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Efficiency Optimization and Control of Permanent Magnet Synchronous Brushless Motors in Three-Phase Pulse Width Modulated Voltage Source Inverter Drives

by

Olusegun Richard Solomon

Dissertation submitted to the
College of Engineering and Mineral Resources at
WEST VIRGINIA UNIVERSITY
in partial fulfillment of the requirements
for the degree of

Doctor of Philosophy
in
Electrical Engineering

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Morgantown, West Virginia

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Abstract

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West Virginia University

Professor Parviz Famouri, Ph.D., Chair

In high performance drives where it is desirable to exploit the usefulness of reluctance torque and machine saliency, permanent magnet synchronous brushless motors are machines of choice. However, speed control of these machines especially in the flux weakening region becomes more complex due to the non-linear coupling among the winding currents as well as the nonlinearity present in the torque. While numerous research efforts in the past have considered control and efficiency improvements of induction motors, and synchronous motors with field windings, research efforts in developing an efficiency optimization and control strategy applicable to all salient-type permanent magnet synchronous brushless motors are still in their infancy.

A traditional control technique that has commonly been employed in efficiency improvement efforts is the stator's zero d-axis current ($i_{ds}=0$) technique. In this method, the rotor flux is aligned with the direct-axis so that the stator's direct-axis current is zero and the torque becomes a linear function of the stator's quadrature-axis current. Although this method achieves decoupling of winding currents and simplicity of control, it does not fully exploit the use of the machine's saliency and reluctance torque, and is also not well-suited for wide-range load operations. The maximum torque per ampere (MTPA) technique is another less complex technique that has been considered which fully exploits the use of machine saliency with motor torque selected along the geometric curve of minimum-amplitude current space vectors for minimum loss operation. The drawback of the MTPA technique is that it does not provide high efficiency performance for synchronous reluctance motors running at low fractional loads.

In this work, the problem of efficiency optimization in the salient-type permanent magnet synchronous brushless motors is investigated. A machine model which includes the effect of core losses is proposed for developing a loss minimization algorithm that dynamically determines the optimal reference currents and voltages required for minimizing the total electrical losses (copper losses and core losses) within the feasible operating regions imposed by the motor and inverter capacities. The loss minimization strategy is implemented within a speed control loop for a synchronous reluctance motor drive and the effectiveness of the proposed scheme is validated by comparing performances with that of the traditional maximum torque per ampere and stator's zero d-axis current vector control methods. It is shown that the proposed scheme offers the advantages of simplicity and superior performance throughout the entire operating range, and also improves motor efficiency to 96% at full load and full-speed operating condition.

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Chapter 1

Introduction

1.1 Motivation

The permanent magnet synchronous motors (PMSMs) are a class of synchronous machines comprising of the interior permanent magnet synchronous motor (IPMSM), the surface permanent magnet synchronous motor (SPMSM), the synchronous reluctance motor (SRM) and the permanent magnet brushless DC motor (PMBDCM). These types of machines differ slightly in physical construction by the position of the permanent magnet with respect to the machine rotor. They are of special interest and are fast gaining prominence in low and medium power variable speed drive systems for automotive, aerospace, industrial and commercial applications because of their inherent advantages such as high torque-to-inertia ratio, quiet operation and lower maintenance cost due to the absence of brushes. Other attractive characteristics of the PMSMs include: superior torque-speed characteristics, higher power density and compactness, and higher efficiency thus making them more attractive than the brushed DC motors and induction motors.

Electric motor driven systems used in industrial processes consume 700 billion kWh or 72 percent of total electricity used in U.S. industry, according to a Department of Energy report published in 2004. With the issue of rising cost of electricity, energy efficiency for electric motors and drives is becoming increasingly relevant and companies have to adopt efficiency improvement policies that will bring about significant energy savings in operations and

processes in order to compete favorably in this environment. This challenge opens opportunities for continuous research in efficiency improvement of motor drives.

In the past twenty years, researchers have made significant progress in the areas of efficiency improvement of motor drives. These efforts were largely directed on efficiency improvement of induction machines which are known to be greatest consumers of electric power. However, there is a growing interest in the permanent magnet synchronous machine drives due to the inherent advantages and features outlined above. Although, the PMSM offer superior performance and higher efficiency than the induction motor of equivalent power rating, there is still a lot of room for improvement. One of the drawbacks of the PMSM is the prohibitive cost of rare earth magnets which limits their use to low power and medium power application. However, research on their uses in very high power applications is still on-going.

1.2 Literature Review

Improvement of efficiency is an important issue for the permanent magnet synchronous motor (IPMSM, SPMSM, SRM and PMDCM) drives especially in applications such as hybrid electric vehicles and compressor drives where drives are powered for continuous operation by power-limited battery sources and fuel cells. Therefore, significant efforts and attention is given to improve the efficiency of these drives in applications where very high efficiency is desirable in addition to fast speed and torque response. An improvement in the efficiency of a motor will lead to significant energy savings and also a reduction in the running cost of the equipment. An approach adopted in [1]-[2] for the permanent magnet synchronous motor focused on the search for the optimum rotor structure. Efficiency improvement using this

approach is very expensive as physical modification has to be made to the rotor design. An alternative is to intervene in the motor operation with automatic control techniques.

Other control strategies that have been proposed to minimize losses in permanent magnet synchronous machine and induction machine drives can be found in [3-6]. Maximum torque per ampere (MTPA) method has been employed in [3] by controlling the armature current vector to minimize copper loss while in [4] a nonlinear torque control method has been employed to reduce copper loss. These control strategies only considered the copper losses, the core losses were not accounted for. Furthermore, in implementation, the control schemes require accurate knowledge of machine parameters which were assumed constant; this is not true in practice as the machine parameters may drift with temperature, saturation and other operating conditions.

In [7] and [8-10], a loss minimization condition for a Loss Model Controller (LMC) determines optimal d-axis component of the armature current in the flux weakening region for vector-controlled induction motor and synchronous motor respectively. The advantages of these control techniques are that they effectively utilize the reluctance torque and d-axis armature reaction to minimize the controllable losses. However, the burden of rigorous mathematical operation using off-line made lookup tables and the difficulty of measuring the parameters especially when in operation makes implementation of these loss model control costly and time-consuming. Search Controller (SC) and Perturbation Controller (PC) methods have been introduced in [6], [11-13], and these do not require knowledge of the motor loss model, but require the measurement of input power and adjustment of the d-axis current or perturbation signal that gives the minimum input power.

The loss minimization control strategy proposed by Kioskeridis and Margaritis [7] has shown great potential for achieving significant energy savings. Result evidence provided in their work has shown that the LMC performance in induction motor drives is superior to that of the search controller or perturbation controller. Their work also confirmed some of the limitations of the search controller approach which include drive's unsatisfactory performance due to the presence of torque disturbances, and the inability of the controller to reach a steady state and find minimum input power that is smooth and flat around the minimum.

An optimal efficiency control strategy based on the loss model controller was also developed for an interior permanent magnet synchronous motor (IPMSM) in [14]. The loss minimization operating point was found by varying the d-axis current and this strategy has been shown to be applicable to the surface permanent magnet synchronous motor and the synchronous reluctance motors as well. The performance of the LMC was shown to be superior to the conventional $i_{ds}=0$, and maximum torque per ampere current control method. The limitation of this approach is that the equivalent excitation current of the permanent magnet has to be known. Furthermore, the loss controller gain which depends on the machine parameter has to be adjusted experimentally to compensate for parameter variation and changing load conditions.

An improvement in the efficiency of the IPMSM when subjected to parameter variation has been presented in [15] by measuring saturation dependent parameters (q and d-axis inductances) on-line. In [16], core losses and effects of saturation dependent parameters are included in the IPMSM model for which a control scheme based on input-output linearization technique is applied to decouple the winding currents, rotor speed and loss minimization dynamics. A loss function which specifies the condition for minimum loss is derived and used

to achieve speed control under a minimum loss strategy. The limitation of this technique is that the model assumed a fairly lossless system (very high core loss resistance) and in the control implementation, the loss function γ is assumed to be zero which does not hold true in practice. This error introduces high harmonic and ripple contents in the machine currents and outputs.

The objective of this work is to develop an optimal efficiency scheme that will mitigate the inherent drawbacks in previous techniques. The development of the proposed loss minimization scheme (LMS) will seek advantages of reduced costs, simplicity and ease of implementation.

Implementation and application of the concepts of Loss Model Strategy (LMS) to the interior permanent magnet synchronous motors (IPMSM), and the synchronous reluctance motor (SRM) is the subject of this research. Our proposed scheme takes advantage of the machine saliency so that the reluctance torque and d-axis armature reaction are effectively used to minimize the total electrical loss while the stray loss is assumed negligible. A loss model controller that satisfies the loss minimization condition is developed for the IPMSM and the SRM; the steady state and dynamic performances are then compared with that obtained using the traditional $i_{ds} = 0$ and maximum torque per ampere current methods.

1.3 Losses in Synchronous Machines

The types of losses in the IPMSM and the PMBDCM are those that are common to synchronous machines and these include copper losses, iron (or core) losses, stray-losses, mechanical losses and harmonic losses. The copper (or ohmic) losses are $i^2 R$ losses due to current flowing through the stator (or armature) winding of the machine. With a very high

current, copper losses are significant and can only be limited by the armature resistance. The expression for copper losses is given as:

$$P_{Cu} = r_s I_s^2 = r_s (I_{qs}^2 + I_{ds}^2) \quad (1.2-1)$$

where, r_s is the stator per phase resistance; I_s is the stator's per phase steady state current magnitude; I_{qs} is the stator's quadrature-axis steady state current; and I_{ds} is the stator's direct-axis steady state current.

The iron losses (also known as core losses) are due to hysteresis and eddy currents. It has been experimentally shown that iron losses vary with frequency and amplitude of the magnetizing flux. The empirical expression for iron losses of the synchronous motor is given by [14], [17]:

$$P_{Fe} = c_{Fe} \omega_e^\beta \Phi_m^2 = c_{Fe} \omega_e^\beta L_{md}^2 (I_{md}^2 + \sigma^2 I_{mq}^2) \quad (1.2-2)$$

where, c_{Fe} and β are experimental constants with β typically ranging between 1.5 and 1.6.

σ is the saliency ratio expressed as:

$$\sigma = \frac{L_{mq}}{L_{md}} \quad (1.2-3)$$

Stray losses are losses on the copper and iron of the motor which also vary with the frequency and amplitude of the armature current. The expression for the stray losses is given by [14], [17]:

$$P_{stray} = c_{stray} \omega_e^2 I_s^2 = c_{stray} \omega_e^2 (I_{qs}^2 + I_{ds}^2) \quad (1.2-4)$$

The mechanical losses are losses due to friction and windage and are proportional to square of rotor speed. The expression for the mechanical losses is given by:

$$P_{mech} = c_{mech} \omega_e^2 \quad (1.2-5)$$

The harmonic losses are losses due to non-sinusoidal stator voltage applied to the synchronous machine. This type of losses could be very significant in drives supplied from switch-mode power converters, phase-controlled rectifiers, six-step and PWM inverters. In PWM control, a non-sinusoidal reference modulating signal will produce output with high harmonic contents which will in turn increase the harmonic losses in the drive. Harmonic components have indirect influence on the machine iron and copper losses. For example, harmonic voltages increase the iron losses while harmonic currents increase stator copper losses of synchronous machines. It would be reasonable to suggest that a highly efficient drive would have little or no harmonic losses hence an efficiency optimization control technique geared at minimizing total electrical losses should also aim at harnessing the indirect benefits of minimizing harmonic losses.

The total electrical losses of the system are those accounted for by copper losses, iron losses and stray losses as these losses contain current variables I_{qs} and I_{ds} which make them amenable to flux weakening control. The stray losses, the windage and frictional losses are usually very small and for simplicity and ease of analysis are assumed negligible. In [8] the total electrical losses are expressed by:

$$P_L = P_{Cu} + P_{Fe} + P_{stray} \quad (1.2-6)$$

Substituting (1.2-1), (1.2-2) and (1.2-4) into (1.2-6)

$$P_L = a(I_{ds}^2 + I_{qs}^2) + b \left[(I_f' + I_{ds}')^2 + \sigma^2 I_{qs}^2 \right] \quad (1.2-7)$$

where

$$a = r_s + c_{stray} \omega_e^2 \quad (1.2-8)$$

and

$$b = c_{Fe} \omega_e^\beta L_{md}^2 \quad (1.2-9)$$

One of the shortcomings of obtaining the total electrical loss using equation (1.2-7) is that the loss constants, c_{Fe} and c_{stray} , and the field current I_f' have to be determined experimentally and these require the knowledge of the machine parameters and machine model. Secondly, the use of these fixed constants in any control scheme may not yield accurate results since machine parameter may vary with temperature or saturation causing a drift in the actual constants under a given operating condition. One of the techniques that render the performance of the drive insensitive to parameter changes is the model reference adaptive control (MRAC) scheme. Although this technique provides robustness to unwanted disturbances and parameter variation, one of the challenges is difficulty to properly define the reference model that represents the desired drive performance specification. Furthermore, in the MRAC, all plant states are required for formulating the feedback control law which makes practical implementation for such systems complex and costly.

1.4 Research Objectives

The development of an optimal efficiency strategy and speed control scheme for a high performance permanent magnet synchronous brushless motor drive is the objective of this research. The proposed system is implemented so that it has little or no sensitivity to changes in machine parameters while maintaining optimum efficiency performance over a wide range

of operation. The use of simple proportional, integral, derivative controller for implementation of the proposed scheme will be considered. With a reduced number of feedback signals, it is expected that the proposed scheme will offer the added advantages of reduced cost, simplicity and ease of implementation which will make our approach more attractive than the elaborate and complex nonlinear control and model reference adaptive control alternatives.

1.5 Research Contribution

The subject of this research is the implementation and application of the concepts of Loss Model Strategy (LMS) for efficiency optimization of the permanent magnet brushless machine. The major contributions are highlighted as follows:

- A model is proposed for a vector controlled permanent magnet synchronous brushless motor drive which takes core losses into account.
- This model is used to study and make quantitative assessment of the influence of core losses on the motor's open-loop performance characteristics such as output torque, stator currents and total electrical losses.
- In order to achieve an improvement in operating efficiency of the motor, a loss minimization strategy (LMS) is developed based on the concept of loss model controller. The minimization strategy is formulated by deriving the loss minimization condition that ensures optimal efficiency for motor operation in the feasible operating regions defined by the system operating constraints.
- The proposed scheme is implemented in closed-loop speed control operating mode. Comparison of steady state and dynamic performances of the system without and with the

LMS strategy is made to show the effectiveness of the proposed strategy. The superiority of the LMS approach is determined from the comparative study of motor efficiency and dynamic performances with that of the conventional zero d-axis ($i_{ds} = 0$) and maximum torque per ampere methods.

- A brief study will be conducted to examine the robustness of the proposed LMS scheme to changes in machine's parameters.
- Simulation and experimental results are presented to validate the proposed loss minimization strategy. The effectiveness of our proposed scheme is determined by comparing performances with that of previous loss minimization techniques.

1.6 Dissertation Organization

In this chapter, an introduction which includes a literature review in areas related to this dissertation work has been given. In Chapter 2, a machine model which includes the effect of core losses for the permanent magnet synchronous motor is proposed and used to derive the necessary condition that minimizes the total electrical losses for the machine. A detailed account on the operation and analysis of a pulse width modulated rectifier-inverter permanent magnet synchronous motor drive is given in Chapter 3. In Chapter 4, the linearized equations and transfer functions of the permanent synchronous motor are derived and application to the state-feedback control design is given. Chapter 5 is devoted to the study of steady state analysis and operating regions of the various permanent magnet synchronous motor types. A detailed description of the implementation of the proposed efficiency optimization strategy as well as system performance under no-load and load conditions is given in Chapter 6. Results that compare the performances of the proposed scheme to conventional and previous efficiency improvement methods are also included in Chapter 6. Finally, Chapter 7 gives the conclusions and future research directions.

Chapter 2

Machine Model Including Effect Of Core-Losses

In this chapter, we present the dynamic model of the family of permanent magnet synchronous motors (PMSMs) energized by balanced three-phase stator voltage with frequency corresponding to the machine rotor speed. The types of PMSM considered are the interior permanent magnet synchronous motor (IPMSM), the surface permanent magnet synchronous motor (SPMSM), the synchronous reluctance motor (SRM) and the permanent magnet brushless DC motor (PMBDCM). Except for the slight differences in the physical structures, the equations which describe the behavior of these machines are similar. It therefore follows that the analysis for a particular type of machine such as the PMBDCM with sinusoidal back emf can be easily extended to the rest of the machines of the same family.

The mathematical model and operating constraint for the PMBDCM with sinusoidal back emf is presented in this chapter. This model includes a core-loss resistance to account for the contribution of core-losses to the machine's total electrical losses. In order to develop a control scheme that minimizes total electrical losses and ultimately improves drive efficiency for a given output power demand, we derive an expression for the total electrical losses as a cost function in terms of the machine state variables. By minimizing the total electrical loss function subject to operating constraints of the machine and inverter, the minimum loss conditions that ensures maximum efficiency of drive is established. Machine equations are expressed in the rotor reference frame rather than the stationary reference frame so as to greatly simplify the analysis.

2.1 Machine Voltage And Torque Equations

The structure of a two-pole PMBM is shown in Fig. 2.1-1. The voltage and torque equations for this machine are similar to that of the synchronous motor (SM). The basic difference is that there are no rotor (or field) windings in the PMBM so that its rotor voltage equation is omitted. The d-and q-axis equivalent circuits for the three-phase PMBM are shown in Fig. 2.1-2. A loss shunt resistance, R_c has been included to account for the core-losses. The relationship between the core-loss resistance and the core-loss follows an inverse law so that the larger the magnitude of the core-loss resistance, the lower the core losses and vice-versa. The neutral of the machine is assumed to be unconnected, so that no zero currents flow in the stator windings. For motoring operation, the direction of stator currents is positive, flowing into the machine. The d-q voltage and flux equations in the rotor reference frame as well as the electromagnetic torque equation are given by [18], [19]:

$$v_{qs} = r_s i_{qs} + \omega_r \lambda_{ds} + p \lambda_{qs} \quad (2.1-1)$$

$$v_{ds} = r_s i_{ds} - \omega_r \lambda_{qs} + p \lambda_{ds} \quad (2.1-2)$$

$$\lambda_{qs} = L_{qs} i'_{qs} \quad (2.1-3)$$

$$\lambda_{ds} = L_{ds} i'_{ds} + \lambda_m \quad (2.1-4)$$

$$T_e = \frac{3P}{4} (\lambda_{ds} i'_{qs} - \lambda_{qs} i'_{ds}) \quad (2.1-5)$$

Figure 2.1-1: Cross sectional view of a permanent magnet synchronous brushless motor.

(i)

(ii)

Figure 2.1-2: Equivalent circuits for a three-phase, symmetrical permanent magnet synchronous brushless motor including core-losses.

where,

$$i_{qs} = i'_{qs} + i_{qc} \quad (2.1-6)$$

$$i_{ds} = i'_{ds} + i_{dc} \quad (2.1-7)$$

and

$$p = \frac{d}{dt}, \quad L_{qs} = L_{ls} + L_{mq}, \quad L_{ds} = L_{ls} + L_{md} \quad (2.1-8)$$

Substituting (2.1-3) and (2.1-4) into (2.1-1) and (2.1-2) respectively, the following machine state equations are obtained:

$$p i'_{qs} = -\frac{r_s i'_{qs}}{L_{qss}} - \frac{\omega_r L_{ds} i'_{ds}}{L_{qs}} + \frac{v_{qs}}{L_{qss}} - \frac{\omega_r \lambda_m}{L_{qs}} \quad (2.1-9)$$

$$p i'_{ds} = \frac{\omega_r L_{qs} i'_{qs}}{L_{ds}} - \frac{r_s i'_{ds}}{L_{dss}} + \frac{v_{ds}}{L_{dss}} \quad (2.1-10)$$

where,

$$L_{qss} = L_{qs} \left(1 + \frac{r_s}{R_c} \right), \quad L_{dss} = L_{ds} \left(1 + \frac{r_s}{R_c} \right) \quad (2.1-11)$$

The expression for the electromagnetic torque T_e is given by:

$$T_e = \frac{3P}{4} \left[\lambda_m i'_{qs} + (L_{ds} - L_{qs}) i'_{qs} i'_{ds} \right] \quad (2.1-12)$$

The core-loss resistance is modeled as a function of the machine rotor speed. The expression is given by the following equation [20]:

$$R_c = k_1 + k_2 \omega_r^n \quad (2.1-13)$$

Equation (2.1-13) shows that the core loss resistance is the result of two components: a fixed component and an exponentially increasing component. The exponential part is a function of the rotor speed raised to the n index power. From the above relationship, constant k_1 accounts for the machine's fixed core losses and this is the core loss at zero speed when the machine is initially energized. The contribution of the fixed core losses can be very significant. Values of k_1 range from tens to thousands of ohms. Constant k_2 is the fraction of the exponential contribution to the total machine core losses and this depends on the core material with values typically ranging from 0.16 to 0.25. The index n ranges from 1.5 to 1.7 with typical value at 1.6. Note that the core-loss resistance remains the same when machine is running at a given rotor speed.

2.2 Total Electrical Loss Equation

As mentioned previously, the total electrical losses include the copper losses and core-losses and an expression for the total electrical losses (P_L) can be derived based on the d-q equivalent model of the PMBDCM. The derived expression will be the objective function for a loss minimization control scheme which that will be presented in the later part of this work. In the derivation of the expression for P_L , stray losses have been omitted as this is assumed to be negligible.

From Fig. 2.1-2, the instantaneous total electrical loss is given as:

$$P_L = P_{Cu} + P_{Fe} \quad (2.2-1)$$

Since the core-losses have been modeled as a resistance R_c ,

$$P_L = \frac{3}{2}r_s(i_{qs}^2 + i_{ds}^2) + \frac{3}{2}R_c(i_{qc}^2 + i_{dc}^2) \quad (2.2-2)$$

At steady state, $pi'_{qs} = pi'_{ds} = 0$ and the motor steady state electrical loss may be written as:

$$P_L = \frac{3}{2}r_s(I_{qs}^2 + I_{ds}^2) + \frac{3}{2}R_c(I_{qc}^2 + I_{dc}^2) \quad (2.2-3)$$

where,

$$I_{qs} = I'_{qs} + I_{qc} \quad (2.2-4)$$

$$I_{ds} = I'_{ds} + I_{dc} \quad (2.2-5)$$

and,

$$I_{qc} = \frac{\omega_r \lambda_{ds}}{R_c} \quad (2.2-6)$$

$$I_{dc} = -\frac{\omega_r \lambda_{qs}}{R_c} \quad (2.2-7)$$

On substituting (2.2-4) through (2.2-7) into (2.2-3), the total electrical loss in terms of the steady state currents, I'_{qs} and I'_{ds} is given as:

$$\begin{aligned} P_L = & \left(\frac{3}{2}\right)r_s(I_{qs}'^2 + I_{ds}'^2) + \left(\frac{3}{2}\right)\frac{(r_s + R_c)(L_{ds}^2 I_{ds}'^2 + L_{qs}^2 I_{qs}'^2)\omega_r^2}{R_c^2} \\ & + \frac{3\omega_r I'_{qs} I'_{ds} (L_{ds} - L_{qs})}{R_c} + 3\omega_r \lambda_m \left(\frac{r_s}{R_c}\right) I'_{qs} \\ & + 3\omega_r^2 \lambda_m L_{ds} \left(\frac{r_s + R_c}{R_c^2}\right) I'_{ds} + \left(\frac{3}{2}\right)\frac{(r_s + R_c)\lambda_m^2 \omega_r^2}{R_c^2} \end{aligned} \quad (2.2-8)$$

On further rearranging (2.2-8),

$$\begin{aligned}
P_L = & \frac{3}{2} \left[r_s + \frac{\omega_r^2 (r_s + R_c) L_{qs}^2}{R_c^2} \right] I_{qs}'^2 + \frac{3}{2} \left[r_s + \frac{\omega_r^2 (r_s + R_c) L_{ds}^2}{R_c^2} \right] I_{ds}'^2 \\
& + 3\omega_r \frac{r_s}{R_c} (L_{ds} - L_{qs}) I_{qs}' I_{ds}' + 3\omega_r \lambda_m \frac{r_s}{R_c} I_{qs}' \\
& + 3\omega_r^2 \lambda_m L_{ds} \left(\frac{r_s + R_c}{R_c^2} \right) I_{ds}' + \left(\frac{3}{2} \right) \frac{(r_s + R_c) \lambda_m^2 \omega_r^2}{R_c^2}
\end{aligned} \tag{2.2-9}$$

Hence,

$$P_L = a I_{qs}'^2 + b I_{ds}'^2 + c I_{qs}' I_{ds}' + d I_{qs}' + e I_{ds}' + f \tag{2.2-10}$$

where,

$$\begin{aligned}
a = & \frac{3}{2} \left[r_s + \frac{\omega_r^2 (r_s + R_c) L_{qs}^2}{R_c^2} \right]; b = \frac{3}{2} \left[r_s + \frac{\omega_r^2 (r_s + R_c) L_{ds}^2}{R_c^2} \right]; c = 3\omega_r \frac{r_s}{R_c} (L_{ds} - L_{qs}) \\
d = & 3\omega_r \lambda_m \frac{r_s}{R_c}; e = 3\omega_r^2 \lambda_m L_{ds} \left(\frac{r_s + R_c}{R_c^2} \right); f = \frac{3}{2} \frac{(r_s + R_c) \lambda_m^2 \omega_r^2}{R_c^2}
\end{aligned} \tag{2.2-11}$$

Note that the above derivations are applicable to any size of permanent magnet brushless motor. In order to prove the effectiveness of our analysis, a 3 hp permanent magnet synchronous brushless motor is considered. The parameters for the machine are shown in Table 1. It is assumed that the machine saliency can be specified by appropriately selecting the values of the mutual inductances, L_{mq} and L_{md} .

Fig. 2.2-1 shows the electrical loss P_L as a function of I_{qs}' and I_{ds}' currents for a given machine with $L_{mq} > L_{md}$ ($\sigma > 1$).

TABLE 2.1: PARAMETERS OF A 3HP PMSM

Parameter	Value	Unit
Resistance R_s	0.9889	Ω
$L_{qs}, L_{ds} : \sigma > 1$	0.031, 0.021	H
$L_{qs}, L_{ds} : \sigma = 1$	0.021, 0.021	H
$L_{qs}, L_{ds} : \sigma < 1$	0.021, 0.1021	H
Flux linkage λ_m	0.063	Wb (V/rad/s)
Number of Poles, P	4	dimensionless
Rotor Inertia, J_m (nominal)	0.00018	Kg. m ²
Damping, B_m (nominal)	0.0000212	N.m/s

Fig. 2.2-1: Total electrical loss as a function of magnetizing currents for $\sigma > 1$

For the size of machine under consideration, a rotor reference speed ω_r fixed at 4 rad/s will give a measurable amount of core-loss. The figure depicts all possible values of P_L for motoring and braking operating modes for the machine.

The corresponding plots for cases of $\sigma = 1$, and $\sigma < 1$ are shown in Fig. 2.2-2 and Fig. 2.2-3 respectively. The shapes of the surfaces in all cases are elliptical. Hence, regardless of the machine saliency (whether $L_{mq} > L_{md}$, $L_{mq} = L_{md}$, $L_{mq} < L_{md}$), the total electrical loss while the machine is running will always be defined on an elliptical surface.

Fig. 2.2-2: Total electrical loss as a function of magnetizing currents for $\sigma = 1$

Fig. 2.2-3: Total electrical loss as a function of magnetizing currents for $\sigma < 1$

2.3 Loss Minimization Condition

The derived expression for the total electrical loss P_L is an elliptical surface and the objective is to minimize P_L to meet a desired load torque and motor speed. The objective function P_L has to be minimized subject to the constraints of fixed load torque $T_e = T_L$ and speed, ω_r .

The loss minimization condition at steady state (T_e and ω_r are held constant) with respect to I'_{ds} is given by:

$$\left. \frac{\partial P_L}{\partial I'_{ds}} \right|_{T_e, \omega_r} = 0 \quad (2.3-1)$$

Using (2.2-10), condition (2.3-1) is satisfied when

$$2aI'_{qs} \frac{\partial I'_{qs}}{\partial I'_{ds}} + 2bI'_{ds} + c \left[I'_{ds} \frac{\partial I'_{qs}}{\partial I'_{ds}} + I'_{qs} \right] + d \frac{\partial I'_{qs}}{\partial I'_{ds}} + e = 0 \quad (2.3-2)$$

$$\left(2aI'_{qs} + cI'_{ds} + d \right) \frac{\partial I'_{qs}}{\partial I'_{ds}} + \left(cI'_{qs} + 2bI'_{ds} + e \right) = 0 \quad (2.3-3)$$

Since the electromagnetic torque T_e is constant

$$\left. \frac{\partial T_e}{\partial I'_{ds}} \right|_{\omega_r} = 0 \quad (2.3-4)$$

On differentiating (2.1-12) with respect to I'_{ds} ,

$$\lambda_m \frac{\partial I'_{qs}}{\partial I'_{ds}} + (L_{ds} - L_{qs}) \left[I'_{ds} \frac{\partial I'_{qs}}{\partial I'_{ds}} + I'_{qs} \right] = 0 \quad (2.3-5)$$

Hence,

$$\frac{\partial I'_{qs}}{\partial I'_{ds}} = \frac{(\sigma - 1)I'_{qs}}{\lambda_{mmd} + (1 - \sigma)I'_{ds}} \quad (2.3-6)$$

where,

$$\lambda_{mmd} = \frac{\lambda_m}{L_{md}} \quad (2.3-7)$$

Substituting (2.3-6) into (2.3-3) and collecting terms, the loss minimization condition is given by:

$$\begin{aligned} & 2a(\sigma - 1)I'^2_{qs} + 2b(\sigma - 1)I'^2_{ds} + 2c(\sigma - 1)I'_{qs}I'_{ds} \\ & + [d(\sigma - 1) + c\lambda_{mmd}]I'_{qs} + [e(\sigma - 1) + 2b\lambda_{mmd}]I'_{ds} + e\lambda_{mmd} = 0 \end{aligned} \quad (2.3-8)$$

Equation (2.3-8) may be rewritten as:

$$\Gamma = X_{11}I'^2_{qs} - X_{22}I'^2_{ds} + X_{12}I'_{qs}I'_{ds} - X_1I'_{qs} - X_2I'_{ds} - \mathcal{G} = 0 \quad (2.3-9)$$

where,

$$\begin{aligned} X_{11} &= 2a(1 - \sigma) & X_{22} &= 2b(1 - \sigma) \\ X_{12} &= 2c(1 - \sigma) & X_1 &= -d(1 - \sigma) + c\lambda_{mmd} \\ \mathcal{G} &= e\lambda_{mmd} & X_2 &= 2b\lambda_{mmd} + e(1 - \sigma) \end{aligned} \quad (2.3-10)$$

The solution of equation (2.3-8) is the intersection or projection of the surface Γ on the $I'_{ds} - I'_{qs}$ plane ($\Gamma=0$). For a salient-pole machine ($\sigma \neq 1$), the loss minimization condition

($\Gamma=0$) as depicted in Fig. 2.3-1 and Fig. 2.3-2 are hyperbolic contour curves which define the optimal efficiency locus of the machine for the saliency condition $\sigma < 1$ and $\sigma > 1$ respectively. The center and slopes of the asymptotes are dependent on machine parameters. Thus a change in machine parameter will cause a shift in the solution locus for optimal efficiency.

The optimal efficiency locus for non-salient machine ($\sigma = 1$) is shown in Fig. 2.3-3. The locus is described by straight lines unlike the salient pole machine above. Hence it is expected that a linear control for loss minimization can be realized for this type of machine at the expense of the reluctance torque component.

Assuming that there is no parameter drift, the optimal operating points for minimum loss can be determined from solution locus for a fixed set of machine parameter under a given load condition.

Fig. 2.3-1: Loss function versus magnetizing current for $\sigma < 1$

Fig. 2.3-2: Loss function versus magnetizing currents for $\sigma > 1$

Fig. 2.3-3: Loss function versus magnetizing currents for $\sigma = 1$

If I'_{qs} is known, I'_{ds} can be determined by solving equation (2.3-9). On rearranging equation (2.3-9),

$$X_{22} I'^2_{ds} + (X_2 - X_{12} I'_{qs}) I'_{ds} + (\mathcal{G} + X_1 I'_{qs} - X_{11} I'^2_{qs}) = 0 \quad (2.3-10)$$

Solving equation (2.3-10),

$$I'_{ds_{1,2}} = \frac{-(X_2 - X_{12} I'_{qs}) \pm \sqrt{(X_2 - X_{12} I'_{qs})^2 - 4X_{22}(\mathcal{G} + X_1 I'_{qs} - X_{11} I'^2_{qs})}}{2X_{22}} \quad (2.3-11)$$

The solution for I'_{ds} given by equation (2.3-11) exists if and only if:

$$(X_2 - X_{12} I'_{qs})^2 - 4X_{22}(\mathcal{G} + X_1 I'_{qs} - X_{11} I'^2_{qs}) \geq 0 \quad (2.3-12)$$

Alternatively, if I'_{ds} is known, I'_{qs} can be determined by solving equation (2.3-9).

Rearranging equation (2.3-9), we obtain

$$X_{11} I'^2_{qs} + (X_1 - X_{12} I'_{ds}) I'_{qs} - (\mathcal{G} + X_2 I'_{ds} + X_{22} I'^2_{ds}) = 0 \quad (2.3-13)$$

The solution of equation (2.3-13) is given by equation (2.3.14):

$$I'_{qs_{1,2}} = \frac{-(X_1 - X_{12} I'_{ds}) \pm \sqrt{(X_1 - X_{12} I'_{ds})^2 + 4X_{11}(\mathcal{G} + X_2 I'_{ds} + X_{22} I'^2_{ds})}}{2X_{11}} \quad (2.3-14)$$

Similarly, the condition that I'_{qs} exists is given by:

$$(X_1 - X_{12} I'_{ds})^2 + 4X_{11}(\mathcal{G} + X_2 I'_{ds} + X_{22} I'^2_{ds}) \geq 0 \quad (2.3-15)$$

The electromagnetic torque can be re-expressed in terms of the saliency factor σ and q'-d' currents as:

$$T_e = \frac{3P}{4} L_{md} [\lambda_{mmd} I'_{qs} + (1 - \sigma) I'_{qs} I'_{ds}] \quad (2.3-16)$$

Hence, with I'_{qs} known, substitution of equation (2.3-11) in (2.3.16) gives the electromagnetic torque for the two solution of I'_{ds} .

Optimal Operation:

If (I_{ds}^*, I_{qs}^*) is a critical point satisfying equation (2.3-1), then this operating point is a relative minimum point if the following condition is satisfied:

$$\left. \frac{\partial^2 P_L}{\partial I_{ds}^2} \right|_{I_{ds}^*, I_{qs}^*} > 0 \quad (2.3-17)$$

Solution for I_{ds}^* in equation (2.3-11) which satisfies equation (2.3-17) is the optimal solution. Note that for motoring operation, the electromagnetic torque T_e is positive and this condition must also be satisfied by the solution of (2.3-11). The above condition, imposes a limit on the range of possible values for d'-axis current I'_{ds} for a given load condition and q'-axis current I'_{qs} . The range for I'_{ds} also varies with the motor saliency.

2.4 Current Transformation To D'-Q' Variable

Our analysis in the previous section considered the formulation of loss optimization condition in terms of the d'-q' current quantities. As was mentioned above, the steady state d'-q' current quantities (I'_{ds}, I'_{qs}) are obtained from the actual steady state d-q currents (I_{ds}, I_{qs}) by

applying necessary transformation or mapping even though the actual d-q currents may not be easily accessible and measurable.

A relationship between the d-q and d'-q' current quantities will be useful for establishing the necessary conditions in terms of the d-q current variables that will ensure optimal efficiency for the brushless motor. In section 2.1, the instantaneous stator machine currents (i_{qs}, i_{ds}) were shown to consist of the core-loss current component (i_{qc}, i_{dc}) and the useful magnetizing currents (i'_{qs}, i'_{ds}) expressed by equations (2.1-6) and (2.1-7) respectively and are repeated below for convenience:

$$i_{qs} = i'_{qs} + i_{qc} \quad (2.4-1)$$

$$i_{ds} = i'_{ds} + i_{dc} \quad (2.4-2)$$

but,

$$i_{qc} = \left(p \frac{L_{qs}}{R_c} \right) i'_{qs} + \frac{\omega_r L_{ds}}{R_c} i'_{ds} + \frac{\omega_r \lambda_m}{R_c} \quad (2.4-3)$$

$$i_{dc} = - \frac{\omega_r L_{qs}}{R_c} i'_{qs} + \left(p \frac{L_{ds}}{R_c} \right) i'_{ds} \quad (2.4-4)$$

By substituting (2.4-3) and (2.4-4) into (2.4-1) and (2.4-2) respectively, we obtain:

$$i_{qs} = \left(1 + p \frac{L_{qs}}{R_c} \right) i'_{qs} + \frac{\omega_r L_{ds}}{R_c} i'_{ds} + \frac{\omega_r \lambda_m}{R_c} \quad (2.4-5)$$

$$i_{ds} = - \frac{\omega_r L_{qs}}{R_c} i'_{qs} + \left(1 + p \frac{L_{ds}}{R_c} \right) i'_{ds} \quad (2.4-6)$$

Equations (2.4-5) and (2.4-6) define the dynamic model for the transformation between the d-q currents and the d'-q' currents. The transformation model is a multi-input multi-output linear time-invariant second order system which deserves careful consideration to determine stability boundaries while ensuring performances are within the system's operating limits.

A close examination of the above equations will reveal that under steady state conditions, the transformation is linear and the signs for the d-q stator currents are primarily determined by the magnitudes of the d'-q' currents, the size of the machine self inductances and on the magnitude of the core loss resistance and the rotor speed. Under steady state condition, the machine core loss remains fixed since the machine is running at a fixed rotor speed. For a high performance drive with negligible core losses (very high R_c), the trajectories of the d-q currents are closely matched with that of the d'-q' currents. The influences of machine saliency, machine core losses, and motor rotor speed on the current transformation are shown in Fig. 2.4-1 through Fig. 2.4-8. The current trajectories start at time t_0 corresponding to machine zero speed and ends at time t_f corresponding to steady state rotor speed. Fig. 2.4-1 to Fig. 2.4-6 show that the steady state operating points for the currents coincide directly with the steady state operating points of the currents under no-load conditions for all types of the permanent magnet synchronous motors.

Figure 2.4-1: Current responses of the SRM drive ($\sigma < 1$) under no-load condition.

Figure 2.4-2: Current responses of the SRM drive ($\sigma < 1$) under step load condition

Figure 2.4-3: Current responses of the IPMSM drive ($\sigma > 1$) under no-load condition

Figure 2.4-4: Current responses of the IPMSM drive ($\sigma > 1$) under step load condition

Figure 2.4-5: Current responses of the SPMSM drive ($\sigma = 1$) under no-load condition.

Figure 2.4-6: Current responses of the SPMSM drive ($\sigma=1$) under step load condition.

Figure 2.4-7: Current responses of the SRM drive at no-load and near standstill conditions.

Figure 2.4-8: Current responses of the SRM under step load and near standstill conditions.

Under step load conditions, the steady state responses for the d-q and d'-q' currents also coincide with each other for the machine types with $\sigma < 1$ and $\sigma = 1$ except in the case of the interior permanent magnet synchronous motor (Fig. 2.4-4) where the $\sigma > 1$. The result shows that the interior permanent magnet motor carries heavy current demand while operating under load conditions. For this type of machine, stator windings with high current rating must be used and appropriate protective fuses incorporated to prevent excessive currents and blow-outs.

The displacement of the steady state operating points between the d-q and currents is more pronounced when the machines are running at very low speed as shown in Fig. 2.4-7 and Fig. 2.4-8. At very low speed, the core loss becomes significant (R_c is smaller) and the ratio of rotor speed to core loss resistance is larger. This factor introduces offset in the transformation between the d-q currents and d'-q' currents thereby causing a mismatch in the steady state operating points.

2.5 Conclusion

In this chapter, the proposed model for the permanent magnet synchronous brushless motor has been presented. A core loss resistance has been included in the d-q model of the machine to account for the effect of core losses. The model has been used to derive the necessary condition that minimizes the total electrical losses of the machine. A relationship that transforms machine currents between the d-q and d'-q' coordinate systems has also been established. This relationship will be used in subsequent implementation scheme.

Chapter 3

The Rectifier-Inverter Permanent Magnet Synchronous Motor Drive

The permanent magnet brushless motors are relatively inexpensive and rugged machine which initially suffered the drawback that its speed was not readily adjustable. The advent of massive improvements in power electronics and consequent improvements in the capabilities of controlled rectifiers and inverters with a reduction in the cost of their manufacture has made the adjustable speed control of permanent magnet synchronous motors more practical.

However, most variable frequency ac drives are complex and it is often difficult to predict their dynamic performances without the aid of a computer. For instance, it has been shown that variable speed permanent magnet motor drives may become unstable due to interaction of the machine and the rectifier-inverter input (front-end) filter. It is therefore important to formulate analytical methods based on model equations to accurately predict system performance over a wide range of operating conditions.

This part of the dissertation presents the model equations of the rectifier-inverter permanent magnet synchronous machine drive. The type of PMSM considered is the interior permanent magnet synchronous motor (IPMSM). By employing and incorporating the concept of switching functions, the equations governing the operation of the rectifier-inverter supply system are formulated and used in conjunction with the q-d voltage model equations of the motor [17] to simulate the complete drive system. Simulation results of dynamic performances under no-load and load conditions are given to illustrate the validity of our approach.

3.1 System Description and Model Equations

A simplified diagram of the rectifier-inverter IPMSM motor drive system is shown in Fig. 3.1-1. This consists of a three-phase power supply, a three-phase voltage rectifier, a filter, a three-phase inverter and a three-phase IPMSM machine as the load. The nonlinear equations which govern the operation of the three-phase voltage rectifier are given as:

$$V_{ao} = V_a - I_a X_{co} \quad (3.1-1)$$

$$V_{bo} = V_b - I_b X_{co} \quad (3.1-2)$$

$$V_{co} = V_c - I_c X_{co} \quad (3.1-3)$$

where X_{co} is the input reactance that limit (or minimizes) the switching harmonics. In this study, we assume the effects of harmonics are negligible and that the voltage supply are balanced, ideal and symmetrical. Hence $X_{co} = 0$, and $V_{ao}=V_a$, $V_{bo}=V_b$, and $V_{co}=V_c$. Then the input phase current corresponding to the phase voltages are I_a , I_b , and I_c .

The state of each switch in the rectifier is described by the following switching functions,

$$S_{xy} = 1 \quad \text{when switch is ON} \quad (3.1-4)$$

$$S_{xy} = 0 \quad \text{when switch is OFF} \quad (3.1-5)$$

$$x = 1,2,3 \quad y = 1,2$$

Fig. 3.1-1: A rectifier front-end three-phase adjustable drive

The constraints on the switching functions of the switches on the upper and lower half of the bridge are given as:

$$S_{11} + S_{21} + S_{31} = 1 \quad (3.1-6)$$

$$S_{12} + S_{22} + S_{32} = 1 \quad (3.1-7)$$

S_{11} , S_{21} and S_{31} are the switching functions of the devices on the upper half of the rectifier while, S_{21} , S_{22} and S_{32} are the corresponding switching functions for the lower half of the rectifier.

The output inductor current, I_{LF} is related to the rectifier phase currents, I_a , I_b and I_c by:

$$I_a = (S_{11} - S_{12})I_{LF} = S_a I_{LF} \quad (3.1-8)$$

$$I_b = (S_{21} - S_{22})I_{LF} = S_b I_{LF} \quad (3.1-9)$$

$$I_c = (S_{31} - S_{32})I_{LF} = S_c I_{LF} \quad (3.1-10)$$

Similarly, the output voltage of the rectifier is given by:

$$V_o = (S_{11} - S_{12})V_a + (S_{21} - S_{22})V_b + (S_{31} - S_{32})V_c = S_a V_a + S_b V_b + S_c V_c \quad (3.1-11)$$

Using the concept of existence function [21], the switching functions S_a , S_b and S_c can be approximated by fundamental and harmonic components as:

$$S_a = \sum_{n=1}^K A_{na} m \cos n(\omega_e t + \psi) \quad (3.1-12)$$

$$S_b = \sum_{n=1}^K A_{nb} m \cos \left(n(\omega_e t + \psi) - \frac{n2\pi}{3} \right) \quad (3.1-13)$$

$$S_c = \sum_{n=1}^K A_{nc} m \cos \left(n(\omega_e t + \psi) + \frac{n2\pi}{3} \right) \quad (3.1-14)$$

Where m is modulation index, ψ is the modulation angle, and ω_e is the frequency of the supply voltage; n is the number of phases (which is 3 for a three phase system).

The output of the rectifier is fed through a series R-L-C filter circuit to remove unwanted harmonics and to also minimize the output ripple. The objective is to obtain a constant dc output V_{dc} across the output capacitor C_{LF} as shown in Fig. 3.1-1.

The equations for the filter circuit are given by:

$$V_o = Z_{LF} I_{LF} + V_{dc} \quad (3.1-15)$$

$$Z_{LF} = R_{LF} + jX_{LF} \quad (3.1-16)$$

It is assumed that the PWM inverter has very high input impedance so that input current into the inverter is negligible. In this case, the inductor current I_{CF} is assumed to be the same as the charging current flowing through the filter capacitors C_{LF} . For simplicity, the filter capacitor is assumed to be large enough so as to ensure a negligible ripple in the charging current. Hence, the dc capacitor voltage V_{dc} (which is the same as the inverter dc rail voltage) may be assumed constant and ripple free.

The capacitor (inductor) charging current I_{CF} is given by:

$$I_{LF} = \int \left(\frac{V_o - V_{dc} - R_{LF} I_{LF}}{L_{LF}} \right) dt \quad (3.1-17)$$

For the PWM inverter, the common PWM techniques employed are the carrier-based PWM technique or the digital PWM technique [22]. In order to reduce the switching losses, a choice of PWM technique is selected from a Generalized Discontinuous PWM strategy (GDPWM). More details on this approach will be discussed in a later section. It suffices to state at this point that a suitable PWM technique is employed for synthesizing the inverter output voltages (supply voltages of the IPMSM). In this work, a balanced three-phase system is assumed so that the resulting inverter output voltages (stator voltages) V_{as} , V_{bs} , and V_{cs} and line voltages, V_{ab} , V_{bc} , and V_{ca} are given by:

$$\begin{bmatrix} V_{as} \\ V_{bs} \\ V_{cs} \end{bmatrix} = \frac{V_{dc}}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} S_{ap} \\ S_{bp} \\ S_{cp} \end{bmatrix} \quad (3.1-18)$$

and

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix}. \quad (3.1-19)$$

In this analysis, it is also assumed that the machine neutral is unconnected.

The voltage and torque equations in the rotor reference frame governing the operation of the IPMSM under balanced conditions with the d'-q' currents (i'_{qs}, i'_{ds}) and rotor speed (ω_r) as state variables may be written in compact form using equations (2.1-9), (2.1-10) and (2.3-16) as:

$$\begin{bmatrix} v_{qs} \\ v_{ds} \end{bmatrix} = \begin{bmatrix} r_s + pL_{qss} & \omega_r L_{dss} \\ -\omega_r L_{qss} & r_s + pL_{dss} \end{bmatrix} \begin{bmatrix} i'_{qs} \\ i'_{ds} \end{bmatrix} + \begin{bmatrix} \omega_r \lambda_{m \text{ mod}} \\ 0 \end{bmatrix} \quad (3.1-20)$$

where

$$\lambda_{m \text{ mod}} = \lambda_m \left(1 + \frac{r_s}{R_c} \right) \quad (3.1-21)$$

For motoring operation, the electromagnetic torque $T_e > 0$ and is given by:

$$T_e = \frac{3P}{4} L_{md} \left[\lambda_{mmd} i'_{qs} + (1 - \sigma) i'_{qs} i'_{ds} \right] \quad (3.1-22)$$

$$T_L = T_e - \left(\frac{2J}{P} \right) p \omega_r \quad (3.1-23)$$

where,

p = operator d / dt

r_s = stator resistance

P = number of poles

ω_r = electrical angular velocity of the rotor

There are no rotor circuits for the IPMSM. Hence, $v_{qr} = v_{dr} = 0$. Also, the machine's rotor runs at the synchronous speed, so that in the rotating reference frame, the rotor speed ω_r is set equal to the electrical angular velocity of the fundamental frequency components of the applied stator voltages, which is denoted as ω_e .

3.2 PWM Classification

Pulse width modulation is performed on voltage and current waveforms to generate an output waveform of desired frequency and magnitude. The purpose is to generate close to an ideal output waveform of desired spectral shape while providing variable-magnitude and variable-frequency operation in ac-dc and dc-ac inverters used in Adjustable Speed Drives (ASDs), active filters, static frequency changers, unified power flow controllers in flexible ac transmission systems, Uninterruptible Power Supplies (UPS) and other applications. A typical example of an application employing PWM control is the ac drive unit previously shown in Fig. 3.1-1. This consists of front-end-line frequency ac-to-dc converter, a dc bus with filter, and an output switch-mode dc-ac inverter. The inverter switches are BJTs or MOSFETs and are controlled using pulse width modulation.

In the past decades, several techniques were proposed and developed to achieve voltage and frequency control. The most popular technique is the carrier-based PWM methods due to its simplicity of implementation. Two main implementation techniques exist under this strategy: the triangle intersection scheme and the direct digital method. In the triangle intersection PWM technique [23], three reference modulating signals V_a , V_b , and V_c are

compared with a triangular carrier signal and the intersections define the switching instants S_{ap} , S_{bp} , and S_{cp} of the controllable devices which are used to synthesize the inverter output waveform.

Of all the triangle intersection techniques, the most common is the traditional Sine wave Pulse Width Modulation (SPWM) scheme as shown in Fig. 3.2-1. Under the SPWM scheme, the inverter switching pulses are generated by comparing sinusoidal reference modulation waves V_a , V_b , and V_c with a high frequency triangular carrier wave to produce the inverter output voltage. This method, however, suffers drawback of low dc bus utilization and presence of low order harmonics. This led to the development of enhanced techniques such as the improved Sine PWM, Third Harmonic Injection PWM (THIPWM) and the Third and Ninth Harmonic Injection PWM (TNHIPWM) [24] which provide unity (full) dc bus utilization and superior output spectrum. Another type of pulse width modulation is the Space Vector Pulse Width Modulation. This has proven to provide superior performance and is well suited for microprocessor-based implementation.

All the above modulating techniques employ the use of continuous modulating signals which are compared with high frequency triangular carrier waves. The larger the frequency of the carrier wave, the better the quality of the inverter output. Unfortunately, the improvement in the quality of the output waveform is at the expense of the inverter efficiency since switching has to be done at a higher frequency resulting in higher switching losses of the inverter.

By employing a discontinuous signal in which part of its cycle is flat, the number of switching pulses is reduced, hence resulting in reduction of the inverter switching losses. In the past, several discontinuous modulation signals were generated by adding a zero sequence to the

Fig. 3.2-1: SPWM Waveforms and Switching Signals in the linear region.

reference modulating waveforms as shown in Fig. 3.2-2. In this technique, the choice of zero sequence provides a degree of freedom which is equivalent to the partitioning of the two zero states in the direct digital implementation [25-26].

The use of discontinuous modulating waveforms in PWM inverters/converters generally results in lower number of switching pulses. The consequence of this is reduced switching losses and hence a better efficiency for the drive as compared to the case of continuous pulse width modulation. This advantage unfortunately is weakened by the increase in distortion of inverter currents in low frequency switching operations, thus making SVPWM more preferable for low modulation range operation in high performance drives because of its superior waveform quality [26]. Discontinuous PWM on the other hand is the preferred choice in high frequency switching and high modulation range operations. Since each of these modulators exhibits different characteristics under varying operating conditions, it is difficult to identify a single “best-in-class” modulator that can provide the best performance over a wide operating range. Combining more than two PWM methods increases algorithm and implementation complexity, whereas combination of just two of the PWM methods does not guarantee an optimal solution [27]. This necessitated the development of a Generalized Discontinuous Pulse Width Modulation (GDPWM) strategy that offers the optimal solution without complex algorithm is suitable for on-line implementation. A detailed account of the development, implementation and application of the GDPWM strategy to the induction motor and permanent magnet synchronous motor drives can be found in [27], [29]. A performance evaluation of the various modulating schemes (sinusoidal and non-sinusoidal signals) has also been presented in [28] for the case of the induction motor drives.

Fig. 3.2-2. Block diagram of the zero Sequence Injection PWM technique.

One of the goals of this section is to present a systematic approach for selecting a PWM scheme that is suitable for implementing the proposed loss minimization and control strategy for the PMBM drive. This chapter presents result that informs the decision on the choice of PWM scheme.

3.3 Development of the GDPWM Scheme Using Space Vectors

A generalized expression for the zero sequence signal V_o that is added to the reference signals V_a^* , V_b^* and V_c^* for generating a variety of non-sinusoidal (and discontinuous modulating signals) is derived using the Space Vector theory [28-30] from this the modulating signal used in the carrier-based Space Vector Pulse Width Modulation (SVPWM) is obtained. In the direct digital implementation of the SVPWM [22], the reference voltage, V_{qd}^* as shown in Fig. 3.3-1 is generated by averaging two adjacent active vectors among the set V_1 through V_6 (corresponding to inverter active states) and the two zero vectors V_0 and V_7 (corresponding to inverter null states) over the switching period T_s . The relationship between time durations t_a and t_b of adjacent modes and t_z comprising of t_0 and t_7 for null modes are given by:

$$\begin{aligned} T_s &= t_a + t_b + t_z \\ t_z &= t_0 + t_7 \\ t_0 &= (1 - \xi)t_z \\ t_7 &= \xi t_z \end{aligned} \tag{3.3-1}$$

ξ is a variable of zero sequence signal called the distribution factor: $0 \leq \xi \leq 1$.

By transforming the states vectors V_0 through V_7 to the qdo reference frame and invoking the Volt-Second balance principle:

$$V_{qdo} T_s = V_{qdoa} t_a + V_{qdob} t_b + V_{qdo0} t_0 + V_{qdo7} t_7 \tag{3.3-2}$$

Fig. 3.3-1: Space Vector Diagram in d-q reference frame

In a balanced system, the zero sequence voltage [28] is the same as the neutral voltage and is given by:

$$\langle V_o \rangle = -[0.5V_d(1-2\xi) + \xi V_{\max} + V_{\min}(1-\xi)] \quad (3.3-3)$$

where,

$$V_{\max} = \max(V_a^*, V_b^*, V_c^*) \text{ and } V_{\min} = \min(V_a^*, V_b^*, V_c^*)$$

Fig. 3.3-2 shows the block diagram for the carrier-based implementation of the GDPWM strategy in which the control parameter ξ is employed to control selection of the zero sequence signal V_o that is added to the three sinusoidal reference signals V_a^* , V_b^* and V_c^* to produce the modified reference waveforms M_a , M_b and M_c respectively. The modified reference signals are compared with high frequency triangular carrier signal to generate the inverter triggering pulses used to synthesize the inverter output line voltages. A range of discontinuous functions are realized by selecting, ξ . Choosing constant values of $\xi=0, 0.5$ and 1.0 generate DPWMMAX, SVPWM and DPWMMIN waveforms respectively. When ξ varies with time:

$$\xi = f(t) = 0.5[1 + \text{sgn}(\cos 3(\omega t - \psi - \phi))] \quad (3.3-4)$$

Setting $\phi=0, \pi/6, \pi/3$, and $\pi/4$ generate the discontinuous modulating waveforms DPWM1, DPWM2, DPWM3, and DPWM4 respectively. At unity power factor, $\psi=0$.

The modulating waveform for the SVPWM is shown in Fig. 3.3-3 while that of the other DPWM schemes are shown in Fig. 3.3-4 (a)-(f). From Fig. 3.3-3 and Fig. 3.3-4, with m and ϕ as the GDPWM control parameters, the following observations are noteworthy:

Fig. 3.3-2: Block diagram of carrier-based implementation of the GDPWM strategy

Fig. 3.3-3: The Space Vector PWM waveforms in the linear modulation region.
(Mod. Index, $m = 0.8$)

(a)

(b)

(c)

(d)

(e)

(f)

Fig. 3.3-4: The DPWM waveforms (a) DPWMAX, (b) DPWMMIN @ $m=1.154$; (c) DPWM1, (d) DPWM2, (e) DPWM3, and (f) DPWM4 @ $m=0.8$

1. The modified modulating signals are clipped for 120° per cycle of the fundamental reference signal.
2. The effect of varying the modulation index, m , is to cause a change in the magnitude of the fundamental of the modified reference signal.
3. Changing ϕ in increments of 120° correspondingly shifts the modified modulating waveform by 120° . Any increments other than 120° will result in a drastic change in waveform type of the modified reference signal thereby altering the modulator type (e.g. from DPWM1 to DPWM3).

3.4 Characteristics of the GDPWM Method

3.4.1 Extended Linearity Gain

The fundamental harmonic amplitudes of the inverter output line-to-line voltage is used to compare inverter gain characteristics for the various modulating schemes. Usually a carrier signal with a high frequency (about 21 times the modulating frequency) is compared with the modulation signal to generate the inverter switching pulses. In Figs. 3.4-1 and 3.4-2, the gain characteristics of the GDPWM is compared with that of the traditional Sinusoidal PWM (SPWM) for the modulating ranges $m \leq 1.155$ and $m \geq 1.155$ respectively. From Fig. 3.4-1, it is obvious that a linear relationship exists between the fundamental amplitude V_1 and the modulation index, m in the linear modulation region. However, this is not the case in Fig. 3.4-2 for the over-modulation region where linearity is lost due to saturation. From Fig. 3.4-1, it is also evident that the GDPWM modulators have extended linearity range up to the theoretical modulation limit of $M_{\max} = 2/\sqrt{3}$ (15% more than SPWM). Note that the theoretical modulation limit for the SPWM, $M_{\max} = 1$.

Fig. 3.4-1: Fundamental harmonic amplitude in the linear modulation region (modulation index, $m \leq M_{max}$), $f_c = 21f_s$

Fig. 3.4-2: Fundamental harmonic amplitude in the over-modulation region (modulation index, $m \geq M_{max}$), $f_c = 21f_s$

3.4.2 Over-modulation Gain

In the over-modulation range (modulation index, $m > M_{\max}$), the inverter gain characteristics for the GDPWM are non-linear and cannot exceed $4\sqrt{3}/(2\pi)$, the maximum gain for the six-step mode. Under this condition, DPWM1 and DPWM2 provide the highest gain while DPWM3 exhibits the least gain as shown in Fig. 3.4-2.

3.4.3 Distortion Factor

Distortion Factor (DF) and Total Harmonic Distortion (THD) provide a measure for the quality of the output voltage waveform. By choosing a carrier frequency $f_c = 21f_s$ and considering harmonic order $n < 19$ to be significant, the amplitudes of the output harmonics C_n can be obtained. Since this is a three phase modulation scheme, all triplen harmonics are eliminated and there are no even harmonics as the carrier frequency is an odd multiple of the supply frequency. The 15th, 17th, and 19th harmonics (C_{15} , C_{17} and C_{19}) dominate the linear region [28] while 5th, 7th and 13th harmonics dominate the over modulation region. The distortion factors for the GDPWM modulators are lowest when the inverter is operating close to the linear modulation limit as shown in Fig. 3.4-3. The claim that discontinuous modulating signals are better suited for operation in the over-modulation region than in the linear region is made valid.

Fig. 3.4-3: Distortion Factor versus modulation index for the GDPWM and SPWM ($f_c=21f_s$)

3.4.4 Simulation and Experimental Results

Simulation waveforms showing the dynamic operation for a three-phase voltage source inverter with input DC voltage of 20V and switching at a frequency of 20 kHz is shown in Fig. 3.4-4. The result shows from top to bottom, the SVPWM modulating waveform M_a at a modulation index of 0.8, phase 'a' voltage of the inverter V_{an} , and the line-to-line voltage V_{ab} . The reference modulating signal is a sinusoidal waveform of 60 Hz. The waveforms of corresponding line currents for the voltage source inverter (VSI) feeding an inductive (RL) load are shown in Fig. 3.4-5. For the inductive load: $R = 15$ ohms and $L = 8.45$ mH. The output line currents are sinusoidal and their algebraic sum is always zero, thus confirming the operation is a balanced three phase system.

The three-phase VSI feeding an inductive load has been successfully implemented in the laboratory using the Analog Devices ADMC 401 and Texas Instruments TMS320LF2407 DSP evaluation boards. The PWM triggering pulses are supplied from the PWM block of the DSP board to control switching of the six inverter switching devices. Fig. 3.4-6 shows experimental waveforms of the phase 'a' voltage and line current respectively. The experimental waveform of the line current is also a sinusoidal waveform similar to the simulation result of Fig. 3.4-5.

The results presented thus far show that space vector PWM and Sinusoidal PWM are the modulation schemes that provide low output distortion as shown in Fig. 3.4-3. These modulation schemes are expected to offer much better output quality than the discontinuous pulse width modulators. For this reason, a current regulated Voltage Source Inverter with SPWM or SVPWM will be employed in the proposed efficiency optimization strategy.

Fig. 3.4-4: Simulation Waveforms of Space Vector PWM voltage source inverter.

Fig. 3.4-5: Simulation waveforms of line currents for the VSI feeding an inductive load.

Fig. 3.4-6: Experimental waveforms for the VSI feeding and inductive load.
Ch1: phase 'a' voltage; Ch2: phase 'a' line current

3.5 Open-Loop Dynamic Performance

It is necessary to validate our model by observing the variables of the interior permanent magnet synchronous motor (IPMSM) under various operating conditions. The nonlinear differential equations which describe the operation of the complete drive (rectifier/filter, inverter and motor) are used to simulate the drive performances under: (a) free (no load) acceleration from stall, (b) step load changes (c) three-phase fault.

The simulation results that will follow are those obtained in the rotor reference frame. This is accomplished by setting the arbitrary reference frame speed ω to ω_r in the machine's non-linear differential equations of previous section.

The parameters of the drive under consideration are given in Table 3.5-1. This includes the parameters of a 4-pole, 60 Hz, three-phase IPMSM. In the table, J is the load inertia which is assumed to be the same as that of the rotor.

Table 3.5-1: Parameters of a 3-phase 3 HP IPMSM drive

r_s	=	0.9889 Ω	Number of poles, P	=	4
L_{ds}	=	0.031 H	Rotor/Load, Inertia, J	=	0.0018 kg.m ²
L_{qs}	=	0.021 H	Rectifier Input Supply Voltage	=	220 V (rms) Line-to-line
λ_m	=	0.063 Wb	Supply frequency, f	=	60 Hz
L_F	=	25 mH	Viscous Friction, B_m	=	0.00212 N.m/s
R_{LF}	=	2.0 Ω	C_{LF}	=	800 μ H

Machine Rated Speed	= 1096 rpm
Inverter SVPWM Modulation Index, m	= 0.866
Inverter Input DC Voltage, V_{dc}	= 34 V (DC)

3.6 Free Acceleration (No-Load) Performance

The torque-speed characteristics during free acceleration or no-load ($T_L=0$) is shown in Fig. 3.6-1. Note that since the machine is supplied from a Space Vector PWM inverter, the characteristics are different from that obtained with the machine fed directly from a purely sinusoidal voltage supply. The result of Fig. 3.6-1 suggests the presence of torque ripple and harmonics. This is confirmed from the plots of Figs. 3.6-2(a) and Fig. 3.6-2(b) which show traces of phase 'a' voltage, currents, torque and speed for the same machine. Fig. 3.6-2(a) depicts the dynamic performance of the machine from stall to steady state; a large starting current (about 10 times rated value) and starting torque is observed when rated voltage is applied to the machine. It is therefore recommended to start this machine with reduced voltage using a compensator or transformer until machine attains 60-80 % full speed whereupon full voltage is applied. The presence of ripple torque is easily observed in Fig. 3.6-2(b) which is an amplified (or magnified) result of Fig. 3.6-2(a). In both figures, friction and windage losses are neglected and it is seen that the machine accelerates to synchronous speed under no-load condition.

Fig. 3.6-1: Torque-speed characteristics during free acceleration for a rectifier-inverter 3 hp interior permanent magnet synchronous motor (IPMSM) drive.

Fig. 3.6-2: Dynamic Performance of IPMSM under no-load condition.
(a) from starting to steady state, (b) at steady state

Fig. 3.6-3 shows waveforms of the rectifier and filter output during free acceleration (no-load) condition from stall to steady-state. The figure contains waveforms of the rectifier output voltage V_o , the inductor current I_{LF} and the filter output voltage V_{dc} . A large instantaneous increase in the inductor charging current is observed at starting and this begins to decline within a fraction of a second as charge continues to build on filter capacitors C_{LF} . A corresponding steady rise in output voltage V_{dc} is observed as the capacitor voltage V_{dc} builds up. At steady state, the charging current declines to zero and the capacitor voltage V_{dc} attains the value of the rectifier output V_o of 27 V.

From Fig. 3.6-3, we see that the inductor current diminishes to zero at about 1.7 seconds after starting. At this point, the system attains steady state. Notice, that the rectifier output voltage remains constant and is not affected by the dynamics of the filter, inverter and motor under no-load condition. Next we examine the drive performance due to sudden change in load torque.

3.7 Performance Under Load Torque

The torque speed characteristics and the dynamic response of the 3 hp IPMSM motor drive during step changes in load torque are shown in Fig. 3.7-1 and 3.7-2 respectively. In Fig. 3.7-2, the machine is initially operating on no-load and is building up to attain the synchronous speed when the load is suddenly stepped to 0.4 N.m at 0.1 seconds after starting and is maintained at this constant level until the motor attains a steady operating condition. During this period, there is an increase in machine currents and a decrease in rotor speed due to step increase in load torque.

Fig. 3.6-3: Dynamic performance of rectifier and filter for the IPMSM drive under no-load condition.

Fig. 3.7-1: Torque-speed characteristics of the rectifier-inverter IPMSM drive during step change in load torque.

Fig. 3.7-2: Dynamic performance of the IPMSM drive during step changes in load torque.

Next, the load torque is stepped down to 0.1 N.m at 0.8 seconds which causes a decrease in machine currents and an increase in rotor speed until the machine establishes another steady state operating condition. In both cases, the machine approaches each new operating point in an over-damped fashion. This is true for most small machines of similar capacities [17]. There is a similarity in the machine's steady-state torque-speed characteristic under step load condition to that of the free acceleration (no-load) characteristics. In fact there is very little difference between the no-load (free acceleration) torque-speed characteristic and the torque-speed characteristic response under constant step load condition.

The influence of changes in load torque on the machine stator voltages is not very conspicuous. A passive observation will suggest that the stator voltage V_{as} is unaffected by the change in load torque. This will be hasty and erroneous conclusion as V_{as} in Fig. 3.7-2 contains PWM harmonic contents which can only be analyzed from the frequency spectrum.

A quicker assessment can be made by examining the waveforms of the rectifier and filter output of Fig. 3.7-3. From Fig. 3.7-3, we see a decrease in the filter capacitor voltage (same as the inverter input dc voltage) V_{dc} when the load is stepped up from zero to 0.4 N.m. During this period there is a corresponding increase in the inductor charging current I_{CF} while the rectifier output voltage V_o remains the same all through the transition.

Since this is an open-loop system and there is no feedback loop, switching functions for synthesizing the inverter output voltages are independent of the machine currents and voltages. Therefore, it can be deduced that a decrease in inverter input voltage V_{dc} will lead to a proportionate decrease in the inverter output voltages (V_{as} , V_{bs} , and V_{cs}). This reasoning can be extended further to conclude that a step increase in the load torque will cause a decrease in the fundamental component of the inverter output voltage.

Fig. 3.7-3: Dynamic performance of rectifier and filter for the IPMSM drive under step load condition.

3.8 Dynamic Performance during Three-Phase Fault.

The dynamic performance of the 3 hp IPMSM drive during and after a three-phase fault at the terminals with constant load torque ($T_L = 0.4$ Nm) is shown in Figs. 3.8-1 and 3.8-2. A constant step load of 0.4 Nm is applied after 0.01 sec and held constant for during the motor operation. A three-phase fault occurred at 1.5 sec after start and cleared at 1.6 sec. This fault is simulated by setting the machine stator voltages V_{as} , V_{bs} and V_{cs} to zero at the instant of fault, then after 0.1 second, the stator voltages are reapplied.

The IPMSM variables before, during and after fault conditions are shown in Fig. 3.8-1. From this figure, it is seen that the fault condition in which the terminal voltages are zero gives rise to decaying offset in both the stator and rotor currents. The transient offsets in the stator currents induce decaying oscillations in the rotor circuits. Similar effect is observed on the stator circuits with the transient offset in the rotor currents inducing decaying oscillations in the stator circuit at a frequency corresponding to the rotor speed. The effect of the transient current is more pronounced with large sized (capacity) machines. Since the machine under consideration is small and highly damped, the effects of stator and rotor transient die out quickly before the fault is cleared and the stator voltages reestablished.

Fig. 3.8-1 shows the effect of the three-phase fault on the machine currents. During the fault condition, the machine swings between braking and regenerative mode. This unpredictable behavior has a great potential to induce instability on system operation. After the fault is cleared, normal motoring operation is restored. It can be inferred from this result that a three-phase fault condition has a great potential to cause system instability. The effect of the fault condition on the rectifier-output filter combination is also shown in Fig. 3.8-2.

Fig. 3.8-1: Dynamic performance of the IPMSM drive during three-phase fault.

Fig. 3.8-2: Performance of rectifier-filter during three-phase fault.

It is seen that a three phase fault causes a fatal dip in the inductor charging current I_{LF} which in turn causes a spike in the capacitor output voltage V_{dc} . The rectifier and filter states are restored after the fault is cleared. Notice, however that the rectifier output voltage V_o remains unchanged and is unaffected by the fault condition. Other fault conditions such as line-to-line short circuit fault, line to ground fault, and open-line fault can be simulated to study the impact of such faults on the overall drive performance. The simulation of fault condition is a very useful tool for determining component ratings in practical design.

3.9 Conclusion

In this dissertation chapter, the model equations governing the operation of the rectifier-inverter interior permanent magnet synchronous motor (IPMSM) drive has been presented. The model equations have been used in computer simulation to study performance of the drive under various operating conditions such as no-load (free acceleration), performances due to step load change and performances during a three-phase fault at the machine terminals. In the analysis, the effects of harmonics have been neglected and it has been shown that the drive model is valid and sufficient to predict the performance of the drive. The drive model presented is therefore an invaluable tool for conducting preliminary control studies. It is also very useful in design for establishing system ratings and operating limits. This tool may be used for stability studies for future research work.

Chapter 4

Linearized Equations of Permanent Magnet Synchronous Motor

The equations which describe the behavior of permanent magnet synchronous motors are nonlinear and are often solved with the aid of computer. Considerable insight can be gained about small-excursion behavior of these machines using linearized or small-displacement version of the equations. Linearization is achieved by applying Taylor's expansion about an operating point to obtain a set of linear differential equations which describe the dynamic behavior of the machine for small excursions. The machine can then be treated as a linear system with regards to small disturbances, whereupon the linearized equations is used to establish transfer functions for designing a simple and low-cost linear controls for the machines.

In this section, the non-linear voltage and torque equations of the permanent magnet brushless motors are expressed in the rotor reference frame and are linearized using Taylor's expansion. The direct-axis and quadrature-axis currents and rotor speed are chosen to be the state variables and it is assumed that during steady state balanced conditions, machine parameters and driving forces are constant and are independent of time.

4.1 Linearized Machine Equations in D'-Q' Variables

The voltage and torque equations for the PMBM under balanced conditions with the d'-q' currents (i'_{qs} , i'_{ds}) and rotor speed (ω_r) as state variables may be written in compact form using equations (2.1-9), (2.1-10) and (2.3-16) as:

$$\begin{bmatrix} v_{qs} \\ v_{ds} \end{bmatrix} = \begin{bmatrix} r_s + pL_{qss} & \omega_r L_{dss} \\ -\omega_r L_{qss} & r_s + pL_{dss} \end{bmatrix} \begin{bmatrix} i'_{qs} \\ i'_{ds} \end{bmatrix} + \begin{bmatrix} \omega_r \lambda_{m \text{ mod}} \\ 0 \end{bmatrix} \quad (4.1-1)$$

$$T_e = \frac{3P}{4} L_{md} \left[\lambda_{mmd} i'_{qs} + (1 - \sigma) i'_{qs} i'_{ds} \right] \quad (4.1-2)$$

$$T_L = T_e - \left(\frac{2J}{P} \right) p \omega_r \quad (4.1-3)$$

$$\lambda_{m \text{ mod}} = \lambda_m \left(1 + \frac{r_s}{R_c} \right) \quad (4.1-4)$$

Equations (4.1-1), (4.1-2) and (4.1-3) describe the dynamics of the motor. Due to the coupling among the state variables, it is very clear that these equations are highly non-linear. A linearized model can be realized by writing the machine variables, i'_{qs} , i'_{ds} and ω_r , in terms of a Taylor's expansion about a fixed operating point $(i'_{qso}, i'_{dso}, \omega_{ro})$. Since each of the voltage and torque equations is a function of the variables, i'_{qs} , i'_{ds} and ω_r , we employ the Taylor's expansion [17] for a function of three variables, $g(f_1, f_2, f_3)$ such that:

$$g(f_1, f_2, f_3) \approx g(f_{1o}, f_{2o}, f_{3o}) + \frac{\partial}{\partial f_1} g(f_{1o}, f_{2o}, f_{3o}) \Delta f_1 + \frac{\partial}{\partial f_2} g(f_{1o}, f_{2o}, f_{3o}) \Delta f_2 + \frac{\partial}{\partial f_3} g(f_{1o}, f_{2o}, f_{3o}) \Delta f_3 + \dots \quad (4.1-5)$$

where,

$$f_i = f_{io} + \Delta f_i \quad (4.1-6)$$

The above Taylor's expansion method is then applied to equations (4.1-1), (4.1-2) and (4.1-3) to obtain a linearized system model. Since this treatment considers only small excursions from a fixed operating point, all terms higher than the first-order are neglected. The linearized system model for the PMBM is given by:

$$\begin{bmatrix} \Delta v_{qs} \\ \Delta v_{ds} \\ \Delta T_L \end{bmatrix} = \begin{bmatrix} r_s + pL_{qss} & \omega_{ro} L_{dss} & \lambda_{m \text{ mod}} + L_{dss} i'_{dso} \\ -\omega_{ro} L_{qss} & r_s + pL_{dss} & -L_{qss} i'_{qso} \\ \frac{3P}{4} L_{md} (\lambda_{mmd} + (1-\sigma) i'_{dso}) & \frac{3P}{4} L_{md} (1-\sigma) i'_{qso} & -\left(\frac{2J}{P}\right) p \end{bmatrix} \begin{bmatrix} \Delta i'_{qs} \\ \Delta i'_{ds} \\ \Delta \omega_r \end{bmatrix} \quad (4.1-7)$$

Since PMSMs are energized by connecting to the electric power system grid, linearization of the system voltage and torque equations have been performed in the synchronously rotating reference frame for convenience. This approach also greatly simplifies our analysis as it is also assumed that under normal and balanced conditions, the rotor of the machine turns at the synchronous speed so that the inductances, L_{qss} and L_{dss} are fixed parameters with fixed values evaluated at the synchronous speed ω_e .

It is convenient to write equation (4.1-7) in the form:

$$Epx = Fx + u \quad (4.1-8)$$

where,

$$(x)^T = \begin{bmatrix} \Delta i'_{qs} & \Delta i'_{ds} & \Delta \omega_r \end{bmatrix} \quad (4.1-9)$$

$$(u)^T = \begin{bmatrix} \Delta v_{qs} & \Delta v_{ds} & \Delta T_L \end{bmatrix} \quad (4.1-10)$$

$$E = \begin{bmatrix} L_{qss} & 0 & 0 \\ 0 & L_{dss} & 0 \\ 0 & 0 & -\left(\frac{2J}{P}\right) \end{bmatrix} \quad (4.1-11)$$

$$F = - \begin{bmatrix} r_s & \omega_{ro} L_{dss} & \lambda_{m \text{ mod}} + L_{dss} i'_{dso} \\ -\omega_{ro} L_{qss} & r_s & -L_{qss} i'_{qso} \\ \frac{3P}{4} L_{md} (\lambda_{mmd} + (1-\sigma) i'_{dso}) & \frac{3P}{4} L_{md} (1-\sigma) i'_{qso} & 0 \end{bmatrix} \begin{bmatrix} \Delta i'_{qs} \\ \Delta i'_{ds} \\ \Delta \omega_r \end{bmatrix} \quad (4.1-12)$$

The linear differential equation (4.1-8) can further be written in the standard state equation form:

$$px = Ax + Bu \quad (4.1-13)$$

Since equation (4.1-8) may be written as:

$$px = (E)^{-1} Fx + (E)^{-1} u \quad (4.1-14)$$

Therefore,

$$A = (E)^{-1} F \quad (4.1-15)$$

$$B = (E)^{-1} \quad (4.1-16)$$

4.2 Transfer Function Formulation

In the previous section, a linearized model of the PMBM was derived using the d'-q' current variables and machine rotor speed. In actual design, it is necessary to formulate transfer function from the linearized state model in terms of the actual d-q stator currents using current transformation.

By following the same procedure outlined in section 4.1, the linearized model for the current transformation from d'-q' to d-q quantities using equations (2.4-5) and (2.4-6) is given by:

$$\begin{bmatrix} \Delta i_{qs} \\ \Delta i_{ds} \\ \Delta \omega_r \end{bmatrix} = \begin{bmatrix} 1 + p \frac{L_{qs}}{R_c} & \frac{\omega_{ro} L_{ds}}{R_c} & \frac{\lambda_m + L_{dss} i'_{dso}}{R_c} \\ -\frac{\omega_{ro} L_{qs}}{R_c} & 1 + p \frac{L_{ds}}{R_c} & -\frac{L_{ds} i'_{qso}}{R_c} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta i'_{qs} \\ \Delta i'_{ds} \\ \Delta \omega_r \end{bmatrix} \quad (4.2-1)$$

Again it is convenient to write equation (4.1-7) in the form:

$$Hpx + Jx = g \quad (4.2-2)$$

where,

$$(x)^T = \begin{bmatrix} \Delta i'_{qs} & \Delta i'_{ds} & \Delta \omega_r \end{bmatrix} \quad (4.2-3)$$

$$(g)^T = \begin{bmatrix} \Delta i_{qs} & \Delta i_{ds} & \Delta \omega_r \end{bmatrix} \quad (4.2-4)$$

$$H = \begin{bmatrix} \frac{L_{qs}}{R_c} & 0 & 0 \\ 0 & \frac{L_{ds}}{R_c} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (4.2-5)$$

$$J = \begin{bmatrix} 1 & \frac{\omega_{ro} L_{ds}}{R_c} & \frac{\lambda_m + L_{ds} i'_{dso}}{R_c} \\ -\frac{\omega_{ro} L_{qs}}{R_c} & 1 & -\frac{L_{ds} i'_{qso}}{R_c} \\ 0 & 0 & 1 \end{bmatrix} \quad (4.2-6)$$

By using equation (4.1-13), equation (4.2-2) may be rewritten as:

$$g = H(Ax + Bu) + Jx \quad (4.2-7)$$

$$g = (HA + J)x + (H B)u \quad (4.2-8)$$

But,

$$X(s) = (sI - A)^{-1} BU(s) \quad (4.2-9)$$

where I is the identity matrix ($n \times n$)

So that,

$$G(s) = (HA + J)(sI - A)^{-1} BU(s) + (H B)U(s) \quad (4.2-10)$$

Hence,

$$G(s) = [(HA + J)(sI - A)^{-1} + H]BU(s) \quad (4.2-11)$$

Combining the above results with that obtained in section 4.1 allows us to formulate a complete state model for the PMSM as a linear multi-input multi-output (MIMO) system described by the following equations:

$$\text{State equation: } \dot{x} = Ax + Bu ; \quad \text{state equation} \quad (4.2-12)$$

$$\text{Output Equation: } y = Cx + Du ; \quad \text{output equation} \quad (4.2-13)$$

where,

$$x = \begin{bmatrix} \Delta i'_{qs} & \Delta i'_{ds} & \Delta \omega_r \end{bmatrix}^T \quad : (3 \times 1) \text{ state vector}$$

$$y = g = \begin{bmatrix} \Delta i_{qs} & \Delta i_{ds} & \Delta \omega_r \end{bmatrix}^T \quad : (3 \times 1) \text{ output vector}$$

$$u = \begin{bmatrix} \Delta v_{qs} & \Delta v_{ds} & \Delta T_L \end{bmatrix}^T \quad : (3 \times 1) \text{ input vector}$$

and,

$$A = (E)^{-1} F \quad : (3 \times 3) \text{ system matrix} \quad C = HA + J \quad : (3 \times 3) \text{ output matrix}$$

$$B = (E)^{-1} \quad : (3 \times 3) \text{ input matrix} \quad D = HB \quad : (3 \times 3) \text{ transmission matrix}$$

The block diagram representation of the state model of the linearized equations for the PMSM is shown in Figure 4.2-1 and the corresponding transfer function block diagram is shown in Figure 4.2-2. These diagrams show the relationship between the output variable being controlled to that of the controlling input variable. The analysis thus far presented permits the design of necessary controls (e.g. speed or torque) for the PMSM in variable-speed drive systems. In control system design, it is usual practice to incorporate full state variable feedback to improve system stability as indicated by the dashed lines in Fig. 4.2-2. However, in many practical situations, not all state variables are accessible for measurement and control purposes; only inputs and outputs are measurable. In such cases, state observers driven by available inputs and outputs are employed to estimate the state vector. An alternative control technique is the use of output feedback control.

In a state feedback control, $u = -Kx$ and pole-placement technique can be used to compute state-feedback gain matrix K such that the eigenvalues of $A - BK$ correspond to those specified by the system poles.

Figure 4.2-1: Block diagram representation of the state model for the linear multi-input-multi-output (MIMO) permanent magnet synchronous motor.

Figure 4.2-2: Transfer function block diagram of a permanent magnet synchronous motor
($u = -Kx + r$)

Define:

$$\tilde{A} = A - BK \quad (4.2-14)$$

The system eigenvalues is specified from the following characteristic equation:

$$|\lambda I - \tilde{A}| = 0 \quad (4.2-5)$$

4.3 Eigen-values and Pole Placement

Consider a synchronous reluctance motor with the following parameters:

$$\begin{array}{lll} r_s = 0.9889 \, \Omega & L_{qs} = 0.021 \, \text{H} & L_{ds} = 0.1021 \, \text{H} \\ \lambda_m = 0.063 \, \text{Wb} & P = 4 & J = 0.00018 \, \text{Kg.m}^2 \end{array}$$

It is desired to operate the machine at the following operating conditions:

$$i'_{qso} = 2 \, \text{A}, \quad i'_{dso} = 0.1 \, \text{A}, \quad \omega_{ro} = 200 \, \text{rad/s}$$

The desired poles of inner current-loop of the system are given by:

$$\lambda_1 = -2 \quad \lambda_2 = -1 + j2 \quad \lambda_3 = -1 - j2$$

The feedback gain matrix that satisfies equation (4.2-5) is given by:

$$K = \begin{bmatrix} 1.0099 & 20.4822 & 0.0733 \\ -4.4086 & 1.0911 & -0.0420 \\ 0.4273 & 0.4866 & -0.0002 \end{bmatrix}$$

Chapter 5

Steady State Analysis And Operating Constraints Of The Permanent Magnet Synchronous Motor

In this chapter the steady-state operation of the PMSM is considered. First it is assumed that the harmonic losses are negligible when the motor is energized by balanced sinusoidal stator voltages. This allows us to study the steady state performance of the system under balanced condition so as to determine the influence of motor variables on the system characteristics. The goal of the steady state analysis is also to determine the feasible operating boundaries and stability limits. The steady state characteristic performances under the conventional and proposed loss minimization schemes will be presented.

5.1 Analysis of Steady State Operation

The machine state equation defined by equations (2.1-9) and (2.1-10) for a general time-invariant system is of the form:

$$\dot{x} = f(x, u) \quad (5.1-1)$$

where,

$$x = \begin{bmatrix} i'_{qs} & i'_{ds} & \omega_r \end{bmatrix}^T \quad (5.1-2)$$

$$u = \begin{bmatrix} v_{qs} & v_{ds} & T_L \end{bmatrix}^T \quad (5.1-3)$$

This equation can be linearized for small variations about an equilibrium point, (x_o, u_o) . The system is in equilibrium when:

$$\dot{x} = f(x_o, u_o) = 0 \quad (5.1-4)$$

Since the derivative of all the state variables are zero at the equilibrium point, this condition corresponds to the steady state condition where the system continues to lie at the equilibrium point unless otherwise perturbed. For a PMSM operating as an autonomous system, the origin is clearly an equilibrium point. Under this condition, the machine is un-energized and no load is applied.

Using the state model for the PMSM, the steady state equations under balanced sinusoidal impressed stator voltages are given by:

$$V_{qs} = r_s I'_{qs} + \omega_r L_{dss} I'_{ds} + \omega_r \lambda_{m \text{ mod}} \quad (5.1-5)$$

$$V_{ds} = -\omega_r L_{dss} I'_{ds} + r_s I'_{ds} \quad (5.1-6)$$

where uppercase letters have been used to denote steady state quantities. Note that all variables have been expressed in rotor reference frame and the rotor magnetic flux $\lambda_{m \text{ mod}}$ will remain constant when the motor has come up to synchronous speed. For a motor with negligible core losses, $\lambda_{m \text{ mod}}$ is approximately the rotor's permanent magnet flux (i.e. $\lambda_{m \text{ mod}} \approx \lambda_m$). The steady state electromagnetic torque is expressed from (4.1-2) as:

$$T_e = \frac{3P}{4} L_{md} \left[\lambda_{mmd} I'_{qs} + (1 - \sigma) I'_{qs} I'_{ds} \right] \quad (5.1-7)$$

Equations (2.4-5) and (2.4-6) defines the transformation for current variables from d'-q' to d-q quantities. At steady state, these equations become linear relationship expresses as:

$$I_{qs} = I'_{qs} + \frac{\omega_r L_{ds}}{R_c} I'_{ds} + \frac{\omega_r \lambda_m}{R_c} \quad (5.1-8)$$

$$I_{ds} = -\frac{\omega_r L_{qs}}{R_c} I'_{qs} + I'_{ds} \quad (5.1-9)$$

The solution for I'_{qs} and I'_{ds} in equations (2.4-5) and (2.4-6) in terms of I_{qs} and I_{ds} are given by:

$$I'_{qs} = \alpha_1 I_{qs} + \alpha_2 I_{ds} + \alpha_3 \quad (5.1-10)$$

$$I'_{ds} = \beta_1 I_{qs} + \beta_2 I_{ds} + \beta_3 \quad (5.1-11)$$

$\alpha_1 = \left(\frac{R_c^2}{R_c^2 + \omega_r^2 L_{qs} L_{ds}} \right)$	$\beta_1 = \left(\frac{\omega_r L_{qs} R_c}{R_c^2 + \omega_r^2 L_{qs} L_{ds}} \right)$	(5.1-12)
$\alpha_2 = -\left(\frac{\omega_r L_{ds} R_c}{R_c^2 + \omega_r^2 L_{qs} L_{ds}} \right)$	$\beta_2 = \left(1 - \frac{\omega_r^2 L_{qs} L_{ds} R_c}{R_c^2 + \omega_r^2 L_{qs} L_{ds}} \right)$	
$\alpha_3 = -\left(\frac{\omega_r \lambda_m R_c}{R_c^2 + \omega_r^2 L_{qs} L_{ds}} \right)$	$\beta_3 = -\left(\frac{\omega_r^2 L_{qs} R_c}{R_c^2 + \omega_r^2 L_{qs} L_{ds}} \right)$	

In equation (5.1-7) the electromagnetic torque is expressed in terms of the steady state d'-q' current quantities. As stated earlier, the d'-q' current quantities are transformed to the d-q quantities by applying the current transformation relationship of equations (5.1-10) and (5.1-11). Using this relationship, the electromagnetic torque may be expressed in terms of the d-q current quantities as:

$$T_e = a_T I_{qs}^2 + b_T I_{ds}^2 + c_T I_{qs} I_{ds} + d_T I_{qs} + e_T I_{ds} + f_T \quad (5.1-13)$$

where,

$$a_T = \frac{3P}{4} L_{md} (1 - \sigma) \alpha_1 \beta_1 \quad (5.1-14)$$

$$b_T = \frac{3P}{4} L_{md} (1 - \sigma) \alpha_2 \beta_2 \quad (5.1-15)$$

$$c_T = \frac{3P}{4} L_{md} (1 - \sigma) (\alpha_1 \beta_2 + \alpha_2 \beta_1) \quad (5.1-16)$$

$$d_T = \frac{3P}{4} L_{md} [(1 - \sigma) (\alpha_1 \beta_2 + \alpha_3 \beta_1) + \lambda_{mmd} \alpha_1] \quad (5.1-17)$$

$$e_T = \frac{3P}{4} L_{md} [(1 - \sigma) (\alpha_2 \beta_3 + \alpha_3 \beta_2) + \lambda_{mmd} \alpha_2] \quad (5.1-18)$$

$$f_T = \frac{3P}{4} L_{md} [(1 - \sigma) \alpha_3 \beta_3 + \lambda_{mmd} \alpha_3] \quad (5.1-19)$$

When the d-axis and q-axis inductances are the same, the electromagnetic torque becomes a linear function of the q-axis and d-axis currents under steady state conditions. The contributions due to machine saliency are lost which consequently diminishes the magnitude of the developed electromagnetic torque. Since the saliency ratio for the surface permanent magnet synchronous motor is unity, this type of motor is most amenable to linear torque control for which the rotor flux is aligned with the stator d-axis so that the d-axis current is zero. Unlike the surface permanent magnet synchronous motor, the saliency ratios for the interior permanent magnet synchronous motor and the synchronous reluctance motor are not unity and for these types of machines, the maximum torque per ampere (MTPA) control are most commonly employed. More details on development of the MTPA and LMS strategies will be discussed shortly.

5.2 Operating Constraints Of The PMSM

A high performance permanent magnet brushless motor drive is often desired to exhibit superior operating performance in terms of fast transient response and high efficiency. Some of the control techniques for achieving these objectives are the zero d-axis ($i_{ds} = 0$), and the maximum torque per ampere (MTPA) control strategies. In the $i_{ds} = 0$ strategy, the machine's rotor flux is aligned with the d-axis so that the stator d-axis current component is zero. This technique, however, does not exploit fully the capability of the reluctance torque. In the MTPA strategy, the reluctance torque is fully utilized for all salient type machines with the objective of maximizing the machine output torque with respect to the current. The MTPA strategy implemented in past research work has shown improved performance and is more widely used in the constant torque limit operating range.

In most practical motor drive, current and voltage constraints are imposed in order to ensure that the machine's operating states stay within the system limits. These are essential to ensure safety and reliability. Therefore implementation of the afore-mentioned control strategies must consider motor and inverter ratings to ensure system operation stays within the system limits imposed by the current and voltage constraints.

In this section, the steps for the development of the $i_{ds} = 0$, and MTPA control strategies are outlined. The operation limits for the proposed loss minimization strategy and the conventional control strategies are clearly established.

5.2.1 Operating Region for the $i_{ds} = 0$ Strategy

The zero d-axis ($i_{ds} = 0$) method is the most common technique among vector controlled schemes for ac machines. By aligning the rotor flux so that the d-axis current is zero, the torque becomes a linear function of the q-axis current (i_{qs}) thereby greatly simplifying the control. Implementation of this control scheme assumes that the machine's core losses are insignificant and are therefore negligible. This assumption corresponds to condition for which an infinitely large core-loss resistance is assumed in our proposed model. With this assumption, the current drop in the core-loss resistance is negligible, and all the d-axis current is available as magnetizing current in the equivalent model presented in the earlier part of this work.

When the $i_{ds} = 0$, then $i'_{ds} = 0$ and total electrical losses may be expressed in terms of steady state variables as:

$$P_L = aI_{qs}^2 + dI_{qs} + f \quad (5.2.1-1)$$

The total electrical loss is no longer a function of the steady state d-axis current component.

Therefore, the optimal value I_{qs}^* that minimizes the total electrical losses is one that satisfies

the condition, $\frac{\partial P_L}{\partial I_{ds}} = 0$, therefore

$$aI_{qs}^2 + dI_{qs} + f = 0 \quad (5.2.1-2)$$

The optimal developed electromagnetic torque is therefore given by:

$$T_e^* = \frac{3P}{4} \lambda_m I_{qs}^* \quad (5.2.1-3)$$

where, during motoring operation:

$$T_e > 0 \quad (5.2.1-4)$$

5.2.2 Operating Region For MTPA Strategy

Under the MTPA strategy the following current and voltage constraints are imposed:

$$I_{qs}^2 + I_{ds}^2 \leq I_m^2 \quad (5.2.2-1)$$

$$V_{qs}^2 + V_{ds}^2 \leq V_m^2 \quad (5.2.2-2)$$

where, I_m and V_m are the system's maximum line-current and the maximum phase-voltage respectively. The current constraint given by (5.2.2-1) on the $I_{qs} - I_{ds}$ plane can be represented by a circular region. Since this circular region maps onto an ellipse in the $I'_{qs} - I'_{ds}$ plane, the transformation is no longer linear and it is imperative that caution be exercised in the procedure for developing the MTPA strategy in the foregoing analysis. Where possible, the d-q quantities are used in the analysis that follows.

Equations (5.1-5), (5.1-6), (5.1-10) and (5.1-11) are applied to equations (5.2.2-1) and (5.2.2-2) above to establish the following boundary conditions:

$$\text{Current constraint:} \quad I_{qs}^2 + I_{ds}^2 = I_m^2 \quad (5.2.2-3)$$

$$\text{Voltage constraint:} \quad a_v I_{qs}^2 + b_v I_{ds}^2 + c_v I_{qs} I_{ds} + d_v I_{qs} + e_v I_{ds} + f_v = 0 \quad (5.2.2-4)$$

Note that, the voltage amplitude V_m is a constant and has been incorporated in the constant, f_v above. The q-axis steady state current is dependent on the d-axis steady state current, $I_{qs} = f(I_{ds})$ from equation (5.2.2-3). It is also clear from above, that the voltage constraint equation (5.2.2-4) defines an elliptical voltage limit for each value of rotor speed.

From (5.2.2-3),

$$\frac{\partial I_{qs}}{\partial I_{ds}} = -\frac{I_{ds}}{I_{qs}} \quad (5.2.2-5)$$

Since electromagnetic torque is kept constant for a given rotor speed,

$$\left. \frac{\partial T_e}{\partial I_{ds}} \right|_{\omega_r} = 0 \quad (5.2.2-6)$$

Using equation (5.1-13), condition (5.2.2-6) is satisfied when,

$$\left(2a_T I_{qs} + c_T I_{ds} + d_T \right) \frac{\partial I_{qs}}{\partial I_{ds}} + \left(c_T I_{qs} + 2b I_{ds} + e_T \right) = 0 \quad (5.2.2-7)$$

Substituting, (5.2.2-5) into (5.2.2-7) and rearranging terms, the condition for maximum torque per ampere (MTPA) strategy is given by:

$$\textbf{MTPA:} \quad c_T I_{qs}^2 - c_T I_{ds}^2 + 2(b_T - a_T) I_{qs} I_{ds} + e_T I_{qs} - d_T I_{ds} = 0 \quad (5.2.2-8)$$

Equation (5.2.2-8) is a hyperbola which defines the maximum torque per ampere curve. The expression shows that the MTPA curve is independent of actual electromagnetic torque developed in the machine. At a first glance at the equation, one may be tempted to suggest that the curve would cave inwards as the machine operating speed increases. However, this is not the case. It will be shown shortly that the MTPA curve for the interior permanent magnet synchronous motor and the synchronous reluctance motors has very low sensitivity to changes in machine operating speed. For most surface permanent magnet motors with saliency ratio of one, the MTPA curve is simply a straight line.

5.2.3 Operating Region for LMS

The procedure for determining the feasible operating region for the proposed loss minimization strategy (LMS) is similar to that followed for the MTPA. In the formulation of the LMS condition, the total electrical loss is minimized with respect to the steady-state stator d-axis current. It is assumed that the machine is running at a constant speed for a given load.

The expression for total electrical losses as a function of the steady-state stator d'-q' current quantities was derived in chapter 2. By using the current transformation equations (5.1-10) and (5.1-11), the equivalent expression for the total electrical losses in terms of the d-q current quantities is given by:

$$P_L = A_{11}I_{qs}^2 + A_{22}I_{ds}^2 + A_{12}I_{qs}I_{ds} + A_1I_{qs} + A_2I_{ds} + A_c \quad (5.2.3-1)$$

where,

$$A_{11} = a + b\beta_1^2 + c\beta_1 \quad (5.2.3-2)$$

$$A_{22} = a\alpha_1^2 + b\beta_2^2 + c\alpha_1\beta_2 \quad (5.2.3-3)$$

$$A_{12} = 2a\alpha_1 + 2b\beta_1\beta_2 + c(\beta_2 + \alpha_1\beta_1) \quad (5.2.3-4)$$

$$A_1 = 2a\alpha_2 + 2b\beta_1\beta_3 + c(\beta_3 + \alpha_2\beta_1) + e\beta_1 + d \quad (5.2.3-5)$$

$$A_2 = 2a\alpha_1\alpha_2 + 2b\beta_2\beta_3 + c(\alpha_1\beta_3 + \alpha_2\beta_2) + d\beta_1 + e\beta_2 \quad (5.2.3-6)$$

$$A_c = a\alpha_2^2 + b\beta_3^2 + c\alpha_2\beta_3 + d\alpha_2 + e\beta_3 + f \quad (5.2.3-7)$$

Since total electrical loss is minimized for a given electromagnetic torque and for a given rotor speed,

$$\left. \frac{\partial P_L}{\partial I_{ds}} \right|_{T_e, \omega_r} = 0 \quad (5.2.3-8)$$

Using equation (5.2.3-1), condition (5.2.3-8) is satisfied when,

$$\left(2A_{11}I_{qs} + A_{12}I_{ds} + A_1 \right) \frac{\partial I_{qs}}{\partial I_{ds}} + \left(A_{12}I_{qs} + 2A_{22}I_{ds} + A_2 \right) = 0 \quad (5.2.3-9)$$

Substituting equation (5.2.3-8) into (5.2.3-9), the condition for minimum loss is given by:

$$\textbf{LMS:} \quad A_{12}I_{qs}^2 - A_{12}I_{ds}^2 + 2(A_{22} - A_{11})I_{qs}I_{ds} + A_2I_{qs} - A_1I_{ds} = 0 \quad (5.2.3-9)$$

The LMS condition describes a hyperbola curve. Just like the MTPA, it can also be shown that the LMS curve has very low sensitivity to changes in the rotor speed. There is just a little difference between the LMS and the MTPA curves when the motor is operating at very high speed. The equations derived in section 5.2 can now be used to define the operating regions and characteristic curves for the all the classes of PMSM.

5.3 Optimal Motor Operation Considering Current And Voltage Limits

The characteristic curves depicting motor operation for the interior permanent magnet synchronous motor ($\sigma > 1$) is shown in Fig. 5.3-1. This figure consists of: (1) four voltage limit ellipses w_1 , w_2 , w_3 and w_4 on the $I_{qs} - I_{ds}$ plane corresponding to four motor speeds such that, $\omega_1 < \omega_2 < \omega_3 < \omega_4$; (2) four positive constant torque curves T_1 , T_2 , T_3 and T_4 corresponding to

$T_{e1} = 0.3 \text{ Nm}$, $T_{e2} = 0.76 \text{ Nm}$, $T_{e3} = 1.5 \text{ Nm}$, and $T_{e4} = 2.3 \text{ Nm}$ respectively; (3) three current-limit circles, I_1 , I_2 , and I_3 of different amplitudes; and (4) the LMS trajectories defined by a bundle of curves AOA'. Also included in the figure, are the trajectories of the corresponding negative constant torques, $-T_1$, $-T_2$, $-T_3$ and $-T_4$, indicated in thin dashed lines. The negative torque curves show the feasible region of machine operation in the regenerative braking mode. Since the primary focus is on motor operation with positive electromagnetic torque, more emphasis will be given to the set of positive constant torques T_1 , T_2 , T_3 , and T_4 in the discussion which follows.

For optimal motor operation, the desired torque is generated with a current space-vector of minimum length. Each constant torque trajectory can be reached with minimum amplitude current space-vectors located on the LMS curve. The geometric location of all the minimum amplitude current space vectors for the LMS strategy coalesce into a bundle of curves AOA' depicted in thick blue in Fig. 5.3-1. On the AOA' curve, maximum positive torque at minimum loss can be generated at operating points a , b , c and d for which negative d-axis currents are applied. Note that AOA' is the collection of trajectories satisfying the LMS conditions. The operating points a , b , c and d are the intersection points of the positive constant torque trajectories with the respective current-limit circles. Point a' , b' , c' and d' are the intersection points of the negative constant torque trajectories with the respective current-limit circles. Because of the symmetry of the constant torque curves and the LMS curves about the I_{ds} axis, the two set of intersection points a , b , c , d and a' , b' , c' , d' are mirror images of each other.

Fig. 5.3-1: Steady-state trajectories in the $I_{qs} - I_{ds}$ plane for the IPMSM drive. ($\sigma = 1.48$, $V_m = 10V$ and showing: **1.** Voltage ellipses, w_n (red): $\omega_1 = 10$ rad/s, $\omega_2 = 50$ rad/s, $\omega_3 = 100$ rad/s, and $\omega_4 = 120$ rad/s; **2.** Current-circles I_m (green): $I_{m1} = 1.8A$, $I_{m2} = 4.0A$ and $I_{m3} = 5.6 A$.; **3.** LMS curves (blue); **4.** Positive constant torque curves, T_n (brown): $T_{e1} = 0.3$ Nm, $T_{e2} = 0.76$ Nm, $T_{e3} = 1.52$ Nm, and $T_{e4} = 2.3$ Nm)

Although, for optimal motor operation the operating points a , b , c and d on the A0A' curve are desired, in order to guarantee a safe and stable operation of the drive, the available operating points for any given speed and torque should be inside the overlapping region bounded by the voltage-limit ellipse w_n and the current-limit circle I_n . In general, for a given current-limiting constraint, the available operating region diminishes as the motor speed increases.

As an illustration, let us consider the characteristics curves in Fig. 5.3-1 for the interior permanent magnet motor. The overlapping region between the current-limit circle I_1 and the voltage-limit ellipse w_1 is greater than the region bounded by I_1 and voltage-limit ellipse w_2 . The region bounded by I_1 and voltage-limit ellipse w_2 is also greater than the region bounded by I_1 and voltage-limit ellipse w_3 . It is obvious that as the motor speed increases, the voltage-limit ellipse shrinks thereby decreasing the area of available operating region of the motor. As the developed electromagnetic torque increases, the trajectory of constant torque recedes farther away from the I_{ds} axis. The non-linear relationship between the machine's core-loss and rotor speed also greatly affects the symmetry of the voltage-limit ellipses with respect to I_{qs} and I_{ds} axes. The effect of the core-losses on the symmetry of the voltage ellipses with respect to the q-axis is more pronounced at low operating speed. This is understandable since the equivalent core-loss resistance R_c increases with higher speed. The effect of the core-losses causes the long axis of the voltage-limit ellipses to lose symmetry about the I_{ds} axis as the rotor speed increases. The effect of this is that the available operating region with respect to the torque speed characteristics becomes non-linear.

This can be further illustrated using Fig. 5.3-1. In this figure, the constant torque T_1 intersects the voltage ellipses w_1 , w_2 , and w_3 at points a_1 , a_2 , and a respectively. These intersection points corresponds to the rotor speeds ω_1 , ω_2 and ω_3 . Since T_1 does not intersect voltage ellipse w_4 , there is no operating point on the constant torque curve T_1 that corresponds to the highest rotor speed ω_4 . However, in the regenerative mode, the negative constant torque curve $-T_1$, intersects the voltage-ellipse w_4 at point a_4 corresponding to the rotor speed, ω_4 . It is therefore obvious that the speed range for the regenerative braking power of the motor is larger than the motoring power.

Although, it is often desired to choose the operating points on the LMS curves (A0A'), at higher rotor speeds this strategy may no longer hold for the interior permanent magnet synchronous motors with $\sigma > 1$. As shown in Fig. 5.3-1, not all the voltage-limit ellipses intersect the A0A' curve. Since the voltage-ellipse corresponding to the highest rotor speed ω_4 does not intersect the A0A' curve and is barely touching the current circle I_1 , motor operation can not be fully sustained at rotor speeds as high as ω_4 when the current-limit I_1 is imposed. However, operation at higher speed can be safely guaranteed by extending the limit of the current constraint. The characteristics curves of Fig. 5.3-1 explain why the interior permanent magnet synchronous motors (IPMSMs) with higher current capability are better suited for high speed operation than other motor types (SRMs and SPMSMs) with lower current capacity.

It is important to note that extending the operating limits of the machine can not be achieved without a price. By selecting an operating point outside the minimum loss A0A' curve, it is possible to sustain motor operation at the voltage limit for a given speed at the expense of the developed output torque or at an additional cost by extending the current limit.

In Fig. 5.3-1, moving the operating point from the optimal location point b on the $A0A'$ curve to another point g along the voltage-limit ellipse w_2 requires more negative d-axis current than necessary for an optimal utilization of the reluctance torque. Therefore, for the operating point to move from point b to g and also stay within the current limit imposed by I_3 , the developed torque is reduced from T_2 to T_1 .

The converse is the case when the operating point moves from point a_2 outside the $A0A'$ curve and along the same voltage-limit ellipse w_2 to the point b on the $A0A'$ curve as shown in Fig. 5.3-1. In this transition, the maximum reachable torque increases from T_1 to T_2 when the current is maintained within the current limit imposed by I_3 . Therefore, when all the conditions imposed by the current and voltage constraints are met, moving the operating point to the LMS curve $A0A'$ will always produce an optimal output in terms of a higher output torque or a higher output power.

Similar insights into the operating limits of the synchronous reluctance motor ($\sigma < 1$) can also be obtained from characteristic curves of Fig. 5.3-2. The centers of the high speed voltage-limit ellipses (w_3 and w_4) are located close to the origin (0) and are also well within the vicinity of the current-limit circles. Therefore, there is less electrical limitation for rotor speed for this type of machine. It is clear that this type of machine may be better suited for high-speed operation in the regenerative mode than the interior permanent magnet synchronous motors. Just like the case of the interior permanent magnet motor, the optimal operating points for the synchronous reluctance machine during motoring are the points a , b , c and d on the $A0A'$ curve. At these optimal points, d-axis currents are positive whereas in the case of the IPMSM, the d-axis currents are negative at the optimal points.

Fig. 5.3-2: Steady-state trajectories in the $I_{qs} - I_{ds}$ plane for the SRM drive. ($\sigma = 0.677$, $V_m = 10V$ and showing: **1.** Voltage ellipses, w_n (red): $\omega_1 = 10$ rad/s, $\omega_2 = 50$ rad/s, $\omega_3 = 100$ rad/s, and $\omega_4 = 120$ rad/s; **2.** Current-circles I_m (green): $I_{m1} = 1.8A$, $I_{m2} = 4.0A$ and $I_{m3} = 5.6$ A.; **3.** LMS curves (blue); **4.** Positive constant torque curves, T_n (brown): $T_{e1} = 0.3$ Nm, $T_{e2} = 0.76$ Nm, $T_{e3} = 1.52$ Nm, and $T_{e4} = 2.3$ Nm.

Fig. 5.3-3 shows the characteristic curves for the surface permanent magnet synchronous motor (SPMSM) with $\sigma = 1$ which also provides rich information about the feasible operating regions during motoring and regenerative braking operations. Fig. 5.3-3 shows that the curves of constant torques and the LMS curves are straight lines which are quite different from that of the IPMSM and the SRM of Figs. 5.3-1 and 5.3-2 respectively.

The SPMSMs provides greater flexibility in operation since it is possible to move between operating points with little or no effect on developed output torque or rotor speed. For example, point a on the $A0A'$ curve is the optimal operating point corresponding to a constant torque T_1 and current limit I_l . If the current constraint is relaxed to I_3 , it is possible to move from a to another point along the same voltage-ellipse (rotor speed) and at the same fixed torque T_1 . The implementation of the LMS control may require moving the system's operating point located outside the $A0A'$ curve to an optimal point on the $A0A'$ trajectory along the same line of constant torque or along the same voltage-limit trajectory. Thus, SPMSM provides various possibilities of operating points for the system. Although, the available region of operation is limited compared to the other two types of machines, it is the most common type of machines employed in applications where simplicity of control and flexibility are the chief attributes. However, since the SPMSM provides no opportunity for exploiting the use of the reluctance torque, opportunities for efficiency improvement is limited.

Fig. 5.3-3: Steady-state trajectories in the $I_{qs} - I_{ds}$ plane for the SPMSM drive. ($\sigma = 1$, $V_m = 10\text{V}$ and showing: **1.** Voltage ellipses, w_n (red): $\omega_1 = 10\text{ rad/s}$, $\omega_2 = 50\text{ rad/s}$, $\omega_3 = 100\text{ rad/s}$, and $\omega_4 = 120\text{ rad/s}$; **2.** Current-circles I_m (green): $I_{m1} = 1.8\text{A}$, $I_{m2} = 4.0\text{A}$ and $I_{m3} = 5.6\text{ A}$; **3.** LMS curves (blue); **4.** Positive constant torque curves, T_n (brown): $T_{e1} = 0.3\text{ Nm}$, $T_{e2} = 0.76\text{ Nm}$, $T_{e3} = 1.52\text{ Nm}$, and $T_{e4} = 2.3\text{ Nm}$.

5.4 Conclusion

In this chapter, steady-state analyses for the permanent magnet synchronous motors have been presented. The three types of permanent magnet synchronous motor considered were: the interior permanent magnet synchronous motor, the synchronous reluctance motor and the surface permanent magnet synchronous motor. For each machine, the conditions for implementing the existing control strategies and the proposed loss minimization strategy were derived. These conditions were used to develop the characteristic curves that show the feasible and available operating regions of the machine under current and voltage constraints. The result presented has helped to identify the merits and demerits as well as areas of application for the types of permanent magnet synchronous motor considered.

Chapter 6

Implementation of the Proposed Loss Minimization Strategy

The permanent magnet synchronous machines are commonly driven from a six-step inverter which converts constant voltage to three-phase voltages with frequency corresponding to the rotor speed. They are more extensively used in industry because of their large torque, fast response, and high power density than the DC machines. In the chapter 5, condition for efficiency optimization for the PMSM has been derived based on the machine's steady-state model which includes the effects of core-losses.

In this chapter, the vector control scheme for implementing the proposed loss minimization strategy is presented. First, the machine model that includes the core-loss resistance is validated by comparing simulation results with experimental data obtained for an interior permanent magnet synchronous motor (IPMSM). Next, the loss minimization strategy is implemented in a speed control scheme for a synchronous reluctance motor drive with saliency factor $\sigma < 1$. The open-loop and closed-loop performances are studied to determine the effectiveness of the proposed strategy under various operating conditions of load torques and motor speeds. Two types of permanent magnet synchronous brushless motors have been considered in the study – the interior permanent magnet synchronous motor and the synchronous reluctance motor to show the applicability of the proposed strategy to all salient-type permanent magnet synchronous motor drives.

6.1 Model Validation

To show the efficacy of our proposed loss minimization strategy, our proposed loss minimization model, which includes the effect of machine core-losses, is validated by comparing efficiency results with that of the conventional MTPA method, stator's zero d-axis current method, and the loss model control technique described in [14]. Simulation and experimental results are compared to prove the effectiveness of our proposed loss minimization strategy. The machine used for validation is a 3.4 kW interior permanent magnet synchronous motor (IPMSM) with saliency factor of 1.69 and parameters corresponding to those given in [14].

Fig. 6.1-1 shows the motor efficiency of our proposed loss minimization strategy and that of the LMC relative to the traditional stator's zero d-axis current ($i_{ds} = 0$) method for motor operation at 4/4, 3/4, 2/4, and 1/4 of full speed. It is clear from this result that although motor efficiency may not consistently increase with load as there are certain loads at which motor efficiency is a minimum, motor efficiency generally increases with motor speed. The results also clearly show that performance of our proposed LMS is superior to that of the LMC and stator's zero d-axis current method.

Fig. 6.1-2 shows corresponding efficiency of our proposed LMS and that of LMC relative to that of the maximum torque per ampere (MTPA) method. Results are shown at operating speeds of 4/4, 3/4, 2/4, and 1/4 of full speed as in the case of the $i_{ds} = 0$ method above. Although motor efficiency as shown in Fig. 6.1-2 improves with speed, the efficiency appears to consistently diminish with increasing load torque. The motor operates at higher efficiency at light loads, and lower efficiency at heavy loads.

Fig. 6.1-1: Relative efficiency of loss minimization techniques to $i_{ds} = 0$ method.

Fig. 6.1-2: Relative efficiency of loss minimization techniques to the MTPA method

These observations are applicable to interior permanent magnet synchronous motor with saliency factor $\sigma > 1$, however, the converse is expected to hold true in the case of the synchronous reluctance motor with saliency factor $\sigma < 1$.

6.2 Motor Open-Loop Dynamic Performance

The machine loss model which includes core-loss has been presented in chapter 2 of this dissertation. The effectiveness of this model is verified by simulating the dynamic performance during free acceleration and step changes in load torque when machine is energized with sinusoidal stator voltages. The machine parameters used in this computer study are shown in Table 6.2-1.

TABLE 6.2-1: PARAMETERS OF A 3HP SRM

Parameter	Value	Unit
Resistance R_s	0.9889	Ω
q-axis Inductance, L_{qs}	0.021	H
d-axis Inductance, L_{ds}	0.1021	H
Flux linkage λ_m	0.063	Wb (V/rad/s)
Number of Poles, P	4	dimensionless
Rotor Inertia, J_m	0.00018	Kg. m ²
Damping, B_m	0.0000212	N.m/s

6.2.1 Free Acceleration (No Load) Response

The free acceleration or no load ($T_L=0$) response of the SRM motor drive with phase voltage $V_s = 11.25$ V (rms), $\sigma < 1$, are shown in Fig. 6.2.1-1 and Fig. 6.2.1-2. In this simulation, $\theta_r(0)=0$, and $\theta_{ev}(0)=0$. This follows from the transformation of the a-b-c voltage quantities to the q-d-0 quantities which is governed the following expressions:

$$v_{qs}^r = \sqrt{2} v_s \cos \theta_{ev}(0) \quad (6.2-1)$$

$$v_{ds}^r = -\sqrt{2} v_s \sin \theta_{ev}(0) \quad (6.2-2)$$

Therefore, by fixing the rotor reference frame $\theta_r(0)$, and the time zero position of applied sinusoidal voltages $\theta_{ev}(0)$ at zero, then v_{ds}^r is always zero and $v_{qs}^r = \sqrt{2} V_s$ with $V_s = 11.25$ V, $v_{qs}^r = 15.9099$ V. This result is confirmed in the plot of Fig. 6.2.1-1 which shows the stator's phase voltage (V_{as}), the stator phase current (I_{as}) and the stator's q-axis equivalent voltage (V_{qs}) all in actual quantities. However, it is more convenient to express the quantities in per unit system using the base values. Usually, the machine's rated values for voltage, current, speed, and power are used as the base values. However, the choice of base values is not restrictive as long as there is consistence in the way by which they are applied. For this reason, most of the results that follow will be given in per unit values where appropriate. The per unit value system is adopted so as to make for easy comparison of results.

Figure 6.2.1-1: Stator's phase 'a' voltage and current of the SRM under open loop condition.

In Fig. 6.2.1-2, the plot of rotor speed ω_r shows that the steady state no-load speed (base speed) of 1.0 p.u. is attained after about 1.2 seconds. This speed corresponds to 243.29 rad/s (1162 rev/min) for a 4-pole machine.

The machine's no-load electrical loss (P_{LOSS}) is also plotted in Fig. 6.2.1-2. In Fig. 6.2.1-2, at steady state, the total electrical loss is 0.38 p.u and this value is negligibly small due to the fact that very small (or negligible) current flows through the stator windings under no-load conditions. The total electrical loss at steady state depends on the magnitude of the input rms supply voltage V_s , the magnitude of core loss resistance R_c and on the machine load conditions. A plot of electromagnetic torque T_e versus rotor speed ω_r under no load condition is also shown in Fig. 6.2.1-3. Note that the dynamic torque-speed characteristic shown in Fig. 6.2.1-3 is significantly different from that of the steady state characteristics. This plot is also different from the case of the non-salient pole machine where the machine saliency, $\sigma = 1$.

6.2.2 Performance Due to Load Torque Change

The open-loop dynamic performance due to step changes in load torque is illustrated in Fig. 6.2.2-1. Fig. 6.2.2-1 shows the plots of applied phase voltage V_{as} , phase current I_{as} , and q-axis voltage v_{qs}^r versus time, while Fig. 6.2.2-2 shows the plots of q-axis current (i_{qs}^r), the d-axis current (i_{ds}^r), the electromagnetic torque (T_e), and rotor speed (ω_r) versus time for a step change in load torque and with core loss resistance R_l . From Fig. 6.2.2-2, the machine is initially operating with $T_L = 0.01$ p.u. After 0.1 second, the machine load torque is suddenly stepped to 0.4 p.u and 0.8 seconds after this change, the load is stepped back down to 0.1 p.u.

Fig. 6.2.1-2: Dynamic performance of the SRM. From Top: (a) stator q-axis current (b) stator d-axis current (c) Electromagnetic torque and reference load torque (d) Rotor Speed (e) Total electrical power loss. (All quantities in p.u).

Fig. 6.2.1-3: Dynamic torque-speed characteristics for the SRM under no-load conditions

From the plot of Fig. 6.2.2-2, the electromagnetic torque T_e is seen to track the step changes in the load torque T_L .

Comparison of Fig. 6.2.2-2 with Fig. 6.2.1-2 suggests that an increase in step load torque causes an increase in the total electrical loss. When a load of 0.4 p.u. is applied, total electrical loss increased to 1.0310 p.u. When the load torque is stepped down to 0.1 p.u, the total electrical loss also dropped to 0.4804 p.u. Therefore, total electrical loss is proportional to the magnitude of applied load. Fig. 6.2.2-2 also shows that a step increase in load torque causes the machine speed to drop until steady state is reached. Similar trend is observed with the machine armature current I_{as} . An increase in load torque causes a corresponding increase the stator current and vice-versa. It should be noted also that the speed of response of the machine to changes in load torque depends on the total inertia of the rotor and load. The greater the inertia, the longer it takes for the machine to reach full (or steady state) speed.

Fig. 6.2.2-3 shows the dynamic performance due to similar step changes in load torque but with core loss resistance $R_c = R_2$. It is assumed that temperature or saturation effect caused the machine core loss resistance R_c to decrease from R_1 to R_2 . This prediction is confirmed by comparing Fig. 6.2.2-3 with Fig. 6.2.2-2. In Fig. 6.2.2-3, with $R_c = R_2$, the total electrical loss is 1.1146 p.u. when the load is stepped to 0.4 p.u, and 0.5723 p.u. when the load torque is stepped down to 0.1 p.u. These values are relatively higher than the case for which $R_c = R_1$. This confirms our earlier prediction that the core loss increases with a decrease in the shunt core loss resistance R_c . Hence, the higher the value of R_c , the better the efficiency assuming all other conditions remain the same.

Figure 6.2.2-1: Stator's phase 'a' voltage and current due to step change in load torque.

Fig. 6.2.2-2: Dynamic performance of the SRM due to a step change in load torque at $R_c = R_l$.

From Top: (a) stator q-axis current (b) stator d-axis current (c) Electromagnetic torque and reference load torque (d) Rotor Speed (e) Total electrical power loss. (All quantities in p.u)

Fig. 6.2.2-3: Dynamic performance of the SRM due to a step change in load torque at $R_c = R_2$

From Top: (a) stator q-axis current (b) stator d-axis current (c) Electromagnetic torque and reference load torque (d) Rotor Speed (e) Total electrical power loss.

6.3 The Proposed Optimal Efficiency System

6.3.1 System Description

The configuration of the proposed optimal efficiency scheme for a SRM drive is shown in Fig. 6.3.1-1. This system includes speed control, the loss minimization algorithm, a PWM current regulated voltage source inverter, and the SRM. The q'-axis reference current command (i_{qs}^*) is obtained from proportional-integral PI controller and filter using the speed error signal which is the difference between the reference speed (ω_r^*) and the detected motor angular speed, (ω_r). The d'-axis reference current command (i_{ds}^*) is calculated by the loss minimization algorithm given by (2.3-11). A current transformation or linear mapping is then applied to the q'-d' reference currents to obtain equivalent q-d reference currents which are also transformed via vector rotation to the three phase reference command currents, i_a^* , i_b^* , and i_c^* . The reference currents and the measured motor currents are compared in the PWM/VSI inverter circuit to synthesize the motor output line (or phase) voltages.

Fig. 6.3.1 is one of the possible configurations for improving the efficiency of a permanent magnet synchronous motor drive in closed-loop speed control. There are other possible arrangements for implementing the proposed efficiency optimization strategy. In Fig. 6.3.1, speed control is implemented in the outer system loop using a combination of PI controller and filter as shown in Fig. 6.3.1-1. The purpose of the filter is to improve the response of the outer speed-loop thereby improving the response of the closed-loop system. Fig. 6.3.1-2 shows the controller structure from which the control signal i_{qs}^* is generated.

Fig. 6.3.1-1: Configuration of the proposed system.

Fig. 6.3.1-2: Structure of the controller

The transfer function of the PI controller is given by:

$$\frac{\omega_r(s)}{\omega_r^*(s)} = \frac{K_P}{s^2 + sK_I + K_P} \quad (6.3-1)$$

Parameters of the PI controller and the filter as shown in Appendix 1 are chosen so that the eigen-values of closed-loop system are located in the left-hand side of the $j\omega$ -axis. The Butterworth method is employed to optimize the process of locating the poles for the PI controller [31] uniformly in the left-half s -plane on a circle of radius ω_o with origin as its center. The Butterworth polynomial for a second-order transfer function is given as:

$$s^2 + 2\xi\omega_o s + \omega_o^2 \quad (6.3-2)$$

Value of the damping factor ξ is chosen as 0.707 for optimal result. Selection of the parameters for the PI controller and filter is further made easier by ensuring that the eigen-values of the inner current loop is a subset of the eigen-values of the closed-loop system. With eigen-values of closed-loop system located on the left hand side of the s -plane, stability of the closed-loop system is guaranteed.

It is expected that this proposed scheme will offer the benefits of reduced cost, simplicity and ease of implementation which are hitherto unavailable with more complex control techniques. Another advantage of the closed-loop system of Fig. 6.3.1-1 is that it is amenable to digital implementation and its validity will be proven by comparing results with the traditional $i_{ds} = 0$ and the maximum torque per ampere techniques.

6.3.2 Closed-Loop Dynamic Performance

Fig. 6.3.2-1 and Fig. 6.3.2-2 show the dynamic speed responses of the motor under the $i_{ds} = 0$ control and the proposed loss minimum strategy respectively. In both cases, the load torque is maintained constant at 0.4 p.u. The command speed ramps up from zero to 0.87 p.u. after 2 seconds. The speed is maintained constant at this value for another second after which it begins to drop linearly. Although, both schemes show good speed responses as the actual speed accurately tracks the reference speed, the total loss under the minimum loss strategy is less than that of the $i_{ds} = 0$ control. It is therefore expected that a significant amount of energy savings can be realized from the proposed LMS under similar operating conditions.

Fig. 6.3.2-3 shows the loss characteristic of the motor under two different load conditions. In the figure, a_1 and a_2 are the optimal (minimum loss) operating points, for load torques of 0.4p.u and 0.8p.u respectively. The optimal total electrical loss at point a_1 is 0.87 p.u which is similar to the result obtained during the step load increase to 0.4 p.u in Fig. 6.3.2-2. Points b_1 and b_2 in Fig. 6.3.2-3 are corresponding locations of the optimal q-axis current components under the proposed LMS. This result corroborates the experimental results obtained in [1]. Fig. 6.3.2-4 shows the motor efficiencies under the proposed LMS, maximum torque per ampere and the $i_{ds} = 0$ control techniques at motor speeds of $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$ and $\frac{4}{4}$ of full load. The corresponding relative efficiencies of the proposed LMS scheme to that of the MTPA and $i_{ds} = 0$ control techniques are shown in Fig. 6.3.2-5. The result shows that the performance of the LMS over the MTPA method is more pronounced at smaller load whereas at higher loads, the efficiency of the LMS gets much better than the $i_{ds} = 0$ method. In overall, the motor efficiency is higher in the LMS than in the $i_{ds} = 0$ and MTPA control techniques.

Fig. 6.3.2-1: Speed regulation of motor drive with $i_{ds} = 0$ control at constant load torque. From top: (a) electromagnetic torque and load torque, (b) actual speed and reference speed (c) total electrical loss. (All quantities in p.u)

Fig. 6.3.2-2: Speed regulation of motor drive with LMS control at nominal values and constant load torque. From top: (a) electromagnetic torque and load torque, (b) actual speed and reference speed, (c) total electrical loss.

(All quantities in p.u).

Fig. 6.3.2-3: Loss characteristics under load conditions. (Reference speed, $\omega_r^* = 0.87$ p.u)

(1) $T_e = 0.4$ p.u (blue), and (2) $T_e = 0.8$ p.u (red). (All quantities in p.u).

Fig. 6.3.2-4: Efficiency of proposed LMS and conventional control techniques.

Fig. 6.3.2-5: Relative efficiency of proposed LMS to conventional control techniques.

This shows the superior performance of our proposed strategy over the existing conventional methods.

To examine the robustness of the proposed system, the motor inertia was reduced by a factor of 0.5 and the viscous damping was increased by a factor of 1.5. The speed regulation at constant load torque of 0.4 p.u and changes to the motor inertia and viscous damping is shown in Fig. 6.3.2-6. The figure shows that there is little or no change on the closed loop performance of the system thus confirming the robustness of the proposed LMS.

6.4. Conclusion

In this chapter, the implementation of the proposed loss minimization strategy for the vector-controlled permanent magnet brushless AC motor has been described. The proposed strategy has been developed based on a machine model that includes effect of machine core losses. The proposed model of the machine has been shown to be effective for the both no-load and load conditions. Simulation results of open-loop dynamic performance of the model show a dependency of machine losses on operating conditions such as load torque, input supply and also on machine parameters. The dynamics of the vector control system under closed-loop speed control has also been presented.

The efficiency of the proposed system has been shown to be higher than that of the zero d-axis ($i_{ds} = 0$) and MTPA strategy. Simulation results have been presented to prove effectiveness of our proposed approach.

Fig. 6.3.2-6: Speed regulation of motor drive with LMS control with changes in motor inertia and viscous damping. From top: (a) electromagnetic torque and load torque, (b) actual speed and reference speed (c) total electrical loss. (All quantities in p.u, and $J=0.5J_n$, $B_m=1.5B_n$)

Chapter 7

Conclusion and Future Work

This chapter summarizes the contributions made by this dissertation. The contributions in different areas are described in separate paragraphs. The previous work done in these areas has been detailed in chapter 1 to establish some background for the work carried out in this dissertation. Future work is discussed and some recommendations are made based on the work developed.

7.1 Conclusions

The model for the permanent magnet synchronous machine which includes the effect of core losses has been presented. Based on this model, an expression for the total controllable electrical losses was formulated. The problem of improving the efficiency of the PMSM drive has been formulated as the minimization of the total controllable electrical losses as the objective function subject to the system operating constraints and operation modes. This allowed us to establish the conditions for implementing the proposed loss minimization control scheme (LMS). A detailed account for formulating the conditions for implementing the well known $i_{ds} = 0$ and maximum torque per ampere (MTPA) strategies was also given. A high performance vector control system for the synchronous reluctance motor (SRM) based on the loss minimization strategy has been demonstrated. The performances of the proposed system for speed and torque control has been compared with that of the stator zero d-axis current

method and the MTPA control techniques. It has been shown that the proposed LMS offers superior performance over that of the traditional control methods.

The choice of the PWM scheme used in the implementation of the loss minimization strategy was also considered. A generalized discontinuous pulse width modulation scheme which includes the space vector PWM (SVPWM) was developed for the three-phase voltage source inverter PMSM drive. An informed choice of PWM technique was made based on performance evaluation of the GDPWM and comparison with that of the traditional sinusoidal pulse width modulation (SPWM) scheme. It was shown that the SVPWM and the SPWM are the suitable PWM methods that ensure a good compromise for minimizing switching losses with less distortion in the output waveform quality.

7.2 Future Work

In this work, an efficiency optimization scheme has been developed and implemented for a three-phase voltage source inverter synchronous reluctance motor drive. The effectiveness of the proposed efficiency optimization strategy has been demonstrated by comparing its performance with that of the traditional zero d-axis current and maximum torque per ampere methods. Although, the robustness of the closed-loop system to slight changes in the rotor inertia, viscous damping and load torque has been shown, the robustness of the closed loop system to changes in machine parameter has not been fully explored. Therefore, a possible areas for future work is robust control schemes for efficiency optimization of permanent magnet synchronous motor drive.

Another possible area for future research is the extension of the proposed strategy to multi-phase system including five and six phases by examining the effect of selecting the active states and zero sequence voltages on real multi-phase machines.

Appendix 1

The structure and coefficients of the PI controller and filter are as follows: PI Controller:

PI Controller: $K_P + K_I/s$

where,

$$K_P = 0.1 \text{ and } K_I = 26.18;$$

Filter: $(s+b)/(s+a)$

where,

$$a = 100, b = 38.197.$$

PWM switching frequency $f_{sw} = 20$ kHz.

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