2013

Analysis of Shale Production Performance Using Decline Curve Methods

Brad Thomas Nelson
West Virginia University

Follow this and additional works at: <https://researchrepository.wvu.edu/etd>

Recommended Citation

Nelson, Brad Thomas, "Analysis of Shale Production Performance Using Decline Curve Methods" (2013). *Graduate Theses, Dissertations, and Problem Reports*. 3397.
<https://researchrepository.wvu.edu/etd/3397>

This Thesis is protected by copyright and/or related rights. It has been brought to you by the The Research Repository @ WVU with permission from the rights-holder(s). You are free to use this Thesis in any way that is permitted by the copyright and related rights legislation that applies to your use. For other uses you must obtain permission from the rights-holder(s) directly, unless additional rights are indicated by a Creative Commons license in the record and/ or on the work itself. This Thesis has been accepted for inclusion in WVU Graduate Theses, Dissertations, and Problem Reports collection by an authorized administrator of The Research Repository @ WVU. For more information, please contact researchrepository@mail.wvu.edu.

Analysis of Shale Production Performance Using Decline Curve Methods

Brad Thomas Nelson

Thesis submitted to the
Benjamin M. Statler College of Engineering and Mineral Resources
at West Virginia University
In Partial fulfillment of the requirements
For the degree of

Master of Science
In
Petroleum and Natural Gas Engineering

Khashayar Aminian, Ph.D., Chair
Samuel Ameri, M.S.
Ebrahim Fathi, Ph.D.

Department of Petroleum and Natural Gas Engineering

Morgantown, West Virginia
2013

Keywords: Decline Curves, Marcellus Shale, Horizontal Wells, Gas Production Prediction
Copyright 2013 Brad Thomas Nelson

ABSTRACT

Analysis of Shale Production Performance Using Decline Curve Methods

Brad Thomas Nelson

With the recent development of shale gas reservoirs such as the Marcellus using horizontal drilling and fracturing, it has become necessary to evaluate the amount of gas recoverable for both economic and operational purposes. As a result of limited production history, the production behavior of horizontal well producing from Marcellus shale has not been well established. A technique in accomplishing the estimation of future production history would be most useful to the industry.

Decline curve analysis (DCA) methods have been utilized successfully in various hydrocarbon plays throughout the world in approximating future production. Several DCA models have been proposed specifically for unconventional gas reservoirs. However, their applicability to production data from Marcellus shale wells has not been attempted. Four sets of simulated Marcellus shale production profiles were generated in this study. They included production from a 3000 feet-long horizontal well containing seven hydraulic fracture stages (a hydraulic fracture spacing of 500 feet) and thirteen fractures (a hydraulic fracture spacing of 250 feet). Two sets were simulated using a dual porosity model with adsorbed gas and two sets were simulated using a dual porosity model without adsorbed gas. The most appropriate DCA models for each set were selected based on the entire production profile (30 years). Subsequently, a technique was developed to predict the long term DCA model parameters based on the limited production history via dimensionless log-log plots. Finally, the developed methodology was applied to the limited field production data from a horizontal well containing eight hydraulic fracture stages (a hydraulic fracture spacing of 429 feet). The comparison of the predicted future production rates with those rates predicted by history matching with a commercial reservoir simulator confirmed the reliability of methodology developed in this study.

Acknowledgements

I would like to thank my advisor, Dr. Aminian, for his guidance during the research process. In addition, I express my gratitude to committee members Prof. Ameri and Dr. Fathi for academic support.

TABLE OF CONTENTS

ABSTRACT.....	ii
Acknowledgements.....	iii
List of Tables	vi
List of Figures	viii
Nomenclature	xi
1. Introduction and Problem Statement	1
2. Literature Review.....	2
2.1 Background:	2
2.2 Flow Regimes:.....	2
2.3 Reserve Estimation:.....	3
2.4 Decline Theories:	3
2.4.1 Arps Decline Model.....	3
2.4.2 Power Law Exponential Decline Model (PLE):.....	5
2.4.3 Stretched Exponential Decline Model (SEPD):	6
2.4.4 Duong Model:.....	7
3. Objective and Methodology.....	9
3.1 Objective:	9
3.2 Methodology:	9
3.2.1 Development of a Marcellus Shale Reservoir Model:.....	10
3.2.2 Application of Decline Curve Analysis Techniques to Simulated Data:	12
3.2.3 Application of Selected DCA Models to Varying Simulated Production Times:	12
3.2.4 Validation Against Field Data:	13
4. Results and Discussion	16
4.1 Development of a Marcellus Shale Reservoir Model:	16
4.2 Application of Decline Curve Analysis Techniques to Simulated Data:	18
4.3 Application of Selected DCA Models to Varying Simulated Production Times:.....	35
4.4 Validation Against Field Data:.....	43
5. Conclusions and Recommendations	60
6. References.....	61

7. Appendix	64
7.1 Correlations and Simulator Match	64
7.2 Well 1 - 13 Fracs (Adsorbed Gas) Predictions.....	90

List of Tables

Table 1: Eclipse Basic Model Parameters	11
Table 2: 7 Fracs (Ads. Gas) Arps Model	37
Table 3: 7 Fracs (Ads. Gas) PLE Model	37
Table 4: 7 Fracs (Ads. Gas) Duong Model	38
Table 5: 13 Fracs (Ads. Gas) Arps Model	38
Table 6: 13 Fracs (Ads. Gas) PLE Model	39
Table 7: 13 Fracs (Ads. Gas) Duong Model	39
Table 8: 7 Fracs (No Ads. Gas) Arps Model	40
Table 9: 7 Fracs (No Ads. Gas) PLE Model	40
Table 10: 7 Fracs (No Ads. Gas) Duong Model	41
Table 11: 13 Fracs (No Ads. Gas) Arps Model	41
Table 12: 13 Fracs (No. Ads Gas) PLE Model	42
Table 13: 13 Fracs (No. Ads Gas) Duong Model	42
Table 14: 7 Fracs (Ads. Gas) Arps Table of Ratios	45
Table 15: 7 Fracs (Ads. Gas) PLE Table of Ratios	45
Table 16: 7 Fracs (Ads. Gas) Duong Table of Ratios	46
Table 17: 7 Fracs (Ads. Gas) Arps Well 1 Ratio Values	52
Table 21: 7 Fracs (Ads. Gas) PLE Well 1 Ratio Values	52
Table 19: 7 Fracs (Ads. Gas) Duong Well 1 Ratio Values	52
Table 20: 7 Fracs (Ads. Gas) Arps Well 1 Actual and Predicted Values	54
Table 21: 7 Fracs (Ads. Gas) PLE Well 1 Actual and Predicted Values	55
Table 22: 7 Fracs (Ads. Gas) Duong Well 1 Actual and Predicted Values	55
Table 23: Simulator Predicted 7 Fracs (Ads. Gas) Arps	87
Table 24: Simulator Predicted 7 Fracs (Ads. Gas) PLE	87
Table 25: Simulator Predicted 7 Fracs (Ads. Gas) Duong	87
Table 26: Simulator Predicted 13 Fracs (Ads. Gas) Arps	87
Table 27: Simulator Predicted 13 Fracs (Ads Gas.) PLE	88
Table 28: Simulator Predicted 13 Fracs (Ads. Gas) Duong	88
Table 29: Simulator Predicted 7 Fracs (No Ads. Gas) Arps	88
Table 30: Simulator Predicted 7 Fracs (No Ads. Gas) PLE	88
Table 31: Simulator Predicted 7 Fracs (No Ads. Gas) Duong	88
Table 32: Simulator Predicted 13 Fracs (No Ads. Gas) Arps	88
Table 33: Simulator Predicted 13 Fracs (No Ads. Gas) PLE	89
Table 34: Simulator Predicted 13 Fracs (No Ads. Gas) Duong	89
Table 35: 13 Fracs (Ads. Gas) Arps Table of Ratios	90
Table 36: 13 Fracs (Ads. Gas) PLE Table of Ratios	90
Table 37: 13 Fracs (Ads. Gas) Duong Table of Ratios	91
Table 38: 13 Fracs (Ads. Gas) Arps Well 1 Ratio Values	96

Table 39: 13 Fracs (Ads. Gas) PLE Well 1 Ratio Values.....	97
Table 40: 13 Fracs (Ads. Gas) Duong Well 1 Ratio Values	97
Table 41: 13 Fracs (Ads. Gas) Arps Well 1 Actual and Predicted Values	97
Table 42: 13 Fracs (Ads. Gas) PLE Well 1 Actual and Predicted Values.....	97
Table 43: 13 Fracs (Ads. Gas) Duong Well 1 Actual and Predicted Values	98

List of Figures

Figure 1: Well 1 Production.....	14
Figure 2: Well 1 History Match (4.277 Years)	14
Figure 3: Well 1 History Match (30 Years)	15
Figure 4: 7 Fracs (Adsorbed Gas): q vs. t	16
Figure 5: 13 Fracs (Adsorbed Gas): q vs. t	17
Figure 6: 7 Fracs (No Adsorbed Gas): q vs. t	17
Figure 7: 13 Fracs (No Adsorbed Gas): q vs. t	18
Figure 8: 7 Fracs (Ads. Gas) Arps q vs. t (30 Years).....	19
Figure 9: 7 Fracs (Ads. Gas) PLE q vs. t (30 Years)	20
Figure 10: 7 Fracs (Ads. Gas) Duong q vs. t (30 Years)	20
Figure 11: 7 Fracs (Ads. Gas) q vs. t Comparison – Years 0-2	21
Figure 12: 7 Fracs (Ads. Gas) q vs. t Comparison – Years 2-5	21
Figure 13: 7 Fracs (Ads. Gas) q vs. t Comparison – Years 5-10	22
Figure 14: 7 Fracs (Ads. Gas) q vs. t Comparison – Years 10-20	22
Figure 15: 7 Fracs (Ads. Gas) q vs. t Comparison – Years 20-30	23
Figure 16: 13 Fracs (Ads. Gas) Arps q vs. t (30 Years).....	23
Figure 17: 13 Fracs (Ads. Gas) PLE q vs. t (30 Years)	24
Figure 18: 13 Fracs (Ads. Gas) Duong q vs. t (30 Years)	24
Figure 19: 13 Fracs (Ads. Gas) q vs. t Comparison – Years 0-2	25
Figure 20: 13 Fracs (Ads. Gas) q vs. t Comparison – Years 2-5	25
Figure 21: 13 Fracs (Ads. Gas) q vs. t Comparison – Years 5-10	26
Figure 22: 13 Fracs (Ads. Gas) q vs. t Comparison – Years 10-20	26
Figure 23: 13 Fracs (Ads. Gas) q vs. t Comparison – Years 20-30	27
Figure 24: 7 Fracs (No Ads. Gas) Arps q vs. t (30 Years).....	27
Figure 25: 7 Fracs (No Ads. Gas) PLE q vs. t (30 Years)	28
Figure 26: 7 Fracs (No Ads. Gas) Duong q vs. t (30 Years).....	28
Figure 27: 7 Fracs (No Ads. Gas) q vs. t Comparison – Years 0-2	29
Figure 28: 7 Fracs (No Ads. Gas) q vs. t Comparison – Years 2-5	29
Figure 29: 7 Fracs (No Ads. Gas) q vs. t Comparison – Years 5-10	30
Figure 30: 7 Fracs (No Ads. Gas) q vs. t Comparison – Years 10-20	30
Figure 31: 7 Fracs (No Ads. Gas) q vs. t Comparison – Years 20-30	31
Figure 32: 13 Fracs (No Ads. Gas) Arps q vs. t (30 Years).....	31
Figure 33: 13 Fracs (No Ads. Gas) PLE q vs. t (30 Years)	32
Figure 34: 13 Fracs (No Ads. Gas) Duong q vs. t (30 Years).....	32
Figure 35: 13 Fracs (No Ads. Gas) q vs. t Comparison – Years 0-2	33
Figure 36: 13 Fracs (No Ads. Gas) q vs. t Comparison – Years 2-5	33
Figure 37: 13 Fracs (No Ads. Gas) q vs. t Comparison – Years 5-10	34
Figure 38: 13 Fracs (No Ads. Gas) q vs. t Comparison – Years 10-20	34

Figure 39: 13 Fracs (No Ads. Gas) q vs. t Comparison – Years 20-30	35
Figure 40: 7 Fracs (Ads. Gas) Arps Ratio of n vs. Time	46
Figure 41: 7 Fracs (Ads. Gas) Arps Ratio of d_i vs. Time	47
Figure 42: 7 Fracs (Ads. Gas) Arps Ratio of q_i vs. Time	47
Figure 43: 7 Fracs (Ads. Gas) PLE Ratio of D_∞ vs. Time	48
Figure 44: 7 Fracs (Ads. Gas) PLE Ratio of n vs. Time	48
Figure 45: 7 Fracs (Ads. Gas) PLE Ratio of q_i vs. Time	49
Figure 46: 7 Fracs (Ads. Gas) PLE Ratio of D_1 vs. Time	49
Figure 47: 7 Fracs (Ads. Gas) Duong Ratio of a vs. Time	50
Figure 48: 7 Fracs (Ads. Gas) Duong Ratio of m vs. Time	50
Figure 49: 7 Fracs (Ads. Gas) Duong Ratio of q_1 vs. Time	51
Figure 50: 7 Fracs (Ads. Gas) Duong Ratio of q_∞ vs. Time	51
Figure 51: Well 1 Arps Actual Match	53
Figure 52: Well 1 PLE Actual Match	53
Figure 53: Well 1 Duong Actual Match	54
Figure 54: Well 1 - 7 Fracs (Ads. Gas) Arps Predicted Match	56
Figure 55: Well 1 - 7 Fracs (Ads. Gas) PLE Predicted Match	56
Figure 56: Well 1 - 7 Fracs (Ads. Gas) Duong Predicted Match	57
Figure 57: 7 Fracs (Ads. Gas) 30 Year Well 1 Predicted Comparison	58
Figure 58: 7 Fracs (Ads. Gas) Arps n Correlation	65
Figure 59: 7 Fracs (Ads. Gas) Arps d_i Correlation	65
Figure 60: 7 Fracs (Ads. Gas) Arps q_i Correlation	66
Figure 61: 7 Fracs (Ads. Gas) PLE D_∞ Correlation	66
Figure 62: 7 Fracs (Ads. Gas) PLE n Correlation	67
Figure 63: 7 Fracs (Ads. Gas) PLE q_i Correlation	67
Figure 64: 7 Fracs (Ads. Gas) PLE D_1 Correlation	68
Figure 65: 7 Fracs (Ads. Gas) Duong a Correlation	68
Figure 66: 7 Fracs (Ads. Gas) Duong m Correlation	69
Figure 67: 7 Fracs (Ads. Gas) Duong q_1 Correlation	69
Figure 68: 7 Fracs (Ads. Gas) Duong q_∞ Correlation	70
Figure 69: 13 Fracs (Ads. Gas) Arps n Correlation	70
Figure 70: 13 Fracs (Ads. Gas) Arps d_i Correlation	71
Figure 71: 13 Fracs (Ads. Gas) Arps q_i Correlation	71
Figure 72: 13 Fracs (Ads. Gas) PLE D_∞ Correlation	72
Figure 73: 13 Fracs (Ads. Gas) PLE n Correlation	72
Figure 74: 13 Fracs (Ads. Gas) PLE q_i Correlation	73
Figure 75: 13 Fracs (Ads. Gas) PLE D_1 Correlation	73
Figure 76: 13 Fracs (Ads. Gas) Duong a Correlation	74
Figure 77: 13 Fracs (Ads. Gas) Duong m Correlation	74
Figure 78: 13 Fracs (Ads. Gas) Duong q_1 Correlation	75

Figure 79: 13 Fracs (Ads. Gas) Duong q_{∞} Correlation	75
Figure 80: 7 Fracs (No Ads. Gas) Arps n Correlation	76
Figure 81: 7 Fracs (No Ads. Gas) Arps d_i Correlation.....	76
Figure 82: 7 Fracs (No Ads. Gas) Arps q_i Correlation	77
Figure 83: 7 Fracs (No Ads. Gas) PLE D_{∞} Correlation	77
Figure 84: 7 Fracs (No Ads. Gas) PLE n Correlation.....	78
Figure 85: 7 Fracs (No Ads. Gas) PLE q_i Correlation	78
Figure 86: 7 Fracs (No Ads. Gas) PLE D_1 Correlation.....	79
Figure 87: 7 Fracs (No Ads. Gas) Duong a Correlation	79
Figure 88: 7 Fracs (No Ads. Gas) Duong m Correlation.....	80
Figure 89: 7 Fracs (No Ads. Gas) Duong q_1 Correlation.....	80
Figure 90: 7 Fracs (No Ads. Gas) Duong q_{∞} Correlation	81
Figure 91: 13 Fracs (No Ads. Gas) Arps n Correlation	81
Figure 92: 13 Fracs (No Ads. Gas) Arps d_i Correlation	82
Figure 93: 13 Fracs (No Ads. Gas) Arps q_i Correlation	82
Figure 94: 13 Fracs (No Ads. Gas) PLE D_{∞} Correlation	83
Figure 95: 13 Fracs (No Ads. Gas) PLE n Correlation.....	83
Figure 96: 13 Fracs (No Ads. Gas) PLE q_i Correlation	84
Figure 97: 13 Fracs (No Ads. Gas) PLE D_1 Correlation.....	84
Figure 98: 13 Fracs (No Ads. Gas) Duong a Correlation	85
Figure 99: 13 Fracs (No Ads. Gas) Duong m Correlation.....	85
Figure 100: 13 Fracs (No Ads. Gas) Duong q_1 Correlation.....	86
Figure 101: 13 Fracs (No Ads. Gas) Duong q_{∞} Correlation	86
Figure 102: 13 Fracs (Ads. Gas) Arps Ratio of n vs. Time	91
Figure 103: 13 Fracs (Ads. Gas) Arps Ratio of d_i vs. Time.....	92
Figure 104: 13 Fracs (Ads. Gas) Arps Ratio of q_i vs. Time	92
Figure 105: 13 Fracs (Ads. Gas) PLE Ratio of D_{∞} vs. Time	93
Figure 106: 13 Fracs (Ads. Gas) PLE Ratio of n vs. Time	93
Figure 107: 13 Fracs (Ads. Gas) PLE Ratio of q_i vs. Time	94
Figure 108: 13 Fracs (Ads. Gas) PLE Ratio of D_1 vs. Time.....	94
Figure 109: 13 Fracs (Ads. Gas) Duong Ratio of a vs. Time	95
Figure 110: 13 Fracs (Ads. Gas) Duong Ratio of m vs. Time	95
Figure 111: 13 Fracs (Ads. Gas) Duong Ratio of q_1 vs. Time.....	96
Figure 112: 13 Fracs (Ads. Gas) Duong Ratio of q_{∞} vs. Time	96
Figure 113: Well 1 - 13 Fracs (Ads. Gas) Arps Predicted Match.....	98
Figure 114: Well 1 - 13 Fracs (Ads. Gas) PLE Predicted Match	99
Figure 115: Well 1 - 13 Fracs (Ads. Gas) Duong Predicted Match.....	99
Figure 116: 13 Fracs (Ads. Gas) 30 Year Well 1 Predicted Comparison.....	100

Nomenclature

D = Initial decline rate, 1/ time

q_0 = Initial rate, volume/time

n = Constant loss ratio or time ratio, dimensionless

G_p = Cumulative gas production, volume

D_∞ = Loss ratio at ($t=\infty$), 1/time

D_1 = Loss ratio at ($t=1$), 1/time

τ = Characteristic time constant, time

Q = Cumulative production, volume

Γ = Gamma function, dimensionless

q = Flow rate, volume/time

a = Vertical axis intercept of q/G_p vs. time plot, dimensionless

m = Slope of log-log plot of q/G_p vs. time plot, dimensionless

$t(a,m)$ = Duong time function, dimensionless

q_1 = Slope of q vs. $t(a,m)$ plot, volume

q_∞ = Vertical axis intercept of q vs. $t(a,m)$ plot, dimensionless

1. Introduction and Problem Statement

With the recent development of shale gas reservoirs such as the Marcellus using horizontal drilling and fracturing, it has become necessary to evaluate the amount of gas recoverable for both economic and operational purposes. With the limited production from the Marcellus, little is known about predicting future reserves. The production behavior from the Marcellus has not been well established. A technique in accomplishing the estimation of future production history would be most useful to the industry.

One way of evaluating recoverable hydrocarbons is through Decline Curve Analysis (DCA) models. Traditional methods for DCA focused mainly on the equations formulated by Arps in 1945 (Kanfar, 2012). The Arps method works well for reservoirs which exhibit boundary dominated flow (BDF) to abandonment (Kanfar, 2012). This becomes problematic in shale gas reservoirs which have long transient flow regimes for most of their production life (Kanfar, 2012). Arps' method will overestimate reserves when applied to shale gas reservoirs under transient flow (Kanfar, 2012). Overestimation of reserves resulted in development of further techniques such as the Power Law Exponential Decline (PLE) and the Duong method which more accurately forecast production during transient flow followed by boundary dominated flow (Joshi, 2013). Other commonly employed models are the Stretched Exponential Decline (SEPD) and the Logistic Growth Model (LGM).

Specifically, future production behavior is to be determined in shale gas using the Arps, PLE, and Duong methods of decline curve analysis for a shale gas reservoir containing both seven and thirteen fractures with and without adsorbed gas.

2. Literature Review

2.1 Background:

The hydraulic fracturing process involves injecting water, proppants, and chemicals into a well under high pressure to increase formation permeability (Hassett, 2013). Proppants remain within the reservoir fractures, holding them open and allowing for greater quantities of gas or oil to escape. Chemicals, such as corrosion inhibitors, are also added to protect equipment in the well bore. Increasing the volume of oil or gas produced from a reservoir contributes to greater economic well-being for all parties involved. Horizontal drilling allows operators to minimize surface impacts in surrounding communities. Using a single well pad, six to eight horizontal wells are often drilled. In the past, each well required a separate pad. As a result of horizontal drilling and fracturing, the global power balance between importing and exporting petroleum producing countries is changing. In addition, burning natural gas is better for the environment, as it emits about one-half the carbon dioxide emissions of coal (Hassett, 2013). The development of the Barnett shale in Texas, USA with these technologies was responsible for the increase in gas production nationwide (Belvalkar, 2010).

2.2 Flow Regimes:

Several flow periods exist during the life of a shale gas well. These include formation linear flow, fracture interference flow, linear flow in un-stimulated matrix, and boundary dominated flow (Joshi, 2013). Determining flow type can be accomplished by fitting a negative half-slope line or a negative quarter slope line to a log-log plot of flow rate versus time (Joshi, 2012). A negative half-slope indicates linear flow and a negative quarter slope indicates bi-linear flow (Joshi, 2012). Formation linear flow occurs when fracture boundaries have not been reached; in many wells lasting the majority of production life (Joshi, 2012). Fracture interference flow is dependent upon fracture spacing and permeability, with many current wells not having

reached this phase (Joshi, 2012). Linear flow in un-stimulated matrix is not usually seen within the economic life of a well, and is similar to formation linear flow (Joshi, 2012). Boundary dominated flow occurs during the end of well life, when the boundaries of the reservoir are encountered (Joshi, 2012).

2.3 Reserve Estimation:

Prior to the development of DCA models, estimation of oil reserves was accomplished by calculating the contents of a reservoir based on saturation and percentage of recoverable oil over a certain known area (Valko, 2010). This resulted in a very rough estimate of recoverable hydrocarbons (Valko, 2010). Decline Curve Analysis (DCA) models are frequently employed by professionals in the petroleum and natural gas industries to determine production from wells with limited production history. A technique for predicting production using DCA models would allow for rapid determination of production characteristics.

2.4 Decline Theories:

Several of the most useful and popular decline curve analysis techniques are presented below. Each model works best for a specific situation during the life of a well.

2.4.1 Arps Decline Model

The following technique developed by J.J. Arps in 1945 to estimate the reserves from a reservoir was based on the ability of past production to be an indicator of future reservoir performance (Valko, 2010). In addition, the Arps model is based on the observation that a well's loss ratio, shown below, is usually constant with time (Arps, 1945; Fetkovitch *et al.* 1996; Joshi, 2010). During BDF, the three types of decline visible in production rates are Exponential, Hyperbolic, and Harmonic (Kanfar, 2012).

The loss ratio is given as,

$$\frac{1}{D} = -\frac{q}{dq/dt} \dots\dots\dots(1)$$

The loss ratio derivative,

$$n = \frac{d}{dt} \left[-\frac{q}{dq/dt} \right] \dots\dots\dots(2)$$

The three declines have “ n ” values from 0 to 1. A “ n ” value of 0 indicates exponential decline. Values $0 < n < 1$ indicate hyperbolic decline, and a “ n ” value equal to 1 indicates harmonic decline. In 1980, Fetkovitch used models initially presented by Arps in 1945 combined with constant –pressure infinite acting radial flow solutions to form a type curve (Kanfar, 2012).

The Arps hyperbolic model,

$$q = q_0(1 + nDt)^{-1/n} \dots\dots\dots(3)$$

Where:

$$q_0 = \text{intial rate, volume/time}$$

$$D = \text{initial decline rate, 1/time}$$

$$n = \text{constant loss ratio, dimensionless}$$

The hyperbolic model is limited to “ n ” values between 0 and 1 (Kanfar, 2012). It is often seen in shale reservoirs that “ n ” values greater than 1 match data (Kanfar, 2012). During transient linear flow, a “ n ” value of 2 can be observed, while during bi-linear flow, a “ n ” value of 4 can be seen (Kanfar, 2012). Periods of extended bi-linear and linear flow regimes as a result of micro and nano Darcy permeability in shale cause “ n ” values to be greater than 1 (Joshi, 2012).

A cumulative production relationship can also be given by the following (Mishra, 2012):

$$G_p = \frac{q_i^n}{D_i(n-1)} \left(q(t)^{(1-n)} - q_i^{(1-n)} \right) \dots\dots\dots(4)$$

Where:

$$G_p = \text{Cumulative production, volume}$$

2.4.2 Power Law Exponential Decline Model (PLE):

Power Law DCA methods were developed specifically for low permeability reservoirs such as shale (Ilk *et al.* 2008; Seshadri, 2010). This is accomplished by matching early transient data but not overestimating reserves (Ilk *et al.* 2008; Seshadri, 2010). By accounting for the change from transient to boundary dominated flow, the PLE model can match many unconventional formations. PLE was developed by Ilk *et al.* (2008) as a modification of Arps' exponential decline (Seshadri, 2010).

The PLE model,

$$q = q_0 \exp(-D_\infty t - \frac{D_1}{n} t^n) \dots \dots \dots (5)$$

Where:

$$q_0 = \text{intial rate, } \frac{\text{volume}}{\text{time}}$$

$$D = \text{Arps' decline constant, } 1/\text{time}$$

$$D_\infty = \text{loss rato at } (t = \infty), 1/\text{time}$$

$$D_1 = \text{loss rato at } (t = 1), 1/\text{time}$$

$$n = \text{time exponent, dimensionless}$$

During early times, the “ $D_\infty t$ ” term is insignificant. This reduces equation (5) to a power law function which tends to better match early production data. At late times, “ $D_\infty t$ ” becomes dominant and the model matches the loss ratio of exponential decline. The “ D_∞ ” term sets a limit on the lowest point of decline which prevents reserve overestimation (Seshadri, 2010). No

cumulative time relation exists for the PLE model because of the introduction of the “ D_∞ ” term (Okouma, 2012).

2.4.3 Stretched Exponential Decline Model (SEPD):

First proposed by Valko (2009) and Valko and Lee (2010), the SEPD model was developed to fit transient flow regimes (Joshi, 2013; Kanfar, 2012). This method has been shown to work well for production data greater than three years in other studies (Joshi, 2013). SEPD does not consider late time behavior like the PLE model (Kanfar, 2012).

The SEPD model,

$$q = q_0 \exp \left[- \left(\frac{t}{\tau} \right)^n \right] \dots\dots\dots (6)$$

Where:

$$q_0 = \text{intial production rate, volume/time}$$

$$\tau = \left(\frac{n}{D_1} \right)^{1/n} = \text{characteristic time constant, time}$$

$$n = \text{exponent, dimensionless}$$

“ τ ” and “ n ” are shape and scale factors, while “ q_0 ” signifies the start of the production curve (Joshi, 2013). A cumulative-time relationship is given by:

$$Q = \frac{q_0 \tau}{n} \left\{ \Gamma \left[\frac{1}{n} \right] - \Gamma \left[\frac{1}{n}, \left(\frac{\tau}{n} \right)^n \right] \right\} \dots\dots\dots (7)$$

Where:

$$Q = \text{cumulative production, volume}$$

$$\Gamma\left[\frac{1}{n}\right] = \text{complete gamma function}$$

$$\Gamma\left[\frac{1}{n}, \left(\frac{\tau}{n}\right)^n\right] = \text{incomplete gamma function}$$

2.4.4 Duong Model:

The basis for the Duong model is that production rate and time would form a straight line, or power law relationship, when plotted on a log-log scale (Kanfar, 2012; Duong 2011). Long duration linear flow is assumed in this model (Joshi, 2013). Two equations are used to calculate the model parameters (Joshi, 2013). Model parameters “ a ” and “ m ” are determined using the following equation:

$$\frac{q}{G_p} = at^{-m} \dots\dots\dots (8)$$

Where:

$$q = \text{flowrate, volume/time}$$

$$G_p = \text{cumulative gas production}$$

$$a = \text{vertical axis intercept of log - log plot of } \frac{q}{G_p} \text{ vs. } t$$

$$m = \text{slope of log - log plot of } \frac{q}{G_p} \text{ vs. } t$$

A plot of “ q ” versus “ $t(a,m)$ ” is generated next in order to determine model parameters “ q_1 ” and “ q_∞ ”, where:

$$t(a, m) = t^{-m} e^{\frac{a}{1-m}(t^{1-m}-1)} \dots\dots\dots (9)$$

After obtaining a straight line for this plot, the slope corresponds to “ q_1 ” while the intercept corresponds to “ q_∞ ” (Kanfar, 2012). The production rate equation can now be given as:

$$q = q_1 t(a, m) + q_\infty \dots\dots\dots(10)$$

The “ q_∞ ” term was added to the equation to account for data which did not run through the origin on a plot of “ q ” versus “ $t(a, m)$ ”, and can be positive or negative (Kanfar, 2012). A cumulative production formula can be used when “ q_∞ ” is equal to zero (Kanfar, 2012). It is given as:

$$G_p = \frac{q_1 t(a, m)}{a t^{-m}} \dots\dots\dots(11)$$

If the above equation (11) is used when “ q_∞ ” is not equal to zero, cumulative production values will be incorrect due to the increasing effect of “ q_∞ ” in the production rate equation (10) at long production times (Kanfar, 2012).

3. Objective and Methodology

3.1 Objective:

Two main objectives are contained within the scope of the following analysis.

- 1) The first is to determine which decline curve analysis model is most appropriate for simulated Marcellus shale data.
- 2) Using the appropriate DCA method, the second objective is to devise a technique for predicting future Marcellus shale production with limited production history.

3.2 Methodology:

The procedure for developing a technique for predicting future production is divided into three steps.

- 1) Simulate thirty-year production history for a horizontal well completed in a shale formation with seven and thirteen hydraulic fracture stages including and excluding adsorbed using the Eclipse reservoir simulation software, creating four possible reservoir scenarios.
- 2) Apply Arps, PLE, SEPD, and Duong decline curve analysis methods to the simulated thirty-year production history in order determine which model best fits the entire production history.
- 3) Apply the decline curve analysis methods to the truncated simulated data and determine DCA model parameters for different of production time.
- 4) Establish a correlation to predict DCA model parameters for thirty-year production based on the short production history. Use this technique to validate field data.

3.2.1 Development of a Marcellus Shale Reservoir Model:

Generating a production profile to accurately represent production data from the Marcellus shale was accomplished with the use of a commercial reservoir simulator (Eclipse by Schlumberger). Production for a 3000 feet long horizontal well completed in an ultra-low permeability shale formation containing multiple fracture stages was simulated using the dual porosity reservoir model. Table 1 summarizes the basic input into the Eclipse reservoir simulation software. Four thirty-year production profiles were simulated. The first contained seven hydraulic fractures stages (a hydraulic fracture spacing of 500 feet). The second contained thirteen hydraulic fracture stages (a hydraulic fracture spacing of 250 feet). The third and fourth contained seven and thirteen hydraulic fracture stages, respectively, but with the exclusion of adsorbed gas from the model.

Table 1: Eclipse Basic Model Parameters (Belyadi, 2010)

Basic Model Parameters	
Reservoir Parameters	
Depth, ft.	7000
Thickness, ft.	75
Length, ft.	4000
Width, ft.	2000
Initial pressure, psia	3000
Water saturation, fraction	0.15
Temperature, °F	120
Rock Properties	
type	Dual Porosity
Fissure Porosity, frac.	0.005
Matrix Porosity, frac.	0.05
Fissure perm, i, j, k, md	0.002, 0.002, 0.0002
Matrix perm, i, j, k, md	0.0004, 0.0004, 0.00004
Fissure Spacing, σ , 1/ft ²	0.0073
Compressibility, 1/psia	1×10^{-6}
Density, lb/ft ³	150
Hydraulic Fracture Properties	
Half length, ft.	500
Width, in	0.01
Top of Fracture, ft.	7000
Bottom of Fracture, ft.	7075
Permeability, md	2000
Porosity, fraction	0.2
Number of fracture stages	7,13
Stage Spacing, ft.	500, 250
Well Production Controls	
P_{wf} , psia	500
Adsorption	
Diffusion Coefficient, ft ² /day	1
Sorption Time, day	62
Langmuir Pressure, psia	635
Langmuir Concentration, MSCF/ton	0.08899

3.2.2 Application of Decline Curve Analysis Techniques to Simulated Data:

The Arps, PLE, SEPD, and Duong methods were first applied to all four sets of simulated Marcellus shale production profiles. Each of the four scenarios was evaluated for applicability by using the nonlinear-regression method in Microsoft Excel as proposed by Towler (2002). A spreadsheet was set up containing cells for each parameter of every DCA model. For every model, a column with the appropriate production rate equation, q' , the errors squared, $(q-q')^2$, and total errors squared, $(q-q_{\text{average}})^2$, was then generated, where “ q ” corresponds to the flow rate from simulated production data. A cell for the sum of errors squared, S_{SE} , and total errors squared, S_{ST} , was also created. The regression coefficient was also generated in a cell by using the following formula:

$$R^2 = 1 - \frac{S_{SE}}{S_{ST}} \dots\dots\dots(12)$$

By using Microsoft Excel’s built-in Solver function and setting the objective cell containing R^2 equal to one while changing the appropriate variable cells for each DCA model, the best fit to simulated production data could be obtained.

3.2.3 Application of Selected DCA Models to Varying Simulated Production Times:

The simulated production profiles for each case were truncated at different production times to generate a series of production profiles with limited production history. Using the DCA models from step two, the model constants for the production profiles with production times of two, three, four, five, seven, ten, fifteen, twenty, twenty-five, and thirty years were estimated. Tables for each DCA model constant were then compiled to study the trend of each constant as a function of production time. Finally, a predictive method was established to estimate long-term DCA model constants from the constant obtained using the limited production history.

3.2.4 Validation Against Field Data:

After obtaining results with simulated data, it becomes necessary to confirm results with actual field data. Using field data extends the application of the procedure outlined thus far.

Marcellus well production history is not readily available with detailed information on fracture spacing. Utilizing an actual Marcellus horizontal well containing seven or thirteen fracture stages would be optimal. However, this data was not available or known to exist. To accomplish this task, data from a well in Upshur County, West Virginia (Well 1), API number 4709703527, was used containing 4.274 years of production history and eight fracture stages. Well 1 is 3000 feet in horizontal length, thus having a fracture spacing of 429 feet. It is assumed that the reservoir from which Well 1 is producing has adsorbed gas, as this is common in the Marcellus shale.

Therefore, only comparisons to the adsorbed gas scenarios from above of seven and thirteen fractures will be shown (500 feet and 250 feet hydraulic fracture spacing, respectively).

Well 1 data was history matched with a dual porosity model using Eclipse in order to determine production at points in time beyond 4.274 years. Well 1 production and the corresponding Eclipse history match can be seen in Figure 1, Figure 2, and Figure 3.

Figure 1: Well 1 Production

Figure 2: Well 1 History Match (4.277 Years)

Figure 3: Well 1 History Match (30 Years)

4. Results and Discussion

4.1 Development of a Marcellus Shale Reservoir Model:

Data generated from Eclipse for the four scenarios is shown below in Figure 4, Figure 5, Figure 6, and Figure 7. Matching DCA models for each scenario are shown in section 4.2.

Figure 4: 7 Fracs (Adsorbed Gas): q vs. t

Figure 5: 13 Fracs (Adsorbed Gas): q vs. t

Figure 6: 7 Fracs (No Adsorbed Gas): q vs. t

Figure 7: 13 Fracs (No Adsorbed Gas): q vs. t

4.2 Application of Decline Curve Analysis Techniques to Simulated Data:

After generating data from Eclipse and plotting the flow rate as a function of time, appropriate DCA models could be applied to the simulator results. The methodology described above was utilized in generating a comparison between the simulator and DCA model. Matching the appropriate DCA model to all simulated data yielded the results shown below seven fractures with gas adsorption as shown in Figure 8 through Figure 10. A detailed comparison can be seen in Figure 11 through Figure 15. DCA models applied to thirteen fractures having adsorbed gas can be seen in Figure 16 through Figure 18, while a detailed comparison can be seen in Figure 19 through Figure 23. Seven fractures having no adsorbed gas DCA matches can be seen in Figure 24 through Figure 26, while a detailed comparison can be seen in Figure 27 through Figure 31.

Finally, thirteen fractures having no adsorbed gas DCA matches can be seen in Figure 32 through Figure 34, while a detailed comparison can be seen in Figure 35 through Figure 39.

Figure 8: 7 Fracs (Ads. Gas) Arps q vs. t (30 Years)

Figure 9: 7 Fracs (Ads. Gas) PLE q vs. t (30 Years)

Figure 10: 7 Fracs (Ads. Gas) Duong q vs. t (30 Years)

Figure 11: 7 Fracs (Ads. Gas) q vs. t Comparison – Years 0-2

Figure 12: 7 Fracs (Ads. Gas) q vs. t Comparison – Years 2-5

Figure 13: 7 Fracs (Ads. Gas) q vs. t Comparison – Years 5-10

Figure 14: 7 Fracs (Ads. Gas) q vs. t Comparison – Years 10-20

Figure 15: 7 Fracs (Ads. Gas) q vs. t Comparison – Years 20-30

Figure 16: 13 Fracs (Ads. Gas) Arps q vs. t (30 Years)

Figure 17: 13 Fracs (Ads. Gas) PLE q vs. t (30 Years)

Figure 18: 13 Fracs (Ads. Gas) Duong q vs. t (30 Years)

Figure 19: 13 Fracs (Ads. Gas) q vs. t Comparison – Years 0-2

Figure 20: 13 Fracs (Ads. Gas) q vs. t Comparison – Years 2-5

Figure 21: 13 Fracs (Ads. Gas) q vs. t Comparison – Years 5-10

Figure 22: 13 Fracs (Ads. Gas) q vs. t Comparison – Years 10-20

Figure 23: 13 Fracs (Ads. Gas) q vs. t Comparison – Years 20-30

Figure 24: 7 Fracs (No Ads. Gas) Arps q vs. t (30 Years)

Figure 25: 7 Fracs (No Ads. Gas) PLE q vs. t (30 Years)

Figure 26: 7 Fracs (No Ads. Gas) Duong q vs. t (30 Years)

Figure 27: 7 Fracs (No Ads. Gas) q vs. t Comparison – Years 0-2

Figure 28: 7 Fracs (No Ads. Gas) q vs. t Comparison – Years 2-5

Figure 29: 7 Fracs (No Ads. Gas) q vs. t Comparison – Years 5-10

Figure 30: 7 Fracs (No Ads. Gas) q vs. t Comparison – Years 10-20

Figure 31: 7 Fracs (No Ads. Gas) q vs. t Comparison – Years 20-30

Figure 32: 13 Fracs (No Ads. Gas) Arps q vs. t (30 Years)

Figure 33: 13 Fracs (No Ads. Gas) PLE q vs. t (30 Years)

Figure 34: 13 Fracs (No Ads. Gas) Duong q vs. t (30 Years)

Figure 35: 13 Fracs (No Ads. Gas) q vs. t Comparison – Years 0-2

Figure 36: 13 Fracs (No Ads. Gas) q vs. t Comparison – Years 2-5

Figure 37: 13 Fracs (No Ads. Gas) q vs. t Comparison – Years 5-10

Figure 38: 13 Fracs (No Ads. Gas) q vs. t Comparison – Years 10-20

Figure 39: 13 Fracs (No Ads. Gas) q vs. t Comparison – Years 20-30

When comparing Figure 8 through Figure 15, the best match was seen using the Arps, PLE, and Duong models for seven fractures with adsorbed gas. Comparing Figure 16 through Figure 23 for thirteen fractures with adsorbed gas had the best match using the Arps, PLE, and Duong models. In Figure 24 through Figure 31 for seven fractures without adsorbed gas, the best match was found with the Arps, PLE, and Duong models. For thirteen fractures without adsorbed gas, Figure 32 through Figure 39 show the best match with the Arps, PLE, and Duong models.

4.3 Application of Selected DCA Models to Varying Simulated Production Times:

After selecting the best models for each of the four scenarios, production times were varied in each case. For seven fractures with adsorbed gas, the Arps, PLE, and Duong models were analyzed. The Arps DCA model depends on three parameters: “ n ”, “ d_i ”, and “ q_i ”. The PLE model depends on four variables: “ D_∞ ”, “ n ”, “ q_i ”, and “ D_1 ”. Two, three, four, five, seven, ten,

fifteen, twenty, twenty-five, and thirty years of production data were analyzed for the Arps and PLE models. The Duong model also depends on four variables: “ a ”, “ m ”, “ q_1 ”, and “ q_∞ ”. Two, five, seven, ten, eleven, twelve, fifteen, twenty, twenty-five, and thirty years of production data were analyzed for the Duong model. Results using the Arps, PLE, and Duong models are shown in Table 2, Table 3, and Table 4, respectively, for seven fractures with adsorbed gas.

For thirteen fractures with adsorbed gas the Arps, PLE, and Duong models were again compared. Two, three, four, five, seven, ten, fifteen, twenty, twenty-five, and thirty years of production data were analyzed for the Arps and PLE models. Two, five, seven, ten, fifteen, twenty, twenty-five, and thirty years of data were compared with the Duong model. The results for the Arps, PLE, and Duong models are shown in Table 5, Table 6, and Table 7, respectively.

Both seven and thirteen fractures without adsorbed gas were again compared using the Arps, PLE, and Duong models. Two, five, seven, ten, twenty, and thirty years of simulated production data were analyzed with Arps. Two, three, four, five, seven, ten, fifteen, twenty, twenty-five, and thirty years of data were analyzed with the PLE and Duong models. Table 8 through Table 10 show results for seven fractures without adsorbed gas and Table 11 through Table 13 show the results for thirteen fractures without adsorbed gas.

Table 2: 7 Fracs (Ads. Gas) Arps Model

7 Fracs (Adsorbed Gas)				
<i>ARPS</i>	<i>n</i>	<i>d_i</i>	<i>q_i</i>	<i>R²</i>
2 Years	2.144855936	0.023323937	6520.488341	0.998834611
3 Years	2.042163839	0.021646549	6476.017301	0.998576706
4 Years	1.962762569	0.02031234	6437.09575	0.998217044
5 Years	1.903433321	0.019303388	6405.356372	0.997875624
7 Years	1.817050449	0.017824757	6354.676966	0.997286163
10 Years	1.733880926	0.016399972	6300.464878	0.996613984
15 Years	1.647740081	0.014933675	6238.021762	0.995750559
20 Years	1.590509014	0.013972554	6192.740423	0.995014608
25 Years	1.547212709	0.013256256	6156.393177	0.994334127
30 Years	1.519207088	0.012799688	6131.925950	0.993825899

Table 3: 7 Fracs (Ads. Gas) PLE Model

7 Fracs (Adsorbed Gas)					
<i>PLE</i>	<i>D_∞</i>	<i>n</i>	<i>q_i</i>	<i>D₁</i>	<i>R²</i>
2 Years	-0.001060577	0.455555	7587.815433	0.059637167	0.999582646
3 Years	-0.000612477	0.411413	7854.131534	0.065066909	0.999539920
4 Years	-0.000419918	0.387526	8038.112229	0.068504608	0.999507431
5 Years	-0.000351313	0.379033	8108.669242	0.069690837	0.999489516
7 Years	-0.000240756	0.360519	8286.192814	0.072832119	0.999487612
10 Years	-0.000171505	0.34550	8468.215065	0.075866152	0.999496479
15 Years	-0.000116956	0.33014	8696.955258	0.079448935	0.999496872
20 Years	-0.000090539	0.321551	8838.912585	0.081584919	0.999478933
25 Years	-0.000072642	0.31490	8959.725708	0.083346378	0.999452656
30 Years	-0.000062095	0.310592	9043.501566	0.084538706	0.999430138

Table 4: 7 Fracs (Ads. Gas) Duong Model

7 Fracs (Adsorbed Gas)					
<i>Duong</i>	<i>a</i>	<i>m</i>	<i>q₁</i>	<i>q_∞</i>	<i>R²</i>
2 Years	1.0771146892	1.069985	7,116.28	-690.322432	0.999200446
5 Years	1.0929486760	1.07651	6,869.91	-483.457811	0.999496556
7 Years	1.1019813153	1.07970	6,773.57	-413.393955	0.999538557
10 Years	1.1140512051	1.08361	6,662.00	-342.999951	0.999746676
11 Years	1.1172691178	1.084627	6,633.15	-326.152667	0.999528545
12 Years	1.1201824032	1.08560	6,611.08	-310.034424	0.999518890
15 Years	1.1293618969	1.088262	6,539.14	-274.123495	0.999488182
20 Years	1.1397740910	1.091312	6,465.78	-236.301148	0.999439289
25 Years	1.1476539303	1.093538	6,415.76	-212.017110	0.999396050
30 Years	1.1538573351	1.095169	6,374.92	-196.847078	0.999363747

Table 5: 13 Fracs (Ads. Gas) Arps Model

13 Fracs (Adsorbed Gas)				
<i>ARPS</i>	<i>n</i>	<i>d_i</i>	<i>q_i</i>	<i>R²</i>
2 Years	1.422064191	0.017251735	11234.63942	0.997048288
3 Years	1.325103771	0.016051587	11155.88033	0.997134777
4 Years	1.277739321	0.015444181	11112.21741	0.997257213
5 Years	1.252331888	0.015108133	11086.7214	0.9973699
7 Years	1.226070863	0.014749212	11058.26552	0.997549767
10 Years	1.208618895	0.014499816	11037.59537	0.997717969
15 Years	1.19369613	0.014276593	11018.37142	0.997854533
20 Years	1.183097781	0.014112406	11003.79476	0.997903495
25 Years	1.174041049	0.013969303	10990.81183	0.997907167
30 Years	1.167637628	0.013867125	10981.408409	0.997890572

Table 6: 13 Fracs (Ads. Gas) PLE Model

13 Fracs (Adsorbed Gas)					
<i>PLE</i>	D_{∞}	n	q_i	D_1	R^2
2 Years	-0.000409892	0.473872	13168.63809	0.055397864	0.999758742
3 Years	-0.000626938	0.491115	13037.16271	0.053729096	0.999798162
4 Years	-0.000761042	0.502337	12951.86106	0.052582096	0.999814484
5 Years	-0.000827258	0.508167	12906.73397	0.051957722	0.999826135
7 Years	-0.000849465	0.510194	12891.02573	0.051734013	0.999844835
10 Years	-0.000774572	0.502046	12963.14091	0.052763787	0.999844686
15 Years	-0.000618370	0.481849	13168.09880	0.055649911	0.999752186
20 Years	-0.000501011	0.463693	13382.83652	0.058576838	0.999589374
25 Years	-0.000417719	0.448829	13582.56459	0.061209155	0.999403020
30 Years	-0.000368812	0.439178	13725.22332	0.063034074	0.999255790

Table 7: 13 Fracs (Ads. Gas) Duong Model

13 Fracs (Adsorbed Gas)					
<i>Duong</i>	a	m	q_1	q_{∞}	R^2
2 Years	1.055818401	1.05438086	15,370.37	-3,978.723716	0.999063438
3 Years	1.103435989	1.07418246	13,413.14	-2,274.862123	0.998695403
4 Years	1.14010187	1.08754631	12,511.28	-1,563.759830	0.998280884
5 Years	1.167973398	1.0970169	11,997.09	-1185.253278	0.997908205
7 Years	1.211459104	1.11065168	11,364.61	-774.743511	0.997289043
10 Years	1.254029268	1.12295323	10,873.86	-502.531506	0.996663122
15 Years	1.294907790	1.1340316	10,479.17	-317.430071	0.996101843
20 Years	1.318141938	1.13998117	10,275.13	-238.005781	0.995832166
25 Years	1.334348771	1.143967	10,143.83	-193.746928	0.995685513
30 Years	1.343002362	1.14606886	10,074.80	-171.696022	0.995611151

Table 8: 7 Fracs (No Ads. Gas) Arps Model

7 Fracs (No Adsorbed Gas)				
ARPS	n	d_i	q_i	R^2
2 Years	2.149795050	0.02920	6695.836929	0.996567215
5 Years	1.870119847	0.023225	6549.526835	0.995936693
7 Years	1.778589557	0.021287	6492.100706	0.995413841
10 Years	1.692905226	0.019494	6433.075136	0.994805695
20 Years	1.546688743	0.016508	6318.684594	0.993204363
30 Years	1.473641425	0.015075	6254.665180	0.991905457

Table 9: 7 Fracs (No Ads. Gas) PLE Model

7 Fracs (No Adsorbed Gas)					
PLE	D_∞	n	q_i	D_1	R^2
2 Years	-0.000565458	0.379411800	8380.688847	0.077233471	0.998813822
3 Years	-0.000358894	0.358422535	8582.247962	0.080325179	0.999022764
4 Years	-0.000251029	0.344871266	8736.963321	0.082614841	0.999126327
5 Years	-0.000198069	0.337175861	8835.187418	0.084031512	0.999191823
7 Years	-0.000149211	0.328941520	8951.357620	0.085680948	0.999278277
10 Years	-0.000119280	0.322889834	9045.995295	0.087011798	0.999350297
15 Years	-0.000091291	0.315982957	9166.027226	0.088684431	0.999402231
20 Years	-0.000071093	0.310079017	9278.875013	0.090228949	0.999413703
25 Years	-0.000055694	0.305002022	9383.508828	0.091630964	0.999406587
30 Years	-0.000046167	0.301581887	9458.065489	0.092611844	0.999395231

Table 10: 7 Fracs (No Ads. Gas) Duong Model

7 Fracs (No Adsorbed Gas)					
<i>Duong</i>	<i>a</i>	<i>m</i>	<i>q₁</i>	<i>q_∞</i>	<i>R²</i>
2 Years	1.0048595176	1.049280450	8,097.03	-1,289.889330	0.998087228
3 Years	1.0153609167	1.055426159	7,761.95	-976.961800	0.998410712
4 Years	1.0253061164	1.060158962	7,560.73	-802.347290	0.998580933
5 Years	1.0311745608	1.062961556	7,453.50	-708.306389	0.998665628
7 Years	1.0438532564	1.068349330	7,277.16	-564.278320	0.998720680
10 Years	1.0604132400	1.074675705	7,102.19	-433.689099	0.998689170
15 Years	1.0782662614	1.081012306	6,947.58	-329.820729	0.998588041
20 Years	1.0902598708	1.084966637	6,859.65	-277.707437	0.998505073
25 Years	1.0994828702	1.087867757	6,798.75	-245.001347	0.998436188
30 Years	1.1055616221	1.089715269	6,760.70	-226.394471	0.998384385

Table 11: 13 Fracs (No Ads. Gas) Arps Model

13 Fracs (No Adsorbed Gas)				
<i>ARPS</i>	<i>n</i>	<i>d_i</i>	<i>q_i</i>	<i>R²</i>
2 Years	1.374379279	0.020422	11471.82352	0.994176469
5 Years	1.206277136	0.017818	11314.63079	0.995422964
7 Years	1.185086745	0.017469	11290.7255	0.995808338
10 Years	1.172206623	0.017247	11274.99281	0.996127757
20 Years	1.149794019	0.016840	11244.79385	0.996479636
30 Years	1.133843025	0.016540	11221.65087	0.996501102

Table 12: 13 Fracs (No. Ads Gas) PLE Model

13 Fracs (No Adsorbed Gas)					
PLE	D_{∞}	n	q_i	D_1	R^2
2 Years	0.000406027	0.384232990	14601.31366	0.073366475	0.998927468
3 Years	-0.000115679	0.429922235	14027.84804	0.067655535	0.999034412
4 Years	-0.000450412	0.459029456	13714.86083	0.064006382	0.999073103
5 Years	-0.000647232	0.476182672	13543.93886	0.061842302	0.999101520
7 Years	-0.000814733	0.491106361	13400.48432	0.059924980	0.999163291
10 Years	-0.000810162	0.490497714	13407.46653	0.060024438	0.999230991
15 Years	-0.000665095	0.473781593	13594.29060	0.062601177	0.999210144
20 Years	-0.000540371	0.456882643	13810.76346	0.065499473	0.999083931
25 Years	-0.000451017	0.443109423	14008.96447	0.068066815	0.998924400
30 Years	-0.000398608	0.434266780	14147.54847	0.069813027	0.998797460

Table 13: 13 Fracs (No. Ads Gas) Duong Model

13 Fracs (No Adsorbed Gas)					
Duong	a	m	q_1	q_{∞}	R^2
2 Years	1.0055464162	1.039769764	17,421.52	-5,413.112895	0.997826101
3 Years	1.0450989975	1.061640607	14,603.78	-2,776.432774	0.997233777
4 Years	1.0777849559	1.076368221	13,522.86	-1,830.089152	0.996580366
5 Years	1.1049010056	1.087271964	12,922.21	-1343.569673	0.996005728
7 Years	1.1449713816	1.102045474	12,268.73	-859.668671	0.995083570
10 Years	1.1844416155	1.115341983	11,786.57	-548.427933	0.994208223
15 Years	1.2211160591	1.126867124	11,425.28	-347.036449	0.993493461
20 Years	1.2422541230	1.133099165	11,242.47	-262.052133	0.993179995
25 Years	1.2559344364	1.137017436	11,133.71	-215.908279	0.993015426
30 Years	1.2640243273	1.139277328	11,071.21	-191.932125	0.992930736

The Duong and Arps models were selected as the best correlated match with simulated data for both seven fractures with and without adsorbed gas (500 feet hydraulic fracture spacing) and thirteen fractures with and without adsorbed gas (250 feet hydraulic fracture spacing). This

was due to the trends of constants plotted versus time as shown in Appendix Figure 58 through Figure 101. Each of the three DCA models matched limited simulated production history accurately. However, when the values of constants obtained for varying production histories were plotted versus time, trends for some models were not visible until later in the production history.

For seven fractures with adsorbed gas, the best trend in simulated data occurs after two years of production history using the Arps and PLE models, while the Duong model trends well with all simulated production history. For thirteen fractures with adsorbed gas, Arps has a trend after a production history of two years, while Duong trends well with all simulated production data and the PLE model has a trend after five years.

Simulated data for seven fractures without adsorbed gas most closely correlated with the Arps and Duong models using all production history, and after two years using the PLE model. Values of “ n ” in the Arps model for seven fractures without adsorbed gas begin at approximately two, indicating linear flow. When applying the Arps model to thirteen fractures without adsorbed gas, a better trend is obtained after two years of simulated production data. The “ n ” values approach a harmonic decline ($n=1$) as time progresses. The PLE model had a trend for the thirteen fractures without adsorbed gas model after five years of production history, while the Duong correlated well with all production history.

4.4 Validation Against Field Data:

The Well 1 data was first matched to each DCA model and values of constants were obtained for this time interval. Next, the correlations derived above were applied at the Well 1 production history time interval of 4.274 years and predicted into the future. Each of the four scenarios yielded significantly different constant values from the actual Well 1 DCA match and

future predictions were inaccurate. In order to overcome this barrier, a dimensionless ratio was used. This allowed for the conversion of constants in each of the four DCA models obtained by simulated data to the Well 1 data. By using the DCA constant values obtained in Table 2 through Table 13, a ratio was obtained by using the following formula:

$$Ratio = \frac{Value\ of\ DCA\ constant\ at\ t = Available\ production\ history}{Value\ of\ DCA\ constant\ at\ t = 30\ years}(13)$$

The ratio was then plotted versus time on a log-log scale for each DCA constant. Log-log plots have successfully been used as a dimensionless conversion between sets of data in other petroleum engineering applications. Reservoir parameters affect the value of dimensionless numbers, such as permeability and size. In this analysis, the reservoir parameters were kept constant in all simulated data. A correlation was then derived for this log-log plot. By entering the production history of Well 1 (4.274 years) into this log-log correlation, a value of the ratio at this point in time was obtained. Dividing the actual Well 1 value of the constant obtained via the nonlinear-regression method in Microsoft Excel at 4.274 years by the value of the ratio at 4.274 years, a predicted value was calculated which was able to determine future reservoir performance. Table 14 through Table 16 show the results for ratio calculations derived from seven fractures having adsorbed gas. Thirteen fractures with adsorbed gas can be found in Table 35 through Table 40, located in the Appendix. Well 1 is an eight fracture horizontal well (a hydraulic fracture spacing of 429 feet) with adsorbed gas, which is most similar to a seven fracture well (a hydraulic fracture spacing of 500 feet) with adsorbed gas.

Table 14: 7 Fracs (Ads. Gas) Arps Table of Ratios

7 Fracs (Adsorbed Gas) Table of Ratios			
<i>ARPS</i>	<i>n</i>	<i>d_i</i>	<i>q_i</i>
2 Years	1.411825914	1.822226936	1.063367104
3 Years	1.344230063	1.691177835	1.056114727
4 Years	1.291965121	1.586940188	1.049767366
5 Years	1.252912348	1.508113884	1.044591279
7 Years	1.196051850	1.392593016	1.036326436
10 Years	1.141306501	1.281279046	1.027485480
15 Years	1.084605314	1.166721769	1.017302201
20 Years	1.046933645	1.091632373	1.009917679
25 Years	1.018434367	1.035670180	1.003990138
All	1.000000000	1.000000000	1.000000000

Table 15: 7 Fracs (Ads. Gas) PLE Table of Ratios

7 Fracs (Adsorbed Gas) Table of Ratios				
<i>PLE</i>	<i>D_∞</i>	<i>n</i>	<i>q_i</i>	<i>D₁</i>
2 Years	17.07980826	1.466729	0.839035121	0.705442158
3 Years	9.863485554	1.324607	0.868483460	0.769670035
4 Years	6.762465855	1.24770	0.888827427	0.810334237
5 Years	5.657635636	1.220355	0.896629384	0.824366027
7 Years	3.877190089	1.160747	0.916259344	0.861523940
10 Years	2.761965094	1.112385	0.936386753	0.897413208
15 Years	1.883486136	1.062935	0.961680074	0.939793602
20 Years	1.458063288	1.035283	0.977377238	0.965059942
25 Years	1.169850138	1.013872	0.990736347	0.985896065
All	1.000000000	1.00000	1.000000000	1.000000000

Table 16: 7 Fracs (Ads. Gas) Duong Table of Ratios

7 Fracs (Adsorbed Gas) Table of Ratios				
Duong	a	m	q_1	q_∞
2 Years	0.933490351	0.97700	1.116292700	3.506897020
5 Years	0.947213007	0.982959	1.077645898	2.456007042
7 Years	0.955041219	0.985877	1.062533762	2.100076659
10 Years	0.965501688	0.989445	1.045032798	1.742469097
11 Years	0.968290519	0.990374	1.040505833	1.656883453
12 Years	0.970815342	0.991262	1.037045089	1.575001402
15 Years	0.978770826	0.993693	1.025760197	1.392570808
20 Years	0.987794640	0.996478	1.014251783	1.200430051
25 Years	0.994623768	0.998511	1.006405524	1.077065060
All	1.000000000	1.00000	1.000000000	1.000000000

Figure 40 through Figure 50 below show plots of the dimensionless ratio versus time when plotted on log-log scale for every constant of the Arps, PLE, and Duong DCA models having seven fractures with adsorbed gas. Thirteen fractures with adsorbed gas can be found in Figure 102 through Figure 112, located in the Appendix.

Figure 40: 7 Fracs (Ads. Gas) Arps Ratio of n vs. Time

Figure 41: 7 Fracs (Ads. Gas) Arps Ratio of d_i vs. Time

Figure 42: 7 Fracs (Ads. Gas) Arps Ratio of q_i vs. Time

Figure 43: 7 Fracs (Ads. Gas) PLE Ratio of D_{∞} vs. Time

Figure 44: 7 Fracs (Ads. Gas) PLE Ratio of n vs. Time

Figure 45: 7 Fracs (Ads. Gas) PLE Ratio of q_i vs. Time

Figure 46: 7 Fracs (Ads. Gas) PLE Ratio of D_1 vs. Time

Figure 47: 7 Fracs (Ads. Gas) Duong Ratio of a vs. Time

Figure 48: 7 Fracs (Ads. Gas) Duong Ratio of m vs. Time

Figure 49: 7 Fracs (Ads. Gas) Duong Ratio of q_1 vs. Time

Figure 50: 7 Fracs (Ads. Gas) Duong Ratio of q_∞ vs. Time

The following results in Table 17 through Table 19 show values of the log-log ratio when computed at 4.274 years of production history using the Arps, PLE, and Duong DCA models derived from seven fractures with adsorbed gas (a hydraulic fracture spacing of 500 feet). Log-

log ratios evaluated at 4.274 years for thirteen fractures with gas adsorption (a hydraulic fracture spacing of 250 feet) can be seen in Table 38 through Table 40, located in the Appendix.

Table 17: 7 Fracs (Ads. Gas) Arps Well 1 Ratio Values

7 Fracs (Adsorbed Gas) Arps Well 1 Ratio Values			
Well 1 (Years)	n	d_i	q_i
4.273972603	1.279229065	1.553561067	1.047229074

Table 18: 7 Fracs (Ads. Gas) PLE Well 1 Ratio Values

7 Fracs (Adsorbed Gas) PLE Well 1 Ratio Values				
Well 1 (Years)	D_∞	n	q_i	D_1
4.273972603	6.86859833	1.265470183	0.88697	0.801196

Table 19: 7 Fracs (Ads. Gas) Duong Well 1 Ratio Values

7 Fracs (Adsorbed Gas) Duong Well 1 Ratio Values				
Well 1 (Years)	q_∞	a	q_1	m
4.273972603	2.563988501	0.946399711	1.082766	0.981634

The actual values of constants using the Arps, PLE, and Duong models were obtained and are shown graphically in Figure 51 through Figure 53.

Figure 51: Well 1 Arps Actual Match

Figure 52: Well 1 PLE Actual Match

Figure 53: Well 1 Duong Actual Match

Actual values are shown numerically in Table 20 through Table 22 along with the predicted values. As stated previously, the predicted values are obtained by dividing the actual value by the ratio value from Table 17 through Table 19 for seven fractures with adsorbed gas. Corresponding tables are shown for thirteen fractures with adsorbed gas in Table 41 through Table 43, found in the Appendix.

Table 20: 7 Fracs (Ads. Gas) Arps Well 1 Actual and Predicted Values

Well 1	Actual	Predicted
n	1.726571773	1.34969711
d_i	0.004360324	0.002806664
q_i	2,171.430282	2073.500761

Table 21: 7 Fracs (Ads. Gas) PLE Well 1 Actual and Predicted Values

Well 1	Actual	Predicted
D_{∞}	-0.000382914	-0.000055749
n	0.506990695	0.400634248
q_i	2627.174227	2961.966806
D_1	0.027735762	0.03461797

Table 22: 7 Fracs (Ads. Gas) Duong Well 1 Actual and Predicted Values

Well 1	Actual	Predicted
a	1.482659542	1.56663144
m	1.113649238	1.134484704
q_1	1,414.02	1305.935384
q_{∞}	-148.7245961	-58.00517283

Predicted values were then extrapolated to thirty years of production. The predictions can be seen graphically in Figure 54 through Figure 56 for the Arps, PLE, and Duong models derived from seven with adsorbed gas. Predictions for thirteen fractures with adsorbed gas be seen in Figure 113 through Figure 115, located in the Appenidx.

Figure 54: Well 1 - 7 Fracs (Ads. Gas) Arps Predicted Match

Figure 55: Well 1 - 7 Fracs (Ads. Gas) PLE Predicted Match

Figure 56: Well 1 - 7 Fracs (Ads. Gas) Duong Predicted Match

To confirm predicted results for Well 1 (hydraulic fracture spacing of 429 feet) obtained via the method of ratios described above, the Well 1 history match data generated from Eclipse earlier was used (also a hydraulic fracture spacing of 429 feet). Comparing the predicted values for Well 1 to the simulated data will enable the determination of which DCA model matches most closely. Figure 57 below shows the comparison between Well 1 predicted values and the Eclipse simulator data from 4.274 years until 30 years of production history. The Well 1 predicted values for thirteen fractures with adsorbed gas can be seen in Figure 116, located in the Appenix.

Figure 57: 7 Fracs (Ads. Gas) 30 Year Well 1 Predicted Comparison

As Figure 57 and Figure 116 show, the best match for Well 1 (429 feet hydraulic fracture spacing) to the Eclipse simulator data (429 feet fracture spacing) occurs with the Arps DCA model derived by the method of ratios from seven fractures and adsorbed gas (500 feet fracture spacing). The PLE and Duong models for both seven and thirteen fractures having adsorbed gas overestimated and underestimated, respectively, when compared to the Eclipse data. The Well 1 predicted value using Arps for thirteen fractures having adsorbed gas also overestimates simulator data to a greater extent than Arps for seven fractures having adsorbed gas.

The results obtained confirm the idea that hydraulic fracture spacing in field data should be closer to correlations and ratios derived from simulator data nearest to that spacing value. In the analysis performed above, the hydraulic fracture spacing of Well 1 was given as 429 feet, which is much closer to the value of the initial Eclipse reservoir model having seven fractures

and a hydraulic fracture spacing of 500 feet than thirteen fracture Eclipse model with hydraulic fracture spacing of 250 feet.

The Arps model when applied to thirteen fractures (a hydraulic fracture spacing of 250 feet) initially matches the additional Eclipse simulator data. However, shortly thereafter the prediction varies significantly. The decrease in spacing between fractures attributes to interference between them, resulting in deviation from simulator predictions. A thirteen fracture reservoir (a hydraulic fracture spacing of 250 feet) will produce gas at a greater rate initially than a seven fracture (a hydraulic fracture spacing of 500 feet) or an eight fracture reservoir (a hydraulic fracture spacing of 429 feet), as seen in both Well 1 and its Eclipse simulator data. After this point, as reservoir boundaries are slowly approached, deviations between a seven, eight, and thirteen fracture reservoir begin to diminish.

5. Conclusions and Recommendations

- 1) The technique of developing a correlation based on dual porosity simulated production history, applying the method of ratios to simulated data, and confirming results with field and separate dual porosity simulator data proved successful in this analysis. Limited Well 1 Marcellus data (a hydraulic fracture spacing of 429 feet) matched most closely with the correlations and ratios derived from seven fractures having adsorbed gas (a hydraulic fracture spacing of 500 feet) using the Arps DCA model than other models for both seven and thirteen fractures (a hydraulic fracture spacing of 250 feet).
- 2) Additional analysis could be performed on actual field production data. The limited availability of Marcellus horizontal well production history is at present a hindrance to histories greater than approximately six years. With time, more production history will enhance and broaden the scope of results and analysis presented within this report.
- 3) Further investigation of actual well data obtained from a geographically diverse area within the Marcellus boundary would increase the applicability of methodologies presented to a wider geological realm. Varying inherent properties of the Marcellus shale will undoubtedly cause results obtained via the method of ratios to differ slightly when applied to different geographical locations. Which specific Marcellus characteristics affect these differences remains to be seen.
- 4) Adjustments to simulator data when obtaining correlations would also have significant impact in results obtained. Narrowing and defining model parameters to more accurately simulate Marcellus wells would improve accuracy. Changing the length of the horizontal fracture spacing and reservoir area dimensions are two such examples.

6. References

Arps, J.J. 1945. Analysis of Decline Curves. Trans. AIME **160**: 228-247.

Belvalkar, R. and Oyewole, S. Development of Marcellus Shale in Pennsylvania. Paper 134852-MS presented at the SPE Annual Technical Conference and Exhibition, Florence, Italy, 19-22 September 2010. doi: 10.2118/134852-MS.

Belyadi, A *et. al.* 2010. Performance of the Hydraulically Fractured Horizontal Wells in Low Permeability Formation. Paper 139082-MS presented at the SPE Eastern Regional Meeting, Morgantown, West Virginia, USA, 12-14 October 2010. doi: 10.2118/139082-MS.

Duong, A.N. 2011. Rate-Decline Analysis for Fracture-Dominated Shale Reservoirs. SPE Reservoir Evaluation & Engineering (3): pp. 377-387. doi: 10.2118/137748-PA.

Fetkovitch, M.J., Fetkovitch, E.J., and Fetkovitch, M.D. 1996. Useful Concepts for Decline Curve Forecasting, Reserve Estimation, and Analysis. *SPE Reservoir Engineering* **11** (1): 13-22.

Hassett, K.A. and Mathur, A. 2013. *Benefits of Hydraulic Fracking*. Oxford Energy Forum. February 2013. pp. 11-13.

Ilk, D. *et. al.* 2008. Exponential vs. Hyperbolic Decline in Tight Gas Sands: Understanding the Origin and Implications for Reserve Estimates Using Arps' Decline Curves. Paper 116731-MS presented at the SPE Annual Technical Conference and Exhibition, Denver, Colorado, USA, 21-24 September 2008. doi: 10.2118/116731-MS.

Joshi, K. and Lee, J. 2013. Comparison of Various Deterministic Forecasting Techniques in Shale Gas Reservoirs. Paper 163870-MS presented at the 2013 SPE Hydraulic Fracturing Technology Conference, The Woodlands, Texas, USA, 04-06 February 2013. doi: 10.2118/163870-MS.

Joshi, K.J. 2012. Comparison of Various Deterministic Forecasting Techniques in Shale Gas Reservoirs with Emphasis on the Duong Method. MS Thesis, Department of Petroleum Engineering, Texas A&M University.

Kanfar, M.S. and Wattenbarger, R.A. 2012. Comparison of Empirical Decline Curve Methods for Shale Wells. Paper SPE 162648-MS presented at the SPE Canadian Unconventional Resources Conference, Calgary, Alberta, Canada, 30 October-01 November 2012. doi: 10.2118/162648-MS.

Mishra, S. 2012. A New Approach to Reserves Estimation in Shale Gas Reservoirs Using Multiple Decline Curve Analysis Models. Paper 161092-MS presented at the SPE Eastern Regional Meeting, Lexington, Kentucky, USA, 03-05 October 2012. doi: 10.2118/161092-MS.

Okouma, V. *et. al.* 2012. Practical Considerations for Decline Curve Analysis in Unconventional Reservoirs --- Application of Recently Developed Rate-Time Relations. Paper 162910-MS presented at the SPE Hydrocarbon Economics and Evaluation Symposium, Calgary, Alberta, Canada, 24-25 September 2012. doi: 10.2118/162910-MS.

Seshadri, J. and Mattar, L. 2010. Comparison of Power Law and Modified Hyperbolic Decline Methods. Paper 137320-MS presented at the Canadian Unconventional Resources and International Petroleum Conference, Calgary, Alberta, Canada, 19-21 October 2010. doi: 10.2118/137320-MS.

Towler, Brian F. 2002. *Fundamental Principles of Reservoir Engineering*. Richardson, Texas: Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers.

Valko, P.P. 2009. Assigning Value to Stimulation in the Barnett Shale: A Simultaneous Analysis of 7000 Plus Production Histories and Well Completion Records. Paper 119369-MS presented at the SPE Hydraulic Fracturing Technology Conference, The Woodlands, Texas, USA, 19-21 January 2009. doi: 10.2118/119369-MS.

Valko, P.P. and Lee, W.J. 2010. A Better Way to Forecast Production from Unconventional Gas Wells. Paper SPE 134231 presented at the SPE Annual Technical Conference and Exhibition, Florence, Italy, 19-22 September. doi: 10.2118/134231-MS.

7. Appendix

7.1 Correlations and Simulator Match

As stated previously, using the appropriate models for each of the four scenarios, the constants in each equation were plotted as a function of time. A linear regression analysis was performed to determine trends. For seven fractures with adsorbed gas, the correlations were obtained in Figure 58 to Figure 60 using Arps, in Figure 61 to Figure 64 using PLE, and in Figure 65 to Figure 68 using Duong.

For thirteen fractures with adsorbed gas, the correlations were obtained as shown in Figure 69 through Figure 71 using Arps, in Figure 72 through Figure 75 using PLE, and in Figure 76 through Figure 79 using Duong.

Using the Arps model for seven fractures and without adsorbed gas, the correlations were obtained in Figure 80 through Figure 82 using Arps, in Figure 83 through Figure 86 using PLE, and in Figure 87 through Figure 90 using Duong.

Finally, using the Arps model for thirteen fractures and without adsorbed gas, the following correlations were obtained in Figure 91 through Figure 93 using Arps, in Figure 94 through Figure 97 using PLE, and in Figure 98 through Figure 101 using Duong.

Figure 58: 7 Fracs (Ads. Gas) Arps n Correlation

Figure 59: 7 Fracs (Ads. Gas) Arps d_i Correlation

Figure 60: 7 Fracs (Ads. Gas) Arps q_i Correlation

Figure 61: 7 Fracs (Ads. Gas) PLE D_∞ Correlation

Figure 62: 7 Fracs (Ads. Gas) PLE n Correlation

Figure 63: 7 Fracs (Ads. Gas) PLE q_i Correlation

Figure 64: 7 Fracs (Ads. Gas) PLE D_1 Correlation

Figure 65: 7 Fracs (Ads. Gas) Duong a Correlation

Figure 66: 7 Fracs (Ads. Gas) Duong m Correlation

Figure 67: 7 Fracs (Ads. Gas) Duong q_1 Correlation

Figure 68: 7 Fracs (Ads. Gas) Duong q_{∞} Correlation

Figure 69: 13 Fracs (Ads. Gas) Arps n Correlation

Figure 70: 13 Fracs (Ads. Gas) Arps d_i Correlation

Figure 71: 13 Fracs (Ads. Gas) Arps q_i Correlation

Figure 72: 13 Fracs (Ads. Gas) PLE D_{∞} Correlation

Figure 73: 13 Fracs (Ads. Gas) PLE n Correlation

Figure 74: 13 Fracs (Ads. Gas) PLE q_i Correlation

Figure 75: 13 Fracs (Ads. Gas) PLE D_1 Correlation

Figure 76: 13 Fracs (Ads. Gas) Duong a Correlation

Figure 77: 13 Fracs (Ads. Gas) Duong m Correlation

Figure 78: 13 Fracs (Ads. Gas) Duong q_1 Correlation

Figure 79: 13 Fracs (Ads. Gas) Duong q_∞ Correlation

Figure 80: 7 Fracs (No Ads. Gas) Arps n Correlation

Figure 81: 7 Fracs (No Ads. Gas) Arps d_i Correlation

Figure 82: 7 Fracs (No Ads. Gas) Arps q_i Correlation

Figure 83: 7 Fracs (No Ads. Gas) PLE D_∞ Correlation

Figure 84: 7 Fracs (No Ads. Gas) PLE n Correlation

Figure 85: 7 Fracs (No Ads. Gas) PLE q_i Correlation

Figure 86: 7 Fracs (No Ads. Gas) PLE D_1 Correlation

Figure 87: 7 Fracs (No Ads. Gas) Duong a Correlation

Figure 88: 7 Fracs (No Ads. Gas) Duong m Correlation

Figure 89: 7 Fracs (No Ads. Gas) Duong q_1 Correlation

Figure 90: 7 Fracs (No Ads. Gas) Duong q_{∞} Correlation

Figure 91: 13 Fracs (No Ads. Gas) Arps n Correlation

Figure 92: 13 Fracs (No Ads. Gas) Arps d_i Correlation

Figure 93: 13 Fracs (No Ads. Gas) Arps q_i Correlation

Figure 94: 13 Fracs (No Ads. Gas) PLE D_{∞} Correlation

Figure 95: 13 Fracs (No Ads. Gas) PLE n Correlation

Figure 96: 13 Fracs (No Ads. Gas) PLE q_i Correlation

Figure 97: 13 Fracs (No Ads. Gas) PLE D_1 Correlation

Figure 98: 13 Fracs (No Ads. Gas) Duong a Correlation

Figure 99: 13 Fracs (No Ads. Gas) Duong m Correlation

Figure 100: 13 Fracs (No Ads. Gas) Duong q_1 Correlation

Figure 101: 13 Fracs (No Ads. Gas) Duong q_∞ Correlation

Using the correlations obtained, predicting the values of constants at a future point in time was performed to confirm their accuracy to simulated Marcellus shale data. Thirty years of simulated production data was chosen. Table 23, Table 24, and Table 25 show results for the Arps, PLE, and Duong methods with seven fractures with adsorbed gas. Table 26, Table 27, and

Table 28 show results for the Arps, PLE, and Duong methods with thirteen fractures with adsorbed gas. Table 29, Table 30, and Table 31 show results for the Arps, PLE, and Duong methods with seven fractures and without adsorbed gas. Finally, Table 32, Table 33, and Table 34 show results for the Arps, PLE, and Duong methods with thirteen fractures without adsorbed gas.

Table 23: Simulator Predicted 7 Fracs (Ads. Gas) Arps

Simulator Predicted 7 Fracs (Adsorbed Gas)				
<i>ARPS</i>	<i>n</i>	<i>d_i</i>	<i>q_i</i>	<i>R² (30 yr. Match)</i>
30 Years	1.500628194	0.01233533	6131.858716	0.993757754

Table 24: Simulator Predicted 7 Fracs (Ads. Gas) PLE

Simulator Predicted 7 Fracs (Adsorbed Gas)					
<i>PLE</i>	<i>D_∞</i>	<i>n</i>	<i>q_i</i>	<i>D₁</i>	<i>R² (30 Yr. Match)</i>
30 Years	-0.000019761	0.30435	9046.679166	0.087700000	0.994403338

Table 25: Simulator Predicted 7 Fracs (Ads. Gas) Duong

Simulator Predicted 7 Fracs (Adsorbed Gas)					
<i>Duong</i>	<i>a</i>	<i>m</i>	<i>q₁</i>	<i>q_∞</i>	<i>R² (30 Yr. Match)</i>
30 Years	1.149915562	1.09489	6355.992317	-160.9148911	0.998362788

Table 26: Simulator Predicted 13 Fracs (Ads. Gas) Arps

Simulator Predicted 13 Fracs (Adsorbed Gas)				
<i>ARPS</i>	<i>n</i>	<i>d_i</i>	<i>q_i</i>	<i>R² (30 yr. Match)</i>
30 Years	1.15532696	0.013538922	10973.229	0.997865002

Table 27: Simulator Predicted 13 Fracs (Ads Gas.) PLE

Simulator Predicted 13 Fracs (Adsorbed Gas)					
<i>PLE</i>	<i>D_∞</i>	<i>n</i>	<i>q_i</i>	<i>D₁</i>	<i>R² (30 Yr. Match)</i>
30 Years	-0.000479641	0.44143893	13,663.13	0.063000	0.953070029

Table 28: Simulator Predicted 13 Fracs (Ads. Gas) Duong

Simulator Predicted 13 Fracs (Adsorbed Gas)					
<i>Duong</i>	<i>a</i>	<i>m</i>	<i>q₁</i>	<i>q_∞</i>	<i>R² (30 Yr. Match)</i>
30 Years	1.358446683	1.149853	9,917.55	-54.314969	0.991845400

Table 29: Simulator Predicted 7 Fracs (No Ads. Gas) Arps

Simulator Predicted 7 Fracs (No Adsorbed Gas)				
<i>ARPS</i>	<i>n</i>	<i>d_i</i>	<i>q_i</i>	<i>R² (30 yr. Match)</i>
30 Years	1.443303049	0.014994	6254.064108	0.991649027

Table 30: Simulator Predicted 7 Fracs (No Ads. Gas) PLE

Simulator Predicted 7 Fracs (No Adsorbed Gas)					
<i>PLE</i>	<i>D_∞</i>	<i>n</i>	<i>q_i</i>	<i>D₁</i>	<i>R² (30 Yr. Match)</i>
30 Years	-0.000059880	0.29867	9442.84519	0.092505987	0.998147686

Table 31: Simulator Predicted 7 Fracs (No Ads. Gas) Duong

Simulator Predicted 7 Fracs (No Adsorbed Gas)					
<i>Duong</i>	<i>a</i>	<i>m</i>	<i>q₁</i>	<i>q_∞</i>	<i>R² (30 Yr. Match)</i>
30 Years	1.105114351	1.09099	6663.113279	-120.315770658	0.990487269

Table 32: Simulator Predicted 13 Fracs (No Ads. Gas) Arps

Simulator Predicted 13 Fracs (No Adsorbed Gas)				
<i>ARPS</i>	<i>n</i>	<i>d_i</i>	<i>q_i</i>	<i>R² (30 yr. Match)</i>
30 Years	1.134254499	0.016419	11222.19857	0.996492721

Table 33: Simulator Predicted 13 Fracs (No Ads. Gas) PLE

Simulator Predicted 13 Fracs (No Adsorbed Gas)					
<i>PLE</i>	<i>D_∞</i>	<i>n</i>	<i>q_i</i>	<i>D₁</i>	<i>R² (30 Yr. Match)</i>
30 Years	-0.000479641	0.43785	14072.76227	0.068988621	0.996208105

Table 34: Simulator Predicted 13 Fracs (No Ads. Gas) Duong

Simulator Predicted 13 Fracs (No Adsorbed Gas)					
<i>Duong</i>	<i>a</i>	<i>m</i>	<i>q₁</i>	<i>q_∞</i>	<i>R² (30 Yr. Match)</i>
30 Years	1.284517032	1.14876	10494.62572	533.4697226	0.820740249

7.2 Well 1 - 13 Fracs (Adsorbed Gas) Predictions

Table 35: 13 Fracs (Ads. Gas) Arps Table of Ratios

13 Fracs (Adsorbed Gas) Table of Ratios			
<i>ARPS</i>	<i>n</i>	<i>d_i</i>	<i>q_i</i>
2 Years	1.217898565	1.244074369	1.023059976
3 Years	1.134858744	1.157528115	1.015887936
4 Years	1.094294403	1.113726250	1.011911860
5 Years	1.072534713	1.089492814	1.009590117
7 Years	1.050043981	1.063609890	1.006998839
10 Years	1.035097590	1.045625226	1.005116553
15 Years	1.022317285	1.029527938	1.003365963
20 Years	1.013240541	1.017687914	1.002038568
25 Years	1.005484083	1.007368313	1.000856304
All	1.000000000	1.000000000	1.000000000

Table 36: 13 Fracs (Ads. Gas) PLE Table of Ratios

13 Fracs (Adsorbed Gas) Table of Ratios				
<i>PLE</i>	<i>D_∞</i>	<i>n</i>	<i>q_i</i>	<i>D₁</i>
2 Years	1.111384885	1.07899338	0.959439202	0.878858218
3 Years	1.699886887	1.11824919	0.949859356	0.852382315
4 Years	2.063494445	1.14381284	0.943640869	0.834189228
5 Years	2.243034026	1.15708648	0.940361644	0.824281769
7 Years	2.303247076	1.1616984	0.939205982	0.820735541
10 Years	2.100181394	1.14314014	0.944450434	0.837076178
15 Years	1.67665366	1.09715735	0.959398974	0.88285719
20 Years	1.358447499	1.05581263	0.975050456	0.929292862
25 Years	1.132610275	1.02197154	0.989614083	0.971055433
All	1.000000000	1.0000000	1.000000000	1.000000000

Table 37: 13 Fracs (Ads. Gas) Duong Table of Ratios

13 Fracs (Adsorbed Gas) Table of Ratios				
Duong	a	m	q_1	q_∞
2 Years	0.786162728	0.92000	1.525624844	23.17306877
3 Years	0.82161880	0.93727566	1.331354809	13.24935838
4 Years	0.848920227	0.94893626	1.241838909	9.107723145
5 Years	0.869673376	0.95720	1.190801855	6.903207581
7 Years	0.902052847	0.96910	1.128023157	4.512297396
10 Years	0.933750605	0.9798305	1.079312544	2.926867500
15 Years	0.964188766	0.98950	1.040136204	1.848791067
20 Years	0.981488920	0.9946882	1.019883966	1.386204405
25 Years	0.993556533	0.99816602	1.006851353	1.128429921
All	1.000000000	1.00000	1.000000000	1.000000000

Figure 102: 13 Fracs (Ads. Gas) Arps Ratio of n vs. Time

Figure 103: 13 Fracs (Ads. Gas) Arps Ratio of d_i vs. Time

Figure 104: 13 Fracs (Ads. Gas) Arps Ratio of q_i vs. Time

Figure 105: 13 Fracs (Ads. Gas) PLE Ratio of D_{∞} vs. Time

Figure 106: 13 Fracs (Ads. Gas) PLE Ratio of n vs. Time

Figure 107: 13 Fracs (Ads. Gas) PLE Ratio of q_i vs. Time

Figure 108: 13 Fracs (Ads. Gas) PLE Ratio of D_1 vs. Time

Figure 109: 13 Fracs (Ads. Gas) Duong Ratio of a vs. Time

Figure 110: 13 Fracs (Ads. Gas) Duong Ratio of m vs. Time

Figure 111: 13 Fracs (Ads. Gas) Duong Ratio of q_1 vs. Time

Figure 112: 13 Fracs (Ads. Gas) Duong Ratio of q_∞ vs. Time

Table 38: 13 Fracs (Ads. Gas) Arps Well 1 Ratio Values

13 Fracs (Adsorbed Gas) Arps Well 1 Ratio Values			
Well 1 (Years)	n	d_i	q_i
4.273972603	1.091093409	1.109860819	1.011518534

Table 39: 13 Fracs (Ads. Gas) PLE Well 1 Ratio Values

13 Fracs (Adsorbed Gas) PLE Well 1 Ratio Values				
Well 1 (Years)	D_{∞}	n	q_i	D_1
4.273972603	3.31373464	1.2406446	0.912950414	0.751755115

Table 40: 13 Fracs (Ads. Gas) Duong Well 1 Ratio Values

13 Fracs (Adsorbed Gas) Duong Well 1 Ratio Values				
Well 1 (Years)	q_{∞}	a	q_1	m
4.273972603	8.582678644	0.85306289	1.262985874	0.949212079

Table 41: 13 Fracs (Ads. Gas) Arps Well 1 Actual and Predicted Values

Well 1	Actual	Predicted
n	1.726571773	1.582423428
d_i	0.004360324	0.003928712
q_i	2,171.43	2146.703407

Table 42: 13 Fracs (Ads. Gas) PLE Well 1 Actual and Predicted Values

Well 1	Actual	Predicted
D_{∞}	-0.000380832	-0.00011493
n	0.506473221	0.40823393
q_i	2,627.55	2878.08502
D_1	0.02777503	0.03694691

Table 43: 13 Fracs (Ads. Gas) Duong Well 1 Actual and Predicted Values

Well 1	Actual	Predicted
a	1.482659542	1.73804249
m	1.113649238	1.17323543
q_1	1,414.02	1119.58657
q_∞	-148.7245961	-17.3284591

Figure 113: Well 1 - 13 Fracs (Ads. Gas) Arps Predicted Match

Figure 114: Well 1 - 13 Fracs (Ads. Gas) PLE Predicted Match

Figure 115: Well 1 - 13 Fracs (Ads. Gas) Duong Predicted Match

Figure 116: 13 Fracs (Ads. Gas) 30 Year Well 1 Predicted Comparison