-n_b-d) \\ y(1) & \dots & y(2-n_a) & u(2-d) & \dots & u(2-n_b-d) \\ \vdots & & \vdots & \vdots & & \vdots \\ y(N-1) & \dots & y(N-n_a) & u(N-d) & \dots & u(2-n_b-d) \end{bmatrix} \begin{bmatrix} -a_1 \\ -a_2 \\ \vdots \\ -a_{n_a} \\ b_1 \\ \vdots \\ b_{n_b} \end{bmatrix} + \begin{bmatrix} \varepsilon(1) \\ \varepsilon(2) \\ \vdots \\ \varepsilon(N) \end{bmatrix}$$

and making,

$$\mathbf{y}(t) = \begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(N) \end{bmatrix},$$

$$\Phi = \begin{bmatrix} y(0) & \dots & y(1-n_a) & u(1-d) & \dots & u(2-n_b-d) \\ y(1) & \dots & y(2-n_a) & u(2-d) & \dots & u(2-n_b-d) \\ \vdots & & \vdots & \vdots & & \vdots \\ y(N-1) & \dots & y(N-n_a) & u(N-d) & \dots & u(2-n_b-d) \end{bmatrix},$$

$$\boldsymbol{\theta}^T = [-a_1 \quad -a_2 \quad \cdots \quad -a_{n_a} \quad b_1 \quad \cdots \quad b_{n_b}],$$

and,

$$\boldsymbol{\varepsilon}^T = [\varepsilon(1) \quad \cdots \quad \varepsilon(N)]$$

then,

$$\mathbf{y} = \boldsymbol{\Phi}\boldsymbol{\theta} + \boldsymbol{\varepsilon} \quad (4.8)$$

and minimizing the sum of square residuals using the least squares algorithm described in [68], and [69] by:

$$\min_{\boldsymbol{\theta}} \sum_{i=1}^N \varepsilon^2(i) = \min_{\boldsymbol{\theta}} \sum (\mathbf{y} - \boldsymbol{\Phi}\boldsymbol{\theta})^T (\mathbf{y} - \boldsymbol{\Phi}\boldsymbol{\theta}), \quad (4.9)$$

The optimum estimation to the undetermined elements in $\boldsymbol{\theta}$ can be written as

$$\boldsymbol{\theta} = [\boldsymbol{\Phi}^T \boldsymbol{\Phi}]^{-1} \boldsymbol{\Phi}^T \mathbf{y} \quad (4.10)$$

This resembles a curve fitting approach. Note that $[\boldsymbol{\Phi}^T \boldsymbol{\Phi}]^{-1}$ might be ill-conditioned if the input excitation signal $u(t)$ does not contain properly uncorrelated data.

The condensed representation of a discrete SISO ARX model is

$$y(k) = \sum_{i=1}^{n_a} b_i u(k-i) - \sum_{i=1}^{n_b} a_i y(k-i) + \varepsilon(k) \quad (4.11)$$

and for the MIMO case is:

$$y(k) = \sum_{i=1}^{n_a} \mathbf{B}_i \mathbf{u}(k-i) - \sum_{i=1}^{n_b} \mathbf{A}_i \mathbf{y}(k-i) + \boldsymbol{\varepsilon}(k) \quad (4.12)$$

where $\mathbf{B}$ and $\mathbf{A}$ are matrices of $n_u \times n_y$ and $n_y \times n_y$ sizes, respectively; n_u is the number of inputs and n_y is the number of outputs; and $\boldsymbol{\varepsilon}$ is an n_y dimensional vector of error in the model

4.4.5.1 HyPer Auto Regressive Exogenous (ARX) Identification.

The ARX model is a linear model. It is classified as a kind of transfer function or polynomial model. Different order structures are tested in the algorithm to select the best combination of retarded outputs and inputs in the model based on the Loss Function and FPE

parameters. The time delays were also estimated in the algorithm in order to get the values that can match the output/input delays

The MIMO system consists of a non-square system of four inputs and two outputs. CA, HA, BA valves, and electric load have been selected as the inputs of the system. Turbine speed and cathode airflow are the outputs. Initially, the batch of data was loaded into the Matlab workspace for identification as an *idata*". After that, the command *arx*" was used for the ARX identification.

the *output order matrix*" selected for the ARX order model was :

$$n_a = \begin{bmatrix} 3 & 3 \\ 2 & 2 \end{bmatrix}$$

the final *input order matrix*" selected was

$$n_b = \begin{bmatrix} 3 & 2 & 2 & 2 \\ 3 & 2 & 2 & 2 \end{bmatrix}$$

The MIMO ARX equation model identified for a set of data with the output/input orders previously selected is shown in equation 4.13. The time delays used in this identification were obtained in section 4.3.1.

$$\begin{aligned} & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} y(t) + \begin{bmatrix} -0.8031 & 260.44 \\ -0 & -0.4484 \end{bmatrix} y(t - 0.4) + \begin{bmatrix} -0.1246 & 175.43 \\ 0 & 0.1964 \end{bmatrix} y(t - 0.8) + \\ & \begin{bmatrix} 0.1942 & -817.77 \\ 0 & 0 \end{bmatrix} y(t - 1.2) = \\ & \begin{bmatrix} 0 & 0 & 34.92 & -15.66 \\ 0 & 0 & -0.0079 & 0.0054 \end{bmatrix} u(t) + \begin{bmatrix} -39.2669 & 80.83 & -43.38 & 0.6802 \\ 0.0108 & -0.0112 & 0.0030 & 0.0034 \end{bmatrix} u(t - 0.4) + \\ & \begin{bmatrix} 7.8021 & 65.69 & 0 & 0 \\ -0.0045 & 0.0087 & 0 & 0 \end{bmatrix} u(t - 0.8) + \begin{bmatrix} -8.3078 & 0 & 0 & 0 \\ 0.0057 & 0 & 0 & 0 \end{bmatrix} u(t - 1.2) \end{aligned} \quad (4.13)$$

where,

$$y(t) = \begin{bmatrix} \text{Turbine speed (rpm)} \\ \text{Cathode airflow}(\frac{kg}{s}) \end{bmatrix},$$

and,

$$u(t) = \begin{bmatrix} CA \text{ valve } (\%) \\ HA \text{ valve } (\%) \\ BA \text{ valve } (\%) \\ Electric \text{ Load } (kW) \end{bmatrix}$$

Figure 4.14 compares the measured and the estimated data obtained by equation 4.9. The elapsed computer time to obtain this identification was 0.123651 seconds. This calculation was performed by using a Intel Core i7 processor running at 2.13 GHz with 3GB of RAM, and 32 bit Operating System. The processing time could be shortened by compiling the code and/or by using fewer data points in the identification. The number N of samples used was 200. It is illustrated in Figure 4.14 that the model tracks the transient excursion of the turbine speed and the cathode airflow data very well. This model is likely to do a good job in predicting the future values of the outputs, given the past input and output measurements.

The Loss Function was 12.03 and the Akaike's Final Prediction Error (FPE) was 16.49. The Loss Function and the FPE are not significant by themselves. But, these values serve as reference to compare other ARX orders structures. For instance, when the same dynamic excursion was used, but the number of data N was decreased to 150 samples. The elapsed computer time was 0.098404 seconds, representing this value a reduction in time of 0.045 seconds or 26.1% of the first computer time. But the Loss function and FPE were 15.9039 17.4791, respectively. These values increase because there are less data for fitting and these parameters are functions of the number of data.

Figure 4.14 Comparison between model vs measured outputs and the input dynamics.

The next several ARX identification examples are done to demonstrate the approach to get sequential identification models. Each model is based on 200 data samples. The model is formed and then the oldest two pieces of data are eliminated and two “newer” pieces of data to the batch. For this sequential demonstration, four identifications were performed starting at 60 seconds and ending at 62.4 seconds. The new identifications were carried out only every other sampling time (i.e. every 0.8 seconds) to reduce the number of plots required to illustrate the results.

The multivariable ARX equation model for $t = 60.0$ second is:

$$\begin{aligned}
 & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} y(t) + \begin{bmatrix} -0.8031 & 260.4427 \\ -0 & -0.4484 \end{bmatrix} y(t - 0.4) + \begin{bmatrix} -0.1246 & 175.43 \\ 0 & 0.1964 \end{bmatrix} y(t - 0.8) + \\
 & \qquad \qquad \qquad \begin{bmatrix} 0.1942 & -817.77 \\ 0 & 0 \end{bmatrix} y(t - 1.2) = \\
 & \begin{bmatrix} 0 & 0 & 34.92 & -15.66 \\ 0 & 0 & -0.0079 & 0.0054 \end{bmatrix} u(t) + \begin{bmatrix} -39.27 & 80.83 & -43.38 & 0.6802 \\ 0.0108 & -0.0112 & 0.0030 & 0.0034 \end{bmatrix} u(t - 0.4) + \\
 & \begin{bmatrix} 7.802 & 65.69 & 0 & 0 \\ -0.0045 & 0.0087 & 0 & 0 \end{bmatrix} u(t - 0.8) + \begin{bmatrix} -8.3078 & 0 & 0 & 0 \\ 0.0057 & 0 & 0 & 0 \end{bmatrix} u(t - 1.2) \quad (4.14)
 \end{aligned}$$

The elapsed computer time was 0.123829 seconds. The Loss function and FPE were 12.031 and 16.493, respectively. Figure 4.15 shows the results that compares the measured data with the estimates of the model.

Figure 4.15 Comparison between model versus measured outputs and the input dynamics at t = 60 seconds.

The multivariable ARX equation model for $t = 60.8$ second is:

$$\begin{aligned}
 & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} y(t) + \begin{bmatrix} -0.7947 & 204.53 \\ 0 & -0.4331 \end{bmatrix} y(t - 0.4) + \begin{bmatrix} -0.1121 & 160.91 \\ -0 & 0.1691 \end{bmatrix} y(t - 0.8) + \\
 & \begin{bmatrix} 0.1655 & -771.36 \\ 0 & 0 \end{bmatrix} y(t - 1.2) = \\
 & \begin{bmatrix} 0 & 0 & 57.41 & -33.01 \\ 0 & 0 & 0.0009 & -0.0013 \end{bmatrix} u(t) + \begin{bmatrix} -60.12 & 91.32 & -52.27 & -12.86 \\ 0.0060 & -0.0064 & 0.0054 & -0.0068 \end{bmatrix} u(t - 0.4) + \\
 & \begin{bmatrix} 13.06 & 71.86 & 0 & 0 \\ -0.0022 & 0.0082 & 0 & 0 \end{bmatrix} u(t - 0.8) + \begin{bmatrix} -11.06 & 0 & 0 & 0 \\ 0.0056 & 0 & 0 & 0 \end{bmatrix} u(t - 1.2) \quad (4.15)
 \end{aligned}$$

The elapsed computer time was 0.120887 seconds. The Loss function and FPE were 9.08877 and 12.4594, respectively. Figure 4.16 shows the results that compare the measured data with the estimates of the model.

Figure 4.16 Comparison between model versus measured outputs and the input dynamics at $t = 60.8$ seconds.

The multivariable ARX equation model for $t = 61.6$ second is:

$$\begin{aligned}
 & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} y(t) + \begin{bmatrix} -0.7528 & 462.60 \\ 0 & -0.4097 \end{bmatrix} y(t - 0.4) + \begin{bmatrix} -0.1402 & 160.15 \\ -0 & 0.2281 \end{bmatrix} y(t - 0.8) + \\
 & \begin{bmatrix} 0.2122 & -375.58 \\ 0 & 0 \end{bmatrix} y(t - 1.2) = \\
 & \begin{bmatrix} 0 & 0 & 21.49 & -4.25 \\ 0 & 0 & -0.0054 & 0.0038 \end{bmatrix} u(t) + \begin{bmatrix} -24.74 & 89.72 & -30.45 & -0.0754 \\ 0.0125 & -0.0066 & 0.0108 & -0.0063 \end{bmatrix} u(t - 0.4) + \\
 & \begin{bmatrix} 5.734 & 69.56 & 0 & 0 \\ -0.0034 & 0.0050 & 0 & 0 \end{bmatrix} u(t - 0.8) + \begin{bmatrix} -7.1781 & 0 & 0 & 0 \\ 0.0066 & 0 & 0 & 0 \end{bmatrix} u(t - 1.2) \quad (4.16)
 \end{aligned}$$

The elapsed computer time was 0.117140 seconds. The Loss function and FPE were 15.2248 and 20.871, respectively. Figure 4.17 shows the results that compares the measured data with the estimated of the model.

Figure 4.17 Comparison between model versus measured outputs and the input dynamics at t = 61.6 seconds.

The multivariable ARX equation model for $t = 62.4$ second is:

$$\begin{aligned}
 & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} y(t) + \begin{bmatrix} -0.7496 & 485.67 \\ 0 & -0.4321 \end{bmatrix} y(t - 0.4) + \begin{bmatrix} -0.1356 & 163.19 \\ 0 & 0.2157 \end{bmatrix} y(t - 0.8) + \\
 & \qquad \qquad \qquad \begin{bmatrix} 0.2078 & -327.76 \\ 0 & 0 \end{bmatrix} y(t - 1.2) = \\
 & \begin{bmatrix} 0 & 0 & 16.56 & -0.2768 \\ 0 & 0 & -0.0027 & 0.0016 \end{bmatrix} u(t) + \begin{bmatrix} -20.36 & 80.52 & -27.97 & 2.0359 \\ 0.0101 & -0.0061 & 0.0092 & -0.0072 \end{bmatrix} u(t - 0.4) + \\
 & \begin{bmatrix} 4.7167 & 68.20 & 0 & 0 \\ -0.0029 & 0.0076 & 0 & 0 \end{bmatrix} u(t - 0.8) + \begin{bmatrix} -6.5022 & 0 & 0 & 0 \\ 0.0063 & 0 & 0 & 0 \end{bmatrix} u(t - 1.2) \quad (4.17)
 \end{aligned}$$

The elapsed computer time was 0.114992 seconds. The Loss function and FPE were 14.6158 and 20.0363, respectively. Figure 4.18 shows the results that compare the measured data with the estimates of the model.

Figure 4.18 Comparison between model versus measured outputs and the input dynamics at $t = 62.4$ seconds.

The ARX identification model has been shown to accurately capture the dynamics occurring in a sequential batch of time domain data. In general, the system dynamics can be obtained from the past input/output data through the present using the ARX identification technique. Although in the examples above the identification was performed (adapted) each two sample periods (0.8 seconds), this time could be extended, i.e. to four sample times. The limitation on how long a model is valid is a function of how rapidly the operating point is changing. So long as the system stays near the same operating point, it is not necessary to make a new model every sample period or every two sample periods. However, since the ARX model can be computed in approximately one-third of a sampling period, it is simpler to just compute a new model at regular, 0.4-second intervals.

The time delay has also been included inside of this elapsed time of model application. The ARX identification demonstrated a relatively low computational time. The examples shown above required computation times of around 0.12 seconds which is comfortably less than the sample time of 0.4 seconds. This time must be added to the time spent by the MPC optimization, which will be determined in a future section. It is also possible to improve the computation time

by compiling the file into a standalone application or even by getting a faster computer exclusively for this application.

4.4.5.2 MPC Design Using On-line ARX Models

The ARX model constructed in the former section is used to design a MPC as described in [84], [85], [86], [87], [88], and [89]. The Matlab MPC Toolbox arises from a State-Space formulation. The ARX model is loaded in the Toolbox and it is internally transformed to a State-Space form.

In order to test the MPC controller, the inputs and outputs were constrained. The cathode air flow (CAF) was changed from the nominal point (0.55 kg/s) to a new set-point value (0.7 kg/s). The set-point for turbine speed was moved slightly from 40200 to 40500 rpm. Ten seconds after the simulation started, a sinusoidal cathode airflow set-point was set to test the controller.

Figure 4.19 shows the plant outputs and inputs behaviors after the set-points were set. In this figure the controller was tested with the default weighting values of the MPC toolbox. In other words, any tuning was done in the controller and all of the weighting matrices kept their default values. In the first three seconds, the valves and the electric load made some moves to implement the new set-points for cathode air flow and turbine speed. At ten seconds, a sinusoidal set-point for cathode airflow was set. It is seen that the cathode air flow is not accurately tracking the set-point.

Figure 4.20 shows the plant outputs and inputs after the controller was tuned using the weighted matrices. For the same output variation, the cathode airflow shows better tracking, but with a one sample delay used in the identification equation. In Figure 4.21 the anticipation feature (explained in section 1.6.1.6) of MPC was used to improve the delay response. The cathode airflow tracking improves using this feature and the oscillations in turbine speed shown in Figure 4.20 were mitigated. It can be seen in Figure 4.21 that the reaction of inputs is carried out seconds before the set-point is set to the sinusoid, as compared with the Figure 4.20 where the inputs reaction was slightly after the set-point change occurred. Also, in Figure 4.21 the movements of the inputs were relatively further from the constraints values compared with Figure 4.20.

In general, the tracking of the turbine speed and cathode airflow was very robust in Figure 4.21. In the figure the inputs moved very quickly to adapt to any change in the set-points. Also the figures show that although the constraints limits were sometimes achieved, the response of the controller and the system has been qualitatively and quantitatively very good, showing the strong characteristic of MPC to work with physical constraints.

Figure 4.19 MPC input and output dynamics with some set-point variation using default values of Matlab weighted matrices (without anticipation) using the ARX identification model.

Figure 4.20 MPC input and output dynamics with some set-point variation after tuning the controller weighted matrices (without anticipation) using the ARX identification model.

Figure 4.21 MPC input and output dynamics with some set-point variation after tuning the controller and using the MPC anticipation feature and the ARX identification model.

4.4.6 State-Space Identification and MPC

State-Space models will also be tested for identification. This method only requires the model order (dimension of the state vector) to be provided for the identification. In general, the State-Space model provides a more complete representation of the system, especially for MIMO systems because the State-Space model is similar to a first principles model. The identification procedure does not involve nonlinear optimization so the estimation reaches a solution regardless of the initial guess. For MIMO systems, if the State-Space and ARX model order is too ~~high~~”, the ARX model would be faster and more efficient. The disadvantages of State-Space models are that for a large number of data points the computations can be slow and require large amount of memory (for more information see [83]). Furthermore, the solution is not unique for the State-Space models.

The State-Space has the advantage of directly generating a model for the disturbances. The ~~pem~~” command is used and allows the user to specify a range of model orders and to evaluate the performance of several State-Space models. This command uses the prediction error method. It means that the minimized error is determined by: $y(t) - \hat{y}(t|t-1)$. This identification included a disturbance model as shown in equation 4.18:

$$\begin{cases} x(t + Ts) = Ax(t) + Bu(t) + Ke(t) \\ y(t) = Cx(t) + Du(t) + e(t) \end{cases} \quad (4.18)$$

Next are presented examples of state space identification models and the MPC design using these particular models. Initially, the batch of data was loaded into the Matlab workspace for identification as an ~~iddata~~”. After that the command ~~pem~~” was used for the State-Space identification. The model order was tested and it was found that the model order 2 was enough to fitting the data. It is important to remember that in State-Space identification, the model order is the size of the state vector.

4.4.6.1 State-Space Identification: Example 1.

Figure 4.22 shows data used to identify the system after the step changes were made in the different inputs. The combustor fuel flow was selected for the identification task as another

input, since there is a direct cause/effect relationship between the fuel flow and other important parameters like turbine speed. For identification, it is irrelevant that the fuel flow is a *disturbance* input; it is still an input from the standpoint of model development. In the MPC design, we will reassign the fuel flow as a measured disturbance. The outputs selected are the turbine speed and the cathode inlet flow.

Figure 4.22 illustrates that the response times for the CA and HA valves are less than 2 seconds to change from 40% to 80% and 50 to 80% open, respectively. The electric load for this case changes from 50 kW to 0 kW in one step. This is a huge power change in the facility that induces many changes in temperatures and instabilities in the compressor.

System identification has the command *–advice*” that takes the data selected for identification and analyzes the *–excitation level*”, *–possibility of feedback*”, and *–possibility of nonlinearities*”. The *–excitation level*” is a measurement of persistent excitation for the inputs. The *–excitation level*” was low for the electric load most of the time, except in the dynamic transitions. The electric load data shows a constant value before and after the reference point was achieved. This noiseless value of the electric load suggested a zero excitation level for identification in steady state. The *–possibility of feedback*” was found in the data. This is adverse in system identification, but does not mean that a good model cannot be achieved. The *–possibility for nonlinearities*” was negative for all cases run in these examples. This is another proof that in the range selected for operation of this controller, the system can be considered linear. Examples of the *–advice*” identification for the four cases of the Figure 4.23 and 4.24 are shown in Appendix M.

Figure 4.23, 4.24, 4.25, and 4.26 show eight sequential plots of the on-line State-Space identification process. The best model was obtained based on the smallest value of the Loss Function and FPE parameters. The model order determines the size of the state vector when state space identification is used. Another parameter showed in the figures is the fit.

Figure 4.22 HyPer Data measured for state space identification.

Figure 4.23, 4.24, 4.25, and 4.26 show the results obtained by the simulation model, the elapsed time employed by the computer to perform the identification, the values of the Loss Function and FPE parameters, and the State-Space equation found in each case. The State-Space

second order model was enough to simulate the dynamics of the system. It is important to notice that the input data employed in the identification model was the same employed to generate the simulation, with all the noise and disturbances included. The number of data points in this identification was 400. The maximum elapsed time spent by the computer to do this calculation was 0.654 seconds. While twice as many data points were used in the State-Space compared with the ARX case, the total identification time was approximately 5 times more for the state-space method. The on-line identification performed here shows good results even when huge changes in valve positions and electric load are performed. In this case, the availability of computational power could be a critical factor.

Based on the values of FPE and Loss Function, the State-Space model is better than ARX because it includes an estimation model of the disturbance. In addition, it was easier to set a generalized model-order for the entire State-Space model compared with the ARX model where each input/output relationship has its own order.

Figure 4.23, 4.24, 4.25, and 4.26 also show that the tracking between the measured data and the simulated model is very good. It is important to highlight that this is a MIMO identification whose representation is typically more difficult to obtain due to the presence of interactions, or nonlinearities, etc. This recursive on-line identification is appropriate for an adaptive model and controller. This allows implementation of a real-time batch least squares identification model to use with a MPC algorithm.

Figure 4.23 Comparison between on-line state-space identification model data and measured output (time 1 and 2).

3. Model Order: 2 Elapsed time: 0.538 (s)
Loss function: 0.0504 and FPE 0.0519

$$A = \begin{bmatrix} 1.0085 & 726.79 \\ -9.5573 \times 10^{-6} & 0.9497 \end{bmatrix} \quad B = \begin{bmatrix} 29.486 & -38.968 & 0.53241 & -0.76763 & 0.002518 \\ -0.00271 & 0.00152 & 0.00570 & -0.0013 & -2.053 \times 10^{-7} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad D = [0] \quad K = \begin{bmatrix} 0.90717 & -764.5 \\ -5.838 \times 10^{-6} & 0.29019 \end{bmatrix} \quad x(0) = \begin{bmatrix} 40406 \\ 0.5999 \end{bmatrix}^T$$

4. Model Order: 2 Elapsed time: 0.562 (s)
Loss function: 0.0582 and FPE: 0.0600

$$A = \begin{bmatrix} 0.9303 & 3180.9 \\ -3.0372 \times 10^{-6} & 0.9101 \end{bmatrix} \quad B = \begin{bmatrix} 7.6465 & 17.27 & 3.5012 & 0.7224 & 0.018614 \\ -0.00256 & 0.00096 & 0.00319 & -0.00117 & -2.536 \times 10^{-7} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad D = [0] \quad K = \begin{bmatrix} 0.6333 & 35.186 \\ -1.0456 \times 10^{-5} & 0.07679 \end{bmatrix} \quad x(0) = \begin{bmatrix} 40413 \\ 0.60272 \end{bmatrix}^T$$

Figure 4.24 Comparison between on-line state-space identification model data and measured output (time 3 and 4).

5. Model Order: 2 Elapsed time: 0.555 (s)
Loss function: 0.0565 and FPE 0.0581

$$A = \begin{bmatrix} 0.93644 & 3068.7 \\ -9.0514 \times 10^{-7} & 0.8471 \end{bmatrix} \quad B = \begin{bmatrix} 4.1822 & 25.132 & -9.7283 & 3.2042 & 0.017996 \\ -0.0021 & 0.000210 & 0.0031 & -0.00113 & -7.115 \times 10^{-7} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad D = [0] \quad K = \begin{bmatrix} 0.67823 & -266.39 \\ -2.0405 \times 10^{-5} & 0.12476 \end{bmatrix} \quad x(0) = \begin{bmatrix} 40570 \\ 0.59788 \end{bmatrix}^T$$

6. Model Order: 2 Elapsed time: 0.624 (s)
Loss function: 0.0577 and FPE 0.0595

$$A = \begin{bmatrix} 0.90517 & 3360.9 \\ 1.9158 \times 10^{-7} & 0.89753 \end{bmatrix} \quad B = \begin{bmatrix} 14.068 & 3.8537 & 13.438 & -3.3453 & 0.027227 \\ -0.0032 & 0.002319 & 0.00113 & -0.00078 & -6.055 \times 10^{-7} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad D = [0] \quad K = \begin{bmatrix} 0.65642 & 5.1743 \\ -1.1987 \times 10^{-5} & 0.0698 \end{bmatrix} \quad x(0) = \begin{bmatrix} 40499 \\ 0.59658 \end{bmatrix}^T$$

Figure 4.25 Comparison between on-line state-space identification model data and measured output (time 5 and 6).

7. Model Order: 2 Elapsed time: 0.611 (s)
Loss function: 0.0597 and FPE: 0.0615

$$A = \begin{bmatrix} 0.89873 & 3616.4 \\ 4.2053 \times 10^{-7} & 0.89965 \end{bmatrix} \quad B = \begin{bmatrix} 12.089 & 9.0121 & 12.547 & -3.3413 & 0.033636 \\ -0.0033 & 0.002483 & 0.00094 & -0.00075 & -6.764 \times 10^{-7} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad D = [0] \quad K = \begin{bmatrix} 0.63895 & 12.613 \\ -1.3419 \times 10^{-5} & 0.038105 \end{bmatrix} \quad x(0) = \begin{bmatrix} 40454 \\ 0.59646 \end{bmatrix}^T$$

8. Model Order: 2 Elapsed time: 0.627 (s)
Loss function: 0.0593 and FPE: 0.0611

$$A = \begin{bmatrix} 0.89444 & 3370.7 \\ 2.0767 \times 10^{-7} & 0.92515 \end{bmatrix} \quad B = \begin{bmatrix} 15.289 & 1.4735 & 19.315 & -4.0187 & 0.037177 \\ -0.0035 & 0.002912 & 0.00071 & -0.00077 & -5.464 \times 10^{-7} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad D = [0] \quad K = \begin{bmatrix} 0.64359 & 98.332 \\ -1.3973 \times 10^{-5} & 0.00635 \end{bmatrix} \quad x(0) = \begin{bmatrix} 40452 \\ 0.59708 \end{bmatrix}^T$$

Figure 4.26 Comparison between on-line state-space identification model data and measured output (time 7 and 8).

4.4.6.2 State-Space Identification: Example 2.

One of the problems of off-line identification is in the required a priori extensive design to accommodate all possible operating points at different conditions in the HyPer envelope. This requires extensive work to insure that the model works at different operating and dynamic conditions. Another disadvantage of the off-line identification model is that any change or deterioration in the equipment is not accounted for by the old identification version. Meanwhile the on-line identification allows the model to be updated continuously. The transfer function model obtained by Tsai is an example of this off-line approach. Limitations of operability of this off-line model have not been studied, but it is a cumbersome problem because some changes have been made in the facility.

This example is presented to show a completely different dynamic response and to test if the identification approach is valid in an extended region of operation. All magnitudes and directions of movements of the inputs/outputs in this example are different from those shown in the former example. Figure 4.27 shows the measured output and input data for a case in which the electric load was increased from 0kW to 25kW in approximately one second. This change in load decreases the turbine speed. Also, changes in the valve settings increased the cathode airflow, as opposed to a decrease in this output in the prior example. The CA valve changes from 80% to 60% in less than two seconds. The HA valve closes by 60% in approximate two seconds. This is a huge and quick change in the HA valve position. The BA valve stayed constant during this simulation.

Figure 4.28, 4.29, and 4.30 show 6 different sequential plots of identification. The plots show very good tracking of the model with the measured data. The maximum elapsed time by the computer was 0.615 seconds. The model order was 2. The model was continuously updated and the tracking was very good in all the subplots shown. The on-line identification performed here shows good results even when huge change in valves position and electric load are performed. In this case, the availability of computational power, as opposed to memory, could be a critical factor.

Figure 4.27 HyPer Data measured for state space identification.

Figure 4.28 Comparison between on-line state-space identification model data and measured output (time 1 and 2).

Figure 4.29 Comparison between on-line state-space identification model data and measured output (time 3 and 4).

5. Model Order: 2 Elapsed time: 0.557 (s)
Loss function: 0.00428 and FPE: 0.0441

$$A = \begin{bmatrix} 0.9542 & -806.88 \\ 3.312 \times 10^{-5} & 0.28797 \end{bmatrix} \quad B = \begin{bmatrix} 11.032 & -24.718 & 36.241 & -35.998 & -0.0049889 \\ -0.0033 & -0.00485 & -0.00469 & -0.00237 & -1.125 \times 10^{-5} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad D = [0] \quad K = \begin{bmatrix} 0.9706 & 430.87 \\ 9.8956 \times 10^{-6} & 0.50897 \end{bmatrix} \quad x(0) = \begin{bmatrix} 40461 \\ 0.37459 \end{bmatrix}^T$$

6. Model Order: 2 Elapsed time: 0.539 (s)
Loss function: 0.0402 and FPE: 0.0415

$$A = \begin{bmatrix} 0.91048 & -495.38 \\ 3.0521 \times 10^{-5} & 0.30434 \end{bmatrix} \quad B = \begin{bmatrix} 9.6672 & -22.034 & 53.558 & -36.276 & -0.00335 \\ -0.003349 & -0.00471 & -0.0037 & -0.0024 & -9.57 \times 10^{-6} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad D = [0] \quad K = \begin{bmatrix} 0.91516 & 310.56 \\ 4.394 \times 10^{-6} & 0.50045 \end{bmatrix} \quad x(0) = \begin{bmatrix} 40411 \\ 0.36259 \end{bmatrix}^T$$

Figure 4.30 Comparison between on-line state-space identification model data and measured output (time 5 and 6).

4.4.6.3 MPC Design Using On-line State-Space Models

The plant models identified in section 4.4.6.1 and 4.4.6.2 were used to design and test an off-line MPC controller as described in [89], and [90]. The State-Space models were imported to the MPC toolbox. The constraints and weight matrices were then tuned to test different plant conditions. Changes in cathode air flow and application of fuel flow disturbances were set. The MPC GUI was updated with the former information and it is shown in Figure 4.31. The MPC relates the inputs (manipulated variables) and the outputs (measured outputs) to the control diagram of the MPC GUI. The MPC GUI assigns unmeasured disturbance models to each output variable of the model automatically. These unmeasured disturbance models are Gaussian white noise models and could be modified in the command line or inside the Simulink MPC block.

Figure 4.31 illustrates the MPC structure. Seven inputs regulate the plant, four of them are manipulated variables; two are unmeasured disturbances, one for each output variable; and one is a measurable disturbance. This plant model does not have unmeasured outputs. Both of the outputs selected for control are measured and are depicted in the MPC structure overview. The name, type, description, units and nominal properties of the inputs and outputs are customized in this window as shown in the Figure 4.31. The fuel flow represents the thermal effluent of the FC, and it is not considered in the real system as a manipulated variable. The thermal effluent is then a measured disturbance because currently the fuel flow is measured in the facility. In a real system, the heat generated by the FC is dispersed in the 3D structure of the FC and in the air flow running through the stack channels. The heat coming out from the unreacted fuel burned in the combustor is also challenging to measure, due to the difficulty to know the exact thermal behavior of the FC as the real fuel utilization and the dynamics of the reactions are carried out inside of each cell. Thus, in real application, this heat could be an unmeasured disturbance.

In this section, the EL is considered a manipulated variable. There is not a unified opinion in the scientific community about this concern. Some researchers consider that the total supplied power of the hybrid system could be split in a way that the turbine EL could be used to control the turbine speed, at least up to a certain point. The EL has been used in the HyPer facility to control the turbine speed in a SISO PID control scheme. Other researchers consider that the EL

is a variable that changes with the external demand for electrical power, and must be considered a measured disturbance. For this example, a controller design will be tested considering the turbine EL as a manipulated variable.

Figure 4.31 Variable type, description, units and nominal values.

The next step is to consider the test scenarios for tuning the controller. The first scenario is set with the default values of the MPC Matlab Toolbox. The results of scenario 1 are shown in Figure 4.32. The test is to initially set a cathode airflow and turbine speed set-point. At ten seconds after, a sinusoidal set-point is set in the cathode airflow for the rest of the simulation. Then, a fuel flow perturbation of 50 grams per minute is included at 20 seconds to simulate a heat effluent change, as shown in the Figure 4.32. The time duration of the simulation is 30 seconds. The prediction (P) and control horizon (M) are 30 and 10 seconds, respectively.

The results for the system output/input response are illustrated in the Figure 4.32. The turbine speed was able to follow the set-points showing robust tracking. The CA and HA valves follow the sinusoidal behavior of the cathode airflow. The EL increases in order to compensate the increased of fuel flow but the hard constraint of 50 kW was achieved. Then, the BA valve opened to compensate for this disturbance. The outputs start the simulation from values calculated from the State-Space model using the *“initial value”* determined in the system identification. The output/input values remained inside the constraint bounds defined in the MPC controller.

Figure 4.33 shows the results after tuning the controller by using the weight matrices entries. The outputs are able to follow the set-points with a delay found for the cathode airflow. The CA and HA valves follow the sinusoidal set-point behavior of cathode airflow. The electric load increased and the BA valve opened in order to compensate for the turbine speed increasing due to the fuel flow disturbance at time 20 seconds. The output/input values remained inside the constraint bounds defined in the MPC controller.

Figure 4.34 shows the results when the anticipation feature is used. Now the tracking of the cathode air flow is improved with the *“prediction”* reaction of the inputs (before 10 seconds). The turbine speed also showed very good tracking and the oscillations shown in Figure 4.33 were mitigated. The CA and BA valves and the electric load achieved the constraint value several times.

Figure 4.32 MPC input and output dynamics with some set-point variation using default values of Matlab weighted matrices (without anticipation) using the State-Space identification model.

Figure 4.33 MPC inputs and outputs dynamics with some set-point variation after tuning the controller weighted matrices (without anticipation) using the State-Space identification model.

Figure 4.34 MPC inputs and outputs dynamics with some set-point variation after tuning the controller and using the MPC anticipation feature and the State-Space identification model.

4.4.7 Transfer Functions Model and MPC

Next is presented an example of the MPC design using the MIMO TFs obtained from the HyPer facility by Tsai [29]. This is another way to implement MPC very quickly. Using these TFs, the necessity of recursive identification is avoided. However, the possibility of large modeling uncertainty can arise, if the model is not accurate enough far away from the operating point established for the identification of these TFs. The TFs equations were derived from a sinusoidal modulation of the CA, HA, BA valves, EL, and fuel valve. TFs were obtained by Tsai [29] for these input/output variables using frequency analysis. Magnitude and phase Bode plots were obtained from the data and TFs were then derived from this information. The outputs of interest for this example are the turbine speed (speed), and cathode airflow (CAF). The turbine speed and the cathode air flow were adjusted to set-point values by the controller.

Transfer functions of turbine speed and cathode airflow as functions of the CA valve opening are:

$$G_{11} = \frac{Speed}{CA} = \frac{-26.68 (s^2 + 0.2s + 0.1494)}{(s + 0.077)(s + 0.082)(s + 2)} e^{-0.66 s}$$

$$G_{21} = \frac{CAF}{CA} = \frac{-0.32 (s + 0.085)}{(s + 0.08)(s + 1.91)} e^{-0.51 s}$$

Transfer functions of turbine speed and cathode airflow as functions of the HA valve opening are:

$$G_{12} = \frac{Speed}{HA} = \frac{2.543 (s + 0.03)}{(s + 0.06)(s + 0.09)} e^{-0.85 s}$$

$$G_{22} = \frac{CAF}{HA} = \frac{-0.04 (s + 0.7)}{(s + 0.91)(s + 2.5)} e^{-0.77 s}$$

Transfer functions of turbine speed and cathode inlet flow as functions of the BA valve opening are:

$$G_{13} = \frac{Speed}{BA} = \frac{-0.227}{(s + 0.046)(s + 0.09)(s + 1)(s + 0.25)} e^{-1.12 s}$$

$$G_{23} = \frac{CAF}{BA} = \frac{-0.000534}{(s + 0.12)(s + 0.32)(s + 0.8)} e^{-2.29 s}$$

Transfer functions of the turbine speed and cathode airflow as functions of electric load are:

$$G_{14} = \frac{Speed}{EL} = \frac{-21.339 (s + 0.02)}{(s + 0.04)(s + 0.06)} e^{-0.153 s}$$

$$G_{24} = \frac{CIF}{EL} = \frac{0.048 * (s + 0.022)(s + 0.3)(s + 1.5)}{(s + 0.043)(s + 0.071)(s + 1)(s + 3.5)} e^{-1.59 s}$$

$$G_{34} = \frac{CIT}{EL} = \frac{-0.001(s + 2.5)^2}{(s + 0.025)(s + 0.1)} e^{-2.71 s}$$

Transfer functions of the turbine speed and cathode airflow as function of the fuel valve percent open are:

$$G_{15} = \frac{Speed}{FV} = \frac{17.495 (s + 0.03)}{(s + 0.04)(s + 0.07)} e^{-0.2 s}$$

$$G_{25} = \frac{CAF}{FV} = \frac{0.041 * (s + 0.022)(s + 0.3)}{(s + 0.033)(s + 0.061)(s + 1)(s + 4)} e^{-1.38 s}$$

The MIMO Transfer Function is condensed in the following matrix representation:

$$\begin{bmatrix} Speed \\ CAF \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} & G_{15} \\ G_{21} & G_{22} & G_{23} & G_{24} & G_{25} \end{bmatrix} \begin{bmatrix} CA \\ HA \\ BA \\ EL \\ FV \end{bmatrix} \quad (4.19)$$

This Linear Time Invariant (LTI) system is transformed to an extended MIMO State-Space representation appropriate to run on the MPC Toolbox as:

$$\begin{cases} x(t + Ts) = Ax(t) + Bu(t) + Ke(t) \\ y(t) = Cx(t) + Du(t) + e(t) \end{cases} \quad (4.20)$$

Figure 4.35 shows the MPC structure and different properties of the input and output signals such as name, type, description, units, and nominal operation points of the system model by the TFs. The fuel valve is not considered an input because it represents in part the heat effluent of the FC, which, in the table, is selected as a measured disturbance. Note that the fuel flow is not included as a manipulated variable but as a measured disturbance. The prediction horizon (P) and the control horizon for testing are 30 and 10 seconds, respectively. Figure 4.36 shows the limits of the input variables and the outputs. As mentioned above, in order to maintain some stall margin, the lower value of the CA valve was limited to 30% open.

Input Signal Properties				
Name	Type	Description	Units	Nominal
CA	Manipulated	Cold Air Valve	% Open	40
HA	Manipulated	Hot Air Valve	% Open	40
BA	Manipulated	Bleed Air Valve	% Open	14
EL	Manipulated	Turbine Electric Load	kW	45
Fuel	Measured Disturb.	Fuel Flow Demand	%	35
Output Signal Properties				
Name	Type	Description	Units	Nominal
Speed	Measured	Turbine Speed	RPM	40500
CAF	Measured	Cathode Airflow	kg/s	0.5

Figure 4.35 MPC structure, inputs and outputs type, units and nominal values (Matlab MPC Toolbox).

Constraints on Manipulated Variables					
Name	Units	Minimum	Maximum	Max Down Rate	Max Up Rate
CA Valve	% Open	30	100	10	10
HA Valve	% Open	0	100	10	10
BA Valve	% Open	0	30	5	5
Electric Load	kW	0	50	10	10

Constraints on Output Variables			
Name	Units	Minimum	Maximum
Turbine Speed	RPM	38000	44000
Cathode Airflow (CAF)	kg/s	0.3	1

Figure 4.36 MPC inputs and outputs constraints (Matlab MPC Toolbox).

4.4.7.1 MPC Design Using Transfer Functions Model

The TFs were used to test controller scenarios. The first scenario is set with the default values of the MPC Matlab Toolbox. The results of scenario 1 are shown in Figure 4.37. The test consists to initially set a cathode airflow and turbine speed set-point. At ten seconds after, a sinusoidal set-point is set in the cathode airflow for the rest of the simulation. Then, 5% of fuel valve percentage opening perturbation is included at 20 seconds, as shown in the Figure 4.37. The time duration of the simulation is 30 seconds. The prediction (P) and control horizon (M) are 30 and 10 seconds, respectively. Disturbance on fuel flow is applied to simulate a heat effluent change.

The results of this first scenario for the system outputs/inputs response are illustrated in the Figure 4.37. The cathode airflow was not able to tracking the initial set-point and even the sinusoidal variation. Initially, the cathode airflow showed very bad oscillations to get the set-point value. The turbine speed was able to follow the set-points showing robust tracking of it. The CA and HA valves follow the sinusoidal behavior of the cathode airflow. The electric load increases in order to compensate the increase of fuel flow but the hard constraint of 50 kW was achieved. Then, the BA valve opened to compensate for this disturbance. The output/input values remained inside the constraint bounds defined in the MPC controller.

Figure 4.38 shows the results after tuning the controller by using the weight matrices entries. The outputs are able to follow the set-points with a delay found for the cathode airflow. The initial cathode airflow oscillations were rejected by the controller. The fuel flow perturbation had some appreciable effect in the turbine speed. The CA and HA valves follow the sinusoidal set-point behavior of cathode airflow. The EL increased and the BA valve opening in order to compensate for the turbine speed increasing due to fuel flow disturbance at time 20 seconds. The output/input values remained inside the constraint bounds defined in the MPC controller.

Figure 4.39 shows the results when the anticipation feature is used. Now the tracking of the cathode air flow is improved with the ahead reaction of the inputs (before 10 seconds). The delay behavior was compensated by the anticipation reaction of the controller. The turbine speed also showed very good tracking. The perturbation in the turbine speed was rejected by the controller. The CA valve and the electric load achieved the constraint value several times. Appendix N shows the Simulink schematic of MPC with the TFs model.

Figure 4.37 MPC inputs and outputs dynamics with some set-point variation using default values of Matlab weighted matrices (without anticipation) using transfer functions model.

Figure 4.38 MPC inputs and outputs dynamics with some set-point variation after tuning the controller weighted matrices (without anticipation) using the transfer functions model.

Figure 4.39 MPC inputs and outputs dynamics with some set-point variation after tuning the controller and using the MPC anticipation feature and the transfer functions model.

Chapter 5

Conclusions

The work presented in this document has been focused on two different research areas. First, a 3^4 factorial design of experiments was performed for characterization of the operating space for the NETL HyPer facility. Four factors were varied simultaneously: CA, HA, BA valves and the electric load. Each factor was changed in steps according to the randomized factorial design. The cathode air mass flow, various pressures, and several temperatures were monitored and the set-points were held until the system achieved steady state values in each treatment of the experiments. Many variables of the facility were recorded and the most important for FC/GT integration were selected for analysis. The objective was to develop a thorough understanding of the steady state behavior of this complex system over a significant range of the possible operating conditions. Analysis of the data gathered from these experiments has provided the first-ever map of most of the state space of the HyPer facility.

Each step change in the inputs also produced transient response data that was useful to fit System Identification structures and to create simplified dynamic models of some of the key interactions within the HyPer facility. In particular, the effects of the manipulation of the three air bypass valves and the electric load on the cathode air flow and the turbine speed were modeled. These models were used to design, develop and analyze a Model Predictive Control strategy for management of those two key operating variables. Several approaches were examined to the development of the controller, including the comparison of model order and structure, the selection of appropriate terms for the optimization cost function and the tuning of weighting parameters in the cost function. Excellent tracking of both simple step changes and more complicated sinusoidal variations in the set-points for the controlled variables was achieved.

More specific observations and some recommendations for future research are provided next.

Design of Experiments

A battery of experiments has been designed, implemented, and analyzed using the HyPer facility. This is the first time that anyone has been able to describe the MIMO operating space of the system using experimental data. It has great importance for building control strategies for this and possibly other configurations of hybrid systems.

The experiments have been focused primarily on the linear region of operation in the HyPer system. The interaction effects among the variables in this region were found to be small compared with the main effects themselves. This is a very important result, because the advantages of each of valve acting independently have been preserved when the valves are manipulated simultaneously. However, the input variables are not completely decoupled. For example, opening the Cold Air valve affects not only the turbine inlet temperature and the mass flow rate of air through the cathode. Therefore some simultaneous manipulation of the bypass valves is necessary.

Although the valves don't create multiplicative interactions at steady state, the relationships between valve positions and many of the other variables of interest (cathode air flow, cathode inlet air temperature and turbine inlet air temperature) are not linear, as was shown in the analysis in Chapter 3. The nonlinearities are not severe, however, and the input/output responses can be modeled using low-order polynomial functions. Example models were derived in this work and by Rosen et.al. [58].

Thus far, only a handful of variables and relationships have been examined from the library of data collected in the experiments. This work has focused on data related to the control of the HyPer system, however the experimental records include data that can provide important insights on nearly any operating parameter in the system. Examples would be turbine exhaust temperature, heat exchanger performance, turbine efficiency, compressor efficiency and many other key variables could be analyzed with the data from the battery of experiments.

Another important result of this work was to expand the potential turndown ratio of the system. Tucker suggested in [3], that the maximum turn down for the system was around 69%

of the FC power. His estimate was based on the amount of air that could be diverted around the cathode through either the HA or CA valves when one was opened 100% with the other valve closed. If EL is removed from the fuel cell and airflow through the cathode remains constant, the FCs could be overcooled. Thus EL turndown is directly related to the ability to bypass air around the cathode. The DoE experiments have shown that with simultaneous operation of the CA and HA valves, the bypass capacity is 85% of the cathode air flow with HA open 80% and CA open 80%. Thus a significant expansion of the operating envelope is possible by using the multivariable MPC controller. While gas turbine cycles offer up to around 60% turndown as shown in Figure 5.1, the expected turndown of the FC/GT hybrid with 85% bypass could be nearly 100%. The implementation of FC/GT hybrid technology could result in more flexible power systems and a significant contribution to energy security in the U.S.

Figure 5.1 Comparison of turndown capability between standard gas turbine generator, extended turndown generator and FC/GT hybrid system.

The CA valve maintained the effect of cooling gas turbine temperatures, even when the other valves and EL were also changing. This effect could be used to maintain gas turbine temperature within a desired range and to enhance the operation life of the turbine. The other important effect is the capacity of this valve to control compressor stall. This valve was limited in this experiment to a lower bound of 40% precisely to avoid compressor stall.

Simultaneous valve operation was shown to improve the ability of the system to absorb and effectively use waste heat from the FC. This ability will be important during transient operations or off-design operation. The coordinated actions of the BA and CA valves with the electric load have a huge effect in the fuel flow-turbine speed control loop. The hybrid FC/GT has in these three actuators a synergistic way to improve thermal management to control turbine speed of the integrated systems.

Model Predictive Control

The absence of a first-principles analytical model of the HyPer facility and the difficulty of building an accurate one is an impediment to the development of traditional control systems for the facility. Numerous attempts to build Matlab™ or Simulink™ models have failed to produce one that can accurately predict the responses of the system measured in verification experiments. This is unfortunate, since most classical or state space control systems are built on some dynamic model of the system.

However, in the last decade, MPC technology has gained popularity in the refinery and petrochemical industry and has started to attract the interest from other process industries. Most often, the MPC controller uses a linear dynamic model of the process that is obtained by way of black-box system identification. However, due to various reasons, the cost of current MPC identification is very high and many trials and errors have to be made on-line by the user. It is believed that process modeling and identification is the most difficult and time consuming part of an MPC project.

Adaptive or recursive identification used here with off-line data was shown in this research to be a good way to find a model to implement a MPC. One of the strengths of MPC is that it can

allow the designer to impose strict limits on inputs and outputs in order to keep the system within known safe bounds. This approach could be implemented to adapt a model in a real-time application that could be used with the MPC technology to insure safe operation of the facility. The results show that MIMO recursive identification can fit an accurate model that can be used to determine the prediction horizon of the MPC strategy.

Two identification structures, ARX and State-Space model, were used to fit the measured data to “*black box*” dynamic models of the HyPer facility. The State-Space identification was very accurate with a second order model. This model was obtained in canonical form making the state variables equal to the output variables, which is convenient for observability and control purposes. Visual inspection of the tracking accuracy shows that the ARX approach was approximately as accurate as the State-Space structure in its ability to reproduce measured data. However, by comparing the Loss Function and the FPE parameters, the State-Space approach gives better results.

The State-Space representation also included a disturbance model in its formulation, and this is likely to be an important capability in the control of hybrid systems that may experience a wider range of ambient temperatures, fuel compositions or other disturbances than were present in the HyPer lab. The ARX structure was less time consuming than the State-Space, averaging 0.12 seconds for the parameter identification, using uncompiled Matlab code running on an Intel Core i7 processor running at 2.13 GHz with 3GB of RAM, and 32 bit Operating System. The time consumption of the State-Space identification was slightly higher than the sampling rate, at approximately 0.6 second using the same processor. However, the execution could easily be improved by compiling the code or by running on a faster computer. Therefore, the State-Space identification is here recommended as an accurate and easy way to obtain an adaptive model of the HyPer system.

The MPC proved to be a good strategy to control the HyPer facility. The CA, HA, BA valves and the EL were used to control the turbine speed and the cathode airflow. Three methods of system identification were used to test and tune the model predictive controller. ARX, State-Space identification, and TF models derived from experimental data were all proven to deliver accurate results. For all of these models the MPC was very robust in tracking set-point

variations. The anticipation feature of the MPC was revealed to be a good tool to compensate time delays in the output variables of the facility or to anticipate eventual set-point moves in order to achieve the objectives very quickly. The MPC also displayed good disturbance rejection on the output variables when the fuel flow was set to simulate FC heat effluent disturbances.

An input analysis was also carried out. It was found that the CA and HA valves can regulate the cathode airflow very well and very quickly, allowing a rapid response of the cathode airflow to changes in FC temperature. The cathode airflow practically follows the rate of change of these two valves. On the other hand, turbine speed is the most rapid variable in the facility. Supplementary fuel flow and the electric load are recommended to control turbine underspeed. Supplementary fuel flow is already used to control turbine speed and it is very accurate and adequate. The BA valve and the EL have the ability to control turbine overspeed. The EL is fast enough to control turbine speed, but the BA valve must be updated for quick response to be a viable option to control turbine overspeed.

Chapter 6

Contribution Overview and Recommendations

The current research effort has been carried out to characterize and provide insight to manage the transient load and heat disturbance rejection capabilities of the HyPer system. The design of experiments performed helped to extend the HyPer operating envelope by more than 20%. The control strategies developed herein are a simple and effective way to maintain the system within operating constraints during significant perturbations and provide excellent tracking of the reference commands by the controlled variables.

Maintaining the FC temperature limits during transients and disturbances is of critical importance to maintain the FC durability. This can be done by monitoring the temperature of the cathode air at the inlet and outlet of the FC stack and by monitoring the EL on the FC. An observer can be constructed to compute cathode airflow set-points for input to the MPC.

Also, to keep constant turbine speed in a synchronized generation is very important. In addition, constant turbine speed helps to mitigate airflow and pressure variation in the turbomachinery and consequently the FC dynamic stable

Contribution Overview

The data gathered from the set of designed experiments represents the first ever experimental characterization of a hybrid generation facility over the “heart” of its operating envelope using a randomized factorial experiment. The factorial experiment design is important, because it can detect and characterize interactions among the manipulated variables that are impossible to see using traditional OFAT methods. The characterization of HyPer is of great importance as it is one of only a handful of hardware-in-the-loop hybrid test facilities in the world and these results will be of interest to researchers around the globe. Several important contributions resulted from this work:

First, the interactions were practically negligible between the different DoE factors. The main effects of the HyPer factors in the OFAT experiments prevail in the results of the design of experiments.

Second, a new extended envelope was presented and analyzed to understand the effects of the simultaneous operation of the bypass valves and electric load on several outputs parameters of the HyPer facility.

Third, hybrid-parameters regression models were obtained to describe the relationships between controlled and manipulated variables. In the range of this performed experiment, the relationships between factors and response variables were shown to be nearly linear.

Fourth, a model-based predictive control strategy based on system identification was tested to show how bypass valves and turbine electric load can be used to control turbine speed and cathode airflow in the HyPer facility.

The objectives proposed and the results obtained in this work help to increase knowledge of the process and to identify new potential methods to regulate airflow, manage thermal gradients, control turbine speed, and mitigate compressor stall and surge during operational transients using a multi-input envelope based on the manipulation of the three bypasses simultaneously. This work was focused primarily on study of how to control the turbine speed and the cathode inlet mass flow, because these parameters play an important role in the integration of the FC/GT hybrid system.

6.1 Future Work and Recommendations

Perform the design of experiment extending the CA, HA, and BA valves limits. Extending the CA and HA valves level to completely opened (100%) and completely closed (0%) can bring more knowledge about the envelope because at the initially opening/closing the valve behavior is more non-linear as shown in [3]. Extending the full range of valve operation also allows the creation of a regression equation that sweeps the complete steady state output variables of the facility. Validation of steady state regression equations must be accomplished to verify the

accuracy of the models using some intermediate points localized among the levels selected on the DoE.

In future studies, the analysis of experiments will consider the integration of the FC numerical model including the anode reactions to investigate the best integrated control for optimal on-design and off-design operating points. Fuel flow variations, fuel utilization, FC efficiency and FC load are some of the variables that can be studied together with each point found in the design of experiments. The relation between the FC temperature of operation and cathode air mass flow has not been identified in order to set the reference point of the flow with respect to optimal FC operation. In other words, anode fuel flow coupled with a cathode inlet flow for a specific electric load should be found in order to maintain safe hybrid operation.

Variations in ambient conditions were relatively low during the design of experiments. The highest difference was 16 °C. This small variation did not have a significant impact on the results of the DoE, as shown in the fractional factorial analysis completed to indicate repetability of the experiments. Ambient conditions have not been accounted in this design of experiments and could be considered as random variable in a future design of experiment for temperatures variation between summer and winter.

The control concepts developed herein require further experimental validation. Such experimental validation will give more confidence in the HyPer system transient capabilities explored and developed. To make the advantages and opportunities of model based control available to industrial turbomachinery configurations, more research, both in simulations and in experiments is required.

It is recommended also to run some experiments to characterize and include the dynamics of the valves in the simulations. The valves have some special dynamics that create limitations in the implementation of the controller. Also, it is recommended to characterize the effect of variation of the turbine speed on airflow, temperatures and pressures at different valve positions and electric load levels.

The recursive on-line identification techniques used in this work should be experimentally validated to verify their ability to accurately accommodate more complex dynamics than the step

changes used in this work. If the linear online methods do not prove adequate, a nonlinear identification (i.e. NARX) model should be implemented and compared with the actual ARX model used here to check for nonlinearities presented in the model.

Additional work must be performed to develop methods for effective use of the electric load for the control of turbine speed, while simultaneously responding to external load changes by the hybrid system. This will include the temporary alteration of “normal” load splits between the FC and the GT to accommodate system load changes while recovering as much energy as possible from the thermal effluent of the FC.

There is ongoing work in the literature concerning the assessment of multivariable controller performance. Early work with SISO models used minimum variance methods (e.g. Goodwin et al [94]). Those methods have been extended to MIMO systems and improved by Harris et al. [95] and Huang et al. [96] to account for time delay and nonminimum phase zeros, both of which were shown by Tsai [26] to be present in the HyPer system. Huang notes in [96] that the optimization process is necessarily application dependent.

The MPC method was shown in a qualitative way to give good tracking performance in the HyPer system simulations. Tsai [26] also showed that the H_∞ approach may hold some promise for control of hybrid systems. However, no quantitative method currently exists to compare the two methods, or even various tunings of either controller realization. Thus it is here recommended that future work include the development of specific performance indices for the HyPer system. The index can be used to test different combinations of modeling approaches, control strategies and controller tuning parameters.

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Appendix A

ASME Fuel Cell Conference Paper

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COMBINATION OF A NONLINEAR STATIC AND A LINEAR DYNAMIC MODEL OF THE NETL HYPER SYSTEM

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Abstract

A nonlinear steady-state thermodynamic model was coupled with linearized dynamic transfer functions to achieve a dynamic description of the NETL HyPer Fuel Cell Gas Turbine (FC/GT) power plant. Nonlinear dynamic models insure accuracy in modeling steady-state behavior over a wide range of operation, but such models are often complex and difficult to implement in real-time using conventional control systems equipment. Conversely, the linearized models provide the ability to predict transient behavior upon which dynamic control systems can be constructed, but are valid only about a narrow operating point. In systems with one or two state variables, it is relatively straightforward to construct controllers that use gain scheduling schemes. But the HyPer system contains many coupled state variables and high degrees of nonlinearity. A method called Real-Time Piecewise Linear Dynamic Modeling (RPLDM) has been implemented to provide both modeling accuracy and real-time performance for the HyPer system over a multi-dimensional hypersurface. Both the nonlinear and the linear constituent models were constructed based on

experimental data collected in tests performed on the HyPer system. The models presently consider only the cathode circuit of the fuel cell and contain a recuperated gas turbine system equipped with an electric generator, a simulated fuel cell cathode and various bypass valves for thermal management and system control. The key variables of air temperature, air pressure and mass flow to the cathode of the fuel cell and the turbomachinery have been predicted to within 2% of measured values. This paper presents the modeling technique and comparisons of the model output with experimental data.

Introduction

Modeling gas turbine dynamics in order to get a good description for control purposes is challenging, especially when the turbine/compressor are coupled to a Solid Oxide Fuel Cell (SOFC). The modeling is more complex due to the insertion of significant amounts of piping and flow restriction between compressor and turbine. While static models are relatively simple, detailed transient thermodynamic models require the solution of differential equations using integrators whose numerical techniques for

solution are time consuming. Such solution methods are often impractical for use in real-time controllers. For instance, a previous transient model developed by Shelton et al. [1] of the HyPer system showed that a real-time process could not be implemented because of the complexity and computation time required for the heat exchanger fluid flow calculations.

In addition, thermodynamic models are typically nonlinear and the methods for nonlinear control are less well-known than those used for linear systems. On the other hand, linear dynamic models of complex systems are often used for control using the state-space formulation. Nonlinear systems are linearized about an operating point or about a set of operating points from models determined by using first-principle approaches. Controllers can then be designed to operate reliably within a narrow range about the linearization point. Alternately, linear representations of a system could be built based on experimental data using system identification.

This paper describes the design and implementation of a set of state space models for different operating points using experimental data obtained from the HyPer Facility at the National Energy Technology Laboratory in Morgantown, WV.

Detailed descriptions of the facility with all its components have been provided in previous papers by Tucker et al. [2,3]. In addition, several mathematical models of the hybrid facility have already been published [1, 3, 4, 5]. Figure 1 shows a layout of the facility used for experimentation. The HyPer facility, as any hybrid system, is highly nonlinear in much of its behavior and it is difficult to model accurately from first principles. Thus, in this paper a set of first order (FO) and second order (SO) state representations is developed to describe the behavior of the system at several operating points. The state space matrices are modeled for different variables such as airflow, pressure and temperature.

Figure 1. Layout of the HyPer Facility.

A method called real-time piecewise linear dynamic modeling (RPLDM) is implemented. The technique is applied and validated, it shows how the nonlinear steady-state approach could be combined with a linear dynamic model which is appropriate for control system implementation with high accuracy and real-time processing.

Nomenclature

English

A	State space matrix
B	Control state matrix
C	Output state matrix
D	Output control matrix
G	Represent a transfer function
N	Number of operation points
P	Pressure
T	Temperature
U	Control vector
V	Cost function
X	State vector
Y	Observable vector
Z	Vector difference between states X^N and X^1
a, b	Transfer function parameters
e	Error
m	Mass flow
s	Laplace domain
t	Time

Greek

η	Interpolation factor
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Superscripts

k	number of operation point
T	Matrix Transpose

Acronyms

APU	Auxiliary Power Unit
CA	Cold Air Valve

ETFE	Empirical Transfer Function Estimates
FC	Fuel Cell
FO	First Order
GT	Gas Turbine
NETL	National Energy Technology Laboratory
LDM	Linear Dynamic Model
MIMO	Multiple Input Multiple Output
NLSM	Non-Linear Static Model
PEM	Predictive Error Minimization Method
RPLDM	Real-Time Piecewise Linear Dynamic Model
SO	Second Order
SOFC	Solid Oxide Fuel Cell
SOFC/GT	SOFC/Gas Turbine Hybrid System
TF	Transfer Function

Process Variables

F ₁₁₀	Airflow measured in the compressor
F ₃₈₀	Airflow measured in the cathode
P ₁₅₁	Pressure measured in the compressor outlet
P ₁₈₀	Pressure measured in the turbine inlet
P ₃₀₅	Pressure measured in the cathode inlet
T ₁₄₇	Temperature measured in the compressor outlet
T ₂₀₂	Temperature measured in the turbine outlet
T ₃₂₆	Temperature measured in the cathode inlet
T ₃₅₀	Temperature measured in the turbine inlet
Comb	Combustor
Elec	Electric
Exp	Experimental data

Methodology

Linear models are computationally faster when they are implemented, but they are less accurate over wide operating ranges than nonlinear models. The HyPer facility dynamics are known to be nonlinear. If the nonlinear model is linearized around an operating point, it is necessary to ensure that the model is valid in a neighborhood large enough about this point. As described by Breikin et al. [6] and by Kulikov and Thompson in [7], a RPLDM combines the accuracy of a nonlinear static model with the speed of the linear dynamic model. This method approximates with smooth functions the derivatives around the operating points eliminating the sharp changes that can appear in standard gain scheduling methods when the system jumps from one operating point to another. The nonlinear static model can be derived from a detailed thermodynamic model or based on experimental data knowing the steady-state points of operation at different conditions. Experimental data is used in this paper.

RPLDM theory posits that the ideal model is a combination of an accurate nonlinear static model with a set of faster but less accurate (15-20% error) dynamic models [7], linearized about a carefully chosen set of operating points. This approach extends the range of application of the specific operating points of linear models to the whole area of system operation. The model is based on a set of linear models coupled with a nonlinear static cycle line. Linear dynamic interpolation of the linear model parameters is performed for the nearest point on the static line during simulation.

The method consists, firstly, in the selection of a set of operating points and determination of unknown parameters based on empirical steady-state operating data. The static operation points are tabulated graphically and are known for any condition of the states. Secondly, based on the experimental data and using system identification, the typical curves of the system in operation are obtained. It is suitable to assume that the dynamics of the system can be represented by first or second order models as revealed from experiments carried out in the facility. In general, it is appropriate use a reduced-order simplified model for control design and monitoring conditions. The transfer function or state space parameters are then determined and stored for different operating points. Finally, interpolation between the static points and updating of the dynamic parameters are performed to determine the dynamic trajectories of the system until the steady-state condition is achieved. Figure 2 shows the block diagram of the three phases required to build the model. This approach is used because of its low complexity, fast execution, high accuracy, and suitability for control development.

Figure 2. Block diagram to build and validate the RPLDM.

Components of RPLDM

Two components are necessary to build a RPLDM: the Non-linear Static Model (NLSM) and the Linear Dynamic Model (LDM).

Non-linear Static Model (NLSM)

When the input is modified, the static operating point (temperature, pressure, and mass flow) moves in the thermodynamic cycle as shown in Figure 3. The steady-state values of the static operating points have been previously determined from experiments, and hence the nonlinear static model can be represented in tabular form (or lookup tables). Figure 4 shows a block diagram of the output variables and the input.

Figure 3. Schematic representation of nonlinearity between input and output for steady-state cycles operation.

Figure 4. HyPer Facility subsystems flow diagram.

Assuming that the characteristic coordinates are known at N operation points,

$$[X^{(k)}, U^{(k)}, Y^{(k)}]$$

where $k=1, \dots, N$ is the operating point number, X is the state vector, U is the control vector, and Y is the output vector. Figures 5 and 6 show the steady-state values for turbine and cathode inlet temperature and pressure, respectively. Notice that equal interval variations in the CA valve have different impacts in temperatures and pressures. For instance, for more than 70% change in the CA valve position, Figures 5 and 6 show negligible changes in temperature and pressure.

Figure 5. Steady-state turbine and cathode inlet temperatures as function of CA valve step changes (position is percentage of opening).

Figure 6. Steady-state turbine and cathode inlet pressures as function of CA valve Step Changes (position is percentage of opening).

Operating Points

It is well known that certain ranges of CA valve opening position have a significant impact on the outputs of the system. In Figure 5, temperature and pressure undergo high changes with respect to CA valve opening in the specific region of 20% to 60%. The operating steady-state points are selected based on this variation. The first six points were established at intervals of 5% opening in the CA valve, the next three points at 10%, and the remaining at 15%. The number of operating points increases the accuracy of the model. The CA points were 15, 20, 25, 30, 35, 40, 50, 60, 70, 85 and 100%. It is important to mention that the experimental data was collected during a period of three days in which the environmental conditions were different.

Linear Dynamic Model (LDM)

System Identification as a Tool for Model Building.

An empirical transfer function estimate (ETFE) is a transfer function that is derived directly from experimental data. The steady-state transfer functions can be obtained using the identification Tools of Matlab. The optimization of the estimators is called the Prediction Error

Minimization method (PEM), in which estimates of $G(s)$ are performed by minimizing the quadratic cost function:

$$V_N(G) = \sum_{t=1}^N e^2(t) \quad (1)$$

Data Set

The data were recorded by the data acquisition system at a sampling interval of 400 ms. Step changes were implemented in the CA valve and different thermodynamic properties (flows, temperatures, and pressures) were measured while the electric load was constant. More details of the experimental procedure are given in [8].

Illustrative Data

Figure 7 shows the profile of the temperature to the cathode as a function of CA valve position for a load of 50 kW. Figure 8 presents a zoom of Figure 7 (the cold air valve changes from 60 to 70% open), and illustrates the main characteristics of the dynamic response such as settling time $t_s = 260$ s, and the output step change $\Delta T = 14.4$ C. The delay time for this particular case was 0.8 seconds.

Figure 7. Cathode inlet temperature as function of CA Valve step changes (Position is percentage of opening).

Figure 8. Temperature response (settling time, time delay and output response) as a function of a step change in the Cold Air valve.

Model Structure

The information obtained from the step change was used to build first order and second order linear dynamic models for all the output variables. The temperature, pressure and airflow were modeled with the following transfer functions:

$$G_1(s) = \frac{\Delta y}{\Delta u} = \frac{K}{s+a} \quad (2)$$

$$G_1(s) = \frac{\Delta y}{\Delta u} = \frac{K}{s^2+as+b} \quad (3)$$

where Δy is the difference between two output operating points, Δu is the CA valve percentage opening step change, and K , a , and b are the parameters of the TFs.

Processing

Figure 9 shows an example of experimental data imported to the identification tools of Matlab in the time domain. Figure 10 illustrates the results for two estimated transfer functions (TF). These TF were obtained with underdamped and overdamped assumptions. In this particular case, the estimation is very similar between the two transfer functions and very close to the experimental data. The fit shown in Figure 10 was calculated based on the following equation:

$$Fit = \left[1 - \frac{\|Y - Y_{hat}\|}{\|Y - \text{mean}(Y)\|} \right] * 100 \quad (4)$$

where Y is the empirical data, and Y_{hat} is the predicted output of the estimated TF.

Figure 9. Example of input (CA valve position) and output (T350) signals after remove means of the data.

Figure 10. Example of measured and simulated model output, following CA step changes.

Equation 5 and 6 show the overdamped and underdamped transfer functions obtained using system identification, respectively. Even though these transfer functions look similar, the underdamped TF was selected for the model since the CA valve bypasses the mass flow directly from the compressor to the turbine without imposing large damping effects. The estimated overdamped and underdamped transfer functions for this case are:

$$G(s)_1 = \frac{-5.1578}{(1+1.9019s)(1+0.001s)} \quad (5)$$

$$G(s)_2 = \frac{-5.1524}{(1+0.94568s+0.001s^2)} \quad (6)$$

Once the non-linear static model and the empirical transfer function have been determined, a model based on the combination of these two components is constructed.

Building a RPLDM

The transfer functions at every operating point were transformed into state space representation. This is appropriate for control design, and allows an easy implementation of parameters and variables in lookup tables. Eleven operation points were selected for the nonlinear static line and for the LDM, between 15-100% open. The 11 CA positions were tested for each of 3 different electric loads: 0, 25, and 50 kW. The electric load is considered a disturbance in the system which was identified based on experimental data as:

$$G_3(s) = \frac{1.003}{0.811s^2 + 10.998s + 1} \quad (7)$$

This TF represents the dynamic response of any variable when a load change occurs in the system. A total of 297 first order transfer functions and the same amount for the second order model were identified.

Output Variables

In total, nine output variables were selected for this study. They were organized in three subsystems. The cathode output vector is represented by the mass flow, pressure, and temperature to the plenum tank which represents the input to the cathode of a fuel cell:

$$\mathbf{Y} = [T_{326} \ P_{305} \ m_{F380}]^T \quad (8)$$

this set of variables is important to control the cathode system, maintain the operation of the fuel cell at the desired conditions, and protect it against thermal and mechanical stresses.

The turbine output vector is represented by turbine temperatures and the turbine inlet pressure:

$$\mathbf{Y} = [T_{350} \ T_{202} \ P_{180}]^T \quad (9)$$

and the compressor output vector is represented by the compressor mass flow, compressor outlet temperature and pressure:

$$\mathbf{Y} = [T_{147} \ P_{151} \ m_{F110}]^T \quad (10)$$

The last two vectors contain important output variables for monitoring the turbomachinery and its components.

All of the above mentioned parameters were modeled as functions of the mass flow (Cold Air Valve) directly piped from the compressor to the turbine bypassing the heat exchanger and the fuel cell:

$$\mathbf{U} = [\mathbf{CA}] \quad (11)$$

It is important to mention that the data was gathered while running the system under speed control for safety reasons and to maintain the required frequency of the generator. The fuel flow, which represents the fuel cell heat effluent, was used to control the speed of the turbomachinery with a proportional-integral control scheme which must be used as part of the system when this model will be implemented for control purposes.

Each state space TF was modeled in Simulink to obtain the corresponding state vector $[\mathbf{X}]$ for each operating point. The state-space representation is presented in an observable canonical companion form and checked for controllability. The LDM at each operating point is represented in state space in the Cauchy form by:

$$\Delta \dot{\mathbf{X}}(t) = \mathbf{A}^{(k)} \Delta \mathbf{X}(t) + \mathbf{B}^{(k)} \Delta \mathbf{U}(t) \quad (12)$$

$$\mathbf{Y}(t) = \mathbf{C}^{(k)} \Delta \mathbf{X}(t) + \mathbf{D}^{(k)} \Delta \mathbf{U}(t) \quad (13)$$

where $\mathbf{A}^{(k)}$, $\mathbf{B}^{(k)}$, $\mathbf{C}^{(k)}$, and $\mathbf{D}^{(k)}$ are the matrices of LDM coefficients; and $\Delta \mathbf{X} = \mathbf{X} - \mathbf{X}^{(k)}$, $\Delta \mathbf{U} = \mathbf{U} - \mathbf{U}^{(k)}$, $\Delta \mathbf{Y} = \mathbf{Y} - \mathbf{Y}^{(k)}$ are deviations of the state vector $\mathbf{X}$, control vector $\mathbf{U}$, and output vector $\mathbf{Y}$ from the k^{th} operating point.

RPLDM Processing

The linear dynamic part of the model was interpolated in the neighborhood of two operating points. In order to determine which of k state space matrices is closer to the nonlinear static point, the “distance” from a required point on the static response curves to the current dynamic point should be a minimum. It is possible to trace a line between a dynamic point and the static point and to optimize for the minimum distance. When the nearest operating points are determined, the linear dynamic parameters are updated and the state space estimation performed.

Operating Parameters Perpendicular to Static Line

The following steps show a simple way to calculate the minimum distance: first, a vector Z must be defined as the difference between the first and the last operating point in the total range of operation of the state vector:

$$\mathbf{Z} = \mathbf{X}^{(N)} - \mathbf{X}^{(1)} \quad (14)$$

The range is determined based on the maximum and minimum position of the CA valve.

Second, the static points are connected by a polyline and the operating parameter $\eta(t)$ is introduced as a scalar product of Z and the dynamic point $\mathbf{X}(t)$:

$$\eta(t) = (\mathbf{Z} \cdot \mathbf{X}(t)) = \sum_{i=1}^n z_i x_i(t) \quad (15)$$

A new dynamic point $\mathbf{X}(t)$ is related to the corresponding nearest static point using this parameter. When the model is running, new values for $\mathbf{X}(t)$ are obtained by the integrator of the state space formulation, and then the associated $\eta(t)$ is updated. Using this new $\eta(t)$, values for the pressure, temperature and mass flow, and new values for $\mathbf{A}(\eta)$, $\mathbf{B}(\eta)$, $\mathbf{C}(\eta)$, and $\mathbf{D}(\eta)$ for the dynamic model are read in the lookup tables:

$$\begin{aligned} \mathbf{A}^{st} &= (1 - p)\mathbf{A}^k + p\mathbf{A}^{k+1} \\ \mathbf{B}^{st} &= (1 - p)\mathbf{B}^k + p\mathbf{B}^{k+1} \\ \mathbf{C}^{st} &= (1 - p)\mathbf{C}^k + p\mathbf{C}^{k+1} \\ \mathbf{D}^{st} &= (1 - p)\mathbf{D}^k + p\mathbf{D}^{k+1} \end{aligned} \quad (16)$$

where p represents the interpolation factor in each lookup table. The flowchart for the RPLDM simulation algorithm is shown in Appendix 1 and was obtained from [7]. Note that η is the adaptive parameter.

Figure 11. Operating parameter η “perpendicular” to static line. Figure reproduced by permission from Thompson [7].

RPLDM Realization

The air mass flow to the cathode is compared in Figures 12 and 13. The relative error between measured and estimated values is less than 2% for the first and second order model. Figure 13 is a zoom of Figure 12 when the CA valve was varied from 50 to 60%. Notice that the dynamic characteristic behavior is completely achieved. In Figure 14 the error for temperature to the cathode was less than 1%, showing the prediction accuracy of the model for first and second order. In addition, Figure 15 shows a good prediction for the steady-state value (error less than 1%), but with some difference in the transient trajectory. Thus, the second order model provides better results for this particular case.

Figure 12. Air mass flow to the cathode as function of cold air valve position (ambient pressure 98.14 kPa, and ambient temperature -28.0°C).

Figure 13. Air mass flow into the cathode as function of cold air valve position (zoom of Figure 12 when CA changes from 50 to 60%) (ambient pressure 98.14kPa, and ambient temperature 28.0°C).

Figure 14. Comparison between experiment and model of temperature as function of cold air valve position. (Amb. Pressure 98.14kPa, and Ambient Temperature 28.0°C).

Figure 15. Comparison between experiment and model of pressure as function of cold air valve position. (Ambient Pressure 98.14kPa, and Ambient Temperature 28.0°C).

Validation and Analysis of Results

The repeatability of the system is dependant partly on the ambient conditions such as temperature, pressure and humidity. The ambient temperature and pressure were recorded and presented in the graphs. However, no information was available concerning the humidity.

In order to verify the accuracy of the model, the outputs were compared with experimental data. The first validation was performed based on data available and detailed in [5]. The data was obtained with the CA valve

position following a sinusoidal variation with amplitude between 30% and 50% at different frequencies. The electric load was set at 45 kW. In this paper, frequencies of 0.001 and 0.01 Hz were considered.

Figures 16 to 18 illustrate the performance of the model tracking the sinusoidal response at 0.001Hz of temperature, pressure and mass flow. In Figure 16, the relative error for the temperature is less than 5% for FO and less than 2% for SO. Figure 17 shows a relative error less than 2% for both FO and SO. And Figure 18 shows better tracking for SO and a relative error less than 3% for both systems.

Figure 16. Comparison between experiment and model of turbine inlet temperature as a function of the sinusoidal CA valve modulation ($f=0.001$ Hz, Ambient Pressure 97.27 kPa, and Ambient Temperature 18.5 °C).

Figure 17. Comparison between experiment and model for cathode inlet temperature as a function of the sinusoidal CA valve modulation ($f=0.001$ Hz, Ambient Pressure 97.27 kPa, and Ambient Temperature 18.5 °C).

valve modulation ($f=0.001$ Hz, Ambient Pressure 97.27 kPa, and Ambient Temperature 18.5 °C).

Figure 18. Comparison between experiment and model of air mass flow to the cathode as a function of the sinusoidal CA valve modulation ($f=0.001$ Hz, Ambient Pressure 97.27 kPa, and Ambient Temperature 18.5 °C).

Figures 19 to 21 show the results when the CA valve was modulated at 0.01 Hz. The mass flow to the compressor and cathode inlet were very well predicted with the SO functions, but with some sharp changes with the FO. The turbine inlet temperature was predicted in similar way by the FO and SO model with an error of less than 2%.

Figure 19. Comparison between experiment and model of Compressor inlet air mass flow as a function of the sinusoidal CA valve modulation ($f=0.01$ Hz, Ambient Pressure 97.446 kPa, and Ambient Temperature 32.8 °C).

Figure 20. Comparison between experiment and model of cathode inlet mass flow as a function of the sinusoidal CA valve modulation ($f=0.01$ Hz, Ambient Pressure 97.446 kPa, and Ambient Temperature 32.8°C).

Figure 21. Comparison between experiment and model of turbine inlet temperature as a function of the sinusoidal CA valve modulation ($f=0.01$ Hz, Ambient Pressure 97.446 kPa, and Ambient Temperature 32.8°C).

Figures 22 and 23 present the validation results using another set of data for the variation in the electric load. These Figures show the turbine inlet temperature (T350) as a function of the small step changes in the electric load. The CA valve position was 15% in both cases. Once again, the SO model predicts with more accuracy the dynamics of the system.

Figure 22. Tracking and prediction of Turbine inlet temperature as function of Electric load variation (Ambient Pressure 98.89 kPa, and Ambient Temperature 23.8 °C).

Figure 23. Tracking and prediction of Turbine inlet temperature as function of Electric load variation (Ambient Pressure 98.89 kPa, and Ambient Temperature 23.8 °C).

Computing Performance

It is important to mention that the model shows a rapid computing performance which is a key aspect for real-time applications. Table 1 presents a comparison between the time consumption of the proposed model and the physically based NETL thermodynamic transient model [1] to simulate 300 seconds of operation. Both models were run in a DELL 2.88 GHz Pentium D microprocessor Desktop Machine with 1.0 Gb of Ram memory.

The results show that the NETL model required a step time of 0.005 s to run in order to avoid a singularity in the

numerical integration, compared with the 0.02 s step time of the RPLDM model. This provides a valuable condition for real-time operations.

Table 1. Computing time for simulation of 300 seconds of operation using the NETL and RPLDM Models.

Model	Run time with $\Delta t = 0.005$ (s)	Run time with $\Delta t = 0.02$ (s)
NETL Model	98	No run
FO Model	23	3.5
SO Model	26	4.5

Conclusion

The behavior of real dynamical systems is often too complex for complete mathematical analysis. The RPLDM used here proves to be a good method for modeling. It is easy to program, runs quickly with good accuracy, is suitable for control strategies in real-time, and could be implemented for monitoring conditions. This approach effects the adaption of a nonlinear system to the total region of operation using a dynamic parameter of linearization. The results show that the error for steady-state prediction is very small and the transient modeling is also well-represented, however the model is open for improvement if more data is available for identification. RPLDM is presented here as a good tool for model and control development. For future work, this model could be coupled to a fuel cell model and used to control the fuel flow, temperature or pressure to the cathode of the fuel cell and to keep the fuel cell working safely. Knowing that the dynamics of the fuel cell are slow compared with the dynamics of the turbomachinery, this model is adequate for implementing an optimal controller that properly determines the best performance of both systems. It can also be used to implement a MIMO model with the Cold

Air valve working together with the HA valve and Bleed Air valve.

Although some variations were found in ambient conditions between the validation data and the data used to build the model, it appears that the model works properly. However, more experiments must be performed under other environmental conditions.

Acknowledgments

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Appendix 1.

Appendix B

Design of Experiments Test Plan Purpose

HYPER Test Plan and Check List

Design of Experiments, July 06, 2010

Purpose: The purpose of the startup test is to bring the turbine up to the operational steady state nominal speed of 40,500rpm, testing the system at different Electrical Load, varying the Cold Air valve (CA), Hot Air valve (HA), and Bleed Air valve (BA) simultaneously. All of them will be combined as detailed below to develop a full factorial experimental design analysis.

The purpose of this operational test is to map an envelope of the system that represents a space of operation of the whole system. All the variables (inputs) are manipulated simultaneously under the limitations impose for the environment, and the equipment itself.

Ambient Temperature and Pressure, Humidity, constant Turbine Speed, and Limited Temperatures of operation will be the constraint of the experiments. This design of experiments will permit us to have a greater visualization of the effects of the variables and their interactions in different states of the system; to have an envelope of the system; and develop strategies for control. That mean, it will permit to take advantage of manipulate all the valves at the same time or one by one. The data obtained in this procedure it will be implement in the Fuel Cell Model off-line in order to study the reaction of different parameters in the Fuel Cell and the complete dynamics of the hybrid system.

The Cold Air and Hot Air Valve will be opened from 40, 60, and 80%, the Bleed Air Valve will be opened up to 10, 12 and 14%, and the Electrical Load will be set for 0, 25 and 50 kW. All of these variables will be set by levels and performed in a full combinatorial factorial design.

Initially, the Hot Air Valve is opened 12% to guarantee functionality after the heating of the system. The levels for the four factors of study are:

Electrical Load: 0, 25, and 50 kW.

Cold Air Valve: 40, 60 and 80 % Opened.

Hot Air Valve: 20, 50 and 80 % Opened.

Bleed Air Valve: 10, 12, 14% Opened.

The order of the treatments is specified in the test plan as well as the specific experiment to be performed. After the each treatment is set, steady state (T344 constant by 5 min.) of the system must be achieved before move to a new treatment.

All the time, the equivalence ratio parameter should be monitored (Eq. ratio < 1) in the GAP system to avoid rich mixture (overheating) in the combustor .

It is important to observe carefully all the time during the experiments. Large change in the parameters must proceed in small steps to protect the equipment from the operational constraints in surge margin as in temperatures limits.

This test will be conducted by Bernardo Restrepo **BR**, William Rosen **WR**, Megan Gorrel **MG**, Dr. Larry Banta **LB**, Dr. Alex Tsai **AT**, Dr. David Tucker **DT**, Roger Lapeer **RL** and David Ruehl **DR**.

All Hyper states will be recorded.

Appendix C

Design of Experiments Results

Compressor Mass Flow (FT110) as function of the treatments.

Bleed and Cold Air Mass Flow (FT162) as function of the treatments.

Compressor Outlet Pressure (PT151) as function of the treatments.

Cathode Inlet Pressure (PT305) as function of the treatments.

Turbine Inlet Pressure (PT180) as function of the treatments.

Appendix D

Fuel Flow Regression Analysis

Regression Analysis: FT432 versus CA, HA, BA, EL

The regression equation is

$$FT432 = 519 + 44.3 \text{ CA} - 11.2 \text{ HA} + 14.2 \text{ BA} + 96.7 \text{ EL}$$

Predictor	Coef	SE Coef	T	P
Constant	518.641	8.118	63.89	0.000
CA	44.321	1.988	22.29	0.000
HA	-11.181	1.988	-5.62	0.000
BA	14.190	1.988	7.14	0.000
EL	96.720	1.988	48.64	0.000

$$S = 14.6120 \quad R\text{-Sq} = 97.5\% \quad R\text{-Sq}(\text{adj}) = 97.4\%$$

General Regression Analysis: FT432 versus CA, HA, BA, EL

Regression Equation

$$\begin{aligned}
 FT432 = & 548.588 + 29.2666 \text{ CA} + 4.40814 \text{ HA} + 1.1776 \text{ BA} + 75.571 \text{ EL} - 8.62916 \\
 & \text{CA*HA} + 2.9484 \text{ CA*BA} + 12.3047 \text{ CA*EL} + 1.04773 \text{ HA*BA} - 4.30976 \text{ HA*EL} \\
 & + 7.80468 \text{ BA*EL} + 1.6926 \text{ CA*HA*BA} + 1.72726 \text{ CA*HA*EL} - 2.96816 \\
 & \text{CA*BA*EL} - 1.37163 \text{ HA*BA*EL}
 \end{aligned}$$

Coefficients

Term	Coef	SE Coef	T	P
Constant	548.588	49.0481	11.1847	0.000
CA	29.267	20.3652	1.4371	0.155
HA	4.408	20.3652	0.2165	0.829
BA	1.178	20.3652	0.0578	0.954
EL	75.571	20.3652	3.7108	0.000
CA*HA	-8.629	7.9800	-1.0813	0.283
CA*BA	2.948	7.9800	0.3695	0.713
CA*EL	12.305	7.9800	1.5419	0.128
HA*BA	1.048	7.9800	0.1313	0.896
HA*EL	-4.310	7.9800	-0.5401	0.591
BA*EL	7.805	7.9800	0.9780	0.332
CA*HA*BA	1.693	2.7107	0.6244	0.535
CA*HA*EL	1.727	2.7107	0.6372	0.526
CA*BA*EL	-2.968	2.7107	-1.0950	0.278
HA*BA*EL	-1.372	2.7107	-0.5060	0.615

Summary of Model

$$\begin{aligned}
 S &= 13.2795 & R\text{-Sq} &= 98.20\% & R\text{-Sq}(\text{adj}) &= 97.81\% \\
 \text{PRESS} &= 17958.4 & R\text{-Sq}(\text{pred}) &= 97.22\%
 \end{aligned}$$

General Regression Analysis: FT432 versus CA, HA, BA, EL

Regression Equation

$$\text{FT432} = 547.275 + 28.2543 \text{ CA} - 7.60245 \text{ HA} + 15.9402 \text{ BA} + 78.8242 \text{ EL} - 1.78944 \text{ CA*HA} + 9.82287 \text{ CA*EL} - 0.874905 \text{ BA*EL}$$

Coefficients

Term	Coef	SE Coef	T	P
Constant	547.275	16.8417	32.4951	0.000
CA	28.254	6.4488	4.3813	0.000
HA	-7.602	4.7321	-1.6066	0.112
BA	15.940	4.7321	3.3685	0.001
EL	78.824	6.4488	12.2231	0.000
CA*HA	-1.789	2.1905	-0.8169	0.417
CA*EL	9.823	2.1905	4.4842	0.000
BA*EL	-0.875	2.1905	-0.3994	0.691

Summary of Model

S = 13.1433 R-Sq = 98.05% R-Sq(adj) = 97.86%
PRESS = 15495.9 R-Sq(pred) = 97.60%

Appendix E

Fuel Flow Standard Deviation

	CA	HA	BA	EL	FT432		CA	HA	BA	EL	FT432		CA	HA	BA	EL	FT432
1	40	20	90	0	3.05259	28	40	20	90	25	3.524338	55	40	20	90	50	2.379415
2	40	20	88	0	3.558938	29	40	20	88	25	4.928543	56	40	20	88	50	5.69301
3	40	20	86	0	2.58887	30	40	20	86	25	5.449499	57	40	20	86	50	5.698624
4	40	50	90	0	3.258972	31	40	50	90	25	4.793165	58	40	50	90	50	4.627871
5	40	50	88	0	5.637024	32	40	50	88	25	5.253649	59	40	50	88	50	4.88254
6	40	50	86	0	3.07379	33	40	50	86	25	4.440131	60	40	50	86	50	6.738208
7	40	80	90	0	2.19907	34	40	80	90	25	4.886762	61	40	80	90	50	6.181336
8	40	80	88	0	5.412418	35	40	80	88	25	5.54473	62	40	80	88	50	5.980085
9	40	80	86	0	2.94331	36	40	80	86	25	3.29697	63	40	80	86	50	6.754064
10	60	20	90	0	3.39096	37	60	20	90	25	4.225674	64	60	20	90	50	4.682718
11	60	20	88	0	2.160679	38	60	20	88	25	5.674318	65	60	20	88	50	5.184014
12	60	20	86	0	2.989896	39	60	20	86	25	5.145892	66	60	20	86	50	6.070021
13	60	50	90	0	3.683156	40	60	50	90	25	6.635302	67	60	50	90	50	6.402795
14	60	50	88	0	5.259935	41	60	50	88	25	5.162205	68	60	50	88	50	6.084905
15	60	50	86	0	5.251809	42	60	50	86	25	6.820744	69	60	50	86	50	6.639791
16	60	80	90	0	5.468972	43	60	80	90	25	4.223258	70	60	80	90	50	5.973651
17	60	80	88	0	4.337533	44	60	80	88	25	4.732748	71	60	80	88	50	6.128573
18	60	80	86	0	4.727941	45	60	80	86	25	6.646183	72	60	80	86	50	6.355789
19	80	20	90	0	3.576203	46	80	20	90	25	5.007448	73	80	20	90	50	8.247859
20	80	20	88	0	8.827534	47	80	20	88	25	9.452677	74	80	20	88	50	8.764368
21	80	20	86	0	3.517622	48	80	20	86	25	6.89682	75	80	20	86	50	6.775112
22	80	50	90	0	5.698656	49	80	50	90	25	5.367087	76	80	50	90	50	5.887082
23	80	50	88	0	5.760705	50	80	50	88	25	6.224336	77	80	50	88	50	4.19796
24	80	50	86	0	8.426376	51	80	50	86	25	7.23654	78	80	50	86	50	4.631864
25	80	80	90	0	6.611716	52	80	80	90	25	3.178175	79	80	80	90	50	15.93891
26	80	80	88	0	7.424662	53	80	80	88	25	3.526829	80	80	80	88	50	9.484302
27	80	80	86	0	7.717014	54	80	80	86	25	4.367858	81	80	80	86	50	4.982603

Appendix F

Cathode Air Flow Regression Analysis

Regression Analysis: FT380 versus CA, HA, EL

The regression equation is

$$FT380 = 1.17 - 0.105 CA - 0.184 HA - 0.0145 EL$$

Predictor	Coef	SE Coef	T	P
Constant	1.16858	0.02562	45.62	0.000
CA	-0.104557	0.007198	-14.53	0.000
HA	-0.184484	0.007198	-25.63	0.000
EL	-0.014477	0.007198	-2.01	0.048

$$S = 0.0528925 \quad R-Sq = 91.9\% \quad R-Sq(adj) = 91.6\%$$

General Regression Analysis: FT380 versus CA, HA, BA, EL

Regression Equation

$$FT380 = 1.36596 - 0.188481 CA - 0.278219 HA + 0.00310798 BA - 0.0304956 EL + 0.0400412 CA*HA - 0.00229745 CA*BA + 0.00261586 CA*EL - 0.00211258 HA*BA + 0.00739518 HA*EL - 0.00250971 BA*EL + 0.00115044 CA*HA*BA - 0.000981965 CA*HA*EL + 0.000632722 CA*BA*EL + 0.000603264 HA*BA*EL$$

Coefficients

Term	Coef	SE Coef	T	P
Constant	1.36596	0.178834	7.63814	0.000
CA	-0.18848	0.074253	-2.53835	0.014
HA	-0.27822	0.074253	-3.74689	0.000
BA	0.00311	0.074253	0.04186	0.967
EL	-0.03050	0.074253	-0.41070	0.683
CA*HA	0.04004	0.029096	1.37618	0.173
CA*BA	-0.00230	0.029096	-0.07896	0.937
CA*EL	0.00262	0.029096	0.08990	0.929
HA*BA	-0.00211	0.029096	-0.07261	0.942
HA*EL	0.00740	0.029096	0.25417	0.800
BA*EL	-0.00251	0.029096	-0.08626	0.932
CA*HA*BA	0.00115	0.009883	0.11640	0.908
CA*HA*EL	-0.00098	0.009883	-0.09936	0.921
CA*BA*EL	0.00063	0.009883	0.06402	0.949
HA*BA*EL	0.00060	0.009883	0.06104	0.952

Summary of Model

$$S = 0.0484184 \quad R-Sq = 94.17\% \quad R-Sq(adj) = 92.94\%$$

General Regression Analysis: FT380 versus CA, HA, BA

Regression Equation

FT380 = 1.30351 - 0.185313 CA - 0.265241 HA - 0.00118578 BA + 0.0403782 CA*HA

Coefficients

Term	Coef	SE Coef	T	P
Constant	1.30351	0.0387382	33.6493	0.000
CA	-0.18531	0.0169260	-10.9485	0.000
HA	-0.26524	0.0169260	-15.6706	0.000
BA	-0.00119	0.0063974	-0.1854	0.853
CA*HA	0.04038	0.0078352	5.1534	0.000

Summary of Model

S = 0.0470112 R-Sq = 93.67% R-Sq(adj) = 93.34%

Appendix G

Cathode Inlet Temperature Regression Analysis

General Regression Analysis: TE_326A versus CA, HA, BA, EL

Regression Equation

$$TE_{326A} = 585.996 - 2.22505 CA - 4.00184 HA + 8.27237 BA + 47.1388 EL$$

Coefficients

Term	Coef	SE Coef	T	P
Constant	585.996	4.44183	131.927	0.000
CA	-2.225	1.08802	-2.045	0.044
HA	-4.002	1.08802	-3.678	0.000
BA	8.272	1.08802	7.603	0.000
EL	47.139	1.08802	43.325	0.000

Summary of Model

$$S = 7.99530 \quad R\text{-Sq} = 96.25\% \quad R\text{-Sq}(\text{adj}) = 96.06\%$$

General Regression Analysis: TE_326A versus CA, HA, BA, EL

Regression Equation

$$TE_{326A} = 599.594 - 12.9571 CA - 10.9647 HA - 0.691198 BA + 59.2023 EL + 7.28747 CA*HA + 4.24521 CA*BA - 3.40327 CA*EL + 3.1153 HA*BA - 6.2528 HA*EL - 0.595591 BA*EL - 2.35914 CA*HA*BA + 0.892373 CA*HA*EL + 0.0850712 CA*BA*EL + 1.1325 HA*BA*EL$$

Coefficients

Term	Coef	SE Coef	T	P
Constant	599.594	27.2823	21.9774	0.000
CA	-12.957	11.3279	-1.1438	0.257
HA	-10.965	11.3279	-0.9679	0.337
BA	-0.691	11.3279	-0.0610	0.952
EL	59.202	11.3279	5.2263	0.000
CA*HA	7.287	4.4388	1.6418	0.105
CA*BA	4.245	4.4388	0.9564	0.342
CA*EL	-3.403	4.4388	-0.7667	0.446
HA*BA	3.115	4.4388	0.7018	0.485
HA*EL	-6.253	4.4388	-1.4087	0.164
BA*EL	-0.596	4.4388	-0.1342	0.894
CA*HA*BA	-2.359	1.5078	-1.5646	0.122
CA*HA*EL	0.892	1.5078	0.5918	0.556
CA*BA*EL	0.085	1.5078	0.0564	0.955
HA*BA*EL	1.133	1.5078	0.7511	0.455

Summary of Model

$$S = 7.38657 \quad R\text{-Sq} = 97.22\% \quad R\text{-Sq}(\text{adj}) = 96.63\%$$

General Regression Analysis: TE_326A versus CA, HA, BA, EL

Regression Equation

$$\begin{aligned} \text{TE_326A} = & 609.115 - 16.8669 \text{ CA} - 19.0642 \text{ HA} - 1.88238 \text{ BA} + 54.4416 \text{ EL} + \\ & 9.07221 \text{ CA*HA} + 4.41535 \text{ CA*BA} - 1.44838 \text{ CA*EL} + 5.3803 \text{ HA*BA} - \\ & 2.20305 \text{ HA*EL} - 2.35914 \text{ CA*HA*BA} \end{aligned}$$

Coefficients

Term	Coef	SE Coef	T	P
Constant	609.115	16.7375	36.3923	0.000
CA	-16.867	7.4093	-2.2765	0.026
HA	-19.064	7.4093	-2.5730	0.012
BA	-1.882	6.9935	-0.2692	0.789
EL	54.442	3.6022	15.1135	0.000
CA*HA	9.072	3.2373	2.8024	0.007
CA*BA	4.415	3.2373	1.3639	0.177
CA*EL	-1.448	1.2236	-1.1837	0.241
HA*BA	5.380	3.2373	1.6619	0.101
HA*EL	-2.203	1.2236	-1.8005	0.076
CA*HA*BA	-2.359	1.4986	-1.5742	0.120

Summary of Model

S = 7.34161 R-Sq = 97.09% R-Sq(adj) = 96.67%

Appendix H

Turbine Inlet Temperature Regression Analysis

Regression Analysis: TE_350A versus CA, HA, BA, EL

The regression equation is

$$TE_{350A} = 797 - 6.08 CA - 2.21 HA + 11.9 BA + 65.9 EL$$

Predictor	Coef	SE Coef	T	P
Constant	796.511	6.555	121.50	0.000
CA	-6.083	1.606	-3.79	0.000
HA	-2.210	1.606	-1.38	0.173
BA	11.879	1.606	7.40	0.000
EL	65.865	1.606	41.02	0.000

$$S = 11.7998 \quad R\text{-Sq} = 95.8\% \quad R\text{-Sq(adj)} = 95.6\%$$

General Regression Analysis: TE_350A versus CA, HA, BA, EL

Regression Equation

$$TE_{350A} = 827.776 - 24.3379 CA - 15.8195 HA - 1.84092 BA + 80.6542 EL + 10.3979 CA*HA + 6.14419 CA*BA - 5.03365 CA*EL + 3.72399 HA*BA - 8.4453 HA*EL + 0.558303 BA*EL - 2.64542 CA*HA*BA + 1.90108 CA*HA*EL - 0.446259 CA*BA*EL + 1.30833 HA*BA*EL$$

Coefficients

Term	Coef	SE Coef	T	P
Constant	827.776	37.7276	21.9408	0.000
CA	-24.338	15.6648	-1.5537	0.125
HA	-15.819	15.6648	-1.0099	0.316
BA	-1.841	15.6648	-0.1175	0.907
EL	80.654	15.6648	5.1487	0.000
CA*HA	10.398	6.1382	1.6940	0.095
CA*BA	6.144	6.1382	1.0010	0.320
CA*EL	-5.034	6.1382	-0.8201	0.415
HA*BA	3.724	6.1382	0.6067	0.546
HA*EL	-8.445	6.1382	-1.3759	0.174
BA*EL	0.558	6.1382	0.0910	0.928
CA*HA*BA	-2.645	2.0850	-1.2688	0.209
CA*HA*EL	1.901	2.0850	0.9118	0.365
CA*BA*EL	-0.446	2.0850	-0.2140	0.831
HA*BA*EL	1.308	2.0850	0.6275	0.533

Summary of Model

$$S = 10.2146 \quad R\text{-Sq} = 97.30\% \quad R\text{-Sq(adj)} = 96.72\%$$

General Regression Analysis: TE_350A versus CA, HA, BA, EL

Regression Equation

$$\begin{aligned} \text{TE_350A} = & 815.389 - 19.5754 \text{ CA} - 15.9758 \text{ HA} + 11.957 \text{ BA} + 74.1665 \text{ EL} + 8.9092 \\ & \text{CA*HA} - 0.0391796 \text{ CA*BA} - 2.12401 \text{ CA*EL} - 2.0265 \text{ HA*EL} \end{aligned}$$

Coefficients

Term	Coef	SE Coef	T	P
Constant	815.389	14.6576	55.6291	0.000
CA	-19.575	6.0192	-3.2521	0.002
HA	-15.976	4.9789	-3.2087	0.002
BA	11.957	3.6535	3.2727	0.002
EL	74.167	4.9789	14.8960	0.000
CA*HA	8.909	1.6913	5.2678	0.000
CA*BA	-0.039	1.6913	-0.0232	0.982
CA*EL	-2.124	1.6913	-1.2559	0.213
HA*EL	-2.026	1.6913	-1.1982	0.235

Summary of Model

S = 10.1476 R-Sq = 97.09% R-Sq(adj) = 96.77%

Appendix I
Replicate using a 1/9 fractional factorial
experiment

Exp.	CA	HA	BA	EL
1	40	20	10	0
2	40	50	14	50
3	40	80	12	25
4	60	20	14	25
5	60	50	12	0
6	60	80	10	50
7	80	20	12	50
8	80	50	10	25
9	80	80	14	0

Appendix J

Matlab Program for State Space Identification

“It is important to remember that any estimated model, no matter how good it looks on your screen, has only picked up a simple reflection of reality. Surprisingly often, however, this is sufficient for rational decision making.” - Ljung, 1987

```
clc
tic
s=0;
x=1+s;
n=90+s;
y=[Speed, Cath_flow];
u=[CA, HA, BA, EL, Fuel_flow];
data1=iddata(y(x:n,:),u(x:n,:),0.4);

num=1;
den=[1 0.01];
filter = {num,den};
data=idfilt(data1,filter);

num1=1;
den1=[1 0.01];
filter1 = {num1,den1};
data=idfilt(data1(x:n,1),filter1);

num2=1;
den2=[1 1];
filter2 = {num2,den2};
data=idfilt(data1(x:n,2),filter2);

Model_1_Canonical = pem(data,2,...
    'ssparameterization','canonical');
toc
present(Model_1_Canonical)    % Displays model properties

delayest(data) just for single ouput
m2 = n4sid(data1, 2);
m3 = n4sid(data1, 3)

X0est = findstates(mCanonical,data)
Y=sim(mCanonical,u(x:n:),'InitialState',X0est);
compare(data,mCanonical)
predict(mCanonical,data,5)

t=0:0.4:(n-x)*0.4;
subplot(2,1,1)
plot(t,y(x:n,1),'r',t,Y(1:end,1),'m','LineWidth',3)
subplot(2,1,2)
plot(t, y(x:n,2),'b', t, Y(1:end,2),'g','LineWidth',3)
```

Appendix K

Table of Equivalence Ratio

Fuel Flow (grams/min), Cathode Air Mass Flow (kg/s)												
$\text{Equival. ratio} = \frac{(FA)_a}{(FA)_i} = 0.0002867 \frac{FT 432 \text{ (g / min)}}{FT 380 \text{ (kg / s)}}$												
Cath Flow	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
Fuel Flow												
600	1.72	0.86	0.57	0.43	0.34	0.29	0.25	0.22	0.19	0.17	0.16	0.14
620	1.78	0.89	0.59	0.44	0.36	0.30	0.25	0.22	0.20	0.18	0.16	0.15
640	1.83	0.92	0.61	0.46	0.37	0.31	0.26	0.23	0.20	0.18	0.17	0.15
660	1.89	0.95	0.63	0.47	0.38	0.32	0.27	0.24	0.21	0.19	0.17	0.16
680	1.95	0.97	0.65	0.49	0.39	0.32	0.28	0.24	0.22	0.19	0.18	0.16
700	2.01	1.00	0.67	0.50	0.40	0.33	0.29	0.25	0.22	0.20	0.18	0.17
720	2.06	1.03	0.69	0.52	0.41	0.34	0.29	0.26	0.23	0.21	0.19	0.17
740	2.12	1.06	0.71	0.53	0.42	0.35	0.30	0.27	0.24	0.21	0.19	0.18
760	2.18	1.09	0.73	0.54	0.44	0.36	0.31	0.27	0.24	0.22	0.20	0.18
780	2.24	1.12	0.75	0.56	0.45	0.37	0.32	0.28	0.25	0.22	0.20	0.19
800	2.29	1.15	0.76	0.57	0.46	0.38	0.33	0.29	0.25	0.23	0.21	0.19
820	2.35	1.18	0.78	0.59	0.47	0.39	0.34	0.29	0.26	0.24	0.21	0.20
840	2.41	1.20	0.80	0.60	0.48	0.40	0.34	0.30	0.27	0.24	0.22	0.20
860	2.47	1.23	0.82	0.62	0.49	0.41	0.35	0.31	0.27	0.25	0.22	0.21
880	2.52	1.26	0.84	0.63	0.50	0.42	0.36	0.32	0.28	0.25	0.23	0.21
900	2.58	1.29	0.86	0.65	0.52	0.43	0.37	0.32	0.29	0.26	0.23	0.22
920	2.64	1.32	0.88	0.66	0.53	0.44	0.38	0.33	0.29	0.26	0.24	0.22
940	2.69	1.35	0.90	0.67	0.54	0.45	0.38	0.34	0.30	0.27	0.24	0.22
960	2.75	1.38	0.92	0.69	0.55	0.46	0.39	0.34	0.31	0.28	0.25	0.23
980	2.81	1.40	0.94	0.70	0.56	0.47	0.40	0.35	0.31	0.28	0.26	0.23
1000	2.87	1.43	0.96	0.72	0.57	0.48	0.41	0.36	0.32	0.29	0.26	0.24
1020	2.92	1.46	0.97	0.73	0.58	0.49	0.42	0.37	0.32	0.29	0.27	0.24
1040	2.98	1.49	0.99	0.75	0.60	0.50	0.43	0.37	0.33	0.30	0.27	0.25
1060	3.04	1.52	1.01	0.76	0.61	0.51	0.43	0.38	0.34	0.30	0.28	0.25
1080	3.10	1.55	1.03	0.77	0.62	0.52	0.44	0.39	0.34	0.31	0.28	0.26
1100	3.15	1.58	1.05	0.79	0.63	0.53	0.45	0.39	0.35	0.32	0.29	0.26
1120	3.21	1.61	1.07	0.80	0.64	0.54	0.46	0.40	0.36	0.32	0.29	0.27
1140	3.27	1.63	1.09	0.82	0.65	0.54	0.47	0.41	0.36	0.33	0.30	0.27
1160	3.33	1.66	1.11	0.83	0.67	0.55	0.48	0.42	0.37	0.33	0.30	0.28
1180	3.38	1.69	1.13	0.85	0.68	0.56	0.48	0.42	0.38	0.34	0.31	0.28
1200	3.44	1.72	1.15	0.86	0.69	0.57	0.49	0.43	0.38	0.34	0.31	0.29
1220	3.50	1.75	1.17	0.87	0.70	0.58	0.50	0.44	0.39	0.35	0.32	0.29
1240	3.56	1.78	1.19	0.89	0.71	0.59	0.51	0.44	0.40	0.36	0.32	0.30
1260	3.61	1.81	1.20	0.90	0.72	0.60	0.52	0.45	0.40	0.36	0.33	0.30
1280	3.67	1.83	1.22	0.92	0.73	0.61	0.52	0.46	0.41	0.37	0.33	0.31
1300	3.73	1.86	1.24	0.93	0.75	0.62	0.53	0.47	0.41	0.37	0.34	0.31
1320	3.78	1.89	1.26	0.95	0.76	0.63	0.54	0.47	0.42	0.38	0.34	0.32
1340	3.84	1.92	1.28	0.96	0.77	0.64	0.55	0.48	0.43	0.38	0.35	0.32
1360	3.90	1.95	1.30	0.97	0.78	0.65	0.56	0.49	0.43	0.39	0.35	0.32
1380	3.96	1.98	1.32	0.99	0.79	0.66	0.57	0.49	0.44	0.40	0.36	0.33
1400	4.01	2.01	1.34	1.00	0.80	0.67	0.57	0.50	0.45	0.40	0.36	0.33
1420	4.07	2.04	1.36	1.02	0.81	0.68	0.58	0.51	0.45	0.41	0.37	0.34
1440	4.13	2.06	1.38	1.03	0.83	0.69	0.59	0.52	0.46	0.41	0.38	0.34
1460	4.19	2.09	1.40	1.05	0.84	0.70	0.60	0.52	0.47	0.42	0.38	0.35
1480	4.24	2.12	1.41	1.06	0.85	0.71	0.61	0.53	0.47	0.42	0.39	0.35
1500	4.30	2.15	1.43	1.08	0.86	0.72	0.61	0.54	0.48	0.43	0.39	0.36

Appendix L

Guidelines for Determining Model Parameters

Labview Guidelines

For the black-box model, determining the delay and model order for the parametric model is typically a trial-and-error process. The following is a useful set of steps that can lead to a suitable model. This is not the only methodology you can use, nor is this a comprehensive procedure.

Obtain useful information about the model order by observing the number of resonance peaks in the nonparametric frequency response function. Normally, the number of peaks in the magnitude response equals half the order of $A(q)F(q)$.

2. Obtain a reasonable estimate of delay using correlation analysis and/or by testing reasonable values in a medium size ARX model. Choose the delay that provides the best model fit based on prediction errors or other fit criterion.
3. Test various ARX model orders with this delay choosing those that provide the best fit.
4. Since the ARX model describes both the system dynamics and noise properties using the same set of poles, the resulting model may be unnecessarily high in order. By plotting the zeros and poles (with the uncertainty intervals) and looking for cancellations you can reduce the model order. The resulting order of the poles and zeros are a good starting point for ARMAX, OE and/or BJ models with these orders used as the B and F model parameters and first or second order models for the noise characteristics.
5. If a suitable model is not obtained at this point attempt to determine if there are additional signals that may influence the output. Measurements of these signals can be incorporated as extra input signals.

If you cannot obtain a suitable model following these steps additional physical insight into the problem might be necessary. Compensating for nonlinear sensors or actuators and handling of

important physical non-linearities are often necessary in addition to using a ready-made model.

From the prediction error standpoint, the higher the order of the model is, the better the model fits the data because the model has more degrees of freedom. However, you need more computation time and memory for higher orders. The parsimony principle advocates choosing the model with the smallest degree of freedom, or number of parameters, if all the models fit the data well and pass the verification test.

Matlab Guidelines

Black-box modeling is useful when your primary interest is in fitting the data regardless of a particular mathematical structure of the model. The toolbox provides several linear and nonlinear black-box model structures, which have traditionally been useful for representing dynamic systems. These model structures vary in complexity depending on the flexibility you need to account for the dynamics and noise in your system. You can choose one of these structures and compute its parameters to fit the measured response data. Black-box modeling is usually a trial-and-error process, where you estimate the parameters of various structures and compare the results. Typically, you start with the simple linear model structure and progress to more complex structures. You might also choose a model structure because you are more familiar with this structure or because you have specific application needs. The simplest linear black-box structures require the fewest options to configure:

- Linear ARX model, which is the simplest input-output polynomial model.
- State-space model, which you can estimate by specifying the number of model states

Estimation of these structures also uses noniterative estimation algorithms, which further reduces complexity.

You can configure a model structure using the *model order*. The definition of model order varies depending on the type of model you select. For example, if you choose a transfer function representation, the model order is related to the number of poles and zeros. For state-space representation, the model order corresponds to the number of states. In some cases, such as for

linear ARX and state-space model structures, you can estimate the model order from the data. If the simple model structures do not produce good models, you can select more complex model structures by:

- Specifying a higher model order for the same linear model structure. Higher model order increases the model flexibility for capturing complex phenomena. However, unnecessarily high orders can make the model less reliable.

- Explicitly modeling the noise:

$$y(t)=Gu(t)+He(t)$$

where H models the additive disturbance by treating the disturbance as the output of a linear system driven by a white noise source $e(t)$. Using a model structure that explicitly models the additive disturbance can help to improve the accuracy of the measured component G. Furthermore, such a model structure is useful when your main interest is using the model for predicting future response values.

- Using a different linear model structure.
- Using a nonlinear model structure.

Nonlinear models have more flexibility in capturing complex phenomena than linear models of similar orders.

Appendix M

System Identification “advice”

Plot 1.

Excitation level in data:

Input number 4 is persistently exciting of order 10.

This means that you will have problems when estimation models of order higher than 10, at least for model parameters associated with this input.

The excitation orders for all the inputs are [50 50 50 10 50].

Possibility of feedback in data:

There is a strong indication of feedback in the data.

You should be careful when interpreting the results of SPA and also interpret the results of output error models with care (Output error models result from the OE command or setting 'DisturbanceModel'= 'None' in state-space models.).

With feedback in data, it is recommended to use estimate a model with large enough disturbance model. For example, use BJ models in place of OE models and estimate state space models using 'DisturbanceModel'= 'Estimate'.

Possibility of nonlinearity:

There is no clear indication of nonlinearities in this data set.

Plot 2

Excitation level in data:

Input number 4 is essentially a constant. Estimating parameters associated with this input will be difficult, unless the other experiments support this input.

Input number 4 is persistently exciting of order 0.

This means that you will have problems when estimation models of order higher than 0, at least for model parameters associated with this input.

The excitation orders for all the inputs are [50 50 50 0 50].

Possibility of feedback in data:

Possibility of feedback could not be determined.

Use the "feedback" command for assessment of feedback in data with more options.

Possibility of nonlinearity:

Input number 5 is binary. Building nonlinear models with this data may be difficult.

There is no clear indication of nonlinearities in this data set.

Plot 3

Excitation level in data:

Input number 4 is persistently exciting of order 12.

This means that you will have problems when estimation models of order higher than 12, at least for model parameters associated with this input.

The excitation orders for all the inputs are [50 50 50 12 50].

Possibility of feedback in data:

There is a very strong indication of feedback in the data.

You should be careful when interpreting the results of SPA and also interpret the results of output error models with care (Output error models result from the OE command or setting 'DisturbanceModel'= 'None' in state-space models.).

With feedback in data, it is recommended to use estimate a model with large enough disturbance model. For example, use BJ models in place of OE models and estimate state space models using 'DisturbanceModel'= 'Estimate'.

Possibility of nonlinearity:

There is no clear indication of nonlinearities in this data set.

Appendix N

Simulink Representation of the MPC and transfer functions Model