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**Development of Type Curves for Gas Production from Hydraulically
Fractured Horizontal Wells in Unconventional Reservoirs**

Faisal N. Alenezi

**Thesis submitted to the
College of Engineering and Mineral Resources
at West Virginia University
in partial fulfillment of the requirements
for the degree of**

**Master of Science
in
Petroleum and Natural Gas Engineering**

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**Morgantown, West Virginia
2011**

**Keywords: Unconventional gas reservoir, Decline analysis, Horizontal
wells, Type Curves, Gas Production Predication, Hydraulic Fracturing.**

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ABSTRACT

Development of Type Curves for Gas Production from Hydraulic Fractured Horizontal Wells in Unconventional Reservoirs

Faisal N. Alenezi

Unconventional gas reservoirs represent a long-term global source of natural gas. Hydraulic fracturing combined with horizontal drilling has turned unproductive unconventional gas reservoirs into the largest natural gas fields in the world. At the early time of production, due to lack of needed variables, using numerical models is challenging, time consuming, and expensive. Production type curves are a dependable tool for predicting the performance of gas reservoirs.

The goal of this research was to develop a simple and reliable tool to predict the performance of the production of hydraulically fractured horizontal wells in unconventional gas reservoirs. A set of production type curves were developed. Two set of type curves were developed using the reservoir model. They represent the two flow regimes associated with the horizontal wells, the early time liner flow and the late time pseudo-radial (elliptical) flow. The dimensionless well length and the ratio of well length to reservoir length were found to influence the type curves significantly.

The impact of some of the reservoir parameters was reviewed. Drainage area, horizontal permeability, and vertical permeability were found not to impact type curves extensively. Reservoir thickness has a minor effect on type curves. Reservoir porosity has no effect on early production but significant effect on late production.

In addition to the reservoir parameters, a range of hydraulic fracture parameters was studied. Number of hydraulic fractures was found to have impact on type curves particularly with very low permeability. Fracture half length, fracture permeability, and fracture width were found to have no major affect on type curves.

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CHAPTER 1

INTRODUCTION

Unconventional gas reservoir is a term commonly used to refer to a low permeability reservoir that produces mainly dry natural gas. Sandstone has many low permeability reservoirs, but important quantities of gas are also produced from low-permeability carbonates, shales, and coalbed methane. Unconventional natural gas production has increased nearly 65% since 1998. Therefore, the future of the gas industry is unconventional gas.

The use of horizontal drilling technology in oil exploration, development, and production operations has grown rapidly over the past 7 years. It is becoming a larger percentage of the development wells. The aim of horizontal drilling is to increase the well productivity by increasing the contact with the reservoir.

Hydraulic fracturing is a common technique and widely accepted application used to stimulate the production of oil and natural gas. This technique creates a conductive flow path by injecting fluids underground at high pressures to increase the productivity. Hydraulic fracturing, combined with horizontal drilling, has turned previously unproductive unconventional gas reservoirs into the largest natural gas fields in the world.

Reservoir simulators are used to predict the wells production performance. But the use of reservoir simulators is not always affordable, especially at the early stage of the well life. Lack of sufficient data makes the use of simulators difficult, time consuming and expensive. Therefore, it was found, from previous studies, that developing type curves for gas production is a simple and reliable tool to evaluate and predict the production through the life of the wells.

This research is an extension to a previous research done by Abdullah Almansour as his MS thesis in 2009. The study title is “Development of Type Curves for Gas Production from Horizontal Wells in Conventional Reservoirs”.

CHAPTER 2

LITERATURE REVIEW

2.1 Decline Curves:

Decline curves are one of the most extensively used forms of data analysis employed in the evaluation of hydrocarbon properties. Most of the existing decline curve analysis techniques are based on the empirical Arps equations: exponential, hyperbolic, and harmonic equations. The Arps decline curve analysis approach was proposed nearly sixty years ago. However a great number of studies on production decline analysis are still based on this empirical method. Many published papers have tried to interpret the Arps decline equation theoretically. The empirical Arps decline equation represents the relationship between production rate and time for hydrocarbon wells during pseudosteadystate period and is shown as follows:

$$q(t) = \frac{q_i}{(1 + bDt)^{\frac{1}{b}}} \dots\dots\dots (2.1)$$

Where $q(t)$ is the oil production rate at time t and q_i is the initial oil production rate. b and Di are two constants.

It is difficult to foresee which equation the reservoir will follow. Each approach has some disadvantages. For example, the exponential decline curve tends to underestimate reserves and production rates; the harmonic decline curve has a tendency

to over-predict the reservoir performance. In some cases, production decline data do not follow any model but cross over the entire set of curves.

Exponential decline is the simplest one of the decline curves and is often used since many wells and fields follow a constant percentage decline over a great portion of their productive life, and only deviate from this behavior at the end of the productive life. As shown in Figure 1, these three forms of decline curves; exponential, harmonic, and hyperbolic have a different shape on Cartesian and Semi-log graphs of gas production (rate, cumulative production) versus time.

Arps equations assume that the well is produced at same flowing pressure P_{wf} , constant reservoir drainage area, constant reservoir permeability and skin factor.

Figure1 Decline curve shapes for Cartesian and Semi-log plots (Lee & Wattenbarger, 1996).

2.2 Type Curves:

Type curves are long-term constant pressure solutions based on theoretical considerations. The type curves are derived from models that simulate the production decline behavior of a gas well against a constant back pressure, P_{wf} (Aminian, 2009). Type curves are derived from solutions to fluid flow equations using specific initial and boundary conditions. The responses usually presented in terms of dimensionless variables, (e.g., dimensionless time t_D). The type curves used in this study represent gas production as function of time.

Fetkovich presented the theoretical basis for Arps' production decline models using the pseudosteady-state flow equation. He also developed decline type curves that not only enable us to forecast well performance but also to estimate reservoir properties (i.e., flow capacity kh) as well as original oil-in-place (OOIP). This classic work by Fetkovich laid the foundation for all the work that followed regarding decline type curves. He combined the analytical constant terminal pressure solutions of the well diffusivity equation with the classical decline curve equation to yield a series of composite log-log dimensionless curves. The type curves developed by Fetkovich are primarily developed for oil wells and, as a result, difficulties may be encountered when they are applied to gas wells. The type curves do not account for the pressure loss due to high gas velocity near wellbore (non-Darcy effects). Also, they do not consider changes in fluid viscosity and compressibility as reservoir pressure is reduced. Figure 2 shows an example of Fetkovich developed type curves. They are the result of the combination of the

empirical back pressure equation given by equation 2.2 and the gas material balance equation, assuming the gas compressibility factor z equals to 1.0, equation 2.3

$$q = C(P_R^2 - P_{wf}^2)^n \dots\dots\dots (2.2)$$

$$P_R = -\left(\frac{P_i}{G_i}\right)G_p + P_i \dots\dots\dots (2.3)$$

The main assumption of these type curves is a constant flowing pressure from a well centered in a circular reservoir with no flow boundaries. These type curves can be used for analyzing long-term gas production data from hydraulically fractured wells during the pseudoradial flow period and once the outer boundaries affect the pressure response.

Figure 2 Fetkovich rate time decline type curves (1980).

In order to account for non-Darcy flow, Smith (1980) extended Fetkovich type curves by varying the exponent “ n ” and generated many sets of type curves. Equation 2.2 is only an empirical relationship and it has been shown that the value of “ n ” does not remain constant over the entire life of the well. When “ n ” is constant, the forecasted flow rate based on type curve was found inaccurate.

At later time, Carter (1984) generated a new set of type curves with a finite-difference reservoir model. His type curves are based on a drawdown parameter, λ that permits good estimate representation of real gas flow with single set of curves. He found that in order to estimate accurate gas in place, data are required over that portion of the producing history corresponding to a t_D value of at least 0.7 to 2. For that, Carter’s type curves are not easy to use.

In 1985, Fraim and Wattendbarger introduced decline curve analysis using type curves with real gas pseudopressure and normalized time. This study was to improve the application of Fetkovich type curves by changing the gas properties with reservoir pressure. This method has the disadvantage of requiring an estimate of gas in place before the normalized time can be calculated.

In 1986, Aminian et al. has developed a set of more representative curves by combining the theoretical stabilized gas flow equation, (equation 2.4) and the material balance for a volumetric gas reservoir, (equation 2.5):

$$P_p(P_R) - P_p(P_{gf}) = aq - bq^2 \dots\dots\dots (2.4)$$

$$G_p = \frac{G_i(B_g - B_{gi})}{B_g} \dots\dots\dots (2.5)$$

All of the authors mentioned earlier have neglected the inclusion of non-Darcy flow where this model accounts for non-Darcy flow and dependency of gas properties on pressure. It also assumes constant reservoir parameters and operating conditions for the life of the well.

In 1987, Aminian et al has developed a model to discuss the abuse of this assumption. The model was modified and utilized to study the decline behavior of gas wells when the drainage area, back pressure, and skin factor changed. A general correlation between initial gas flow rate and back pressure was established. Also, a general correlation between initial gas flow and skin factor has been established.

Aminian et al. (1989) have developed type curves for predicting horizontal well production performance. The type curves developed are for low permeability gas reservoirs. He discussed the effect of the ratio of the horizontal well length to the reservoir length. Also, he found that the permeability anisotropy has a significant effect on performance of the horizontal well.

To make a forecast using type curves: first, the history of gas production is matched with type curves until one is found; second, the future production rates, gas reserve, and reservoir parameters are evaluated from the type curve. Figure 3 explains an

example of type curve matching. If the production history is available, type curve can be used to determine reservoir parameters.

Figure 3 Graphic example of the type curve matching.

2.3 Unconventional Reservoirs

An unconventional reservoir is one that cannot be produced at economic flow rates or that does not produce economic volumes of oil and gas without assistance from massive stimulation treatments or special recovery processes and technologies, such as

hydraulic fracturing. Typical unconventional reservoirs are tight-gas sands, coal-bed methane, heavy oil, and gas shales.

Tight reservoirs (which are considered in my study) are those which have low permeability, often quantified as less than 0.1 millidarcies. Poor permeability is primarily due to fine-grained nature of the sediments, compaction, or infilling of pore spaces by carbonate or silicate cements precipitated from water within the reservoir (Canadian Centre for Energy Information).

Figure 4 a Thin section of a conventional sandstone reservoir that has been injected with blue epoxy. The blue areas are pore space and would contain natural gas in a producing gas field. The pore space can be seen to be interconnected so gas is able to flow easily from the rock. (G. C. Naik, Tight Gas Reservoirs – An Unconventional Natural Energy Source for the Future)

Figure 4 b Thin section Photo of a tight gas sandstone. The blue areas are pores. The pores are irregularly distributed through the reservoir and the porosity of the rock can be seen to be much less than the conventional reservoir. (G. C. Naik, Tight Gas Reservoirs – An Unconventional Natural Energy Source for the Future)

2.4 Horizontal Wells

Horizontal wells are one of the most important strategic tools in petroleum exploitation. In recent years, horizontal wells have been very successful in increasing productivity, adding reserves, and improving the overall cost-effectiveness of field operations. As a result of the advances in drilling and completion technologies in the last two decades, the efficiency and economy of horizontal wells have significantly increased. Today, horizontal well technology is applied more often and in many different types of formations. The state of the art applications of horizontal well technology require better completion designs to optimize production rates, long-term economics, and ultimate producible reserves.

According to DOE, horizontal well remediation systems are usually faster, cheaper, and more effective than the baseline technology of vertical wells. They provide:

- 1) Improved access to contaminants at sites with surface restrictions (e.g., buildings, tanks),
- 2) Improved hydraulic control along leading edge of contaminant plume or at property boundary,
- 3) Minimal surface disturbance because fewer wellheads may be required,
- 4) Ability to monitor beneath contaminant sources (e.g., tanks, pits, lagoons),
- 5) Increased surface-area contact with contaminants,
- 6) Reduced operating expenses because fewer wells may be required, and
- 7) Access to off-site contamination to be treated by on-site operations.

Figure 5 Applications for horizontal wells (G. C Thakur, Chevron PTC)

Joshi (1988) found that horizontal wells are not effective in very thick reservoirs (500 to 600 ft) and in a formation with low vertical permeability. A decrease in vertical permeability results in an increase in vertical flow resistance for horizontal wells and a corresponding decrease in oil or gas production.

The horizontal well technology has three major disadvantages (Joshi, 2003):

- 1) High cost as compared to a vertical well. In the U.S., a new horizontal well drilled from the surface, costs 1.5 to 2.5 times more than a vertical well. A re-entry horizontal well costs about 0.4 to 1.3 times a vertical well cost.
- 2) Generally only one zone at a time can be produced using a horizontal well. If the reservoir has multiple pay-zones, especially with large differences in vertical

depth, or large differences in permeabilities, it is not easy to drain all the layers using a single horizontal well.

- 3) The overall current commercial success rate of horizontal wells in the U.S. appears to be 65%. (This success ratio improves as more horizontal wells are drilled in the given formation in a particular area.) This means, initially it is probable that only 2 out of 3 drilled wells will be commercially successful.

2.5 Hydraulic Fracturing

Hydraulic fracturing, commonly referred to as fracing, is a proven technological advancement which allows natural gas producers to safely recover natural gas from low permeability formations. Hydraulic fracturing has been used by the oil and gas industry since the 1940s and has become a key element of natural gas development worldwide.

Hydraulic fracturing was first used more than 100 years ago in 1903, but the first commercial fracturing treatment was performed in 1949. By some accounts it took more than 40 years for geologists and engineers to perfect the process, but since then, the pay-off has been extraordinary. Its efficacy in bringing new life to old wells quickly made it an integral part of our nation's energy strategy, and by 1988, it had been applied more than one million times. As technology improved, hydraulic fracturing's applications did, as well. Now, fracturing is used not only to stimulate production in old wells, but to jump start the production process in unconventional formations and in unfavorable locations (Energy in Depth).

The main idea of hydraulic fracturing is to create a highly conductive flow path which extends far beyond any damage zone around the wellbore and therefore attracts fluid from the undisturbed parts of the reservoir. A hydraulic fracture is formed by pumping the fracturing fluid into the wellbore at a rate sufficient to increase the pressure downhole to a value in excess of the fracture gradient of the formation rock. To keep this fracture open after the injection stops, a solid proppant, commonly sieved round sand, is added to the fracture fluid. The propped hydraulic fracture then becomes a high permeability conduit through which the formation fluids can flow to the well.

Figure 6 Hydraulic Fracturing operations

2.5.1 Hydraulic Fracture Types

Depending on well orientation, with respect to the minimum horizontal stress, and length of the perforated interval, either a transverse or longitudinal fracture may be created.

If the horizontal well is drilled parallel to the minimum horizontal stress, and the perforated interval is shorter than four times the wellbore diameter, it is expected that the

created fractures will be perpendicular to the horizontal well; i.e., transverse fractures will be created. If the horizontal well is drilled perpendicular to the minimum horizontal stress, the created fracture will be longitudinal. These two cases represent the two limiting and recommended cases.

2.5.1 Dimensionless Fracture Conductivity (FCD)

If F_{CD} is more than 100 it is considered an infinite, if F_{CD} is less than 100 it is considered a finite.

The diagram shows the equation for Dimensionless Fracture Conductivity (F_{CD}) with labels for its components:

$$F_{CD} = \frac{k_f \times w_f}{k \times X_f}$$

- Fracture Permeability** points to k_f .
- Fracture Width** points to w_f .
- Formation Permeability** points to k .
- Fracture Half-Length** points to X_f .

Below the equation, it states: $F_{CD} > 100 \rightarrow \text{Infinite Conductivity}$

Figure 7 Fracture conductivity Equation.

Figure 8 Infinite-conductivity and finite-conductivity fractures.

2.6 Horizontal Wells Flow Regimes

There are usually several flow regimes with different durations because of the partially penetrated nature of the horizontal wells and multiple boundary effects. In general, horizontal wells may observe three radial (pseudoradial) flow regimes for a

single layer horizontal well. Figure 9 shows the flow regimes in horizontal well (Kuchuk, 1995).

The first radial flow regime is the radial flow around the wellbore and it may continue until the effect of the nearest boundary is reached. This flow regime may not develop if the anisotropy ratio, K_H/K_V is large. The second radial flow regime is a hemicylindrical flow regime that follows the first radial flow. It may occur when well is not centered with respect to the no-flow top and bottom boundaries. Intermediate time linear flow regime may occur if the horizontal well is much longer than the formation thickness. This flow regime develops after the effects of the upper and lower boundaries are felt. When the top and bottom boundaries are felt, a third radial flow will develop.

Figure 9 Radial flow regimes for a horizontal well (Kuchuk, 1995).

2.7 Hydraulic Fractured Horizontal Well Flow Regimes

Okzan et al. (2006) has presented a summary of the flow regimes of fractured horizontal wells. The following flow regimes have been identified:

Characteristics of fractured horizontal wells:

1. Fracture-storage induced flow regimes:

- i) Fracture radial flow,
- ii) Radial-linear flow
- iii) Bilinear flow

2. Reservoir flow regimes:

- i) Early-time linear flow normal to fractures
- ii) Intermediate-time pseudoradial flow around individual fractures,
- iii) Intermediate-time linear flow normal to horizontal well axis (compound linear flow),
- iv) Late-time pseudoradial flow around the horizontal well (compound pseudoradial flow)

Not all flow regimes exist for all fractured horizontal wells. Figure 10 shows the sketched flow regimes.

Figure 10 Potential flow regimes for fractured horizontal wells (Okzan, 2006)

He also (Okazn, 2006) has concluded that the fracture geometry, varying fracture properties, and non-Darcy flow significantly influence the flow regimes. Figure 11 show the effect of fracture geometry on potential flow regimes.

Figure 11 Effect of fracture geometry on potential early time flow regimes
(Okzan, 2006).

2.8 Dimensionless Variables

Dimensional analysis is routinely used to check the plausibility of derived equations and computations. The dimensionless variables that can affect the production type curves are: 1) the dimensionless well length (L_D) and 2) the dimensionless well radius (r_{wD}).

The dimensionless well length is related to vertical and horizontal permeabilities (K_V and K_H), formation thickness (h), and the well length (L). The dimensionless well

radius is related to the size of the wellbore (r_w) to the well length. Aminian et al. and Ameri et al. (1989) have defined these dimensionless variables as:

$$L_D = [(L/2h)\sqrt{k_v/k_H}] \dots\dots\dots (2.6)$$

$$r_{wD} = (2r_w)/L \dots\dots\dots (2.7)$$

In 1989, Aminian and Ameri have predicted horizontal well production performance by developing type curves. The well productivity was presented in terms of dimensionless cumulative gas production and dimensionless time for finite reservoirs.

$$G_{pDA} = [(9T)/(h\phi\mu_i c_u A \Delta m(p))] \times G_p \dots\dots\dots (2.8)$$

$$t_{DA} = [(0.006328k)/(\phi\mu_i c_u A)] \times t \dots\dots\dots (2.9)$$

The influence of the ratio of horizontal well length to reservoir length, penetration ratio ($L/2X_e$) into the long term production behavior of horizontal wells has been shown in figure 12.

Figure 12 Effect of horizontal well penetration on type curves (Aminian and Ameri, 1989).

The comparison between the responses for rectangular and square areas is shown in figure 13 with L_D and $L/2X_e$ constant. The performance of a horizontal well in a rectangular drainage area is improved over a square area.

Figure 13 Effect of the drainage area shape on type curves (Aminian and Ameri, 1989).

The horizontal well productivity was presented in terms of dimensionless cumulative gas production and dimensionless time for infinite reservoir.

$$G_{pDL} = \left[\frac{(36T)}{(h\phi\mu_i c_n L^2 \Delta m(p))} \right] \times G_p \dots\dots\dots (2.10)$$

$$t_{DL} = \left[\frac{(0.02532k)}{(\phi\mu_i c_n L^2)} \right] \times t \dots\dots\dots (2.11)$$

Large pressure difference at early times can be induced with long wells thinner reservoirs. As shown in figure 14, greater L_D (more than 10) the influence of the top and bottom boundaries becomes small and performance of a horizontal well approaches that of fully penetrating infinite conductivity fracture.

Figure 14 Type curves for various L_D values for infinite reservoirs (Aminian and Ameri, 1989).

2.9 Boundary Conditions

Two boundary conditions on the well surface can be considered: infinite conductivity and uniform flux (Okzan, 1987). For the uniform flux, the production rate is constant with pressure varying along the length of the well. The assumption of the infinite conductivity is a constant pressure in the horizontal well bore. For a single horizontal well or drainhole, the infinite conductivity idealization is the only viable boundary condition.

2.10 Development of Type Curves for Gas Production from Horizontal Wells in Conventional Reservoirs

Abdulla Almansour (2009) has developed type curves to evaluate gas production from horizontal wells in conventional reservoirs. Also, these type curves can be used to predict the production performance during the life of the wells. Almansour has created two set of type curves using a finite-difference multi-layers reservoir model. They were presented in terms of dimensionless gas production and dimensionless time. Drainage shape was assumed to be rectangle since it is the more effective drainage area for horizontal wells. Almansour found that two dimensionless parameters to control the type curves, the dimensionless well length (L_D) and the pressure drawdown dimensionless (X_i). They represent the two flow regimes associated with the horizontal wells, the early time liner flow and the late time pseudo-radial (elliptical) flow. Figure 15 shows the impact of L_D on type curves and figure 16 shows the impact of ratio of rate of penetration ($L/2X_e$) and X_i on the type curves.

Figure 15 Impact of L_D on the type curves (Almansour, 2009).

Figure 16 Impact of $L/2X_e$ and X_i on the type curves (Almansour, 2009).

Almansour has reviewed different parameters that affecting the performance of the horizontal wells such as horizontal permeability, porosity, thickness and drainage area.

Almansour has concluded that drainage area and porosity had no effect on type curves for both early and late production periods. He also found that reservoir thickness and horizontal permeability changes had a minimal effect on type curves. Appendix show the impact of these parameters resulted from his study on type curves.

CHAPTER 3

OBJECTIVE AND METHDOLOGY

The goal of this research was to develop a simple and reliable tool to predict the performance of the production of hydraulically fractured horizontal wells. More specifically, the objective is to develop production type curves for hydraulically fractured wells in unconventional gas reservoirs.

In order to achieve the goal of this study, a methodology consisting of the following procedures was used:

1. Acquire unconventional reservoir parameters from the literature.
2. Acquire hydraulically fracture parameters.
3. Develop a basic model to predict gas production profiles for hydraulically fractured horizontal wells.
4. Evaluate the impact of various reservoir parameters and the impact of various hydraulic fracture parameters on the type curves.
5. Develop a set of production type curves for hydraulically fractured horizontal wells.

3.1 Numerical Models

Production decline curves are usually used to forecast the recovery factor, future revenues, and well performance. The performance of gas production of hydraulically fractured reservoirs can be best predicted by using numerical reservoir systems that

account for various mechanisms that control gas production. The gas simulator is a multi dimensional model that solves one, two or three dimensional problems. Cartesian or polar can be specified in the simulator. The boundary conditions are flexible in that any pressure or rate, as a function of time, may be operated in a radial mode to simulate a single production well.

After the literature review was conducted and the parameters established, the best model to use was the Schlumberger Eclipse Reservoir Simulator. Although the “Eclipse Office” CBM template is not a dual porosity model, it models a single porosity reservoir. This tool allows the user to easily generate the reservoir model and evaluate results quickly. It can be used to study the comparative value of simple completions, hydraulic fracture enhancements, and single or multi lateral horizontal completions. Figure 17 shows the template model workflow.

Figure 17 Example of template model flow chart.

3.2 Base Model Parameters and Assumptions

Defining the reservoir geometry, hydraulic fracture geometry, initial and boundary conditions is needed for a hydraulically fractured horizontal well production responses development. The base model was developed for hydraulically fractured reservoir with the well configuration as shown in Figure 18.

In order to establish the unique type curves, the impact of the reservoir parameters were investigated. The parameters used to develop the base model were selected from a previous study (Almansour, 2009). Base model developed in this study using a rectangular coordinate system. The solutions assume that the horizontal well is located in the center of the reservoir ($h/2$) and parallel to the top and bottom of the reservoir. A homogenous, single phase gas flow, and single porosity multi layer reservoir is considered in this study. For the base case, the permeability values are different at a constant ratio K_z/K_x 1:3. The K_x and K_y values are equal. The list of parameters used in the base case can be seen in Table 1.

This investigation assumes that the horizontal well was hydraulically fractured at the center of the well length ($L/2$) with one fracture. The main fracture parameters used are; fracture half length (X_f) = 500 ft, the fracture width (W_f)=0.1 in and the fracture porosity is 10%. The fracture parameters were changed in some cases of the cases for comparison. Also, more than one fracture were added to the model in one of the cases to see how the results may change based on the number of the fractures in the horizontal well. Table 2 lists the base case hydraulic fracture parameters.

Figure 18 Basic Case Model Schematic

Table 1 Parameters and values used in the base model.

Parameters	Values
Period of production (years)	30
Fluid Type	Dry Gas
Porosity Model	Single Porosity
Porosity (%)	5
Model Geometry	Multilayer Reservoir (5 layers)
Grid Size (ft)	100 x 100
Reservoir Area (acers)	46
Shape	Rectangular
Reservoir depth (ft)	3,000
Reservoir Thickness (ft)	20
Reservoir Length (2Xe) (ft)	2,000
Reservoir Width (Ye) (ft)	1,000
Well Length (ft)	1,500
X-direction Permeability (mD)	0.1
Y-direction Permeability (mD)	0.1
Z-direction Permeability (mD)	0.03
Reservoir Pressure (psia)	1,500
Bottom Hole Flowing Pressure (psia)	500
Temerture (°F)	100
Gas Gravity	0.72
Wellbore Diameter (ft)	0.5

Table 2 Fracture Parameters in the base model

Parameters	Values
Half Length (ft)	500
Width (in)	0.1
Top of the fracture (ft)	3,000
Bottom of the fracture (ft)	3,020
X-center (ft)	1,000
Y-center (ft)	500
Permeability (mD)	20000
Porosity (%)	10

3.3 Dimensionless Groups for Type Curve Development

In order to have the unique type curves, the assumption is that two different dimensionless groups will be needed. The dimensionless well length (L_D) and the well penetration ratio ($L/2X_e$) are the main parameters in the production analysis development. Different reservoir parameters were investigated to evaluate their impact on type curves. Two different sets of type curves are needed due to the two flow regimes that the horizontal well encounters during production. The first dimensionless group represents the linear flow regime where the second dimensionless group represents the elliptical/radial flow regime.

The dimensionless well length values (L_D) used in this study: 10, 25, 37.5, 50, and 100.

The well penetration ratios ($L/2X_e$) used: 0.75, 0.6, 0.3, and 0.18. Table 3 show the ranges of values used in this study.

Table 3 Ranges of values used in the model

Parameters	Ranges	Used Values
Reservoir Shape	Rectangular	Rectangular
Area	46 – 103	46, 69 & 103
Well Penetration Ratio ($L/2X_e$)	0.18 – 0.75	0.18, 0.3, 0.6 & 0.75
Dimensionless Well Length (L_D)	10 – 100	10, 25, 37.5, 50 & 100
Reservoir Thickness	15 – 45	15, 20, 30 & 45
Horizontal Permeability (mD)	0.001 – 0.1	0.001, 0.01, 0.05, 0.07 & 0.1
Vertical Permeabilty (mD)	0.02 – 0.03	0.02 & 0.03
Porosity (%)	0.03 – 0.1	0.03, 0.05 & 0.1
Fracture Permeability (mD)	10000 - 40000	10000, 20000, 30000 & 40000
Fracture half length (X_f)	200 – 500	200 & 500
Fracture Width (in)	0.01 – 0.1	0.01 & 0.1
Fractures Number	1 - 9	1, 2, 3,4 and 9

3.4 Gas Production Predication for Type Curve Development

Two different sets of type curves will be needed to represent the hydraulically fractured horizontal well production responses. The first sets of type curves will represent the early part of well production. The second set of type curves represent the late time of the life of the well. Equations 2.6, 2.7, 2.11, and 3.1 were used to develop the type curves.

$$q_D = [(1424T)/(kh\Delta m(p))] \times q \dots\dots\dots (3.1)$$

To develop the type curves for the early time of production, the dimensionless well length was kept constant and one of the reservoir parameters was changed in different values. For example, to investigate the impact of the reservoir drainage area, the dimensionless well length (L_D) was kept in a constant value of 37.5 and the drainage area was varied (46 acres, 69 acres, and 103 acres). The same technique was followed for the other reservoir parameters (as shown in Table 3).

The same procedures were used to generate type curves for the late part time of the production. The constant dimensionless variable in this part is the rate of penetration ratio ($L/2X_e$). The ($L/2X_e$) ratio was kept at a constant value (0.75) and one reservoir parameter was varied in every model run.

CHAPTER 4

RESULTS AND DISCUSSION

This research objective is to develop a set of type curves to predict gas production performance in low permeability reservoir. To achieve this goal, a basic model was built using a numerical simulator. Two different dimensionless groups have been evaluated for type curves development, the dimensionless well length (L_D) and the rate of penetration ratio ($L/2X_e$). Also, a large number of simulations have been run to investigate the reservoir parameters (drainage area, thickness, porosity, horizontal permeability, and vertical permeability) and the fracture parameters (number of fractures, fracture half length, and fracture width). The results are shown below.

4.1 The Evaluation of Different Dimensionless Groups for Type Curves

4.1.1 Impact of Well Length Dimensionless (L_D)

It has been observed that the performance of the hydraulically fractured horizontal well is significantly affected by the well length dimensionless (L_D). When the number of the L_D increases, less impact will be on type curves. As shown on figure 19, different L_D s were investigated on type curves; 10, 25, 50, and 100. It is clear from the figure that when the L_D reaches the value of 50, no impact observed on type curves.

4.1.2 Impact of Rate of Penetration Ratio ($L/2X_e$)

A range of values of rate of penetration ratios are used in the study; 0.75, 0.6, 0.3, and 0.18. For the early production, no effect on production responses is found from the variation of the rate of penetration ratio. Figure 20 shows the type curves at different rate of penetration ratios for early production. The same values are used for the late production period. When the ratio is high, the impact on type curves is obvious. The impact of the ($L/2X_e$) on type curves decreases when the ratio values decreases. Figure 21 shows the impact of ($L/2X_e$) for late production.

4.2 Impact of Different Reservoir Parameters on Type Curves

As stated before, different reservoir parameters are investigated. The following paragraphs are discussing the impact of these parameters on type curves.

4.2.1 Impact of Drainage Area

The drainage area was examined for both early time and late time of the life of the well. The values used for variation are 46, 69, and 103. No effect was noticed on type curves by changing the drainage area values for both the early time and the late time of the well production. Figure 22 and 23 show the production performance for the early and late times.

Figure 19 Impact of different values of L_D (early production).

Figure 20 Impact of $(L/2Xe)$ (early production)

Figure 21 Impact of $(L/2Xe)$ (late production)

Figure 22 Impact of drainage area for $L_D = 37.5$ (early production)

Figure 23 Impact of drainage area for $L/2X_e = 0.75$ (late production)

4.2.2 Impact of Horizontal Permeability

The horizontal permeability was varied in the range of 0.001 mD to 0.1 mD. It was found that horizontal permeability has a very minor effect on type curves for the early production. Figure 24 shows the production performance for the early time. Figure 25 shows no effect of the horizontal permeability on type curves for the late production.

Figure 24 Impact of horizontal permeability for $L_D = 37.5$ (early production)

Figure 25 Impact of horizontal permeability for $L/2X_e = 0.75$ (late production)

4.2.3 Impact of Vertical Permeability

To examine the effect of the vertical permeability on type curves, two values are used. $K_v = 0.02$ mD and $K_v = 0.03$ mD. As figure 26 illustrates, there is no effect on type curves by changing the value of vertical permeability.

4.2.4 Impact of Reservoir Thickness

Figure 27 shows the impact of reservoir thickness on type curves for early production. The reservoir thickness was varied from 15 ft to 45 ft. It was found that a slight effect occur to the type curves at early production. As shown in figure 28, reservoir thickness has less effect at late time when varying reservoir thickness.

Figure 26 Impact of vertical permeability for $L_D = 55$ and $L/2X_e = 0.75$

Figure 27 Impact of reservoir thickness for $L_D = 37.5$ (early production)

Figure 28 Impact of reservoir thickness for $L/2X_e = 0.75$ (late production)

4.2.5 Impact of Porosity

The porosity was varied from a range of 3% to 10%. Figure 29 shows that there is no effect of porosity on production responses at early production. The results indicate that the porosity effects the late production. As shown in figure 30, when porosity decreases the type curve shifts to the right of the plot.

Figure 29 Impact of porosity for $L_D = 25$ (early production)

Figure 30 Impact of porosity for $L/2X_e = 0.5$ (late production)

4.3 Impact of Hydraulic Fracture Parameters on Type Curves

Number of fractures, fracture permeability (K_f), fracture half length (X_f), and fracture width (W_f) have been considered in this study. The subsections below show the results of these parameters.

4.3.1 The Impact of the Number of Fractures

Different numbers of the hydraulic fractures have been added to the base model. It is found that the very low permeability value the more impact of the number of the fractures on type curves. The permeabilities of 0.1 and higher, the number of fractures

has less impact on type curves. For the permeabilities less than 0.1 mD, the number of fractures has more impact on type curves. Figures 31, 32, and 33 illustrate the results.

4.3.2 The Impact of Fracture Permeability and Fracture Half Length

It is found that the fractures half length and the fractures permeability have no effect on type curves. The values range used for both fracture half length and the fracture permeability are shown in figure 33.

Figure 31 Impact of the number of fractures (K= 0.001 mD).

Figure 32 Impact of the number of fractures (K= 0.01 mD).

Figure 33 Impact of the number of fractures (K= 0.1 mD).

Figure 34 The impact of X_f and K_f on type curves.

4.3.3 The Impact of Fracture Width

Two different values of fracture width (W_f) are tested in this study 0.01 in and 0.1 in. The results indicate that the effect of variation on the fracture width on type curves is minimal at the early part. Figure 34 shows the production responses of the fracture width change for the early production time.

Figure 35 The impact of W_f for $L_D = 50$ (early production)

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

The objective of this research was to develop a set of type curves that could be used to evaluate and predict production data for unconventional gas horizontal wells. The research took into account reservoir parameters and fracture parameters to determine the impacts on production performance. Based on the results, the following conclusions were made:

1. It was found that two dimensionless groups are needed for developing type curves for gas horizontal wells.
2. Two dimensionless variables have been shown to influence the gas production type curves, the dimensionless well length L_D (early production) and the well penetration ratio $L/2X_e$ (late production).
3. Drainage area, horizontal permeability, and vertical permeability were found not to impact type curves significantly for both early and late production.
4. Reservoir thickness has a minor effect on type curves.
5. Reservoir porosity has no effect on early production but significant effect on late production.
6. The number of hydraulic fractures does affect type curves particularly with very low permeability.
7. In low permeability reservoirs, more hydraulic fractures will be needed to improve the production performance.

8. Fracture half length, fracture permeability, and fracture width were found to have no major affect on type curves.

It is recommended to extend this study to evaluate the production performance of gas wells in naturally fractured reservoirs. Also, future study is needed for the unconventional gas in shale reservoirs.

NOMENCLATURE

P_R = Reservoir pressure (psia)

P_i = Initial Reservoir Pressure, (psia)

P_{wf} = Bottom-hole flowing pressure (psia)

P = Pressure (psia)

P = Pseudo-pressure, psi^2/cp

a = Darcy flow coefficient, $\text{psi}^2/(\text{cp})(\text{Mscf/D})$

b = Non- Darcy flow coefficient, $\text{psi}^2/(\text{cp})(\text{Mscf/D})^2$

q_D = Dimensionless gas rate

q = Gas rate (Mscf/day)

t_D = Dimensionless time

t_{DL} = Dimensionless time with length

t_{DA} = Dimensionless time with area

t = Time (days)

$2X_e$ = Width of reservoir (ft)

Y_e = Length of reservoir (ft)

L = Length of lateral (ft)

L_D = Dimensionless length

G_p = Cumulative gas production (Mscf)

G_D = Dimensionless cumulative gas produced

G_{DL} = Dimensionless cumulative gas produced with length

G_{DA} = Dimensionless cumulative gas produced with area

A = Area (ft^2)

$m(p)$ = Real gas potential

h = Thickness (ft)

k_H = Horizontal permeability in x and y direction (mD)

k_v = Vertical permeability in z direction (mD)

k = Horizontal permeability (mD)

μ_i = Initial Viscosity (cp)

ϕ = Porosity (%)

C_{ti} = Total initial compressibility (psi^{-1})

T = Temperature ($^{\circ}\text{R}$)

r_w = Wellbore radius (ft)

r_{wD} = Dimensionless wellbore radius

D_t = Decline constant (days^{-1})

b = Arps decline – curve constant

z = Gas deviation factor, dimensionless

X_f = Fracture half length (ft)

W_f = Fracture width (in)

K_f = Fracture permeability (mD)

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Appendix A

Impact of reservoir parameters on type curves for horizontal gas wells (Almansour, 2009)

Figure A.1 Impact of LD on type curves

Figure A.2 Impact of drainage area for LD = 10 (early production)

Figure A.3 Impact of drainage area for $L/2X_e = 0.75$ (late production)

Figure A.4 Impact of horizontal permeability for LD = 10 (early production)

Figure A.5 Impact of horizontal permeability for $L/2X_e = 0.75$ (late production)

Figure A.6 Impact of thickness for LD = 10 (early production)

Figure A.7 Impact of thickness for $L/2X_e = 0.75$ (late production)