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Modeling of Roughness and Dissipation of Actuated Polymer Films

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Modeling of Roughness and Dissipation of Actuated Polymer Films

by

Joaquín María Gutiérrez

Thesis submitted to the
College of Engineering and Mineral Resources
at West Virginia University
in partial fulfillment of the requirements
for the degree of

Master of Science
in
Mechanical Engineering

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2011

Keywords: Dissipation, Actuated Polymer Films, Finite Element, ABAQUS[®] ,
MATLAB[®] , Hyperelasticity, Viscoelasticity, Bergstrom - Boyce Model

Actuated polymer films (APF) are a novel kind of material with several possible applications (e.g., dynamic roughness for aerodynamic flow control, tactile responsive touch panels, dust mitigation systems for solar panels and other devices, and tunable hydrophobicity surfaces). The development of the required analytical and numerical tools for the design and analysis of APF's is addressed in this research.

A hyperelastic hysteretic material model (Bergstrom-Boyce) is used to model the APF. A program to calibrate the Bergstrom-Boyce material parameters from experimental data is developed using MATLAB[®]. The soundness of this calibration tool is assessed using experimental data from the literature.

To calculate the response of the APF and the power requirement to actuate the film, a program is written. The performance of this program is validated with experimental results from the literature. Two- and three-dimensional models are analyzed and compared using ABAQUS[®].

A procedure for the determination of the volume fraction of the actuators embedded in the APF is proposed using digital image processing techniques and Voronoi diagrams.

Dedication

To my wife, Sofia.

Acknowledgments

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Table of Symbols and Notation

Tensors

B	:	Left Cauchy-Green Deformation Tensor
D	:	Rate of Deformation Tensor
F	:	Deformation Gradient Tensor
I	:	Identity Tensor
L	:	Velocity Gradient Tensor
P	:	First Piola-Kirchhoff Stress Tensor
S_B	:	Deviatoric Cauchy Stress Tensor
U	:	Displacement Field Tensor / Right Stretch Tensor
W	:	Spin Tensor
σ	:	Cauchy Stress Tensor

Scalars

J	:	Jacobian
λ	:	Stretch

Acronyms

APF	:	Actuated Polymer Film
BB Model	:	Bergstrom-Boyce Model
FE	:	Finite Element
ITO	:	Indium Tin Oxide
PBC	:	Periodic Boundary Conditions
PVA	:	Polyvinyl Alcohol
RVE	:	Representative Volume Element

Boldface letters denote tensors.

Chapter 1

Introduction

1.1 Background

The goal of the present work is to develop a set of tools for the design and analysis of actuated polymer films (APF). In the context of this work, the actuator polymer films considered are made of electrically responsive liquid crystal ellipsoids embedded in a polymeric film. When these actuators are subjected to an electric field, they try to align with the field gradient.

A possible application of these novel materials is in the aerodynamic flow control field. It has been studied that flow separation in airfoils can be reduced when the leading edge can change its roughness at specific frequencies and amplitudes (sub-boundary layer) [33, 39, 28]. Reductions in the flow separation can result in lift enhancement, drag reduction or both. A crucial factor in the feasibility of the application of dynamic roughness for flow control, is that the power savings associated with the improved aerodynamics must be greater than the power required to actuate the APF. Therefore, the ability to calculate the power consumption and the roughness achieved when the APF is actuated is of paramount importance.

Other possible applications of the actuated polymer films suggested by Morris et al. [42], include tactile responsive touch panels, dust mitigation systems for solar panels and other devices, and tunable hydrophobicity surfaces.

As it was mentioned in the previous paragraphs, it is very important to be able to calculate *how much* will the surface of the actuated polymer film change its geometry and at *what cost*, power-wise speaking. Although this could be determined with physical testing, it is of special interest to have the analytical and numerical tools to do it virtually. Since the actuated polymer film technology is new and still under development, the capacity of doing virtual testing allows to explore different scenarios in advance. For example, currently, the liquid crystal actuators are embedded in polyvinyl alcohol (PVA) [42], but because PVA is soluble in water [48], it could not be used for dynamic roughness applications in aircraft wings. Therefore, different materials (possibly rubbery elastomers) should be analyzed as possible candidates. Furthermore, many achievable geometric configurations (varying the actuator's geometry, film thickness, etc) can be tested using a parametric analysis, creating savings in time and money.

A procedure for the calculation of the power required to actuate the APF and the roughness obtained in the surface of the film are at the core of the present work. Other related

tasks, like the fit of experimental data to a particular material model and the determination of the volume fraction of the actuators in the APF are also developed in this document.

1.2 Literature Review

The key component in the actuated polymer films are the actuators themselves. Responsive liquid crystal rods have been developed by Cairns et al. [21, 44]. The first potential applications sought for these actuators were active mixing in microfluidic channels and electro-rheological fluids [21].

More recently, the idea of embedding these actuators in a polymeric film was investigated by Morris et al. [42]. Currently the electric field is applied to the film when it is placed between two plates of indium-tin oxide (ITO) coated glass. For applications like aerodynamic flow control, at least one of the plates must be removed and the external surface of the APF should be coated with an electrically conductive thin layer in order to apply the voltage differential [20].

Since the possible applications of the actuated polymer films require a measurable change in the roughness of the surface, the deformations are expected to be finite. Therefore the infinitesimal theory shall not be used, and a hyperelastic model should be used.

A relatively new constitutive model for large strain, time-dependent, elastomeric materials has been developed by Bergstrom and Boyce [11, 13]. This model has the ability to capture the viscous rate-dependence and the non-linear relationship between stress and strain. This model was later slightly modified by Bergstrom and Boyce to improve the hysteretic behavior of the model under cyclic loading [16]. The Bergstrom-Boyce model has been also successfully used to model soft biological tissues [16] and filled elastomers [14, 15].

The hyperelastic model that will be used in conjunction with the Bergstrom-Boyce model in the present work is the model proposed by Arruda and Boyce [4].

In the recent years, some authors have worked on the finite element implementation of the Bergstrom-Boyce model [23, 3, 24]. The Bergstrom-Boyce model has been implemented in general purpose commercial finite element packages like ABAQUS® and ANSYS®. For ABAQUS®, at least until version 6.9 there is a flaw in the calculation of the dissipated energy (see Appendix B). ANSYS® introduced the Bergstrom-Boyce model in the 12.0 release. The author found inconsistencies in the results from the implementation used in ANSYS® 12.0, and reported this fact to the company's Technical Support [34]. Later, ANSYS® issued a Defect Analysis Report confirming the problem [1].

Johansson et al. report a calibration procedure for the Bergstrom-Boyce model using a gradient-based optimization method [35, 36] for the uniaxial stress state. In the present work, a gradient-free optimization method will be used to do the calibration of the Bergstrom-Boyce model. The gradient-free method is easier to implement because the analytical derivatives of the constitutive equation with respect to the parameters and the flow rule are not needed, but a higher computational cost must be paid, for the same accuracy in the results.

1.3 Thesis Outline

In Chapter 2, the theoretical aspects that will be used in the following Chapters are developed from first principles. A brief description of some of the different hyperelastic models is given, and the necessity for such models is outlined. Some basic equations of finite kinematics are derived. The Bergstrom-Boyce model is introduced and the particular case of uniaxial deformation is analyzed.

A procedure to fit the parameters of the Bergstrom-Boyce model is developed for uniaxial loading, assuming incompressibility of the material in Chapter 3. This procedure is employed to fit experimental data for rubbery materials, available from the literature, and the goodness of fit is assessed.

Finite element models of the actuated polymer films are developed in Chapter 4. A program is written to calculate the response of the APF and the power requirement to actuate the film. The performance of this program is validated with experimental results from the literature.

Models to compare with the laboratory are analyzed in the first part of Chapter 4. In these models, the electrically conductive thin coating is not included.

Then, two- and three-dimensional models including the top film are analyzed. These models represent what the real application of the actuated polymer films (APF) would be. The power required to actuate the films is calculated and the response of the top surface, as well. The correlation of the dissipation power and roughness between the two- and three-dimensional models is computed.

At the end of the Chapter 4, a parametric analysis is done on a two-dimensional model. The geometry, loading and mesh are varied to analyze their impact in the results. A mesh convergence study is also carried out.

Finally, a novel methodology to calculate the volume fraction of the actuators embedded in the APF is proposed. The input for this method is a digital image of the APF. A series of image enhancement techniques are applied to the original image and the area of each actuator is digitally measured. The domain is partitioned using Voronoi graphs and the volume fraction is statistically calculated.

Chapter 2

Theory

2.1 Hyperelastic Materials

Hyperelastic materials are derived from a postulated Helmholtz free energy potential Ψ . If the free energy function is a function of the strain only, then it is referred as the strain energy function [38, 31].

$$\Psi = \Psi(\mathbf{F}) \quad (2.1)$$

The formulation of hyperelastic materials is needed to describe the behavior of materials under finite deformations.

2.1.1 Classification of Hyperelastic Models

The hyperelastic material models can be classified in two groups,

1. Phenomenological Models
2. Mechanistic or Statistical Models

The phenomenological models describe the material behavior from a macroscopic point of view, fitting the parameters of postulated equations to experimental data [31]. Some examples of phenomenological models are Saint Venant-Kirchhoff, Odgen, Mooney-Rivlin and Yeoh.

The mechanistic or statistical models focus on the deformation mechanism and its relation with the microscopic structure of the material. Rubbery materials are made of a very large number of extremely long polymer chains. Due to the large number of possible configurations on each chain and the large number of chains, it is reasonable to use statistic methods to derive the elasticity equations for these materials [38, §8.1],[46, §3.3]. Some examples of mechanistic models are Neo-Hookean, Arruda-Boyce, Treloar, Anand and Flory. In the present work the Arruda-Boyce hyperelastic model will be used.

2.2 Mullins Effect

During the first loading cycles many elastomers present a stress softening behavior. This behavior is associated with damage. After reaching a certain strain level, subsequent loading and unloading up to that strain will not cause further softening after a few cycles [31]. In the present work this phenomena is not going to be analyzed, but it is important to bear in mind that material samples should be conditioned before testing to remove this effect [11].

2.3 True Stress - True Strain

It is important to understand the difference between true stresses and strains and their engineering counterparts. The engineering stress is defined as the force acting per *undeformed* unit area. The true stress is the force acting per unit area in the *deformed* configuration. If the strains are infinitesimal the difference is minimal, but this is not true as the strains are finite.

The engineering strain is defined as

$$\epsilon^{\text{eng}} = \frac{\Delta L}{L} = \frac{\ell - L}{L} = \frac{\ell}{L} - 1 \quad (2.2)$$

where L and ℓ are the initial and final lengths of the fiber, respectively.

Rearranging (2.2), the stretch is defined as

$$\lambda = \frac{\ell}{L} = \epsilon^{\text{eng}} + 1 \quad (2.3)$$

The true, or logarithmic, strain ϵ is obtained by integration of the incremental strain $\delta\epsilon$,

$$\begin{aligned} \delta\epsilon &= \frac{\delta\ell}{\ell} \\ \int \delta\epsilon &= \int_L^\ell \frac{1}{\ell} \delta\ell \\ \epsilon &= \ln(\ell) - \ln(L) = \ln\left(\frac{\ell}{L}\right) \end{aligned} \quad (2.4)$$

From (2.3) and (2.4),

$$\epsilon = \ln(\lambda) = \ln(1 + \epsilon^{\text{eng}}) \quad (2.5)$$

The relationship between the normal true stress and the nominal or engineering stress is given by,

$$\sigma = \sigma^{\text{eng}} \lambda \quad (2.6)$$

this equation will be derived at the end of Section 2.7.

2.4 Finite Deformation Kinematics

The purpose of this section is to write down some basic equations of Continuum Mechanics that will be used for the development of the forthcoming sections. Some derivations are included for the sake of clarity, but by no means the intention of this chapter is to develop all the equations that are derived in any Continuum Mechanics book.

If a continuum medium is subjected to a motion field, the displacement field $\mathbf{U}$ can be represented as

$$\mathbf{U}(\mathbf{X}, t) = \mathbf{x}(\mathbf{X}, t) - \mathbf{X} \quad (2.7)$$

where $\mathbf{x}$ is the current position and $\mathbf{X}$ is the position in the reference configuration [31, 38]. The gradient of the the displacement field with respect $\mathbf{X}$ gives,

$$\frac{\partial \mathbf{U}}{\partial \mathbf{X}} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}} - \mathbf{I} \quad (2.8)$$

The deformation gradient tensor $\mathbf{F}$ is defined as,

$$\mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}} \quad (2.9)$$

and is a two-point tensor since references to the deformed and the undeformed configurations.

Rearranging (2.8) and using (2.9), the deformation gradient can be expressed as a function of the displacement field.

$$\mathbf{F} = \frac{\partial \mathbf{U}}{\partial \mathbf{X}} + \mathbf{I} \quad (2.10)$$

The determinant of the deformation gradient tensor $\mathbf{F}$ is known as the Jacobian,

$$J = \det(\mathbf{F}) \quad (2.11)$$

and represents the change of volume.

Applying the polar decomposition to the deformation gradient $\mathbf{F}$,

$$\mathbf{F} = \mathbf{R} \cdot \mathbf{U} = \mathbf{v} \cdot \mathbf{R} \quad (2.12)$$

where $\mathbf{U}, \mathbf{v}^1$ are the right and left stretch tensors, respectively, and $\mathbf{R}$ is the rotation tensor. Because $\mathbf{U}, \mathbf{v}$ are symmetric and positive definite second order tensors, it is possible to diagonalize them by using the coordinate transformation defined by their respective eigenvectors. The values of the diagonal terms are defined by the associated eigenvalues. This procedure is also known as spectral decomposition.

When the left stretch tensor $\mathbf{v}$ is transformed to principal coordinates, takes the form

$$\mathbf{v} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \quad (2.13)$$

¹Note that the displacement field and the right stretch tensors share the same symbol, $\mathbf{U}$, although they represent different variables. From here on, $\mathbf{U}$ will be used to represent the right stretch tensor.

where λ_1, λ_2 and λ_3 are the principal stretches, which are calculated as the eigenvalues of $\mathbf{v}$.

It can be proved that

$$\det(\mathbf{R}) = 1 \quad (2.14)$$

Recalling the following property from determinants,

$$\det(\mathbf{A} \cdot \mathbf{B}) = \det(\mathbf{A}) \det(\mathbf{B}) \quad (2.15)$$

and, substituting (2.12) and (2.14) in (2.11), it can be seen that

$$J = \det(\mathbf{F}) = \det(\mathbf{v} \cdot \mathbf{R}) = \det(\mathbf{v}) \det(\mathbf{R}) = \det(\mathbf{v}) = \lambda_1 \lambda_2 \lambda_3 \quad (2.16)$$

The deformation gradient $\mathbf{F}$ can be decomposed into volume-changing (dilatational) and volume-preserving (distortional) parts using the following multiplicative decomposition,

$$\mathbf{F} = (J^{1/3} \mathbf{I}) \mathbf{F}^* = J^{1/3} \mathbf{F}^* \quad (2.17)$$

where $\mathbf{F}^*$ is the distortional part and $(J^{1/3} \mathbf{I})$ is the dilatational part.

It also will be useful to introduce the velocity gradient $\mathbf{L}$,

$$L_{i,j} = \frac{\partial v_i}{\partial x_j} \quad (2.18)$$

where the velocity $\mathbf{v}$ is given by,

$$v_i = \frac{\partial x_i}{\partial t} \quad (2.19)$$

Using the chain-rule, it is possible to express (2.18) as

$$L_{i,j} = \frac{\partial v_i}{\partial X_a} \frac{\partial X_a}{\partial x_j} \quad (2.20)$$

Replacing v by its definition (2.19), and using Clairaut's theorem,

$$\frac{\partial v_i}{\partial X_a} = \frac{\partial}{\partial X_a} \left(\frac{\partial x_i}{\partial t} \right) = \frac{\partial}{\partial t} \left(\frac{\partial x_i}{\partial X_a} \right) = \frac{\partial \mathbf{F}}{\partial t} = \dot{\mathbf{F}} \quad (2.21)$$

And noting that

$$\frac{\partial X_a}{\partial x_j} = \mathbf{F}^{-1}, \quad (2.22)$$

it is possible to express $\mathbf{L}$ in terms of $\mathbf{F}$ and its time derivative [31, Equation 2.139].

$$\mathbf{L} = \dot{\mathbf{F}} \cdot \mathbf{F}^{-1} \quad (2.23)$$

Any second order tensor can be decomposed into a symmetric and anti-symmetric (skew-symmetric) parts. For the velocity gradient tensor $\mathbf{L}$, this additive decomposition yields

$$\mathbf{L} = \mathbf{D} + \mathbf{W} \quad (2.24)$$

with

$$\mathbf{D} = \frac{1}{2}(\mathbf{L} + \mathbf{L}^T) \quad (2.25)$$

$$\mathbf{W} = \frac{1}{2}(\mathbf{L} - \mathbf{L}^T) \quad (2.26)$$

where $\mathbf{D}$, $\mathbf{W}$ are the rate of deformation and spin tensors, respectively.

Furthermore, the total deformation gradient $\mathbf{F}$ can be decomposed using a multiplicative decomposition

$$\mathbf{F} = \mathbf{F}^e \cdot \mathbf{F}^p \quad (2.27)$$

where $\mathbf{F}^e$ is the elastic component and $\mathbf{F}^p$ is the viscous or plastic component.

The Left Cauchy-Green deformation tensor (also known as Finger tensor) $\mathbf{B}$ is written in terms of the spatial (deformed) coordinates as

$$\mathbf{B} = \mathbf{F} \cdot \mathbf{F}^T \quad (2.28)$$

Note that it is named as the *left* deformation tensor because the deformation gradient $\mathbf{F}$ is written unaltered to the *left* of the dot in the right hand side of (2.28).

The left Cauchy-Green deformation tensor of the distortional part of the deformation $\mathbf{B}^*$ can be written in terms of $\mathbf{F}^*$ as

$$\mathbf{B}^* = \mathbf{F}^* \cdot \mathbf{F}^{*T} = J^{-2/3} \mathbf{F} \cdot \mathbf{F}^T \quad (2.29)$$

Any tensor can be decomposed into a deviatoric part and a spherical part using an additive decomposition.

$$\mathbf{A} = \alpha \mathbf{I} + \text{dev}[\mathbf{A}] \quad (2.30)$$

$$\alpha = \frac{\text{tr}[\mathbf{A}]}{3} \quad (2.31)$$

The deviatoric part is noted using the deviatoric operator $\text{dev}[\]$ and from (2.30) and (2.31),

$$\text{dev}[\] = [\] - \frac{\text{tr}[\]}{3} \mathbf{I} \quad (2.32)$$

2.5 Stress Measures

The Cauchy stress tensor $\boldsymbol{\sigma}$ is also known as the *true* stress tensor because the traction vectors obtained when multiplied by a normal vector, represent the force applied per unit area in the *deformed* configuration. In the other hand, the first Piola-Kirchhoff stress tensor $\mathbf{P}$ is

Figure 2.1: BB Model Scheme

known as the *nominal* stress tensor because considers the *undeformed* area. The relationship between both can be calculated using Nanson's formula and is given by,

$$\mathbf{P} = J\boldsymbol{\sigma} \cdot \mathbf{F}^{-T} \quad (2.33)$$

2.6 Bergström - Boyce Material Model

The Bergström - Boyce Material Model (BB Model) has the ability to reproduce the hyperelastic and rate-dependent viscous behavior of rubber-like polymers. The most simple viscoelastic models, e.g. Prony Series, are lineal models. Thus, they can not capture rate dependency in the viscous behavior. One of the principal objectives of the present work is to calculate the dissipation of the Actuated Polymer Film as a function of the frequency of the excitation. Therefore, a material model with a hysteretic behavior sensitive to the frequency is needed.

The BB model is relatively new and has been implemented in general purpose Finite Element Analysis packages like ABAQUS® and ANSYS® with different success [1].

2.6.1 Constitutive and Kinematic Model

The BB model can be conceptually thought as a two networks in parallel. Network A characterizes the response when the viscous effects are not present (equilibrium), while Network B carries the time dependent component (non-equilibrium). A schematic representation of this model is given in Figure 2.1.

The dials in the springs and dashpot in Figure 2.1 reflect that the response of these elements is not linear. In order to account for finite strains, the spring elements should be modeled with a hyperelastic model. Different models could be used in the networks A and B, but they are usually modeled using the same kind of hyperelastic model.

Because of the parallel nature of the BB model, both branches are subjected to the same deformation gradient $\mathbf{F}$ [13, 17],

$$\mathbf{F} = \mathbf{F}_A = \mathbf{F}_B \quad (2.34)$$

Given (2.34) is possible to apply the decomposition given by (2.27) to $\mathbf{F}_B$

$$\mathbf{F}_B = \mathbf{F}_B^e \cdot \mathbf{F}_B^p \quad (2.35)$$

In a parallel system, all the branches are subjected to the same deformation gradient. But each branch can, in general, have a different stress. Therefore the resulting stress will be sum of the stresses from each branch. For the BB model there are only two networks, thus the resulting stress will be

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_A + \boldsymbol{\sigma}_B \quad (2.36)$$

where $\boldsymbol{\sigma}$ is the Cauchy (true) stress tensor.

The next steps will be to find the strain-stress relationships of networks A and B.

Network A

Here, a relationship between the elastic part of the deformation gradient $\mathbf{F}^e$ and the Cauchy stress tensor σ_A must be defined. For this purpose any hyperelastic model can be used.

In the present work the path followed by Bergstrom and Boyce [13, 11, 15, 16] will be taken, adopting the eight-chain network model proposed by Arruda and Boyce [4]. In this model, the Cauchy stress tensor is given by

$$\boldsymbol{\sigma}_A = \frac{\mu_A}{J\bar{\lambda}^*} \frac{\mathcal{L}^{-1}(\bar{\lambda}^*/\lambda_A^{\text{lock}})}{\mathcal{L}^{-1}(1/\lambda_A^{\text{lock}})} \text{dev}[\mathbf{B}^*] + \kappa(J-1)\mathbf{I} \quad (2.37)$$

where $\mu_A, \lambda_A^{\text{lock}}, \kappa$ are the hyperelastic parameters of network A (initial shear modulus, limiting chain stretch and bulk modulus, respectively) and $\mathbf{B}_A^* = \mathbf{B}_B^* = \mathbf{B}^*$. $\mathcal{L}^{-1}(x)$ is the inverse Langevin function and $\bar{\lambda}^*$ is the effective distortional chain stretch given by

$$\bar{\lambda}^* = \sqrt{\text{tr}[\mathbf{B}^*]/3} \quad (2.38)$$

The Langevin function is given by

$$\mathcal{L}(\beta) = \coth \beta - \frac{1}{\beta} \quad (2.39)$$

The following approximation to the inverse Langevin function is proposed by Bergstrom [12],

$$\mathcal{L}^{-1}(x) = \begin{cases} 1.31446 \tan(1.58986x) + 0.91209x, & \text{if } |x| < 0.84136 \\ 1/(\text{sign}(x) - x), & \text{if } 0.84136 \leq |x| < 1 \end{cases} \quad (2.40)$$

In this manuscript, the series expansion approximation (2.41) of the inverse Langevin will be used [45, 37].

$$\mathcal{L}^{-1}(x) \approx 3x + \frac{9}{5}x^3 + \frac{297}{175}x^5 + \frac{1539}{875}x^7 + \frac{126117}{67375}x^9 + \dots \quad (2.41)$$

Although (2.40) has been reported to perform slightly better than (2.41) [12], the latter will be used because is what ABAQUS[®] uses in its formulation [26, Section 4.6.1]. Therefore, it is sound to use the same expression both in the fitting of the material parameters and in the finite element simulations.

Network B

Within network B, the elastic and viscous elements are laid in series. Thus, they share the same stress but the deformation in each element might be different. The stress on the network B is calculated in a similar fashion as it was done for network A, using the hyperelastic 8-chain model by Arruda and Boyce, but in this case only the elastic part of the deformation gradient $\mathbf{F}_B^e$ should be used,

$$\boldsymbol{\sigma}_B = \frac{\mu_B}{J_B^e \bar{\lambda}_B^{e*}} \frac{\mathcal{L}^{-1}(\bar{\lambda}_B^{e*}/\lambda_B^{\text{lock}})}{\mathcal{L}^{-1}(1/\lambda_B^{\text{lock}})} \text{dev}[\mathbf{B}_B^{e*}] + \kappa[J_B^e - 1]\mathbf{I} \quad (2.42)$$

where μ_B and λ_B^{lock} are the parameters of the hyperelastic part of network B. It is common to express μ_B as a function of μ_A , introducing the parameter s

$$\mu_B = \mu_A s \quad (2.43)$$

And usually, it is assumed that the bulk modulus κ and the limiting chain stretch λ^{lock} is the same for both networks,

$$\lambda^{\text{lock}} = \lambda_A^{\text{lock}} = \lambda_B^{\text{lock}} \quad (2.44)$$

The time dependent part of network B is analyzed introducing its velocity gradient $\mathbf{L}_B$. Substituting (2.27) in (2.23),

$$\begin{aligned} \mathbf{L}_B &= \dot{\mathbf{F}}_B \cdot \mathbf{F}_B^{-1} \\ &= \overline{(\mathbf{F}_B^e \cdot \mathbf{F}_B^p)} \cdot (\mathbf{F}_B^e \cdot \mathbf{F}_B^p)^{-1} \\ &= \left(\mathbf{F}_B^e \cdot \dot{\mathbf{F}}_B^p + \dot{\mathbf{F}}_B^e \cdot \mathbf{F}_B^p \right) \cdot \mathbf{F}_B^{p-1} \cdot \mathbf{F}_B^{e-1} \\ &= \mathbf{F}_B^e \cdot \dot{\mathbf{F}}_B^p \cdot \mathbf{F}_B^{p-1} \cdot \mathbf{F}_B^{e-1} + \dot{\mathbf{F}}_B^e \cdot \mathbf{F}_B^p \cdot \mathbf{F}_B^{p-1} \cdot \mathbf{F}_B^{e-1} \\ &= \mathbf{F}_B^e \cdot \mathbf{L}_B^p \cdot \mathbf{F}_B^{e-1} + \dot{\mathbf{F}}_B^e \cdot \mathbf{I} \cdot \mathbf{F}_B^{e-1} \\ &= \underbrace{\mathbf{F}_B^e \cdot \mathbf{L}_B^p \cdot \mathbf{F}_B^{e-1}}_{\widetilde{\mathbf{L}}_B^p} + \mathbf{L}_B^e \\ &= \widetilde{\mathbf{L}}_B^p + \mathbf{L}_B^e \end{aligned} \quad (2.45)$$

Using (2.24) it is possible to decompose the tensor $\widetilde{\mathbf{L}}_B^p$ into the rate of deformation and spin tensors,

$$\widetilde{\mathbf{L}}_B^p = \widetilde{\mathbf{D}}_B^p + \widetilde{\mathbf{W}}_B^p \quad (2.46)$$

In the derivation of the BB model it is *assumed* that the spin tensor is equal to zero, $\widetilde{\mathbf{W}}_B^p = 0$. Under this assumption, it is possible to rewrite (2.45) as

$$\mathbf{L}_B = \widetilde{\mathbf{D}}_B^p + \mathbf{L}_B^e \quad (2.47)$$

The rate of deformation of the network B is assumed to follow the flow rule given by,

$$\widetilde{\mathbf{D}}_B^p = \dot{\gamma}_B \frac{\mathbf{S}_B}{\tau_B} \quad (2.48)$$

where $\dot{\gamma}_B$ is the effective viscous strain rate in network B, $\mathbf{S}_B$ is the deviatoric part of the Cauchy stress tensor $\boldsymbol{\sigma}_B$ acting in the network B and τ_B is an effective stress in network B defined by the Frobenius norm of $\mathbf{S}_B$.

$$\tau_B = \|\mathbf{S}_B\|_F = \sqrt{\text{tr}[\overline{\mathbf{S}}_B^T \cdot \mathbf{S}_B]} \quad (2.49)$$

where $\overline{\mathbf{S}}_B^T$ is the conjugate transpose of $\mathbf{S}_B$. Being $\mathbf{S}_B$ a real and symmetric second order tensor,

$$\tau_B = \sqrt{\text{tr}[\mathbf{S}_B \cdot \mathbf{S}_B]} = \sqrt{\mathbf{S}_B : \mathbf{S}_B} \quad (2.50)$$

The effective viscous strain rate in network B $\dot{\gamma}_B$ is driven by

$$\dot{\gamma}_B = \left(\frac{\dot{\gamma}_0}{\tau_{\text{base}}^m} \right) \left(\overline{\lambda}_B^p - 1 + E \right)^c \tau_B^m \quad (2.51)$$

where $\dot{\gamma}_0/\tau_{\text{base}}^m$, c and m are material parameters, E is a constant added to improve the numerical convergence and $\overline{\lambda}_B^p$ is the chain stretch given by

$$\overline{\lambda}_B^p = \sqrt{\frac{\text{tr}[\mathbf{F}_B^{pT} \cdot \mathbf{F}_B^p]}{3}} \quad (2.52)$$

The left hand side of the flow rule (2.48) can be replaced by $\mathbf{F}_B^e \cdot \dot{\mathbf{F}}_B^p \cdot \mathbf{F}_B^{p-1} \cdot \mathbf{F}_B^{e-1}$ from (2.45),

$$\mathbf{F}_B^e \cdot \dot{\mathbf{F}}_B^p \cdot \mathbf{F}_B^{p-1} \cdot \mathbf{F}_B^{e-1} = \dot{\gamma}_B \frac{\mathbf{S}_B}{\tau_B} \quad (2.53)$$

Solving for $\dot{\mathbf{F}}_B^p$,

$$\begin{aligned} \dot{\mathbf{F}}_B^p &= \frac{\dot{\gamma}_B}{\tau_B} \mathbf{F}_B^{e-1} \cdot \mathbf{S}_B \cdot \underbrace{\mathbf{F}_B^e \cdot \mathbf{F}_B^p}_{\mathbf{F}_B = \mathbf{F}_A = \mathbf{F}} \\ &= \frac{\dot{\gamma}_B}{\tau_B} \mathbf{F}_B^{e-1} \cdot \mathbf{S}_B \cdot \mathbf{F} \end{aligned} \quad (2.54)$$

2.7 Uniform Uniaxial Deformation and Incompressible Material

In this section the uniform uniaxial deformation will be developed. This type of deformation is very important because it corresponds to the case of tensile/compressive testing. Later

Figure 2.2: Uniaxial Deformation

on, it will be used for fitting material parameters to experimental data. Many rubber-like materials and elastomers show an incompressible or near incompressible behavior, allowing to introduce the *incompressibility* hypothesis. If the material it is assumed to incompressible, then

$$J = \det(\mathbf{F}) = 1 \quad (2.55)$$

Under this assumption the following relationship must be satisfied,

$$\lambda_1 \lambda_2 \lambda_3 = 1 \quad (2.56)$$

In uniaxial deformation, two of the stretches are equal between them [30]. Arbitrary picking $\lambda_2 = \lambda_3$ and replacing in (2.56), it is possible to solve for λ_2

$$\lambda_2 = \sqrt{\frac{1}{\lambda_1}} \quad (2.57)$$

Also, in uniaxial deformation there are no rotations. Therefore,

$$\mathbf{R} = \mathbf{I} \quad \Rightarrow \quad \mathbf{F} = \mathbf{v} \quad (2.58)$$

and the deformation gradient $\mathbf{F}$ becomes

$$\mathbf{F} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & 1/\sqrt{\lambda} & 0 \\ 0 & 0 & 1/\sqrt{\lambda} \end{bmatrix} \quad (2.59)$$

Replacing (2.59) into (2.29),

$$\mathbf{B}^* = \begin{bmatrix} \lambda^2 & 0 & 0 \\ 0 & 1/\lambda & 0 \\ 0 & 0 & 1/\lambda \end{bmatrix} \quad (2.60)$$

With the explicit form of $\mathbf{B}^*$ for the uniaxial and incompressible case, it is possible to write the equations for the effective distortional chain stretch $\bar{\lambda}^*$,

$$\bar{\lambda}^* = \sqrt{\text{tr}[\mathbf{B}^*]/3} = \sqrt{\frac{1}{3} \left(\lambda^2 + \frac{2}{\lambda} \right)} \quad (2.61)$$

and for the deviatoric part of the distortional deformation $\text{dev}[\mathbf{B}^*]$,

$$\begin{aligned} \text{dev}[\mathbf{B}^*] &= \mathbf{B}^* - \frac{1}{3} \text{tr}[\mathbf{B}^*] \mathbf{I} \\ \text{dev}[\mathbf{B}^*] &= \begin{bmatrix} \lambda^2 & 0 & 0 \\ 0 & 1/\lambda & 0 \\ 0 & 0 & 1/\lambda \end{bmatrix} - \frac{1}{3} \left(\lambda^2 + \frac{2}{\lambda} \right) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned} \quad (2.62)$$

The state of stress for the uniaxial deformation is given by,

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (2.63)$$

The relationship between the true stress and the nominal stress for the uniaxial case can be calculated using (2.33),

$$\mathbf{P} = J \begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1/\lambda & 0 & 0 \\ 0 & \sqrt{\lambda} & 0 \\ 0 & 0 & \sqrt{\lambda} \end{bmatrix} = \begin{bmatrix} \sigma_{11}/\lambda & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (2.64)$$

Therefore, the relationship between the nominal and true stress in the loading direction is

$$P_{11} = \frac{\sigma_{11}}{\lambda} \quad (2.65)$$

as seen in (2.6).

2.7.1 Application to the BB Model

Replacing (2.62) in (2.37) it is possible to express the Cauchy stress tensor of the network A as a function of the stretch in the direction of the uniaxial strain λ ,

$$\boldsymbol{\sigma}_A = \frac{\mu_A}{J\bar{\lambda}^*} \frac{\mathcal{L}^{-1}(\bar{\lambda}^*/\lambda_A^{\text{lock}})}{\mathcal{L}^{-1}(1/\lambda_A^{\text{lock}})} \left\{ \begin{bmatrix} \lambda^2 & 0 & 0 \\ 0 & 1/\lambda & 0 \\ 0 & 0 & 1/\lambda \end{bmatrix} - \frac{1}{3} \left(\lambda^2 + \frac{2}{\lambda} \right) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\} + \kappa(J-1) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2.66)$$

Although the factor $(J-1)$ in the last term of (2.66) is equal to zero because incompressibility was assumed ($J=1$), it will be useful to keep it.

Under the uniaxial deformation assumption and for an isotropic material, the only non-zero term of the stress tensor is σ_{A11} . Writing explicitly the stress component σ_{A22} ,

$$\sigma_{A22} = \frac{\mu_A}{J\bar{\lambda}^*} \frac{\mathcal{L}^{-1}(\bar{\lambda}^*/\lambda_A^{\text{lock}})}{\mathcal{L}^{-1}(1/\lambda_A^{\text{lock}})} \left\{ \frac{1}{3} \left(\frac{1}{\lambda} - \lambda^2 \right) \right\} + \kappa(J-1)1 \quad (2.67)$$

and equating it to zero $\sigma_{A22} = 0$, it is possible to solve for $\kappa(J-1)$,

$$\kappa(J-1) = \frac{\mu_A}{J\bar{\lambda}^*} \frac{\mathcal{L}^{-1}(\bar{\lambda}^*/\lambda_A^{\text{lock}})}{\mathcal{L}^{-1}(1/\lambda_A^{\text{lock}})} \left\{ \frac{1}{3} \left(\lambda^2 - \frac{1}{\lambda} \right) \right\} \quad (2.68)$$

The result obtained would be the same if instead of choosing σ_{A22} , σ_{A33} was used.

Lastly, replacing (2.68) in (2.66) the only non-zero stress component σ_{A11} is expressed in terms of the stretch in the loading direction [35].

$$\sigma_{A11} = \frac{\mu_A}{\bar{\lambda}^*} \frac{\mathcal{L}^{-1}(\bar{\lambda}^*/\lambda_A^{\text{lock}})}{\mathcal{L}^{-1}(1/\lambda_A^{\text{lock}})} \left(\lambda^2 - \frac{1}{\lambda} \right) \quad (2.69)$$

Similarly, for the stress component σ_{B11} in network B,

$$\sigma_{B11} = \frac{\mu_B}{\bar{\lambda}_B^{e*}} \frac{\mathcal{L}^{-1}(\bar{\lambda}_B^{e*}/\lambda_B^{\text{lock}})}{\mathcal{L}^{-1}(1/\lambda_B^{\text{lock}})} \left(\lambda_B^{e2} - \frac{1}{\lambda_B^e} \right) \quad (2.70)$$

where the effective distortional chain stretch in the elastic part of network B is given by

$$\bar{\lambda}_B^{e*} = \sqrt{\frac{1}{3} \left((\lambda_B^{e*})^2 + \frac{2}{\lambda_B^{e*}} \right)} \quad (2.71)$$

For the uniaxial state,

$$\mathbf{S}_B = \text{dev}[\boldsymbol{\sigma}_B] = \frac{\sigma_{B11}}{3} \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad (2.72)$$

$$\tau_B = \sqrt{\text{tr}[\mathbf{S}_B \cdot \mathbf{S}_B]} = |\sigma_{B11}| \sqrt{\left(\frac{2}{3}\right)^2 + 2\left(\frac{-1}{3}\right)^2} = \sqrt{\frac{2}{3}} |\sigma_{B11}| \quad (2.73)$$

$$\mathbf{F}_B^{e-1} = \begin{bmatrix} 1/\lambda_B^e & 0 & 0 \\ 0 & \sqrt{\lambda_B^e} & 0 \\ 0 & 0 & \sqrt{\lambda_B^e} \end{bmatrix} \quad (2.74)$$

Substituting (2.72), (2.73) and (2.74) in the flow rule (2.54),

$$\dot{\mathbf{F}}_B^p = \frac{\dot{\gamma}_B}{\sqrt{\frac{2}{3}} |\sigma_{B11}|} \begin{bmatrix} 1/\lambda_B^e & 0 & 0 \\ 0 & \sqrt{\lambda_B^e} & 0 \\ 0 & 0 & \sqrt{\lambda_B^e} \end{bmatrix} \cdot \left(\frac{\sigma_{B11}}{3} \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \right) \cdot \begin{bmatrix} \lambda & 0 & 0 \\ 0 & 1/\sqrt{\lambda} & 0 \\ 0 & 0 & 1/\sqrt{\lambda} \end{bmatrix} \quad (2.75)$$

Writing the component in the loading direction $\dot{\mathbf{F}}_{B11}^p$, and replacing $\dot{\gamma}_B$ by (2.51),

$$\dot{\lambda}_B^p = \left(\frac{\dot{\gamma}_0}{\tau_{\text{base}}^m} \right) \left(\sqrt{\frac{1}{3} \left((\lambda_B^p)^2 + \frac{2}{\lambda_B^p} \right)} - 1 + E \right)^c \left(\sqrt{\frac{2}{3}} |\sigma_{B11}| \right)^m \left[\sqrt{\frac{2}{3}} \text{sign}(\sigma_{B11}) \right] \lambda_B^p \quad (2.76)$$

The above equation is a first-order ordinary differential equation (ODE) in λ_B^p . Therefore, only one initial condition is needed to solve the ODE.

Chapter 3

BB Model Parameters Fit

The parameters of the BB Model can be fit to uniaxial experimental data using a MATLAB[®] program written for this purpose. The program is used to fit available uniaxial experimental data from Reference [10] for Nitrile Rubber (NR), Acrylate-Butadiene Rubber (ABR), Silicone Rubber (SR) and Viton.

The theory behind the BB Model and the particular case of uniaxial deformation have been developed in Chapter 2.

3.1 Program Structure

This program is divided in three files, `fit10.m`, `bb10.m` and `bb10_error.m`. In addition to these files, the non built-in minimization function `FMINSEARCHCON.m` [27] is used to fit the material model to the experimental data and it should be in the same directory or in the path.

The variables `Y_data`, `X_data`, `DATA_SETS`, `ROW_INDEX` and `LENGTH_DATA_SETS` are defined as *global* and they will be used to pass information between the three parts of the program.

The code listing for this program is included in the Appendix A. The code is thoroughly commented.

3.1.1 Main Program

Experimental Data

The main file of the program is `fit10.m`. In this MATLAB[®] file the experimental data is loaded. The program allows to input the results of more that one experiment. Each set of data is stored in a cell inside the variable `M`. The variables `DATA_SETS`, `ROW_INDEX` and `LENGTH_DATA_SETS` are used to keep track of how many experiments will be used and how many data points each experiment has.

The experimental data needed to do the uniaxial fit for the BB model is composed of time, strain and stress. The stresses and strains that are input to this program *must* be *true* values, not *nominal* values. If the experimental data is available in the latter fashion, it should be converted to *true* values using (2.5) and (2.6).

The true stress σ will be considered to be a function of time t and the true strain ϵ history. Therefore, $\sigma = \sigma(t, \epsilon(t))$.

The BB model has the ability to capture the rate dependent behavior, characteristic of many elastomers. But in order to do a successful fit of the parameters that define this model, it is imperative that the experimental data is sampled, at least, for two different strain rates.

Initial Values and Bounds

The initial values for the parameters must be defined in this section. These initial values will be used as a starting point by the solver, and they are stored in the array `beta0`. The function `FMINSEARCHCON` allows the use of bound constraints for the parameters to fit. This feature is used in this problem because the value of some of the parameters of the BB model are restricted to certain ranges. For example, the initial shear modulus of network A, μ_A must be positive. Also the creep strain exponent c must be in the $[-1, 0]$ interval. The lower and upper bounds for the parameters are stored in the `LB` and `UB` arrays, respectively. For unbounded values, the bound is set as ∞ using the `inf` command in MATLAB®.

Set the minimization algorithm options

The options for the termination of `FMINSEARCHCON` must be specified here.

TolFun Tolerance on the function to minimize. If the difference between two consecutive iterations of the value of the function to minimize is less than `TolFun` the iteration process will be terminated.

TolX Tolerance on the parameters. Similar to `TolFun`, but the difference is evaluated on the value of the parameters that are being fitted.

MaxFunEvals Maximum number of evaluations of the function to minimize.

MaxIter Maximum number of iterations to be used during the minimization.

Usually the set of points to be fit is very large. Therefore, the computational time required to do this kind of non-linear minimization is considerable. It was found that the most effective parameter to control the time spent on the optimization is `MaxIter`. The improvement on the goodness of fit parameter R^2 , changes marginally when `MaxIter` is changed from 10 to 100. The results that will be presented later in this Chapter have been calculated using `MaxIter` = 10.

Call the minimization algorithm

Once the initial values and bounds of the parameters and the options for the minimization algorithm have been set, the program actually calls to `FMINSEARCHCON`.

A function that quantifies the difference between the *experimental* values and the *predicted* values must be defined. The error function is the function that will be minimized by `FMINSEARCHCON`. This function will be described in Section 3.1.3.

Output

Once the minimization algorithm has finished, either because it has converged to the defined criteria or because it has exceeded the maximum number of iterations, the main program will show the estimated parameters and it will calculate the goodness of fit R^2 .

3.1.2 Constitutive Equation Function

In this Section the constitutive equation for the uniaxial loading case of the Bergstrom-Boyce material model will be numerically implemented. Given a strain-time curve, the constitutive equation will calculate the stress. In order to do this, the ordinary differential equation (2.76) must be solved. In this program the solution of the ODE is done using MATLAB®'s built-in `ode23` solver.

At the beginning of each experimental data set, the strain and stress are known and equal to zero. Therefore, the total stretch is equal to the elastic and viscous stretches, $\lambda = \lambda_B^e = \lambda_B^p = 1$. This will be the initial condition for the ODE solver for the first time step of every experimental data set. The ODE solver will integrate the flow rule (2.76) from the current time step to the following. Once it solves for viscous stretch λ_B^p for the following time step, the elastic stretch λ_B^e is calculated using the multiplicative decomposition given by (2.35), particularized for the uniaxial case ($\lambda = \lambda_B^e \lambda_B^p$).

Having the elastic stretch for network B λ_B^e and the total stretch λ , it is possible to find the stress in both networks employing (2.69) and (2.70).

At this point, all the ingredients required to start the next iteration are available. The viscous stretch λ_B^p calculated at this step will become the initial condition to solve the ODE for the following step.

3.1.3 Error Function

The error function uses the constitutive equation function developed in the previous section and calculates the difference between the stress calculated by the constitutive equation and the experimental stress. The smaller the difference, the better the fit is. There are different functions that can be used to measure the error. In the present work the Normalized Median Absolute Difference will be used.

$$\text{ERROR} = 100 \frac{\sum_{i=1}^n |\sigma_i^{\text{exp}} - \sigma_i^{\text{fit}}|}{\sum_{i=1}^n |\sigma_i^{\text{exp}}|} \quad (3.1)$$

where n is the number of experimental data points, σ^{exp} is vector containing all the measured stresses and σ^{fit} is the vector containing the stresses calculated using the constitutive law and the current value of the parameters. The minimization process is driven by reducing the error given by (3.1).

Figure 3.1: Acrylate-Butadiene Rubber (ABR)

3.2 Examples

In this Section the parameters for the BB model are calculated for four different materials. The experimental data was obtained from the Dr. Bergstrom's website [10]. The calculated parameters and the coefficient of determination R^2 are included in Table 3.1.

For the four cases the maximum number of iterations (`MaxIter`) was set to 10. For each material two or three experimental data sets were available, except for Viton. For this material only two data sets were available, but one of them could not be used because some inconsistencies in the data. For the other three cases, the coefficients of determination are higher than 0.95.

The stress-strain plots of the experimental data and the predicted values are included in Figures 3.1 - 3.4.

Material	R^2	μ_A	λ_{lock}	s	$\dot{\gamma}_0/\tau_{base}^m$	c	m	MaxIter	Data Sets
ABR	0.971	0.910	2.073	2.251	0.686	-0.471	2.406	10	Compression1 and 2
NR	0.976	1.130	6.382	2.300	0.540	-0.990	4.500	10	Nitrile01, 02 and 03
Viton	0.898	1.633	1.282	1.736	0.398	-0.909	9.852	10	Viton_slow ¹
SR	0.952	0.830	5.224	2.636	0.527	-0.624	4.393	10	Silicone01, 02 and 03

Table 3.1: Parameters fit for the BB material model

^aThe second data set available, `Viton_fast.txt` could not be used in the fit because, for some unknown reason, there were several duplicated times with different values of stress and strain.

Figure 3.2: Nitrile Rubber

Figure 3.3: Viton

Figure 3.4: Silicone Rubber

Chapter 4

Finite Element Analysis

4.1 Introduction

In this Chapter four different types of models will be analyzed. In this Section the parameters that are common to all the models will be introduced.

4.1.1 Units

The base units used in the models included in this Chapter are newtons (N) and millimeters (mm). Thus, the stresses and modulus are measured in mega-pascals (MPa).

4.1.2 Geometry

The dimensions of each model will be described in the respective sections. It will be considered that the *width* of the RVE is along the x direction, the *thickness* is along the y direction and that the *depth* runs along the z direction. The x, y, z coordinate system and a schematic drawing of the RVE are included in Figure 4.10.

4.1.3 Material Models and Properties

In the development of the finite element models, three materials are defined. One for the actuator (ellipse in two-dimensional models and ellipsoid in three-dimensional models), other for the base film where the actuator is embedded and lastly, one for the top film. The top film must be electrically conductive, in order to apply the voltage potential to actuate the ellipses/ellipsoids¹.

Actuator

The actuator is modeled using a linear elastic material model, with Young's modulus $E = 3000$ MPa and Poisson ratio $\nu = 0.3$. The Young's modulus used is based on an upper value

¹The words ellipsoid and ellipse will be used interchangeably, although strictly speaking ellipsoid should be used only for three-dimensional geometries while ellipse for two-dimensional geometries.

estimation [20]. The effect of changing the actuator Young's modulus will be studied in Section 4.3.6.

Base Film

The base film is modeled using the Bergstrom-Boyce Model (BB Model) [13, 14]. The equilibrium (relaxed) response follows the hyperelastic model proposed by Arruda and Boyce [4] with material parameters fit to DuPont's Hytrel 4056 (TPE) elastomer. The equilibrium (relaxed) properties were obtained from CAMPUS Database [19] and are listed in Table 4.1.

The base film material will be *assumed* to be incompressible, a reasonable hypothesis for most elastomers. Under the assumption of incompressibility many simplifications can be done in the kinematic equations but some special considerations must be taken when applying the finite element method.

It is important to bear in mind that this material was chosen as a starting point in order to build the models. A thorough analysis should be made to assess which material has the optimum properties (mechanical, optical, processing, etc.) for a particular application. Additionally, it should be noted that the analyses have been done at a single arbitrary temperature (23 °C). Shall the intended application will be used at a different temperature (or at variable temperatures), this should be taken into account in the model. This can be done using the time-temperature superposition [8].

The hyperelastic model can be changed if other model fits better the available data. For example, some trials have been done using the Polynomial Strain Energy Potential (degree 2). But as Hytrel 4056 is being used just as an example and it is not the material that will be used in the application, there is no point of choosing a particular model at this stage.

The parameters of the Arruda-Boyce hyperelastic model that fit the data in Table 4.1 were calculated using ABAQUS® and are $\mu_A = 4.68$ MPa, $\lambda_A^{\text{lock}} = 5311.53$ and $d = 0.00/\text{MPa}$, where μ_A is the shear modulus of network A, λ_A^{lock} is the limiting chain stretch and d is the compressibility parameter.

There are four remaining parameters of the BB Model that characterize the time dependent and hysteretic behavior of the material. Because the data required to fit these parameters is not available from the literature, these parameters were taken from the literature as usual values for polymers. The *adopted* values are $s = 2.0$, $\dot{\gamma}_0/\tau_{\text{base}}^m = 0.5/(s \text{ MPa}^m)$, $m = 4.5$ and $c = -1.0$, where s is the stress scaling factor (2.43), $\dot{\gamma}_0/\tau_{\text{base}}^m$ is the creep parameter (2.51), m is the effective stress exponent (2.51) and c is the creep strain exponent (2.51).

The procedure developed in Chapter 3 can be used to fit *all* the required parameters of the BB model from experimental data for *any* desired material.

Top Film

The top film is modeled using a linear elastic material model, with Young's modulus $E = 70\,000$ MPa and Poisson ratio $\nu = 0.3$. The Young's modulus used is based on an upper value estimation [20]. The effect of changing the top film modulus will be addressed in Section 4.3.6.

ϵ^{eng} []	σ^{eng} [MPa]		ϵ^{eng} []	σ^{eng} [MPa]
0.047	2.83		1.050	8.69
0.083	4.29		1.080	8.73
0.127	5.50		1.130	8.79
0.163	6.20		1.160	8.84
0.207	6.83		1.210	8.91
0.243	7.21		1.240	8.96
0.286	7.55		1.290	9.03
0.323	7.74		1.320	9.09
0.366	7.92		1.370	9.17
0.402	8.02		1.400	9.22
0.446	8.09		1.450	9.30
0.482	8.15		1.480	9.37
0.525	8.18		1.530	9.45
0.562	8.20		1.560	9.52
0.605	8.23		1.610	9.60
0.641	8.25		1.640	9.68
0.685	8.29		1.680	9.78
0.721	8.32		1.720	9.86
0.765	8.36		1.760	10.00
0.801	8.39		1.800	10.10
0.844	8.44		1.840	10.20
0.881	8.48		1.880	10.20
0.924	8.53		1.920	10.40
0.960	8.57		1.960	10.50
1.000	8.62		2.000	10.60

Table 4.1: Stress-Strain Curve for Hytrel 4056 (TPE) at 23 °C [19]

Figure 4.1: Stress-Strain Curves for Hytrel 4056 (TPE) at 23 °C

4.1.4 Hybrid Elements

When a material is incompressible or near incompressible, the traditional displacement based finite element formulations (where all the unknowns of the problems are kinematic variables) fail because of a numerical singularity (volumetric locking). The cause of this problem is that very small changes in the displacements (associated with dilatational deformation) cause very large changes in the hydrostatic pressure. The solution is to treat the pressure as an independent unknown. The basis for this approach is given by the *mixed* or *hybrid* finite element formulations. In the *Hellinger-Reissner* mixed method, the displacements and stresses are treated as unknowns. The more general method by *Hu-Washizu*, also treats the strains as basic variables, in addition to the displacements and stresses [47, 18].

4.1.5 Periodic Boundary Conditions

All the models developed in this Chapter, represent a portion of the whole structure. It is *assumed* that this portion or substructure repeats itself periodically. If the geometry, material properties, boundary conditions and load are periodic, the solution has to be periodic too. When modeling the substructure it is necessary to recover this periodic behavior on the boundaries of the model. This is achieved using periodic boundary conditions [7, §2.3.4 and §6.2].

4.1.6 Geometric Non-Linearity

In the models analyzed in this Chapter the deformations are *finite*. Therefore, the *current* or *deformed* configurations are no longer similar to the *reference* configuration, as in the infinitesimal deformation theory [49, §5]. This distinction makes the problem non-linear and the superposition principle is no longer applicable.

4.1.7 Post-Processing

Dissipated Energy

Although ABAQUS® has the option to calculate the creep dissipated energy in the model (History Output ALLCD), the results are not accurate. Therefore, a PYTHON script is used to do this calculation. A simple example showing this flaw is included in Appendix B. For this analyses, the dissipated energy at one point is defined as the difference between the recoverable elastic energy and the integral between the stress-strain curve. Inertial forces, and therefore kinetic energy, are neglected. For the whole model, the problem reduces to do this integration at each integration point and then to integrate over the volume of model. If the resulting dissipation energy is divided by the duration of the analysis, then a *dissipation power* can be calculated. Lastly, for application purposes, it will be useful to divide the dissipation power by the width and depth of the RVE, which will define the *surface dissipation power*.

Roughness

The *roughness* will be defined as the difference between the maximum and minimum vertical displacement of the top surface of the RVE. For time dependent loadings, a difference can be calculated for each time frame in the solution process. In this case, the roughness will be the maximum of these differences over all the time frames.

4.2 Models to Compare with the Laboratory Experiments

At this time, December 2010, the research group that has developed the actuated polymer films is researching how to apply the electrically conductive thin coating (top film) to the base film. In the meantime, the electric potential is being applied by placing the film sample between opposing pieces of indium-tin oxide (ITO) glass, materializing the electrodes for the application of a electric potential [42].

4.2.1 Geometry

The models will be two-dimensional. Plane strain will be assumed. The RVE will include two ellipses and the top film will not be included. Table 4.2 has the basic dimensions and Figure 4.2 shows a schematic representation.

Figure 4.2: Two-Dimensional, Two-Actuators RVE Geometry

Component	Width [μm]	Thickness [μm]	Depth [μm]
Ellipse	60	10	∞
Base Film	240	30	∞

Table 4.2: Two Actuators RVE Two-Dimensional Geometry

4.2.2 Material Properties

The material properties of the actuator and base film are the ones in Section 4.1.3. For these analyses the viscous behavior is not included. Therefore the base film is modeled using only the hyperelasticity model, without hysteresis.

4.2.3 Load

These analyses will be considered as static, this implies that the potential differential should be applied and kept for the duration of the experiment, giving enough time to relax the viscous part of the stress.

The top and bottom surfaces of each ellipse are loaded, respectively, in opposite directions with a uniform distributed load applied in the x direction (parallel to the ellipse major axis) to generate the torque. The magnitude of the load does not change if the surfaces change their size during the deformation. It could be argued that the magnitude of the load should change when the actuator rotates with respect to the electric field, but the rotations are so small that $\cos \alpha = 0.9998$, where α is the maximum rotation angle. The angle was calculated from the model “3D-2” listed in Table 4.4. Also, it is assumed that the direction does not change when the ellipses deform (no *following forces*).

The magnitude of the distributed traction that generates the torque is *assumed* to be 10 MPa. This value was adopted because the resultant displacements from the models are in good agreement (same order of magnitude) with experimental results done in an alike actuated polymer film [42]. Furthermore, it is *assumed* that the actuators generate torque by a volume and not by an area effect. Cairns et al. propose an equation where the generated torque due to the applied electric field is proportional to the volume V of the actuator, both for cylindrical [21, Equation 1] and spherical [22, Equation 1] geometries. In the latter reference, the torque per unit volume Γ is explicitly defined.

The relationship between the loading for the two- and three-dimensional cases is discussed in Appendix C. In two models the polarity of both ellipses will be the same, while opposite in two other models.

For the finite element models where the top glass is included, a single load (surface traction) magnitude will be analyzed. The effect of changing the magnitude of the surface traction will be studied in Section 4.3.6, where in addition to 10 MPa, loads of 5 MPa and 15 MPa will be analyzed.

4.2.4 Contact

In these models the effect of the glass resting on the top surface will be included. The top glass will be idealized as a infinitely rigid surface. The kinematics of any rigid body in a two-dimensional space can be described with the three degrees of freedom of any point. This reference point on the analytic rigid surface (top glass) is set at its geometric center. Its mass will be taken as 1.25 g/cm^2 , excerpting a pressure of $1.23 \times 10^{-4} \text{ MPa}$. The resulting force is applied at the reference point.

A surface to surface, finite sliding algorithm was used in these models. The model was also analyzed using small sliding and both results are in good agreement. The interface between the APF and the glass is *assumed* to be frictionless. In the surface to surface formulation, some small penetration may occur, but large penetrations of nodes from the master surface into the slave surface are unlikely to occur.

When analyzing contact problems it is important to pay special attention to the following points,

Master-Slave Surfaces The master surface must always be the one that is made of a stiffer material. If both materials have similar stiffness, the surface which has a coarser mesh should be defined as the master surface to minimize the penetration into the slave surface. Following the first criteria, in these examples the analytical rigid surface *must* be the master.

Normals The normals within each surface should be consistent. The normals of the master surface must point towards the slave surface, and vice versa.

Contact Initiation If the contact between two parts is generated by a force and not by a displacement, it is advisable to create a load step before applying the force where the contact will be generated by a prescribed displacement.

Solver Type When using the finite sliding contact algorithm it is strongly recommended to use a general un-symmetric solver.

4.2.5 Mesh

The mesh is composed of 15713 linear, quadrilateral, continuum plane strain elements (See Figure 4.3). The base film is meshed with reduced integration, hybrid elements (CP4RH). Hybrid elements must be used when simulating incompressible materials. ABAQUS® Theory Manual [26, §3.2.4] also suggests that reduced integration elements should be used when

modeling incompressible materials at finite strain. The actuator is meshed using reduced integration (CP4R) elements, because it is a cost-effective method if the solution is expected to be smooth or if the stresses and/or strains in the region are not of concern.

4.2.6 Partially Constrained Top Glass Model

In this Section two different models will be analyzed, none of them having the conductive top film. The purpose of these models is to develop the numerical tools required to, eventually, do a comparison with the experimental results. Each model will include two actuators in the RVE. The difference between the two models is the polarization of both actuators.

Here, the term polarization is used to describe in which direction the actuator will rotate when trying to align with the electric field. The polarization will be considered to be positive when the actuator rotates counter clock-wise from the observer and negative when it does it in a clock-wise fashion. In a sample of APF the polarization of each actuator is random. Therefore it is important to analyze the response of the RVE that includes two actuators with the same or opposite polarity.

The PYTHON script used to create and analyze this model in ABAQUS® is listed in the Appendix E.1.

Boundary Conditions

Base Film The base film is modeled as glued to the bottom glass (not included in the model). Therefore both translational degrees of freedom are set to zero on the bottom surface of the base film. Periodic boundary conditions are applied to the vertical edges of the base film.

Top Glass It is *assumed* that a glass would be lifted evenly by the combined effect of all the actuators in a sample relatively large. Therefore the rotation of the glass is set to zero. Also the displacement in the horizontal direction is set to zero, leaving only the vertical displacement as an active degree of freedom.

Results

As it can be seen in Figure 4.4, the difference between the maximum and minimum displacements at the top surface, both for the same and opposite polarity cases, is about the same. For the case in which both actuators have the same polarity, the roughness is $1.89\text{ }\mu\text{m}$ and for the case in which the polarity of each actuator is different, $1.85\text{ }\mu\text{m}$. Thus, it could be said that the roughness in both cases is very similar. But in the opposite polarity case, the valley in the center of the RVE is noticeably narrower and more shallow.

Figures 4.5 and 4.6 show the contour plots of the vertical displacement for the same and opposite polarities, respectively. In these plots it can also be seen the analytic rigid surface that models the top glass, in light orange color.

If in the application of the APF, the actuators are close as in these examples and the polarity of each ellipse is random, the overall response of the film would be the average of

Figure 4.3: Model 4.2 Mesh

Figure 4.4: Vertical displacement of the top surface of the base film

the two cases treated in this section. If the actuators are far from each other, there will be no neighboring effect and the problem should be modeled as *diluted*².

4.2.7 Totally Constrained Top Glass Model

The objective by fixing the top glass is to quantify the vertical reaction that the actuated polymer film generates against the top glass.

Boundary Conditions

Base Film The boundary conditions on the base film are the same as in the previous model, where both translational degrees of freedom are set to zero on the bottom surface of the base film. Also, periodic boundary conditions are applied to the vertical edges of the base film.

Top Glass In these models, the top glass is assumed to have its three degrees of freedom forced to be null. Here, the vertical displacement is *assumed* to be kept in place by means of a testing device set to displacement control.

Results

For these models, where the top glass is fully constrained, the reaction force of the rigid analytic surface is very similar regardless the polarization of the two actuators. Therefore,

²In a *diluted* model, the separation between the actuators is large enough to consider that the interaction with other actuators is negligible.

Figure 4.5: Vertical displacement contour plot, actuators with same polarity

Figure 4.6: Vertical displacement contour plot, actuators with opposite polarity

Figure 4.7: Vertical reaction of the top glass as a function of the base film compressibility

only the results for same polarization case will be discussed.

The restriction on the displacement of the top glass, in addition to the glued boundary condition on the bottom of the film and the periodic boundary conditions on the sides of the RVE, highly confines the APF in the model. Under this condition the numerical solution of the problem is highly dependent on the bulk modulus of the material κ [16, Appendix A]. The bulk modulus of the material κ is related to the compressibility parameter d by

$$d = \frac{2}{\kappa} \quad (4.1)$$

In Figure 4.7, the vertical reaction force at the top glass reference point is plotted against the compressibility parameter d . It can be seen that the reaction force is highly dependent on the compressibility parameter. Therefore, the bulk modulus of the base film should be measured accurately in order to compare the reaction force obtained numerically with the experimental measurement.

4.3 Models of the Application

In this Section, models of the intended application are analyzed. For some possible applications of the actuated polymer films, like tunable roughness coatings, the electric field is supposed to be applied through an electrically conductive thin film acting as a electrode on the top surface and the substrate where the APF is glued to. Therefore, the top film must be included in the models that will analyze the response of the APF for applications like

aerodynamic flow control or dust mitigation systems.

It is of special concern to develop the framework that will allow to analyze the response of the actuated polymer films and to calculate the required power to actuate the films. The latter requires to include a hysteretic material model.

In Sections 4.3.3 and 4.3.4 diluted two- and three-dimensional models are developed, respectively. These models have only one actuator embedded in the base film, which is modeled with a width of five times the actuator's length. This is purposely done to analyze the case where the actuators are far from each other (diluted case). The results from these models will be used later, in Section 4.3.5, to correlate the roughness and dissipation between the two- and three-dimensional models, as a function of the volume fraction V_f .

Lastly, in Section 4.3.6, a series of parametric analyses will be done using a two-dimensional model. The top surface response (roughness) and dissipation power will be analyzed varying the geometry and frequency of excitation. It will be *assumed* that the three-dimensional behavior can be extrapolated from the two-dimensional results by employing the outcome of Section 4.3.5.

4.3.1 Material Properties

The material properties of the actuator, top film and base film are the ones in Section 4.1.3.

4.3.2 Load

For the application models the load definition is similar to Section 4.2.3, but in this case the uniform distributed load has a sinusoidal function of time,

$$F_x(t) = A \sin(B t) \quad (4.2)$$

where A is the amplitude and $2\pi B$ is the period of the excitation.

The magnitude A of the surface traction is *assumed* to be 10 MPa, for all but a few cases. In Section 4.3.6, magnitudes of 5 MPa and 15 MPa will be also considered.

The solver is set to a time increment size equal or smaller than $1/120$ of the excitation period. The duration of the analysis matches the excitation period. Therefore, at least 120 steps are used to solve the problem. Such a fine time discretization is done because the results from each time step will be used to integrate the dissipation of the model during the post-processing.

4.3.3 Two-Dimensional Diluted Model

Geometry

This model is two-dimensional. Plane strain is assumed. The RVE includes one actuator and the top film. Table 4.3 has the basic dimensions and Figure 4.8 shows a schematic representation of the RVE.

Figure 4.8: Two-Dimensional, One-Actuator Application RVE Geometry

Component	Width [μm]	Thickness [μm]	Depth [μm]
Ellipse	60	10	∞
Base Film	300	30	∞
Top Film	300	0.03	∞

Table 4.3: Two-Dimensional, One Actuator Application RVE Geometry

Load

The frequency of the load applied by the actuator is 200 Hz. For some of the applications of the APF, the required excitation frequency is expected to be in the 30 Hz to 200 Hz range [32]. Furthermore, the required frequency is inversely proportional to the roughness. Because the calculated roughness from the finite element analysis results is smaller than the optimum, the top of the frequency range is chosen. In Section 4.3.6, a parametric analysis varying the magnitude and frequency of the excitation will be done. The results are shown in Figures 4.15, 4.16, 4.17 and 4.18.

Boundary Conditions

The base film is modeled as glued to the bottom glass (not included in the model). Therefore, both translational degrees of freedom are set to zero on the bottom surface of the base film. Periodic boundary conditions are applied to the vertical edges of the RVE.

Mesh

The elements used are the same type as in the model developed in Section 4.2. For this analysis, only a few (314 out of 13621) constant strain triangular elements are used to improve mesh transitions. The triangular elements used are CPE3 and CPE3H, the former in the actuator and the latter in the base film.

Results

The dissipation power per unit surface and roughness for this analysis are included in Table 4.5.

Figure 4.9: Symmetry on the RVE

Figure 4.10: Three-Dimensional, One-Actuator Application RVE Geometry

4.3.4 Three-Dimensional Diluted Model

These models are three-dimensional. The RVE includes one actuator and the top film. As it can be seen in Figure 4.9, the RVE presents symmetry planes. Symmetry is exploited, so only one half of the RVE is modeled. Table 4.4 has the basic dimensions and Figure 4.10 shows a schematic representation of the RVE.

The PYTHON script used to analyze these models is listed in the Appendix E.3.

Geometry

Four different volume fractions (0.70 %, 1.39 %, 1.92 % and 3.32 %) will be analyzed, by changing the depth of the RVE (5.25 μm , 9.81 μm , 12.50 μm and 25.00 μm). With the current process used to create the actuated polymer films there is a minimum theoretical separation

Component	Width [μm]	Thickness [μm]	Depth [μm]
Ellipsoid	60	10	5
Base Film	300	30	Variable: 5.25 - 25
Top Film	300	0.03	Variable: 5.25 - 25

Table 4.4: Three-Dimensional, One Actuator Application RVE Geometry

between actuators in the direction of the depth of the RVE (See Appendix D), given by

$$\text{min DEPTH} = a\sqrt[3]{\text{AR}} \quad (4.3)$$

where a is the minor radii of the actuator and AR is the aspect ratio (4.5). For these examples, $a = 5\mu\text{m}$ and $\text{AR} = 6$. Therefore, the minimum theoretical depth is $9.1\mu\text{m}$. Three of the four cases span between the minimum depth and the equivalent to a separation of 5 minor diameters between the ellipsoids centers. The fourth case ($\text{DEPTH} = 5.25\mu\text{m}$) is included to have a solution close to what it would be the plane strain condition, used in Section 4.3.3.

Load

For the three-dimensional models, only one excitation frequency will be analyzed. The load applied on the actuator has, as it was pointed out in Section 4.3.2, a frequency of 200 Hz. A parametric analysis varying the excitation magnitude and frequency will be done in a two-dimensional model in Section 4.3.6. The results from the two-dimensional model can be correlated with the three-dimensional case using the ratios shown in Figure 4.14, as a first approximation.

Boundary Conditions

The base film is modeled as glued to the substrate where the APF is applied. Therefore, the three translational degrees of freedom are set to zero on the bottom surface of the base film. The two side surfaces (the surfaces that have the x axis a normal, have periodic boundary conditions enforced between them. Lastly, the front and back surfaces (z axis as normal), have symmetry boundary conditions ($\text{UZ}=\text{ROTX}=\text{ROTY}=0$).

Mesh

The mesh is composed of linear, hexahedral, continuum elements. The number of elements varies for each volume fraction V_f analyzed (See Table 4.5). All the elements of the mesh use a reduced integration scheme. In the top film this is done to avoid shear locking, using (C3D8R) elements. The base film is meshed with reduced integration, hybrid elements (C3D8RH). Hybrid elements must be used when simulating incompressible materials. ABAQUS® Theory Manual [26, §3.2.4] also suggests that reduced integration elements should be used when modeling incompressible materials at finite strain. The actuator is meshed using reduced integration (C3D8R) elements, because it is an cost-effective method if the solution is expected to be smooth or if the stresses and/or strains in the region are not of concern.

The RVE is partitioned into twelve cells to allow for structured meshing. Along the width of the RVE, the element size tapers towards the sides of the RVE, where the actuator's effect is negligible. The elements of the top film have a very high aspect ratio, something that is not desirable, but they do not cause an error flag during the analysis. The cause, is that the thickness of the top film is extremely small compared to the two other dimensions. Better shaped elements could be obtained by making a finer mesh, which should have a transition within the base film, significantly increasing the computational cost of the analysis.

Figure 4.11: Three-dimensional model mesh

Model	Volume Fraction	RVE Depth	Dissipation Power	Surf. Dissip. Power	Roughness	Elements
	[%]	[μm]	[mW]	[mW/mm ²]	[μm]	[]
2D-1	5.23	∞	0.012638 ¹	0.0421	0.9440	13621
3D-1	3.32	5.25	0.0000496	0.0314	0.8344	12482
3D-2	1.92	9.1	0.0000310	0.0114	0.5146	16630
3D-3	1.39	12.5	0.0000294	0.0078	0.4512	25970
3D-4	0.70	25	0.0000264	0.0035	0.4487	33976

Table 4.5: Results for the two- and three-dimensional diluted models

Figure 4.3 shows the mesh for the deepest (25 μm) model.

Results

The computational effort to solve the three-dimensional models is considerable. For example, the largest model took 3 h 10 min to solve and post-process in a PC system with a 4-core CPU and 4 GB RAM. The dissipation power, dissipation power per unit surface and roughness for these analyses are included in Table 4.5. As the depth of the RVE is increased, the dissipation power per unit area decreases, not only because the neighboring effect is reduced but mostly because the dissipation caused by the actuator is divided by a larger area. In the limit, the surface dissipation tends to zero when the volume fraction goes to zero, as it can be seen in Figure 4.12. In Figure 4.13, it can be seen that although the dissipation power is reduced for lower volume fractions (due to the neighboring effect), the extrapolation to $V_f = 0$ yields a dissipation power of 2.5×10^{-5} mW. This limiting case could be considered as a single actuator embedded in an infinite medium.

Figure 4.12: Surface dissipation power as a function of the volume fraction for the three-dimensional models

4.3.5 Correlation between 2D and 3D

Based on the results from Sections 4.3.3 and 4.3.4, a correlation between the two- and three-dimensional response based on the volume fraction V_f is proposed. For the two-dimensional model the volume fraction V_f is equal to the area fraction A_f , because the model is assumed to be in plane strain. The surface dissipation power and roughness from the two-dimensional model are set as the reference values and the ratios of three-dimensional to two-dimensional results are calculated as a function of the volume fraction, as follows:

$$r_{[\bullet]}(V_f) = \frac{[\bullet]_{3D}(V_f)}{[\bullet]_{2D}(A_f)} \quad (4.4)$$

The calculated ratios r_R, r_D for the roughness and dissipation, respectively, are included in Figure 4.14.

It is important to bear in mind that these results were calculated for a sinusoidal load with 10 MPa amplitude and 200 Hz frequency for an RVE width of 300 μm , resulting in $A_f = V_f = 5.23\%$ for the two-dimensional case.

¹Because this result is from a plane strain model, the units are per mm depth. Thus, they should not be compared with the other results under this column.

Figure 4.13: Dissipation power as a function of the volume fraction for the three-dimensional models

Figure 4.14: Ratios for roughness and surface dissipation power

4.3.6 Two Dimensional - Parametric Analysis

In this section a two-dimensional parametric analysis is done on a RVE with one actuator. Because periodic boundary conditions are enforced, it is *assumed* that all the actuators are *coherent* or *polarized* (i.e. *all* the right tips will go up and *all* the left tips will go down). The problem with two neighboring actuators having opposite polarization has been treated in Section 4.2.

The parameters that will be analyzed are: frequency and magnitude of the excitation, volume fraction (changing the width of the film), vertical location of the actuator within the base film, aspect ratio of the actuator and the Young's modulus of the top film and the actuator. Also a mesh dependency analysis will be done to show that the results are convergent.

The model 2D-P-01 in Table 4.6 is set as a reference for comparison purposes in the parametric analyses.

The PYTHON script used to analyze these models is the same as in Section 4.3.3, which is listed in the Appendix E.2.

All the results of this section are listed in Table 4.6.

Excitation Frequency

Here, a fixed width ($120\text{ }\mu\text{m}$) of the RVE will be considered. The frequency of the excitation will be changed from 20 Hz to 200 Hz in increments of 30 Hz. The results are shown in Figures 4.15 and 4.16, and they are based on the models 2D-P-01 to 2D-P-07.

Torque Magnitude

As the torque generated by actuators depends on the intensity of the electric field applied, and the magnitude of the torque has not been experimentally determined yet (see Section 6.1), a parametric study will be done to assess its influence on the APF. In the present work, the torque generated by the actuators is materialized by the application of a surface traction with the same magnitude but opposite directions in the top and bottom surfaces of the actuators. Here, the magnitude of the traction will be increased and reduced to study the response of the APF. Being the torque proportional to the applied traction, if the latter is changed, the former will also change accordingly.

The results are shown in Figures 4.17 and 4.18. The models used to create these figures are 2D-P-01, 2D-P-20 and 2D-P-21, which are listed in Table 4.6. As it can be seen in these figures, the response is non-linear, as it was expected because the base film material is not linear (see Figure 4.1).

Actuator Location

In this Section the vertical location of the ellipse is modified and the response is analyzed. Figures 4.19 and 4.20 show the results of the models 2D-P-01, 2D-P-08 and 2D-P-09. It is noticeable how, both surface dissipation and roughness, increase as the actuator is closer to the top surface. If the roughness needs to be increased and power consumption is not a concern, moving the actuators towards the top surface could be a good option.

Model	Vol. Frac.	RVE Width	Act. Width	Act. Thick.	Act. Offset	Frequency	Traction	E Actuator	E Top Film	Elements	Surf. Dis. Pwr.	Roughness
	[]	[mm]	[mm]	[mm]	[mm]	[Hz]	[MPa]	[GPa]	[GPa]	[]	[mW/mm ²]	[mm]
2D-P-01	0.1308	0.12	0.0600	0.0100	0.000	200	10	3	70	8040	0.1072	0.000959
2D-P-02	0.1308	0.12	0.0600	0.0100	0.000	170	10	3	70	8040	0.0961	0.000980
2D-P-03	0.1308	0.12	0.0600	0.0100	0.000	140	10	3	70	8040	0.0842	0.001005
2D-P-04	0.1308	0.12	0.0600	0.0100	0.000	110	10	3	70	8040	0.0709	0.001038
2D-P-05	0.1308	0.12	0.0600	0.0100	0.000	80	10	3	70	8040	0.0560	0.001083
2D-P-06	0.1308	0.12	0.0600	0.0100	0.000	50	10	3	70	8040	0.0388	0.001151
2D-P-07	0.1308	0.12	0.0600	0.0100	0.000	20	10	3	70	8040	0.0179	0.001285
2D-P-08	0.1308	0.12	0.0600	0.0100	-0.005	200	10	3	70	8809	0.0268	0.000327
2D-P-09	0.1308	0.12	0.0600	0.0100	0.005	200	10	3	70	8426	0.1935	0.001904
2D-P-10	0.1744	0.09	0.0600	0.0100	0.000	200	10	3	70	4428	0.1647	0.001084
2D-P-11	0.1046	0.15	0.0600	0.0100	0.000	200	10	3	70	5292	0.0829	0.000940
2D-P-12	0.0523	0.30	0.0600	0.0100	0.000	200	10	3	70	13621	0.0421	0.000944
2D-P-13	0.1308	0.12	0.0600	0.0100	0.000	200	10	3	70	1931	0.0951	0.000917
2D-P-14	0.1308	0.12	0.0600	0.0100	0.000	200	10	3	70	17394	0.1072	0.000959
2D-P-15	0.1308	0.12	0.0600	0.0100	0.000	200	10	3	70	4915	0.1052	0.000958
2D-P-16	0.0824	0.12	0.0238	0.0159	0.000	200	10	3	70	4101	0.0238	0.015874
2D-P-17	0.1602	0.12	0.0900	0.0082	0.000	200	10	3	70	4239	0.0274	0.000622
2D-P-18	0.1308	0.12	0.0600	0.0100	0.000	200	10	3	70	8830	0.1059	0.000956
2D-P-19	0.1308	0.12	0.0600	0.0100	0.000	200	10	3	70	8830 ¹	0.1078	0.000962
2D-P-20	0.1308	0.12	0.0600	0.0100	0.000	200	5	3	70	8830	0.0089	0.000377
2D-P-21	0.1308	0.12	0.0600	0.0100	0.000	200	15	3	70	8830	0.3405	0.001704
2D-P-22	0.1308	0.12	0.0600	0.0100	0.000	200	10	2	70	8830	0.1084	0.000968
2D-P-23	0.1308	0.12	0.0600	0.0100	0.000	200	10	1	70	8830	0.1168	0.000990
2D-P-24	0.1308	0.12	0.0600	0.0100	0.000	200	10	3	50	8830	0.1061	0.000954
2D-P-25	0.1308	0.12	0.0600	0.0100	0.000	200	10	3	30	8830	0.1065	0.000950
2D-P-26	0.1308	0.12	0.0600	0.0100	0.000	200	10	3	30	9070 ²	0.1065	0.000950

Table 4.6: Results for the two-dimensional parametric models

^aThis model has been solved using quadratic elements (CPE8R and CPE8RH).^bTwo elements were used through the thickness of the top film in this model.

Figure 4.15: Roughness as a function of the excitation frequency

Figure 4.16: Surface Dissipation Power as a function of the excitation frequency

Figure 4.17: Roughness as a function of the surface traction magnitude

Figure 4.18: Surface dissipation power as a function of the surface traction magnitude

Figure 4.19: Roughness as a function of the actuator's vertical offset

Figure 4.20: Surface Dissipation Power as a function of the actuator's vertical offset

Figure 4.21: Roughness as a function of the actuator's aspect ratio

Actuator Geometry

In this Section the geometry of the actuator is changed and the response is analyzed. Three models (2D-P-01, 2D-P-16 and 2D-P-17) are created where the geometry of the actuator is changed under the constraint of keeping the volume of the ellipsoid constant (not the area). The roughness and surface dissipation power are plotted against the actuator's aspect ratio AR in Figures 4.21 and 4.22, respectively. The aspect ratio is defined as,

$$AR = \frac{c}{a} \quad (4.5)$$

where c, a are the major and minor radii, respectively. And the volume of a prolate spheroid is given by,

$$Vol = \frac{4}{3}\pi a^2 c \quad (4.6)$$

An effort to analyze actuators with aspect ratio equal or close to 1 (circle) was done without results. The mesh gets highly distorted and the solver does not converge. A possible solution, beyond the scope of the present work, could be a coupled Eulerian-Lagrangian analysis [25, §13.1.1].

Volume Fraction

To study the effect of the volume fraction V_f , the width of the RVE is changed and the response is analyzed. The results from models 2D-P-01 and 2D-P-10 to 2D-P-12 are plotted

Figure 4.22: Surface Dissipation Power as a function of the actuator's aspect ratio

in Figures 4.23 and 4.24.

Note that the roughness as defined in this work, is a local measure of the deformation around the actuator. Therefore, the increase of the roughness with the volume fraction shown in Figure 4.23, shows the interaction effect with the neighboring actuators.

Actuator and Top Film Modulus

In this Section, the actuator and top film Young's modulus will be changed and the impact on the response will be assessed. In the models 2D-P-22 and 2D-P-23, the actuator's modulus will be set to 2000 MPa and 1000 MPa, respectively, while keeping all the other parameters equal to the ones of the reference model 2D-P-01. The top film modulus will be reduced from 70 000 MPa to 50 000 MPa and 30 000 MPa in the models 2D-P-24 and 2D-P-25, respectively. For the two latter models, all the other parameters are the same as in model 2D-P-01.

The surface dissipation power and roughness results for all the aforementioned models are included in Table 4.6 and shown in Figures 4.25 to 4.28. As it can be seen in these figures, the impact on the results of changes in the order of 300 % in the actuator Young's modulus and 233 % in the top film Young's modulus is very small. The most noticeable change is in the surface dissipation power, when the actuators Young's modulus is reduced to 1000 MPa, as it is shown in Figure 4.26. The change in the response due to changes in the top film modulus are negligible (see Figures 4.27 and 4.28). This latter fact might happen because the top film, although it has a higher modulus than the base film, it is 1000 times thinner; and the bending stiffness of a plate is proportional to its thickness cubed.

Figure 4.23: Roughness as a function of the volume fraction

Figure 4.24: Surface Dissipation Power as a function of the volume fraction

Figure 4.25: Roughness as a function of the actuator Young's modulus

Figure 4.26: Surface dissipation power as a function of the actuator Young's modulus

Figure 4.27: Roughness as a function of the top film Young's modulus

Figure 4.28: Surface dissipation power as a function of the top film Young's modulus

Figure 4.29: Roughness as a function of the number of elements in the mesh

Mesh Dependency Study

h-Refinement In the *h-refinement* method, the same type of elements is used, but the size of the elements is changed [50]. For this case, complete regeneration will be used. From a computational point of view this is very expensive, but for this academic problem it is feasible to do it.

Meshes with different element sizes will be generated and the results will be compared. If the problem is mesh independent, the results should converge asymptotically as the element size is reduced.

In Figures 4.29 and 4.30, it can be seen that the results are very similar when the number of elements is increased from 8040 to 17394. The aforementioned figures are based on the results of models 2D-P-01, 2D-P-13, 2D-P-14 and 2D-P-15.

p-Refinement The idea behind the *p-refinement* method is to keep the original mesh, but the degree of the elements is changed. ABAQUS® has two types of elements available, linear and quadratic. In order to compare the results, two models are analyzed; one with linear (4 noded), plane strain continuum elements and the other with quadratic (8 noded) elements. The ratio between the roughness and surface dissipation power between both meshes are 1.0063 and 1.0176, respectively. The results for these models are listed in Table 4.6 as 2D-P-18 and 2D-P-19. Being the latter the one meshed with quadratic elements.

Figure 4.30: Surface Dissipation Power as a function of the number of elements in the mesh

Chapter 5

Volume Fraction Calculation

5.1 Introduction

In this Chapter a tool that will help to define the volume fraction V_f and the dimensions of the representative volume element (RVE) will be developed. The aim is to determine which is the typical volume fraction and RVE based in an digital image of the actuated polymer film (APF) obtained with optical microscopy.

This will be done in a MATLAB[®] script where digital image processing, computational geometry and statistics will be used. MATLAB[®] Image Processing Toolbox will be used. The whole process will be explained in the next sections using an example.

5.2 Program Structure

5.2.1 Image Processing

The analysis begins with a color image obtained from an optical microscope (Leitz Laborlux 12 ME) of the APF. Figures 5.1 and 5.2, show two regions of an APF sample. As it can be seen in these pictures, the size distribution of the ellipsoids is not very uniform. Also, the size of the ellipses is smaller that what is expected to be used in the applications of the APF's. At the present time, the research group that has developed the APF is investigating for a method to control the size of the droplets that, after stretching, will morph to ellipsoids. It is expected that for the application the size of the actuators will be larger and more uniform. The aforementioned Figures will be used to explain the process proposed for the calculation of the volume fraction.

The first goal is to convert Figures 5.1 and 5.2 into binary images. In order to do this the color images are first converted to gray scale. Then, adaptive histogram equalization (`adapthisteq` command in MATLAB[®]) is applied to improve the image contrast. This type of equalization is called adaptive because instead of taking the image as a whole, it operates on different areas improving the contrast locally. This is useful if the image has non-uniform illumination [29]. After improving the contrast, the threshold that will define whether a pixel will be black or white is calculated using the `graythresh` command. Then, the gray scale image is converted to black and white employing the `im2bw` command and the

Figure 5.1: Image A of the actuators embedded in PVA (by Morris [41])

Figure 5.2: Image B of the actuators embedded in PVA (by Morris [41])

Figure 5.3: Image A converted to black and white

threshold calculated in the previous step.

At this point, the original images are now converted to a black and white images as it can be seen in Figures 5.3 and 5.4. In these Figures, it can be seen that there are some white spots in the interior of a few ellipses. Thus, the next step will be to fill the interior of the regions using the command `imfill`.

After filling the interior of the regions, the images are morphologically opened and closed using the commands `imopen` and `imclose`, respectively. The combination of these two operations will smoothen the contours of the regions and will break thin connections between neighboring regions [29].

As it can be seen in Figures 5.5 and 5.6, the boundaries are smoother, the interior regions are full and some of small regions were eliminated.

At this stage, all the modifications to the original images are done. The next step is to detect all the connected regions in the binary images, using MATLAB[®] 's command `bwconncomp`. Then the command `regionprops` is used to analyze the properties of all the regions found. In this script, the area, minor and major axes, eccentricity, orientation, centroid and bounding box for each region are calculated. A detailed list of all the possible properties to be analyzed can be found in MATLAB[®] Documentation [40].

A criteria is set to discard all the regions that should not be considered as actuators. This script will discard all the regions for which their areas are outside a specified range and all the regions for which the eccentricity is smaller than a specified value. The eccentricity is given by,

$$e = \sqrt{1 - \left(\frac{2a}{2c}\right)^2} \quad (5.1)$$

where $2c, 2a$ are the major and minor axis of the region, respectively. In the examples, some small regions are discarded because their area is smaller than the minimum and/or

Figure 5.4: Image B converted to black and white

Figure 5.5: Image A after being filled and morphologically opened and closed

Figure 5.6: Image B after being filled and morphologically opened and closed

because their eccentricity is close to 1. Obviously, the area and eccentricity criteria must be consistent with the image analyzed and it is likely that they will have to be modified to work correctly if the shape of the “typical” actuator changes.

After removing the regions that did not pass the filter, the resulting images are shown in Figures 5.7 and 5.8. At this point, all the image processing is done.

5.2.2 Computational Geometry

In this Section, the Voronoi diagram theory [9, §7] will be used to divide the image into cells. Each cell is associated with a site or point, that in this manuscript is the centroid of each actuator. The region defined by each Voronoi cell has the property of being the set of points that is closer to the Voronoi site associated with the cell than to any other Voronoi site. Figures 5.9 and 5.10 show the division of the images into the Voronoi cells associated with each actuator. In MATLAB[®], the functions `voronoin` and `voronoi` calculate the vertexes of the cells and plot the Voronoi diagram, respectively.

5.2.3 Area Fraction

The area fraction for each cell can be easily calculated, as the ratio between the area of the actuator and the area of the respective Voronoi cell. In the present work, the area fraction will be calculated *only* for the interior cells because for the cells that are on the boundaries of the image, nothing can be told about how far is the closest actuator. This explains why is so important to have a picture big enough to show *several* actuators.

Figure 5.7: Image A after removal of the filtered regions

Figure 5.8: Image B after removal of the filtered regions

Figure 5.9: Image A - Actuators and their associated Voronoi cells

Figure 5.10: Image B - Actuators and their associated Voronoi cells

Figure 5.11: Image A - Bounding boxes for the interior Voronoi cells

5.2.4 Volume Fraction

From a two-dimensional image it is not possible to calculate the volume fraction V_f without additional information. In order to estimate the volume fraction, the thickness of the film must be given and *assumed* to be constant. Furthermore, a three-dimensional geometry of the actuator must be *assumed*. For these examples, the three-dimensional geometry is considered to be a prolate spheroid and the major and minor radii for this geometry are *assumed* to be one half of the major and minor axes of the connected region (which were obtained from the binary images in Section 5.2.1).

5.2.5 RVE dimensions

In this Section, an approximated method to calculate the “typical” width and depth of the representative volume element (RVE) will be developed. These dimensions, and the thickness of the APF, are needed to analyze the APF using the finite element method and periodic boundary conditions, as it was done in Section 4. Based on the vertexes of the polygon of each Voronoi cell, an upper bound on the dimensions of the RVE can be taken as the bounding box that encloses all the vertexes for each cell.

For the examples that are being used in this Chapter, the bounding boxes for the interior cells are displayed in Figures 5.11 and 5.12. Here, the boxes are assumed to be aligned with the $x - y$ directions. Hence, it is expected that the x direction in the original image is parallel to the direction in which the APF was stretched.

As it was mentioned before, the dimensions of the RVE calculated here, constitute an upper bound. As it can be seen in Figures 5.11 and 5.12, there is some overlap between the bounding boxes, so the dimensions of the true RVE are smaller.

A lower bound on the RVE dimensions could be built considering the largest rectangle inscribed inside each cell. Although this problem has known solutions [2, 43], it is far from

Figure 5.12: Image B - Bounding boxes for the interior Voronoi cells

		$\bar{x}$	s
Actuator Area	$[\mu\text{m}^2]$	30.1664	31.4807
Area Fraction	$[\]$	0.0632	0.0523
Volume Fraction	$[\]$	0.0191	0.0209
RVE Width	$[\mu\text{m}]$	74.9373	10.1906
RVE Depth	$[\mu\text{m}]$	68.4344	11.5257

Table 5.1: Results for the mean value $\bar{x}$ and standard deviation s estimators

simple to implement, and therefore will not be included in the present work.

5.3 Results

The results for Image B (Figure 5.2) will be included in this Section. The source code of the complete program is included in Appendix F. The thickness of the APF sample depicted in Figures 5.1 and 5.2 is $36\mu\text{m}$ [41]. This thickness will be used in the calculation of the volume fraction of the present example.

The mean value and standard deviation estimations for different variables are listed in Table 5.1. The estimates on the area of the actuators are based on a sample size of 26 (all the actuators in the picture), while the volume fraction, area fraction and RVE dimensions are based on a sample size of 10 (only the interior cells).

Chapter 6

Conclusions and Future Work

6.1 Future Work

6.1.1 Temperature Dependence

In the present work it was assumed that the process is isothermal. If desired, the mechanical properties of the materials used in the model can be made temperature dependent. A thermal analysis should be done to evaluate the change of the temperature within the RVE due to the hysteresis cycles.

6.1.2 Torque Measurement

The torque generated by the actuators when they are subjected to an electric field must be assessed. Although an analytical expression has been proposed [22, 21, Equation 1], this equation can not be immediately used because one of the parameters, the dielectric anisotropy $\Delta\epsilon$ is not known due to non-uniformly aligned regions of the liquid crystals in the actuators [21]. The determination of the torque generated by the actuators by indirect measurement will be necessary to input the magnitude A in (4.2). Some of the possible techniques are discussed in this section.

Optical Perfilometer

Once the material science group develops the process to apply a conductive coating to the top of the film, the following experiment could be done. A sample of the APF is actuated, keeping the potential differential fixed. The surface of the APF is analyzed with an optical perfilometer, which requires no physical contact with the sample. When the displacements stabilize, the profile of surface is recorded. Then, the load in the FE model is changed until the magnitude of the displacements is similar to the experimental observation. This method is advantageous because it would involve measuring the response of the film as close as it would be in the final application of the APF. The major drawback is that the application of the conductive coating has not been successfully developed, yet. In addition, the College of Engineering and Mineral Resources at West Virginia University does not own this piece of equipment at the present time.

Fringe Pattern

A sample of the APF is placed between two indium tin oxide (ITO) coated glasses, the potential differential is applied and depth of the valleys is measured using optical interferometry. Once the displacements are known, the applied torque in the FE model could be varied until a similar response is observed. If one of the surfaces is glued the glass, it might help to avoid interference between the fringe patterns created by the each tip. Using this technique, the measurement is possible but not very easy to do. This APF would lack of the conductive thin film, which modifies the mechanical behavior of the film. Furthermore, the APF used in this case would be different to the one that is intended for the application, for aerodynamic flow control for example, where the conductive coating is required.

Micro Indenter - Force Control

The bottom of a medium sized sample (e.g. 25 mm by 25 mm) of the APF is glued to an indium tin oxide (ITO) coated glass, and another ITO coated glass is laid on top of the film. The micro indenter tip is moved until it makes contact with the top glass. Then, the micro indenter is set to force control. The force is set to zero and the displacement of the top glass is recorded, while the voltage differential is applied to the coated glasses.

If the mechanical properties of the base film are known, a FE analysis can be done using the script listed in Appendix E.1 and the applied load can be back-calculated so that the displacement of the analytic rigid surface matches the experimental displacement of the top glass.

Contact Perfilometer

As for the method that employs an Optical Perfilometer, the procedure for the application of the conductive coating should be established, before the following experiment can be done. The electric potential is applied, and after waiting until the displacements stabilize (viscous effects), a replica of the surface is made. This can be done applying a release agent and casting an UV curable resin, for example, on the top surface of the deformed APF. The necessity of the replica is because the perfilometer sensor could be damaged if it gets in contact with the conductive coating, when the potential differential is applied. The surface of the replica can be later analyzed with a contact perfilometer. Once the displacements are known, the applied torque in the FE model could be varied until a similar response is observed.

6.1.3 Base film mechanical properties

For the calculation of the dissipation and response of the APF, the base film material mechanical properties should be determined for the selected or candidate materials. The parameters of the BB model have been calculated for some elastomers using experimental data from the literature [10] and the program developed in Chapter 3. The calculated parameters and the goodness of fit are listed in Table 3.1.

In order to fit the parameters of the BB Model the following tests are suggested by Bergstrom [10],

- Uniaxial tensile testing at three different true strain rates [6].
- Uniaxial compressive testing at three different true strain rates [5].

To achieve a constant true strain rate, the testing machine must be set to displacement control and the cross-head displacement must follow an exponential law. The samples should be conditioned to remove the Mullins effect, introduced in Section 2.2.

Currently, the APF has the actuators embedded in polyvinyl alcohol (PVA). Cairns [20] indicates that it is unlikely to get reliable testing data for PVA because its mechanical properties are highly dependent on the degree of polymerization (DP) and the humidity of the sample. Furthermore, it is improbable that for most of the possible applications, where the APF would be exposed to the environment (e.g. aerodynamic flow control), the base film will be PVA due to its solubility in water [48].

6.2 Conclusions

In this research effort, a set of analytical and numerical tools for the design and analysis of actuated polymer films has been developed. It is important to understand the difference between *analysis* and *design*. At the *design* stage there are, usually, several unknowns in terms of the geometry, material properties and loads. Therefore, the tools used for design should be able to cope with many different cases with enough simplicity that allows to study a large number of combinations of the uncertain variables in a reasonable amount of time. Sometimes, some simplifications must be done, for the sake of speed. On the other hand, the *analysis* tools must be as accurate and precise as possible, because in the analysis stage a great part of the original unknowns that were present at the design stage have been already quantified and the number of possible combinations has been narrowed.

Sophisticated three-dimensional models like the ones presented in Section 4.3.4 give the most accurate and precise results, while simplified two-dimensional models, like the ones presented in Section 4.3.6, allowed to run a large number of cases (about twenty-five in this manuscript) in an affordable amount of time. Although there are still many unknowns of the actuated polymer films, in the present work the three-dimensional models have been used to define a ratio between the results of these and a two-dimensional model. This ratio can be used as a *design* tool, to extrapolate the results of the simplified two-dimensional models to the three-dimensional case, as a function of the volume fraction.

A remarkable fact from the results of the three-dimensional models (Table 4.5), is that as the volume fraction V_f decreases, both the total dissipation power and the roughness reach threshold values. For the dissipation power, it can be clearly seen in Figure 4.13 that the dissipation should be close to 2.5×10^{-5} mW, if the volume fraction is extrapolated to zero. Conversely, the *surface* dissipation power, tends to zero as the volume fraction tends to zero (Figure 4.12). The latter conclusion is physically sound, because if a single actuator, which has a finite dissipation power, is embedded in a infinite medium, the dissipation power per unit surface should be, in the limit, equal to zero.

A similar behavior can be observed in Figure 4.14, for the roughness and surface dissipation ratios as a function of the volume fraction. While the roughness ratio has a plateau

for volume fractions smaller than 1.5 %, the ratio for the surface dissipation power is smaller than 0.1 for $V_f = 0.7$ %.

Several interesting observations can be extracted from the parametric analyses done in Section 4.3.6. It is worth to mention that while the roughness decreases with increasing excitation frequencies, the surface dissipation power follows the opposite conduct (Figures 4.15 and 4.16, respectively). This behavior arises from the viscous network of the base film material model. For a creeping material it is expected that the strains will grow with time, if the load is kept along the time. In the case of the actuated polymer films, the higher the frequency, the less time that the base film will have to deform. Thus, the roughness is reduced at higher frequencies. Regarding the surface dissipation power, higher frequencies of excitation will cause more displacement reversal cycles in the base film. From basic mechanics, it is known that the dissipation in a dash-pot or damper is associated to the first derivative of the displacement with respect to time, which for a sinusoidal movement becomes maximum when the displacement is reversed from positive to negative or vice versa.

The roughness and the surface dissipation power were higher, when the applied torque was increased (see Figures 4.17 and 4.18). It is important to notice that in order to increase the roughness by a 78 %, the dissipation is triplicated. This fact puts in evidence the non-linear behavior of the viscous effects in the analyzed models.

When the vertical position of the actuator was changed, by translating it closer to the top surface, both the dissipation and the roughness increased. The opposite response was obtained when the actuator was moved closer to the bottom surface.

Another geometric parameter that was varied to study its effect in the results, is the aspect ratio of the actuator. Although only three different aspect ratios were investigated, the roughness and the surface dissipation power seem to follow a decaying exponential law, as it is shown in Figures 4.21 and 4.22.

From the mesh dependency analysis, it can be concluded that for a two-dimensional model with an RVE width of 120 μm and thickness of 30.03 μm , the change in the results is marginal for meshes with more than 4900 elements (e.g. less than 0.1 % change of the roughness). Also, the use of quadratic elements showed a very small variation in the results (about 0.4 % roughness difference).

Large changes in the top film and actuator modulus did not show a significant impact in the results (Figures 4.25 to 4.28). Therefore, it could be said that the response of the APF model has a very small sensitivity to these parameters.

A parametric study of the base film material properties was not done because the material was purposely chosen to be very compliant. Furthermore, the base film material is characterized by several parameters and in contrast to geometric parameters, the material parameters can not be tailored as desired. In this work, the parameters for the hyperelastic behavior were fit from experimental data available from the literature for Hytrel 4056. The parameters that characterize the viscous behavior were chosen to be similar to the ones of Nitrile Rubber (Section 3).

For the dissipation calculation, a custom script was written in Python. Appendix B showed that ABAQUS[®] built in variable for this purpose yields an incorrect result, by estimating about twice of the true dissipation. The proposed method was compared with experimental data from Bergstrom [16], showing very good agreement.

As it can be seen in Figure B.1, the solution obtained from the Finite Element model is

very close to the experimental results. This result supports the fact that the FE solution using the Bergstrom-Boyce material model is representative of the real behavior of elastomeric materials under cyclic loading.

For all the models analyzed in Chapter 4, the material properties for the base film were obtained from the literature for a chosen material candidate. But it is important to bear in mind that a procedure to fit *any* desired material, from experimental data, was developed in Chapter 3. The performance of this procedure was assessed by fitting several materials from experimental data available in the literature and calculating the goodness of fit.

Lastly, a novel approach was used in Chapter 5 for the determination of the volume fraction from digital images of the APF. The proposed technique also allowed to calculate the geometry of the “typical” RVE, which is very useful to analyze the APF using finite element analysis with periodic boundary conditions. For illustration purposes, an example was done using a digital image of an APF sample.

In conclusion, a varied set of powerful tools for the design and analysis of actuated polymer films were developed and implemented in this work. It is expected that these procedures will be of valuable help in the process of filling the gap from the laboratory, where the actuated polymer films are being developed, to their successful application.

Appendix A

Code of the program to fit the BB model

A.1 Main Program

```
1 %-----%
2 %   Joaquin M. Gutierrez - 2010   %
3 %   West Virginia University   %
4 %-----%
5
6 function y=fit10
7 clc
8 clear all
9
10 %this global definition is needed when using fminsearch
11 global Y_data X_data DATA_SETS ROW_INDEX LENGTH_DATA_SETS
12
13
14 %Store the data of experiment in a different cell of M
15 %Each cell of M contains an array with the experimental data with the
16 %following layout:
17     %column 1: time [s]
18     %column 2: uniaxial true strain []
19     %column 3: uniaxial true stress [MPa]
20
21 M{1}=dlmread('ABR_compression1.txt');
22 M{2}=dlmread('ABR_compression2.txt');
23
24 %Calculate the number of experiments (number of cells of M) and save it as
25 %the variable "DATA_SETS"
26 DATA_SETS=size(M,2);
27
28 %Initialize the Array "LENGTH_DATA_SETS" for better performance
29 LENGTH_DATA_SETS(DATA_SETS)=0;
30
31 %The TRUE STRESS will be considered a function of the TIME and TRUE STRAIN
32 %history. Therefore, Y=Y[X], where Y=True Stress (Scalar) and X=[time, True
33 %Strain] (vector).
```

```

34
35 Y_data=[];
36 X_data=[];
37
38 %Merge the data of all the experiments (stored in the cells of M) into two
39 %arrays, Y (True Stress) and X (Time, True Strain).
40 %Create a list (LENGTH_DATA_SETS) where each element has the number of
41 %datapoints for each experiment.
42 for K=1:1:DATA_SETS
43     LENGTH_DATA_SETS(K)=size(M{K},1);
44     Y_data=[Y_data;M{K}(:,3)];
45     X_data=[X_data;M{K}(:,1),M{K}(:,2)];
46 end
47 %Initialize the list "ROW_INDEX" for better performance
48 ROW_INDEX(DATA_SETS)=0;
49 %The list (1D Array) "ROW_INDEX" will store the row number of the Y,X
50 %arrays corresponding to the beginning of each experiment.
51 ROW_INDEX(1)=1;
52 for K=2:1:DATA_SETS
53     ROW_INDEX(K)=ROW_INDEX(K-1)+LENGTH_DATA_SETS(K-1);
54 end
55
56
57
58 %Define the initial guess for the parameters
59 %      [mu_a    l_lock    s      gamma_0    C      m]
60 beta0 = [    0.77,      3.3,      2.5,      0.6,      -0.5,      1.5];
61
62 %Define the Lower Bound for Parameters
63 %      [mu_a    l_lock    s      gamma_0    C      m]
64 LB=[          0.0,      1.0,      0.0,      0.0,      -1.0,      0.0];
65
66 %Define the Upper Bound for Parameters
67 %      [mu_a    l_lock    s      gamma_0    C      m]
68 UB=[          inf,     10.0,     inf,      inf,      0.0,     100];
69
70
71 %Using Fminsearchcon to do the parameters fit
72 %Set the OPTIONS of fminsearchcon
73 options = optimset('Display','iter','TolFun', 0.01, 'TolX', 0.01,...
74 'MaxFunEvals',10000,'MaxIter',10);
75
76 %Call the function FMINSEARCHCON, indicating the function to minimize, the
77 %initial values for the parameters, lower and upper bounds for the
78 %parameters, and lastly the options to be used.
79 [beta,fval,exitflag,output]=...
80     fminsearchcon('bb10_error',beta0,LB,UB,[],[],[],options);
81
82 %Display the solution found
83 disp('MU_A')
84 beta(1)
85 disp('L_LOCK')
86 beta(2)
87 disp('s')

```

```

88 beta(3)
89 disp( 'GAMMA0')
90 beta(4)
91 disp( 'C')
92 beta(5)
93 disp( 'm')
94 beta(6)
95
96 % %Display the solver stats
97 % fval
98 % exitflag
99 % output
100
101 %Goodness of Fit
102
103 %Here, using the independent variables (time, True Strain) stored in X, and
104 %the parameters fitted (stored in beta), the stresses will be calculated
105 %and stored as "yfit".
106 yfit = bb10(beta);
107 %The correlation coefficient is calculated between the experimental values
108 %of the true stress and the ones calculated using the fitted parameters.
109 C = corrcoef(Y_data, yfit);
110 disp( 'Goodness_of_Fit_-_R_Squared: ')
111 rsq1 = C(1,2)^2
112 rsq2 = 1 - sum((Y_data-yfit).^2) / sum((Y_data - mean(Y_data)).^2)
113
114
115 %Plot Results
116 figure(1)
117 set(gca, 'FontName', 'Arial', 'FontSize', 12); %important
118 plot(X_data(:,2), Y_data, '-k', 'Linewidth', 2, 'MarkerFaceColor', 'k') %(solid) k(
    black line)
119 hold all
120 plot(X_data(:,2), yfit, '—k', 'Linewidth', 2, 'MarkerFaceColor', 'k') %(solid) k(
    black line)
121 xlabel( 'True_Strain', 'fontsize', 14);
122 ylabel( 'True_Stress_[MPa]', 'fontsize', 14);
123 legend( 'Experiment', 'Prediction', 'Location', 'best')
124 grid on;
125 text( 'units', 'normalized', 'pos', [0.65 0.1], 'fontsize', 14, 'EdgeColor', 'black', '
    BackgroundColor', 'white', 'string', [ 'R^2= ', num2str(rsq2) ])
126 saveas(gca, './Fig_Ch03_Figure01_ABS-REV00.eps'); %.eps for LaTeX
127 hold off
128 end

```

A.2 Constitutive Equation Function

```

1 function [T]=bb10(beta)
2 global Y_data X_data DATA_SETS ROW_INDEX LENGTH_DATA_SETS
3
4 %Notes

```

```

5  %Incompressible Version
6  %The output stresses are TRUE STRESSES
7  %Joaquin M. Gutierrez - 2010 - West Virginia University
8
9  %Print the values of the parameters stored in beta at the current iter.
10 beta
11
12 mu_a=beta(1);
13 l_lock=beta(2);
14 %mu_b=s*mu_a
15 mu_b=mu_a*beta(3);
16 %Make l_lock_a=l_lock_b=l_lock
17 %l_lock_b=beta(4);
18 gamma_0=beta(4);
19 C=beta(5);
20 m=beta(6);
21
22 %Coefficient to remove singularity in the flow rule.
23 var_epsilon=0.01; % Set TO 0.01. See Bergstrom 2001a
24
25 %ODE Solver Options
26 options_ode=odeset('RelTol',0.01,'AbsTol',0.001,'NormControl','on',...
27     'Stats','off','MaxStep',0.5);
28
29 % %Number of steps to solve
30 n_steps=size(X_data,1);
31
32 %Stretch as a function of true strain (x(:,2))
33 lambda=exp(X_data(:,2));
34 %Time
35 t=X_data(:,1);
36
37 %Pre-Allocate Memory
38 lambda_v=zeros(1,n_steps); %Viscous Stretch
39 lambda_e=zeros(1,n_steps); %Elastic Stretch
40 T_a=zeros(1,n_steps); %TRUE Stress in Network A
41 T_b=zeros(1,n_steps); %TRUE Stress in Network B
42 T=zeros(n_steps,1); %TRUE Total Stress (T_a+T_b)
43
44 %Initial Values %!!!!IT is important to set these conditions at the
45 %beginning of each each experiment data
46 for I_data=1:1:DATA_SETS
47 %At the beginning of EACH experiment lambda=lambda_v=lambda_e=1
48 lambda_v(ROWINDEX(I_data))=1;
49 lambda_e(ROWINDEX(I_data))=1;
50 %At the beginning of EACH experiment the stress=0
51 T_a(ROWINDEX(I_data))=0;
52 T_b(ROWINDEX(I_data))=0;
53 T(ROWINDEX(I_data),1)=0;
54 for K=ROWINDEX(I_data)+1:1:ROWINDEX(I_data)+LENGTH(DATA_SETS(I_data))-1
55     %Calculate the TRUE tress in Network A as a function of the total
56     %stretch
57     T_a(K) = sigma_a(lambda(K),mu_a,l_lock);
58     %Prepare the function to be passed to the ODE solver

```

```

59     f_handle = @(t_ode,l_v)flow_rule(t_ode,l_v,var_epsilon,T_b,...
60         gamma_0,C,m,K);
61     %      %For debugging purposes
62     %      disp('Solving ODE for time span between:')
63     %      t(K-1)
64     %      t(K)
65     %Call the ODE solver and solve for lambda_v:
66     %Use the function prepared before, solve the time span [t(K-1),t(K)],
67     %use the previous value of lambda_v as the initial condition, and solve
68     %using the options for the ODE solver defined in the header of this
69     %file
70     [t_ode,Y] = ode23(f_handle,[t(K-1),t(K)],lambda_v(K-1),options_ode);
71     %Store the calculated value Y in lambda_v
72     lambda_v(K) = Y(size(Y,1));
73     %Find the elastic stretch using the total stretch and the viscous
74     %stretch (multiplicative decomposition).
75     lambda_e(K) = lambda(K)/lambda_v(K);
76     %Calculate the TRUE Stress in Network B as a function of the elastic
77     %stretch.
78     T_b(K) = sigma_b(lambda_e(K),mu_b,l_lock);
79     %Calculate the Total TRUE stress as T=T_a+T_b
80     T(K,1) = T_a(K)+T_b(K);
81 end
82 end
83
84 end
85
86 %Auxiliary Functions
87
88
89 %-----
90 %Effective distortional chain stretch for the uniaxial case
91     function y = j(x)
92         y=sqrt((x^2+2/x)/3);
93     end
94 %-----
95 %Stress in the "A" Network as a function of lambda (total stretch)
96 %Here the Arruda-Boyce Hyperelastic Potential is used
97     function y = sigma_a(lambda,mu_a,l_lock)
98         y=(mu_a/inv_langevin(1/l_lock)) *...
99         ((lambda^2-1/lambda)/j(lambda)) *...
100         inv_langevin(j(lambda)/l_lock)*1/1;
101     end
102 %-----
103 %Stress in the "B" Network as a function of lambda_e (elastic stretch)
104 %Here the Arruda-Boyce Hyperelastic Potential is used
105     function y = sigma_b(lambda_e,MU_B,JLOCK_B)
106         y=(MU_B/inv_langevin(1/JLOCK_B)) *...
107         ((lambda_e^2-1/lambda_e)/j(lambda_e)) *...
108         inv_langevin(j(lambda_e)/JLOCK_B)*1/1;
109
110     end
111 %-----
112 %Inverse Langevin Function

```

```

113 function f = inv_langevin(x)
114 %      if abs(x)<0.84136
115 %          f=1.31446*tan(1.58986*x)+0.91209*x;
116 %      elseif (0.84136 <= abs(x)) && (abs(x) < 1)
117 %          f=1/(sign(x)-x);
118 %      else
119 %          %Use the Ing. Langevin function used by ABAQUS (5 terms series)
120 %          %[21] Goy Teck Lim. Scratch Behavior Of Polymers. PhD thesis,
121 %          %Texas A&M University, 2005. [24] L.R.G. Treloar. Physics of
122 %          %Rubber Elasticity (3rd Edition). Oxford University Press,
123 %          %2009.
124
125          f=3*x+9/5*x^3+297/175*x^5+1539/875*x^7+126117/67375*x^9;
126      %end
127 end
128 %-----
129 %This is the ODE in lambda_v (viscous stretch)
130 function [d_lambda_v_dt] = flow_rule(t_ode,l_v,var_epsilon,T_b,...
131     gamma_0,C,m,K)
132     d_lambda_v_dt=gamma_0*(j(l_v)- 1 + var_epsilon)^C*...
133     (sqrt(2/3)*abs(T_b(K-1)))^m * sqrt(2/3)*sign(T_b(K-1))*l_v;
134 end
135 %-----

```

A.3 Error Function

```

1 function [error]=bb10_error(beta)
2 global Y_data X_data DATA_SETS ROW_INDEX LENGTH_DATA_SETS
3
4 %Notes
5 %Incompressible Version
6 %The output stresses are TRUE STRESSES
7 %Joaquin M. Gutierrez - 2010 - West Virginia University
8
9 %Print the values of the parameters stored in beta at the current iter.
10 beta
11
12 mu_a=beta(1);
13 l_lock=beta(2);
14 %mu_b=s*mu_a
15 mu_b=mu_a*beta(3);
16 %Make l_lock_a=l_lock_b=l_lock
17 %l_lock_b=beta(4);
18 gamma_0=beta(4);
19 C=beta(5);
20 m=beta(6);
21
22 %Coefficient to remove singularity in the flow rule.
23 var_epsilon=0.01; % Set TO 0.01. See Bergstrom 2001a
24
25 %ODE Solver Options

```

```

26 options_ode=odeset('RelTol',0.01,'AbsTol',0.001,'NormControl','on',...
27     'Stats','off','MaxStep',0.5);
28
29 % %Number of steps to solve
30 n_steps=size(X_data,1);
31
32 %Stretch as a function of true strain (x(:,2))
33 lambda=exp(X_data(:,2));
34 %Time
35 t=X_data(:,1);
36
37 %Pre-Allocate Memory
38 lambda_v=zeros(1,n_steps); %Viscous Stretch
39 lambda_e=zeros(1,n_steps); %Elastic Stretch
40 T_a=zeros(1,n_steps); %TRUE Stress in Network A
41 T_b=zeros(1,n_steps); %TRUE Stress in Network B
42 T=zeros(n_steps,1); %TRUE Total Stress (T_a+T_b)
43
44 %Initial Values %!!!!IT is important to set these conditions at the
45 %beginning of each each experiment data
46 for I_data=1:1:DATA_SETS
47 %At the beginning of EACH experiment lambda=lambda_v=lambda_e=1
48 lambda_v(ROWINDEX(I_data))=1;
49 lambda_e(ROWINDEX(I_data))=1;
50 %At the beginning of EACH experiment the stress=0
51 T_a(ROWINDEX(I_data))=0;
52 T_b(ROWINDEX(I_data))=0;
53 T(ROWINDEX(I_data),1)=0;
54 for K=ROWINDEX(I_data)+1:1:ROWINDEX(I_data)+LENGTH_DATA_SETS(I_data)-1
55 %Calculate the TRUE tress in Network A as a function of the total
56 %stretch
57 T_a(K) = sigma_a(lambda(K),mu_a,l_lock);
58 %Prepare the function to be passed to the ODE solver
59 f_handle = @(t_ode,l_v)flow_rule(t_ode,l_v,var_epsilon,T_b,...
60     gamma_0,C,m,K);
61 % %For debugging purposes
62 % disp('Solving ODE for time span between:')
63 % t(K-1)
64 % t(K)
65 %Call the ODE solver and solve for lambda_v:
66 %Use the function prepared before, solve the time span [t(K-1),t(K)],
67 %use the previous value of lambda_v as the initial condition, and solve
68 %using the options for the ODE solver defined in the header of this
69 %file
70 [t_ode,Y] = ode23(f_handle,[t(K-1),t(K)],lambda_v(K-1),options_ode);
71 %Store the calculated value Y in lambda_v
72 lambda_v(K) = Y(size(Y,1));
73 %Find the elastic stretch using the total stretch and the viscous
74 %strecth (multiplicative decomposition).
75 lambda_e(K) = lambda(K)/lambda_v(K);
76 %Calculate the TRUE Stress in Network B as a function of the elastic
77 %stretch.
78 T_b(K) = sigma_b(lambda_e(K),mu_b,l_lock);
79 %Calculate the Total TRUE stress as T=T_a+T_b

```



```

80     T(K,1) = T_a(K)+T_b(K);
81 end
82 end
83 %Calculate the error (difference) between the experimental data (Total True
84 %Stress from Testing) and the predicted Total True Stress using the fitted
85 %parameters.
86
87 %Definition of the error,
88
89 %OPTION "A": abs. val. of the difference between the prediction and the
90 %measurement
91 %error=norm( Y_data-T);
92
93 %OPTION "B": MSD (Mean square deviation)
94 %error=100*sum(( Y_data-T).^2)/length(T);
95
96 %OPTION "C": NMAD (Normalized Median Absolute Difference)
97 error=100*sum(abs( Y_data-T))/sum(abs( Y_data));
98 end
99
100 %Auxiliary Functions
101
102
103 %-----
104 %Effective distortional chain stretch for the uniaxial case
105     function y = j(x)
106         y=sqrt((x^2+2/x)/3);
107     end
108 %-----
109 %Stress in the "A" Network as a function of lambda (total stretch)
110 %Here the Arruda-Boyce Hyperelastic Potential is used
111     function y = sigma_a(lambda,mu_a,l_lock)
112         y=(mu_a/inv_langevin(1/l_lock)) *...
113           ((lambda^2-1/lambda)/j(lambda)) *...
114           inv_langevin(j(lambda)/l_lock)*1/1;
115     end
116 %-----
117 %Stress in the "B" Network as a function of lambda_e (elastic stretch)
118 %Here the Arruda-Boyce Hyperelastic Potential is used
119     function y = sigma_b(lambda_e,MU_B,J_LOCK_B)
120         y=(MU_B/inv_langevin(1/J_LOCK_B)) *...
121           ((lambda_e^2-1/lambda_e)/j(lambda_e)) *...
122           inv_langevin(j(lambda_e)/J_LOCK_B)*1/1;
123
124     end
125 %-----
126 %Inverse Langevin Function
127     function f = inv_langevin(x)
128         % if abs(x)<0.84136
129         %     f=1.31446*tan(1.58986*x)+0.91209*x;
130         % elseif (0.84136 <= abs(x)) && (abs(x) < 1)
131         %     f=1/(sign(x)-x);
132         % else
133         %Use the Ing. Langevin function used by ABAQUS (5 terms series)

```

```

134      % [21] Goy Teck Lim. Scratch Behavior Of Polymers. PhD thesis,
135      % Texas A&M University, 2005. [24] L.R.G. Treloar. Physics of
136      % Rubber Elasticity (3rd Edition). Oxford University Press,
137      % 2009.
138
139      f=3*x+9/5*x^3+297/175*x^5+1539/875*x^7+126117/67375*x^9;
140      %end
141  end
142  %-----
143  % This is the ODE in lambda_v (viscous stretch)
144  function [d_lambda_v_dt] = flow_rule(t_ode, l_v, var_epsilon, T_b, ...
145      gamma_0, C, m, K)
146      d_lambda_v_dt = gamma_0 * (j(l_v) - 1 + var_epsilon)^C * ...
147      (sqrt(2/3) * abs(T_b(K-1)))^m * sqrt(2/3) * sign(T_b(K-1)) * l_v;
148  end
149  %-----

```

Appendix B

Dissipation Calculation

B.1 Comparison

The aim of this Section is to show that ABAQUS[®] built-in creep energy dissipation calculation (ALLCD) has a flaw, and therefore should not be used.

A simple uniaxial compression model will be simulated. The geometry, material properties, boundary conditions and loading are the same that were used in Reference [16, Figure 2].

The geometry is a disc with diameter $\phi = 28$ mm and height $h = 13$ mm. The material properties used are $\mu_A = 1.31$ MPa, $\lambda_A^{\text{lock}} = \lambda_B^{\text{lock}} = 3.00$, $d = 2/\kappa = 0.004/\text{MPa}$, $s = 3.37$, $\dot{\gamma}_0/\tau_{\text{base}}^m = 0.33/(s \text{ MPa}^m)$, $m = 5.21$ and $c = -1.0$.

The bottom surface of the disk is restrained from moving in the vertical direction and the top surface will have an imposed displacement. In Reference [16], the loading is done by imposing a constant compressive *true* strain rate. In order to reproduce this in the present work, an amplitude defined by a table is used in the ABAQUS[®] model. Given the true strain rate $\dot{\epsilon}$ and the undeformed height of the sample h , the displacement of the top surface during the loading phase is described by,

$$U_2(t) = -h(1 - \exp(\dot{\epsilon} t)) \quad (\text{B.1})$$

The maximum true strain to be achieved is -0.8. Using a strain rate of $-0.05/\text{s}$, it takes 16 s to reach the maximum strain, and further 16 s to unload. The table used to define the displacement in ABAQUS[®] is listed in Table B.1.

Because this loading condition creates a uniform state of stress (the stress tensor is the same for *all* the points in the continuum), analyzing only one point is enough to understand the behavior of the whole. In Figure B.1 the true stress-strain¹ curve in the loading direction is plotted for one of the integration points in model (line with markers). These results are in very good agreement with the experimental results of Bergstrom and Boyce [16, Figure 2], which are also shown in Figure B.1.

The area enclosed by the experimental stress-strain curve at a strain rate equal to $0.05/\text{s}$ was calculated in AutoCAD, resulting in a dissipation energy density of 0.5878 MPa. The area enclosed by the stress-strain curve represents the dissipated energy density for the

¹ABAQUS[®] refers to the true strain as logarithmic strain.

Time [s]	U_2 [mm]
0	0.000
1	-0.634
2	-1.237
3	-1.811
4	-2.357
5	-2.876
6	-3.369
7	-3.839
8	-4.286
9	-4.711
10	-5.115
11	-5.500
12	-5.865
13	-6.213
14	-6.544
15	-6.859
16	-7.159
17	-6.859
18	-6.544
19	-6.213
20	-5.865
21	-5.500
22	-5.115
23	-4.711
24	-4.286
25	-3.839
26	-3.369
27	-2.876
28	-2.357
29	-1.811
30	-1.237
31	-0.634
32	0.000

Table B.1: Vertical displacement of the top surface as a function of time

Figure B.1: Stress-Strain curve for strain rate 0.05 (Experimental data from [16, Fig. 2])

loading-unloading cycle due to the hysteretic behavior of the material. For the numerical example done in this work, the dissipated energy density u is equal to 0.6501 MPa, which is in good agreement with the experimental results by Bergstrom and Boyce (0.5878 MPa).

To calculate the energy dissipated in the whole model a simplification will be introduced, by multiplying the dissipated energy density by the *undeformed* volume of the element. Although this is not true because the Cauchy stress belongs to the *deformed* configuration, due to the low compressibility of the material used ($d = 0.004/\text{MPa}$), the change in volume of the model is small. Therefore, the total energy dissipated in the sample used by Bergstrom and Boyce can be approximated by

$$\mathcal{U}_{\text{creep}} \approx \pi \frac{\phi^2}{2} h u = 8005 \text{ mm}^3 0.5878 \text{ MPa} = 4705 \text{ mJ} \quad (\text{B.2})$$

On the other hand, when the total creep-dissipated energy is queried at the last time step through the history output variable ALLCD, the result given by ABAQUS® is $\text{ALLCD}|_{t=32\text{s}} = 10\,578.7 \text{ mJ}$. The evolution of the variable ALLCD is included in Figure B.2.

The dissipation energy calculated by ABAQUS® is over twice as the energy calculated in (B.2) for the experimental sample. When the dissipation is calculated by numerical integration using the post-processing script developed by the author and presented in the following Section, the dissipated energy obtained is 5255.1 mJ, which is in very good agreement with the experimental data. Thus, the dissipation calculation for the BB model can not be done

Figure B.2: Creep dissipation energy (**ALLCD**) for the whole model

using ABAQUS® built-in method and must be done using the script proposed by the author.

B.2 PYTHON Script

In order to calculate the dissipated energy correctly, a PYTHON script was developed. This script is ran after the analysis is done and it uses the output database from ABAQUS®. The field outputs that this script must read are

LE Logarithmic Strain Tensor

S Cauchy Stress Tensor

IVOL Integration Point Volume

The script must be able also to query the history output **ALLSE**, the total recoverable (elastic) strain energy at the last frame (time step) of the analysis.

The total strain energy can be calculated using numerical integration as

$$\mathcal{W} = \sum_{k=2}^n \sum_{l=1}^m \sum_{i=1}^p \left(\text{LE}_i^{k,l} - \text{LE}_i^{k-1,l} \right) \frac{\left(S_i^{k,l} + S_i^{k-1,l} \right)}{2} \frac{(\text{IVOL}^{k,l} + \text{IVOL}^{k-1,l})}{2} \quad (\text{B.3})$$

where n is the number of frames in the analysis (time steps), m is the number of integration points in the model and p is the number of independent components of the stress and strain tensors in the analysis. For example, in a three-dimensional analysis $p = 6$, while in a plane strain analysis $p = 3$. The first (outer) summation loops over the frames of the analysis, the second over integration points and the third over the components of the stress and strain tensors.

To save computational time, the integration can be carried out only in the regions made of hysteretic material. For an elastic material, the strain energy is equal to zero when the material is unloaded. Viscous materials exhibit a phenomena where the strains lag behind the stresses. This means that even when the stress is removed, the strain might not be equal to zero. Therefore, the script must subtract the recoverable elastic strain energy from the total strain energy in order to calculate the dissipated energy.

$$\mathcal{U}_{\text{creep}} = \mathcal{W} - \text{ALLSE}|_{k=n} \quad (\text{B.4})$$

where $\mathcal{W}$ is given by (B.3) and $\text{ALLSE}|_{k=n}$ is the value of the recoverable strain energy at the last frame for the region where $\mathcal{W}$ was calculated.

A temporary variable is created to hold the frame repository. This noticeably speeds up the process. Further performance improvements could be done by accessing the output database using C++ instead of PYTHON.

B.3 Code Listing

The following lines of code will be included in most of the scripts used to analyze the different models in Chapter 4. They are included here again, independently of the other code, for the sake of clarity.

```

1 #Initialize the variable where the dissipation calculated will be stored
2 dissipation = 0.0
3
4 #Begin the Post-Processing
5
6 #Open The Output Data Base of the Current Job
7 odb = openOdb(path='Job-1.odb')
8 myAssembly = odb.rootAssembly
9
10 #Integrate ONLY through the elements made of PolymerMatrix
11 BASEFILM = odb.rootAssembly.instances['RVE-1'].elementSets['BASEFILM']
12
13 #Creating the repository to speeds up the process
14 frameRepository = odb.steps['SinusoidalTorque'].frames
15 frameLE=[]
16 frameS=[]
17 frameIVOL=[]
18 frameU=[]
19 for i in range(len(frameRepository)-1):
20     frameLE.insert(i,frameRepository[i].fieldOutputs['LE'].
21         getSubset(region=BASEFILM,position=INTEGRATION_POINT))
22     frameS.insert(i,frameRepository[i].fieldOutputs['S'].
```

```

23         getSubset(region=BASEFILM, position=INTEGRATION_POINT))
24     frameIVOL.insert(i, frameRepository[i].fieldOutputs['IVOL']).
25         getSubset(region=BASEFILM, position=INTEGRATION_POINT))
26     frameU.insert(i, frameRepository[i].fieldOutputs['U']).
27         getSubset(region=TOP, position=NODAL))
28
29 #Number of frames in the step to be integrated
30 N_frames=len(frameLE)
31
32 #Initialize the following temporary variable
33 dummy=0
34
35 #Do the integration. The index II loops over the integration points, JJ loops
over the frames, KK over the different components of the stress/strain
tensors.
36 for II in range(0, len(frameLE[0].values)):
37     for JJ in range(0+1, N_frames):
38         x_c=frameLE[JJ].values[II].data
39         x_w=frameLE[JJ-1].values[II].data
40         y_c=frameS[JJ].values[II].data
41         y_w=frameS[JJ-1].values[II].data
42         v_c=frameIVOL[JJ].values[II].data
43         v_w=frameIVOL[JJ-1].values[II].data
44         dummy=0.25*(x_c-x_w)*(y_c+y_w)*(v_c+v_w)
45         dummy_2=0
46         for KK in range(0, len(dummy)):
47             dummy_2=dummy_2+dummy[KK]
48         dissipation=dissipation+dummy_2
49
50 #Subtract the Recoverable elastic strain in the THE BASE FILM (Polymer Matrix
Set)
51 dissipation=dissipation-odb.steps['SinusoidalTorque'].historyRegions['
    ElementSet_BASEFILM'].historyOutputs['ALLSE'].data[-1][1]
52 print dissipation

```


Appendix C

Relationship between the two- and three-dimensional loads

Cairns et al. suggest that the actuators generate torque by volume [21, 22]. In the following Sections, the relationship between the surface traction that must be applied to generate torque by volume for the two- and three-dimensional cases will be developed.

C.1 Three-dimensional case

In the three-dimensional case the geometry of the actuator is a *prolate spheroid*, a particular case of an ellipsoid. The equation that describes the boundary of a prolate spheroid is given by,

$$\frac{x^2 + y^2}{a^2} + \frac{z^2}{c^2} = 1, \quad c > a \quad (\text{C.1})$$

where a, c are the minor and major radii, respectively.

The torque generated by a uniform traction acting in the surface of the actuator in the z direction, can be calculated considering only 1/8 of the actuator due to symmetry conditions.

If the differential torque is generated by a traction F_z acting on the surface of the ellipsoid, times the arm, the differential torque is proportional to the volume. The arm is given by the height y from the xz plane to the surface where the traction is applied, as it can be seen in Figure C.1.

From (C.1), for the first octant, the distance y to the xz plane can be written as,

$$y(x, z) = \sqrt{\left(1 - \frac{z^2}{c^2}\right) a^2 - x^2} \quad (\text{C.2})$$

Therefore, for every point in the xz plane, the corresponding arm is given by (C.2). The torque generated by the actuator can be calculated as

$$\text{Torque} = 8 \int \int_{\Omega} y(x, z) F_z dx dz \quad (\text{C.3})$$

Figure C.1: Three-dimensional torque differential

As the surface traction F_z is independent of x, z , it can be pulled out of the integral. The remaining integral multiplied by eight, is equivalent to the volume of the prolate spheroid.

$$\text{Torque} = F_z 8 \iint_{\Omega} y(x, z) dx dz = F_z \text{Volume} \quad (\text{C.4})$$

Therefore, it can be said that the relation between the torque generated by the actuator and its volume is constant and equal to the value of the surface traction F_z ,

$$\frac{\text{Torque}}{\text{Volume}} = F_z \quad (\text{C.5})$$

C.2 Two-dimensional case

For the two-dimensional case, the geometry of the actuator is an *elliptic cylinder*. In this case, symmetry dictates that the calculation of the torque can be split in four parts.

$$\text{Torque} = 4 \int_0^L \int_0^c y(z) F_z dz dx \quad (\text{C.6})$$

where L is the depth of the model.

Again, the surface traction F_z is independent of x, z , it can be pulled out of the integral,

$$\text{Torque} = F_z L 4 \int_0^c y(z) dz \quad (\text{C.7})$$

where $L 4 \int_0^c y(z) dz$ is the Volume of the actuator. Therefore, it is possible to express the relation between the torque generated by the elliptical cylinder shaped actuator and its volume as,

Figure C.2: Two-dimensional torque differential

$$\frac{\text{Torque}}{\text{Volume}} = F_z \quad (\text{C.8})$$

From (C.5) and (C.8) it can be concluded that for a given surface traction F_z , the generated torque per unit volume of actuator is the same for the two- and three-dimensional cases.

Appendix D

Minimum RVE depth

The process currently used to create the actuated polymer film, consists in the dispersion of liquid crystal droplets in a base film made of polyvinyl alcohol (PVA). After the PVA has dried, the film is stretched. In this stretched configuration, the film is exposed to ultra violet light to cure the liquid crystal actuators [42].

Assuming that the droplets were perfect spheres before the stretching and that when stretched and cured they become prolate spheroids, it is possible to calculate the minimum separation, in the depth direction, that two parallel actuators can have (See Figure D.1). An additional assumption must be made, that the volume of the liquid crystal does not changes when the film is stretched.

The volume of the initial spherical droplet is given by,

$$\text{Volume}_{\text{sphere}} = \frac{4}{3}\pi r^3 \quad (\text{D.1})$$

where r is the radius of the sphere.

The volume of a prolate spheroid is given by,

$$\text{Volume}_{\text{p.sph.}} = \frac{4}{3}\pi a^2 c \quad (\text{D.2})$$

where a, c are the minor and major radii, respectively.

Figure D.1: Spherical droplets become prolate spheroids when stretched

Figure D.2: Minimum theoretical depth of the RVE

If the volume of the liquid crystal droplet does not change during the stretching,

$$\text{Volume}_{\text{p.sph.}} = \text{Volume}_{\text{sphere}} \quad (\text{D.3})$$

Therefore,

$$r^3 = a^2 c \quad (\text{D.4})$$

Defining the aspect ratio AR of the prolate spheroid as,

$$\text{AR} = \frac{c}{a} \quad (\text{D.5})$$

and replacing this definition in (D.4), it is possible to find the radius of the original sphere as a function of the prolate spheroid minor radius a and its aspect ratio AR,

$$r = a \sqrt[3]{\text{AR}} \quad (\text{D.6})$$

Under the aforementioned assumptions, in the limit case where two spherical droplets were cast in such way that they are tangential, the minimum depth for the RVE in the stretched configuration is given by (D.6), as it can be seen in Figure D.2.

Appendix E

ABAQUS Scripts

E.1 Contact Model - Partially Constrained Top Glass

```
1 print '#####'
2 print '#-----Joaquin M. Gutierrez-----#'
3 print '#West Virginia University--2010--#'
4 print '#####'
5
6
7 from part import *
8 from material import *
9 from section import *
10 from assembly import *
11 from step import *
12 from interaction import *
13 from load import *
14 from mesh import *
15 from job import *
16 from sketch import *
17 from visualization import *
18 from connectorBehavior import *
19
20 #####
21 session.journalOptions.setValues(recoverGeometry=COORDINATE)
22 #session.journalOptions.setValues(recoverGeometry=INDEX)
23 #session.journalOptions.setValues(recoverGeometry=COMPRESSEDINDEX)
24
25 #This sets the recover and journal files to record geometry commands in the
26 #form of coordinates instead of geometric indices. This step is very helpful
27 #if any type of parameterization will be used with the script.
28 #####
29
30 #####
31 # Variables
32 #####
33 #Script Name (It is important to keep it updated, for proper documentation of
34 #the results)
35 scriptName='CONTACT02c.py' #!!!!KEEP ME UPDATED!!!!
```

```

36 #Geometry
37 BF_W=120e-3          #Base Film Width [mm]
38 BF_T=30e-3          #Base Film Thickness [mm]
39 #TF_T=0.03e-3      #Top Film Thickness [mm]
40 EL_W=60e-3          #Ellipse Width [mm]
41 EL_T=10e-3          #Ellipse Thickness [mm]
42 EL_Y=BF_T*0.5      #Y Coordinate of the Ellipse Center [mm]
43
44
45 #Glass Weight (per unit depth) (From Nick's Email 2010-Dec-10)
46 W_Glass=1.226E-4*(2*BF_W) #[MPa]*mm=N/mm
47
48
49
50 #Duration of the ellipses loading
51 T_load_2=1.0
52
53 #Define the magnitude of the surface traction (load) to be applied [MPa]
54 magnitudeSurfaceTraction=10.00
55 #Left Ellipse Polarization (+1 for CCW rotation, -1 for CW rotation)
56 LE_pol=+1.00
57
58 #Right Ellipse Polarization (+1 for CCW rotation, -1 for CW rotation)
59 RE_pol=-1.00
60
61 #Mesh Seeds
62 S_HE_TF_BF=120      #Seeds @ Horizontal Edges of the Top Film and Base Film
63 S_HE_VE_BF=30       #Seeds @ Vertical Edges of the Base Film
64 S_E=120              #Seeds @ Ellipse Edges
65
66 #For Student Edition (1000 nodes max) NO WAY!
67 #WITH COARSE MESHES DOES NOT CONVERGES!
68
69
70 #####
71 #      Geometry
72 #####
73 mdb.models['Model-1'].ConstrainedSketch(name='__profile__', sheetSize=4*BF_W)
74 mdb.models['Model-1'].sketches['__profile__'].sketchOptions.setValues(
75     decimalPlaces=3)
76
77 #Base Film Rectangle (BF_W x BF_T)
78 mdb.models['Model-1'].sketches['__profile__'].rectangle(point1=(-BF_W*1, 0.0),
79     point2=(BF_W*1, BF_T))
80
81 #Create Part (2D Planar Deformable Body)
82 mdb.models['Model-1'].Part(dimensionality=TWO_D_PLANAR, name='Part-1', type=
83     DEFORMABLE_BODY)
84
85 #Create Base Shell and Delete Profile used to build the Rectangle
86 mdb.models['Model-1'].parts['Part-1'].BaseShell(sketch=
87     mdb.models['Model-1'].sketches['__profile__'])
88 del mdb.models['Model-1'].sketches['__profile__']
89

```

```

90 #Partition of the LEFT and RIGHT Ellipses from the Rectangle Begins
91 mdb.models[ 'Model-1' ]. ConstrainedSketch( gridSpacing=0.006, name='__profile__',
92     sheetSize=0.243, transform=
93     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. MakeSketchTransform(
94     sketchPlane=mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces[ 0 ],
95     sketchPlaneSide=SIDE1, sketchOrientation=RIGHT, origin=(0.00, 0.00, 0.0)))
96 mdb.models[ 'Model-1' ]. sketches[ '__profile__' ]. sketchOptions.setValues(
97     decimalPlaces=6)
98 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. projectReferencesOntoSketch( filter=
99     COPLANAR_EDGES, sketch=mdb.models[ 'Model-1' ]. sketches[ '__profile__' ])
100 #Sketch Ellipse
101 mdb.models[ 'Model-1' ]. sketches[ '__profile__' ]. EllipseByCenterPerimeter(
102     axisPoint1=(BF_W*0.5+EL_W*0.5, EL_Y), axisPoint2=(BF_W*0.5, EL_T*0.5+EL_Y),
103     center=(BF_W*0.5, EL_Y))
104 mdb.models[ 'Model-1' ]. sketches[ '__profile__' ]. EllipseByCenterPerimeter(
105     axisPoint1=(-BF_W*0.5+EL_W*0.5, EL_Y), axisPoint2=(-BF_W*0.5, EL_T*0.5+EL_Y)
106     ), center=(-BF_W*0.5, EL_Y))
107 #Do the actual Partition
108 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. PartitionFaceBySketch( faces=
109     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces[ 0 ], sketch=mdb.models[ 'Model-1' ]
110     ). sketches[ '__profile__' ])
111 del mdb.models[ 'Model-1' ]. sketches[ '__profile__' ]
112 #Partition of the LEFT and RIGHT Ellipses Ends
113
114 #Partition of the LEFT Ellipse
115 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. PartitionEdgeByParam( edges=
116     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges.findAt((( -BF_W*0.5, EL_T*0.5+
117     EL_Y, 0.0), ),
118     ), parameter=0.5)
119 #Create of the LEFT Ellipse Top and Bottom Surfaces
120 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. Surface( name='Ellipse-Left-Top',
121     side2Edges=mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges.findAt((( -BF_W*0.5,
122     EL_T*0.5+EL_Y, 0.0), ), ))
123 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. Surface( name='Ellipse-Left-Bottom',
124     side2Edges=mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges.findAt((( -BF_W*0.5,
125     -EL_T*0.5+EL_Y, 0.0), ), ))
126 #Partition of the RIGHT Ellipse
127 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. PartitionEdgeByParam( edges=
128     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges.findAt((( BF_W*0.5, EL_T*0.5+
129     EL_Y, 0.0), ),
130     ), parameter=0.5)
131 #Create of the RIGHT Ellipse Top and Bottom Surfaces
132 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. Surface( name='Ellipse-Right-Top',
133     side2Edges=mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges.findAt((( BF_W*0.5,
134     EL_T*0.5+EL_Y, 0.0), ), ))
135 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. Surface( name='Ellipse-Right-Bottom',
136     side2Edges=mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges.findAt((( BF_W*0.5,
137     -EL_T*0.5+EL_Y, 0.0), ), ))
138
139 #Instance Part-1-1 in the Assembly
140 mdb.models[ 'Model-1' ]. rootAssembly. DatumCsysByDefault( CARTESIAN)
141 mdb.models[ 'Model-1' ]. rootAssembly. Instance( dependent=ON, name='Part-1-1',

```



```

139     part=mdb.models[ 'Model-1' ]. parts[ 'Part-1' ])
140 mdb.models[ 'Model-1' ]. rootAssembly. regenerate()
141
142
143
144 #Define Left and Right Surfaces
145 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. Surface(name='LeftSurface', side1Edges=
146     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges.findAt((( -BF_W, BF_T*0.5, 0.0)
147         ), ))
148 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. Surface(name='RightSurface', side1Edges=
149     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges.findAt((( BF_W, BF_T*0.5, 0.0),
150         ), ))
151 #Define TOP Surface
152 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. Surface(name='Top', side1Edges=
153     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges.findAt((( -BF_W*0.99, BF_T,
154         0.0), ), ))
155
156 #Create Sets TOP, Bottom, Left, Right, PolymerMatrix
157 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. Set(faces=
158     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces.findAt(((0, BF_T*0.5,
159         0.0), (0.0, 0.0, 1.0)), ), name='PolymerMatrix')
160 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. Set(edges=
161     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges.findAt((( -BF_W, BF_T*0.5, 0.0)
162         ), ), name='Left')
163 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. Set(edges=
164     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges.findAt((( BF_W, BF_T*0.5, 0.0),
165         ), ), name='Right')
166 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. Set(edges=
167     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges.findAt(((0, BF_T, 0.0), ), ),
168     name='Top')
169 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. Set(edges=
170     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges.findAt(((0.0, 0.0, 0.0), ), ),
171     name='Bottom')
172
173 mdb.models[ 'Model-1' ]. rootAssembly. regenerate()
174
175
176 #####
177 # Displacement BC's
178 #####
179
180 #Displacement BC's
181 #Bottom Edge (UX=UY=0)
182 mdb.models[ 'Model-1' ]. DisplacementBC(amplitude=UNSET, createStepName='Initial',
183     distributionType=UNIFORM, fieldName='', localCsys=None, name='Bottom',
184     region=Region(
185     edges=mdb.models[ 'Model-1' ]. rootAssembly. instances[ 'Part-1-1' ]. edges.findAt
186         (((0.0,

```

```

186     0.0, 0.0), ), ), u1=SET, u2=SET, ur3=UNSET)
187
188 # mdb.models['Model-1'].DisplacementBC(amplitude=UNSET, createStepName='
    Initial',
189     # distributionType=UNIFORM, fieldName='', localCsys=None, name='Bottom',
190     # region=Region(
191     # edges=mdb.models['Model-1'].rootAssembly.instances['Part-1-1'].edges.
        getSequenceFromMask(
192     # mask=('[#4 ]', ), ), u1=UNSET, u2=SET, ur3=UNSET)
193
194 mdb.models['Model-1'].boundaryConditions['Bottom'].setValues(region=
195     mdb.models['Model-1'].rootAssembly.instances['Part-1-1'].sets['Bottom'])
196
197     # #Bottom Left Corner (On the Master Surface)
198 # mdb.models['Model-1'].DisplacementBC(amplitude=UNSET, createStepName='
    Initial',
199     # distributionType=UNIFORM, fieldName='', localCsys=None, name=
200     # 'Left Bottom Corner', region=Region(
201     # vertices=mdb.models['Model-1'].rootAssembly.instances['Part-1-1'].
        vertices.findAt((-BF_W,
202     # 0.0, 0.0), ), ), u1=SET, u2=UNSET, ur3=UNSET)
203
204 # mdb.models['Model-1'].DisplacementBC(amplitude=UNSET, createStepName='
    Initial',
205     # distributionType=UNIFORM, fieldName='', localCsys=None, name=
206     # 'Left Bottom Corner', region=Region(
207     # vertices=mdb.models['Model-1'].rootAssembly.instances['Part-1-1'].
        vertices.getSequenceFromMask(
208     # mask=('[#4 ]', ), ), u1=SET, u2=UNSET, ur3=UNSET)
209
210 #####
211 #                                     Periodic Boundary Conditions—>Constrains—>
    Tie
212 #####
213 mdb.models['Model-1'].Tie(adjust=OFF, master=
214     mdb.models['Model-1'].rootAssembly.instances['Part-1-1'].surfaces['
        LeftSurface']
215     , name='PBC', positionTolerance=BF_W*2.1, positionToleranceMethod=SPECIFIED
        ,
216     slave=
217     mdb.models['Model-1'].rootAssembly.instances['Part-1-1'].surfaces['
        RightSurface']
218     , thickness=ON, tieRotations=ON)
219
220 #####
221 #                                     MATERIALS
222 #####
223 #Create Rod Material
224 mdb.models['Model-1'].Material(name='Rod')
225 mdb.models['Model-1'].materials['Rod'].Elastic(table=((3000.0, 0.3), ))
226 # #Define Polymer Matrix Material Hysteresis Params.
227 # mdb.models['Model-1'].Material(name='BaseFilm')
228 # #Define Polymermatrix Materia using Coeff.
229

```

```

230 # mdb.models['Model-1'].materials['BaseFilm'].Hyperelastic(n=2, table=((
231     # -30.8732, 41.5025, 10.3921, -32.8395, 36.4617, 0.0, 0.0), ), testData=OFF
232     ,
233     # type=POLYNOMIAL, volumetricResponse=VOLUMETRICDATA)
234 # mdb.models['Model-1'].materials['BaseFilm'].hyperelastic.Hysteresis(
235     # table=((2.0, 0.5, 4.5, -1.0), ))
236
237 #Define Base Film Material using Coefficients for ARRUDA-BOYCE MODEL (8-Chain)
238 #Hytrel 4056 (TPE)
239 mdb.models['Model-1'].Material(name='BaseFilm')
240 #Units in N,mm
241 mdb.models['Model-1'].materials['BaseFilm'].Hyperelastic(table=((
242     4.68032146, 5311.53, 0.0), ), testData=OFF, type=ARRUDA_BOYCE,
243     volumetricResponse=VOLUMETRICDATA)
244 # mdb.models['Model-1'].materials['BaseFilm'].hyperelastic.UniaxialTestData(
245     # table=((2.83, 0.0471), (4.29, 0.0833), (5.5, 0.127), (6.2, 0.163),
246         (6.83,
247         # 0.207), (7.21, 0.243), (7.55, 0.286), (7.74, 0.323), (7.92, 0.366),
248         (8.02,
249         # 0.402), (8.09, 0.446), (8.15, 0.482), (8.18, 0.525), (8.2, 0.562),
250         (8.23,
251         # 0.605), (8.25, 0.641), (8.29, 0.685), (8.32, 0.721), (8.36, 0.765),
252         (8.39,
253         # 0.801), (8.44, 0.844), (8.48, 0.881), (8.53, 0.924), (8.57, 0.96),
254         (8.62,
255         # 1.0), (8.69, 1.05), (8.73, 1.08), (8.79, 1.13), (8.84, 1.16), (8.91,
256         1.21),
257         # (8.96, 1.24), (9.03, 1.29), (9.09, 1.32), (9.17, 1.37), (9.22, 1.4),
258         (9.3,
259         # 1.45), (9.37, 1.48), (9.45, 1.53), (9.52, 1.56), (9.6, 1.61), (9.68,
260         1.64),
261         # (9.78, 1.68), (9.86, 1.72), (10.0, 1.76), (10.1, 1.8), (10.2, 1.84),
262         (10.2,
263         # 1.88), (10.4, 1.92), (10.5, 1.96), (10.6, 2.0)))
264 # #BB Model - Network B Parameters (Hysteretic Behavior) !VERY IMPORTANT The
265     second constant HAS units!!! 1/(sec * MPa**m)
266 # NO VISCOUS MODEL FOR THIS ANALYSIS, JUST HYPERELASTIC!
267 # mdb.models['Model-1'].materials['BaseFilm'].hyperelastic.Hysteresis(table=((
268     # 2.0, 0.5, 4.5, -1.0), ))
269
270 #####
271 #
272 #####
273
274 #Create Section "RoD"
275 mdb.models['Model-1'].HomogeneousSolidSection(material='Rod', name='Rod',
276     thickness=None)
277 #Create Section "BaseFilm"
278 mdb.models['Model-1'].HomogeneousSolidSection(material='BaseFilm', name=
279     'BaseFilm', thickness=None)
280
281
282

```

```

273 #Assign Rod Section
274 mdb.models['Model-1'].parts['Part-1'].SectionAssignment(offset=0.0,
275     offsetField='', offsetType=MIDDLE_SURFACE, region=Region(
276     faces=mdb.models['Model-1'].parts['Part-1'].faces.findAt((( -BF.W*0.5,
277     EL.Y, 0.0), (0.0, 0.0, 1.0)), ((BF.W*0.5, EL.Y, 0.0), (0.0, 0.0,
278     1.0))), sectionName='Rod')
279
280
281 #Assign BaseFilm Section
282 mdb.models['Model-1'].parts['Part-1'].SectionAssignment(offset=0.0,
283     offsetField='', offsetType=MIDDLE_SURFACE, region=Region(
284     faces=mdb.models['Model-1'].parts['Part-1'].faces.findAt(((0,
285     EL.Y, 0.0), (0.0, 0.0, 1.0))), sectionName='BaseFilm')
286
287
288 #####
289 # MESH
290 #####
291
292 #SEEDS
293
294 #Seed Horizontal Edges of the Top Film and Base Film
295 mdb.models['Model-1'].parts['Part-1'].seedEdgeByNumber(edges=
296     mdb.models['Model-1'].parts['Part-1'].edges.getSequenceFromMask(['#85_'],
297     ), ), number=S_HE_TF_BF)
298
299 mdb.models['Model-1'].parts['Part-1'].seedEdgeByNumber(edges=
300     mdb.models['Model-1'].parts['Part-1'].edges.findAt(((0, 0.0, 0.0), ), ),
301     number=S_HE_TF_BF*2)
302 mdb.models['Model-1'].parts['Part-1'].seedEdgeByNumber(edges=
303     mdb.models['Model-1'].parts['Part-1'].edges.findAt(((0, BF.T, 0.0), ),
304     ), number=S_HE_TF_BF*2)
305
306 #Seed Vertical Edges of the Base Film
307 mdb.models['Model-1'].parts['Part-1'].seedEdgeByNumber(edges=
308     mdb.models['Model-1'].parts['Part-1'].edges.findAt(((BF.W, BF.T*0.5, 0.0),
309     ),
310     ), number=S_HE_VE_BF)
311 mdb.models['Model-1'].parts['Part-1'].seedEdgeByNumber(edges=
312     mdb.models['Model-1'].parts['Part-1'].edges.findAt((( -BF.W, BF.T*0.5, 0.0)
313     , ),
314     ), number=S_HE_VE_BF)
315
316 #Seed Ellipse Edges
317 mdb.models['Model-1'].parts['Part-1'].seedEdgeByNumber(edges=
318     mdb.models['Model-1'].parts['Part-1'].edges.findAt(((BF.W*0.5,
319     EL.T*0.5+EL.Y, 0.0), ),
320     ), number=S_E)
321 mdb.models['Model-1'].parts['Part-1'].seedEdgeByNumber(edges=
322     mdb.models['Model-1'].parts['Part-1'].edges.findAt(((BF.W*0.5,
323     -EL.T*0.5+EL.Y, 0.0), ),
324     ), number=S_E)
325 mdb.models['Model-1'].parts['Part-1'].seedEdgeByNumber(edges=
326     mdb.models['Model-1'].parts['Part-1'].edges.findAt((( -BF.W*0.5,
327     EL.T*0.5+EL.Y, 0.0), ),

```

```

325     ), number=S_E)
326 mdb.models['Model-1'].parts['Part-1'].seedEdgeByNumber(edges=
327     mdb.models['Model-1'].parts['Part-1'].edges.findAt((( -BF_W*0.5,
328     -EL_T*0.5+EL_Y, 0.0)), ),
329     ), number=S_E)
330
331
332
333
334
335 #MESH CONTROL
336
337
338 #Mesh Controls on Base Film, Actuator)
339 mdb.models['Model-1'].parts['Part-1'].setMeshControls(allowMapped=True,
340     regions=mdb.models['Model-1'].parts['Part-1'].faces.findAt((( -BF_W*0.5,
341     EL_Y, 0.0), (0.0, 0.0, 1.0)), ((BF_W*0.5,
342     EL_Y, 0.0), (0.0, 0.0,
343     1.0))), ))
344 mdb.models['Model-1'].parts['Part-1'].setMeshControls(allowMapped=True,
345     regions=mdb.models['Model-1'].parts['Part-1'].faces.findAt(((0,
346     EL_Y, 0.0), (0.0, 0.0, 1.0))), ))
347
348
349
350 #ELEMENT TYPES
351
352 #Element Type CPE4R, CPE3 @ Mask #6: Faces 1,2 Top Film, Actuator
353 #Plane Strain Elements, Reduced Integration
354 mdb.models['Model-1'].parts['Part-1'].setElementType(elemTypes=(ElemType(
355     elemCode=CPE4R, elemLibrary=STANDARD, secondOrderAccuracy=OFF,
356     hourglassControl=DEFAULT, distortionControl=DEFAULT), ElemType(
357     elemCode=CPE3, elemLibrary=STANDARD, distortionControl=DEFAULT)), regions=(
358     mdb.models['Model-1'].parts['Part-1'].faces.findAt((( -BF_W*0.5,
359     EL_Y, 0.0), (0.0, 0.0, 1.0)), ((BF_W*0.5,
360     EL_Y, 0.0), (0.0, 0.0,
361     1.0))), ), ))
362
363 #Element Type CPE4RH, CPE3H @ Mask #1: Face 0 (Base Film)
364 #Hybrid, Plane Strain Elements, Reduced Integration
365 #NOTE: Hybrid elements are needed for INCOMPRESSIBLE material
366 mdb.models['Model-1'].parts['Part-1'].setElementType(elemTypes=(ElemType(
367     elemCode=CPE4RH, elemLibrary=STANDARD, secondOrderAccuracy=OFF,
368     hourglassControl=DEFAULT), ElemType(elemCode=CPE3H, elemLibrary=STANDARD)),
369     regions=(mdb.models['Model-1'].parts['Part-1'].faces.findAt(((0,
370     EL_Y, 0.0), (0.0, 0.0, 1.0))), ), ))
371
372 #CREATE DATUM PLANES FOR FURTHER PARTITIONS TO ALLOW STRUCTURED MESHING
373 mdb.models['Model-1'].parts['Part-1'].DatumPlaneByPrincipalPlane(offset=BF_T
374     *0.5,
375     principalPlane=XZPLANE)
376 mdb.models['Model-1'].parts['Part-1'].DatumPlaneByPrincipalPlane(offset=BF_W
377     *0.5,
378     principalPlane=YZPLANE)

```

```

377 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. DatumPlaneByPrincipalPlane( offset==BF_W
    *0.5,
378     principalPlane=YZPLANE)
379 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. DatumPlaneByPrincipalPlane( offset=0.0,
380     principalPlane=YZPLANE)
381
382 #Switch to COMPRESSED INDEX to do the partitions...these could be defined
    using "findat" if needed
383 session.journalOptions.setValues(recoverGeometry=COMPRESSEDINDEX)
384
385 #CREATE FURTHER PARTITIONS TO ALLOW STRUCTURED MESHING
386 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. PartitionFaceByDatumPlane( datumPlane=
387     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. datums[32], faces=
388     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces.getSequenceFromMask(( '[#7_]' ,
389     ), ))
390 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. PartitionFaceByDatumPlane( datumPlane=
391     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. datums[34], faces=
392     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces.getSequenceFromMask(( '[#3f_]' ,
393     ), ))
394 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. PartitionFaceByDatumPlane( datumPlane=
395     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. datums[35], faces=
396     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces.getSequenceFromMask(( '[#3ff_]' ,
397     ), ))
398 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. PartitionFaceByDatumPlane( datumPlane=
399     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. datums[33], faces=
400     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces.getSequenceFromMask(( '[#fff_]' ,
401     ), ))
402
403 #SET STRUCTURED MESHING
404 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. setMeshControls( regions=
405     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces.getSequenceFromMask((
406     '[#ffff_]' , ), ), technique=STRUCTURED)
407
408 #GENERATE MESH
409 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. generateMesh()
410 mdb.models[ 'Model-1' ]. rootAssembly.regenerate()
411
412
413
414 #####
415 # Create Analytical Surface (TOP GLASS)
416 #####
417
418 mdb.models[ 'Model-1' ]. ConstrainedSketch( name='__profile__' , sheetSize=0.5)
419 mdb.models[ 'Model-1' ]. sketches[ '__profile__' ]. Line( point1=(-BF_W*1.1, 0.0) ,
    point2=
420     (BF_W*1.1, 0.0) )
421 mdb.models[ 'Model-1' ]. sketches[ '__profile__' ]. HorizontalConstraint( entity=
422     mdb.models[ 'Model-1' ]. sketches[ '__profile__' ]. geometry.findAt((0.0, 0.0) ,
423     ))
424 mdb.models[ 'Model-1' ]. Part( dimensionality=TWO_D_PLANAR, name='Part-2' , type=
425     ANALYTIC_RIGID_SURFACE)

```

```

426 mdb.models[ 'Model-1' ]. parts[ 'Part-2' ]. AnalyticRigidSurf2DPlanar ( sketch=
427     mdb.models[ 'Model-1' ]. sketches[ ' _-profile-_' ])
428 del mdb.models[ 'Model-1' ]. sketches[ ' _-profile-_' ]
429 #Define the Reference Point
430 mdb.models[ 'Model-1' ]. parts[ 'Part-2' ]. ReferencePoint ( point=(0.0, 0.0, 0.0))
431
432 #Define Surface
433 mdb.models[ 'Model-1' ]. parts[ 'Part-2' ]. Surface ( name='Glass-Surface ', side2Edges
    =
434     mdb.models[ 'Model-1' ]. parts[ 'Part-2' ]. edges.findAt (((-0.5*BF.W, 0.0, 0.0),
    ),
435     ))
436 mdb.models[ 'Model-1' ]. rootAssembly . regenerate ()
437
438 #Move Instance to the top of the film
439 mdb.models[ 'Model-1' ]. rootAssembly . Instance ( dependent=ON, name='Part-2-1',
440     part=mdb.models[ 'Model-1' ]. parts[ 'Part-2' ])
441 mdb.models[ 'Model-1' ]. rootAssembly . translate ( instanceList=('Part-2-1', ),
442     vector=(0.0, BF.T*1.20, 0.0))
443 #Constrain the glass to be in contact with the film
444 mdb.models[ 'Model-1' ]. rootAssembly . EdgeToEdge ( clearance=0.0, fixedAxis=
445     mdb.models[ 'Model-1' ]. rootAssembly . instances[ 'Part-1-1' ]. edges.findAt ((
446     -0.9*BF.W, BF.T, 0.0), ), flip=OFF, movableAxis=
447     mdb.models[ 'Model-1' ]. rootAssembly . instances[ 'Part-2-1' ]. edges.findAt ((
448     -0.9*BF.W, BF.T*1.20, 0.0), ))
449
450
451 #Apply BC to the Top Glass (UX=ROTZ=0)
452 mdb.models[ 'Model-1' ]. DisplacementBC ( amplitude=UNSET, createStepName='Initial'
    ,
453     distributionType=UNIFORM, fieldName='', localCsys=None, name='BC-3',
454     region=Region ( referencePoints=(
455     mdb.models[ 'Model-1' ]. rootAssembly . instances[ 'Part-2-1' ]. referencePoints
    [2],
456     )), u1=SET, u2=UNSET, ur3=SET)
457
458 #Create Step-1 : Where the glass weight will be applied
459 mdb.models[ 'Model-1' ]. StaticStep ( description='Apply_Glass_Weight ', name=
460     'Step-1', nlgeom=ON, previous='Initial')
461 #Apply Glass Weight Load
462 mdb.models[ 'Model-1' ]. ConcentratedForce ( cf2=-1.0*W_Glass, createStepName=
463     'Step-1', distributionType=UNIFORM, field='', localCsys=None, name='Load-1'
    ,
464     , region=Region ( referencePoints=(
465     mdb.models[ 'Model-1' ]. rootAssembly . instances[ 'Part-2-1' ]. referencePoints
    [2],
466     )))
467
468 #Create Step-2 : Where the ellipses are loaded (constant load, DC voltage)
469 mdb.models[ 'Model-1' ]. StaticStep ( description=
470     'Apply_Constant_Load_to_the_Actuators', initialInc=T_load_2/100.00, maxInc
    =T_load_2,
471     minInc=1/1000000.00*T_load_2, name='Step-2', previous='Step-1', timePeriod
    =T_load_2)

```

```

472
473 #Apply Load on Ellipse-Left-Top
474 mdb.models[ 'Model-1' ]. SurfaceTraction (createStepName='Step-2', directionVector
    =
475     ((0.0, 0.0, 0.0), (1.0, 0.0, 0.0)), distributionType=UNIFORM, field='',
476     follower=OFF, localCsys=None, magnitude=-1.00*LE_pol*
        magnitudeSurfaceTraction, name=
477     'Load-Ellipse-Left-Top', region=
478     mdb.models[ 'Model-1' ]. rootAssembly. instances[ 'Part-1-1' ]. surfaces[ 'Ellipse
        -Left-Top' ]
479     , traction=GENERAL)
480 #Apply Load on Ellipse-Left-Bottom
481 mdb.models[ 'Model-1' ]. SurfaceTraction (createStepName='Step-2', directionVector
    =
482     ((0.0, 0.0, 0.0), (1.0, 0.0, 0.0)), distributionType=UNIFORM, field='',
483     follower=OFF, localCsys=None, magnitude=1.00*LE_pol*
        magnitudeSurfaceTraction, name=
484     'Load-Ellipse-Left-Bottom', region=
485     mdb.models[ 'Model-1' ]. rootAssembly. instances[ 'Part-1-1' ]. surfaces[ 'Ellipse
        -Left-Bottom' ]
486     , traction=GENERAL)
487 #Apply Load on Ellipse-Right-Top
488 mdb.models[ 'Model-1' ]. SurfaceTraction (createStepName='Step-2', directionVector
    =
489     ((0.0, 0.0, 0.0), (1.0, 0.0, 0.0)), distributionType=UNIFORM, field='',
490     follower=OFF, localCsys=None, magnitude=-1.00*RE_pol*
        magnitudeSurfaceTraction, name=
491     'Load-Ellipse-Right-Top', region=
492     mdb.models[ 'Model-1' ]. rootAssembly. instances[ 'Part-1-1' ]. surfaces[ 'Ellipse
        -Right-Top' ]
493     , traction=GENERAL)
494 #Apply Load on Ellipse-Right-Bottom
495 mdb.models[ 'Model-1' ]. SurfaceTraction (createStepName='Step-2', directionVector
    =
496     ((0.0, 0.0, 0.0), (1.0, 0.0, 0.0)), distributionType=UNIFORM, field='',
497     follower=OFF, localCsys=None, magnitude=1.00*RE_pol*
        magnitudeSurfaceTraction, name=
498     'Load-Ellipse-Right-Bottom', region=
499     mdb.models[ 'Model-1' ]. rootAssembly. instances[ 'Part-1-1' ]. surfaces[ 'Ellipse
        -Right-Bottom' ]
500     , traction=GENERAL)
501
502
503 #Create Interaction Property for the Contact between Glass and Film (
    Frictionless)
504 mdb.models[ 'Model-1' ]. ContactProperty ( 'NOFric' )
505 mdb.models[ 'Model-1' ]. interactionProperties[ 'NOFric' ]. TangentialBehavior (
    formulation=FRICTIONLESS)
506
507
508 #Define Contact Properties
509 #WARNING: There are many parameters to play with here!!!
510 # mdb.models[ 'Model-1' ]. SurfaceToSurfaceContactStd (adjustMethod=NONE,
511     # clearanceRegion=None, createStepName='Initial', datumAxis=None,
    enforcement=

```



```

512     # SURFACE_TO_SURFACE, initialClearance=OMIT, interactionProperty=
513     # 'NOFric', master=
514     # mdb.models['Model-1'].rootAssembly.instances['Part-2-1'].surfaces['Glass
      -Surface']
515     # , name='Film-Glass', slave=
516     # mdb.models['Model-1'].rootAssembly.instances['Part-1-1'].surfaces['Top
      '],
517     # sliding=FINITE, surfaceSmoothing=NONE, thickness=ON)
518
519     mdb.models['Model-1'].SurfaceToSurfaceContactStd(adjustMethod=NONE,
520     clearanceRegion=None, createStepName='Initial', datumAxis=None,
      enforcement=
521     SURFACE_TO_SURFACE, initialClearance=OMIT, interactionProperty=
522     'NOFric', master=
523     mdb.models['Model-1'].rootAssembly.instances['Part-2-1'].surfaces['Glass-
      Surface']
524     , name='Film-Glass', slave=
525     mdb.models['Model-1'].rootAssembly.instances['Part-1-1'].surfaces['Top'],
526     sliding=SMALL, surfaceSmoothing=NONE, thickness=ON,
527     bondingSet=None, contactTracking=ONE_CONFIG)
528
529     #Create Job
530     mdb.Job(atTime=None, contactPrint=OFF, description='', echoPrint=OFF,
531     explicitPrecision=SINGLE, getMemoryFromAnalysis=True, historyPrint=OFF,
532     memory=50, memoryUnits=PERCENTAGE, model='Model-1', modelPrint=OFF,
533     multiprocessingMode=DEFAULT, name='Job-1', nodalOutputPrecision=SINGLE,
534     numCpus=1, numDomains=1, parallelizationMethodExplicit=DOMAIN, queue=None,
535     scratch='', type=ANALYSIS, userSubroutine='', waitHours=0, waitMinutes=0)
536
537     #Submit Job
538     mdb.jobs['Job-1'].submit(consistencyChecking=OFF)
539
540     mdb.jobs['Job-1'].waitForCompletion()
541
542     #####
543     # SOLUTION
544     #####
545
546
547     # Paste Post Processing Macro
548     print '#####'
549     print '#-----Joaquin M. Gutierrez-----#'
550     print '#_West_Virginia_University_-_2010_#'
551     print '#####'
552
553     # To make the Odb commands available to your script, you first need to import
      the odbAccess module using the following statements:
554     from odbAccess import *
555     from abaqusConstants import *
556
557     # To make the material and section Odb commands available to your script, you
      also need to import the relevant module using the following statements:
558     from odbMaterial import *
559     from odbSection import *

```

```

560
561 # Import Libraries
562 import datetime
563 import csv
564
565 # Prepare Name for the output file
566 now = datetime.datetime.now()
567 print now
568 filename='2D_contact_output'+'.csv'
569
570 # Open the output file and write headers
571 fOut = open(filename, 'ab') #was 'wb', try 'a' to append on the old
    file
572 writer = csv.writer(fOut)
573 writer.writerow([
574 'Date', 'Load_Magnitude_[MPa]', 'Left_El._Pol.', 'Right_El._Pol.',
575 'Maximum_U2_[mm]', 'Minimum_U2_[mm]',
576 'Bump_[mm]', '#_of_Elements', 'Base_Film_Width_[mm]', 'Base_Film_Thickness_[mm]',
577 'Ellipse_Width_[mm]', 'Ellipse_Thickness_[mm]', 'Script',
578 'Seeds_@_Horizontal_Edges_of_the_Base_Film',
579 'Seeds_@_Vertical_Edges_of_the_Base_Film',
580 'Seeds_@_Ellipse_Edges',
581 'Hyperelastic_Model', 'Degree', 'Hyperelastic_Parameters'])
582
583
584 ##Define Output Requests for the Model
585 # mdb.models['Model-1'].fieldOutputRequests['F-Output-1'].setValues(variables
    =
586 # 'U'))
587
588
589 # Begin the Post-Processing
590
591 # Open The Output Data Base of the Current Job
592
593 odb = openOdb(path='Job-1.odb')
594
595 myAssembly = odb.rootAssembly
596
597
598 # Integrate ONLY through the elements made of PolymerMatrix
599 polymerMatrix = odb.rootAssembly.instances['PART-1-1'].elementSets[ '
    POLYMERMATRIX' ]
600 topSurface = odb.rootAssembly.instances['PART-1-1'].nodeSets['TOP']
601
602
603 # Creating a temporary variable to hold the frame repository provides the same
    functionality and speeds up the process
604 frameU=odb.steps['Step-2'].frames[-1].fieldOutputs['U'].getSubset(region=
    topSurface, position=NODAL)
605
606 # Lookup for the maximum and minimum vertical displacement (U2) in the top
    surface
607 maximo=0.0

```

```

608 minimo=0.0
609 for JJ in range(0,len(frameU.values)):    #Loop over nodes
610     if frameU.values[JJ].data[1]>maximo:
611         maximo=frameU.values[JJ].data[1]
612
613
614 for JJ in range(0,len(frameU.values)):    #Loop over nodes
615     if frameU.values[JJ].data[1]<minimo:
616         minimo=frameU.values[JJ].data[1]
617
618 print maximo
619 # print minimo
620
621 now = datetime.datetime.now()
622 print now
623
624 # Get Number of elements in the model
625 numberElements=mdb.models['Model-1'].rootAssembly.verifyMeshQuality(
        ANALYSIS_CHECKS)['numElements']
626 # Write the results for the current frequency to the output file
627 writer.writerow([
628     now.strftime("%Y-%m-%d_%H:%M%S"), str(magnitudeSurfaceTraction),str(LE_pol),
        str(RE_pol),str(maximo),str(minimo),str(maximo-minimo),
629     numberElements,BF_W,BF_T,EL_W,EL_T,scriptName,
630     S_HE_TF_BF,S_HE_VE_BF,S_E,
631     odb.materials['BASEFILM'].hyperelastic.type,odb.materials['BASEFILM'].
        hyperelastic.n,
632     odb.materials['BASEFILM'].hyperelastic.table])
633 # Close the ODB for the current frequency
634 odb.close()
635
636 # Close The output file
637 fOut.close()
638
639 print "#####"
640 print "#_Analysis_Done_#"
641 print "#####"

```

E.2 Two-Dimensional Model

```

1 #VISCO17
2 #DATE: 2010-12-20
3 #17-a) Automatic incrementation is set with a maximum step size of 1/120
4
5 #VISCO16_refined_mesh
6 #DATE: 2010-12-20
7 #16_refined_mesh-a) Change meshing controls on the base film: free-quad-adv.
        front-use mapped
8 #VISCO16
9 #DATE: 2010-12-20

```

```

10 #16-a) Meshing Strategy: Create Partitions and mesh EVERYTHING as structured
    QUADS but, ONE by ONE in the following order: TFL,TFR,ATL,ATR,ABL,ABR,BFTL
    ,BFTR,BFBL,BFBR
11 #TF: top film, BF: base film, A: actuator, L,R:left/right, T,B: top, bottom
12 #16-b) Change name pattern of the ODBs
13 #16-c) Switch back to fixed incrementation. (1/120)
14 #16-d) Tie Constraint: positionTolerance=BF_W*1.2
15 #16-e) Parallel Solving, 85% Allocation Memory and Processor.
16 #16-f) Change to SEED by SIZE instead of by NUMBER (except for TF_T).
17
18 #VISCO15_autoinc
19 #DATE: 2010-12-19
20 #15_autoinc-a) Automatic incrementation is set
21 #VISCO15
22 #DATE: 2010-12-19
23 #Modifications:
24 #15-a) Indentation in the FOR loops
25 #15-b) Mesh Actuator and Base Film with Structued Mesh->Quads Only+Medial Axis
    Strategy
26
27 from part import *
28 from material import *
29 from section import *
30 from assembly import *
31 from step import *
32 from interaction import *
33 from load import *
34 from mesh import *
35 from job import *
36 from sketch import *
37 from visualization import *
38 from connectorBehavior import *
39
40 #####
41 #
42 #####
43 #Script Name (It is important to keep it updated, for proper documentation of
44 #the results)
45 scriptName='VISCO17.py' #!!!!KEEP ME UPDATED!!!!
46 #Geometry
47 BF_W=120e-3 #Base Film Width [mm]
48 BF_T=30e-3 #Base Film Thickness [mm]
49 #TF_T=0.03e-3 #Top Film Thickness [mm]
50 EL_W=60e-3 #Ellipse Width [mm]
51 EL_T=10e-3 #Ellipse Thickness [mm]
52 #EL_Y=BF_T*0.5 #Y Coordinate of the Ellipse Center [mm] !!!HAVE TO MOVE IT
    WITHIN THE BF_T LOOP!!!
53
54 # #Loop over different Base Film Thicknesses
55 # baseFilmThicknesses=[30e-3]#,40e-3]
56
57 #Loop over different Ellipse Centers
58 ellipseCenters=[BF_T*0.5]#[10.0e-3,20.0e-3]#
59

```

```

60 #Loop over different Top Film Thicknesses
61 topFilmThicknesses=[0.03e-3]
62
63 # #Loop over different Base Film Width
64 # baseFilmWidth=[120.0e-3]#[300.0e-3]# [90.0e-3,120e-3,150.0e-3,180.0e-3] #
65
66 #Excitation Frequencies
67 f=[200.00]#[80.0,110.0,140.0,170.0,200.0]
68 #Number of cycles to analyze
69 numberCycles=1.00
70 #Define the magnitude of the surface traction (load) to be applied
71 magnitudeSurfaceTraction=10.00
72
73 #Mesh Seeds
74 # S_VE_TF=1                                #Seeds @ Vertical Edges of the Top
      Film
75 # S_HE_TF_BF=100                            #Seeds @ Horizontal Edges of the Top Film and
      Base Film
76 # S_HE_VE_BF=15                            #Seeds @ Horizontal Edges of the Base Film
77 # S_E=50                                    #Seeds @ Ellipse Edges
78
79 S_VE_TF=1                                #Seeds @ Vertical
      Edges of the Top Film
80 #Switch these for by SIZE seeding instead of by NUMBER
81 S_HE_TF_BF=0.5e-3                        #Seeds @ Horizontal Edges of the Top Film and
      Base Film
82 S_HE_VE_BF=1e-3                            #Seeds @ Horizontal Edges of the Base Film
83 S_E=1e-3                                    #Seeds @
      Ellipse Edges
84
85 for EL_Y in ellipseCenters:
86     for TF_T in topFilmThicknesses:
87         #EL_Y=BF_T*0.5 #Y Coordinate of the Ellipse Center [mm]
88
89         #####
90         #                                Geometry
91         #####
92         mdb.models['Model-1'].ConstrainedSketch(name='__profile__',
          sheetSize=2*BF_W)
93         mdb.models['Model-1'].sketches['__profile__'].sketchOptions.
          setValues(
94             decimalPlaces=3)
95
96         #Base Film Rectangle (BF_W x BF_T)
97         mdb.models['Model-1'].sketches['__profile__'].rectangle(point1
          =(-BF_W*0.5, 0.0),
98             point2=(BF_W*0.5, BF_T+TF_T))
99
100        #Create Part (2D Planar Deformable Body)
101        mdb.models['Model-1'].Part(dimensionality=TWO_D_PLANAR, name='
          Part-1', type=
102            DEFORMABLE_BODY)
103

```

```

104      #Create Base Shell and Delete Profile used to build the
105      Rectangle
106      mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. BaseShell( sketch=
107      mdb.models[ 'Model-1' ]. sketches[ '__profile__' ])
108      del mdb.models[ 'Model-1' ]. sketches[ '__profile__' ]
109
110      #Partition of the Top Film Begins
111      #Create Sketch on the rectangle created before (Should be Face
112      "0")
113      mdb.models[ 'Model-1' ]. ConstrainedSketch( gridSpacing=0.006,
114      name='__profile__',
115      sheetSize=2*BF.W, transform=
116      mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. MakeSketchTransform(
117      sketchPlane=mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces[0],
118      sketchPlaneSide=SIDE1, sketchOrientation=RIGHT, origin=(0.00,
119      0.00,
120      0.0))
121      mdb.models[ 'Model-1' ]. sketches[ '__profile__' ]. sketchOptions.
122      setValues(
123      decimalPlaces=6)
124      mdb.models[ 'Model-1' ]. parts[ 'Part-1' ].
125      projectReferencesOntoSketch( filter=
126      COPLANAREDGES, sketch=mdb.models[ 'Model-1' ]. sketches[ '
127      __profile__' ])
128      #Sketch Line
129      mdb.models[ 'Model-1' ]. sketches[ '__profile__' ]. Line( point1=(-
130      BF.W*0.5, BF.T),
131      point2=(BF.W*0.5, BF.T))
132      #Do the Actual Partition
133      mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. PartitionFaceBySketch(
134      faces=
135      mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces[0], sketch=mdb.
136      models[ 'Model-1' ]. sketches[ '__profile__' ])
137      del mdb.models[ 'Model-1' ]. sketches[ '__profile__' ]
138      #Partition of the Top Film Ends
139
140      #Partition of the Ellipse from the Rectangle Begins
141      mdb.models[ 'Model-1' ]. ConstrainedSketch( gridSpacing=0.006,
142      name='__profile__',
143      sheetSize=0.243, transform=
144      mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. MakeSketchTransform(
145      sketchPlane=mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces[0],
146      sketchPlaneSide=SIDE1, sketchOrientation=RIGHT, origin=(0.00,
147      0.00, 0.0))
148      mdb.models[ 'Model-1' ]. sketches[ '__profile__' ]. sketchOptions.
149      setValues(
150      decimalPlaces=6)
151      mdb.models[ 'Model-1' ]. parts[ 'Part-1' ].
152      projectReferencesOntoSketch( filter=
153      COPLANAREDGES, sketch=mdb.models[ 'Model-1' ]. sketches[ '
154      __profile__' ])
155      #Sketch Ellipse
156      mdb.models[ 'Model-1' ]. sketches[ '__profile__' ].
157      EllipseByCenterPerimeter(

```

```

axisPoint1=(EL_W*0.5, EL_Y), axisPoint2=(0.0, EL_T*0.5+EL_Y),
    center=(0.0, EL_Y))
#Do the actual Partition
mdb.models['Model-1'].parts['Part-1'].PartitionFaceBySketch(
    faces=
    mdb.models['Model-1'].parts['Part-1'].faces[0], sketch=mdb.
        models['Model-1'].sketches['__profile__'])
del mdb.models['Model-1'].sketches['__profile__']
#Partition of the Ellipse Ends

#Instance Part-1-1 in the Assembly
mdb.models['Model-1'].rootAssembly.DatumCsysByDefault(
    CARTESIAN)
mdb.models['Model-1'].rootAssembly.Instance(dependent=ON, name=
    'Part-1-1',
    part=mdb.models['Model-1'].parts['Part-1'])
mdb.models['Model-1'].rootAssembly.regenerate()

#Partition the Ellipse Edge in halves
mdb.models['Model-1'].parts['Part-1'].PartitionEdgeByParam(
    edges=
    mdb.models['Model-1'].parts['Part-1'].edges.
        getSequenceFromMask(('[#10_]',
    ), ), parameter=0.5)
mdb.models['Model-1'].parts['Part-1'].regenerate()

#Define Ellipse-BottomSurface and TopSurface
mdb.models['Model-1'].parts['Part-1'].Surface(name='Ellipse-
    BottomSurface', side2Edges=
    mdb.models['Model-1'].parts['Part-1'].edges.
        getSequenceFromMask(('[#10_]',
    ), ))
mdb.models['Model-1'].parts['Part-1'].Surface(name='Ellipse-
    TopSurface',
    side2Edges=mdb.models['Model-1'].parts['Part-1'].edges.
        getSequenceFromMask(
    ('[#20_]', ), ))

#Define Left and Right Surfaces
mdb.models['Model-1'].parts['Part-1'].Surface(name='
    LeftSurface', side1Edges=
    mdb.models['Model-1'].parts['Part-1'].edges.
        getSequenceFromMask(('[#102_]',
    ), ))
mdb.models['Model-1'].parts['Part-1'].Surface(name='
    RightSurface', side1Edges=
    mdb.models['Model-1'].parts['Part-1'].edges.
        getSequenceFromMask(('[#48_]',
    ), ))

#Create Sets TOP, Bottom, Left, Right, PolymerMatrix
mdb.models['Model-1'].parts['Part-1'].Set(edges=

```

```

181         mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges .
182             getSequenceFromMask(( '[#4_]' ,
183         ), ), name='Bottom' )
184     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. Set( edges=
185         mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges .
186             getSequenceFromMask(( '[#80_]' ,
187         ), ), name='Top' )
188     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. Set( edges=
189         mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges .
190             getSequenceFromMask(( '[#102_]' ,
191         ), ), name='Left' )
192     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. Set( edges=
193         mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges .
194             getSequenceFromMask(( '[#48_]' ,
195         ), ), name='Right' )
196     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. Set( faces=
197         mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
198             getSequenceFromMask(( '[#1_]' ,
199         ), ), name='PolymerMatrix' )
200     mdb.models[ 'Model-1' ]. rootAssembly . regenerate()
201
202     #Redefine Surfaces , Sets Using Geometry Variables and findAt
203     #Left Set
204     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. Set( edges=
205         mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges . findAt((( -BF.W/2 ,
206             BF.T/2 , 0.0) ,
207         ), ), name='Left' )
208     #Right Set
209     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. Set( edges=
210         mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges . findAt((( BF.W/2 ,
211             BF.T/2 , 0.0) , ),
212         ), name='Right' )
213     #Top Set
214     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. Set( edges=
215         mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges . findAt((( 0.0 ,
216             BF.T+TF.T , 0.0) ,
217         ), ), name='Top' )
218     #Ellipse Bottom Surface
219     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. Surface( name=' Ellipse -
220         BottomSurface' ,
221         side2Edges=mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges . findAt
222             ((( 0.0 ,
223                 EL.Y-EL.T/2 , 0.0) , ), ) )
224     #Ellipse Top Surface
225     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. Surface( name=' Ellipse -
226         TopSurface' ,
227         side2Edges=mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges . findAt
228             ((( 0.0 ,
229                 EL.Y+EL.T/2 , 0.0) , ), ) )
230     #Left Surface
231     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. Surface( name='
232         LeftSurface' , side1Edges=
233         mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges . findAt((( -BF.W/2 ,
234             BF.T/2 , 0.0) ,

```



```

221         ), ))
222     #Right Surface
223     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. Surface( name=
224         'RightSurface', sideEdges=
225         mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges. findAt( ((BF_W/2,
226         BF_T/2, 0.0), ),
227     ))
228     #####
229     # Displacement BC's
230     #####
231     #Displacement BC's
232     #Bottom Edge (Glued -> UX=UY=0)
233     mdb.models[ 'Model-1' ]. DisplacementBC( amplitude=UNSET,
234         createStepName='Initial',
235         distributionType=UNIFORM, fieldName='', localCsys=None, name=
236         'Bottom',
237         region=Region(
238             edges=mdb.models[ 'Model-1' ]. rootAssembly. instances[ 'Part-1-1'
239             ]. edges. getSequenceFromMask(
240             mask=('[#4_]', ), ), u1=SET, u2=SET, ur3=UNSET)
241     mdb.models[ 'Model-1' ]. boundaryConditions[ 'Bottom' ]. setValues(
242         region=
243         mdb.models[ 'Model-1' ]. rootAssembly. instances[ 'Part-1-1' ]. sets
244         [ 'Bottom' ])
245     #Bottom Left Corner (On the Master Surface)
246     mdb.models[ 'Model-1' ]. DisplacementBC( amplitude=UNSET,
247         createStepName='Initial',
248         distributionType=UNIFORM, fieldName='', localCsys=None, name=
249         'Left_Bottom_Corner', region=Region(
250         vertices=mdb.models[ 'Model-1' ]. rootAssembly. instances[ 'Part
251         -1-1' ]. vertices. getSequenceFromMask(
252         mask=('[#4_]', ), ), u1=SET, u2=UNSET, ur3=UNSET)
253     #####
254     # Periodic Boundary Conditions
255     -> Constrains -> Tie
256     #####
257     mdb.models[ 'Model-1' ]. Tie( adjust=OFF, master=
258     mdb.models[ 'Model-1' ]. rootAssembly. instances[ 'Part-1-1' ].
259     surfaces[ 'LeftSurface' ]
260     , name='PBC', positionTolerance=BF_W*1.2,
261     positionToleranceMethod=SPECIFIED,
262     slave=
263     mdb.models[ 'Model-1' ]. rootAssembly. instances[ 'Part-1-1' ].
264     surfaces[ 'RightSurface' ]
265     , thickness=ON, tieRotations=ON)
266     #####
267     # MATERIALS
268     #####

```

```

#Create Top Film Material
mdb.models[ 'Model-1' ]. Material(name='TopFilm')
mdb.models[ 'Model-1' ]. materials[ 'TopFilm' ]. Elastic(table
    =((70000.0, 0.3), ))
#Create Rod Material
mdb.models[ 'Model-1' ]. Material(name='Rod')
mdb.models[ 'Model-1' ]. materials[ 'Rod' ]. Elastic(table=((3000.0,
    0.3), ))
#Define Polymer Matrix Material Hysteresis Params.
mdb.models[ 'Model-1' ]. Material(name='BaseFilm')
#Define Polymermatrix Materia using Coeff.
# mdb.models[ 'Model-1' ]. materials[ 'BaseFilm' ]. Hyperelastic(n
    =2, table=((
# -30.8732, 41.5025, 10.3921, -32.8395, 36.4617, 0.0, 0.0), )
    , testData=OFF,
# type=POLYNOMIAL, volumetricResponse=VOLUMETRICDATA)
# mdb.models[ 'Model-1' ]. materials[ 'BaseFilm' ]. hyperelastic .
    Hysteresis(
# table=((2.0, 0.5, 4.5, -1.0), ))
#Define Base Film Material using Coefficients for ARRUDA-BOYCE
    MODEL (8-Chain)
#Hytrel 4056 (TPE)
mdb.models[ 'Model-1' ]. Material(name='BaseFilm')
#Units in N,mm
mdb.models[ 'Model-1' ]. materials[ 'BaseFilm' ]. Hyperelastic(table
    =((
4.68032146, 5311.53, 0.0), ), testData=OFF, type=
    ARRUDA-BOYCE,
    volumetricResponse=VOLUMETRICDATA)
# mdb.models[ 'Model-1' ]. materials[ 'BaseFilm' ]. hyperelastic .
    UniaxialTestData(
# table=((2.83, 0.0471), (4.29, 0.0833), (5.5, 0.127), (6.2,
    0.163), (6.83,
# 0.207), (7.21, 0.243), (7.55, 0.286), (7.74, 0.323),
    (7.92, 0.366), (8.02,
# 0.402), (8.09, 0.446), (8.15, 0.482), (8.18, 0.525), (8.2,
    0.562), (8.23,
# 0.605), (8.25, 0.641), (8.29, 0.685), (8.32, 0.721),
    (8.36, 0.765), (8.39,
# 0.801), (8.44, 0.844), (8.48, 0.881), (8.53, 0.924),
    (8.57, 0.96), (8.62,
# 1.0), (8.69, 1.05), (8.73, 1.08), (8.79, 1.13), (8.84,
    1.16), (8.91, 1.21),
# (8.96, 1.24), (9.03, 1.29), (9.09, 1.32), (9.17, 1.37),
    (9.22, 1.4), (9.3,
# 1.45), (9.37, 1.48), (9.45, 1.53), (9.52, 1.56), (9.6,
    1.61), (9.68, 1.64),
# (9.78, 1.68), (9.86, 1.72), (10.0, 1.76), (10.1, 1.8),
    (10.2, 1.84), (10.2,
# 1.88), (10.4, 1.92), (10.5, 1.96), (10.6, 2.0)))
#BB Model - Network B Parameters (Hysteretic Behavior) !VERY
    IMPORTANT The second constant HAS units!!! 1/(sec * MPa**m
    )

```

```

294         mdb.models[ 'Model-1' ]. materials[ 'BaseFilm' ]. hyperelastic .
295             Hysteresis( table=((
296                 2.0, 0.5, 4.5, -1.0), ) )
297
298     #####
299     #
300     SECTIONS
301     #####
302     #Create Section "Top Film"
303     mdb.models[ 'Model-1' ]. HomogeneousSolidSection( material='
304         TopFilm', name='TopFilm',
305         thickness=None)
306     #Create Section "RoD"
307     mdb.models[ 'Model-1' ]. HomogeneousSolidSection( material='Rod',
308         name='Rod',
309         thickness=None)
310     #Create Section "BaseFilm"
311     mdb.models[ 'Model-1' ]. HomogeneousSolidSection( material='
312         BaseFilm', name=
313         'BaseFilm', thickness=None)
314
315     #Assign TopFilm Section
316     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. SectionAssignment( offset
317         =0.0,
318         offsetField='', offsetType=MIDDLE_SURFACE, region=Region(
319             faces=mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
320             getSequenceFromMask(
321                 mask=('[#4┐', ), ), ), sectionName='TopFilm')
322
323     #Assign BaseFilm Section
324     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. SectionAssignment( offset
325         =0.0,
326         offsetField='', offsetType=MIDDLE_SURFACE, region=Region(
327             faces=mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
328             getSequenceFromMask(
329                 mask=('[#1┐', ), ), ), sectionName='BaseFilm')
330     #Assign Rod Section
331     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. SectionAssignment( offset
332         =0.0,
333         offsetField='', offsetType=MIDDLE_SURFACE, region=Region(
334             faces=mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
335             getSequenceFromMask(
336                 mask=('[#2┐', ), ), ), sectionName='Rod')
337
338     #####
339     #
340     #####
341
342     #SEEDS
343     #Seed Vertical Edges of the Top Film
344     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. seedEdgeByNumber( edges=

```

MESH

```

337         mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges .
338             getSequenceFromMask(( '[#140_]' ,
339             ), ), number=S_VE_TF)
339         #Seed Horizontal Edges of the Top Film and Base Film
340         mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. seedEdgeBySize(edges=
341             mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges .
342             getSequenceFromMask(( '[#85_]' ,
343             ), ), size=S_HE_TF_BF)
343         #Seed Vertical Edges of the Base Film
344         mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. seedEdgeBySize(edges=
345             mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges .
346             getSequenceFromMask(( '[#a_]' ,
347             ), ), size=S_HE_VE_BF)
347         #Seed Ellipse Edges
348         mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. seedEdgeBySize(edges=
349             mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. edges .
350             getSequenceFromMask(( '[#30_]' ,
351             ), ), size=S_E)
352
353         #MESH CONTROL
354         #Mesh Control @ Top Film
355         mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. setMeshControls(
356             elemShape=QUAD, regions=
357             mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
358             getSequenceFromMask(( '[#4_]' ,
359             ), ), technique=STRUCTURED)
359
360         #Mesh Controls on Mask #3: Faces 0,1 (Base Film, Actuator)
361         # mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. setMeshControls (
362             allowMapped=True,
363             # regions=mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
364             getSequenceFromMask ((
365             # '[#3 ]', ), ))
366         mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. setMeshControls(
367             algorithm=MEDIAL_AXIS,
368             elemShape=QUAD, regions=
369             mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
370             getSequenceFromMask(( '[#3_]' ,
371             ), ))
372
373         #ELEMENT TYPES
374
375         #Element Type CPE4R, CPE3 @ Mask #6: Faces 1,2 Top Film,
376         Actuator
377         #Plane Strain Elements, Reduced Integration
378         mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. setElementType(elemTypes
379             =(ElemType(
380             elemCode=CPE4R, elemLibrary=STANDARD, secondOrderAccuracy=OFF
381             ,
382             hourglassControl=DEFAULT, distortionControl=DEFAULT),
383             ElemType(
384             elemCode=CPE3, elemLibrary=STANDARD, distortionControl=
385             DEFAULT)), regions=(

```

```

376     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
377         getSequenceFromMask(( '[#6_]' ,
378     ), ), ))
379
380     #Element Type CPE4RH, CPE3H @ Mask #1: Face 0 (Base Film)
381     #Hybrid, Plane Strain Elements, Reduced Integration
382     #NOTE: Hybrid elements are needed for INCOMPRESSIBLE material
383     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. setElementType( elemTypes
384         =(ElemType(
385             elemCode=CPE4RH, elemLibrary=STANDARD, secondOrderAccuracy=
386             OFF,
387             hourglassControl=DEFAULT), ElemType(elemCode=CPE3H,
388             elemLibrary=STANDARD)),
389         regions=(mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
390             getSequenceFromMask((
391             '[#1_]' , ), ), ))
392
393     #GENERATE MESH
394     # #Generate Mesh on Mask#4:Face 2 (Top Film)
395     # mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. generateMesh( regions=
396     # mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
397         getSequenceFromMask(( '[#4 ]' ,
398     # ), ))
399
400     # #Generate Mesh on Mask#2:Face 1 (Actuator)
401     # mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. generateMesh( regions=
402     # mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
403         getSequenceFromMask(( '[#1 ]' ,
404     # ), ))
405
406     # #Generate Mesh on Mask#1:Face 0 (Base Film)
407     # mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. generateMesh( regions=
408     # mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
409         getSequenceFromMask(( '[#1 ]' ,
410     # ), ))
411
412     #mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. deleteMesh ( )
413     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ].
414         DatumPlaneByPrincipalPlane( offset=EL_Y,
415             principalPlane=XZPLANE)
416     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ].
417         DatumPlaneByPrincipalPlane( offset=0.0,
418             principalPlane=YZPLANE)
419     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ].
420         PartitionFaceByDatumPlane( datumPlane=
421             mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. datums[33] ,
422             faces=
423             mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
424             getSequenceFromMask(( '[#7_]' ,
425             ), ))
426     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ].
427         PartitionFaceByDatumPlane( datumPlane=

```

```

415         mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. datums[32] ,
416         faces=
417         mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
418         getSequenceFromMask(( '[#3f_]' ,
419         ), )
420     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. setMeshControls( regions=
421     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
422     getSequenceFromMask(( '[#210_]' ,
423     ), ), technique=STRUCTURED)
424     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. setMeshControls( regions=
425     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
426     getSequenceFromMask(( '[#80_]' ,
427     ), ), technique=STRUCTURED)
428     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. setMeshControls( regions=
429     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
430     getSequenceFromMask(( '[#1_]' ,
431     ), ), technique=STRUCTURED)
432     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. setMeshControls( regions=
433     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
434     getSequenceFromMask(( '[#40_]' ,
435     ), ), technique=STRUCTURED)
436     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. setMeshControls( regions=
437     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
438     getSequenceFromMask(( '[#8_]' ,
439     ), ), technique=STRUCTURED)
440     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. setMeshControls( regions=
441     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
442     getSequenceFromMask(( '[#100_]' ,
443     ), ), technique=STRUCTURED)
444     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. setMeshControls( regions=
445     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
446     getSequenceFromMask(( '[#20_]' ,
447     ), ), technique=STRUCTURED)
448     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. setMeshControls( regions=
449     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
450     getSequenceFromMask(( '[#2_]' ,
451     ), ), technique=STRUCTURED)
452     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. setMeshControls( regions=
453     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
454     getSequenceFromMask(( '[#4_]' ,
455     ), ), technique=STRUCTURED)
456     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. generateMesh( regions=
457     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
458     getSequenceFromMask(( '[#210_]' ,
459     ), )
460     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. generateMesh( regions=
461     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
462     getSequenceFromMask(( '[#126_]' ,
463     ), )
464     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. generateMesh( regions=
465     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
466     getSequenceFromMask(( '[#c9_]' ,
467     ), )

```

```

455 #Mesh the base film using free meshing
456 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. setMeshControls(
      algorithm=ADVANCINGFRONT
457     , elemShape=QUAD, regions=
458     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
      getSequenceFromMask(( '[#c9_]' ,
459     ), ), technique=FREE)
460 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. generateMesh( regions=
461     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
      getSequenceFromMask(( '[#80_]' ,
462     ), ) )
463 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. generateMesh( regions=
464     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
      getSequenceFromMask(( '[#40_]' ,
465     ), ) )
466 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. generateMesh( regions=
467     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
      getSequenceFromMask(( '[#1_]' ,
468     ), ) )
469 mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. generateMesh( regions=
470     mdb.models[ 'Model-1' ]. parts[ 'Part-1' ]. faces .
      getSequenceFromMask(( '[#8_]' ,
471     ), ) )
472
473 mdb.models[ 'Model-1' ]. rootAssembly . regenerate ()
474
475 #####
476 #
      SOLUTION
477 #####
478
479 #Paste Post Processing Macro
480 print '#####'
481 print '#_Joaquin_M._Gutierrez_#_'
482 print '#_West_Virginia_University_-_2010_#_'
483 print '#####'
484
485
486
487 #To make the Odb commands available to your script, you first
      need to import the odbAccess module using the following
      statements:
488 from odbAccess import *
489 from abaqusConstants import *
490
491 #To make the material and section Odb commands available to
      your script, you also need to import the relevant module
      using the following statements:
492 from odbMaterial import *
493 from odbSection import *
494
495 #Import Libraries
496 import datetime
497 import csv

```

```

498      #Prepare Name for the output file
499      now = datetime.datetime.now()
500      print now
501      filename='2D_1E_output'+'.csv'
502
503      #Open the output file and write headers
504      fOut = open(filename, 'ab')          #was 'wb', try 'a' to
505          append on the old file
506      writer = csv.writer(fOut)
507
508      # The HEADER writing is commented to avoid rewriting it in
509      # each analysis.
510      # If a new output file is created, it should be uncommented
511      # for the first analysis.
512
513      # writer.writerow([
514      # 'Date', 'Frequency [Hz]', 'Cycles []', 'Analysis Length [s]', '
515      # Load Magnitude [N/m]', 'Dissipation [mJ/mm]',
516      # 'Maximum U2 [mm]', 'Minimum U2 [mm]', 'Lin. Dissip. Power [mW/
517      # mm]', 'Surf. Dissip. Power [mW/mm2]',
518      # 'Bump [mm]', '# of Elements', 'Base Film Width [mm]', 'Base
519      # Film Thickness [mm]', 'Top Film Thickness [mm]',
520      # 'Ellipse Width [mm]', 'Ellipse Thickness [mm]', 'Y Coordinate
521      # of the Ellipse Center [mm]', 'Script',
522      # 'Seeds @ Vertical Edges of the Top Film',
523      # 'Seeds @ Horizontal Edges of the Top Film and Base Film',
524      # 'Seeds @ Horizontal Edges of the Base Film',
525      # 'Seeds @ Ellipse Edges',
526      # 'Hyperelastic Model', 'Degree', 'Hyperelastic Parameters', '
527      # Hysteresis Parameters'])
528
529      #Begin FOR loop for frequencies
530      for freqExcit in f:
531          periodExcit=1.00/freqExcit          #Excitation
532              Period
533          analysisPeriod=periodExcit*numberCycles #Length of
534              the Analysis
535          angularFreqExcit=2.00*3.1416*freqExcit #Angular
536              Frequency of the Excitation
537          inInc=periodExcit/(120.00)          #Time Step
538              Size
539
540          #Reset Dissipation to Zero at the begining of each
541          loop
542          dissipation = 0.0
543
544          #Create Amplitude
545          mdb.models['Model-1'].PeriodicAmplitude(a_0=0.0, data
546             =((0.0, 1.0), ),
547              frequency=angularFreqExcit, name='Sine', start
548              =0.0, timeSpan=STEP)
549
550          #Create Step 'Load' to Apply Load

```



```

537     mdb.models[ 'Model-1' ]. StaticStep ( description='Apply _
        Load', initialInc=inInc , maxNumInc=int (200*
538         numberCycles), name='OneCycle', nlgeom=ON,
            noStop=OFF, previous='Initial',
            timeIncrementationMethod=FIXED, timePeriod
            =analysisPeriod)
539     # WARNING: Changed to Automatic Incrementation. There
        will no longer be frames at fixed incremental
        time. The solver will choose.
540     mdb.models[ 'Model-1' ]. steps[ 'OneCycle' ]. setValues (
        timeIncrementationMethod=
541         AUTOMATIC)
542     mdb.models[ 'Model-1' ]. steps[ 'OneCycle' ]. setValues (
        initialInc=
543         inInc , maxInc=inInc , maxNumInc=250, minInc=
            inInc /1000.0)
544
545     #Load-One Cycle
546     #WARNING: check the stepnames
547     mdb.models[ 'Model-1' ]. SurfaceTraction (amplitude='Sine
        ', createStepName=
548         'OneCycle', directionVector=((0.0, 0.0, 0.0),
            (1.0, 0.0, 0.0)),
549         distributionType=UNIFORM, field='', follower=
            OFF, localCsys=None,
550         magnitude=magnitudeSurfaceTraction , name='
            TopLoad', region=
551         mdb.models[ 'Model-1' ]. rootAssembly . instances [ '
            Part-1-1' ]. surfaces [ 'Ellipse-TopSurface' ]
552         , resultant=ON, traction=GENERAL)
553     mdb.models[ 'Model-1' ]. Load(name='BottomLoad',
        objectToCopy=
554         mdb.models[ 'Model-1' ]. loads[ 'TopLoad' ])
555     mdb.models[ 'Model-1' ]. loads[ 'BottomLoad' ]. setValues (
        magnitude=-1.00*magnitudeSurfaceTraction , region=
556         mdb.models[ 'Model-1' ]. rootAssembly . instances [ '
            Part-1-1' ]. surfaces [ 'Ellipse-BottomSurface
            ' ])
557
558     #Regenerate Assembly
559     mdb.models[ 'Model-1' ]. rootAssembly . regenerate ()
560
561     #Create Job
562     # jobname=now. strftime ("%Y-%m-%d-%H-%M-%S")
563     # mdb.Job (atTime=None, contactPrint=OFF, description
        = '', echoPrint=OFF,
564         # explicitPrecision=SINGLE,
            getMemoryFromAnalysis=True, historyPrint=
            OFF,
565         # memory=50, memoryUnits=PERCENTAGE, model='
            Model-1', modelPrint=OFF,
566         # multiprocessingMode=DEFAULT, name=jobname ,
            nodalOutputPrecision=SINGLE,

```

```

567         # numCpus=1, numDomains=1,
568           parallelizationMethodExplicit=DOMAIN,
569           queue=None,
570         # scratch='', type=ANALYSIS, userSubroutine
571           ='', waitHours=0, waitMinutes=0)
572
573     # Parallel Solving, Allocate More CPU and Memory
574     jobname=now.strftime("%Y-%m-%d-%H-%M-%S")
575     mdb.Job(atTime=None, contactPrint=OFF, description='',
576           , echoPrint=OFF,
577           explicitPrecision=SINGLE,
578           getMemoryFromAnalysis=True, historyPrint=
579             OFF,
580           memory=85, memoryUnits=PERCENTAGE, model='
581             Model-1', modelPrint=OFF,
582           multiprocessingMode=THREADS, name=jobname,
583           nodalOutputPrecision=SINGLE,
584           numCpus=4, numDomains=4,
585           parallelizationMethodExplicit=DOMAIN,
586           queue=None,
587           scratch='', type=ANALYSIS, userSubroutine='',
588           waitHours=0, waitMinutes=0)
589
590     #Define Output Requests for the Model
591     mdb.models['Model-1'].fieldOutputRequests['F-Output-1
592       '].setValues(variables=(
593         'IVOL', 'LE', 'S', 'U'))
594     #Request Recoverable Strain Energy (SE) at the last
595       increment for the PolymerMatrix Set
596     mdb.models['Model-1'].HistoryOutputRequest(
597       createStepName='OneCycle',
598       frequency=LAST_INCREMENT, name='H-Output-2', rebar=
599         EXCLUDE, region=
600     mdb.models['Model-1'].rootAssembly.instances['Part
601       -1-1'].sets['PolymerMatrix']
602     , sectionPoints=DEFAULT, variables=('ALLSE', ))
603
604     #Submit Job
605     job=mdb.jobs[jobname]
606     job.submit(consistencyChecking=OFF)
607     job.waitForCompletion()
608     print"Frequency Solved"
609
610     #Begin the Post-Processing
611
612     #Open The Output Data Base of the Current Job
613
614     odb = openOdb(path=jobname+'.odb')
615
616     myAssembly = odb.rootAssembly

```

```

605 #Integrate ONLY through the elements made of
606 PolymerMatrix
606 polymerMatrix = odb.rootAssembly.instances[ 'PART-1-1'
        ].elementSets[ 'POLYMERMATRIX' ]
607 topSurface = odb.rootAssembly.instances[ 'PART-1-1' ].
        nodeSets[ 'TOP' ]
608
609 #step = odb.steps[ 'OneCycle' ]
610
611 #Creating a temporary variable to hold the frame
repository provides the same functionality and
speeds up the process
612
613 frameRepository = odb.steps[ 'OneCycle' ].frames
614 frameLE=[]
615 frameS=[]
616 frameIVOL=[]
617 frameU=[]
618 for i in range(len(frameRepository)-1):
619     frameLE.insert(i, frameRepository[i].fieldOutputs[ '
        LE' ].getSubset(region=polymerMatrix, position=
        INTEGRATION_POINT))
620     frameS.insert(i, frameRepository[i].fieldOutputs[ 'S'
        ].getSubset(region=polymerMatrix, position=
        INTEGRATION_POINT))
621     frameIVOL.insert(i, frameRepository[i].fieldOutputs[
        'IVOL' ].getSubset(region=polymerMatrix, position
        =INTEGRATION_POINT))
622     frameU.insert(i, frameRepository[i].fieldOutputs[ 'U'
        ].getSubset(region=topSurface, position=NODAL))
623
624 #Number of frames in the step to be integrated
625 N_frames=len(frameLE)
626 dummy=0
627
628 for II in range(0, len(frameLE[0].values)):
629     for JJ in range(0+1, N_frames):
630         x_c=frameLE[JJ].values[II].data
631
632         x_w=frameLE[JJ-1].values[II].data
633
634         y_c=frameS[JJ].values[II].data
635
636         y_w=frameS[JJ-1].values[II].data
637
638         v_c=frameIVOL[JJ].values[II].data
639         v_w=frameIVOL[JJ-1].values[II].data
640
641         dummy=0.25*(x_c-x_w)*(y_c+y_w)*(v_c+v_w)
642         dummy_2=0
643         for KK in range(0, len(dummy)):
644             dummy_2=dummy_2+dummy[KK]
645         dissipation=dissipation+dummy_2
646

```

```

647     print dissipation
648
649     #Subtract the Recoverable elastic strain in the THE
        BASE FILM (Polymer Matrix Set)
650     dissipation=dissipation-odb.steps['OneCycle'].
        historyRegions['ElementSet_POLYMERMATRIX'].
        historyOutputs['ALLSE'].data[-1][1]
651     print dissipation
652
653     #Lookup for the maximum and minimum vertical
        displacement (U2) in the top surface
654     maximo=0.0
655     minimo=0.0
656     frameMaximo=0
657     for II in range(0,N_frames):                                #Loop
        over frames
658         for JJ in range(0,len(frameU[0].values)):                #Loop
        over nodes
659             if frameU[II].values[JJ].data[1]>maximo:
660                 maximo=frameU[II].values[JJ].data[1]
661                 frameMaximo=II
662
663     for JJ in range(0,len(frameU[0].values)):                    #Loop
        over nodes for the frame where the maximum
        occurred
664         if frameU[frameMaximo].values[JJ].data[1]<
            minimo:
665             minimo=frameU[frameMaximo].values[JJ].data
                [1]
666
667     print freqExcit
668     print maximo
669     print minimo
670
671     now = datetime.datetime.now()
672     print now
673
674     #Get Number of elements in the model
675     numberElements=mdb.models['Model-1'].rootAssembly.
        verifyMeshQuality(ANALYSIS_CHECKS)['numElements']
676     #Write the results for the current frequency to the
        output file
677     writer.writerow([
678         now.strftime("%Y-%m-%d_%H:%M:%S"),str(freqExcit),
        str(numberCycles),str(analysisPeriod),
679         str(magnitudeSurfaceTraction),str(dissipation),str(
        maximo),str(minimo),
680         str(dissipation/analysisPeriod),str(dissipation/
        analysisPeriod/BF_W),str(maximo-minimo),
681         numberElements,BF_W,BF_T,TF_T,EL_W,EL_T,EL_Y,
        scriptName,
682         S_VE_TF,S_HE_TF_BF,S_HE_VE_BF,S_E,
683         odb.materials['BASEFILM'].hyperelastic.type,odb.
        materials['BASEFILM'].hyperelastic.n,

```

```

684         odb.materials[ 'BASEFILM' ]. hyperelastic.table ,
685         odb.materials[ 'BASEFILM' ]. hyperelastic.hysteresis .
           table
686     ])
687     #Close the ODB for the current frequency
688     odb.close()
689
690     #End FOR Loop
691
692 #Close The output file
693 fOut.close()
694
695 print "#####"
696 print "#_Analysis_Done_#"
697 print "#####"
```

E.3 Three-Dimensional Model

```

1  #REV 05d
2  #DATE= 2010-12-18
3  #Correction on the calculation of the volume fraction (just post-proc), doesn'
   t affect previous results.
4  #REV 05c
5  #DATE= 2010-12-11
6  #Switch back to millimeters to avoid problems with the hysteresis material
7
8  print '#####'
9  print '#_Joaquin_M._Gutierrez_#_'
10 print '#_West_Virginia_University_-_2010_#_'
11 print '#####'
12
13
14 from part import *
15 from material import *
16 from section import *
17 from assembly import *
18 from step import *
19 from interaction import *
20 from load import *
21 from mesh import *
22 from job import *
23 from sketch import *
24 from visualization import *
25 from connectorBehavior import *
26
27 #####
28 session.journalOptions.setValues(recoverGeometry=COORDINATE)
29 #session.journalOptions.setValues(recoverGeometry=INDEX)
30 #session.journalOptions.setValues(recoverGeometry=COMPRESSEDINDEX)
31
32 #This sets the recover and journal files to record geometry commands in the
```

```

33 #form of coordinates instead of geometric indices. This step is very helpful
34 #if any type of parameterization will be used with the script.
35 #####
36
37
38 #####
39 #Variables
40 #####
41 #Script Name (It is important to keep it updated, for proper documentation of
42 #the results)
43 scriptName='3DMODEL05d.py' #!!!!KEEP ME UPDATED!!!!
44 #Geometry (USE .0 to make these figures FLOATS and not INTEGERS!!!!)
45 BF_W=60.0E-3*5.0          #Base Film Width [mm]
46 BF_T=30.0E-3             #Base Film Thickness [mm]
47 BF_D=5.0E-3*2.5          #Base Film Depth [mm] (In the extrusion direction)
48 TF_T=0.03E-3             #Top Film Thickness [mm]
49 EL_W=60.0E-3             #Ellipsoid Major Diameter [mm]
50 EL_T=10.0E-3             #Ellipsoid Minor Diameter [mm]
51
52 #Mesh Seeds (Input INTEGERS for Number of Seeds and FLOATS for Element Sizes
53 #!!!)
54 # #These seeds are to be used in the 6.10 Student Edition (1000 nodes limit)
55 # S_D=2.0                #Element size along the DEPTH of the RVE
56 # S_T=2.0                #Element size along the THICKNESS of the BF
57 # S_TF=1                 #Seeds along the THICKNESS of the TF (USE INTEGER!!!)
58 # S_W_C=2.0              #Element size along the CENTRAL WIDTH of the RVE
59 # S_W_O_min=2.0          #Min Element Size along the OUTER WIDTH of the RVE (
60 #   BIASED)
61 # S_W_O_max=10.0         #Max Element Size along the OUTER WIDTH of the RVE
62 # S_E=2.0                #Element size along the ELLIPSE
63 # S_E_dev=0.1            #Deviator factor for curvature control allong
64 #   the ELLIPSE EDGES
65
66
67
68
69
70
71
72
73 S_D=1.17857E-3           #2.0E-3                      #Element size
74 #   along the DEPTH of the RVE
75 S_T=2.0E-3               #Element size along the THICKNESS of
76 #   the BF
77 S_TF=1                   #Seeds along the
78 #   THICKNESS of the TF (USE INTEGER!!!)
79 S_W_C=1.0E-3             #Element size along the CENTRAL WIDTH
80 #   of the RVE
81 S_W_O_min=1.0E-3         #Min Element Size along the OUTER
82 #   WIDTH of the RVE (BIASED)
83 S_W_O_max=20.0E-3        #Max Element Size along the OUTER WIDTH of the RVE
84 S_E=1.0E-3               #Element size along the
85 #   ELLIPSE
86 S_E_dev=0.1              #Deviator factor for
87 #   curvature control allong the ELLIPSE EDGES
88
89
90
91
92
93 #Definie the magnitude of the surface traction (load) to be applied (MPa)
94 magnitudeSurfaceTraction=10.00
95 #Excitation Frequency
96 freqExcit=200.00

```

```

77  #Number of cycles to analyze
78  numberCycles=1.00
79
80  #####
81  #Geometry
82  #####
83
84  #####
85  #BEGIN Create Block for Base Film+TopFilm
86  mdb.models[ 'Model-1' ]. ConstrainedSketch( name='__profile__', sheetSize=BF.W*2)
87  #Create Point at 0,0
88  mdb.models[ 'Model-1' ]. sketches[ '__profile__' ]. Spot( point=(0.0, 0.0))
89  #Create Rectangle Sketh
90  mdb.models[ 'Model-1' ]. sketches[ '__profile__' ]. rectangle( point1=(-BF.W*0.5, -
    BF.T*0.5),
91      point2=(BF.W*0.5, BF.T*0.5+TF.T))
92  mdb.models[ 'Model-1' ]. Part( dimensionality=THREE_D, name='BaseFilm', type=
93      DEFORMABLE_BODY)
94  #Extrude
95  mdb.models[ 'Model-1' ]. parts[ 'BaseFilm' ]. BaseSolidExtrude( depth=BF.D, sketch=
96      mdb.models[ 'Model-1' ]. sketches[ '__profile__' ])
97  del mdb.models[ 'Model-1' ]. sketches[ '__profile__' ]
98  #END Create Block for Base Film
99  #####
100 #BEGIN Create Ellipse
101 mdb.models[ 'Model-1' ]. ConstrainedSketch( name='__profile__', sheetSize=EL.W*2)
102
103 mdb.models[ 'Model-1' ]. sketches[ '__profile__' ]. ConstructionLine( angle=0.0,
104     point1=(0.0, 0.0))
105
106 mdb.models[ 'Model-1' ]. sketches[ '__profile__' ]. EllipseByCenterPerimeter(
107     axisPoint1=(-EL.W*0.5, 0.0), axisPoint2=(0.0, EL.T*0.5), center=(0.0, 0.0)
108 )
109
110 mdb.models[ 'Model-1' ]. sketches[ '__profile__' ]. autoTrimCurve( curve1=
111     mdb.models[ 'Model-1' ]. sketches[ '__profile__' ]. geometry.findAt( (0.0,
112     EL.T*0.5), ), point1=(0.0, EL.T*0.5))
113
114 mdb.models[ 'Model-1' ]. sketches[ '__profile__' ]. Line( point1=(-EL.W*0.5, 0.0),
115     point2=
116     (EL.W*0.5, 0.0))
117
118 mdb.models[ 'Model-1' ]. Part( dimensionality=THREE_D, name='Ellipsoid', type=
119     DEFORMABLE_BODY)
120
121 mdb.models[ 'Model-1' ]. parts[ 'Ellipsoid' ]. BaseSolidRevolve( angle=180.0,
122     flipRevolveDirection=OFF, sketch=
123     mdb.models[ 'Model-1' ]. sketches[ '__profile__' ])
124
125 del mdb.models[ 'Model-1' ]. sketches[ '__profile__' ]
126 #END Create Ellipse
127 #####
128
129 #####
130 #Instance the Base Film, Top Film and Ellipsoid in order to do the BOOLEAN
131 #The MESH will be INDEPENDENT of the PARTS, therefore it must be done at the
132 #ASSEMBLY level, not at the PART level.

```

```

128 mdb.models[ 'Model-1' ].rootAssembly.DatumCsysByDefault (CARTESIAN)
129 mdb.models[ 'Model-1' ].rootAssembly.Instance(dependent=OFF, name='BaseFilm-1',
130     part=mdb.models[ 'Model-1' ].parts[ 'BaseFilm' ])
131 mdb.models[ 'Model-1' ].rootAssembly.Instance(dependent=OFF, name='Ellipsoid-1',
132     part=mdb.models[ 'Model-1' ].parts[ 'Ellipsoid' ])
133 mdb.models[ 'Model-1' ].rootAssembly.InstanceFromBooleanMerge (domain=GEOMETRY,
134     instances=(mdb.models[ 'Model-1' ].rootAssembly.instances[ 'BaseFilm-1' ],
135     mdb.models[ 'Model-1' ].rootAssembly.instances[ 'Ellipsoid-1' ]),
136     keepIntersections=ON, name='RVE', originalInstances=SUPPRESS)
137 #####
138
139 #####
140 #BEGIN Create Datum Planes and do Partitions on the RVE
141 mdb.models[ 'Model-1' ].parts[ 'RVE' ].DatumPlaneByPrincipalPlane( offset=0.0,
142     principalPlane=XZPLANE)
143 mdb.models[ 'Model-1' ].parts[ 'RVE' ].DatumPlaneByPrincipalPlane( offset=0.0,
144     principalPlane=YZPLANE)
145 mdb.models[ 'Model-1' ].parts[ 'RVE' ].DatumPlaneByPrincipalPlane( offset=ELW
146     *0.75,
147     principalPlane=YZPLANE)
148 mdb.models[ 'Model-1' ].parts[ 'RVE' ].DatumPlaneByPrincipalPlane( offset=-ELW
149     *0.75,
150     principalPlane=YZPLANE)
151 mdb.models[ 'Model-1' ].parts[ 'RVE' ].DatumPlaneByPrincipalPlane( offset=BF_T*0.5,
152     principalPlane=XZPLANE)
153 #####
154 #Switch to COMPRESSEDINDEX notation
155 session.journalOptions.setValues (recoverGeometry=COMPRESSEDINDEX)
156 #####
157 #Partitions using COMPRESSED INDEXES
158 mdb.models[ 'Model-1' ].parts[ 'RVE' ].PartitionCellByDatumPlane ( cells=
159     mdb.models[ 'Model-1' ].parts[ 'RVE' ].cells.getSequenceFromMask(( '[#3_]' , ),
160     )
161     , datumPlane=mdb.models[ 'Model-1' ].parts[ 'RVE' ].datums[3])
162 mdb.models[ 'Model-1' ].parts[ 'RVE' ].PartitionCellByDatumPlane ( cells=
163     mdb.models[ 'Model-1' ].parts[ 'RVE' ].cells.getSequenceFromMask(( '[#f_]' , ),
164     )
165     , datumPlane=mdb.models[ 'Model-1' ].parts[ 'RVE' ].datums[2])
166 mdb.models[ 'Model-1' ].parts[ 'RVE' ].PartitionCellByDatumPlane ( cells=
167     mdb.models[ 'Model-1' ].parts[ 'RVE' ].cells.getSequenceFromMask(( '[#88_]' , ),
168     )
169     , datumPlane=mdb.models[ 'Model-1' ].parts[ 'RVE' ].datums[6])
170 mdb.models[ 'Model-1' ].parts[ 'RVE' ].PartitionCellByDatumPlane ( cells=
171     mdb.models[ 'Model-1' ].parts[ 'RVE' ].cells.getSequenceFromMask(( '[#62_]' , ),
172     )
173     , datumPlane=mdb.models[ 'Model-1' ].parts[ 'RVE' ].datums[5])
174 mdb.models[ 'Model-1' ].parts[ 'RVE' ].PartitionCellByDatumPlane ( cells=
175     mdb.models[ 'Model-1' ].parts[ 'RVE' ].cells.getSequenceFromMask(( '[#1028_]' ,
176     )
177     , ), datumPlane=mdb.models[ 'Model-1' ].parts[ 'RVE' ].datums[4])
178 #END Create Datum Planes and do Partitions on the RVE
179 #####
180
181 #####
182 #SETS
183 #####

```



```

178 #Bottom
179 mdb.models['Model-1'].parts['RVE'].Set(faces=
180     mdb.models['Model-1'].parts['RVE'].faces.getSequenceFromMask((
181         '[#10010_#200000_#1_]', ), ), name='Bottom')
182 #Symmetry Plane (where the ellipse is)
183 mdb.models['Model-1'].parts['RVE'].Set(faces=
184     mdb.models['Model-1'].parts['RVE'].faces.getSequenceFromMask((
185         '[#410a0848_#19c00a02_#4_]', ), ), name='Symm-1')
186 #Second Symmetry Plane
187 mdb.models['Model-1'].parts['RVE'].Set(faces=
188     mdb.models['Model-1'].parts['RVE'].faces.getSequenceFromMask((
189         '[#20c40620_#89001_#2_]', ), ), name='Symm-2')
190 #Top
191 mdb.models['Model-1'].parts['RVE'].Set(faces=
192     mdb.models['Model-1'].parts['RVE'].faces.getSequenceFromMask((
193         '[#200100_#40100000_]', ), ), name='Top')
194
195 #BaseFilm
196 mdb.models['Model-1'].parts['RVE'].Set(cells=
197     mdb.models['Model-1'].parts['RVE'].cells.getSequenceFromMask(( '[#991b_]',
198         ), ), name='BaseFilm')
199
200 #####
201 #SURFACES
202 #####
203 #Left
204 mdb.models['Model-1'].parts['RVE'].Surface(name='Left', side1Faces=
205     mdb.models['Model-1'].parts['RVE'].faces.getSequenceFromMask((
206         '[#10000000_#80000400_]', ), ))
207 #Right
208 mdb.models['Model-1'].parts['RVE'].Surface(name='Right', side1Faces=
209     mdb.models['Model-1'].parts['RVE'].faces.getSequenceFromMask((
210         '[#0_#20000104_]', ), ))
211 #Ellipse Top
212 mdb.models['Model-1'].parts['RVE'].Surface(name='Ellipse-Top', side1Faces=
213     mdb.models['Model-1'].parts['RVE'].faces.getSequenceFromMask((
214         '[#0_#4004000_]', ), ))
215 #Ellipse Bottom
216 mdb.models['Model-1'].parts['RVE'].Surface(name='Ellipse-Bottom', side1Faces=
217     mdb.models['Model-1'].parts['RVE'].faces.getSequenceFromMask((
218         '[#0_#2002000_]', ), ))
219
220 #####
221 #MATERIALS
222 #####
223 mdb.models['Model-1'].Material(name='TopFilm')
224 mdb.models['Model-1'].materials['TopFilm'].Elastic(table=((70000.0, 0.3),
225     ))
226 mdb.models['Model-1'].Material(name='Actuator')
227 mdb.models['Model-1'].materials['Actuator'].Elastic(table=((3000.0, 0.3),
228     ))
229
230 # #Define Base Film Material using Coefficients for ARRUDA-BOYCE MODEL (8-
    Chain)

```

```

231 # mdb.models['Model-1'].Material(name='BaseFilm ')
232 # mdb.models['Model-1'].materials['BaseFilm'].hyperelastic.setValues(table=((
233     # 4680320.25, 7083.00147, 0.0), ))
234 # #BB Model - Network B Parameters (Hysteretic Behavior)
235 # mdb.models['Model-1'].materials['BaseFilm'].hyperelastic.Hysteresis(table=((
236     # 2.0, 0.5, 4.5, -1.0), ))
237
238 #Define Base Film Material using Coefficients for ARRUDA-BOYCE MODEL (8-Chain)
239 #Hytrel 4056 (TPE)
240 mdb.models['Model-1'].Material(name='BaseFilm')
241 # mdb.models['Model-1'].materials['BaseFilm'].Hyperelastic(table=((
242     # 4680320.25, 7083.00147, 0.0), ), testData=OFF, type=ARRUDA_BOYCE,
243     # volumetricResponse=VOLUMETRIC_DATA)
244 #Units in N,mm
245 mdb.models['Model-1'].materials['BaseFilm'].Hyperelastic(table=((
246     4.68032146, 5311.53, 0.0), ), testData=OFF, type=ARRUDA_BOYCE,
247     volumetricResponse=VOLUMETRIC_DATA)
248 # mdb.models['Model-1'].materials['BaseFilm'].hyperelastic.UniaxialTestData (
249     # table=((2.83, 0.0471), (4.29, 0.0833), (5.5, 0.127), (6.2, 0.163),
250         (6.83,
251         # 0.207), (7.21, 0.243), (7.55, 0.286), (7.74, 0.323), (7.92, 0.366),
252         (8.02,
253         # 0.402), (8.09, 0.446), (8.15, 0.482), (8.18, 0.525), (8.2, 0.562),
254         (8.23,
255         # 0.605), (8.25, 0.641), (8.29, 0.685), (8.32, 0.721), (8.36, 0.765),
256         (8.39,
257         # 0.801), (8.44, 0.844), (8.48, 0.881), (8.53, 0.924), (8.57, 0.96),
258         (8.62,
259         # 1.0), (8.69, 1.05), (8.73, 1.08), (8.79, 1.13), (8.84, 1.16), (8.91,
260         1.21),
261         # (8.96, 1.24), (9.03, 1.29), (9.09, 1.32), (9.17, 1.37), (9.22, 1.4),
262         (9.3,
263         # 1.45), (9.37, 1.48), (9.45, 1.53), (9.52, 1.56), (9.6, 1.61), (9.68,
264         1.64),
265         # (9.78, 1.68), (9.86, 1.72), (10.0, 1.76), (10.1, 1.8), (10.2, 1.84),
266         (10.2,
267         # 1.88), (10.4, 1.92), (10.5, 1.96), (10.6, 2.0)))
268 #BB Model - Network B Parameters (Hysteretic Behavior) !VERY IMPORTANT The
269     second constant HAS units!!! 1/(sec * MPa**m)
270 mdb.models['Model-1'].materials['BaseFilm'].hyperelastic.Hysteresis(table=((
271     2.0, 0.5, 4.5, -1.0), ))
272
273 #####
274 #
275 #####
276 mdb.models['Model-1'].HomogeneousSolidSection(material='TopFilm', name=
277     'TopFilm', thickness=None)
278 mdb.models['Model-1'].HomogeneousSolidSection(material='Actuator', name=
279     'Actuator', thickness=None)
280 mdb.models['Model-1'].HomogeneousSolidSection(material='BaseFilm', name=
281     'BaseFilm', thickness=None)
282
283
284

```

```

274 # mdb.models['Model-1'].parts['RVE'].SectionAssignment(offset=0.0, offsetField
    ='',
275 # , offsetType=MIDDLE_SURFACE, region=Region(
276 # cells=mdb.models['Model-1'].parts['RVE'].cells.getSequenceFromMask(mask
    =(
277 # ['#e4'], ), ), sectionName='TopFilm', thicknessAssignment=FROM_SECTION
    )
278
279 mdb.models['Model-1'].parts['RVE'].SectionAssignment(offset=0.0, offsetField='
    ,
280 # , offsetType=MIDDLE_SURFACE, region=Region(
281 # cells=mdb.models['Model-1'].parts['RVE'].cells.getSequenceFromMask(mask=(
282 # ['#e4_'], ), ), sectionName='TopFilm')
283
284 # mdb.models['Model-1'].parts['RVE'].SectionAssignment(offset=0.0, offsetField
    ='',
285 # , offsetType=MIDDLE_SURFACE, region=Region(
286 # cells=mdb.models['Model-1'].parts['RVE'].cells.getSequenceFromMask(mask
    =(
287 # ['#6600'], ), ), sectionName='Actuator', thicknessAssignment=
288 # FROM_SECTION)
289 mdb.models['Model-1'].parts['RVE'].SectionAssignment(offset=0.0, offsetField='
    ,
290 # , offsetType=MIDDLE_SURFACE, region=Region(
291 # cells=mdb.models['Model-1'].parts['RVE'].cells.getSequenceFromMask(mask=(
292 # ['#6600_'], ), ), sectionName='Actuator')
293
294
295 # mdb.models['Model-1'].parts['RVE'].SectionAssignment(offset=0.0, offsetField
    ='',
296 # , offsetType=MIDDLE_SURFACE, region=Region(
297 # cells=mdb.models['Model-1'].parts['RVE'].cells.getSequenceFromMask(mask
    =(
298 # ['#991b'], ), ), sectionName='BaseFilm', thicknessAssignment=
299 # FROM_SECTION)
300
301 mdb.models['Model-1'].parts['RVE'].SectionAssignment(offset=0.0, offsetField='
    ,
302 # , offsetType=MIDDLE_SURFACE, region=Region(
303 # cells=mdb.models['Model-1'].parts['RVE'].cells.getSequenceFromMask(mask=(
304 # ['#991b_'], ), ), sectionName='BaseFilm')
305
306
307 #####
308 #MESH Module
309 #####
310 #Make the RVE Instance as Independent
311 mdb.models['Model-1'].rootAssembly.makeIndependent(instances=(
312     mdb.models['Model-1'].rootAssembly.instances['RVE-1'], ))
313
314
315 #SEEDS
316 #Using CompressedIndexes, won't work if the geometry is changed its entities,
    not its dimensions

```

```

317
318 #Biased SEEDS along the WIDTH
319 mdb.models['Model-1'].rootAssembly.seedEdgeByBias(biasMethod=SINGLE,
320     constraint=FINER, end1Edges=
321     mdb.models['Model-1'].rootAssembly.instances['RVE-1'].edges.
322     getSequenceFromMask(
323     ('[#a100000_#2800_#40204_]', ), ), end2Edges=
324     mdb.models['Model-1'].rootAssembly.instances['RVE-1'].edges.
325     getSequenceFromMask(
326     ('[#41400_#14040000_#10000001_]', ), ), maxSize=S_W_O_max, minSize=
327     S_W_O_min)
328
329
330 #Seed the CENTRAL WIDTH
331 mdb.models['Model-1'].rootAssembly.seedEdgeBySize(constraint=FINER,
332     deviationFactor=0.1, edges=
333     mdb.models['Model-1'].rootAssembly.instances['RVE-1'].edges.
334     getSequenceFromMask(
335     ('[#5141a000_#a0920000_#40000400_]', ), ), size=S_W_C)
336
337 mdb.models['Model-1'].rootAssembly.Set(edges=
338     mdb.models['Model-1'].rootAssembly.instances['RVE-1'].edges.
339     getSequenceFromMask(
340     ('[#5141a000_#a0920000_#40000400_]', ), ), name='Edge_Seeds-CentralWidth')
341
342 #Seed along the DEPTH of the RVE
343 mdb.models['Model-1'].rootAssembly.seedEdgeBySize(constraint=FINER,
344     deviationFactor=0.1, edges=
345     mdb.models['Model-1'].rootAssembly.instances['RVE-1'].edges.
346     getSequenceFromMask(
347     ('[#200095_#800122a_#10010b0_]', ), ), size=S_D)
348
349 mdb.models['Model-1'].rootAssembly.Set(edges=
350     mdb.models['Model-1'].rootAssembly.instances['RVE-1'].edges.
351     getSequenceFromMask(
352     ('[#200095_#800122a_#10010b0_]', ), ), name='Edge_Seeds-Depth')
353
354 #Seed the BASE FILM THICKNESS
355 mdb.models['Model-1'].rootAssembly.seedEdgeBySize(constraint=FINER,
356     deviationFactor=0.1, edges=
357     mdb.models['Model-1'].rootAssembly.instances['RVE-1'].edges.
358     getSequenceFromMask(
359     ('[#200a0b60_#805c0_#c00902_]', ), ), size=S_T)
360
361 mdb.models['Model-1'].rootAssembly.Set(edges=
362     mdb.models['Model-1'].rootAssembly.instances['RVE-1'].edges.
363     getSequenceFromMask(
364     ('[#200a0b60_#805c0_#c00902_]', ), ), name='Edge_Seeds-BaseFilmThickness')
365
366 #Seed the Ellipse
367 mdb.models['Model-1'].rootAssembly.seedEdgeBySize(constraint=FINER,
368     deviationFactor=S_E_dev, edges=

```

```

360     mdb.models[ 'Model-1' ].rootAssembly.instances[ 'RVE-1' ].edges.
        getSequenceFromMask(
361         ( '[#0_#2408000_#2e1bc000_]' , ) , ) , size=S_E)
362 mdb.models[ 'Model-1' ].rootAssembly.Set(edges=
363     mdb.models[ 'Model-1' ].rootAssembly.instances[ 'RVE-1' ].edges.
        getSequenceFromMask(
364         ( '[#0_#2408000_#2e1bc000_]' , ) , ) , name='Edge_Seeds-Ellipse' )
365
366 #SEED the vertical edges (along the thickness) of the TOP FILM
367 mdb.models[ 'Model-1' ].rootAssembly.seedEdgeByNumber(constraint=FINER, edges=
368     mdb.models[ 'Model-1' ].rootAssembly.instances[ 'RVE-1' ].edges.
        getSequenceFromMask(
369         ( '[#8400000a_#40000014_#200048_]' , ) , ) , number=S_TF)
370 mdb.models[ 'Model-1' ].rootAssembly.Set(edges=
371     mdb.models[ 'Model-1' ].rootAssembly.instances[ 'RVE-1' ].edges.
        getSequenceFromMask(
372         ( '[#8400000a_#40000014_#200048_]' , ) , ) , name=
373         'Edge_Seeds-TopFilmThickness' )
374
375 #Change the Element Type for the BaseFilm to Hybrid Formulation
376 #This needed in order to use incompressible Hyperlastic Material Model
377  #(C3D8RH): (C)ontinuum (3D) (8) Node (R)educed Integration (H)ybrid
378 mdb.models[ 'Model-1' ].rootAssembly.setElementType(elemTypes=(ElemType(
379     elemCode=C3D8RH, elemLibrary=STANDARD, secondOrderAccuracy=OFF,
380     kinematicSplit=AVERAGESTRAIN, hourglassControl=DEFAULT), ElemType(
381     elemCode=C3D6H, elemLibrary=STANDARD), ElemType(elemCode=C3D4H,
382     elemLibrary=STANDARD)), regions=(
383     mdb.models[ 'Model-1' ].rootAssembly.instances[ 'RVE-1' ].cells.
        getSequenceFromMask(
384         ( '[#991b_]' , ) , ) , ))
385
386 #####
387 #MESH the RVE (@INSTANCE level)
388 mdb.models[ 'Model-1' ].rootAssembly.generateMesh(regions=(
389     mdb.models[ 'Model-1' ].rootAssembly.instances[ 'RVE-1' ], ))
390
391 #####
392 #BC's
393 #####
394 #Bottom (Glued -> UX=UY=UZ=0)
395 mdb.models[ 'Model-1' ].DisplacementBC(amplitude=UNSET, createStepName='Initial'
396     ,
397     distributionType=UNIFORM, fieldName='', localCsys=None, name='Bottom' ,
398     region=mdb.models[ 'Model-1' ].rootAssembly.instances[ 'RVE-1' ].sets[ 'Bottom'
399     ]
400     , u1=SET, u2=SET, u3=SET, ur1=UNSET, ur2=UNSET, ur3=UNSET)
401 #Symm-1 ZSYMM (U3=UR1=UR2=0)
402 mdb.models[ 'Model-1' ].ZsymmBC(createStepName='Initial' , name='Symm-1' , region=
403     mdb.models[ 'Model-1' ].rootAssembly.instances[ 'RVE-1' ].sets[ 'Symm-1' ])
404 #Symm-2 ZSYMM (U3=UR1=UR2=0)
405 mdb.models[ 'Model-1' ].ZsymmBC(createStepName='Initial' , name='Symm-2' , region=
406     mdb.models[ 'Model-1' ].rootAssembly.instances[ 'RVE-1' ].sets[ 'Symm-2' ])

```

```

407 #####
408 #Periodic Boundary Conditions—>Constrains—>Tie
409 #WARNING 1: It is important to set the offset value correctly! (Base Film
      Width: BF_W)
410 #WARNING 2: SURFACE_TO_SURFACE might be a feature new to ABAQUS 6.10
411 #####
412 mdb.models[ 'Model-1' ]. Tie(adjust=OFF, constraintEnforcement=SURFACE_TO_SURFACE
      ,
413     master=
414     mdb.models[ 'Model-1' ]. rootAssembly.instances[ 'RVE-1' ]. surfaces[ 'Left' ],
415     name='PBC', positionTolerance=BF_W*1.10, positionToleranceMethod=SPECIFIED
      ,
416     slave=
417     mdb.models[ 'Model-1' ]. rootAssembly.instances[ 'RVE-1' ]. surfaces[ 'Right' ],
418     thickness=ON, tieRotations=OFF)
419
420 #####
421 #LOAD
422 #####
423 periodExcit=1.00/freqExcit                                #Excitation Period
424 analysisPeriod=periodExcit*numberCycles                   #Length of the Analysis
425 angularFreqExcit=2.00*3.1416*freqExcit                     #Angular Frequency of the Excitation
426 inInc=periodExcit/(120.00)                                #Time Step Size
427
428 #Create Amplitude
429 mdb.models[ 'Model-1' ]. PeriodicAmplitude(a_0=0.0, data=((0.0, 1.0), ),
430     frequency=angularFreqExcit, name='Sine', start=0.0, timeSpan=STEP)
431
432 #Create Step 'SinusoidalTorque' to Apply Load
433 mdb.models[ 'Model-1' ]. StaticStep(description='Apply Sinusoidal Load',
      initialInc=inInc,
434     maxNumInc=int(200*numberCycles), name='SinusoidalTorque', nlgeom=ON,
435     noStop=OFF, previous='Initial', timeIncrementationMethod=FIXED,
436     timePeriod=analysisPeriod)
437
438 #Apply Surface Traction on the Top Half of the Ellipsoid (Positive in the X
      direction)
439 mdb.models[ 'Model-1' ]. SurfaceTraction(amplitude='Sine', createStepName=
440     'SinusoidalTorque', directionVector=((0.0, 0.0, 0.0), (1.0, 0.0, 0.0)),
441     distributionType=UNIFORM, field='', follower=OFF, localCsys=None,
442     magnitude=magnitudeSurfaceTraction, name='Ellipse-Top', region=
443     mdb.models[ 'Model-1' ]. rootAssembly.instances[ 'RVE-1' ]. surfaces[ 'Ellipse-
      Top' ]
444     , resultant=ON, traction=GENERAL)
445 #Apply Surface Traction on the Bottom Half of the Ellipsoid (Negative in the X
      direction)
446 mdb.models[ 'Model-1' ]. SurfaceTraction(amplitude='Sine', createStepName=
447     'SinusoidalTorque', directionVector=((0.0, 0.0, 0.0), (1.0, 0.0, 0.0)),
448     distributionType=UNIFORM, field='', follower=OFF, localCsys=None,
449     magnitude=-magnitudeSurfaceTraction, name='Ellipse-Bottom', region=
450     mdb.models[ 'Model-1' ]. rootAssembly.instances[ 'RVE-1' ]. surfaces[ 'Ellipse-
      Bottom' ]
451     , resultant=ON, traction=GENERAL)
452

```

```

453 #Define Output Requests for the Model
454 mdb.models[ 'Model-1' ].fieldOutputRequests[ 'F-Output-1' ].setValues( variables=(
455     'IVOL', 'LE', 'S', 'U'))
456
457 #Request Recoverable Strain Energy (SE) at the last increment for the BaseFilm
   Set
458 mdb.models[ 'Model-1' ].HistoryOutputRequest( createStepName='SinusoidalTorque',
459     frequency=LAST_INCREMENT, name='H-Output-2', rebar=EXCLUDE, region=
460     mdb.models[ 'Model-1' ].rootAssembly.instances[ 'RVE-1' ].sets[ 'BaseFilm'
        ],
461     sectionPoints=DEFAULT, variables=('ALLSE', ))
462
463 #Create Job
464 mdb.Job(atTime=None, contactPrint=OFF, description='', echoPrint=OFF,
465     explicitPrecision=SINGLE, getMemoryFromAnalysis=True, historyPrint=OFF,
466     memory=90, memoryUnits=PERCENTAGE, model='Model-1', modelPrint=OFF, name=
467     'Job-1', nodalOutputPrecision=SINGLE, queue=None, scratch='', type=
        ANALYSIS
468     , userSubroutine='', waitHours=0, waitMinutes=0)
469 #Submit
470 mdb.jobs[ 'Job-1' ].submit()
471 #Wait for finish
472 mdb.jobs[ 'Job-1' ].waitForCompletion()
473
474
475 #####
476 #Post-Processing
477 #####
478 #Reset Dissipation to Zero at the beginning of each loop
479 dissipation = 0.0
480
481 #Begin the Post-Processing
482
483 #Open The Output Data Base of the Current Job
484
485 odb = openOdb(path='Job-1.odb')
486
487 myAssembly = odb.rootAssembly
488
489
490 #Integrate ONLY through the elements made of PolymerMatrix
491 BASEFILM = odb.rootAssembly.instances[ 'RVE-1' ].elementSets[ 'BASEFILM' ]
492 TOP = odb.rootAssembly.instances[ 'RVE-1' ].nodeSets[ 'TOP' ]
493
494 #Creating a temporary variable to hold the frame repository provides the same
   functionality and speeds up the process
495
496 frameRepository = odb.steps[ 'SinusoidalTorque' ].frames
497 frameLE=[]
498 frameS=[]
499 frameIVOL=[]
500 frameU=[]
501 for i in range(len(frameRepository)-1):

```

```

502     frameLE.insert(i, frameRepository[i].fieldOutputs['LE'].getSubset(
        region=BASEFILM, position=INTEGRATION_POINT))
503     frameS.insert(i, frameRepository[i].fieldOutputs['S'].getSubset(region=
        BASEFILM, position=INTEGRATION_POINT))
504     frameIVOL.insert(i, frameRepository[i].fieldOutputs['IVOL'].getSubset(
        region=BASEFILM, position=INTEGRATION_POINT))
505     frameU.insert(i, frameRepository[i].fieldOutputs['U'].getSubset(region=
        TOP, position=NODAL))
506
507 #Number of frames in the step to be integrated
508
509 N_frames=len(frameLE)
510 dummy=0
511
512 for II in range(0, len(frameLE[0].values)):
513     for JJ in range(0+1, N_frames):
514         x_c=frameLE[JJ].values[II].data
515         x_w=frameLE[JJ-1].values[II].data
516         y_c=frameS[JJ].values[II].data
517         y_w=frameS[JJ-1].values[II].data
518         v_c=frameIVOL[JJ].values[II].data
519         v_w=frameIVOL[JJ-1].values[II].data
520         dummy=0.25*(x_c-x_w)*(y_c+y_w)*(v_c+v_w)
521         dummy_2=0
522         for KK in range(0, len(dummy)):
523             dummy_2=dummy_2+dummy[KK]
524         dissipation=dissipation+dummy_2
525 #Subtract the Recoverable elastic strain in the THE BASE FILM (Polymer Matrix
    Set)
526 #
527
    WARNING: name of historyRegions
528 dissipation=dissipation-odb.steps['SinusoidalTorque'].historyRegions['
    ElementSet_BASEFILM'].historyOutputs['ALLSE'].data[-1][1]
529 print dissipation
530
531 #Lookup for the maximum and minimum vertical displacement (U2) in the top
    surface
532 maximo=0.0
533 minimo=0.0
534 frameMaximo=0
535 for II in range(0, N_frames): #Loop over frames
536     for JJ in range(0, len(frameU[0].values)): #Loop over nodes
537         if frameU[II].values[JJ].data[1]>maximo:
538             maximo=frameU[II].values[JJ].data[1]
539             frameMaximo=II
540 for JJ in range(0, len(frameU[0].values)): #Loop over nodes for the frame
    where the maximum occurred
541     if frameU[frameMaximo].values[JJ].data[1]<minimo:
542         minimo=frameU[frameMaximo].values[JJ].data[1]
543
544 print maximo
545 print minimo

```



```

546
547 #Prepare Post-Processing Results Output to File
548 #Import Libraries
549 import datetime
550 import csv
551
552 #Prepare Name for the output file
553 now = datetime.datetime.now()
554 print now
555 #filename='output-'+now.strftime("%Y-%m-%d")+'.csv'
556 filename='3D-output'+'.csv'
557
558 #Open the output file and write headers
559 fOut = open(filename, 'ab') #was 'wb', try 'a' to append on the old
    file
560
561 writer = csv.writer(fOut)
562
563 #This block is commented to avoid writing the headers for each analysis. If a
    new results file is created, it should be uncommented for the first run.
564 # writer.writerow([
565 # 'Date', 'Vol.Frac.', 'Frequency [Hz]', 'Cycles []', 'Analysis Length [s]', 'Load
    Magnitude [N/mm2]', 'Dissipation [mJ]',
566 # 'Maximum U2 [mm]', 'Minimum U2 [mm]', 'Surf. Dissip. Power [mW/mm2]',
567 # 'Bump [mm]', '# of Elements', 'Base Film Width [mm]', 'Base Film Thickness [mm
    ]', 'Base Film Depth [mm]', 'Top Film Thickness [mm]',
568 # 'Ellipse Width [mm]', 'Ellipse Thickness [mm]', 'Script',
569 # 'Hyperelastic Model', 'Degree', 'Hyperelastic Parameters', 'Hysteresis
    Parameters '])
570
571 now = datetime.datetime.now()
572 print now
573
574 #Get Number of elements in the model
575 numberElements=mdb.models['Model-1'].rootAssembly.verifyMeshQuality(
    ANALYSIS_CHECKS)['numElements']
576 #Write the results for the current frequency to the output file
577 writer.writerow([
578 now.strftime("%Y-%m-%d_%H:%M%S"),
579 3.1416/12.0*EL_W*EL_T**2/(BF_W*BF_D*(BF_T+TF_T)),
580 str(freqExcit), str(numberCycles), str(analysisPeriod),
581 str(magnitudeSurfaceTraction), str(dissipation), str(maximo), str(minimo),
582 str(dissipation/analysisPeriod/BF_W/BF_D), str(maximo-minimo),
583 numberElements, BF_W, BF_T, BF_D, TF_T, EL_W, EL_T, scriptName,
584 odb.materials['BASEFILM'].hyperelastic.type, odb.materials['BASEFILM'].
    hyperelastic.n,
585 odb.materials['BASEFILM'].hyperelastic.table,
586 odb.materials['BASEFILM'].hyperelastic.hysteresis.table
587 ])
588 #Close the ODB for the current frequency
589 odb.close()
590
591
592 #Close The output file (Keep outside the looping structure)

```

593 `fOut.close()`

Appendix F

Code of Volume Fraction Calculation Script

```
1 %% Joaquin M. Gutierrez
2 %%2011-01-12
3 %%West Virginia University
4 clc
5 clear all
6 close all
7
8 %Input the Filename to be analyzed
9 Img_File='Ellipsoids200x6-2-CROP';
10 %Input how many lenght units is the width of the image representing
11 %Real_Width=310
12 Real_Width=304*(320/780);
13 %Input the smallest MAJOR Diameter of the ellipses that will be counted
14 d_major=2;
15 %Input the smallest MINOR Diameter of the ellipses that will be counted
16 d_minor=1;
17 %Input the smallest ASPECT RATIO (AR) of the ellipses that will be counted
18 %AR=d_Major/d_Minor
19 AR=1.2;
20
21 %OPTIONAL: Input Film Thickness
22 Thickness=36;
23
24 %Minimum and Maximum Area that will count as an ellipse
25 %This values can be modified by a factor to account for errors in the
26 %measurement of the areas.
27 Area_min= (pi * d_major/2 * d_minor/2) * 1
28 Area_max= (Area_min) * 100
29 %Input the minimum excentricity to count as an ellipse
30 %This values can be modified by a factor to account for errors in the
31 %measurement of ellipses radii.
32 %e=\sqrt{1-\left(\frac{b}{a}\right)^2}
33 %Excen_min=(sqrt(1-(d_minor/d_major)^2)) * 0.5
34 Excen_min=(sqrt(1-(1/AR)^2))
35
36 %Input the maximum angle (degrees) that a region can have with respect to
```

```

37 %the x axis to count as an ellipse
38 Orient_max=20;
39
40 % Read and show the Original Image
41 I = imread([Img_File, '.jpg']);
42 iptsetpref('ImshowAxesVisible','on')
43 imshow(I, 'InitialMagnification','fit')
44 saveas(gca,['./Fig_Ch05_',Img_File, '_Original_REV00.eps']);
45 hold off
46 %Get Image Information
47 Info=imfinfo([Img_File, '.jpg']);
48 disp(['File_Name:',Info.FileName])
49 disp(['Image_Width:',num2str(Info.Width)])
50
51 %Convert from color to grayscale
52 I_gr = rgb2gray(I);
53 %imshow(I_gr)
54
55 %imadjust increases the contrast of the image by mapping the values of the
56 %input intensity image to new values such that, by default, 1% of the data
57 %is saturated at low and high intensities of the input data.
58
59 %histeq performs histogram equalization. It enhances the contrast of images
60 %by transforming the values in an intensity image so that the histogram of
61 %the output image approximately matches a specified histogram (uniform
62 %distribution by default).
63
64 %adapthisteq performs contrast-limited adaptive histogram equalization.
65 %Unlike histeq, it operates on small data regions (tiles) rather than the
66 %entire image. Each tile's contrast is enhanced so that the histogram of
67 %each output region approximately matches the specified histogram (uniform
68 %distribution by default). The contrast enhancement can be limited in order
69 %to avoid amplifying the noise which might be present in the image.
70
71
72 % ImAdjust
73 %imadjust increases the contrast of the image by mapping the values of the
74 %input intensity image to new values such that, by default, 1% of the data
75 %is saturated at low and high intensities of the input data.
76
77 I_gr_adj=imadjust(I_gr,[0.5 0.625],[0 1],1.1);
78 %I_gr=imadjust(I_gr);
79 figure
80 imshow(I_gr_adj)
81 I_gr=I_gr_adj;
82
83 %% histeq
84 %histeq performs histogram equalization. It enhances the contrast of images
85 %by transforming the values in an intensity image so that the histogram of
86 %the output image approximately matches a specified histogram (uniform
87 %distribution by default).
88 % I_gr=histeq(I_gr);
89 % imshow(I_gr)
90 %% adapthisteq

```

```

91 %adapthisteq: Contrast-limited adaptive histogram equalization (CLAHE)
92
93 %adapthisteq performs contrast-limited adaptive histogram equalization.
94 %Unlike histeq, it operates on small data regions (tiles) rather than the
95 %entire image. Each tile's contrast is enhanced so that the histogram of
96 %each output region approximately matches the specified histogram (uniform
97 %distribution by default). The contrast enhancement can be limited in order
98 %to avoid amplifying the noise which might be present in the image.
99
100 I_eq = adapthisteq(I_gr);
101 %I_eq = adapthisteq(I_gr, 'NumTiles',[100 100]);
102
103 %Get Threshold to convert to Black and White from Gray Scale
104 level = graythresh(I_eq);
105 %% Convert to B&W
106 bw = im2bw(I_eq, level);
107 %User Input threshold level
108 %bw = im2bw(I_eq, 120/256);
109 bw = im2bw(I_eq, level*0.95);
110
111 figure
112 imshow(bw)
113 saveas(gca,['./Fig_Ch05_',Img_File,'_BW_REV00.eps']);
114 hold off
115 %% Fill the parts inside the ellipse boundaries, in order to compute
116 %correctly the area.
117 bw = imfill(~bw,'holes');
118
119 figure
120 imshow(~bw);
121 saveas(gca,['./Fig_Ch05_',Img_File,'_BWF_REV00.eps']);
122 hold off;
123 %% Opening and Closing (Img-Processing Techniques)
124 %Might help to split siameses
125 se = strel('disk',2);
126 bw=imopen(bw,se);
127
128 bw=imclose(bw,se);
129 figure
130 imshow(~bw)
131 saveas(gca,['./Fig_Ch05_',Img_File,'_BW_MORPH_REV00.eps']);
132 hold off;
133 %% Find and Label Connected Elements
134 %[L, Ne]=bwlabel(bw);
135 CC=bwconncomp(bw,4);
136
137 %Get the requested stats on the connected regions
138 STATS = regionprops(CC, 'Area', 'Eccentricity', 'Orientation', ...
139     'MajorAxisLength', 'MinorAxisLength', 'Image', 'BoundingBox', 'Centroid');
140
141 %Calculate Scale
142 Scale=1*(Info.Width/Real.Width)
143
144 %Initialize Counter and Geometric variables

```

```

145 ell_num=0;
146 Areas=0;
147 Axes_Major=0;
148 Axes_Minor=0;
149 Orientations=0;
150 %Create an empty image
151 bw_filtered=ones(Info.Height,Info.Width);
152 % bw_filtered_overlay=rgb2gray(I);
153 %Loop over all the connected regions found and filter by area an
154 %eccentricity (the filter criteria should be changed as needed)
155 for n=1:size(STATS,1)
156     if (STATS(n).Area>Area_min*(Scale^2)) &&...
157         (STATS(n).Area<Area_max*(Scale^2)) &&...
158         (STATS(n).Eccentricity>Excen_min) &&...
159         (abs(STATS(n).Orientation)<Orient_max)
160     % disp('-----')
161     % disp(['Region ID: ',num2str(n)])
162     % disp(['Area: ',num2str(STATS(n).Area)])
163     % disp(['MajorAxisLength: ',num2str(STATS(n).MajorAxisLength)])
164     % disp(['MinorAxisLength: ',num2str(STATS(n).MinorAxisLength)])
165     % disp(['Eccentricity: ',num2str(STATS(n).Eccentricity)])
166     % disp(['Orientation: ',num2str(STATS(n).Orientation)])
167     % hold all
168     % %Show Bounding Box
169     % rectangle('Position',STATS(n).BoundingBox,'LineStyle','- ',...
170     % 'EdgeColor','g','LineWidth',1);
171     % %Show the filled ellipse
172     % imshow(~STATS(n).Image,'Xdata',[0,STATS(n).BoundingBox(1)],'Ydata',
173     % [0,STATS(n).BoundingBox(2)]);
174     %Write the filtered images to the file
175     bw_filtered(round(STATS(n).BoundingBox(2)) : ...
176     round(STATS(n).BoundingBox(2))+STATS(n).BoundingBox(4)-1,...
177     round(STATS(n).BoundingBox(1)) : ...
178     round(STATS(n).BoundingBox(1))+STATS(n).BoundingBox(3)-1)=...
179     ~STATS(n).Image;
180     %Write the filtered images on top of the original image
181     % bw_filtered_overlay(round(STATS(n).BoundingBox(2)) : ...
182     % round(STATS(n).BoundingBox(2))+STATS(n).BoundingBox(4)-1,...
183     % round(STATS(n).BoundingBox(1)) : ...
184     % round(STATS(n).BoundingBox(1))+STATS(n).BoundingBox(3)-1)=...
185     % STATS(n).Image;
186     %Increase Counter by 1
187     ell_num=ell_num+1;
188     %Save the coordinates of the centroids,Area and axes for the
189     %ellipses within the criteria
190     Centroids(ell_num,:)=STATS(n).Centroid;
191     Areas(ell_num)=STATS(n).Area;
192     Axes_Major(ell_num)=STATS(n).MajorAxisLength;
193     Axes_Minor(ell_num)=STATS(n).MinorAxisLength;
194     Orientations(ell_num)=STATS(n).Orientation;
195 end
196 %Create new figure to show only the regions that passed the filter
197 % figure

```

```

198 % imshow(bw_filtered_overlay)
199 figure
200 imshow(bw_filtered)
201 hold all
202 saveas(gca,[ './Fig_Ch05_',Img_File , '_BW_Filtered_REV00.eps' ]);
203 hold off
204
205 disp( '_____')
206 disp([ 'Ellipses Found: ',num2str(ell_num)])
207 disp( '_____')
208
209 %It is important to bear in mind that the areas calculated by MATLAB are
210 %based in a number of pixels from the the binary image. Caution must be
211 %taken we transforming to surface area and the scale is introduced. (The
212 %same applies for lengths).
213
214 %Area Stats
215 Area_mean=mean(Areas)/(Scale^2)
216 Area_std=std(Areas)/(Scale^2)
217 %Axes Stats
218 Axis_Major_mean=mean(Axes_Major)/(Scale)
219 Axis_Major_std=std(Axes_Major)/(Scale)
220 Axis_Minor_mean=mean(Axes_Minor)/(Scale)
221 Axis_Minor_std=std(Axes_Minor)/(Scale)
222 %Orientation Stats
223 Orientation_mean=mean(Orientations)
224 Orientation_std=std(Orientations)
225
226
227 %% Voronoi Diagram
228 [V,C] = voronoin(Centroids);
229 figure
230 imshow(bw_filtered);
231 hold on
232 voronoi(Centroids(:,1),Centroids(:,2),'-k');
233 saveas(gca,[ './Fig_Ch05_',Img_File , '_Voronoi_REV00.eps' ]);
234 hold off
235
236 % %To display the verteces numbers of each cell
237 % for i=1:length(C), disp(C{i}), end
238
239 %Create a list with all the vertices that are outside the original picture
240 counter=1;
241 for i=1:size(V,1)
242     if V(i,1)<0 || V(i,1)>Info.Width || V(i,2)<0 || V(i,2)>Info.Height
243         Outsiders(counter)=i;
244         counter=counter+1;
245     end
246 end
247
248 %% %Check that the Centroids and Inner Cells numbering are direct
249 % figure
250 % k=20
251 % plot(Centroids(k,1),Centroids(k,2),'*')

```

```

252 % hold all
253 % XY=[];
254 % for i=1:length(C{k})
255 %     XY(i,:)=V(C{k}(i),:);
256 % end
257 % plot(XY(:,1),XY(:,2))
258
259
260 %Create Bounding Boxes and Area Fraction for the inner cells
261 counter=1;
262 for i=1:size(C,1)
263     if sum(ismember(C{i},Outsiders),2)==0
264         %Create a list with the inner cells
265         InnerCells(counter)=i;
266         %Bounding Boxes
267         X_min(counter)=Info.Width;
268         X_max(counter)=0;
269         Y_min(counter)=Info.Height;
270         Y_max(counter)=0;
271         for j=1:size(C{i},2)
272             if V(C{i}(j),1)< X_min(counter)
273                 X_min(counter)=V(C{i}(j),1);
274             end
275             if V(C{i}(j),1)> X_max(counter)
276                 X_max(counter)=V(C{i}(j),1);
277             end
278             if V(C{i}(j),2)< Y_min(counter)
279                 Y_min(counter)=V(C{i}(j),2);
280             end
281             if V(C{i}(j),2)> Y_max(counter)
282                 Y_max(counter)=V(C{i}(j),2);
283             end
284         end
285         counter=counter+1;
286     end
287 end
288
289 %% Area Fraction
290 %Initialize Area Fraction Vector A_f
291 A_f=zeros(1,size(InnerCells,2));
292 for j=1:size(InnerCells,2)
293     k=InnerCells(j);
294     XY=[];
295     for i=1:length(C{k})
296         XY(i,:)=V(C{k}(i),:);
297     end
298     A_f(j)=Areas(k)/polyarea(XY(:,1),XY(:,2));
299 end
300 %Calculate Area Fraction STATS
301 Area_f_mean=mean(A_f)
302 Area_f_std=std(A_f)
303 Area_f_COV=Area_f_std/Area_f_mean
304
305 %% Calculate Volume Fraction STATS

```



```

306 %Here it will be assumed that the 3D shape of the actuators is a prolate
307 %spheroid and that the major and minor axis of the regions are the minor
308 %and major axis of an ellipse. Thickness must be given.
309 %Initialize Area Fraction Vector A_f
310 V_f=zeros(1,size(InnerCells,2));
311 for j=1:size(InnerCells,2)
312     k=InnerCells(j);
313     XY=[];
314     for i=1:length(C{k})
315         XY(i,:)=V(C{k})(i,:);
316     end
317     V_f(j)=4/3*pi*(Axes_Minor(k)*0.5)^2*(Axes_Major(k)*0.5)/...
318         (polyarea(XY(:,1),XY(:,2))*Thickness);
319 end
320 %Calculate Volume Fraction STATS
321 Volume_f_mean=mean(V_f)
322 Volume_f_std=std(V_f)
323 Volume_f_COV=Volume_f_std/Volume_f_mean
324
325
326
327 %% Plot Bounding Boxes of the RVE's (Upper Limit-Overlap may occur)
328 %[X_min;X_max;Y_min;Y_max]'
329
330 %Calculate the width and height of the box
331 box_width=X_max-X_min;
332 box_height=Y_max-Y_min;
333
334 % figure
335 % imshow(bw_filtered & rgb2gray(I));
336
337 figure
338 imshow(bw_filtered);
339 set(gca,'FontName','Arial','FontSize',12); %important
340 axis([0.0 Info.Width 0.0 Info.Height])%[x- x+ y- y+] limites on axes
341 %title(Img_File)
342 for i=1:size(X_max,2)
343     rectangle('Position',[X_min(i), Y_min(i), box_width(i), ...
344         box_height(i)], 'EdgeColor','k', 'LineWidth',2);
345 end
346 saveas(gca,['./Fig_Ch05_',Img_File,'_RVE_REV00.eps']);
347
348 %Calculate RVE Width STATS
349 RVE_w_mean=mean(box_width)
350 RVE_w_std=std(box_width)
351 RVE_w_COV=RVE_w_std/RVE_w_mean
352
353 %Calculate RVE Depth STATS
354 RVE_d_mean=mean(box_height)
355 RVE_d_std=std(box_height)
356 RVE_d_COV=RVE_d_std/RVE_d_mean
357
358 %Basis Value, Coverage 50%, Confidence 95%
359 Volume_f_mean - (0.6198*Volume_f_std)

```

Appendix G

Companion CD Contents

Folder	File Name	Description
Fit	fit10.m	MATLAB program listed in Appendix A.1
Fit	bb10.m	MATLAB program listed in Appendix A.2
Fit	bb10_error.m	MATLAB program listed in Appendix A.3
Fit	fminsearchcon.m	MATLAB Add-on Minimization Program (by J. D'Errico)
AppxB	dissipation.py	Python Script for the dissipation calculation (Appendix B.3)
AppxE	CONTACT02c.py	Python Script for ABAQUS - Contact Model (Appendix E.1)
AppxE	VISCO17.py	Python Script for ABAQUS - 2D Model (Appendix E.2)
AppxE	3DMODEL05d.py	Python Script for ABAQUS - 3D Model (Appendix E.3)
Ch05-REV01/Image A	EC-REV09-1-200x-CROP-B.m	MATLAB program used for the image processing of Image A
Ch05-REV01/Image A	1-200x-CROP-B.jpg	Image A
Ch05-REV01/Image B	EC_REV09_6_2_CROP.m	MATLAB program used for the image processing of Image B
Ch05-REV01/Image B	Ellipsoids200x6-2-CROP.jpg	Image B

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