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Estimate at Completion for Construction Projects

Yi Fang

**Problem Report submitted to the
College of Engineering and Mineral Resources
at West Virginia University
in partial fulfillment of the requirements
for the degree of**

**Master of Science
in
Industrial Engineering**

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2008

**Keywords: Estimate at Completion, Earned Schedule
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ABSTRACT

Estimate at Completion for Construction Projects

Yi Fang

Construction projects are complex and risky because there are so many variables that may affect their profitability especially in terms of their cost and schedule. The information provided by the measurement of progress, the expended dollars and the forecasting of the outcome of a construction project will provide valuable information as to how to proceed, the impacts, corrective action required, and the trade-offs necessary to maximize profits and minimize risk. This study focused on using different indicators to measure the schedule performance and cost performance of construction projects, and to predict final cost and duration by the moving average and exponential smoothing forecasting techniques. The recommended indicator for schedule predication was the cumulative schedule performance index obtained by using earned schedule. The recommended indicator for estimating final cost was the cumulative cost performance index. These two indicators always produced acceptable results for the given data sets and were the simplest methods to use.

ACKNOWLEDGMENT

First of all, I would like to express my deepest appreciation to my advisor, Dr. Robert C. Creese. He has always been helpful and patient throughout my whole study here. His valuable guidance, incisive comments, and continuous motivation gave me the greatest courage to finish my report. The support he gave me was so enormous that I cannot thank him enough.

Secondly, I would like to thank my committee members, Dr. Majid Jaraiedi and Dr. Feng Yang, who gave me lots of constructive suggestions to improve my research and this report.

Also, I would like to thank my family members, my parents, my sister and my husband for their endless assistance and love.

Finally, I would like to express my special thanks to Dr. Wafik Iskander and Dr. Alan R. McKendall Jr. for their constant support and advice.

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LIST OF NOMENCLATURE

ACWP: Actual Cost of Work Performed

AD: Actual Duration

AF_c: Adjustment Factor for Cost

AF_s: Adjustment Factor for Schedule

AT: Actual Time

BAC: Budget at Completion

BCWP: Budgeted Cost of Work Performed

BCWS: Budgeted Cost of Work Schedule

CI: Confidence Interval

CPI: Cost Performance Index

C/SCSC: Cost/Schedule Control Systems Criteria

CV: Cost Variance

EAC: Estimate at Completion

EAC(*t*): Estimate Duration at Completion

ES: Earned Schedule

EV: Earned Value

EVM: Earned Value Management

MAPE: Mean Absolute Percent Error

PD: Planned Duration

PERT: Program Evaluation and Review Technique

PMB: Performance Measurement Baseline

PV: Planned Value

SCI: Schedule Cost Index

SD (σ): Standard Deviation

SPI: Schedule Performance Index

SPI(t): Schedule Performance Index obtained by using Earned Schedule

SV: Schedule Variance

SV(t): Schedule Variance in terms of time

TCPI: To-complete Performance Index

TCSPI: To-Complete Schedule Performance Index

VAC: Variance at Completion

CHAPTER 1

INTRODUCTION

1.1 Construction projects

Construction is a process that consists of the building or assembling of infrastructure. In general, there are three types of construction: building construction, industrial construction, and heavy/highway construction. Building construction is the process of adding structure to real property. The majority of building construction projects are small renovations.

Industrial construction, though a relatively small part of the entire construction industry, is an important component. These projects can be found in such industries as medicine, petroleum, chemical, power generation, and manufacturing etc. Owners of these projects are usually large, for-profit corporations. Heavy/civil construction is the process of adding infrastructure to our environment. Owners of these projects are usually government agencies. Being different from building and industrial construction, heavy/civil construction projects are not usually undertaken for profit, but to service the public interest. However heavy/civil construction projects are also undertaken by large private corporations. The cases studied in this research belong to the heavy/civil construction. However, the techniques used can be applied to all types of construction projects, as long as necessary data are recorded and maintained.

According to Sullivan (2008), total construction came in at \$611.2 billion for the full year 2007, which is a huge segment of the United States economy. It contributes 14% of the US Gross National Product. There are many uncertain factors may affect the outcome of a

construction project. Laufer and Tucker (1987) noted that cost and time are two major goals of a construction project which receive more attention than quality. The goal is to find good techniques to measure the performance of construction projects and predict the final costs and durations. Many studies have been done for measuring and predicting cost, therefore to find good schedule indices and applying these indices to predict final schedule are addressed. Concepts of construction cost elements, and an introduction of cost and schedule management are presented.

1.2 Cost elements of construction projects

Cost elements may vary widely for different projects, as well as their importance. Cost elements of a construction project include construction work cost, facilities and equipment cost, engineering cost, overhead cost, interest, contingency, and profit. Figure 1.1 (adopted from Ostwald [32], 2001) displays these cost elements as a layer chart.

Figure 1.1: Cost elements of construction projects [32]

Construction work cost consists of direct labor, direct material, tools, equipment, and

subcontract materials. Figure 1.2 (taken from Ostwald [32], page 240) is a layer chart illustrating the elements of construction work cost.

Figure 1.2: Elements of construction work cost [32]

Facilities and equipment are also direct materials but have a different character: delivered and erected equipment, such as large storage tanks or field-fabricated vessels. Facilities and equipment cost can include landing, building and processing costs.

Engineering costs are those incurred for design, specifications, or reports. Engineering costs include salaries and overhead for engineering, administration, CAD, estimating, and drawing reproductions.

Overhead, which makes up an important part of construction price, is an estimate of the contractor's cost that cannot be clearly associated with particular jobs, projects, or systems and must be prorated among all work on some arbitrary basis. Overhead for construction projects is of two types: office and job. Office overhead includes general business expenses, such as home office rent, office insurance, heat, light, supplies, furniture, telephone, legal expenses, travel, advertising, bidding expenses, and salaries of the executives and office employees. Note that office overhead is not incurred for a single project, but the owner's or contractor's overall business (i.e., several projects). Some methods used to calculate office

overhead can be found in Ostwald ([32], page 126). Job overhead, opposed to office overhead, pertains to the project. Typical items of job overhead include permits and fees, insurance, electricity at job location, job office expense, water at job location etc.

Because construction projects are large financial undertakings, interest costs are usually charged against the contractor or owner while construction is in progress.

Contingency has different meanings to different estimators, contractors, and owners. A comprehensive definition of contingency given by Cost Engineers' Notebook [2] is: contingency is a cost element of an estimate to cover a probability of the occurrence of unforeseeable elements of cost within the defined project scope due to a combination of uncertainties, intangibles, and unforeseen/highly unlikely occurrences of future events, based on management decisions to assume certain risks (for the occurrence of those events). Contingency reflects a judgment by management to make an allowance to avoid project cost overrun within the parameters of risk assumed [3]. At the same time, contingency should not be too high to create an unrealistic estimate.

Profit represents the excess of revenue over cost. It is an accounting approximation of the earnings of a company after taxes, cash and accrued expenses, and certain tax-deductible noncash expenses, such as depreciation. Profit is always calculated upon the project cost estimate, which is represented in Figure 1.1 from layer of construction work cost to layer of contingency.

1.3 Cost management

Cost management is the process of estimating, controlling, and data analysis to establish

a continuous cycle of information for the efficient implementation of projects (Hamilton [17], 2004). Cost management has become an increasingly important part of business. All types of projects can benefit from the appropriate application of cost management techniques.

The cost management cycle is illustrated in Figure 1.3, which is adopted from Hamilton [17]. Cost estimation is the determination of quantity and the predicting or forecasting, within a defined scope, of the cost required to construct and equip a facility. Cost control is the application of procedures to monitor expenditures and performance against progress of projects; to measure variance from authorized budgets and allow effective action to be taken to achieve minimum cost [1].

Figure 1.3: Cost management cycle [17]

1.4 Schedule management

While cost management is an important part for construction project success, the same can be said for construction schedule management. In today's fast-paced construction environment, project owners are increasingly placing greater demand on contractors to complete projects in record time. Perhaps the emphasis in schedule management of a

construction project is because of the realization by the contractor and the owner of the severe implications of schedule overruns (Kog *et al.* [24], 1999). Owners will most likely suffer the loss of expected profits as a direct result of delays in putting the facility into service. Consequently, contractors typically face liquidated damages for finishing late.

1.5 Forecasting

Forecasting is an estimate and prediction of future conditions and events based on information and knowledge available at the time of the forecast [2]. According to Brown [6], there are two key forecasts for contractors: the Estimate at Completion (EAC) and the forecasting of Cash Flow Expenditure. This research is focusing on the EAC.

EAC is the result of a disciplined, logical analysis using project data and forecasting methodologies to establish the expected cost at completion and the expected completion date. The highlight of trends and potential budget and duration deviations allows project team to take actions to minimize or avoid cost overruns and schedule delays.

1.6 Objectives of the research

The main objectives of the research include:

1. Using different techniques to forecast total cost, also called estimate at completion (EAC) cost, of construction projects during the construction stage;
2. Using different techniques to forecast final duration, also called estimate duration at completion ($EAC(t)$), of construction projects during the construction stage;
3. Comparing the results obtained by different techniques and making recommendations;

4. Conducting risk analysis for cost and schedule.

1.7 Organization of the report

In Chapter 2, a literature review for the techniques used for forecasting final cost, duration is given. In Chapter 3, the metrics used to measure the performance of construction projects, and the methodologies for forecasting cost and duration are presented. In Chapter 4, proposed methods of EAC are applied to three projects from the literature. Finally, the conclusions of the research and recommendations for future study are presented in Chapter 5.

CHAPTER 2

LITERATURE REVIEW

Earned value management (EVM) is the technique that has been extensively applied to measure the performance of projects and conducting the estimate at completion. EVM is applied throughout the research, therefore a brief literature review for the EVM technique will be given in this chapter. A literature review for the estimated cost at completion and estimated duration at completion will be presented in more detail.

2.1 Earned value management

A basic form of the EVM can be traced back to industrial engineers on the factory floor in the late 1800s (Fleming and Koppelman [14]). Around 1967, EVM was introduced by agencies of the U.S. federal government as an integral part of the Cost/Schedule Control Systems Criteria (C/SCSC) and was used in large acquisition programs. To encourage wider use of EVM in the private sector, the U.S. federal government decided to discard C/SCSC by the end of 1996 and turned toward a more flexible EVM system, also called the earned value project management system (Anbari [4]). Project Management Institute's *A Guide to the Project Management Body of Knowledge* [48] provided the simplified EVM terminology and formulas.

Lipke [26] criticizes that EVM measures schedule performance not in units of time, but rather in cost. More importantly, the scheduling indicators of EVM fail to provide good information over the final third of the project. The author then introduces the concept of "Earned Schedule" to solve the problem. It is shown that the associated schedule indicators

behave appropriately throughout the entire period of project performance and is analogous to the cost indicators.

2.2 Estimate cost at completion

Once the performances have been identified using EVM, the project team can further assess the project by predicting EAC. There have been many studies in the literature dealing with the forecasting of project total cost.

Christensen [7] studied multiple EVM indices for their accuracy in forecasting final cost in defense acquisition. He classified the performance indices into four groups: the cost performance index (CPI), the schedule performance index (SPI), the schedule cost index (SCI), and the composite index. The indices can be based on monthly, cumulative, or averaged data. Based on a review of 25 EAC related studies, the author concluded that the accuracy of regression-based models over index-based formulas has not been established; the accuracy of index-based formulas depends on the type of system, the stage and the phase of the contract, no one formula is always the best; the long-asserted accuracy of the composite index with a 20/80 percent weighting on SPI and CPI was not supported by the evidence; averaging over short periods (e.g., three months) is more accurate than averaging over longer periods (e.g., 6-12 months); comparing the to-complete performance index (TCPI) with the cumulative CPI, and comparing the cumulative cost variance with the variance at completion are useful for evaluating the reasonableness of the contractor's EAC.

Christensen [8] used the EVM to evaluate the EAC. The author recommended that three comparisons should be made to evaluate the reasonableness of the EAC. First, the overrun to date (cost variance) should be compared to the estimated final overrun; second, the CPI

should be compared to the TCPI; the third comparison involves generating a range of “independent” EACs using generic formula. Four performance indices, the CPI, SPI, SCI, and the weighted composite index of $0.8CPI+0.2SPI$, were used to determine the EAC. The author concluded that the EAC derived from the CPI is a reasonable floor to the final cost and the EAC based on the product of the CPI and SPI is usually quite large, since most defense contracts finish behind schedule and over budget.

Anbari [4] reviewed and simplified several common methods to forecast EAC, with different assumptions for each method. When current analysis shows that the original estimate is flawed or no longer applicable due to changed conditions, a revised estimation for the remaining work must be developed. When current analysis shows that past performance is not a good predictor and that future performance will parallel the original plan, the EAC is the sum of cost incurred and the original budget for the remaining work. When current analysis shows that past performance is a good predictor of future performance, the EAC is the sum of cost incurred and the original budget for the remaining work divided by a performance factor. The performance factor is usually the cumulative CPI or SCI.

Stephenson [40] presented a paper of using EAC to identify risks and opportunities. Eight methods were applied to derive EAC and were grouped into four categories: non-performance methods, performance methods, composite methods, and extrapolation method. Then for each cost item there were eight EACs, among the eight values the one using the extrapolation method was taken as the most likely value. By utilizing the Program Evaluation and Review Technique (PERT), the expected value and standard deviation for each cost item can be identified. Finally, a range of the EAC with desired probability can be

obtained.

Singh [39] presented a model that incorporates the EVM, index analysis, moving average, linear and non-linear regression, and probabilistic confidence analysis to predict the final cost at project completion. The author concluded that the confidence level used in each reporting period should decrease as the project progresses. The limitation of the paper was that only the condition where actual total cost equals budgeted cost was evaluated.

Teicholz [43] developed a general approach to forecast the final cost and variance of a construction project based on the periodic data collected. Two methods were tested against a large sample of 121 projects completed over 15 years. Analysis of the results indicated that the sliding moving average method yielded forecasts of final cost with improved accuracy, timeliness, and consistency. To calculate the final variance, it was necessary to forecast the final budget, this was done by using a linear projection of the to-date change in the current total budget. The results showed a significant improvement in accuracy and timeliness.

2.3 Estimate duration at completion

Compared with the studies that have been done for estimating the total cost, there are only a few papers deal with the estimate duration at completion ($EAC(t)$). According to Vandevorde and Vanhoucke [45] only three project duration forecasting methods have been presented in literature.

Anbari [4] extended the assumptions and logic of forecasting final cost to predict the $EAC(t)$. When current analysis shows that past performance is not a good predictor and that future performance will parallel the original plan, the $EAC(t)$ is the sum of actual duration to-

date and the original scheduled time for the remaining work. When current analysis shows that past schedule performance is a good predictor of future performance, the EAC is the sum of actual duration to-date and the original scheduled time for the remaining work divided by a performance factor. The performance factor is usually the cumulative SPI or SCI.

Jacob [20] described an earned duration method to forecast the project duration. The earned duration is the product of the actual duration and SPI. The performance factor is used to adapt the future performance to the past performance. Three performance indices, which reflect the same three situations as stated by Anbari [4] are used to project the final duration.

Henderson [18] applied the earned schedule method to forecast project duration, which is an extension of the work done by Lipke [26]. The author has illustrated the validity of the earned schedule concept by applying it on a portfolio of six projects and on a small scale but time critical information technology software development project.

Vandevoorde and Vanhoucke [45] compared three different project duration methods using earned value metrics and evaluated them on fictive and real-life project data. The authors presented a generic formula to forecast the duration of a project and linked them to different project situations. Each method was further sub-divided into three different forecasting models as a function of the project situation. Each method was applied on a fictive single-activity project with linear and non-linear increasing periodic values reflecting the absence or presence of learning curves as well as three real-life projects from Fabricom Airport Systems, Belgium. The results show a similar forecasting accuracy for each method in the linear planned value case. However, the introduction of learning curves, which is much more realistic, resulted in a different forecasting accuracy for the three methods. The

three real-life projects revealed that the earned schedule method was the only method which showed satisfying and reliable results during the whole project duration. The results obtained by the planned value and earned duration method were unreliable at the end of the project.

CHAPTER 3

ESTIMATE AT COMPLETION

3.1 Introduction

In construction projects, the profitability of the project is directly dependent upon the ability of the contractor to control project cost and schedule [15]. Therefore, it is important for contractor to know if unfavorable variances of cost and schedule have incurred or will happen at any point during the construction stage. Forecasting will allow contractor to foresee what will probably happen and make decisions with some degree of confidence. The various techniques used to obtain EAC are presented.

3.2 Earned value management

Earned value management (EVM) technique has been used extensively on numerous projects and in many industries. In the construction industry, EVM is a powerful tool to measure the cost and schedule performances of projects, as well as the variances from the project plan or baselines. Once the performances and variances have been identified, the contractor can further assess the project by estimating the total cost.

3.2.1 Earned value management key components

In the late 1960s the US Department of Defense (DoD) adopted the Cost/Schedule Control System Criteria (C/SCSC), which were revised by industry, accepted by the government, and re-named as EVM Systems Criteria in 1996 [8]. Although there are several terminologies used in EVM, the three basic data elements central to planning, measurement,

and analysis are: BCWS, BCWP, and ACWP.

The Budgeted Cost of Work Scheduled (BCWS) is the budget for work scheduled to be completed. It can be either periodic (weekly or monthly) or cumulative. As a weekly/monthly amount, it represents the amount of work scheduled to be completed for that week/month. As a cumulative amount, it represents the amount of work scheduled to be completed to date. BCWS is also known as “planned value”.

The Budgeted Cost of Work Performed (BCWP) is the budget for the completed work. Similar to BCWS, BCWP also can be either periodic (weekly or monthly) or cumulative. Weekly/monthly BCWP represents the amount of work completed during a week/month; cumulative BCWP represents the amount of work completed to date. BCWP is also known as “earned value”.

The Actual Cost of Work Performed (ACWP) is the actual cost incurred in accomplishing the work within a given time period. To perform meaningful comparisons, the ACWP should be recorded in the same time period as BCWP for a given element of work.

Regardless of the timing of the work, a budget in terms of hours, dollars, or other measurable units is assigned to each work and planning package. By summing their budgets, a time-phased budgetary baseline for the entire project is defined. This baseline, known as the Performance Measurement Baseline (PMB), represents the standard or plan is compared with BCWP and ACWP.

Figure 3.1 taken from Christensen [8] illustrate a typical condition of many projects: behind schedule and over budget. The PMB represents the plan. Because less work has been completed (in terms of money) than planned during given period, an unfavorable schedule

variance (the vertical difference between BCWS and BCWP) is identified. The schedule variance (SV) can also be represented in term of time (the horizontal difference between BCWS and BCWP), which will be discussed in details in Section 3.2.3. Also, since actual cost exceeds the budget for work completed, an unfavorable cost variance (CV, i.e., the vertical difference between ACWP and BCWP) is identified.

Figure 3.1: Performance Measurement Baseline [8]

3.2.2 Project performance measure

There are two indices commonly used to measure the performance of a project at any time during the construction stage: CPI and SPI.

The cost performance index (CPI) is the ratio of the budget of the earned work to the actual money spent and can be represented by Equation 3-1:

$$\text{CPI} = \text{BCWP} / \text{ACWP} \quad (3-1)$$

When CPI is less than 1.0, an unfavorable cost variance has incurred.

The schedule performance index (SPI) is the ratio of earned value to the planned value and can be expressed by Equation 3-2:

$$SPI = BCWP/BCWS \quad (3-2)$$

Similarly, when SPI is less than 1, the BCWP is below the PMB line, which indicates an unfavorable schedule variance.

Other indices such as schedule cost index (SCI) and variations of SCI are also used as indicators of project performance ([8] and [40]). The schedule cost index also called composite index is applied when there is correlation between the cost performance of the project and the schedule performance of the project. The two most popular composite indices are expressed by Equations 3-3 and 3-4.

$$SCI = CPI \times SPI \quad (3-3)$$

$$SCI = 0.8 \times CPI + 0.2 \times SPI \quad (3-4)$$

A more general form of Equation 3-4 can be represented by Equation 3-5:

$$SCI = w_1 \times CPI + w_2 \times SPI \quad (3-5)$$

Where w_1 and w_2 are weights for CPI and SPI respectively, and the sum of w_1 and w_2 must equal to 1. Also, the baseline of 1.0 is used for comparison purposes. However, these equations must be used with caution when determining overall performance of the project.

For example, when CPI is 1.15 and SPI is 0.88, the SCI is 1.01 (using Equation 3-3) or 1.10 (using Equation 3-4). These two values indicate that the project is performing well.

However, the project may be in jeopardy of delay. Therefore, in order to apply the composite indices, the project team must understand the sensitivities surrounding the cost impact and/or the schedule impact of the project. Once the sensitivities have been defined, the weights can be adjusted to reflect the current project needs [40].

3.2.3 Earned schedule

The interpretation and the behavior of the schedule performance indicators SPI and SV over time have been criticized by different authors [26]. First, the SV is measured in monetary units and not in time units, which makes it difficult to understand. Secondly, towards the end of the project, the SV always converges to 0 indicating a perfect performance even if the project is late. Similarly, the SPI always converges to 1 towards the end of the project, indicating a 100% schedule efficiency even if the project is late. As a result, at a certain point in time, the SV and SPI become unreliable indicators. Lipke [26] pointed out that the “grey time area” where these indicators lose their predictive ability usually occurs over the last third of the project (expressed in percentage completion).

In order to overcome the problem, Lipke [26] introduced the concept of earned schedule (ES). In this method, the earned value (BCWP) at a certain point of time is traced forwards or backwards to the PMB (BCWS). This intersection point is moved downwards on the X-axis (the time scale) to calculate the ES (see Figure 3.2). Hence, ES translates the EV into time increments and measures the real project performance in comparison to its expected time performance. The corresponding schedule performance indices are:

$$SV(t) = ES - AT \quad (3-6)$$

$$SPI(t) = ES/AT \quad (3-7)$$

where AT is Actual Time.

Figure 3.2: SV vs. $SV(t)$ [22]

In contrast to the SV , the $SV(t)$ is expressed in time units, which makes it easier to interpret. When $SV(t) < 0$, it indicates that the project lags its expected schedule; on the other hand, when $SV(t) > 0$, the project is ahead of its schedule. The behavior of $SV(t)$ over time results in a final $SV(t)$ that equals exactly the real time difference at completion, while the SV always equals zero at the end of the project. Similarly, the $SPI(t)$ has a final value reflecting the final project schedule performance, while the SPI always equals 1. The ES can be mathematically expressed as:

$$ES = N + (EV - PV_N) / (PV_{N+1} - PV_N) \quad (3-8)$$

where N is the time increment of the PV that is less than current EV , PV_N is the planned value at time N , and PV_{N+1} is the planned value at time $N+1$. To show the calculation of ES , $SV(t)$, and $SPI(t)$, an example adopted from Lipke [26] is shown in Table 3.1. Taking an instance, during October 2001, the EV (or $BCWP$) is 2,275. According to Equation (3-8) N is equal to 8, since the PV (or $BCWS$) of Aug is 2,135, which is the nearest lower value of the current EV . As a result, the ES of Oct is calculated as: $ES(10) = 8 + (2,275 - 2,135) /$

$$(2,435 - 2,135) = 8.467.$$

Table 3.1: Calculation of ES, SV(t), and SPI(t)

	Year 2001												Year 2002		
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar
BCWS	105	200	515	845	1175	1475	1805	2135	2435	2665	2760	2823	--	--	--
BCWP	95	180	470	770	1065	1315	1610	1900	2150	2275	2425	2555	2695	2770	2823
SV	-10	-20	-45	-75	-110	-160	-195	-235	-285	-390	-335	-268	-128	-53	0
SPI	0.905	0.900	0.913	0.911	0.906	0.892	0.892	0.890	0.883	0.854	0.879	0.905	0.955	0.981	1.000
Month Count	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
ES	0.905	1.789	2.857	3.773	4.667	5.467	6.409	7.288	8.050	8.467	8.967	9.522	10.316	11.159	12.000
SV(t)	-0.095	-0.211	-0.143	-0.227	-0.333	-0.533	-0.591	-0.712	-0.950	-1.533	-2.033	-2.478	-2.684	-2.841	-3.000
SPI(t)	0.905	0.895	0.952	0.943	0.933	0.911	0.916	0.911	0.894	0.847	0.815	0.793	0.794	0.797	0.800

Correspondingly, $SV(10) = 8.467 - 10 = -1.533$ and $SPI(10) = 8.467 / 10 = 0.847$. All of these three measurements indicate an unfavorable schedule performance. It can be seen from Table 3.1 that at the end of the project (Mar 2002) the SPI in terms of money equals 1, even though the project has lagged its schedule for 3 months; opposed to SPI, $SPI(t)$ is 0.8 at the end of the project and $SV(t)$ is -3 months, which is the difference between actual schedule and planned schedule. Figure 3.3 gives a comparison between SPI and $SPI(t)$, it can be seen clearly from the figure that during the first two third period, both indices give the similar results. However, after the 10th month, SPI has the trend to converge to 1, while $SPI(t)$ remains at a lower level, reflecting the actual situation.

Figure 3.3: SPI vs. SPI(t)

3.3 Estimate at completion

As soon as the project has begun, there is a requirement to measure the project against the cost and schedule baselines. The information provided by the measurement of progress and the expended dollars will provide valuable information as to how to proceed, the impacts, corrective action required, and the trade-offs necessary in order to maximize profits and minimize risk [40].

The EVM indices discussed above provide early indications about project performance, thus it allows project team to adjust project strategy and take corrective actions. However, the ultimate goal for every project team is to finish the project within planned budget and schedule. Therefore, it is important for project manager to predict what would be the project final cost and duration, or called estimate at completion (EAC) from both cost and duration point of view. The following sections will discuss the methods for forecasting EAC. In order to distinguish the two terms, EAC is used to refer to estimate cost at completion, while $EAC(t)$ refers to estimate duration at completion.

3.3.1 Estimate cost at completion

Estimate cost at completion (EAC) can be defined as the forecasting of total project costs [40]. Noted that this forecasting is different from budgeting, which is always stipulated in the contract before the execution stage begins. The difference between the budget, or called budget at completion (BAC) and the EAC is called the variance at completion (VAC).

Figure 3.4 illustrates the relationships between BAC, EAC, and VAC. In this graph, an unfavorable cost overrun at the end of the project is indicated due to the EAC is higher than

the original budget (BAC). Since the EAC not only happens at the end of the project, but throughout the whole construction stage, there is forecasting of final cost for every reporting period. The EAC value can be developed by applying mathematical formulas or by extrapolation method, which is based on project team's past experience and review of project data. In this research, only mathematical models are considered.

Figure 3.4: Estimate at Completion and Variance at Completion

Since there are numbers of combinations of project performance indices, there is literally an infinite number of EAC formulas [7]. However, the generic index-based formula can be represented as:

$$EAC = ACWP_c + (BAC - BCWP_c)/Index \quad (3-9)$$

The subscript “c” indicates cumulative data. The index could be individual performance index or combination of them; it also could be cumulative index, most recent period index, average index, or index obtained by using other forecasting techniques (such as moving average, exponential smoothing etc.). Therefore, many formulas can be derived from this generic formula. The following methods were studied:

$$EAC_1 = ACWP_c + [(BAC - BCWP_c)/CPI_c] \quad (3-10)$$

$$EAC_2 = ACWP_c + [(BAC - BCWP_c)/(SPI(t)_c \times CPI_c)] \quad (3-11)$$

$$EAC_3 = ACWP_c + [(BAC - BCWP_c)/(w_1SPI(t)_c + w_2CPI_c)] \quad (3-12)$$

$$EAC_4 = ACWP_c + [(BAC - BCWP_c)/CPI_m] \quad (3-13)$$

$$EAC_5 = ACWP_c + [(BAC - BCWP_c)/(SPI(t)_m \times CPI_m)] \quad (3-14)$$

$$EAC_6 = ACWP_c + [(BAC - BCWP_c)/(w_1SPI(t)_m + w_2CPI_m)] \quad (3-15)$$

$$EAC_7 = ACWP_c + [(BAC - BCWP_c)/CPI_{MA}] \quad (3-16)$$

$$EAC_8 = ACWP_c + [(BAC - BCWP_c)/(SPI(t)_{MA} \times CPI_{MA})] \quad (3-17)$$

$$EAC_9 = ACWP_c + [(BAC - BCWP_c)/(w_1SPI(t)_{MA} + w_2CPI_{MA})] \quad (3-18)$$

$$EAC_{10} = ACWP_c + [(BAC - BCWP_c)/CPI_{EX}] \quad (3-19)$$

$$EAC_{11} = ACWP_c + [(BAC - BCWP_c)/(SPI(t)_{EX} \times CPI_{EX})] \quad (3-20)$$

$$EAC_{12} = ACWP_c + [(BAC - BCWP_c)/(w_1SPI(t)_{EX} + w_2CPI_{EX})] \quad (3-21)$$

The subscript "c" indicates the indices are obtained by using cumulative data; the subscript "m" means that the most recent month indices are used; the subscript "MA" indicates the moving average forecasting technique is used to obtain indices; and the subscript "EX" indicates the exponential smoothing technique is used to obtain indices.

One way to evaluate if the predicted EAC is reasonable is to use the To-Complete Performance Index (TCPI) [7]. The TCPI is the ratio of work remaining to the money remaining and can be mathematically represented as:

$$TCPI_{BAC} = (BAC - BCWP_c) / (BAC - ACWP_c) \quad (3-22)$$

The TCPI represents the level of efficiency at which the contractor must operate from time now to the end of the contract to achieve the BAC. For example, at certain point of time the TCPI is 1.15, in order to complete the remaining work within the BAC the contract will have to earn \$1.15 for every dollar spent.

Because many contracts will overrun the BAC, another useful form of the TCPI is shown as:

$$TCPI_{EAC} = (BAC - BCWP_c) / (EAC - ACWP_c) \quad (3-23)$$

Christensen [7] stated a “10 percent rule”, that is, once a contract is twenty percent complete, the CPI_c does not vary from its value at that point by more than 10 percent; in fact, it tends to worsen. This rule indicates that if the contractor’s TCPI exceeds the CPI_c by more than 10 percent, and the contract is more than 20 percent complete, then the contractor’s EAC is understated. For example, at 25 percent completion point, the CPI_c and $TCPI_{EAC}$ are 0.90 and 1.2 respectively, it will be extremely unlikely that the contractor will be able to achieve the EAC for the remainder of the contract since the $TCPI_{EAC}$ exceeds CPI_c by 33%. Therefore, the EAC is too optimistic.

3.3.2 Estimate duration at completion

Estimate duration at completion $EAC(t)$ is to forecast the final project duration. Although EVM can be used to forecast both the project’s final cost and duration, most of the studies deal with cost. However, the total project duration is critical for a project, since a cost penalty will be incurred due to project delay. Similar to the EAC, the $EAC(t)$ should be reported periodically as soon as the execution of the project begins.

Because of the same logic, there are also infinite number of formulas can be applied to obtain $EAC(t)$. A generic model can be represented as:

$$EAC(t) = AD + (PD - ES) / \text{Index} \quad (3-24)$$

Where AD is the actual duration to date, PD is the total project planned duration, and ES is the earned schedule to date. The index could be individual performance index or combination of them; it also could be cumulative index, most recent period index, average index, or index obtained by using other forecasting techniques (such as moving average, exponential smoothing etc.). The following methods were studied:

$$EAC(t)_1 = AD + [(PD - ES)/SPI(t)_c] \quad (3-25)$$

$$EAC(t)_2 = AD + [(PD - ES)/(SPI(t)_c \times CPI_c)] \quad (3-26)$$

$$EAC(t)_3 = AD + [(PD - ES)/(w_1 SPI(t)_c + w_2 CPI_c)] \quad (3-27)$$

$$EAC(t)_4 = AD + [(PD - ES)/SPI(t)_m] \quad (3-28)$$

$$EAC(t)_5 = AD + [(PD - ES)/(SPI(t)_m \times CPI_m)] \quad (3-29)$$

$$EAC(t)_6 = AD + [(PD - ES)/(w_1 SPI(t)_m + w_2 CPI_m)] \quad (3-30)$$

$$EAC(t)_7 = AD + [(PD - ES)/SPI(t)_{MA}] \quad (3-31)$$

$$EAC(t)_8 = AD + [(PD - ES)/(SPI(t)_{MA} \times CPI_{MA})] \quad (3-32)$$

$$EAC(t)_9 = AD + [(PD - ES)/(w_1 SPI(t)_{MA} + w_2 CPI_{MA})] \quad (3-33)$$

$$EAC(t)_{10} = AD + [(PD - ES)/SPI(t)_{EX}] \quad (3-34)$$

$$EAC(t)_{11} = AD + [(PD - ES)/(SPI(t)_{EX} \times CPI_{EX})] \quad (3-35)$$

$$EAC(t)_{12} = AD + [(PD - ES)/(w_1 SPI(t)_{EX} + w_2 CPI_{EX})] \quad (3-36)$$

The subscript "c" indicates that the indices are obtained by using cumulative data; the subscript "m" means that the most recent month indices are used; the subscript "MA" indicates the moving average forecasting technique is used to obtain indices; and the subscript "EX" indicates the exponential smoothing technique is used to obtain indices.

The To-Complete Schedule Performance Index (TCSPI) can be calculated as:

$$\text{TCSPI}_{\text{PD}} = (\text{PD} - \text{ES}) / (\text{PD} - \text{AD}) \quad (3-37)$$

$$\text{or } \text{TCSPI}_{\text{EAC}} = (\text{PD} - \text{ES}) / (\text{EAC}(t) - \text{AD}) \quad (3-38)$$

The first equation measures the additional effort to finish the project within the planned duration; and the second equation measures the additional effort to finish the project within the latest revised (predicted) duration.

The proposed methods for forecasting total cost and duration can be applied to manage many types of projects, such as IT projects, military projects, etc. The construction projects are mainly studied in this report.

3.4 EAC risk assessment

3.4.1 Expected value and standard deviation of cost and duration

Cooper and Davidson [9] introduced a method to determine the expected value and the standard deviation of the EAC and $\text{EAC}(t)$. The formulas assuming a normal distribution and using an 80 percent confidence level are represented by Equation 3-39 and 3-40 as follows:

$$E(\text{EAC}) = [\text{Lowest value} + 2 * \text{Most likely value} + \text{Highest value}] / 4 \quad (3-39)$$

$$SD = (\text{Highest value} - \text{Lowest value})/2.65 \quad (3-40)$$

where $E(\text{EAC})$ = expected value of EAC

SD = standard deviation of EAC

The same equations can be applied to $\text{EAC}(t)$, therefore at the end of reporting period, expected values for final cost and duration can be obtained, as well as the standard deviation.

The studies by Lipke [27] have shown that the natural logarithms of cumulative CPI and $\text{SPI}(t)$ are normally distributed. As a result, the logarithm of the cumulative index was used to approximate the mean value, and the logarithms of the periodic values of CPI_m and $\text{SPI}(t)_m$ were used to estimate the standard deviations, which is expressed by Equation 3-41 [28].

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (\ln \text{index}(i)_m - \ln \text{index}_c)^2}{n-1}} \quad (3-41)$$

Where $\text{index}(i)_m$ represents the periodical index of month i , and index_c represents the cumulative index up to date. Since the statistical method assumes the population examined is infinite, adjustment factors were given by the authors for cost and schedule and are expressed as Equation 3-42 and 3-43.

$$\text{AF}_c = \sqrt{(\text{BAC} - \text{EV})/(\text{BAC} - (\text{EV}/n))} \quad (3-42)$$

$$\text{AF}_s = \sqrt{(\text{PD} - \text{ES})/(\text{PD} - (\text{ES}/n))} \quad (3-43)$$

As a result, the general equation for the confidence interval is

$$\text{CI} = \ln \text{index}_c \pm (Z_{\alpha/2} * \sigma / \sqrt{n}) * \text{AF} \quad (3-44)$$

Where $Z_{\alpha/2}$ is the critical value for a $(1-\alpha)\%$ confidence limit.

3.4.2 Joint probability and conditional probability model for cost-schedule

By applying the equations given in previous section, a range with desired probability can be calculated for EAC and EAC(t) individually. However, decision makers often require understanding how uncertainties between cost and schedule interact. A decision maker might bet on a high-risk schedule in hopes of keeping the system's cost within requirements. On the other side, the decision maker might be will to spend more money in hopes of a schedule being not delayed ([16], page 309). This is a common tradeoff faced by decision makers.

When a system's cost is the sum of many independent cost elements, normal distributions can arise. Similarly, if a system's schedule is the sum of many independent activities, normal distribution can arise ([16], page 318). Therefore, if normal distributions characterize a system's cost and schedule, then the bivariate normal distribution could serve as an assumed model of their joint distribution.

Assuming cost is the variable called X_1 and schedule is the variable called X_2 , then the mathematical equations are:

$$E(X_1) = \mu_{X_1} = \mu_1 \quad (3-45)$$

$$\text{Var}(X_1) = \sigma_{X_1}^2 = \sigma_1^2 \quad (3-46)$$

$$E(X_2) = \mu_{X_2} = \mu_2 \quad (3-47)$$

$$\text{Var}(X_2) = \sigma_{X_2}^2 = \sigma_2^2 \quad (3-48)$$

$$E(X_1|x_2) = \mu_1 + \frac{\sigma_1}{\sigma_2} \rho(x_2 - \mu_2) \quad (3-49)$$

$$Var(X_1|x_2) = \sigma_1^2(1 - \rho^2) \quad (3-50)$$

$$E(X_2|x_1) = \mu_2 + \frac{\sigma_2}{\sigma_1} \rho(x_1 - \mu_1) \quad (3-51)$$

$$Var(X_2|x_1) = \sigma_2^2(1 - \rho^2) \quad (3-52)$$

Where μ_1 is the mean of EAC; σ_1^2 is the variance of EAC; μ_2 is the mean of EAC(t); σ_2^2 is the variance of EAC(t); ρ is the coefficient of correlation between EAC and EAC(t), and the admissible values for ρ are given by the interval $-1 < \rho < 1$. The equation of calculating ρ is given by Equation 3-53.

$$\rho = \frac{N \sum x_1 x_2 - \sum x_1 \sum x_2}{\sqrt{(N \sum x_1^2 - (\sum x_1)^2)(N \sum x_2^2 - (\sum x_2)^2)}} \quad (3-53)$$

where N is the number of periods up to date.

With the conditional mean and variance of cost (x_1) given schedule (x_2), the conditional probability can be determined. Similarly for conditional probability of schedule given cost. The joint probability for given cost and schedule ranges can be obtained by using Equation 3-54.

$$P(a_1 \leq X_1 \leq b_1 \text{ and } a_2 \leq X_2 \leq b_2) = \int_{a_2}^{b_2} \int_{a_1}^{b_1} \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} e^{-\frac{1}{2}w} dx_1 dx_2 \quad (3-54)$$

$$\text{where } w = \frac{1}{1-\rho^2} \left\{ \left(\frac{x_1 - \mu_1}{\sigma_1} \right)^2 - 2\rho \left(\frac{x_1 - \mu_1}{\sigma_1} \right) \left(\frac{x_2 - \mu_2}{\sigma_2} \right) + \left(\frac{x_2 - \mu_2}{\sigma_2} \right)^2 \right\}$$

CHAPTER 4

COMPUTATIONAL RESULTS

4.1 Data sets

To study the 12 different methods presented in the previous chapter, three project data sets adopted from Vandevoorde and Vanhoucke [45] were analyzed. All three projects were about an airport luggage handling systems at Fabricom Airport Systems in Brussels (Belgium). Each project had a different performance. The first project was to revamp different check-in islands. The planned duration was nine months, with a BAC of €360,738. The project was delivered four months later than expected but under budget. The second project was to link two piers with fully automated baggage conveying lines. The planned duration was nine months, with a BAC of €2,875,000. The project took 12 months and had a cost overrun. The third project was to renovate the transfer baggage conveying system due to changed baggage flows and security issues. This project had a planned duration of 10 months, with a BAC of €906,000. The project finished one month earlier with a cost overrun. The three data sets are given in Appendix I and the S-curves for three projects are represented in Figure 4.1. S-curve is the graphic representative of the cumulative earned value against time. It was observed for the graph that the shape for the third project was close to the standard S-curve, while the other two projects (Project 1 and Project 2) showed some linear patterns.

Figure 4.1: S-curves for three projects

4.2 Calculation of monthly performance indices

The costs given by Vandevoorde and Vanhoucke [45] are all cumulative data, therefore the cumulative performance indices can be calculated directly using Equation 3-1 to 3-5. In order to obtain monthly individual indices, cumulative costs were transformed to monthly data. This was done by deducting the cumulative cost of previous month from the cumulative cost of current month. Using the same equation, CPI_m can be calculated for each month. The SPI_m is obtained by transforming cumulative ES to monthly ES. Since the denominator is 1 (month), the monthly ES is the SPI_m . Taking the first project as an example, during the first two months the $BCWP_c$ are 25,645 and 68,074 respectively, and the $ACWP_c$ are €25,567 and €66,293 respectively. Then the monthly values for the $BCWP_m$ are €25,645 and €42,429; and the $ACWP_m$ are €25,567 and €40,726. The CPI_m for the first two months are calculated as 1.003 ($BCWP_m / ACWP_m = 25,645 / 25,567 = 1.003$) and 1.042 ($BCWP_m / ACWP_m = 42,429 / 40,726 = 1.042$). As expressed by Equation 3-7, the formula to calculate

SPI(t) is ES/AT. Using the same logic, the generic equation to obtain monthly SPI(t)_m can be stated as:

$$\text{SPI}(t)_m = (\text{ES}_{n+1} - \text{ES}_n) / (\text{AT}_{n+1} - \text{AT}_n) \quad (4-1)$$

Where ES_n means the earned schedule of month n . Since the difference between actual time of month $n+1$ and month n is always 1, the denominator of Equation 4-1 equals 1. For the same example showed above, the cumulative ES has been obtained as 0.885 and 1.742 for the first two months, therefore the SPI(t)_m of month 1 and 2 are 0.885 and 0.857

($\frac{1.742 - 0.885}{2 - 1} = 0.857$). Repeating the same calculations, all monthly indices were obtained.

4.3 Forecasting using moving average

The problem of using monthly performance indices to predict EAC is that when project performance is not stable, that is the project performs well during certain period(s) but performs badly during other period(s), the forecasting of EAC based on monthly performance would deviate dramatically from its true value. One solution to this problem could be using the indices obtained by applying moving average to predict EAC. Moving average is one of the simplest forecasting methods, which averages the last m observations. In this research, three-month ($m=3$) moving average was used. There are two reasons for choosing the three-month window: (1) the 3 projects studied are short durations; (2) based on the conclusion given by Christensen (1993) averaging over short periods (e.g., three months) is more accurate than averaging over longer periods (e.g., 6-12 months). The three-month moving average indices were calculated using the mean value of current month and previous two month indices. Figure 4.2 shows the monthly CPI and three month average CPI for the

first project, it can be seen that the performance indices averaging over three months are smoother throughout the project than the monthly indices.

Figure 4.2: Monthly CPI vs. three month average CPI

4.4 Forecasting using exponential smoothing

In addition to using moving average, another method for forecasting could be the use of exponential smoothing. Exponential smoothing is a sophisticated weighted moving average method. Instead of giving the same weight for current and previous data, the idea of exponential smoothing method is to assign more weight to the latest observation and less weight to the older observations. The equation of performance indices by using exponential smoothing can be expressed as:

$$\text{Index}_t = \alpha A_t + (1 - \alpha) \text{Index}_{t-1} \quad (4-2)$$

Where Index_t is the index used to obtain EAC at the end of month t ; A_t is the actual performance index during the t month; and α is a parameter ($0 \leq \alpha \leq 1$) called the smoothing constant. Further extension of formula 4-2 can be displayed as:

$$\begin{aligned}
\text{Index}_t &= \alpha A_t + (1 - \alpha) [\alpha A_{t-1} + (1 - \alpha) \text{Index}_{t-2}] \\
&= \alpha A_t + \alpha (1 - \alpha) A_{t-1} + (1 - \alpha)^2 [\alpha A_{t-2} + (1 - \alpha) \text{Index}_{t-3}] \\
&= \alpha A_t + \alpha (1 - \alpha) A_{t-1} + \alpha (1 - \alpha)^2 A_{t-2} + (1 - \alpha)^3 \text{Index}_{t-3} \\
&\vdots \\
&= \alpha \sum_{k=0}^{t-1} (1 - \alpha)^k A_{t-k} + (1 - \alpha)^t \times \text{Index}_0 \quad (4-3)
\end{aligned}$$

From Equation 4-3, it can be seen that the most recent performance is given the most weight α , the period before is given the weight of $\alpha (1 - \alpha)$, and $\alpha (1 - \alpha)^2$ for 2 periods before, and so on. In order to calculating the index for each month, a starting value of Index is required. Since no historical data are available, 1 is used as the initial value. Taking the first project as an example, it has been calculated that the monthly CPI_m for the first and second month are 1.003 and 1.042 respectively, then the index used to predict EAC at the end of month 2 can be calculated using equation 4-2 (supposing α equals 0.1).

$$\begin{aligned}
\text{Index}_2 &= 0.1 \text{CPI}_2 + 0.1 * (1 - 0.1) \text{CPI}_1 + (1 - 0.1)^2 \times 1 \\
&= 0.1 * 1.042 + 0.1 * 0.9 * 1.003 + 0.9^2 \times 1 = 1.004
\end{aligned}$$

1.004 is then used as the CPI at the end second month to predict EAC (by using Equation 3-19 to Equation 3-21).

4.5 Computational results

4.5.1 Computational results of estimate cost at completion

Detail calculations and results are given in Appendix II to Appendix VII. Figure 4.3 displays the comparison of EAC obtained by using different indices. It is obvious from the

graphs that the EAC by using monthly indices deviate the most from actual value, which indicates that the monthly project performances fluctuate dramatically over periods. From Appendix II, the range of monthly CPI_m is from 0.485 to 2.725. The mean absolute percent error (MAPE) was used to compare different methods. The MAPE can be defined by:

$$MAPE = \frac{100 \sum_{i=1}^T |Actual\ cost - EAC_i| / Actual\ cost}{T} \quad (4-4)$$

Where actual cost is equal to the final cost of the project, which is a constant value for each project. EAC_i is the estimate of final cost/schedule at the end of period i ; $i = 1, \dots, T$, and T is the final length of the project. The MAPEs for the first project using the 12 methods (Equation 3-10 to Equation 3-21) are summarized in Table 4.1. Among the 12 methods, the EACs calculated by using exponential smoothing indices are slightly better than the EACs obtained by using cumulative performance indices. It is also observed that using SCI as the index produces higher EACs, this is due to the unfavorable schedule performance, which makes the product of SPI and CPI low.

Table 4.1: MAPE of the EAC for the first project

Method		Method	
<i>Cumulative index</i>	<i>MAPE</i>	<i>Moving average</i>	<i>MAPE</i>
1. CPI	1.70%	7. CPI	4.73%
2. CPI*SPI (SCI)	4.46%	8. CPI*SPI (SCI)	5.88%
3. Weighted SCI	1.70%	9. Weighted SCI	3.67%
<i>Monthly index</i>	<i>MAPE</i>	<i>Exponential smoothing</i>	<i>MAPE</i>
4. CPI	7.51%	10. CPI	1.65%
5. CPI*SPI (SCI)	18.98%	11. CPI*SPI (SCI)	4.30%
6. Weighted SCI	7.54%	12. Weighted SCI	1.68%

(a) EAC using CPI as performance index

(b) EAC using SCI as performance index

(c) EAC using weighted SCI as performance index

◆ Using cumulative indices ◆ Using monthly indices ▲ Using 3 month average indices
 ■ Using exponential smoothing indices — Actual cost — Planned budget

Figure 4.3: Comparison of EAC for the first project

The selection of weights for SPI (w_1) and weights for CPI (w_2) should be based on project team's experience and reflect current project needs. Therefore, they should be input parameters to the proposed composite indices methods. The main objective of this study is to compare different forecasting techniques, the values for w_1 and w_2 are obtained by using Microsoft Excel Solver, such that the MAPE is minimized. The smoothing constant α is also selected by using the solver after obtaining w_1 and w_2 . The optimal values of α are given in Appendix II to Appendix VII for each of three project. The resulting EACs for project 2 and 3 are presented by Figure 4.4 and 4.5 respectively; and the corresponding MAPE are summarized in Table 4.2 and 4.3.

Table 4.2: MAPE of the EAC for the second project

Method		Method	
<i>Cumulative index</i>		<i>Moving average</i>	
	MAPE		MAPE
1. CPI	3.31%	7. CPI	3.41%
2. CPI*SPI (SCI)	6.60%	8. CPI*SPI (SCI)	7.10%
3. Weighted SCI	2.20%	9. Weighted SCI	3.23%
<i>Monthly index</i>		<i>Exponential smoothing</i>	
4. CPI	3.84%	10. CPI	3.52%
5. CPI*SPI (SCI)	16.86%	11. CPI*SPI (SCI)	3.52%
6. Weighted SCI	3.58%	12. Weighted SCI	3.01%

For the second project, the method of using cumulative indices produces the smallest error, and the method of exponential smoothing is the second best. Note that the second project is both cost overrun and schedule delay, the unfavorable CPI ($CPI < 1$) and the SPI ($SPI < 1$) produce lower SCI, therefore it can be seen from the graph that the results obtained by using SCI have the greatest variation.

(a) EAC using CPI as performance index

(b) EAC using SCI as performance index

(c) EAC using weighted SCI as performance index

◆ Using cumulative indices ★ Using monthly indices ▲ Using 3 month average indices
 ■ Using exponential smoothing indices — Actual cost — Planned budget

Figure 4.4: Comparison of EAC for the second project

(a) EAC using CPI as performance index

(b) EAC using SCI as performance index

(c) EAC using weighted SCI as performance index

◆ Using cumulative indices ◆ Using monthly indices ▲ Using 3 month average indices
 ● Using exponential smoothing indices — Actual cost — Planned budget

Figure 4.5: Comparison of EAC for the third project

Table 4.3: MAPE of the EAC for the third project

Method		Method	
<i>Cumulative index</i>		<i>Moving average</i>	
1. CPI	MAPE 2.07%	7. CPI	MAPE 1.22%
2. CPI*SPI (SCI)	4.21%	8. CPI*SPI (SCI)	2.66%
3. Weighted SCI	2.86%	9. Weighted SCI	2.89%
<i>Monthly index</i>		<i>Exponential smoothing</i>	
4. CPI	1.80%	10. CPI	1.81%
5. CPI*SPI (SCI)	4.43%	11. CPI*SPI (SCI)	4.40%
6. Weighted SCI	2.19%	12. Weighted SCI	2.29%

For the third project, using moving average gives the smallest error. The results calculated by SCI still show the greatest variation, but this time instead of higher than the actual value, most of the results are lower than the actual cost. This is because the third project had cost overrun but was finished 1 month earlier than the schedule, the product of SPI and CPI produces higher value than CPI alone, therefore the forecasting of EAC by using SCI gives lower values.

4.5.2 Computational results of duration estimate at completion

Figure 4.6 to Figure 4.8 display the comparison of $EAC(t)$ using 12 methods for the 3 projects. And Table 4.4 to Table 4.6 give the corresponding MAPE.

For the first project, all 12 methods produce relatively large error compared to their cost forecast and the other two projects duration forecast. Observation of the monthly schedule performance indices reveals that the project had little schedule lag during the first half of its construction; however the schedule performance was extremely poor during the second half of the project. This extreme case leads to optimistic forecasting of duration at first and pessimistic forecasting of duration later. This trend can be seen from Figure 4.6. In this case, using the exponential smoothing of monthly SPI gives the best result.

(a) $EAC(t)$ using CPI as performance index

(b) $EAC(t)$ using SCI as performance index

(c) $EAC(t)$ using weighted SCI as performance index

Figure 4.6: Comparison of $EAC(t)$ for the first project

Table 4.4: MAPE of the $EAC(t)$ for the first project

Method		Method	
<i>Cumulative index</i>		<i>Moving average</i>	
	MAPE		MAPE
1. SPI	14.03%	7. SPI	15.96%
2. CPI*SPI (SCI)	15.18%	8. CPI*SPI (SCI)	15.55%
3. Weighted SCI	14.03%	9. Weighted SCI	13.01%
<i>Monthly index</i>		<i>Exponential smoothing</i>	
4. SPI	20.34%	10. SPI	13.22%
5. CPI*SPI (SCI)	27.04%	11. CPI*SPI (SCI)	15.97%
6. Weighted SCI	16.75%	12. Weighted SCI	19.39%

Table 4.5: MAPE of the $EAC(t)$ for the second project

Method		Method	
<i>Cumulative index</i>		<i>Moving average</i>	
	MAPE		MAPE
1. SPI(t)	7.54%	7. SPI(t)	7.35%
2. CPI*SPI(t) (SCI)	6.02%	8. CPI*SPI(t) (SCI)	7.68%
3. Weighted SCI	7.50%	9. Weighted SCI	7.30%
<i>Monthly index</i>		<i>Exponential smoothing</i>	
4. SPI(t)	13.29%	10. SPI(t)	7.25%
5. CPI*SPI(t) (SCI)	14.76%	11. CPI*SPI(t) (SCI)	5.09%
6. Weighted SCI	8.38%	12. Weighted SCI	10.42%

For the second project, using the exponential smoothing of SCI gives the smallest error. However, there are relatively small differences between methods of cumulative indices, moving average indices and exponential smoothing indices.

(a) $EAC(t)$ using CPI as performance index

(b) $EAC(t)$ using SCI as performance index

(c) EAC using weighted SCI as performance index

Figure 4.7: Comparison of $EAC(t)$ for the second project

◆ Using cumulative indices ◆ Using monthly indices ◆ Using 3 month average indices
 ◆ Using exponential smoothing indices — Actual cost

Figure 4.8: Comparison of EAC(t) for the third project

Table 4.6: MAPE of the $EAC(t)$ for the third project

Method		Method	
<i>Cumulative index</i>		<i>Moving average</i>	
	MAPE		MAPE
1. SPI	4.64%	7. SPI	3.97%
2. CPI*SPI (SCI)	5.17%	8. CPI*SPI (SCI)	4.69%
3. Weighted SCI	4.64%	9. Weighted SCI	3.97%
<i>Monthly index</i>		<i>Exponential smoothing</i>	
4. SPI	5.02%	10. SPI	4.64%
5. CPI*SPI (SCI)	4.93%	11. CPI*SPI (SCI)	3.48%
6. Weighted SCI	4.97%	12. Weighted SCI	4.64%

For the third project, the best result is obtained by using exponential smoothing of weighted SCI. Using cumulative indices also gives good results.

4.6 Risk assessment

4.6.1 Confidence interval by using Cooper's method

By applying Equation 3-39 and 3-40, at the end of each period expected value and standard deviation can be obtained for EAC and $EAC(t)$. For example, at the end of the 4th month, the EACs and $EAC(t)$ s for the first project are calculated by using the 12 methods and are summarized in Table 4.7.

Table 4.7: EAC and $EAC(t)$ for the first project at the end of 4th month

	Cost (Thousands of €)		Duration (Month)
EAC_1	357,365	$EAC(t)_1$	9.69
EAC_2	375,225	$EAC(t)_2$	9.64
EAC_3	357,365	$EAC(t)_3$	9.69
EAC_4	422,639	$EAC(t)_4$	9.45
EAC_5	431,824	$EAC(t)_5$	10.91
EAC_6	419,706	$EAC(t)_6$	9.55
EAC_7	321,103	$EAC(t)_7$	9.60
EAC_8	332,904	$EAC(t)_8$	8.69
EAC_9	349,023	$EAC(t)_9$	9.36
EAC_{10}	349,379	$EAC(t)_{10}$	9.77
EAC_{11}	374,641	$EAC(t)_{11}$	9.32
EAC_{12}	354,981	$EAC(t)_{12}$	9.01

The lowest and highest values for EAC are 321,103 and 431,824 respectively; the lowest and highest values for EAC(*t*) are 8.69 and 10.91 respectively. The most likely value used in Equation 3-39 should be based on past experience and review of project data (Stephenson, 2004). In the calculations below, 357,000 for cost and 9.6 for schedule, which are close to the median values were assumed to be the most likely values to calculate the mean and the standard deviation.

$$E(EAC) = (321,103 + 2 \times 357,000 + 431,824) / 4 = 366,732$$

$$SD(EAC) = (431,824 - 321,103) / 2.65 = 41,782$$

$$E(EAC(t)) = (8.69 + 2 \times 9.6 + 10.91) / 4 = 9.7$$

$$SD(EAC(t)) = (10.91 - 8.69) / 2.65 = 0.84$$

Therefore, the 95.5% confidence interval for EAC estimated at the end of month 4 is [283,168 450,296], and the 95.5% confidence interval for EAC(*t*) estimated at the end of month four is [8.02 11.38].

Confidence intervals were calculated for each project at completion of 25%, 50%, 75%, and 90%, which are the ratio of earned value to date to the planned value. The results are summarized in Table 4.8 to Table 4.10.

Table 4.8: Confidence interval of EAC and EAC(*t*) of the first project

Percent Completion	25%	50%	75%	90%	100% (Actual Value)
E(EAC)	307,312	343,124	355,521	351,041	349,379
SD(EAC)	49,260	13,935	7,074	11,209	--
95.5% CI of cost	[208,791 to 405,833]	[315,254 to 370,995]	[341,372 to 369,670]	[328,623 to 373,459]	--
E(EAC(<i>t</i>))	8.68	9.06	9.64	12.73	13.00
SD(EAC(<i>t</i>))	1.27	0.39	0.29	0.95	--
95.5% CI of duration	[6.13, 11.22]	[8.28, 9.84]	[9.05, 10.23]	[10.82, 14.63]	--

Table 4.9: Confidence interval of EAC and EAC(*t*) of the second project

Percent Completion	25%	50%	75%	90%	100% (Actual Value)
E(EAC)	3,079	3,183	3,475	3,347	3,247
SD(EAC)	297	154	350	95	--
95.5% CI of cost	[2,484 to 3,674]	[2,876 to 3,490]	[2,775 to 4,176]	[3,158 to 3,536]	--
E(EAC(<i>t</i>))	9.76	10.58	12.52	11.88	12.00
SD(EAC(<i>t</i>))	1.10	0.52	1.38	0.44	--
95.5% CI of duration	[7.56, 11.96]	[9.54, 11.62]	[9.76, 15.27]	[11.01, 12.75]	--

Table 4.10: Confidence interval of EAC and EAC(*t*) of the third project

Percent Completion	25%	50%	75%	90%	100% (Actual Value)
E(EAC)	935	912	916	941	952
SD(EAC)	22	22	11	9	--
95.5% CI of cost	[890, 979]	[867, 956]	[894, 937]	[923, 958]	--
E(EAC(<i>t</i>))	9.90	9.57	9.20	8.75	9.00
SD(EAC(<i>t</i>))	0.35	0.21	0.14	0.21	--
95.5% CI of duration	[9.19, 10.60]	[9.15, 9.98]	[8.92, 9.49]	[8.33, 9.17]	--

4.6.2 Confidence interval by using Lipke's method

By applying Equation 3-41 to Equation 3-43, confidence intervals can be obtained at the end of each period. In Lipke's model, since the confidence intervals are logarithms of the cumulative indices, the conversion is required to calculate the bounds for final cost and duration. To show the application of Lipke's method, the calculations for the first project at the end of month 4 are illustrated. The cumulative and monthly values for performance indices have been obtained for the first 4 months, with which the logarithms for the monthly indices can be easily calculated. Table 4-11 summaries these values.

Table 4.11: Logarithms of monthly indices for the first 4 months of the first project

Month	1	2	3	4
CPI_c	1.0031	1.0269	1.1385	1.0094
CPI_m	1.0031	1.0418	1.7551	0.7888
$Ln(CPI_m)$	0.0030	0.0410	0.5625	-0.2373
$SPI(t)_c$	0.8851	0.8709	0.9151	0.9289
$SPI(t)_m$	0.8851	0.8568	1.0036	0.9702
$Ln[SPI(t)_m]$	-0.1221	-0.1546	0.0036	-0.0303

To calculate standard deviations, logarithms of cumulative indices are required. For cost, the value is 0.0094 ($\ln 1.0094 = 0.0094$); and -0.0738 ($\ln 0.9289 = -0.0738$) for schedule.

Then by using Equation 3-41, standard deviations can be obtained.

$$\begin{aligned}
\sigma_{cost} &= \sqrt{\frac{\sum_{i=1}^4 (\ln CPI(i)_m - \ln CPI_c)^2}{4-1}} \\
&= \sqrt{\frac{(0.003 - 0.0094)^2 + (0.041 - 0.0094)^2 + (0.5625 - 0.0094)^2 + (-0.2373 - 0.0094)^2}{4-1}} \\
&= 0.3502 \\
\sigma_{schedule} &= \sqrt{\frac{\sum_{i=1}^4 (\ln SPI(i)_m - \ln SPI_c)^2}{4-1}} = \\
&= \sqrt{\frac{(-0.1221 + 0.0738)^2 + (-0.1546 + 0.0738)^2 + (0.0036 + 0.0738)^2 + (-0.0303 + 0.0738)^2}{4-1}} = 0.0747
\end{aligned}$$

Adjusted factors can be calculated by using Equation 3-42 and 3-43.

$$AF_c = \sqrt{(BAC - EV_4)/(BAC - (EV_4/4))} = \sqrt{(360,738 - 125,244)/(360,738 - (125,244/4))} = 0.8455$$

$$AF_s = \sqrt{(PD - ES_4)/(PD - (ES_4/n))} = \sqrt{(9 - 3.716)/(9 - (3.716/4))} = 0.8091$$

Then by using Equation 3-44, 95.5% confidence interval can be obtained.

$$CI_1 = \ln CPI_c \pm Z^* \sigma_{cost} / \sqrt{4} * AF_{cost} = 0.0094 \pm 2*0.3502/2*0.8455 = 0.0094 \pm 0.2961$$

$$CI_2 = \ln SPI(t)_c \pm Z^* \sigma_{schedule} / \sqrt{4} * AF_{schedule} = -0.0738 \pm 2*0.0747/2*0.8091 = -0.0738 \pm 0.0604$$

Finally, by raising the natural number “e” to the power of the extreme values of indices, the bounds for final cost and duration can be obtained.

$$EAC_H = BAC / e^{-0.2867} = 360,738 / 0.7507 = 480,512$$

$$EAC_L = BAC / e^{0.3055} = 360,738 / 1.3573 = 265,775$$

$$EAC(t)_H = PD / e^{-0.1342} = 9 / 0.8744 = 10.29$$

$$EAC(t)_L = PD / e^{-0.0134} = 9 / 0.9867 = 9.12$$

To compare with Cooper’s method, confidence intervals were calculated for each project by applying Lipke’s method. The results are represented in Table 4.12 to Table 4.14.

Table 4.12: Confidence interval by using Lipke’s method of the first project

	25%	50%	75%	90%	100% (Actual Value)
In index _c	0.130	0.038	0.034	0.035	
Sigma	0.325	0.298	0.267	0.458	
Adjusted Factor	0.906	0.710	0.540	0.317	
95.5% CI of cost	[225,547 445,139]	[287,453 419,681]	[310,028 392,349]	[317,715 381,915]	349,379
In index _c	-0.089	-0.050	-0.069	-0.331	
Sigma	0.084	0.083	0.088	0.558	
Adjusted Factor	0.880	0.726	0.649	0.468	
95.5% CI of duration	[9.03, 10.71]	[8.97, 9.98]	[9.20, 10.10]	[10.62, 14.78]	13.00

Table 4.13: Confidence interval by using Lipke’s method of the second project

	25%	50%	75%	90%	100% (Actual Value)
In index _c	-0.080	-0.072	-0.113	-0.131	
Sigma	0.031	0.105	0.115	0.120	
Adjusted Factor	0.902	0.742	0.513	0.344	
95.5% CI of cost	[3,013 3,217]	[2,883 3,313]	[3,086 3,354]	[3,194 3,365]	3,247
In index _c	-0.126	-0.166	-0.255	-0.267	
Sigma	0.510	0.369	0.333	0.330	
Adjusted Factor	0.885	0.764	0.584	0.404	
95.5% CI of duration	[6.06, 17.18]	[8.25, 13.67]	[10.12, 13.61]	[10.81, 12.80]	12.00

Table 4.14: Confidence interval by using Lipke's method of the third project

	25%	50%	75%	90%	100% (Actual Value)
In index _c	-0.029	-0.025	-0.026	-0.041	
Sigma	0.044	0.035	0.050	0.066	
Adjusted Factor	0.931	0.791	0.467	0.292	
95.5% CI of cost	[890, 978]	[903, 955]	[912, 947]	[930, 958]	952
In index _c	0.018	0.040	0.079	0.125	
Sigma	0.042	0.071	0.079	0.130	
Adjusted Factor	0.879	0.807	0.627	0.483	
95.5% CI of duration	[9.41, 10.25]	[9.07, 10.17]	[8.87, 9.62]	[8.42, 9.25]	9.00

4.6.3 Comparison of Cooper's method and Lipke's method

The comparisons between Cooper's method and Lipke's method to predict the confidence interval of EAC were graphed using Figure 4.9 to Figure 4.11.

Figure 4.9: Comparison of confidence interval of EAC for the first project **Figure 4.10:** Comparison of confidence interval of EAC for the second project

Figure 4.11: Comparison of confidence interval of EAC for the third project

It can be seen from above figures that the predicted confidence intervals are more accurate when the projects close to the end. However, neither method is consistently better than the other. Lipke's method gave better result for the 2nd and the 3rd projects, but not for the 1st project.

The comparisons between Cooper's method and Lipke's method to predict the confidence interval of $EAC(t)$ were given by Figure 4.12 to Figure 4.14.

Figure 4.12: Comparison of confidence interval of $EAC(t)$ for the first project

Figure 4.13: Comparison of confidence interval of $EAC(t)$ for the second project

Figure 4.14: Comparison of confidence interval of $EAC(t)$ for the third project

4.6.4 Joint and conditional probability for cost and schedule

Often times, decision makers may want to know “What is the chance the project can be delivered within given cost and schedule?” or “What the cost might be for a given schedule?”. The bivariate normal distribution equations presented previously can be used to answer these questions. For example, at the end of the 4th month, the project manager wants to know what the probability would be if the given cost and schedule are less than €370,000 and 10 months respectively, and what the expected cost might be if the project should be finished with the duration of 10 months. By applying Equation 3-54 and 3-49 these two question can be

answered.

$$P(X_I \leq 370000 \text{ and } X_2 \leq 10) = \int_{a_2}^{b_2} \int_{a_1}^{b_1} \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} e^{-\frac{1}{2}w} dx_1 dx_2$$

$$= \int_0^{10} \int_0^{350000} \frac{1}{2\pi * 41728 * 0.84\sqrt{1-0.5^2}} e^{-\frac{1}{2}w} dx_1 dx_2$$

$$\text{where } w = \frac{1}{1-\rho^2} \left\{ \left(\frac{x_1 - \mu_1}{\sigma_1} \right)^2 - 2\rho \left(\frac{x_1 - \mu_1}{\sigma_1} \right) \left(\frac{x_2 - \mu_2}{\sigma_2} \right) + \left(\frac{x_2 - \mu_2}{\sigma_2} \right)^2 \right\}$$

In section 4.6.1, it has been calculated that $\mu_1=366,732$, $\mu_2=9.7$, $\sigma_1=41,782$, and $\sigma_2=0.84$.

The calculation of ρ (Equation 3-53) at the end of month 4 for the first project is shown in below table.

Table 4.15: Calculation of correlation between cost and schedule

	Actual cost (x_1)	Duration (x_2)	x_1^2	x_2^2	$x_1 * x_2$
	25,567	1	653671489	1	25,567
	66,293	2	4.395E+09	4	132,586
	78,293	3	6.13E+09	9	234,879
	124,073	4	1.539E+10	16	496,292
Total	294,226	10	2.657E+10	30	889,324

By applying Equation 3-53:

$$\rho = \frac{N \sum x_1 x_2 - \sum x_1 \sum x_2}{\sqrt{(N \sum x_1^2 - (\sum x_1)^2)(N \sum x_2^2 - (\sum x_2)^2)}} = \frac{4 * 889,324 - 294,226 * 10}{\sqrt{(42.657E+10 - (294,226)^2) * (4 * 30 - (10)^2)}} = 0.9793$$

$$\text{Therefore, } w = \frac{1}{1-0.979^2} \left\{ \left(\frac{x_1 - 366732}{41782} \right)^2 - 2 * 0.979 * \left(\frac{x_1 - 366732}{41782} \right) \left(\frac{x_2 - 9.7}{0.84} \right) + \left(\frac{x_2 - 9.7}{0.84} \right)^2 \right\}$$

The probability is obtained as $P(X_1 \leq 380,000 \text{ and } X_2 \leq 10) = 60.08\%$.

The conditional mean and variance are calculated as:

$$\begin{aligned} E(X_1 | x_2 = 10) &= \mu_1 + \frac{\sigma_1}{\sigma_2} \rho (x_2 - \mu_2) \\ &= 366,732 + (41,782/0.84) * 0.979 * (10 - 9.7) = \text{€}381,341 \end{aligned}$$

$$\sigma(X_1 | x_2 = 10) = \sigma_1 \sqrt{1 - \rho^2} = 41,782 * 0.204 = 8,524$$

$$P(X_1 | x_2=10) = 44\%$$

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

To predict the final cost and duration of projects throughout the whole construction stage, 12 methods have been applied and compared. These 12 methods include the technique of using cumulative indices, periodic indices, composite indices, moving average, and exponential smoothing. For the given three project data sets, it was observed that no one method constantly outperformed the other methods. However, it was found that using monthly periodic indices to predict cost and duration gave the least accurate results for all three projects. Three out of six best results were obtained by using exponential smoothing; two out of six best results were obtained by using moving average; and one out of six best results were obtained by using cumulative index. Although by using cumulative indices only produced 17% of the best results, it was found that the results were near the best one. In addition, applying cumulative indices is the simplest method to obtain an acceptable result. Therefore, when a quick and simple method is what the project team desires, it is recommended to use cumulative CPI to predict final cost:

$$EAC = ACWP_c + [(BAC - BCWP_c)/CPI_c]$$

and cumulative SPI(t) to predict final duration:

$$EAC(t) = AD + [(PD - ES)/SPI(t)_c]$$

In the situation where more accurate result is required, more sophisticated techniques such as exponential smoothing might produce better estimates. Without calculating joint and

conditional probability for cost and schedule, Lipke's method is recommended for obtaining confidence intervals, since it only requires cumulative index and periodic indices to calculate mean and standard deviation. The results obtained by using Lipke's method were slightly better than Cooper's method for the given data sets. By using Cooper's method, there were 7 points outside the confidence limits; by using Lipke's method, there were 6 points outside the confidence limits.

5.2 Recommendations for future study

- a. The conclusions given were based on the study of only three projects given in the literature, which is a small sample. The proposed method can be investigated using more real project data;
- b. The simple exponential smoothing was used, which is applicable to constant processes. However, the monthly indices may not be constant over periods, higher order exponential smoothing can be studied for the given data sets;
- c. To further study the relationship between cost and schedule;
- d. To develop a program that will allow users to input different project data and obtain results.

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APPENDIX I - DATA SETS

Project 1 – Re-vamp check-in islands

	Jun-02	Jul-02	Aug-02	Sep-02	Oct-02	Nov-02	Dec-02	Jan-03	Feb-03	Mar-03	Apr-03	May-03	Jun-03
BCWSc	28,975	81,681	91,681	138,586	218,141	302,478	323,632	345,876	360,738	360,738	360,738	360,738	360,738
BCWPc	25,645	68,074	89,135	125,244	198,754	268,763	292,469	306,725	312,864	327,694	338,672	349,861	360,738
ACWPc	25,567	66,293	78,293	124,073	191,367	259,845	285,612	290,843	303,489	316,431	320,690	336,756	349,379

Project 2 – Link lines (costs in thousands of €)

	Sep-03	Oct-03	Nov-03	Dec-03	Jan-04	Feb-04	Mar-04	Apr-04	May-04	Jun-04	Jul-04	Aug-04
BCWSc	375	525	850	1,355	1,768	2,125	2,452	2,625	2,875	2,875	2,875	2,875
BCWPc	325	427	735	1,025	1,453	1,774	2,024	2,190	2,356	2,565	2,735	2,875
ACWPc	344	452	796	1,056	1,562	1,922	2,256	2,451	2,676	2,925	3,138	3,247

Project 3 – Transfer lines (costs in thousands of €)

	Mar-03	Apr-03	May-03	Jun-03	Jul-03	Aug-03	Sep-03	Oct-03	Nov-03	Dec-03
BCWSc	34	87	157	373	549	673	798	842	876	906
BCWPc	36	93	169	402	597	735	839	887	906	906
ACWPc	35	95	174	412	623	754	874	932	952	952

APPENDIX II – PROJECT 1 ESTIMATE COST AT COMPLETION

Planned Budget 360,738
Actual Duration 13
Actual Cost 349,379

$$\alpha = 0.08$$

	Jun-02	Jul-02	Aug-02	Sep-02	Oct-02	Nov-02	Dec-02	Jan-03	Feb-03	Mar-03	Apr-03	May-03	Jun-03
BCWS _c	28,975	81,681	91,681	138,586	218,141	302,478	323,632	345,876	360,738	360,738	360,738	360,738	360,738
BCWP _c	25,645	68,074	89,135	125,244	198,754	268,763	292,469	306,725	312,864	327,694	338,672	349,861	360,738
ACWP _c	25,567	66,293	78,293	124,073	191,367	259,845	285,612	290,843	303,489	316,431	320,690	336,756	349,379
AD	1	2	3	4	5	6	7	8	9	10	11	12	13
PD	9	9	9	9	9	9	9	9	9	9	9	9	9
ES	0.885	1.742	2.745	3.716	4.756	5.600	5.881	6.201	6.491	7.183	7.676	8.268	9.000
CPI _c	1.003	1.027	1.138	1.009	1.039	1.034	1.024	1.055	1.031	1.036	1.056	1.039	1.033
SPI _c	0.885	0.833	0.972	0.904	0.911	0.889	0.904	0.887	0.867	0.908	0.939	0.970	1.000
SPI(t) _c	0.885	0.871	0.915	0.929	0.951	0.933	0.840	0.775	0.721	0.718	0.698	0.689	0.692
SCI(t) _c	0.888	0.894	1.042	0.938	0.988	0.965	0.860	0.817	0.743	0.744	0.737	0.716	0.715
Weighted SCI(t) _c (w ₁ =0, w ₂ =1)	1.003	1.027	1.138	1.009	1.039	1.034	1.024	1.055	1.031	1.036	1.056	1.039	1.033
CPI _m	1.003	1.042	1.755	0.789	1.092	1.022	0.920	2.725	0.485	1.146	2.578	0.696	0.862
SPI _m	0.885	0.857	1.004	0.970	1.041	0.844	0.281	0.319	0.290	0.692	0.494	0.592	0.732
SCI _m	0.888	0.893	1.761	0.765	1.137	0.863	0.259	0.871	0.141	0.793	1.272	0.412	0.631
Weighted SCI _m (w ₁ =0.2, w ₂ =0.8)	0.998	1.034	1.723	0.797	1.090	1.015	0.892	2.622	0.477	1.126	2.488	0.692	0.856
CPI _{MA}			1.267	1.195	1.212	0.968	1.012	1.556	1.377	1.452	1.403	1.473	1.379
SPI _{MA}			0.915	0.943	1.005	0.952	0.722	0.481	0.297	0.434	0.492	0.592	0.606
SCI _{MA}			1.159	1.128	1.218	0.921	0.730	0.749	0.409	0.630	0.690	0.873	0.835
Weighted SCI _{MA} (w ₁ =0.2, w ₂ =0.8)			1.059	1.047	1.090	0.958	0.841	0.923	0.740	0.852	0.866	0.954	0.923
CPI _{EX}	1.003	1.006	1.068	1.045	1.049	1.047	1.036	1.177	1.119	1.121	1.242	1.197	1.169
SPI _{EX}	0.885	0.883	0.893	0.899	0.911	0.905	0.854	0.809	0.766	0.760	0.738	0.726	0.726
SCI _{EX}	0.888	0.888	0.954	0.940	0.956	0.948	0.885	0.952	0.857	0.852	0.917	0.869	0.849
Weighted SCI _{EX} (w ₁ =0.2, w ₂ =0.8)	0.998	0.985	1.038	1.020	1.025	1.022	1.005	1.113	1.058	1.059	1.155	1.115	1.092
EAC ₁	359,641	351,300	316,859	357,365	347,331	348,768	352,280	342,059	349,928	348,339	341,584	347,226	349,379
EAC ₂	403,020	393,543	338,983	375,225	355,322	355,116	364,961	356,920	367,879	360,855	350,632	351,951	349,379
EAC ₃	359,641	351,300	316,859	357,365	347,331	348,768	352,280	342,059	349,928	348,339	341,584	347,226	349,379
EAC ₄	359,641	347,210	233,045	422,639	339,654	349,809	359,816	310,662	402,107	345,268	329,251	352,374	349,379
EAC ₅	403,020	394,177	232,495	431,824	333,847	366,446	549,603	352,886	643,310	358,125	338,036	363,138	349,379
EAC ₆	367,689	357,558	247,539	409,510	341,068	353,062	371,785	314,912	410,733	347,751	330,902	352,857	349,379
EAC ₇			292,719	321,103	325,010	354,878	353,099	325,558	338,258	339,185	336,418	344,139	349,379
EAC ₈			312,604	332,904	324,368	359,710	379,095	362,943	420,591	368,889	352,671	349,219	349,379
EAC ₉			305,320	329,768	329,742	355,197	357,199	331,121	344,727	342,898	338,766	345,141	349,379
EAC ₁₀	359,641	357,134	332,498	349,379	345,765	347,699	351,485	336,749	346,264	345,897	338,452	345,843	349,379
EAC ₁₁	403,020	395,782	363,851	375,158	361,297	357,079	362,713	347,664	359,162	355,091	344,755	349,247	349,379
EAC ₁₂	367,689	363,497	340,540	355,285	349,603	349,918	353,593	339,591	348,835	347,704	339,896	346,542	349,379

APPENDIX III – PROJECT 1 ESTIMATE DURATION AT COMPLETION

Planned Budget 360,738
Actual Duration 13
Actual Cost 349,379

$\alpha = 0.19$

	Jun-02	Jul-02	Aug-02	Sep-02	Oct-02	Nov-02	Dec-02	Jan-03	Feb-03	Mar-03	Apr-03	May-03	Jun-03
BCWS _c	28,975	81,681	91,681	138,586	218,141	302,478	323,632	345,876	360,738	360,738	360,738	360,738	360,738
BCWP _c	25,645	68,074	89,135	125,244	198,754	268,763	292,469	306,725	312,864	327,694	338,672	349,861	360,738
ACWP _c	25,567	66,293	78,293	124,073	191,367	259,845	285,612	290,843	303,489	316,431	320,690	336,756	349,379
AD	1	2	3	4	5	6	7	8	9	10	11	12	13
PD	9	9	9	9	9	9	9	9	9	9	9	9	9
ES	0.885	1.742	2.745	3.716	4.756	5.600	5.881	6.201	6.491	7.183	7.676	8.268	9.000
CPI _c	1.003	1.027	1.138	1.009	1.039	1.034	1.024	1.055	1.031	1.036	1.056	1.039	1.033
SPI _c	0.885	0.833	0.972	0.904	0.911	0.889	0.904	0.887	0.867	0.908	0.939	0.970	1.000
SPI(t) _c	0.885	0.871	0.915	0.929	0.951	0.933	0.840	0.775	0.721	0.718	0.698	0.689	0.692
SCI(t) _c	0.888	0.894	1.042	0.938	0.988	0.965	0.860	0.817	0.743	0.744	0.737	0.716	0.715
Weighted SCI(t) _c (w ₁ =0.9, w ₂ =0.1)	1.003	1.027	1.138	1.009	1.039	1.034	1.024	1.055	1.031	1.036	1.056	1.039	1.033
CPI _m	1.003	1.042	1.755	0.789	1.092	1.022	0.920	2.725	0.485	1.146	2.578	0.696	0.862
SPI _m	0.885	0.857	1.004	0.970	1.041	0.844	0.281	0.319	0.290	0.692	0.494	0.592	0.732
SCI _m	0.888	0.893	1.761	0.765	1.137	0.863	0.259	0.871	0.141	0.793	1.272	0.412	0.631
Weighted SCI _m (w ₁ =0.4, w ₂ =0.6)	0.998	1.034	1.723	0.797	1.090	1.015	0.892	2.622	0.477	1.126	2.488	0.692	0.856
CPI _{MA}			1.267	1.195	1.212	0.968	1.012	1.556	1.377	1.452	1.403	1.473	1.379
SPI _{MA}			0.915	0.943	1.005	0.952	0.722	0.481	0.297	0.434	0.492	0.592	0.606
SCI _{MA}			1.159	1.128	1.218	0.921	0.730	0.749	0.409	0.630	0.690	0.873	0.835
Weighted SCI _{MA} (w ₁ =0.8, w ₂ =0.2)			1.059	1.047	1.090	0.958	0.841	0.923	0.740	0.852	0.866	0.954	0.923
CPI _{EX}	1.003	1.006	1.068	1.045	1.049	1.047	1.036	1.177	1.119	1.121	1.242	1.197	1.169
SPI _{EX}	0.885	0.883	0.893	0.899	0.911	0.905	0.854	0.809	0.766	0.760	0.738	0.726	0.726
SCI _{EX}	0.888	0.888	0.954	0.940	0.956	0.948	0.885	0.952	0.857	0.852	0.917	0.869	0.849
Weighted SCI _{EX} (w ₁ =0.2, w ₂ =0.8)	0.998	0.985	1.038	1.020	1.025	1.022	1.005	1.113	1.058	1.059	1.155	1.115	1.092
EAC(t) ₁	10.2	10.3	9.8	9.7	9.5	9.6	10.7	11.6	12.5	12.5	12.9	13.1	13.0
EAC(t) ₂	10.1	10.1	9.0	9.6	9.3	9.5	10.6	11.4	12.4	12.4	12.8	13.0	13.0
EAC(t) ₃	10.2	10.3	9.8	9.7	9.5	9.6	10.7	11.6	12.5	12.5	12.9	13.1	13.0
EAC(t) ₄	10.2	10.5	9.2	9.4	9.1	10.0	18.1	16.8	17.6	12.6	13.7	13.2	13.0
EAC(t) ₅	10.1	10.1	6.6	10.9	8.7	9.9	19.1	11.2	26.8	12.3	12.0	13.8	13.0
EAC(t) ₆	10.0	10.3	8.8	9.6	9.1	9.9	16.0	13.0	17.1	12.5	12.9	13.2	13.0
EAC(t) ₇			9.8	9.6	9.2	9.6	11.3	13.8	17.5	14.2	13.7	13.2	13.0
EAC(t) ₈			8.4	8.7	8.5	9.7	11.3	11.7	15.1	12.9	12.9	12.8	13.0
EAC(t) ₉			9.4	9.4	9.1	9.6	11.0	12.2	14.2	13.0	13.1	13.0	13.0
EAC(t) ₁₀	10.2	10.3	9.9	9.8	9.5	9.7	10.9	12.0	13.0	12.8	13.2	13.2	13.0
EAC(t) ₁₁	10.1	10.2	9.0	9.3	9.2	9.4	10.7	10.9	12.3	12.4	12.5	12.9	13.0
EAC(t) ₁₂	10.0	9.3	8.6	9.0	9.0	9.2	10.1	10.2	11.3	11.7	12.0	12.6	13.0

APPENDIX IV - PROJECT 2 ESTIMATE COST AT COMPLETION

Planned Budget 2875
Actual Duration 12
Actual Cost 3247

$\alpha = 0.07$

	Sep-03	Oct-03	Nov-03	Dec-03	Jan-04	Feb-04	Mar-04	Apr-04	May-04	Jun-04	Jul-04	Aug-04
BCWS _c	375	525	850	1355	1768	2125	2452	2625	2875	2875	2875	2875
BCWP _c	325	427	735	1025	1453	1774	2024	2190	2356	2565	2735	2875
ACWP _c	344	452	796	1056	1562	1922	2256	2451	2676	2925	3138	3247
AD	1	2	3	4	5	6	7	8	9	10	11	12
PD	9	9	9	9	9	9	9	9	9	9	9	9
ES	0.87	1.35	2.65	3.35	4.24	5.02	5.72	6.20	6.71	7.65	8.44	9.00
CPI _c	0.945	0.945	0.923	0.971	0.930	0.923	0.897	0.894	0.880	0.877	0.872	0.885
SPI _c	0.867	0.813	0.865	0.756	0.822	0.835	0.825	0.834	0.819	0.892	0.951	1.000
SPI(t) _c	0.867	0.673	0.882	0.837	0.847	0.836	0.817	0.775	0.745	0.765	0.767	0.750
SCI(t) _c	0.819	0.636	0.814	0.812	0.788	0.772	0.733	0.692	0.656	0.671	0.669	0.664
Weighted SCI(t) _c (w ₁ =0.3, w ₂ =0.7)	0.923	0.869	0.912	0.933	0.907	0.899	0.875	0.861	0.843	0.846	0.843	0.848
CPI _m	0.945	0.944	0.895	1.115	0.846	0.892	0.749	0.851	0.738	0.839	0.798	1.284
SPI _m	0.867	0.480	1.299	0.700	0.891	0.780	0.700	0.482	0.508	0.947	0.787	0.560
SCI _m	0.819	0.453	1.163	0.781	0.753	0.695	0.524	0.410	0.375	0.795	0.628	0.719
Weighted SCI _m (w ₁ =0.1, w ₂ =0.9)	0.933	0.876	0.955	1.054	0.852	0.875	0.741	0.797	0.704	0.855	0.796	1.177
CPI _{MA}			0.928	0.985	0.952	0.951	0.829	0.830	0.779	0.809	0.792	0.974
SP _{MA}			0.882	0.827	0.964	0.790	0.790	0.654	0.563	0.645	0.747	0.765
SCI _{MA}			0.819	0.814	0.917	0.751	0.655	0.543	0.439	0.522	0.591	0.745
Weighted SCI _{MA} (w ₁ =0.7, w ₂ =0.3)			0.894	0.867	0.961	0.831	0.800	0.699	0.618	0.687	0.758	0.818
CPI _{EX}	0.945	0.945	0.941	0.954	0.946	0.942	0.928	0.922	0.909	0.904	0.896	0.924
SPI _{EX}	0.867	0.839	0.872	0.860	0.862	0.856	0.845	0.818	0.796	0.807	0.805	0.787
SCI _{EX}	0.819	0.792	0.821	0.820	0.815	0.806	0.784	0.755	0.723	0.729	0.722	0.728
Weighted SCI _{EX} (w ₁ =0.2, w ₂ =0.8)	0.933	0.926	0.929	0.937	0.931	0.927	0.913	0.904	0.889	0.887	0.880	0.901
EAC ₁	3043	3043	3114	2962	3091	3115	3205	3218	3265	3279	3299	3247
EAC ₂	3458	4301	3424	3334	3366	3349	3417	3440	3467	3387	3347	3247
EAC ₃	3107	3268	3143	3038	3129	3147	3229	3247	3292	3291	3304	3247
EAC ₄	3043	3044	3186	2715	3243	3157	3393	3256	3379	3294	3313	3247
EAC ₅	3458	5852	2635	3424	3449	3506	3880	4122	4062	3315	3361	3247
EAC ₆	3076	3247	3037	2811	3230	3180	3404	3311	3413	3287	3314	3247
EAC ₇			3102	2934	3055	3080	3283	3276	3342	3308	3315	3247
EAC ₈			3410	3328	3112	3387	3556	3713	3859	3518	3375	3247
EAC ₉			3190	3190	3042	3247	3320	3432	3516	3376	3323	3247
EAC ₁₀	3043	3043	3070	2996	3065	3091	3173	3194	3247	3268	3294	3247
EAC ₁₁	3458	3542	3403	3312	3306	3288	3342	3359	3394	3350	3332	3247
EAC ₁₂	3076	3095	3099	3029	3089	3110	3188	3209	3260	3275	3297	3247

Note: costs in thousands of €

APPENDIX V - PROJECT 2 ESTIMATE DURATION AT COMPLETION

Budget 2875
Duration 12
Cost 3247

$\alpha = 0.15$

	Sep-03	Oct-03	Nov-03	Dec-03	Jan-04	Feb-04	Mar-04	Apr-04	May-04	Jun-04	Jul-04	Aug-04
BCWS _c	375	525	850	1355	1768	2125	2452	2625	2875	2875	2875	2875
BCWP _c	325	427	735	1025	1453	1774	2024	2190	2356	2565	2735	2875
ACWP _c	344	452	796	1056	1562	1922	2256	2451	2676	2925	3138	3247
AD	1	2	3	4	5	6	7	8	9	10	11	12
PD	9	9	9	9	9	9	9	9	9	9	9	9
ES	0.87	1.35	2.65	3.35	4.24	5.02	5.72	6.20	6.71	7.65	8.64	9.00
CPI _c	0.945	0.945	0.923	0.971	0.930	0.923	0.897	0.894	0.880	0.877	0.872	0.885
SPI _c	0.867	0.813	0.865	0.756	0.822	0.835	0.825	0.834	0.819	0.892	0.951	1.000
SPI(t) _c	0.867	0.673	0.882	0.837	0.847	0.836	0.817	0.775	0.745	0.765	0.785	0.750
SCI(t) _c	0.819	0.636	0.814	0.812	0.788	0.772	0.733	0.692	0.656	0.671	0.684	0.664
Weighted SCI(t) _c (w ₁ =0.9, w ₂ =0.1)	0.878	0.712	0.888	0.856	0.859	0.849	0.828	0.792	0.765	0.781	0.797	0.769
CPI _m	0.945	0.944	0.895	1.115	0.846	0.892	0.749	0.851	0.738	0.839	0.798	1.284
SPI _m	0.867	0.480	1.299	0.700	0.891	0.780	0.700	0.482	0.508	0.947	0.983	0.364
SCI _m	0.819	0.453	1.163	0.781	0.753	0.695	0.524	0.410	0.375	0.795	0.784	0.468
Weighted SCI _m (w ₁ =0.4, w ₂ =0.6)	0.915	0.765	1.051	0.955	0.863	0.848	0.730	0.709	0.649	0.881	0.869	0.930
CPI _{MA}			0.928	0.985	0.952	0.951	0.829	0.830	0.779	0.809	0.792	0.974
SPI _{MA}			0.882	0.827	0.964	0.790	0.790	0.654	0.563	0.645	0.812	0.765
SCI _{MA}			0.819	0.814	0.917	0.751	0.655	0.543	0.439	0.522	0.643	0.745
Weighted SCI _{MA} (w ₁ =0.8, w ₂ =0.2)			0.890	0.854	0.962	0.818	0.797	0.684	0.600	0.673	0.809	0.800
CPI _{EX}	0.945	0.945	0.937	0.964	0.946	0.938	0.911	0.902	0.878	0.872	0.861	0.923
SPI _{EX}	0.867	0.810	0.882	0.855	0.860	0.849	0.827	0.776	0.737	0.768	0.799	0.736
SCI _{EX}	0.819	0.765	0.827	0.824	0.814	0.796	0.753	0.700	0.647	0.670	0.688	0.679
Weighted SCI _{EX} (w ₁ =0.2, w ₂ =0.8)	0.915	0.921	0.928	0.945	0.931	0.923	0.896	0.880	0.853	0.854	0.851	0.891
EAC(t) ₁	10.4	13.4	10.2	10.8	10.6	10.8	11.0	11.6	12.1	11.8	11.5	12.0
EAC(t) ₂	10.9	14.0	10.8	11.0	11.0	11.2	11.5	12.0	12.5	12.0	11.5	12.0
EAC(t) ₃	10.3	12.7	10.2	10.6	10.5	10.7	11.0	11.5	12.0	11.7	11.5	12.0
EAC(t) ₄	10.4	17.9	7.9	12.1	10.3	11.1	11.7	13.8	13.5	11.4	11.4	12.0
EAC(t) ₅	10.9	18.9	8.5	11.2	11.3	11.7	13.3	14.8	15.1	11.7	11.5	12.0
EAC(t) ₆	9.9	12.0	9.0	9.9	10.5	10.7	11.5	12.0	12.5	11.5	11.4	12.0
EAC(t) ₇			10.2	10.8	9.9	11.0	11.2	12.3	13.1	12.1	11.4	12.0
EAC(t) ₈			10.8	10.9	10.2	11.3	12.0	13.2	14.2	12.6	11.6	12.0
EAC(t) ₉			10.1	10.6	10.0	10.9	11.1	12.1	12.8	12.0	11.5	12.0
EAC(t) ₁₀	10.4	11.4	10.2	10.6	10.5	10.7	11.0	11.6	12.1	11.8	11.5	12.0
EAC(t) ₁₁	10.9	12.0	10.7	10.9	10.8	11.0	11.4	12.0	12.5	12.0	11.5	12.0
EAC(t) ₁₂	9.9	10.3	9.8	10.0	10.1	10.3	10.7	11.2	11.7	11.6	11.4	12.0

Note: costs in thousands of €

APPENDIX VI - PROJECT 3 ESTIMATE COST AT COMPLETION

Planned Budget 906
Actual Duration 9
Actual Cost 952

$\alpha = 0.83$

	Mar-03	Apr-03	May-03	Jun-03	Jul-03	Aug-03	Sep-03	Oct-03	Nov-03	Dec-03
BCWS _c	34	87	157	373	549	673	798	842	876	906
BCWP _c	36	93	169	402	597	735	839	887	906	906
ACWP _c	35	95	174	412	623	754	874	932	952	952
AD	1	2	3	4	5	6	7	8	9	
PD	9	9	9	9	9	9	9	9	9	
ES	1.04	2.09	3.06	4.16	5.39	6.50	7.93	9.32	10.00	
CPI _c	1.029	0.979	0.971	0.976	0.958	0.975	0.960	0.952	0.952	0.952
SPI _c	1.059	1.069	1.076	1.078	1.087	1.092	1.051	1.053	1.034	1.000
SPI(t) _c	1.038	1.043	1.019	1.041	1.077	1.083	1.133	1.165	1.111	
SCI(t) _c	1.067	1.021	0.989	1.016	1.032	1.055	1.088	1.109	1.057	
Weighted SCI(t) _c (w ₁ =0.3, w ₂ =0.7)	1.031	0.997	0.984	0.994	0.991	1.005	1.008	1.011	0.996	
CPI _m	1.029	0.950	0.962	0.979	0.924	1.053	0.867	0.828	0.950	
SPI _m	1.038	1.048	0.970	1.109	1.222	1.109	1.436	1.392	0.676	
SCI _m	1.067	0.996	0.933	1.086	1.130	1.168	1.244	1.152	0.643	
Weighted SCI _m (w ₁ =0.1, w ₂ =0.9)	1.030	0.964	0.963	0.998	0.968	1.062	0.951	0.911	0.910	
CPI _{MA}			0.980	0.964	0.955	0.986	0.948	0.916	0.881	
SPI _{MA}			1.019	1.042	1.100	1.147	1.256	1.312	1.168	
SCI _{MA}			0.998	1.004	1.051	1.130	1.191	1.202	1.029	
Weighted SCI _{MA} (w ₁ =0.7, w ₂ =0.3)			1.009	1.022	1.064	1.106	1.178	1.212	1.095	
CPI _{EX}	1.029	0.963	0.962	0.976	0.933	1.033	0.894	0.839	0.931	
SPI _{EX}	1.038	1.046	0.983	1.088	1.200	1.124	1.384	1.390	0.795	
SCI _{EX}	1.067	1.008	0.945	1.062	1.119	1.162	1.238	1.166	0.741	
Weighted SCI _{EX} (w ₁ =0.2, w ₂ =0.8)	1.030	0.978	0.966	0.996	0.979	1.049	0.979	0.935	0.908	
EAC ₁	881	925	933	929	945	929	944	952	952	
EAC ₂	850	891	919	908	922	916	936	949	952	
EAC ₃	879	911	923	919	935	924	940	951	952	
EAC ₄	881	951	940	927	957	916	951	955	952	
EAC ₅	850	912	964	876	897	900	928	948	952	
EAC ₆	880	938	939	917	942	915	944	953	952	
EAC ₇			926	935	947	928	945	953	952	
EAC ₈			912	914	917	905	930	948	952	
EAC ₉			905	905	914	909	931	948	952	
EAC ₁₀	881	939	940	928	954	919	949	955	952	
EAC ₁₁	850	902	954	886	899	901	928	948	952	
EAC ₁₂	880	927	937	918	939	917	942	952	952	

Note: costs in thousands of €

APPENDIX VII - PROJECT 3 ESTIMATE DURATION AT COMPLETION

Planned Budget 906
Actual Duration 9
Actual Cost 952

$\alpha = 0.01$

	Mar-03	Apr-03	May-03	Jun-03	Jul-03	Aug-03	Sep-03	Oct-03	Nov-03	Dec-03
BCWS _c	34	87	157	373	549	673	798	842	876	906
BCWP _c	36	93	169	402	597	735	839	887	906	906
ACWP _c	35	95	174	412	623	754	874	932	952	952
AD	1	2	3	4	5	6	7	8	9	
PD	10	10	10	10	10	10	10	10	10	
ES	1.04	2.09	3.06	4.16	5.39	6.50	7.93	9.32	10.00	
CPI _c	1.029	0.979	0.971	0.976	0.958	0.975	0.960	0.952	0.952	0.952
SPI _c	1.059	1.069	1.076	1.078	1.087	1.092	1.051	1.053	1.034	1.000
SPI(t) _c	1.038	1.043	1.019	1.041	1.077	1.083	1.133	1.165	1.111	
SCI(t) _c	1.067	1.021	0.989	1.016	1.032	1.055	1.088	1.109	1.057	
Weighted SCI(t) _c (w ₁ =1, w ₂ =0)	1.036	1.034	1.012	1.032	1.060	1.067	1.108	1.135	1.088	
CPI _m	1.029	0.950	0.962	0.979	0.924	1.053	0.867	0.828	0.950	
SPI _m	1.038	1.048	0.970	1.109	1.222	1.109	1.436	1.392	0.676	
SCI _m	1.067	0.996	0.933	1.086	1.130	1.168	1.244	1.152	0.643	
Weighted SCI _m (w ₁ =0.8, w ₂ =0.2)	1.032	0.988	0.965	1.029	1.039	1.075	1.086	1.045	0.845	
CPI _{MA}			0.980	0.964	0.955	0.986	0.948	0.916	0.881	
SPI _{MA}			1.019	1.042	1.100	1.147	1.256	1.312	1.168	
SCI _{MA}			0.998	1.004	1.051	1.130	1.191	1.202	1.029	
Weighted SCI _{MA} (w ₁ =1, w ₂ =0)			1.012	1.029	1.076	1.119	1.203	1.244	1.119	
CPI _{EX}	1.029	1.000	0.986	0.983	0.962	0.995	0.948	0.904	0.921	
SPI _{EX}	1.038	1.042	1.015	1.050	1.113	1.112	1.231	1.290	1.064	
SCI _{EX}	1.067	1.041	1.001	1.032	1.070	1.107	1.167	1.166	0.980	
Weighted SCI _{EX} (w ₁ =1, w ₂ =0)	1.032	1.007	0.991	0.995	0.988	1.016	0.997	0.971	0.946	
EAC(t) ₁	9.6	9.6	9.8	9.6	9.3	9.2	8.8	8.6	9.0	
EAC(t) ₂	9.4	9.8	10.0	9.7	9.5	9.3	8.9	8.6	9.0	
EAC(t) ₃	9.6	9.6	9.8	9.6	9.3	9.2	8.8	8.6	9.0	
EAC(t) ₄	9.6	9.6	10.2	9.3	8.8	9.2	8.4	8.5	9.0	
EAC(t) ₅	9.4	9.9	10.4	9.4	9.1	9.0	8.7	8.6	9.0	
EAC(t) ₆	9.7	9.7	10.2	9.4	9.0	9.2	8.6	8.5	9.0	
EAC(t) ₇			9.8	9.6	9.2	9.1	8.6	8.5	9.0	
EAC(t) ₈			10.0	9.8	9.4	9.1	8.7	8.6	9.0	
EAC(t) ₉			9.8	9.6	9.2	9.1	8.6	8.5	9.0	
EAC(t) ₁₀	9.6	9.6	9.7	9.6	9.4	9.4	9.0	8.7	9.0	
EAC(t) ₁₁	9.4	9.4	9.5	9.5	9.3	9.3	8.9	8.6	9.0	
EAC(t) ₁₂	9.6	9.6	9.7	9.6	9.4	9.4	9.0	8.7	9.0	

Note: costs in thousands of €