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Rehan Fazal

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Robust Control Design for Non-Linear Loads in AC-DC System

by
Rehan Fazal

Thesis submitted
to Statler College of Engineering and Mineral Resources
at West Virginia University

in partial fulfillment of the requirements
for the degree of

Master’s of Science
in
Electrical Engineering

Committee Members:
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Morgantown, West Virginia
2014

Keywords: synergetic control, feedback linear design, FACTS, dynamic stability, non-linear modeling of power system, non-linear loads

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Abstract

Robust Control Design for Non-Linear Loads in AC-DC System

by

Rehan Fazal

Master’s of Science in Electrical Engineering

West Virginia University

Prof. Muhammad Akram Choudhry Ph.D., Chair

Electric power system is complex interconnected system that is subjected to rapid change in its structure. Addition of new equipment, operation close to stability limits and variation in load in summer and winter are some factors that make power system unstable and introduce electro-mechanical oscillations in the system.

Electro-mechanical oscillations of small magnitude often persist for long periods of time and restrict the power transfer capacity. These oscillations can further increase with the variation in load and system operating points. To stabilize the power system different control strategies are used to damp the oscillations. In the past, the dynamic stability of power system was modeled using linearized modeling of power system at operating points. Linear eigenvalue controllers damp the oscillation to some extent but are not robust for disturbance and variation in system operating points and therefore can render system unstable.

Non-linear controller design has found extensive application in robotics for stabilizing system. Synergetic Controller based on Synergetic Control Theory, is a non-linear controller that forces the system towards global stability and has extensively been used in power electronics.

In this research, a Single Machine Infinite Bus (SMIB) with HVDC link is considered for comparing the performance of linear and non-linear controllers. The test system has Static Var Compensator (SVC) connected on load bus. Reactive modulation is performed through SVC to damp oscillations in the system. The Synergetic Controller is designed for full transient model and the classical model of the synchronous machine.

The performance of both non-linear controller and linear controller is compared for different disturbance and variation in system parameters. Results show the robustness of non-linear controller which damps the oscillations effectively.


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## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>i</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>ii</td>
</tr>
<tr>
<td>Contents</td>
<td>iii</td>
</tr>
<tr>
<td>List of Figures</td>
<td>vii</td>
</tr>
<tr>
<td>List of Tables</td>
<td>ix</td>
</tr>
<tr>
<td><strong>1 Introduction</strong></td>
<td>1</td>
</tr>
<tr>
<td>1.1 Overview</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Problem Statement</td>
<td>1</td>
</tr>
<tr>
<td>1.3 Approach</td>
<td>3</td>
</tr>
<tr>
<td>1.3.1 Synergetic Control Theory</td>
<td>3</td>
</tr>
<tr>
<td>1.4 Scope of Thesis</td>
<td>4</td>
</tr>
<tr>
<td><strong>2 Literature Review</strong></td>
<td>5</td>
</tr>
<tr>
<td>2.1 Literature Survey</td>
<td>5</td>
</tr>
<tr>
<td>2.2 Research Objective</td>
<td>9</td>
</tr>
<tr>
<td><strong>3 Linear Modeling of Power System Components</strong></td>
<td>11</td>
</tr>
<tr>
<td>3.1 Component Connection Model</td>
<td>11</td>
</tr>
<tr>
<td>3.2 Eigenvalue Assignment Technique</td>
<td>13</td>
</tr>
<tr>
<td>3.2.1 Overview</td>
<td>13</td>
</tr>
<tr>
<td>3.2.2 Single Input Single Output System</td>
<td>14</td>
</tr>
<tr>
<td>3.2.3 Eigenvalue Assignment of Multi-variable System</td>
<td>15</td>
</tr>
<tr>
<td>3.2.3.1 Solution Technique</td>
<td>16</td>
</tr>
<tr>
<td><strong>4 Modeling of Interconnected Power System</strong></td>
<td>18</td>
</tr>
<tr>
<td>4.1 Power System Model</td>
<td>18</td>
</tr>
<tr>
<td>4.2 AC/DC Power System Component Model</td>
<td>19</td>
</tr>
<tr>
<td>4.2.1 Synchronous Generator Models</td>
<td>20</td>
</tr>
<tr>
<td>4.2.1.1 Fifth Order Model</td>
<td>20</td>
</tr>
<tr>
<td>4.2.1.2 Third Order Model</td>
<td>20</td>
</tr>
<tr>
<td>4.2.2 Excitation and Turbine Governor Model</td>
<td>20</td>
</tr>
</tbody>
</table>
4.2.3 Static Var Compensator Model ........................................ 21
4.2.4 AC Network Model .................................................. 22
4.2.5 HVDC Link and Converter Control Model ......................... 22
  4.2.5.1 Converter Control Model .................................... 23
  4.2.5.2 Converter Current Control .................................. 23
  4.2.5.3 Converter Configuration .................................... 23
4.2.6 AC/DC Interface Equation ......................................... 24

4.3 Non-linear Modeling of Power System ................................. 24
  4.3.1 For Third order system .......................................... 25
    4.3.1.1 Transmission Line connecting nodes i,j .................. 25
    4.3.1.2 Machine Current ........................................ 26
  4.3.2 For Full Order System .......................................... 27
    4.3.2.1 Bus Voltage ............................................ 27

5 Non-Linear Controller Design ........................................... 29
  5.1 Overview ............................................................. 29
  5.2 Theoretical Background ............................................ 29
    5.2.1 Regulation Problems ......................................... 30
      5.2.1.1 Asymptotic Stabilizing Problem ....................... 30
    5.2.2 Tracking Problem ............................................ 30
      5.2.2.1 Asymptotic Tracking Problem ......................... 30
    5.2.3 Available Methods for Non-linear Controller ............... 31
      5.2.3.1 Trial and Error ......................................... 31
      5.2.3.2 Feedback Linearization ............................... 31
      5.2.3.3 Robust Control ....................................... 31
      5.2.3.4 Adaptive Control ................................... 32
      5.2.3.5 Gain-Scheduling ...................................... 32
  5.3 Synergetic Control Theory ......................................... 33
  5.4 Synergetic Controller using Classical Model ...................... 36
  5.5 Synergetic Controller using full model .......................... 40

6 Analysis and Simulation .................................................. 45
  6.1 Dynamic Compensator using Eigenvalue Assignment ............... 45
    6.1.1 For classical Model ......................................... 45
    6.1.2 For fifth order Model ...................................... 47
  6.2 Synergetic Controller .............................................. 53
    6.2.1 Third Order Model .......................................... 53
    6.2.2 Effect of disturbance ....................................... 55
      6.2.2.1 Change in $P_m$ .................................... 56
      6.2.2.2 Fault on AC-line ................................... 56
    6.2.3 Effect of change in $K_\lambda$ of Controller ............... 57
  6.3 Synergetic Controller from Classical Model in Full order model . 58
  6.4 Synergetic Controller with Full Order of Synchronous Generator . 59
    6.4.1 Hydro Generator ........................................... 60
    6.4.2 Steam Generator ........................................... 62
    6.4.3 Comparison with Dynamic Compensator ...................... 62
6.4.4 Effect of Disturbance .......................... 63
  6.4.4.1 Change in $P_m$ .......................... 63
  6.4.4.2 Fault on AC Line ......................... 64
6.4.5 Variation in Gain 'K' of Synergetic Controller .......... 65
6.4.6 Low Generation ................................ 66

7 Conclusion and Future work 69
  7.1 Future Work .................................. 69

A Non-Linear Modeling 71
  A.1 Static Exciter .................................. 71
  A.2 Turbine Governor ............................... 71
  A.3 DC Converter .................................. 72
    A.3.1 Rectifier .................................. 72
    A.3.2 Inverter .................................. 72
  A.4 Synchronous Generator ......................... 73
    A.4.1 Classical Model ............................ 73
    A.4.2 Synchronous Generator .................... 74
  A.5 Static Load ................................... 75
  A.6 Static Var Compensator ....................... 76

B Linear Modeling of Components 78
  B.1 Synchronous Machine Model (Fifth Order) .......... 78
    B.1.1 Stator equations in Synchronous Reference Frame 78
    B.1.2 Rotor equations in Rotor Reference Frame .... 78
    B.1.3 Prime-mover equations ..................... 78
    B.1.4 Stator flux in Rotor Reference frame .......... 79
    B.1.5 Mutual Flux ................................ 79
    B.1.6 Stator currents in rotor-reference frame .... 79
    B.1.7 Stator currents in synchronous reference frame 79
    B.1.8 Torque Equation ............................ 79
  B.2 Synchronous Generator (Third Order Model) ........ 79
    B.2.1 Field flux Linkage ......................... 79
    B.2.2 Prime Mover Equation ........................ 80
    B.2.3 Stator currents in Rotor Reference frame .... 80
    B.2.4 Voltage back of q-axis synchronous reactance 80
    B.2.5 Stator voltage in RRF ........................ 80
    B.2.6 Stator current in Rotor Reference Frame .... 80
  B.3 Static Exciter .................................. 84
  B.4 Governor - Turbine Model ....................... 84
  B.5 Static Load ................................... 84
  B.6 Rectifier Model ............................... 85
  B.7 Inverter Model ................................ 86
  B.8 Static Var Compensator Model ................... 87

C Component Connection Modeling 88
List of Figures

4.1 Single Machine Infinite bus (SMIB) system model ........................................... 18
4.2 Block Diagram for Interconnected System .................................................. 19
4.3 Block Diagram for Synchronous machine ................................................... 20
4.4 Block Diagram for Synchronous machine (third order) ................................ 21
4.5 HVDC Block Diagram .................................................................................. 22
4.6 HVDC Converter Current Regulator) ............................................................ 23
4.7 Flow Chart for simulation (Third Order) ....................................................... 26
4.8 Flow Chart for simulation (Fifth Order) ......................................................... 28
5.1 System Response for example system ............................................................ 36
5.2 Geometric interpretation of control law in phase plane for 5th order ............. 41
6.1 Variation of Gain $K_z$ for different values of $A_p$ for same $\lambda_{9,10}$ ............. 49
6.2 Variation of Time Constant $T_z$ for different values of $A_p$ for same $\lambda_{9,10}$ 50
6.3 Variation of Gain $K_z$ and Time Constant $T_z$ of dynamic compensator at different values of imaginary part of critical mode $\lambda_{9,10}$ ............................ 50
6.4 Variation of Gain $K_z$ and Time Constant $T_z$ of dynamic compensator at different values of imaginary part of critical mode $\lambda_{9,10}$ ......................... 51
6.5 Speed Response of synchronous generator $\Delta \omega/\omega_b$ for a 5% change in Input Torque with Dynamic Compensator Design 1 ................................. 52
6.6 Speed Response of synchronous generator $\Delta \omega/\omega_b$ for a 5% change in Input Torque with Dynamic Compensator Design 3 ................................. 53
6.7 Speed Deviation of Generator for $A_p, A_q = 2$ ........................................ 54
6.8 Speed Deviation of Generator for $A_p = 0$ and $A_q = 2$ ............................ 55
6.9 Speed Deviation for $A_p, A_q = 2$ for 5% change in $P_m$ at $t= 5.0s$ ............ 56
6.10 Speed Deviation for fault on AC - bus at $t = 2s$ for $A_p, A_q = 2$ ............... 57
6.11 Speed Deviation for different values of $K$ and $\lambda$ for Synergetic Controller 58
6.12 Speed Deviation for Synergetic Controller for classical model in fifth order machine SMIB ................................................................. 59
6.13 Speed Deviation for Synergetic Controller for Hydro Generator for $A_p, A_q = 2$ ................................................................. 60
6.14 Speed Deviation for Synergetic Controller for Hydro Generator for load variation ................................................................. 61
6.15 Speed Deviation for steam generator for $A_p, A_q = 2$ ............................. 62
6.16 Speed Deviation of Generator for $A_p, A_q = 2$ ........................................ 63
6.17 Speed Deviation of Generator for 5% change in $P_m$ at $t = 6.0s$ ............... 64
6.18 Speed Deviation of Generator for fault at $t = 8.0s$ for 0.01s ..................... 65
6.19 Speed Deviation of Generator for different controller values of $K$ ............. 66
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.20</td>
<td>Speed Deviation of Generator for low generation</td>
<td>67</td>
</tr>
<tr>
<td>6.21</td>
<td>Speed Deviation for low generation with Dynamic Compensator</td>
<td>68</td>
</tr>
<tr>
<td>A.1</td>
<td>Static Exciter</td>
<td>71</td>
</tr>
<tr>
<td>A.2</td>
<td>Turbine-Governor Model</td>
<td>71</td>
</tr>
<tr>
<td>A.3</td>
<td>DC Converter - Rectifier</td>
<td>72</td>
</tr>
<tr>
<td>A.4</td>
<td>DC Converter - Inverter</td>
<td>73</td>
</tr>
<tr>
<td>A.5</td>
<td>Phasor Diagram for equivalent machine Model</td>
<td>74</td>
</tr>
<tr>
<td>A.6</td>
<td>Static Var Compensator Model</td>
<td>76</td>
</tr>
</tbody>
</table>
List of Tables

6.1 Eigenvalues for SMIB with 3rd Order Synchronous Machine and Constant Impedance Load \( A_p, A_q = 2 \) .................................................. 46
6.2 Critical Eigenvalue corresponding to \( \Delta \delta \) and \( \Delta \omega \) for different designs of dynamic compensator \( A_q = 2 \) .................................................. 47
6.3 Open-loop eigenvalue with SVC for Constant Impedance Load \( A_p, A_q = 2 \) for 5th Order of machine .......................................................... 48
6.4 Close-loop eigenvalue for 5th Order of machine with SVC for Constant Impedance Load \( A_p, A_q = 2 \) .................................................. 48
6.5 Critical Eigenvalue \( \lambda_{9,10} \) corresponding to \( \Delta \delta \) and \( \Delta \omega \) for different designs of dynamic compensator \( A_q = 2 \) .................................................. 51
6.6 System Data for Hydro and Steam Generator [1] .......................... 61

C.1 Component Connection Table for the DC system .......................... 88
C.2 Connection Values for the DC system (Rectifier + DC Line + Inverter) 89
C.3 Connection Table for the Synchronous Generator (Fifth order) ....... 90
C.4 Connection values for the AC synchronous Generator (Fifth order) 91
C.5 Connection Table for the Overall System .................................. 92
C.6 Connection Values for the overall system .................................. 93

D.1 Data for Synchronous Machine (Full Order) ............................... 94
D.2 Data for Synchronous Machine (Third Order) ............................. 95
D.3 Data for DC Converter ......................................................... 95
Chapter 1

Introduction

1.1 Overview

Electro-mechanical oscillations of very small magnitude exist in power system and often persist for long periods of time and restrict the power transfer capacity. These oscillations can further increase with the variation in load if the controller is not properly designed. Design of controller by implementing nonlinear control techniques has gained significant influence due to its inherent ability to improve control performance beyond what can be achieved by linear controllers. Linear analysis of complex nonlinear power system may fail in capturing some dynamic behaviors of the system especially in the events of critical faults or major disturbances.

Advances in computer systems and signal processing allow the practical implementation of the nonlinear controls. Robust and adaptive control theory has been used extensively for solving control theory problems. These controllers have been extensively used in Robotics but are also used in Power System to enhance the dynamic stability of nonlinear power systems.

1.2 Problem Statement

The electric power system is known to be a complex interconnected systems that constantly changes in structure as a result of disturbances, load changes, faults and addition of new devices or equipment. Due to stringent regulation reforms and more demand for electricity, the complexity of electric power system has increased tremendously. Thus
forcing the power system to operate at points close to their stability limits. The recent fast developments in consumer electronics pose a challenge on conventional power system.

Reliability and Stability are the main concerns of electric grid. To achieve this, the electric grid must remain rigid and unperturbed to a variety of disturbances. This has been a crucial concern when considering or evaluating the behavior of a power system.

According to IEEE / CIGRE joint task on Stability Terms and Definitions, Power System Stability is defined as the ability of an electric power system, for a given operating condition, to recover or regain a state of operating equilibrium after subjected to a physical disturbance, with most system variables bounded so that the entire system remains intact. These disturbances might be due to load changes, loss of long transmission line or trip of large generator(s) from the electric grid [6].

The ability of the power system to maintain synchronism when exposed to a severe transitory disturbance is known as transient stability [8]. Stability depends on both the initial operating state of the system and the severity of the disturbance. Transient stability analysis examines the dynamic behavior of power system electrical networks, electrical loads, and electro-mechanical equations of motion of the interconnected generators for as much as several seconds following a disturbance.

To analyze the system, a mathematical formulation is used based on algebraic differential equations that represent the electro-mechanical dynamics in connection with the grid. Under normal operating conditions, an electrical power system is near equilibrium, with only minor deviations from true-steady state conditions caused by small variations in the loads.

Large disturbances on the power system, such as a short circuit, cause almost instantaneous variations in the rotor speeds of some generators in the system. The power swings produce oscillations in voltages and currents which can lead to damage of equipment or disruption in monitoring devices if the oscillations are sufficiently large. Oscillations in voltages can also lead to voltage limit violations causing protective devices to trip and forcing equipment outages [29].

As the electric power system is growing rapidly, the dynamic performance of the controls, and the method of analyzing the stability of the system is also evolving. The design and implementation of control techniques to improve the stability of large scale power systems is becoming more intriguing and challenging. Design of controller using nonlinear control techniques is becoming popular as a result of its ability to improve control performance beyond what can be accomplished with linear control analysis.
1.3 Approach

In this research the power system is analyzed using both linear analysis technique and non-linear analysis technique. To stabilize the system under study, reactive modulation is achieved using Static Var Compensator (SVC). The linear controller designed is based on Eigen-value Assignment (EVA) technique. The dynamic compensator designed using this technique guarantees stability of the system for given operating points of the system. However, the controller is not robust for variation in load and system operating points. To solve this problem, a non-linear model of power system is implemented using algebraic differential equations and a non-linear controller is implemented based on Synergetic Control Theory.

1.3.1 Synergetic Control Theory

Synergetic Control Theory was developed by Kolesnikov; a Russian scientist [16]. It is a progressive approach in the area of control systems. The approach is often referred to as the stabilization and control via system restriction and manifold invariance [29]. The major idea of the concept is to confine the motion or trajectories of the system to a manifold (or hyperplane). This includes forcing the system onto the manifold in case it is not on the manifold already. The control problem addressed in this work is basically stability (i.e. damping any oscillation that may arise due to disturbance in the system) and since the system is forced by the synergetic controller to take the characteristics of the manifold, the manifold has to be constructed so that the closed loop system behavior is stable.

The design approach can thus be said to comprise of two major steps: the construction of a stable manifold (or hyperplane); and the synthesis or design of a controller so that the trajectories of the system are forced onto, and subsequently remain on the hyperplane. The objective of the controller can be seen as the one that causes the manifold to be invariant and attractive.

The technique of synergetic control is similar to that of Sliding Mode Control (SMC) in the way the manifold is being constructed. However, it differs from SMC technique in the manner the system is forced to reach the manifold. In the case of SMC, the system is forced to reach the manifold in a finite period of time which introduces some form of discontinuities in the control action and thus creating chattering on the manifold. This chattering can cause wears and tears in the actuating components of the system. But in the case of synergetic control, the system is caused to reach the manifold in an exponential manner and thus removing chattering effect. Synergetic Control Theory and the process for designing the controller is discussed in Chapter 5.
1.4 Scope of Thesis

- Chapter 2, provides a literature survey of the previous work done in this domain.

- In Chapter 3, the formulation of linearized state variable model for interconnected power system is outlined. Component Connection Technique to obtain the system state variable co-efficient matrices is outlined. The mathematical development for using Eigenvalue Assignment Technique is detailed.

- In Chapter 4, non-linear modeling of power system is detailed. Two different approaches are detailed for modeling the non-linear power system using MATLAB/Simulink.

- In Chapter 5, Synergetic Control Theory is described in detail. Design methodology for designing synergetic controller using Static Var Compensator (SVC) for different synchronous machine models is described.

- Chapter 6 describes the results and simulation for linear and non-linear analysis of power system. Effect of disturbance is considered on the system and performance of the controller is compared.

- Chapter 7 gives a conclusion and recommendations for future work.
Chapter 2

Literature Review

In this chapter, a literature survey of previous studies relating to damping of oscillation in electric power systems is presented and the objective of this research is discussed.

2.1 Literature Survey

As the electric power system is growing rapidly, the dynamic performance of the controllers and the method of analyzing the stability of the system is also evolving. The design and implementation of control techniques to improve the stability of large scale power systems is becoming more intriguing and challenging. In the modern control theory, the complex problem of effective control of electric power generation is one of the fundamental problems.

The most widespread generating components of power supply systems are generators. Modern electric power plants are equipped with groups of turbo-generators operating in parallel and interacting via the common electric network. The main components are a turbine and a synchronous generator mounted on one shaft. The synchronous generators are equipped with exciters or Automatic Voltage Regulators (AVR). Conventional controllers do not go beyond the framework of proportional integro-differential (PID) controllers: for complex generators these are automatic voltage controllers (AVCs), which are referred to differential (proportional differential (PD)) controllers, turbine rotation frequency controllers, which are referred as proportional-integral (PI) or PID controllers [2].

In the past few decades, considerable efforts have been devoted to the enhancement of power system stability. Classical automatic control systems contain fixed tuning parameters and have rigid structure. Various uncertainties in the control plant, variations of the network configurations (connection or disconnection of consumers or synchronous
generators), and emergency situations determined the requirement for designing adaptive control systems in order to reduce uncertainties of the real process, which implement the flexibility to parameter variation and the action of external perturbations. Today, nonlinear control theory forms a modern discipline that provides the tools necessary to improve dynamic performance so as to provide better quality and more secure power supply [3].

Transients in the power system are analyzed using many levels of modeling details. Mathematical formulation is governed by differential algebraic equations that describe the dynamics of the generators in connection with the grid. These generators are nonlinear electro-mechanical systems that run synchronously. Following large system disturbances, some synchronous generators may swing enough to lose synchronism with the system or become transiently unstable [4], [5]. Control is vital element to maintain the stability of the interconnected power system. Because of this need, control structures are becoming more pervasive and numerous, guaranteeing the stability of system over the wide range of operating conditions. They can be installed on generators, transmission lines, and distribution side.

Two distinct types of oscillations are already identified: local mode oscillation (in the range of 1–2 Hz), which is associated with generators at a generating station swinging with respect to the rest of the power system, and inter-area mode oscillation (in the range of 0.2–1 Hz), which is associated with the swinging of many machines in one area of the system against machines in other areas [6]. Complex interactions among the various machines and excitation systems and other control equipment can under certain conditions, cause the oscillations to grow resulting in system instability [7].

Under certain conditions, certain controller and system parameter can introduce negative damping into the system. To avoid this and improve the stability of the system in general, certain supplementary signals are added. Stabilizing/supplementary signals are introduced to the point where the reference voltage and signal proportional to terminal voltage are compared to obtain the error signal [8].

One way of improving system stability is via excitation system. The exciter provides the required Reactive power to damp the oscillations. The excitation system can regulate the terminal voltage of the generator when it is used with an automatic voltage regulator (AVR). Another approach is via Power System Stabilizer (PSS) [8]. By optimizing the gains of the PSS and bringing the stabilizing signal, the oscillations can be effectively damped. This approach has the advantage that the state variable for the machine are easily accessible.

Another approach of enhancing system stability is via the use of Flexible Alternating Current Transmission (FACTS) devices [9], [10].

In [11] a systematic procedure for the synthesis of a supplementary damping controller for Static Var Compensators (SVC) in multi-machine power systems using structured
sufficient value (SSV or $\mu$) is proposed as the measure of control performance. The said controller effectively enhances the damping of the inter-area oscillations, providing robust stability and good performance characteristics both in frequency and in time domain [12].

FACTS (Flexible AC Transmission System) become a means of equal importance to enhance stability of power system. The range of FACTS devices include thyristor based applications, e.g. Thyristor-Controlled Series Capacitor (TCSC) and the conventional High-Voltage Direct Current (HVDC) transmission systems [13], and Gate Turn-off (GTO) based applications, e.g., Static Synchronous Compensator (STATCOM). In [14] a controller for reactive power modulation is proposed using multi-terminal HVDC system for multi-machine case. The reactive power modulation effectively damps the system oscillations.

Static Var Compensator (SVC) are placed in the power grid with the main purpose of voltage support. These devices have several advantages including reduction of operation and transmission investment cost, increasing system security and reliability, increase power transfer capabilities, and overall enhancing a better quality of power and voltage [14], [15].

SVCs are shunt FACTS devices that can provide continuous and rapid control of reactive power and voltage, enhancing several aspects of transmission system performance. These aspects can be listed as prevention of voltage collapse, transient stability enhancement during system oscillations. Generally, voltage regulation is the primary mode of operation, which improves voltage stability. However, the function of SVC to damp system oscillations alone is not enough. Hence, for SVC damping controller based on stabilizing signal is required to further enhance the damping of the system.

Centralized type of controller requires remote information from each area. Conventional technique for designing these damping controllers are based on linear analysis based on one operating point which is not valid for wide range of operations. High degree of nonlinearity in power system and changes in operating conditions make the situation more challenging.

Design of linear feedback compensators which makes use of the available outputs of the system is fairly easy task. A large class of these compensators are dynamic and are designed to achieve eigenvalue assignment in the augmented closed-loop system consisting of the plant and the compensator [17],[18]. Most control techniques available for designing supplementary controller of power systems are based on linear model of the system around an operating point. A drawback to this approach is that significant large disturbance can make the system unstable because the non-linearity in the system is not properly compensated. Frequency-domain method allow designer the complete freedom
in selecting the compensator eigenvalues and uses only the numerator parameters of the compensator parameters of the compensator transfer function to assign the eigenvalues of the augmented system. The method not only ensures that the required dynamic compensator is stable, but also that its eigenvalues are at pre-specified location in the complex plane.

In [19], a lead-lag dynamic compensator is designed for Static Var Compensator for linearized system. The supplementary signal for reactive power modulation is achieved by designing lead lag compensator using eigenvalue assignment technique [20]. The values of gain and zero of dynamic compensator are designed based on desired complex-conjugate eigenvalues. Simulations of linear system show that dynamic compensator designed at constant impedance load significantly improves the system damping. The sensitivity analysis [21] of this open loop system indicate significant movement of imaginary part of this mode, i.e. the damping present in this mode will be significantly affected if the load varies [13].

Complete centralized control scheme is also unfeasible due to difficulties in transfer of information among various components of the system because of the physical distance, which may lead to unnecessary increase in the cost of transmission of electricity.

In [22][3], a methodology for designing decentralized controller using Extended Backstepping Control is proposed. The controller is designed for both exciter and SVC. The controller proposed is optimized using Particle Swarm Optimization (PSO). A strategy for co-ordination between the SVC and backstepping controller is also proposed. Other methods for damping the oscillations use learning algorithms. In [23], [24], [25], genetically optimized controllers are proposed. One drawback of using this approach is the tuning of the controller. Also, the variation of system parameters can affect the performance of these controllers.

Design of controller using nonlinear control techniques is becoming popular as a result of its ability to improve control performance beyond what can be accomplished with linear control techniques. In [7] a non-linear power system stabilizer is designed for single machine infinite bus system and the effect of fault on transmission lines is considered. The controller is designed using Synergetic Control Theory [9], [16]. Also in [26] a non-linear excitation controller is designed for damping system oscillations for Single Machine Infinite Bus system. The non-linear controllers proposed are robust and can manage contingencies in the system effectively.

For solving the control problems, nonlinear effects will be significant in the dynamics and nonlinear control may be necessary to achieve the desired performance. Generally, the tasks of control systems can be divided into two categories: stabilization (or regulation) and tracking (or servo) [27]. The first deals with a control system, called a stabilizer to be designed so that the state of the closed-loop system will be stabilized around an equilibrium point e.g. position control of robot arms.
On other hand, in tracking control problems the design objective is to construct a controller, called a tracker, so that the system output tracks a given time-varying trajectory. For example making an aircraft fly along a specified path or making a robot hand draw straight lines or circles are typical tracking control tasks. Analysis of non-linear control systems has rich collection of alternative and complementary techniques each applicable to particular case of non-linear control problems. Some of the most common methods for solving Control problems are Trial and Error, Feedback Linearization [13], Robust Control [21] and Adaptive Control. Two major and complementary approaches to dealing with model uncertainty are robust control and adaptive control. A simple approach to robust control is so-called Sliding Control methodology [28]. In sliding mode control, a sliding control surface is proposed and the error of the signal is forced to reach zero. The control approach has been effective in solving certain control problems but has a disadvantage of chattering [29] which can cause wear and tear in electro-mechanical systems.

2.2 Research Objective

The objective of this research can be summarized as follows:

- Linear modeling of Power System using a modeling technique called Component Connection Technique to represent the overall power system. Modeling of HVDC link and other components for the power system and the electro-mechanical system is done by linearizing the system at the operating point. Eigenvalue Assignment (EVA) Technique is used to design dynamic compensator for Static Var Compensator (SVC).

- Model the effect of load variation on the system performance. The load model used is Static non-linear load model. With the recent advancement in consumer electronics and electric vehicles, the electrical load varies and introduces oscillations in power system.

- Non-linear nature of power system makes it difficult to model and simulate the power system. Previously power system was modeled using linear models and linear controllers were proposed to damp oscillations. The linear model of power system is not sufficient to model uncertainties of the system.

- Non-linear simulation for power system is developed using MATLAB/Simulink. The algorithm for modeling approach is discussed. Different models for synchronous machines are considered and the controllers are proposed for each case.
The models used in this research are Classical Model and the detailed Model of Synchronous Machine.

- Non-linear controller design is discussed in detail and a controller is designed based on Synergetic Control Theory. Controller is designed for transient and classical model of the synchronous machine using different sliding surfaces.

- The performance of the controller is tested for different disturbances. The performance is compared with linear Eigenvalue controller and the results are detailed for different test cases.
Chapter 3

Linear Modeling of Power System Components

The power system is composed of different components and requires a technique to model the components in Power System. To study the dynamic analysis of power system, it is modeled using differential equations. However due to computation complexities, the system is modeled using its linearized models. Linearized models are obtained from the non-linear system by linearizing them at the operating point. The power system model consist of various components and numerous techniques are available to model the system. Numerous techniques have been mentioned in [30], [31] and [18] to model the large scale power system. Most of the techniques involved are based on the linear model of components. One such technique is the Component Connection Model, and has been used here to model the interconnection of different components of the system due to its simplicity and easiness.

3.1 Component Connection Model

Most large systems can be modeled in component connection form. The Component Connection Model of linear dynamical system consists of set of two vector matrix equations separately describing component dynamics and component interconnections. Each component has a non-linear state model

\[ x_i = f(x_i, a_i) \]
\[ b_i = g(x_i, a_i) \]

\[ i = 1, 2, \ldots, n \] (3.1)

where,
\[ x_i : \text{Component state variable vector} \]
Chapter 3. \textit{Linear Modeling of Power System Components}

\(a_i\) : Component input vector

\(b_i\) : Component output vector

\(n\) : Number of Components

Let the \(i^{th}\) component have input vector \(a_i\), output vector \(b_i\), and state vector \(x_i\). The linear state model for the \(i^{th}\) component may be written by,

\[
\dot{x}_i = A_i x_i + B_i a_i \\
b_i = C_i x_i + D_i a_i
\]

(3.2)

The composite component model is constructed by stacking the 'n' component equations together. The composite component state model may be expressed as:

\[
\dot{x} = Ax + Ba \\
b = Cx + Da
\]

(3.3)

where,

\(x = [x_1, ..., x_n]^T\), Component state vector

\(a = [a_1, ..., a_n]^T\), Component input vector

\(A = \text{Block diagonal}[A_1, ..., A_n]\)

\(B = \text{Block diagonal}[B_1, ..., B_n]\)

\(n\) is the number of components and in general not equal to the number of states.

The generalized Connection equation takes the form,

\[
a = L_{11} b + L_{12} u \\
y = L_{21} b + L_{22} u
\]

(3.4)

where,

\(a = [a_1, a_2, ..., a_n]^T\) and \(b = [b_1, b_2, ..., b_n]^T\)

\(y\) is the system output vector

\(u\) is the system input vector

\(L_{ij}\) are the connection matrices. These matrices are usually sparse.

Using the Component Connection technique, the composite system equations become

\[
\dot{X} = \overline{A}X + \overline{B}u \\
Y = \overline{C}X + \overline{D}u
\]

(3.5)

Equation 3.5 holds if and only if \([I - L_{11}D]^{-1}\) exists. The matrix \([I - L_{11}D]^{-1}\) exists generically, i.e. for almost all matrices \(L_{11}\) [30].
Equation 3.5 defines the overall state model for the large scale system. The $\bar{A}$, $\bar{B}$, $\bar{C}$ and $\bar{D}$ can be described as:

\begin{align*}
\bar{A} &= A + B[I - L_{11}D]^{-1}L_{11}C & (3.6) \\
\bar{B} &= B[I - L_{11}D]^{-1}L_{12} & (3.7) \\
\bar{C} &= L_{21}C + L_{21}D[I - L_{11}D]^{-1}L_{11}C & (3.8) \\
\bar{D} &= L_{21}D[I - L_{11}D]^{-1}L_{12} + L_{22} & (3.9)
\end{align*}

### 3.2 Eigenvalue Assignment Technique

#### 3.2.1 Overview

Design of feedback compensators which make use of the available outputs of the system has received considerable attention in recent years. A large class of these compensators are dynamic and are designed to achieve eigenvalue assignment in the augmented closed-loop system consisting of the plant and the compensator.

Some of the compensator design techniques for eigenvalue assignment concentrate on the augmented system as a whole and consequently overlook the stability properties of the compensator as a distinct and a separate dynamic system [32]. In these methods, all parameters of the compensator including the coefficients of its characteristic polynomial which determine its stability are calculated so as to position the eigenvalue of the augmented system. This can result in unstable controller design which is clearly undesirable.

In [33], a simple frequency-domain method is described in which the designer has complete freedom in choice of the compensator eigenvalues and uses only the numerator parameters of the compensator parameters of the compensator transfer function to assign the eigenvalues of the augmented system. This method not only ensures that the required dynamic compensator is stable, but also that its eigenvalues are at pre-specified location in the complex plane.

Also, there exists a class of physical system whose transient performance is characterized predominantly by a pair of complex conjugate eigenvalues; typical example is AC/DC power systems. For such systems it seems desirable to shift only the dominant pair of eigenvalues to desired locations.

In [Toliyat 8] a new method for the design of dynamic compensator with fixed pole(s) to assign a pair of complex eigenvalues of the power system to the prescribed value is presented.
3.2.2 Single Input Single Output System

Consider a controllable and observable single-input and single-output $n^{th}$ order linear system.

\[
\dot{X} = AX + BU \\
B = [0 \ldots b]^T \\
Y = X_1
\]  

(3.10)

where,

$X$ : $n \times 1$ state vector  
$U$ : scalar control input  
$Y$ : scalar control output

Let the first element $X_1$ of $X$ be $Y$, then 3.10 represents a system whose pair of dominant eigenvalues $(s_d, s_d^*)$ is to be shifted to desired values.

In the frequency domain, we can write equation 3.10 as:

\[
sX(s) = AX(s) + BU(s) \\
Y(s) = X_1(s)
\]  

(3.11)

The transfer function of $m^{th}$ order dynamic compensator is:

\[
\frac{U(s)}{Y(s)} = K\left[1 + \frac{sT_z}{1 + sT_p}\right]^m \triangleq F(s)
\]  

(3.12)

To have a stable compensator, the poles are pre-specified. In other words, $T_p$ is known and the objective is to find $K$ and $T_z$ such that $(s_d, s_d^*)$ have desired location in the complex plane.

Substituting equation 3.11 to 3.12,

\[
sX(s) = [A + E(s)]X(s)
\]  

(3.13)

where,

$e_{ij} = 0$ for all $i$ and $j$ except

\[
e_{n1} = bF(s)
\]  

(3.14)

Partitioning matrices $A$ and $E$ as follows:

\[
A = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\]  

(3.15)
The characteristic equation of the closed loop system of equation 3.13 can then be given by:

\[ \Delta(s) = det[sI - (A + E)] = 0 \]  

(3.17)

Using equation 3.16 and matrix reduction technique, equation 3.17 becomes:

\[ \Delta(s) = s - A_{11} - A_{12}(sI - A_{22})^{-1}(A_{21} + [0 \ldots 0 bF(s)]^T) = 0 \]  

(3.18)

No, if we substitute for the desired values of the dominant pair of complex eigenvalues \((s_d, s_d^*)\) in equation 3.18 then:

\[ s_d - C_1 - C_2 bF(s) = 0 \]  

(3.19)

or

\[ K \left[ \frac{1 + s_d T_z}{1 + s_d T_p} \right] = \frac{s_d - C_1}{C_2 b} \]  

(3.20)

where,

\begin{align*}
C1 &= A_{11} + A_{12}(s_d I - A_{22})^{-1} A_{21} \\
C2 &= A_{12}(s_d I - A_{22})^{-1} \text{ (the last element of row vector)} \\
b &= \text{the last value of the vector}
\end{align*}

From equation 3.20 it is obvious that in order to have the desired pole assignment the lowest possible value of \(m\) is one. Higher values is possible based on experience.

3.2.3 Eigenvalue Assignment of Multi-variable System

A multi-variable linear system consisting of \(N\) subsystems may be expressed as

\[ \dot{x}_i = A_i x_i + B_i U_i + \sum_{j=1, j \neq i}^{n} A_{ij} x_j \]  

(3.21)

where, \(y_i = [1 \ 0 \ldots 0] x_i\), and \(B_i = [0 \ldots 0 b_i]^T\)

whose stability is governed predominately by \(N\) pairs of complex-conjugate eigenvalues and each of these pairs of eigenvalues originates from each system.

It is desired to improve system stability by relocating all its \(N\) dominant pairs of complex
Chapter 3. *Linear Modeling of Power System Components*

conjugate eigenvalues to prescribed locations \((s_d, s_d^*)\) using N local compensators where,

\[
s_i, s_i^* = -\zeta \omega_i \pm \omega_i \sqrt{1 - \zeta_i^2}
\]  

(3.22)

Let the \(i^{th}\) local compensator be of the form,

\[
\frac{y_i}{U_i} = K_i \frac{1 + s T_{zi} m_i}{1 + s T_{pi}} \triangleq F_i(s)
\]  

(3.23)

In order to ensure the stability of each local compensator, \(T_{pi}\) is pre-specified \([33]\). The order of \(m_i\) is specified.

The design problem is now to find the N compensator parameters \((K_i, T_{zi})\) such that,

\[
\Delta(s_i) = 0, \quad \text{for } i = 1, \ldots, N
\]  

(3.24)

where,

\(\Delta(s) = 0\) is the closed loop system characteristic equation.

**3.2.3.1 Solution Technique**

Using the matrix reduction technique described in previous section, each equation in 3.24 is reduced to the form,

\[
s_i + C_{1i} = C_{2i} b_i F_i(s_i),
\]  

for \(i = 1, \ldots, N\)

(3.25)

where, \((C_{1i}, C_{2i})\) are functions of \(s_i\) and \((K_j, T_{zi})\) for all \(j \neq i\) due to mutual interaction between subsystems.

A numerical technique is required to solve equation 3.25. In Appendix B, the component connection technique for the Single Machine Infinite (SMIB) is detailed.

Different designs have been proposed depending on orders of the synchronous generators in Chapter 6. For solving, the matrices A,B,C have to be re-arranged depending upon the state used for the controller signal. Equation 3.18 can be solved by comparing the real and imaginary parts. The Gain \((K)\) and \(T_z\) are then calculated by;

\[
K = C_{real} - \left( \frac{C_{imag} \times S_{real}}{S_{imag}} \right)
\]  

\[
T_z = \frac{C_{imag}}{K \times S_{imag}}
\]  

(3.26)

where,

\[
C_{real} = Re \left( \frac{s_d - C_i}{C_2 \times \delta} \times (1 + T_p \times s_d) \right)
\]
\[ C_{\text{imag}} = \text{Im} \left( \frac{s_d - C_1}{s_d - C_2} \times (1 + T_p \times s_d) \right) \]
\[ S_{\text{real}} = \text{Re}(s_d) \text{ (Real part of } s_d) \]
\[ S_{\text{imag}} = \text{Im}(s_d) \text{ (Imaginary part of } s_d) \]

Once the controller parameters \( K \) and \( T_z \) are calculated using the above procedure, the controller is added to the existing system using the CCM technique with the proper input and output state.
Chapter 4

Modeling of Interconnected Power System

This chapter details the modeling of interconnected power system and methods for setting up the non-linear simulation of Power System. The detailed power system modeling equation are detailed in Appendix A.

4.1 Power System Model

The test system used in this research is the Single Machine Infinite Bus (SMIB) system. The system is shown in Fig. 4.1. The system model consists of DC link, AC link, Synchronous generator equipped with exciter and turbine governor. The system has Static load connected to on of the bus (bus 2). The infinite bus (Bus 1) is supplying power to the load through both HVDC link and AC line. Reactive modulation is done through Static Var Compensator (SVC). The following sections discuss in detail the

![Image of Single Machine Infinite bus (SMIB) system model]

**Figure 4.1:** Single Machine Infinite bus (SMIB) system model
modeling of each component used in Power System.

4.2 AC/DC Power System Component Model

In this section the linearized model describing the performance for each component of AC/DC is presented. The models are taken from appropriate references. The system equation are arranged in matrix form for modeling using the Component Connection Technique.

The block diagram for interconnected power system for component connection is shown in Fig. 4.2. There are three types of buses shown in figure 4.1

- Generator bus
- Load bus
- DC link terminal bus

Although only one bus or bus pair for each type is shown in figure 4.2, there can be large number of generators, loads and DC links in a given system.
4.2.1 Synchronous Generator Models

A general model based on Park’s equations for synchronous generators is adequately documented in literature and has been formulated in many papers [34], [5] and [1]. Throughout this research, two separate models of synchronous generators are used.

4.2.1.1 Fifth Order Model

In this model the stator transient terms are neglected. The model may be adapted into the CCM framework as shown in block diagram of Fig. 4.3. The linearized equations describing the model are given in Appendix B.

4.2.1.2 Third Order Model

In this model, the damper winding effects and the armature flux derivative are neglected. The third order model due to its simplicity has been used by many [35],[36],[37] to analyze and design generator excitation systems under a variety of conditions. The model may be adopted in the CCM framework as shown in block diagram of Fig. 4.4. The linearized equations describing the performance of the model are given in Appendix B.

4.2.2 Excitation and Turbine Governor Model

The excitation and governor-turbine control systems used in synchronous generator fall into standard categories compiled in IEEE committee reports. The turbine-governor
model used is recommended for steam and hydro-electric generators. The block diagrams for excitation system is given in Appendix A. The linearized equations are given in Appendix B.

### 4.2.3 Static Var Compensator Model

A Static Var Compensator (SVC) with thyristor controlled reactor is used. The model used has been taken from [38]. The thyristor controlled reactor is represented by inertia-less voltage source $E_s$ behind a fixed reactance $X_s$.

The magnitude of $E_s$ is controlled by the controllers of SVC and is always in phase with the terminal voltage $V_t$ and is given by:

$$E_s = f(\alpha_s)V_t$$  \hspace{1cm} (4.1)

where,

$$f(\alpha_s) = \frac{2\alpha_s - \sin 2\alpha_s}{\pi} - 1; \quad \frac{\pi}{2} \leq \alpha_s \leq \pi$$  \hspace{1cm} (4.2)

$\alpha_s$ is the firing angle. The detailed derivation for SVC is provided in Appendix A.
4.2.4 AC Network Model

The AC network is represented in synchronous rotating frame. The network is assumed to be completely described by the modal admittance matrix equations:

\[ T^e = Y V^e \]  \hspace{1cm} (4.3)

The matrix is formulated in AC/DC load flow. Equation 4.3 can be written as:

\[ V^e = Z T^e \]  \hspace{1cm} (4.4)

where,

\[ Z = Y^{-1} \]

\[ V^e = V_q^e - j V_d^e \]

\[ T^e = I_q^e - j I_d^e \]

4.2.5 HVDC Link and Converter Control Model

Representation of HVDC link and control associated with converter for power system stability analysis has been widely researched. Fig. 4.5 illustrates the model of HVDC link with converter control and AC/DC interface block.

---

**Figure 4.5:** Block Diagram for HVDC Controller
4.2.5.1 Converter Control Model

The converter is represented by moving average model for the converter. The converter voltage $V_{dc}$ is represented as a function of control angle $(\alpha, \gamma)$ and direct current $I_{dc}$.

$$V_{dc} = e_c - R_c I_{dc}$$

where,

$$e_c \leq V_{ac} T \cos \alpha : \text{Rectifier}$$

$$e_c \geq V_{ac} T \cos \gamma : \text{Inverter}$$

$T$ : p.u tap ratio

$R_c$ : Commutating Resistance

The model given neglects harmonics but is suitable for lower frequency phenomena like electro-mechanical oscillations.

4.2.5.2 Converter Current Control

The current controller at the rectifier and inverter use difference of reference current $I_{ref}$ and the measured dc current ($I_{dc}$) to derive the control circuitry which determines appropriate control delays. The control system is represented in Fig. 4.6. DC modulation can be implemented by adding a modulating signal to reference current ($I_{ref}$). The signal may be derived from synchronous generator speed or other signal [39],[40].

![Figure 4.6: HVDC Converter Current Regulator](image)

4.2.5.3 Converter Configuration

In this thesis, the converter connected to the infinite bus is a Rectifier and is operated in Constant Current (CC) Control mode. The converter connected to load bus is operated
as Inverter in Constant Extinction Angle (CEA) mode [41]. Reactive modulation can be done using the HVDC link but in this research the controllers are not providing the reactive modulation.

The non-linear equations describing the converter are detailed in Appendix A. Linearized model equations are presented in Appendix B.

### 4.2.6 AC/DC Interface Equation

The DC system representation may be interfaced with a reference frame model of the AC system by relating the direct current $I_{dc}$ with the AC current $I_e^q$ and $I_e^d$ in synchronous reference frame [42]. The fundamental power factor of each converter terminal is given by:

$$\cos \phi = \frac{V_{dc}}{T_a V_{ac}} \quad (4.6)$$

and peak fundamental AC current is given by:

$$I_m = T_a I_{dc} \quad (4.7)$$

The angle between the converter AC bus voltage and the AC synchronous reference frame is also given by:

$$\cos \theta = \frac{V_q^e}{V_{ac}} \quad (4.8)$$

and

$$V_{ac} = \sqrt{V_q^{e2} + V_d^{e2}} \quad (4.9)$$

Therefore, the AC current injected from each DC converter terminal is expressed as:

$$I_q^e = I_m \cos(\phi + \theta) \quad (4.10)$$

$$I_d^e = I_m \sin(\phi + \theta) \quad (4.11)$$

### 4.3 Non-linear Modeling of Power System

In order to model the non-linear power system the general differential-algebraic model of any power system can be written as [29]:

$$\dot{x} = f(x, v) \quad (4.12)$$

$$YV = I(x, V) \quad (4.13)$$

where the parameters are represented as:
• \( x \): state vector of power system model
• \( V \): bus voltage of the system
• \( I \): current injection vector into the system
• \( Y \): admittance matrix, including constant impedance loads and the modification due to faults in the system

Both functions \( f(x, V) \) and \( I(x, V) \) are nonlinear and their values can be obtained if the operating condition is given. The initial values of 4.12 and 4.13 are \( x_0 \) and \( I(x_0, V_0) \) respectively. A multi-machine power system consists of several synchronous generators connected through transmission lines. To design a controller, the machine needs to be modeled as a decoupled subsystem. One approach is to model the impact of overall system as disturbance to the subsystem.

In this work, however, a non-linear simulation for SMIB is developed using MATLAB/Simulink. Instead of using a decoupled approach, the system is modeled altogether by representing the infinite bus and the generator and HVDC link. The following subsection details the implementation of non-linear simulation for different orders of synchronous generators.

### 4.3.1 For Third order system

The modeling of differential equations for SMIB system with classical model of synchronous generator is detailed in this section. The simulation set up depends on the way the machine is modeled. The classical model is represented in such a way that the terminal voltages are calculated first. The generator supplied current is calculated from the resultant of currents in the interconnected system. The block diagram for simulation is given in Fig. 4.7. All the power system components are modeled such that the voltage for the bus is given and the current is taken as output.

#### 4.3.1.1 Transmission Line connecting nodes \( i,j \)

The power system is modeled in \( qdo \) system. The current injected between nodes \( i, j \) is given by 4.14 and 4.15,

\[
\begin{align*}
i_{qij} &= \frac{1}{x_{ij}} \left( -V_{d,i}^e + V_{d,j}^e \right) \\
i_{dij} &= \frac{1}{x_{ij}} \left( V_{q,i}^e - V_{q,j}^e \right)
\end{align*}
\]
4.3.1.2 Machine Current

For the bus connected to synchronous machine, DC converter and Static Load the current on \(i^{th}\) node for the synchronous generator are:

\[
\begin{align*}
  i_{q,i}^e &= \sum_{j \in \Omega} \frac{1}{X_{i,j}} (-V_{d_i}^e + V_{d_j}^e) + i_{q,c,i}^e \quad (4.16) \\
  i_{d,i}^e &= \sum_{j \in \Omega} \frac{1}{X_{i,j}} (-V_{q_i}^e + V_{q_j}^e) + i_{d,c,i}^e \quad (4.17)
\end{align*}
\]

where, \(\Omega\) is the set of adjacent nodes with lines connected to bus \(i\).
4.3.1.3 Bus Voltage

For bus with DC converter and load the voltages are given by;

\[ V_{q,i}^e = R_{L,i} \left( -i_{q,c,i}^e + \sum_{j \in \Omega} i_{q,j,i}^e - \frac{1}{X_{c,i}} V_{d,i}^e + \frac{1}{X_{L,i}} V_{d,i}^e \right) \]  \hspace{1cm} (4.18)

\[ V_{d,i}^e = R_{L,i} \left( -i_{d,c,i}^e + \sum_{j \in \Omega} i_{d,j,i}^e + \frac{1}{X_{c,i}} V_{q,i}^e - \frac{1}{X_{L,i}} V_{q,i}^e \right) \]  \hspace{1cm} (4.19)

4.3.2 For Full Order System

The simulation for SMIB with fifth order model of synchronous generator is slightly different as compared to the modeling strategy used in SMIB system with classical model of synchronous generator. The block diagram describes the implementation of this system. Necessary transformation from electrical to rotor reference frame is done where required. For setting up simulation, the components are modeled with initial estimate of bus voltage. The currents for each component are calculated using the equations given in Appendix A.

Fig. 4.8 represents the flow chart for the simulation of SMIB for fifth order synchronous generator.

4.3.2.1 Bus Voltage

To calculate the bus voltage connected to converter, machine and the load 4.21 is used;

\[ V_{q_2}^e = V_{q_1}^e - I_q^e \times X_{ac} \]  \hspace{1cm} (4.20)

\[ V_{d_2}^e = V_{d_1}^e + I_q^e \times X_{ac} \]  \hspace{1cm} (4.21)

where,

*bus2*: load bus

*bus1*: infinite bus

*\( X_{ac} \)*: reactance of AC line.
Figure 4.8: Flow Chart for simulation (Fifth Order)
Chapter 5

Non-Linear Controller Design

5.1 Overview

The objective of given controller design is to derive a physical system to the specifications of its desired behavior, construct a feedback control law to make the closed-loop system display the desired behavior.

The non-linear control problems are described to be:

- Nonlinear regulation
- Nonlinear tracking

This chapter discusses both the problems and the control theory proposed to solve these problems. Also, Synergetic control Theory is detailed and methodology for designing controller using Synergetic Control Theory is discussed.

5.2 Theoretical Background

If the task of a control system involves large range and/or high speed motions, nonlinear effects will be significant in the dynamics and nonlinear control may be necessary to achieve the desired performance.

Generally, the tasks of control systems can be divided into two categories: stabilization (or regulation) and tracking (or servo) [27].
5.2.1 Regulation Problems

In stabilization problems, a control system, called a stabilizer (or a regulator), is to be designed so that the state of the closed-loop system will be stabilized around an equilibrium point. Typical examples of stabilization tasks are temperature control of refrigerators, altitude control of aircraft and position control of robot arms.

5.2.1.1 Asymptotic Stabilizing Problem

Given a non-linear dynamic system described by
\[ \dot{x} = f(x, u, t) \] (5.1)

Then the asymptotic stabilizing problem is finding a control law \( u \), starting from anywhere in region \( \Omega \), such that the state \( x \) tends to 0 as \( t \to \infty \).

Thus, the objective of the control law is to derive the state to some non-zero set point \( x_d \), we can simply transform the problem into a zero-point regulation problem by taking \( x - x_d \) as the state.

5.2.2 Tracking Problem

In tracking control problems, the design objective is to construct a controller called tracker, so that the system output tracks a given time-varying trajectory. Problems such as making an aircraft fly along a specified path or making a robot hand draw straight lines or circles are typical tracking control tasks.

5.2.2.1 Asymptotic Tracking Problem

Given a non-linear dynamic system described by
\[ \dot{x} = f(x, u, t) \] (5.2)
\[ y = h(x) \] (5.3)

Asymptotic Tracking Problem is to get a desired output trajectory \( y_d \), for a control input \( u \) such that starting from any initial state in region \( \Omega \), the tracking errors \( y(t) - y_d(t) \) goes to zero, while the whole system \( x \) remains bounded.
5.2.3 Available Methods for Non-linear Controller

Non-linear control analysis has rich collection of alternative and complementary techniques each applicable to particular case of non-linear control problems. Some of the widely used methods are briefly discussed.

5.2.3.1 Trial and Error

One can use trial and error to synthesize controllers e.g. linear lead-lag controller design based on Bode plots. The phase plane method, the describing function method and Lyapunov analysis can all be applied for this purpose. This methods requires experience and intuition and therefore it fails often for complex systems.

5.2.3.2 Feedback Linearization

For designing a control system the most significant part is to derive a meaningful model of plant. Feedback linearization deals with techniques for transforming original system models into equivalent models of simpler form. The basic idea is to transform a non-linear system into a fully or partial linear system and then use linear design techniques to complete control design. Chapter 3 details the method for applying this technique for SMIB system in this research.

5.2.3.3 Robust Control

In model-based nonlinear control, the control law is designed based on a nominal model of the physical system. The control system behavior in the presence of model uncertainties is not clear at the design stage.

In robust nonlinear control (such as, e.g., sliding control), the controller is designed based on the consideration of both the nominal model and some characterization of the model uncertainties. Robust nonlinear control techniques have proven very effective and generally require state measurements.

Sliding Mode Control

As discussed earlier, modeling inaccuracies can have adverse effects on nonlinear control systems. Therefore, any practical design must address them explicitly. Two major and complementary approaches to dealing with model uncertainty are robust control and adaptive control.
A simple approach to robust control is so-called sliding control methodology. Intuitively, it is based on the remark that it is much easier to control 1st-order systems (i.e., systems described by 1st-order differential equations), be they nonlinear or uncertain, than it is to control general \( i \)-th-order systems (i.e., systems described by \( n \)-th-order differential equations). Accordingly, a notational simplification is introduced, which, in effect, allows \( n \)-th-order problems to be replaced by equivalent 1st-order problems.

Consider the single input dynamic system

\[
x^n = f(x) + b(x)u
\]

The control problem is to get the state \( x \) to track a specific time varying state \( x_d = [x_d \ x_d \ldots x_d^{(n-1)}]^T \) in the presence of model imprecision on \( f(x) \) and \( b(x) \) Let us define a time varying surface \( S(t) \) in the state space \( R^n \) by the scalar equation \( s(x : t) = 0 \).

Thus the sliding-mode control scheme involves

- Selection of a hyper-surface or a manifold (i.e., the sliding surface) such that the system trajectory exhibits desirable behavior when confined to this manifold.
- Finding feedback gains so that the system trajectory intersects and stays on the manifold.

Because sliding mode control laws are not continuous, it has the ability to drive trajectories to the sliding mode in finite time (i.e., stability of the sliding surface is better than asymptotic). However, once the trajectories reach the sliding surface, the system takes on the character of the sliding mode (e.g., the origin \( x = 0 \) may only have asymptotic stability on this surface) [43].

5.2.3.4 Adaptive Control

Adaptive control is an approach to dealing with uncertain systems or time-varying systems. Although the term "adaptive" can have broad meanings, current adaptive control designs apply mainly to systems with known dynamic structure, but unknown constant or slowly-varying parameters. Adaptive controllers, whether developed for linear systems or for nonlinear systems, are inherently nonlinear.

5.2.3.5 Gain-Scheduling

The idea of gain-scheduling is to select a number of operating points which cover the range of the system operation. Then, at each of these points, the designer makes a linear
time-invariant approximation to the plant dynamics and designs a linear controller for each linearized plant. Between operating points, the parameters of the compensators are then interpolated, or scheduled, thus resulting in a global compensator. Gain scheduling is conceptually simple, and, indeed, practically successful for a number of applications. The main problem with gain scheduling is that has only limited theoretical guarantees of stability in nonlinear operation, but uses some loose practical guidelines such as "the scheduling variables should change slowly" and "the scheduling variables should capture the plant’s non-linearities". Another problem is the computational burden involved in a gain-scheduling design, due to the necessity of computing many linear controllers.

### 5.3 Synergetic Control Theory

Currently non-linear controller methods use simplified models to decrease complexity of algorithm. Taking into consideration the complexity of power system, more realistic method with less computation is required. One of the recent methods for designing non-linear control is based on Synergetic Control Theory developed by Russian scientists [16].

The synergetic control design procedure defines control laws which ensure that the system reaches the specified set of manifold. This results in a dynamic non-linear regulator which ensures asymptotic stability of the steering actuator, robustness against variation of load coefficient, and insensitivity to uncertain external disturbances [44]. The theory is similar to Sliding Mode Control [45] but has no chattering issue.

The design procedure for synergetic control theory follows the Analytic Design of Aggregated Regulators (ADAR) method. Suppose the system to be controlled is described by set of non-linear equations.

\[
\dot{x} = f(x, u, t) \tag{5.5}
\]

where, \( u \) is the control input vector and \( x \) is the state of system. The controller produces the control vector \( u \), which is used to force the system to operate in a desired manner. The synergetic synthesis of the controller begins by defining a macro-variable given by

\[
\varphi = \varphi(x, t) \tag{5.6}
\]

where \( \varphi \) is the macro-variable and \( \varphi(x, t) \) is a user-defined function of system state variables and time. The objective of the synergetic controller is to direct the system to operate on the manifold such that;

\[
\varphi = 0 \tag{5.7}
\]
Thus the problem of synthesis of the control laws which suppress perturbations and provide the given dynamical properties of the closed system is formulated: it is required to determine a control vector \( u(x_1, \ldots, x_n) \) that ensures the transition of the image point of the extended plant from the arbitrary initial state (in some admissible domain) first to some manifolds \( \varphi(x_1 \ldots x_n) = 0 \), and then to the given state \([28]\). The designer can select the characteristics of the macro-variable according to the control specifications (e.g. limitation in the control output, and so on). In the trivial case the macro-variable can be a simple linear combination of the state variables. The same process can be repeated, defining as many macro-variables as control inputs. The desired dynamic evolution of the macro-variable is defined as:

\[
K \dot{\varphi} + \varphi = 0 \quad K \geq 0
\]  

(5.8)

where \( K \) is a design parameter specifying the convergence speed to the manifold specified by the macro-variable. Equation 5.8 is a differential equation whose solution is given by:

\[
\varphi = \varphi(0)e^{-\frac{1}{K}t}
\]

As \( t \rightarrow \infty \) the solution converges to \( \phi = 0 \). In summary, the manifold introduces a new constraint on the state space domain and reduces the order of the system, working in the direction of global stability. The minimum of the following optimizing functional is reached on the trajectories of motion of the closed-loop system and the asymptotic stability of the system in some domain.\([46]\)

\[
J = \int_0^\infty \left( \sum_{k=1}^{m} K_k^2 \varphi_k(t)^2 + \varphi_k(t)^2 \right) dt
\]

(5.9)

The procedure summarized here can be easily implemented as a computer program for automatic synthesis of the control law.

Consider a linear time invariant system whose dynamics are defined by the state space given below:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
b_1 \\
b_2
\end{bmatrix} u
\]

(5.10)

\[
y = x_1
\]

The initial operating points of the system are \( x_{1o}, x_{2o} \). To regulate the system defined by 5.10 to a final stable point of particular interest \( (x_{1f}, x_{2f}) \), we can define error variables as \( z_1 = x_1 - x_{1f} \) and \( z_2 = x_2 - x_{2f} \). Upon substitution of error variable to the system in 5.10,

\[
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2
\end{bmatrix} = \begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix} \begin{bmatrix}
z_1 \\
z_2
\end{bmatrix} + \begin{bmatrix}
b_1 \\
b_2
\end{bmatrix} u + \begin{bmatrix}
f_1 \\
f_2
\end{bmatrix}
\]

(5.11)
where,

\[ f_1 = a_{11}x_1 + a_{12}x_2 \]  \hspace{1cm} (5.12)

\[ f_2 = a_{21}x_1 + a_{22}x_2 \]  \hspace{1cm} (5.13)

As described, the choice of manifold \( \phi \) is arbitrary. For the problem to be regulatory, the controller proposed by Slotine and Li is used. The manifold is based on integro-differential equation and depends on the error variable \( z \). The manifold is given by;

\[ \varphi = \left( \frac{d}{dt} + \lambda \right)^{n-1} z \]  \hspace{1cm} (5.14)

where, \( n \) is the number of states of system From, equation 5.14

\[ \varphi = \dot{z} + \lambda z \]

The solution to the manifold is given by;

\[ \varphi = \varphi(0)e^{-\frac{1}{K}t} \]

For the manifold defined to be stable, \( \lambda > 0 \). The solution will produce, \( z = 0 \) and thus the control input is said to have forced the system to the manifold.

**Example System**

The methodology described in the previous section is applied to a sample test system. Consider the system to be controlled is given by;

\[ \dot{x} - x^3 + x^2 = u \]  \hspace{1cm} (5.15)

Let \( x_1 = x \) and \( x_2 = \dot{x}_1 \) and \( \dot{x}_2 - x_1^3 + x_1^2 = u \)

Defining the problem as regulating problem, the error variable is defined as \( \tilde{x}_1 = x_1 - x_{1o} \), the manifold is defined as;

\[ \varphi = \left( \frac{d}{dt} + \lambda \right)\tilde{x}_1 \]  \hspace{1cm} (5.16)

The dynamic evolution of synergetic controller is given by;

\[ K\dot{\varphi} + \varphi = 0 \]

\[ \dot{x}_1 + \lambda x_1 = \frac{-1}{K}(\dot{x}_1 + \lambda x_1) \]
Rearranging and substituting $x_1 = x$ and $x_2 = x'_1$ and $x'_2 - x_3^2 + x_1^2 = u$,

$$u = -x_3^2 + x_1^2 - \lambda x_2 - \frac{1}{K}(x_2 + \lambda x_1) \quad (5.17)$$

To solve the system, MATLAB ode solver is used. Figure 5.1 shows the response of $x_1$, $x_2$ for different set of controller values. The effect of controller parameters is obvious. Variation in $K$ varies the settling time of the system to the equilibrium points. $\lambda$ affects the overshoot of the system response. Thus the variation in these parameters tremendously affects the closed loop system response.

**5.4 Synergetic Controller using Classical Model**

For designing the controller, the main design issue is choosing suitable manifold. In [26],[47], a polynomial differential equation for the error is used for making small, slowly varying tracking errors decay. The controller is based on approach used by Slotine and Li [27]. The appropriate quantity to be monitored used to be the integral of the error $z$. By use of a symmetric positive definite matrix, first order integro–differential equation can be obtained for the error $\varphi$. The degree of the polynomial depends on order of the system to be controlled. In our case, the macro variable is taken from [27] and is defined as

$$\varphi = (\frac{d}{dt} + \lambda)^{n-1}z \quad (5.18)$$
where $n$, is is the order of the system to be controlled and $z$ is the error variable. For $n = 3$,

$$
\delta(t) = \omega_o(\omega(t) - 1) \tag{5.19}
$$

$$
\omega(t) = \frac{-D}{2H}(\omega(t) - 1) - \frac{1}{2H}(P_e(t) - P_m(t)) \tag{5.20}
$$

$$
E_q'(t) = \frac{1}{\tau_{D_o}}(E_{fd} - (X_d - X_d')I_d(t) - E_q'(t)) \tag{5.21}
$$

$$
E_{fd} = K_c u_f(t)
$$

The dynamic of the evolution of the manifold is given by:

$$
K \dot{\varphi} + \varphi = 0 \tag{5.22}
$$

$$
\varphi = \left(\frac{d}{dt} + \lambda\right)^2 x_1 \tag{5.23}
$$

$$
\varphi = \left(\frac{d^2}{dt^2} + \lambda^2 + 2\lambda \frac{d}{dt}\right)x_1 = \dot{x}_1 + 2\lambda \dot{x}_1 + \lambda^2 x_1 \tag{5.24}
$$

$$
\varphi = \omega_o \dot{x}_2 + 2\lambda \omega_o x_2 + \lambda^2 x_1 \tag{5.25}
$$

Defining the state variables, $x_1 = \delta(t)$, $x_2 = \omega(t) - 1$ and $x_3 = P_e(t) - P_m(t)$ and substituting,

$$
\dot{x}_1 = \omega_o \dot{x}_2 \quad \dot{x}_2 = \frac{-D}{2H} x_2 - \frac{1}{2H} \dot{x}_3 \quad \dot{x}_2 = \frac{-D}{2H} \dot{x}_2 - \frac{1}{2H} \dot{x}_3
$$

$$
\varphi = -\frac{\omega_o D}{2H} x_2 - \frac{\omega_o}{2H} x_3 + 2\lambda \omega_o x_2 + \lambda^2 x_1
$$

The manifold is given as:

$$
\varphi = \lambda^2 x_1 + \omega_o (2\lambda - \frac{D}{2H}) x_2 - \frac{\omega_o}{2H} x_3 \tag{5.26}
$$

Taking derivative of equation 5.26,

$$
\dot{\varphi} = \lambda^2 \dot{x}_1 + \omega_o (2\lambda - \frac{D}{2H}) \dot{x}_2 - \frac{\omega_o}{2H} \dot{x}_3 \tag{5.27}
$$

Substituting equation 5.26 and 5.27 in 5.22 and re-arranging,

$$
\lambda^2 \dot{x}_1 + \omega_o (2\lambda - \frac{D}{2H}) \dot{x}_2 - \frac{\omega_o}{2H} \dot{x}_3 = -\frac{1}{K}(\omega_o \dot{x}_2 + 2\lambda \omega_o x_2 + \lambda^2 x_1)
$$

$$
\frac{\omega_o}{2H} \dot{x}_3 = \lambda^2 \dot{x}_1 + \omega_o (2\lambda - \frac{D}{2H}) \dot{x}_2 + \frac{1}{K}(\omega_o \dot{x}_2 + 2\lambda \omega_o x_2 + \lambda^2 x_1)
$$
\[ \dot{x}_3 = 2H\lambda^2 x_2 + 2H(2\lambda - \frac{D}{2H})\dot{x}_2 + \frac{2H}{K\omega_o}(\omega_o \dot{x}_2 + 2\lambda \omega_o x_2 + \lambda^2 x_1) \]

Re-arranging to get \( \dot{x}_3 \):

\[ \dot{x}_3 = 2H\lambda^2 x_2 + 2H(2\lambda - \frac{D}{2H})\dot{x}_2 + \frac{2H}{K} \dot{x}_2 + \frac{4H\lambda}{K} x_2 + \frac{2H\lambda^2}{K\omega_o} x_1 \]

Re-arranging,

\[ \dot{x}_3 = 2H(2\lambda - \frac{D}{2H} + \frac{1}{K})\dot{x}_2 + 2H(\lambda^2 + \frac{2\lambda}{K}) x_2 + \frac{2H\lambda^2}{K\omega_o} x_1 \]  

(5.28)

\[ P_c(t) = E'_q(t)I_q(t) \]

Using chain rule,

\[ \dot{P}_c(t) = E'_q(t)I_q(t) + E'_q(t)I'_q(t) \]  

(5.29)

where \( I_q(t) \) and \( I_d(t) \) is given by:

\[ I_q(t) = \frac{V_\infty \sin(\delta(t))}{X_{ds}'} , \quad I_d(t) = \frac{E'_q(t) - V_\infty \cos(\delta(t))}{X_{ds}'} \]

and

\[ X_{ds}' = X_d' + X_L - X_d' X_L B_{SVC}(\alpha(t)) , \text{ where } X_L = X_{AC}||X_{DC} \]  

(5.30)

Taking derivative, using the quotient rule \( \frac{d}{dt}\left[ \frac{f}{g} \right] = \frac{g \dot{f} - f \dot{g}}{g^2} \) to find \( I_q'(t) \)

\[ I_q'(t) = \frac{V_\infty (X_{ds}' \cos(\delta(t)) \dot{\delta}(t) - \sin(\delta(t)) X_{ds}' X_L B_{SVC}(\alpha(t)))}{(X_{ds}')^2} \]  

(5.31)

Plugging in equation 5.21 and 5.31 in 5.29

\[ \dot{P}_c(t) = \frac{1}{\tau_{Di}} (E_{fd} - (X_d' - X_d')I_d(t) - E'_q(t))I_q(t) + \]

\[ E'_q(t) \left( \frac{V_\infty (X_{ds}' \cos(\delta(t)) \dot{\delta}(t) - \sin(\delta(t)) X_{ds}' X_L B_{SVC}(\alpha(t)))}{(X_{ds}')^2} \right) \]  

(5.32)

Re-arranging, and substituting \( \dot{\delta}(t) = \omega_o(\omega(t) - 1) \)
\[ P_e(t) = \frac{1}{\tau_{Do}} (K_e u_f(t) I_q(t) - (X_d - X'_d)I_q(t)I_d(t) - E'_q(t)I_q(t)) + \]
\[ \frac{E'_q(t)V_{\infty}\cos(\delta(t))\omega_o(\omega(t) - 1)}{X'_d} - \frac{E'_q(t)V_{\infty}\sin(\delta(t))}{X'_d} X'_d X_L B_{SVC}(\alpha(t)) \] (5.33)

where,
\[ P_e(t) = \frac{E'_q(t)V_{\infty}\sin(\delta(t))}{X'_d}, \quad Q_e(t) = \frac{(E'_q)^2 - E'_q(t)V_{\infty}\cos(\delta(t))}{X'_d} \]

\[ P_e(t) = \frac{1}{\tau_{Do}} (K_e u_f(t) I_q(t) - (X_d - X'_d)I_q(t)I_d(t) - x_3(t) - P_e(t)) + \]
\[ \left( \frac{(E'_q)^2}{X'_d} - Q_e(t) \right) \omega_o(\omega(t) - 1) - P_e(t) \frac{X'_d X_L B_{SVC}(\alpha(t))}{X'_d} \] (5.34)

In terms of state variables \( x_1, x_2 \) and \( x_3 \) equation (5.34) is written as:

\[ x_3'(t) = \frac{1}{\tau_{Do}} (K_e u_f(t) I_q(t) - (X_d - X'_d)I_q(t)I_d(t) - x_3(t) - P_m(t)) + \left( \frac{(E'_q)^2}{X'_d} - Q_e(t) \right) \omega_o(x_2(t)) \]
\[ - x_3(t) \frac{X'_d X_L B_{SVC}(\alpha(t))}{X'_d} \] (5.35)

Plugging equation (5.28) in (5.34) to get \( u_f(t) \)

\[ 2H(2\lambda - \frac{D}{2H} + \frac{1}{K})x_2 + 2H(\lambda^2 + \frac{2\lambda}{K})x_2 + 2H\lambda^2 x_1 = \left( \frac{1}{\tau_{Do}} (K_e u_f(t) I_q(t) - (X_d - X'_d)I_q(t)I_d(t) \right) \]
\[ - x_3(t) - P_m(t)) + \left( \frac{(E'_q)^2}{X'_d} - Q_e(t) \right) \omega_o(x_2(t)) - x_3(t) \frac{X'_d X_L B_{SVC}(\alpha(t))}{X'_d} \]

\[ \frac{K_e I_q(t)}{\tau_{Do}} u_f(t) = 2H(2\lambda - \frac{D}{2H} + \frac{1}{K})x_2 + 2H(\lambda^2 + \frac{2\lambda}{K})x_2 + 2H\lambda^2 x_1 \]
\[ + \left( \frac{(E'_q)^2}{X'_d} - Q_e(t) \right) \omega_o x_2(t) + x_3(t) \frac{X'_d X_L B_{SVC}(\alpha(t))}{X'_d} \]
\[ u_f(t) = \frac{1}{KcI_q(t)} \left( (X_d - X'_d)I_q(t)I_d(t) + x_3(t) + P_m(t) - \tau_{D_o}\omega_o \left( \frac{(E'_q)^2}{X'_{ds}} - Q_e(t) \right) x_2(t) + \frac{X'_dX_LB_{SVC}(\alpha)}{X'_{ds}}x_3(t) \right) \]

\[ \frac{2H\tau_{D_o}}{KcI_q(t)} \left( (2\lambda - D \frac{1}{2H} + \frac{1}{K})\dot{x}_2 + (\lambda^2 + 2\frac{\lambda}{K})x_2 + \frac{\lambda^2}{K\omega_o}x_1 \right) \]

\[ u_f(t) = \frac{1}{KcI_q(t)} \left( (X_d - X'_d)I_q(t)I_d(t) + x_3(t) + P_m(t) - \tau_{D_o}\omega_o \left( \frac{(E'_q)^2}{X'_{ds}} - Q_e(t) \right) x_2(t) + \frac{X'_dX_LB_{SVC}(\alpha)}{X'_{ds}}x_3(t) \right) \]

\[ \tau_{D_o} \left( 2H(2\lambda - D \frac{1}{2H} + \frac{1}{K}) \frac{1}{2H} (-Dx_2(t) - x_3(t)) \right) \]

\[ + 2H(\lambda^2 + 2\frac{\lambda}{K})x_2 + \frac{2H\lambda^2}{K\omega_o}x_1 \] (5.36)

\[ u_f(t) = \frac{1}{KcI_q(t)} \left( (X_d - X'_d)I_q(t)I_d(t) + x_3(t) + P_m(t) - \tau_{D_o}\omega_o \left( \frac{(E'_q)^2}{X'_{ds}} - Q_e(t) \right) x_2(t) + \frac{X'_dX_LB_{SVC}(\alpha)}{X'_{ds}}x_3(t) \right) \]

\[ \tau_{D_o} \left( \frac{D^2}{2H} - 2\lambda D - \frac{D}{K} + 2H(\lambda^2 + \frac{2\lambda}{K}) \right) x_2(t) - (2\lambda - D \frac{1}{2H} + \frac{1}{K}) x_3(t) + \frac{2H\lambda^2}{K\omega_o}x_1(t) \] (5.37)

Equation 5.37 gives the supplementary control signal for synergetic controller. For Eq. 5.37 to hold \( I_q \neq 0 \)

### 5.5 Synergetic Controller using full model

For the full order model, the stator transients are neglected. The detailed equations for the synchronous generators are given in Appendix A.

For designing the controller a different manifold is selected. The manifold proposed by Slotine and Li and used in [29] cannot be used for this model of the synchronous generator. The order of the manifold for fifth order system is 4, and it becomes very troublesome to derive the controller in that case.
For the manifold, the approach used is similar to the approach used in [48].

\[
\varphi = K_1(\omega - \omega_{ref}) - (P_e - P_{ref})
\]  

(5.38)

Here \(\omega_{ref}\) and \(P_{ref}\) are the reference speed and power respectively. The sliding surface can be visualized in Fig. 5.2.

![Figure 5.2: Geometric interpretation of control law in phase plane for 5th order](image-url)

Using the approach for designing synergetic controller discussed in Chapter 5,

\[
K \dot{\varphi} + \varphi = 0
\]

(5.39)

Taking derivative of 5.38,

\[
\dot{\varphi} = K_1 \dot{\omega} + \dot{P}_e
\]

(5.40)

Re-arranging the equation for manifold,

\[
\dot{\varphi} = -\frac{1}{K} \varphi
\]

And,

\[
K_1 \dot{\omega} - \dot{P}_e = -\frac{1}{K} (K_1(\omega - \omega_{ref}) - (P_e - P_{ref})
\]

(5.41)

Let \(\tilde{\omega} = \omega - \omega_{ref}\) and \(\tilde{P}_e = P - P_{ref}\) Then,

\[
K_1 \tilde{\omega} - \dot{\tilde{P}}_e = -\frac{1}{K} (K_1(\tilde{\omega}) - \tilde{P}_e)
\]

(5.42)

\[
\dot{\tilde{P}}_e = \frac{1}{K} \left( K_1(\tilde{\omega}) - \tilde{P}_e \right) + K_1 \tilde{\omega}
\]

(5.43)

\[
\delta(t) = \omega_r - \omega_e
\]

(5.44)

\[
\omega(t) = -\frac{1}{2H} (P_e(t) - P_m(t))
\]

(5.44)

where, \(P_e = \omega_b T_e\)
For Single Machine Infinite Bus (SMIB) system, the active power flow is given by;
\[ P_e = \frac{E_q'V_\infty \sin\delta}{X'_{ds}} \] (5.45)

The reactive power is given by;
\[ Q_e = \frac{(E_q')^2 - E_q'V_\infty \cos\delta}{X'_{ds}} \] (5.46)

Taking derivative using chain rule,
\[ \dot{P}_e = E_q'V_\infty \sin\delta \frac{\dot{\delta}}{X'_{ds}} + E_q V_\infty \sin\delta \frac{\dot{X}'_{ds}}{X'_{ds}} \] (5.47)

where,
\[ I_q = \frac{V_\infty \sin\delta}{X'_{ds}} \] (5.48)

and
\[ I_d(t) = \frac{E_q' - V_\infty \cos(\delta(t))}{X'_{ds}} \] (5.49)

Taking derivative, using the quotient rule \[ \dot{\bigg[\frac{f}{g}\bigg]} = \frac{g\dot{f} - f\dot{g}}{g^2} \] to find \( I_q(t) \)
\[ I_q(t) = V_\infty \left( \frac{(X'_{ds}\cos(\delta(t))\dot{\delta}(t) + \sin(\delta(t))X_A X_B B_{SVC}(\alpha(t)))}{(X'_{ds})^2} \right) \] (5.50)

Substituting equation 5.50 in equation 5.47
\[ \dot{P}_e = E_q'V_\infty \sin\delta \frac{\dot{\delta}}{X'_{ds}} + E_q'V_\infty \cos(\delta(t))\dot{\delta}(t) + \sin(\delta(t))X_A X_B B_{SVC}(\alpha(t)) \] \[ \frac{\dot{X}'_{ds}}{(X'_{ds})^2} \]

Re-arranging,
\[ \dot{P}_e = E_q'V_\infty \sin\delta \frac{\dot{\delta}}{X'_{ds}} + \frac{E_q'V_\infty X_A X_B B_{SVC}(\alpha(t))}{(X'_{ds})^2} + E_q V_\infty \sin(\delta(t))X_A X_B B_{SVC}(\alpha(t)) \] \[ \frac{\dot{X}'_{ds}}{(X'_{ds})^2} \]

and
\[ \dot{P}_e = E_q'V_\infty \sin\delta \frac{\dot{\delta}}{X'_{ds}} + \frac{E_q'V_\infty \cos(\delta(t))\dot{\delta}(t)}{X'_{ds}} + \frac{E_q V_\infty \sin(\delta(t))X_A X_B B_{SVC}(\alpha(t))}{(X'_{ds})^2} \]
Substituting for $P_e$ and $Q_e$

$$\dot{P}_e = \dot{E}_q I_q + \frac{(E_q')^2}{X_{ds}'} - Q_e(t)\delta(t) + P_e \frac{X_A X_B B_{SVC}(\alpha(t))}{X_{ds}'}$$

(5.51)

For fifth order model of synchronous generator, $E_q'$ is proportional to the flux in the field winding $\psi_{fd}$.

From [8], [1], for steady state analysis of the full order of synchronous machine,

$$E_q' = \frac{X_{md}}{X_{fd}} \psi_{fd}'$$

(5.52)

where, $X_{fd} = X_{1fd} + X_{md}$

Taking derivative of equation 5.52, to find $\dot{E}_q'$

$$\dot{E}_q' = \frac{X_{md}}{X_{fd}} \psi_{fd}'$$

(5.53)

where the flux linkage for field winding is given by,

$$\psi_{fd}' = \omega b \left( \frac{r_{fd}}{X_{md}} E_{fd} + \frac{r_{fd}}{X_{1fd}} (\psi_{md}' - \psi_{fd}') \right)$$

(5.54)

Substituting,

$$\dot{E}_q' = \frac{X_{md}}{X_{fd}} \omega b \left( \frac{r_{fd}}{X_{md}} E_{fd} + \frac{r_{fd}}{X_{1fd}} (\psi_{md}' - \psi_{fd}') \right)$$

(5.55)

and $E_{fd} = K_c u_f$,

$$\dot{E}_q' = \frac{\omega b r_{fd}}{X_{fd}} K_c u_f + \frac{X_{md}}{X_{fd}} \omega b \left( \frac{r_{fd}}{X_{1fd}} (\psi_{md}' - \psi_{fd}') \right)$$

(5.56)

Substituting eq. 5.56 in eq. 5.51

$$\dot{P}_e = \frac{\omega b r_{fd}}{X_{fd}} K_c u_f I_q + \frac{X_{md}}{X_{fd}} \omega b \left( \frac{r_{fd}}{X_{1fd}} (\psi_{md}' - \psi_{fd}') \right) I_q$$

$$+ (Q_e + \frac{V_\infty}{X_{ds}'} \delta(t) + P_e \frac{X_A X_B B_{SVC}(\alpha(t))}{X_{ds}'}$$

(5.57)

Substituting 5.57 in the manifold eq. 5.42,

$$\frac{\omega b r_{fd}}{X_{fd}} K_c u_f I_q + \frac{X_{md}}{X_{fd}} \omega b \left( \frac{r_{fd}}{X_{1fd}} (\psi_{md}' - \psi_{fd}') \right) I_q$$

$$+ \left( \frac{(E_q')^2}{X_{ds}'} - Q_e(t)\delta(t) + P_e \frac{X_A X_B B_{SVC}(\alpha(t))}{X_{ds}'} \right) = \frac{1}{K} \left( K_1 \dot{\omega} - \dot{P}_e \right) + K_1 \dot{\omega}$$

(5.58)
Re-arranging to find $u_f$

\[
\frac{\omega_b r_{fd}}{X_{fd}} K_c u_f I_q = - \frac{X_{md}}{X_{fd}} \omega_b \left( \frac{r_{fd}}{X_{ld}} (\psi^r_{md} - \psi^r_{fd}) \right) I_q - \frac{(E_q')^2}{X_{ds}} - Q_e(t) \dot{\delta}(t) - P_e \frac{X_A X_B B_{SVC}(\alpha(t))}{X_{ds}'} + \frac{1}{K} \left( K_1 (\dot{\omega}) - \dot{P}_e \right) + K_1 \dot{\omega}
\]

Then,

\[
u_f = \frac{X_{fd}}{\omega_b r_{fd}} \frac{1}{K_c I_q} \left[ - \frac{X_{md}}{X_{fd}} \omega_b \left( \frac{r_{fd}}{X_{ld}} (\psi^r_{md} - \psi^r_{fd}) \right) I_q - \frac{(E_q')^2}{X_{ds}} - Q_e(t) \dot{\delta}(t) - P_e \frac{X_A X_B B_{SVC}(\alpha(t))}{X_{ds}'} + \frac{1}{K} \left( K_1 (\dot{\omega}) - \dot{P}_e \right) + K_1 \dot{\omega} \right]
\]

Equation 5.59 gives the supplementary control signal for synergetic controller. For eq. 5.59 to hold $I_q \neq 0$
Chapter 6

Analysis and Simulation

In this chapter, results for both linear and non-linear controller for different order of synchronous generators are provided. Results are compared and performance of the controllers for different scenarios are compared. First the analysis of linearized model of SMIB is presented with different models of synchronous machines. Then the results of non-linear controller are presented and compared with dynamic compensator.

6.1 Dynamic Compensator using Eigenvalue Assignment

A Static Var Compensator (SVC) with thyristor controlled reactor is considered for reactive power modulation [19], [49].

As described, a single machine-infinite bus system is subjected to eigenvalue analysis. The system is represented by its linearized equations of system including synchronous machine, excitation system and supplementary controls plus DC converter.

6.1.1 For classical Model

Chapter 3 discussed the methodology for designing linearized lead-lag compensator. The dynamic compensator is designed using pole placement technique. For designing, the system is first linearized at the operating point. For the feedback signal, the speed deviation ($\Delta \omega$) is used as a stabilizing signal. The linearized equations for power system model are presented in Appendix B. The transfer function for the lead - lag compensator is given by:

\[
Transfer \ function = K_z \left[ \frac{1 + sT_z}{1 + sT_p} \right]
\]  

(6.1)
Table 6.1: Eigenvalues for SMIB with 3rd Order Synchronous Machine and Constant Impedance Load \((A_p, A_q = 2)\)

<table>
<thead>
<tr>
<th></th>
<th>Close loop</th>
<th>Open loop</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\lambda_1)</td>
<td>(-163188.77 + 0.00i)</td>
<td>(-163188.77 + 0.00i)</td>
</tr>
<tr>
<td>(\lambda_2)</td>
<td>(-56.427 + 1611.93i)</td>
<td>(-56.428 + 1611.932i)</td>
</tr>
<tr>
<td>(\lambda_3)</td>
<td>(-56.427 - 1611.93i)</td>
<td>(-56.428 - 1611.93i)</td>
</tr>
<tr>
<td>(\lambda_4)</td>
<td>(-51.711 + 414.210i)</td>
<td>(-51.738 + 414.1i)</td>
</tr>
<tr>
<td>(\lambda_5)</td>
<td>(-51.711 - 414.210i)</td>
<td>(-51.738 - 414.1i)</td>
</tr>
<tr>
<td>(\lambda_6)</td>
<td>(-94.149 + 0.00i)</td>
<td>(-93.284 + 0.000i)</td>
</tr>
<tr>
<td>(\lambda_7)</td>
<td>(-33.895 + 0.00i)</td>
<td>(-33.661 + 0.000i)</td>
</tr>
<tr>
<td>(\lambda_8)</td>
<td>(-1.0370 + 7.32i)</td>
<td>(-0.2815 + 6.96i)</td>
</tr>
<tr>
<td>(\lambda_9)</td>
<td>(-1.0370 - 7.32i)</td>
<td>(-0.2815 - 6.96i)</td>
</tr>
<tr>
<td>(\lambda_{10})</td>
<td>(-1.4848 + 0.00i)</td>
<td>(-1.2505 + 0.00i)</td>
</tr>
<tr>
<td>(\lambda_{11})</td>
<td>(-0.9700 + 0.00i)</td>
<td>(-0.9551 + 0.00i)</td>
</tr>
<tr>
<td>(\lambda_{12})</td>
<td>(-9.9999 + 0.00i)</td>
<td>(-10.243 + 0.00i)</td>
</tr>
</tbody>
</table>
| \(\lambda_{13}\) | \(-7.4409 + 0.00i\) |}

where, \(T_p\) is the pole of system. A typical value selected for power system is selected to be 0.1 [14].

The Gain \((K_z)\) and zero \((T_z)\) are designed by solving the equations provided in Chapter 3. The eigenvalues of the overall system are calculated for the system. The system stability can be predicted by using the eigenvalues. The design method for dynamic compensator for SMIB system under study is independent of the order of synchronous generator. Different designs for dynamic compensator for classical model of synchronous generator are used for different values of \(\zeta\). By selecting the desired pair of eigenvalues for constant impedance load \((A_p, A_q = 2)\), the given system can be stabilized. Table 6.1, gives the original eigenvalues of the system and also the eigen values of system with dynamic compensator. It can be seen that the eigenvalues \(\lambda_{8,9}\) are complex pair and represent synchronous machine \(\omega\) and \(\delta\).

The eigenvalues of the closed-loop system are given. Using the matrix partitioning technique only critical eigenvalues related to the electro-mechanical oscillations of the machine are indicated in Table 6.2.

Table 6.2 lists two designs for stabilizing the AC/DC system using dynamic compensator for different damping ratio \(\zeta\).

The primary function of SVC is to control the reactive power and stabilize the commutating bus voltage. The auxiliary stabilizing signal is added to the main controller to improve the dynamic performance of the high voltage AC system.
In 6.2, it can be clearly seen that the dynamic compensator tends to make the system unstable with variation in load parameter ($A_p$).

### 6.1.2 For fifth order Model

The generator is modeled by a fifth order model and is equipped with static exciter and governor-turbine control system. The static-var-compensator is connected at the generator bus. The system data is provided in Appendix D.

The performance of the system depends on several factors including steady state and dynamic stability of a synchronous generator. In addition to reactances, time constants and inertia of the machine, the operating conditions, the impedance of the load, the voltage regulator and the governor have significant affects on the stability [17],[40],[50].

For eigenvalue analysis of the steady state and dynamic stability of synchronous machine system, the non-linear differential equations are linearized at the operating points calculated by the AC/DC load flow and then transforming the reference plane.

The eigenvalues of the overall system are calculated for the system. The system stability can be predicted by using the eigenvalues. The eigenvalues of the open-loop system are given in Table 6.3. Using the participation factor technique it was found that $\lambda_9$, $\lambda_{10}$ are related to the electro-mechanical oscillations of the machine.

---

**Table 6.2: Critical Eigenvalue corresponding to $\Delta\delta$ and $\Delta\omega$ for different designs of dynamic compensator $A_q = 2$**

<table>
<thead>
<tr>
<th></th>
<th>Design 1</th>
<th>Design 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_z$</td>
<td>33.0048</td>
<td>−15.8557</td>
</tr>
<tr>
<td>$T_z$</td>
<td>0.1239</td>
<td>0.000179</td>
</tr>
<tr>
<td>$A_p = 0$</td>
<td>$-2.538 \pm 6.733i$</td>
<td>$1.743 \pm 5.452i$</td>
</tr>
<tr>
<td>$A_p = 1$</td>
<td>$-1.804 \pm 7.087i$</td>
<td>$0.568 \pm 10.849i$</td>
</tr>
<tr>
<td>$A_p = 2$</td>
<td>$-1.037 \pm 7.32i$</td>
<td>$-1.186 \pm 9.500i$</td>
</tr>
<tr>
<td>$A_p = 3$</td>
<td>$-0.259 \pm 7.44i$</td>
<td>$-1.594 \pm 5.133i$</td>
</tr>
<tr>
<td>$A_p = 4$</td>
<td>$0.5167 \pm 7.462i$</td>
<td>$-1.038 \pm 3.915i$</td>
</tr>
</tbody>
</table>
Table 6.3: Open-loop eigenvalue with SVC for Constant Impedance Load $A_p, A_q = 2$ for 5th Order of machine

<table>
<thead>
<tr>
<th>$\lambda_i$</th>
<th>Open loop eigenvalues</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>$-163188.8 + i0$</td>
</tr>
<tr>
<td>$\lambda_{2,3}$</td>
<td>$-23.43 \pm i1609.4$</td>
</tr>
<tr>
<td>$\lambda_{4,5}$</td>
<td>$-7.435 \pm i431.14$</td>
</tr>
<tr>
<td>$\lambda_6$</td>
<td>$-57.407 + i0.0$</td>
</tr>
<tr>
<td>$\lambda_7$</td>
<td>$-33.848 + i0.0$</td>
</tr>
<tr>
<td>$\lambda_8$</td>
<td>$-10.1979 + i0.0$</td>
</tr>
<tr>
<td>$\lambda_{9,10}$</td>
<td>$-0.436 \pm i12.719$</td>
</tr>
<tr>
<td>$\lambda_{11}$</td>
<td>$-5.539 + i0.0$</td>
</tr>
<tr>
<td>$\lambda_{12}$</td>
<td>$-2.519 + i0.0$</td>
</tr>
<tr>
<td>$\lambda_{13}$</td>
<td>$-0.306 + i0.0$</td>
</tr>
<tr>
<td>$\lambda_{14}$</td>
<td>$-1.015 + i0.0$</td>
</tr>
</tbody>
</table>

In order to increase the damping, the Eigenvalue Assignment (EVA) technique discussed in Chapter 3 is used. It is easily seen in Table 6.4 the damping of the critical mode $\lambda_{9,10}$ is significantly improved by reactive power modulation by using $\Delta \omega$ as stabilizing signal.

Table 6.4: Close-loop eigenvalue for 5th Order of machine with SVC for Constant Impedance Load $A_p, A_q = 2$

<table>
<thead>
<tr>
<th>$\lambda_i$</th>
<th>Open loop eigenvalues</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>$-163188.8 + i0$</td>
</tr>
<tr>
<td>$\lambda_{2,3}$</td>
<td>$-23.43 \pm i1609.4$</td>
</tr>
<tr>
<td>$\lambda_{4,5}$</td>
<td>$-7.430 \pm i431.109$</td>
</tr>
<tr>
<td>$\lambda_6$</td>
<td>$-48.287 + i0.0$</td>
</tr>
<tr>
<td>$\lambda_7$</td>
<td>$-32.873 + i0.0$</td>
</tr>
<tr>
<td>$\lambda_8$</td>
<td>$-18.609 + i0.0$</td>
</tr>
<tr>
<td>$\lambda_{9,10}$</td>
<td>$-1.1305 \pm i10.35$</td>
</tr>
<tr>
<td>$\lambda_{11}$</td>
<td>$-5.875 + i0.0$</td>
</tr>
<tr>
<td>$\lambda_{12}$</td>
<td>$-2.505 + i0.0$</td>
</tr>
<tr>
<td>$\lambda_{13}$</td>
<td>$-0.291 + i0.0$</td>
</tr>
<tr>
<td>$\lambda_{14}$</td>
<td>$-1.013 + i0.0$</td>
</tr>
<tr>
<td>$\lambda_{15}$</td>
<td>$-10.00 + i0.0$</td>
</tr>
</tbody>
</table>
Figure 6.1 and 6.2 shows the variation of parameters of dynamic compensator, $K_z$ and $T_z$ at different values of load characteristics $A_p$ for constant value of $A_q = 2$ for same value of critical mode $\lambda_{9,10}$. It can be seen that the variation of Gain and Time Constant is small at higher values of $A_p$.

**Figure 6.1:** Variation of Gain $K_z$ for different values of $A_p$ for same $\lambda_{9,10}$
Figure 6.2: Variation of Time Constant $T_z$ for different values of $A_p$ for same $\lambda_9,10$

Figure 6.3: Variation of Gain $K_z$ and Time Constant $T_z$ of dynamic compensator at different values of imaginary part of critical mode $\lambda_{9,10}$
The dynamic compensator designs in Table 6.5 are tested for different load $A_p$ and for disturbance.

TABLE 6.5: Critical Eigenvalue $\lambda_{9,10}$ corresponding to $\Delta \delta$ and $\Delta \omega$ for different designs of dynamic compensator $A_q = 2$

<table>
<thead>
<tr>
<th></th>
<th>Design 1</th>
<th>Design 2</th>
<th>Design 3</th>
<th>Design 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_z$</td>
<td>$-15.1655$</td>
<td>$-1.5225$</td>
<td>$4.1717$</td>
<td>$9.401$</td>
</tr>
<tr>
<td>$T_z$</td>
<td>$-0.243$</td>
<td>$-1.031$</td>
<td>$0.2622$</td>
<td>$0.0794$</td>
</tr>
</tbody>
</table>

$A_p = 0$ $0.594 \pm 12.939i$ $0.038 \pm 12.613i$ $-0.079 \pm 12.474i$ $-0.160 \pm 12.34i$
$A_p = 1$ $-0.545 \pm 11.769i$ $-0.57 \pm 12.306i$ $-0.605 \pm 12.455i$ $-0.642 \pm 12.57i$
$A_p = 2$ $-1.130 \pm 10.350i$ $-1.13 \pm 11.920i$ $-1.130 \pm 12.400i$ $-1.130 \pm 12.80i$
$A_p = 3$ $-1.165 \pm 9.1349i$ $-1.61 \pm 11.457i$ $-1.653 \pm 12.305i$ $-1.623 \pm 13.01i$
$A_p = 4$ $-0.981 \pm 8.2615i$ $-2.00 \pm 10.936i$ $-2.172 \pm 12.166i$ $-2.123 \pm 13.23i$

Speed Response of synchronous generator $\Delta \omega/\omega_b$ for a 5% change in Input Torque of Prime mover with Dynamic Compensator Design 1 from Table 6.5 is shown in Fig. 6.5. It can be clearly seen that dynamic compensator design renders the system unstable for $A_p = 0$. 

**Figure 6.4:** Variation of Gain $K_z$ and Time Constant $T_z$ of dynamic compensator at different values of imaginary part of critical mode $\lambda_{9,10}$
Figure 6.5: Speed Response of synchronous generator $\Delta \omega/\omega_b$ for a 5% change in Input Torque with Dynamic Compensator Design 1

Speed Response of synchronous generator $\Delta \omega/\omega_b$ for a 5% change in Input Torque of Prime mover with Dynamic Compensator Design 3 from Table 6.5 is shown in Fig. 6.6. It can be clearly seen that dynamic compensator design damps the oscillations for load variation.
6.2 Synergetic Controller

6.2.1 Third Order Model

The performance of synergetic controller designed in Chapter 5 is analyzed with dynamic compensator. Reactive modulation is done through Static-Var Compensator connected on load bus.
Chapter 6. Analysis and Simulation

Figure 6.7: Speed Deviation of Generator for $A_p, A_q = 2$

The performance of the synergetic controller is analyzed by varying the load. From Fig. 6.7, it can be seen that both Dynamic Compensator and Synergetic Controller damp the system’s oscillation. For the dynamic compensator, Design # 2 is used from Table 6.2.

Figure 6.8 shows the performance of dynamic compensator and synergetic controller for variation in load parameter $A_p$ with $A_q$ fixed. Now, $A_p = 0$ and $A_q = 2$ and it can be seen in Fig. 6.8 that the dynamic compensator renders the system unstable which is clearly undesirable whereas Synergetic controller works well in this case and damps the oscillation in the system.
6.2.2 Effect of disturbance

To check the robustness of the proposed synergetic controller different disturbances are considered in the system.
6.2.2.1 Change in $P_m$

![Graph showing speed deviation](image)

**Figure 6.9:** Speed Deviation for $A_p, A_q = 2$ for 5% change in $P_m$ at $t=5.0s$

For a 5% change in Mechanical Power of the turbine governor ($P_m$), the performance of the controller is observed. The controller is robust enough to damp the uncertainty in system.

6.2.2.2 Fault on AC-line

In this case, a 3-phase fault is considered on AC-line. At $t = 2.0s$ the AC-line is disconnected from the system. The transmission line is connected back at $t = 2.01s$. 
The system is stable before the fault was simulated. Once there is a fault the synergetic controller performs well to damp the oscillations.

6.2.3 Effect of change in $K, \lambda$ of Controller

For this case, the Gain $K$, and $\lambda$ of the controller is varied and the effect on system damping is shown.
From fig. 6.11, the load coefficients are $A_p, A_q = 2$. The parameter $K$ effects the decay of the output signal.

6.3 Synergetic Controller from Classical Model in Full order model

The Synergetic controller designed for classical model of synchronous generator is used for SMIB with fifth order synchronous generator. The parameters of fifth order of synchronous generator are different as compared to the classical model of generator. To transform the machine parameters,

\[ X_q = X_{ls} + X_{mq} \]  \hspace{1cm} (6.2)

\[ X_d = X_{ls} + X_{md} \]  \hspace{1cm} (6.3)

\[ X'_d = X_{ls} + \frac{X_{md}X_{lf_2d}}{X_{lf_2d} + X_{md}} \]  \hspace{1cm} (6.4)

The open circuit time constant is given by;

\[ T_{do}' = \frac{1}{\omega_b r_{fd}}(X_{lf_2d} + X_{md}) \]  \hspace{1cm} (6.5)
Using the equations \((6.2 - 6.6)\) the parameters of fifth order synchronous machine were calculated.

\[
E'_{q} = V_{q}^{r} + r_{a}i_{q}^{r} + i_{d}X'_{d}
\]  

\[(6.6)\]

The controller is then used in the non-linear simulation developed for SMIB with fifth order of synchronous generator. It can be seen that the controller is able to damp the oscillation to a certain extent only. This may be because the classical model of the synchronous machine neglects the transients and also the amortisseur winding. Thus when used with full model of synchronous generator, it wasn’t able to damp the system.

\section{6.4 Synergetic Controller with Full Order of Synchronous Generator}

As seen from the previous section, the controller for classical model was not able to damp the oscillations of the machine. Using the procedure discussed in Chapter 5, the controller is designed for higher order of the machine. This section discusses the result of using Synergetic Controller for fifth order of synchronous generator. Two different scenarios are considered for the SMIB system. The scenarios are;
Chapter 6. Analysis and Simulation

- Steam Generator
- Hydro Generator

The performance of the controller is compared with these two types of generators first. Then the controller is compared with linearized eigenvalue controller designed using methodology in Chapter 3. Then the effect of disturbance is checked on the system.

6.4.1 Hydro Generator

Figure 6.13, shows the performance of the synergetic controller. The data for system is given in Table 6.6. The parameters are converted to per unit using [1].

![Figure 6.13: Speed Deviation for Synergetic Controller for Hydro Generator for $A_p, A_q = 2$](image)

It can be seen that the synergetic controller damps the oscillations of the system effectively. Now the load is varied in fig. 6.14 and the performance for the controller is observed.
Table 6.6: System Data for Hydro and Steam Generator [1]

<table>
<thead>
<tr>
<th></th>
<th>Hydro Generator</th>
<th>Steam Generator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rating</td>
<td>325 MVA</td>
<td>835 MVA</td>
</tr>
<tr>
<td>poles</td>
<td>32</td>
<td>2</td>
</tr>
<tr>
<td>$H$</td>
<td>7.5</td>
<td>5.6</td>
</tr>
<tr>
<td>$J$</td>
<td>35.1e6</td>
<td>0.0658e6</td>
</tr>
<tr>
<td>$r_s$</td>
<td>0.0019</td>
<td>0.003</td>
</tr>
<tr>
<td>$X_{ls}$</td>
<td>0.12</td>
<td>0.19</td>
</tr>
<tr>
<td>$X_q$</td>
<td>0.48</td>
<td>1.8</td>
</tr>
<tr>
<td>$X_d$</td>
<td>0.85</td>
<td>1.0</td>
</tr>
<tr>
<td>$r_{fd}$</td>
<td>0.00041</td>
<td>0.000929</td>
</tr>
<tr>
<td>$X_{fd}$</td>
<td>0.2049</td>
<td>0.1414</td>
</tr>
<tr>
<td>$r_{kq1}$</td>
<td></td>
<td>0.00178</td>
</tr>
<tr>
<td>$r_{kq2}$</td>
<td>0.0136</td>
<td>0.00841</td>
</tr>
<tr>
<td>$r_{kd}$</td>
<td>0.0141</td>
<td>0.01334</td>
</tr>
<tr>
<td>$X_{ld}$</td>
<td>0.16</td>
<td>0.08125</td>
</tr>
<tr>
<td>$X_{lkq1}$</td>
<td></td>
<td>0.8125</td>
</tr>
<tr>
<td>$X_{lkq2}$</td>
<td>0.1029</td>
<td>0.0939</td>
</tr>
</tbody>
</table>

Figure 6.14: Speed Deviation for Synergetic Controller for Hydro Generator for load variation
6.4.2 Steam Generator

Figure 6.15, shows the performance of the synergetic controller. The data for low generation is given in Table 6.6.

![Figure 6.15: Speed Deviation for steam generator for $A_p, A_q = 2$](image)

6.4.3 Comparison with Dynamic Compensator

The performance of the controller is compared with dynamic compensator. Figure 6.16 shows the effect of variation of load on both controllers. It can be seen that with the variation in load the linear controller is not robust enough to handle the uncertainty in the system.
6.4.4 Effect of Disturbance

Two type of disturbance are considered on the system;

6.4.4.1 Change in $P_m$

In this case once the system is stabilized, a 5% change in prime mover speed is produced and the performance of controller is observed.
6.4.4.2 Fault on AC Line

In this case once the system is stabilized, a fault is produced on AC at $t = 8.0$ s and cleared at $t = 8.01$ s and the performance of controller is observed.
6.4.5 Variation in Gain ’K’ of Synergetic Controller

For the full order of synchronous generator, the manifold selected is different as compared to the one proposed for classical model. The affect of varying the gain ’K’ of the controller is observed.
The response of the system is highly dependent on suitable value of $K$. From Chapter 5, $K$ is responsible for exponential decay of the system. This phenomenon is obvious in fig. 6.19.

### 6.4.6 Low Generation

The synergetic controller designed is now tested for different operating points of the system. The generation capability $P_G$ of the synchronous machine is reduced from 1.0$p.u$ to 0.3$p.u$. The controller is tested by varying $A_p$. As seen from Figure 6.20, the controller was able to damp the oscillations but the response is not as good as the case when $P_G$ was 1.0$p.u$.
Figure 6.20: Speed Deviation of Generator for low generation

The dynamic compensator however performs poorly in this case and renders the system unstable for different values of $A_p$. 
Figure 6.21: Speed Deviation for low generation with Dynamic Compensator
Chapter 7

Conclusion and Future work

The work provided in this thesis covers the basics for linear and non-linear modeling of power system. Various components for the power system have been modeled and implemented using MATLAB/Simulink. Also, the affect of load variation has been considered on the stability of power system. A non-linear controller design has been proposed for different orders of the synchronous generator. The controllers are designed for both linear and non-linear system design. This thesis proposes a synergetic control design to improve the transient stability of power system subject to contingencies. The generator in the system is considered as a system and a nonlinear synergetic controller is designed for it. The proposed synergetic controller enhances the dynamic performance greatly.

7.1 Future Work

- Addition of dynamic load i.e. induction machine and simulate the effect of dynamic load on the system along with the controller.

- The work in this thesis presents a methodology for doing reactive modulation using Static Var Compensator. Previous work discusses the modulation techniques using Static exciter to perform reactive modulation. It would be very interesting to design a controller for coordinated modulation.

- The research work presented in this thesis covers the Robust analysis technique for solving non-linear control problems. One future recommendation would be to compare Adaptive Control, Gain scheduling and Linear analysis techniques for the SMIB system.
Publication

Appendix A

Non-Linear Modeling

A.1 Static Exciter

\[
\dot{E}_{fd} = -\frac{1}{T_a} E_{fd} + \frac{K_c}{T_a} (V_{ref} - V_t)
\]  \hspace{1cm} (A.1)

A.2 Turbine Governor
\begin{equation}
\dot{P}_m = \frac{1}{T_l} P_v - \frac{1}{T_l} P_m \tag{A.2}
\end{equation}

\begin{equation}
\dot{P}_v = \frac{1}{T_g} (P_c - P_v) - \frac{1}{RT_g} (\omega(t) - 1) \tag{A.3}
\end{equation}

### A.3 DC Converter

![DC Converter Diagram]

**Figure A.3:** DC Converter (Rectifier)

#### A.3.1 Rectifier

\begin{equation}
\dot{e}_c = K (I_{ref} - I_{dc}) \tag{A.4}
\end{equation}

\begin{equation}
I_{dc} = \frac{6\sqrt{3}\omega_b}{\pi X_{sr}} e_c - \frac{6X_l}{\pi X_{sr}} I_{dc} - \frac{\omega_b}{X_{sr}} V_t \tag{A.5}
\end{equation}

\begin{equation}
\dot{V}_t = \omega_b X_c I_e \quad I_c = I_{dc} - I_t \tag{A.6}
\end{equation}

\begin{equation}
I^e_q = I_m \cos(\delta + \phi) \quad I^e_d = I_m \sin(\delta + \phi) \quad I_m = \frac{4\sqrt{3}}{\pi} I_{dc} T_a \tag{A.7}
\end{equation}

\begin{equation}
\cos \phi = \frac{\pi V_{dc}}{6\sqrt{3}\pi T E_m} \tag{A.8}
\end{equation}

\begin{equation}
\tan \delta = \frac{V^e_d}{V^e_q} \tag{A.9}
\end{equation}

\begin{equation}
E_m = \sqrt{V^e_q^2 + V^e_d^2} \tag{A.10}
\end{equation}

#### A.3.2 Inverter

\begin{equation}
I_{dc} = \frac{6X_l}{\pi X_{sr}} I_{dc} + \frac{\omega_b}{X_{sr}} V_t - \frac{6\sqrt{3}\omega_b}{\pi X_{sr}} T E_m \cos \gamma_o \tag{A.11}
\end{equation}
Appendix A. Non-linear Modeling

A.4 Synchronous Generator

A.4.1 Classical Model

\[ \dot{E}_q = \frac{1}{\tau_{do}} (E_{fd} - E - f) \quad \therefore E_{fd} : from\ Exciter \] (A.17)

\[ V_q = E'_q - X_d' I_d - r I_q \] (A.18)

\[ V_d = X_q I_q - r I_d \] (A.19)

\[ E_q = V_q + X_d I_d + r I_q \] (A.20)

\[ E_f = E_q + (X_d - X_q) I_d \] (A.21)

\[ P_e = E'_q I_q \] (A.22)

\[ \dot{\delta}(t) = \omega_0 (\omega(t) - 1) \] (A.23)

\[ \omega(t) = -\frac{D}{2H} (\omega(t) - 1) - \frac{1}{2H} (P_e(t) - P_m(t)) \] (A.24)
\[ \dot{E}_q'(t) = \frac{1}{\tau_{D_o}}(E_{fd} - (X_d - X_d')I_d(t) - E_q'(t)) \]  

(A.25)

A.4.2 Synchronous Generator

\[ \dot{\psi}_{kq}^r = -\frac{\omega_b}{X_{lkq}} \psi_{kq}^r + \frac{\omega_b}{X_{lkq}} \psi_{mq}^r \]  

(A.26)

\[ \dot{\psi}_{kd}^r = -\frac{\omega_b}{X_{lkq}} \psi_{kd}^r + \frac{\omega_b}{X_{lkd}} \psi_{md}^r \]  

(A.27)

\[ \dot{\psi}_{fd}^r = -\frac{\omega_b}{X_{lfq}} \psi_{fd}^r + \frac{\omega_b}{X_{lfq}} \psi_{qd}^r + \frac{\omega_b}{X_{lfq}} E_{fd} \]  

(A.28)

\[ \dot{\delta}(t) = \omega_r - \omega_e \]  

(A.29)

\[ \frac{\dot{\omega}(t)}{\omega_b} = -\frac{1}{2H}(T_e(t) - T_l(t)) \]  

(A.30)

\[ \dot{\psi}_{qs}^r = \omega_b \left( V_{qs}^r - \frac{\omega_r}{\omega_b} \psi_{qs}^r + \frac{r_s}{X_{ls}} (\psi_{mq}^r - \psi_{qs}^r) \right) \]  

(A.31)

\[ \dot{\psi}_{qs}^d = \omega_b \left( V_{qs}^d + \frac{\omega_r}{\omega_b} \psi_{qs}^d + \frac{r_s}{X_{ls}} (\psi_{md}^r - \psi_{qs}^d) \right) \]  

(A.32)
\[ i_{qs}^r = -\frac{1}{X_{ls}}(\psi_{qs}^r - \psi_{mq}^r) \]  
\[ i_{ds}^r = -\frac{1}{X_{ls}}(\psi_{ds}^r - \psi_{md}^r) \]  
\[ \psi_{mq}^r = X_{aq}(\frac{\psi_{qs}^r}{X_{ls}} + \frac{\psi_{kq}^r}{X_{lkq}}) \]  
\[ \psi_{md}^r = X_{ad}(\frac{\psi_{ds}^r}{X_{ls}} + \frac{\psi_{fd}^r}{X_{lfq}} + \frac{\psi_{kd}^r}{X_{lkd}}) \]  
\[ T_e = \psi_{ds}^r i_{qs}^r - \psi_{qs}^r i_{ds}^r \]  
\[ P_L = P_{Lo}(\frac{V}{V_o})^{A_p} \]  
\[ Q_L = Q_{Lo}(\frac{V}{V_o})^{A_q} \]  
\[ V = V_q^r - jV_d^r \]  
\[ V_o = \sqrt{V_q^r + V_d^r} \]  
\[ I_L = I_q^r - jI_d^r \]  
\[ S_L = V I_L^* = (V_q^r - jV_d^r)(I_q^r + jI_d^r) \]  
\[ S_L = V_q^r I_q^r - jV_d^r I_q^r + jV_d^r I_q^r + V_d^r I_d^r \]  
\[ S_L = P_L + jQ_L = (V_q^r I_q^r + V_d^r I_d^r) + j(V_q^r I_d^r - V_d^r I_q^r) \]  
\[ P_L = V_q^r I_q^r + V_d^r I_d^r \]  
\[ Q_L = V_q^r I_d^r - V_d^r I_q^r \]  

**A.5 Static Load**

Substituting equation A.38 in eq. A.40,

\[ P_{Lo}(\frac{V}{V_o})^{A_p} = V_q^r I_q^r + V_d^r I_d^r \]  

Substituting equation A.39 in eq. A.40,

\[ Q_{Lo}(\frac{V}{V_o})^{A_q} = V_q^r I_d^r - V_d^r I_q^r \]  

Multiplying equation A.41 by \(V_q^r\) and eq. A.42 by \(V_d^r\) and subtracting,

\[ V_q^r P_{Lo}(\frac{V}{V_o})^{A_p} - V_d^r Q_{Lo}(\frac{V}{V_o})^{A_q} = V_q^r I_q^r + V_d^r I_d^r \]
Appendix A. Non-linear Modeling

Re-arranging, $I^r_q$ is given by,

$$I^r_q = \frac{1}{V^2_o} [V^r_q P_L o (\frac{V}{V_o})^{A_p} - V^r_d Q_L o (\frac{V}{V_o})^{A_q}]$$  \hspace{1cm} (A.43)

Multiplying equation A.41 by $V^r_d$ and A.42 by $V^r_q$ and adding,

$$V^r_d P_L o (\frac{V}{V_o})^{A_p} + V^r_q Q_L o (\frac{V}{V_o})^{A_q} = V^r_q I^r_d + V^r_d I^r_q$$

Re-arranging, $I^r_d$ is given by,

$$I^r_d = \frac{1}{V^2_o} [V^r_q Q_L o (\frac{V}{V_o})^{A_q} + V^r_d P_L o (\frac{V}{V_o})^{A_p}]$$  \hspace{1cm} (A.44)

A.6 Static Var Compensator

![Static Var Compensator Model](image)

**Figure A.6:** Static Var Compensator Model

$$B_{svc}(\alpha) = \frac{2\alpha - \sin(2\alpha)}{\pi} - 1, \quad \frac{\pi}{2} \leq \alpha \leq \pi$$  \hspace{1cm} (A.45)

$$I_s = \frac{V_t - E_s}{jX_s}, \quad E_s = B_{svc}(\alpha)V_t$$

$$I_s = \frac{V_t - B_{svc}(\alpha)V_t}{jX_s} = \frac{(1 - B_{svc}(\alpha)) \times V_t}{jX_s}$$  \hspace{1cm} (A.46)

Substituting $V_t = V^r_q - jV^r_d$ and $B_{svc}(\alpha)$

$$I_s = \frac{(1 - (\frac{2\alpha - \sin(2\alpha)}{\pi} - 1)) \times (V^r_q - jV^r_d)}{jX_s}$$

$$I_s = \frac{1}{jX_s} (1 - (\frac{2\alpha - \sin(2\alpha)}{\pi} - 1)) \times (V^r_q - jV^r_d)$$
Appendix A. Non-linear Modeling

\[ I_s = \frac{1}{j\pi X_s} (2\pi - 2\alpha + \sin(2\alpha)) \times (V_q^r - jV_d^r) \]
\[ I_s = -\frac{j}{\pi X_s} (2\pi - 2\alpha + \sin(2\alpha))V_q^r - \frac{1}{\pi X_s} (2\pi - 2\alpha + \sin(2\alpha))V_d^r \]

Substituting \( I_s = I_q^r - jI_d^r \) and comparing,

\[ I_q^r - jI_d^r = -\frac{j}{\pi X_s} (2\pi - 2\alpha + \sin(2\alpha))V_q^r - \frac{1}{\pi X_s} (2\pi - 2\alpha + \sin(2\alpha))V_d^r \] (A.47)

Comparing both sides \( I_s q^r \) can be written as

\[ I_s q^r = -\frac{1}{\pi X_s} (2\pi - 2\alpha + \sin(2\alpha))V_d^r \] (A.48)

Comparing both sides \( I_s d^r \) can be written as

\[ I_s d^r = \frac{1}{\pi X_s} (2\pi - 2\alpha + \sin(2\alpha))V_q^r \] (A.49)

Also, \( \alpha \) is given by:

\[ \alpha = \frac{K_a}{1 + sT_{\alpha}} (V - V_{ref\_svc}) \] (A.50)
\[ \alpha + \dot{\alpha} T_{\alpha} = K_a (V - V_{ref\_svc}) \]
\[ \dot{\alpha}(t) = -\frac{1}{T_{\alpha}} \alpha(t) + \frac{K_a}{T_{\alpha}} (V - V_{ref\_svc}) \] (A.51)
Appendix B

Linear Modeling of Components

B.1 Synchronous Machine Model (Fifth Order)

B.1.1 Stator equations in Synchronous Reference Frame

\[ \Delta \psi_q^e = -r_s \Delta i_q^d - \Delta V_d^e \]
\[ \Delta \psi_d^e = r_s \Delta i_q^d - \Delta V_q^e \]

B.1.2 Rotor equations in Rotor Reference Frame

\[ \dot{\Delta} \psi_{kq}^r = -\omega_b \frac{r_{kq}}{X_{lkq}} \Delta \psi_{kq}^r + \frac{\omega_b r_{kq}}{X_{lkq}} \Delta \psi_{mq}^r \]
\[ \dot{\Delta} \psi_{kd}^r = -\omega_b \frac{r_{kd}}{X_{lkd}} \Delta \psi_{kd}^r + \frac{\omega_b r_{kd}}{X_{lkd}} \Delta \psi_{md}^r \]
\[ \dot{\Delta} \psi_{fd}^r = -\frac{\omega_b r_{fd}}{X_{lfkd}} \Delta \psi_{fd}^e + \frac{\omega_b r_{fd}}{X_{lfkd}} \Delta \psi_{md}^e + \frac{\omega_b r_{fd}}{X_{md}} \Delta E_{fd} \]

B.1.3 Prime-mover equations

\[ \dot{\Delta} \frac{\omega}{\omega_b} = -\frac{D}{2H} \frac{\omega}{\omega_b} + \frac{1}{2H} (-\Delta T_e + \Delta T_m) \]
\[ \dot{\Delta} \delta = \omega_b \Delta \delta \]
B.1.4 Stator flux in Rotor Reference frame

\[
\Delta \psi^r_q = \cos \delta \Delta \psi^e_q - \sin \delta \Delta \psi^e_d - \psi^r_d \Delta \delta \\
\Delta \psi^r_d = \sin \delta \Delta \psi^e_q + \cos \delta \Delta \psi^e_d - \psi^r_q \Delta \delta
\]

B.1.5 Mutual Flux

\[
\Delta \psi^r_{mq} = \frac{X_{aq}}{X_{la}} \Delta \psi^r_q + \frac{X_{ad}}{X_{lkq}} \Delta \psi^r_{kq} \\
\Delta \psi^r_{md} = \frac{X_{ad}}{X_{la}} \Delta \psi^r_d + \frac{X_{ad}}{X_{lkq}} \Delta \psi^r_{kd} + \frac{X_{ad}}{X_{lfq}} \Delta \psi^r_{fd}
\]

B.1.6 Stator currents in rotor-reference frame

\[
\Delta i^r_q = \frac{1}{X_{la}} (\Delta \psi^r_{mq} - \Delta \psi^r_q) \\
\Delta i^r_d = \frac{1}{X_{la}} (\Delta \psi^r_{md} - \Delta \psi^r_d)
\]

B.1.7 Stator currents in synchronous reference frame

\[
\Delta i^r_q = \cos \delta + \Delta i^r_q \sin \delta \Delta i^r_d + \Delta i^r_q \Delta \delta \\
\Delta i^r_d = -\sin \delta + \Delta i^r_d \cos \delta \Delta i^r_d + \Delta i^r_q \Delta \delta
\]

B.1.8 Torque Equation

\[
T_e = \psi^r_{dq} \Delta i^r_q - \psi^r_{dq} \Delta i^r_d - \Delta \psi^r_q \Delta i^r_q + \Delta \psi^r_q \Delta i^r_d
\]

B.2 Synchronous Generator (Third Order Model)

B.2.1 Field flux Linkage

\[
\Delta \dot{E}_q = \frac{1}{T'_{D_o}} (-\Delta V^r_q - r_a \Delta i^r_q - X_d \Delta i^r_d + \Delta E_{fd})
\]
B.2.2 Prime Mover Equation

\[
\frac{\Delta \dot{\omega}}{\omega_b} = -\frac{1}{2H} \Delta T_e - \frac{1}{2H} (D \Delta \frac{\omega}{\omega_b} - \Delta T_m) \\
\Delta T_e = E_{q_o} \Delta i^r_q + i^r_{q_o} \Delta E_q
\]

B.2.3 Stator currents in Rotor Reference frame

\[
\begin{align*}
\Delta i^r_q &= \frac{r_a}{X} \Delta V^r_q - \frac{X}{X} \Delta V^r_d - \frac{r_a}{X} \Delta E^r_q \\
\Delta i^r_d &= \frac{X}{X} \Delta V^r_q + \frac{r_a}{X} \Delta V^r_d - \frac{X}{X} \Delta E^r_q \\
X &= -X_q X_d - r_a^2
\end{align*}
\]

B.2.4 Voltage back of q-axis synchronous reactance

\[
\Delta E_q = \Delta V^r_q + r_a \Delta i^r_q + X_q \Delta i^r_d
\]

B.2.5 Stator voltage in RRF

\[
\begin{align*}
\Delta V^r_q &= \cos \delta_o \Delta V^e_q - \sin \delta_o \Delta V^e_d - V^e_{d_o} \Delta \delta \\
\Delta V^r_d &= \sin \delta_o \Delta V^e_q + \cos \delta_o \Delta V^e_d + V^e_{d_o} \Delta \delta
\end{align*}
\]

B.2.6 Stator current in Rotor Reference Frame

\[
\begin{align*}
\Delta i^e_q &= \cos \delta_o \Delta i^e_q + \sin \delta_o \Delta i^e_d + i^e_{d_o} \Delta \delta \\
\Delta i^e_d &= -\sin \delta_o \Delta i^e_q + \cos \delta_o \Delta i^e_d - i^e_{d_o} \Delta \delta
\end{align*}
\]
Appendix B. Linear Modeling of Components

Synchronous Machine (Fifth Order) - Matrix Form

\[ \omega_b = 2\pi f \]
\[ x_{mq} = x_q - x_{ls} \]
\[ x_{md} = x_d - x_{ls} \]
\[ x_{lfd} = x_{ffd} - x_{md} \]
\[ x_{lkq} = x_{kkq} - x_{mq} \]
\[ x_{lkd} = x_{kkd} - x_{md} \]
\[ x_{mq} = \frac{1}{x_{mq} + \frac{1}{x_{ls}} + \frac{1}{x_{lkq}}} \]
\[ x_{md} = \frac{1}{x_{md} + \frac{1}{x_{ls}} + \frac{1}{x_{lfd}}} \]
\[ i_{fd} = \frac{E_x}{x_{md}} \]
\[ \psi_{mq} = -i_q r x_{mq} \]
\[ \psi_{md} = (i_d r + i_{fd}) x_{md} \]
\[ \psi_{d} = -x_{ls} i_q r + \psi_{md} \]
\[ \psi_{q} = -x_{ls} i_d r + \psi_{mq} \]
\[ \psi_{kq} = \psi_{mrq} \]
\[ \psi_{kd} = \psi_{md} \]

\[ A = \begin{bmatrix} -\omega_b r_{kq} & 0 & 0 & 0 & 0 \\ x_{lkq} & 0 & -\omega_b r_{kd} & 0 & 0 \\ 0 & 0 & -\omega_b r_{jd} & 0 & 0 \\ 0 & 0 & 0 & -\frac{D}{2\pi} & 0 \\ 0 & 0 & 0 & 0 & \omega_b \end{bmatrix} \]  

\[ B = \begin{bmatrix} \omega_b r_{kq} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega_b r_{kd} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & w_b \cdot r_{fd} & \omega_b r_{fd} & 0 & 0 & 0 & 0 & -\frac{1}{2\pi} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2\pi} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]  

\[ C = \]
\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\] (B.4)
\[
\begin{bmatrix}
0 & -r & s & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-r & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
\( (B.5) \)
B.3 Static Exciter

\[
\begin{bmatrix}
E_{fd}
\end{bmatrix} = \begin{bmatrix}
-1
\end{bmatrix} \begin{bmatrix}
E_{fd}
\end{bmatrix} + \begin{bmatrix}
-K_e V_{qo} \\
-K_e V_{do} \\
K_e T_e \\
K_e T_e
\end{bmatrix} \begin{bmatrix}
\Delta V_{qo} \\
\Delta V_{do} \\
\Delta V_{ref} \\
\Delta V_{ps}
\end{bmatrix}
\]

\[
E_{fd} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix}
\Delta V_{qo} \\
\Delta V_{do} \\
\Delta V_{ref} \\
\Delta V_{ps}
\end{bmatrix}
\]

B.4 Governor - Turbine Model

\[
\begin{bmatrix}
\Delta T_m \\
\Delta T_v
\end{bmatrix} = \begin{bmatrix}
-\frac{1}{T_t} & -\frac{1}{T_t} \\
0 & -\frac{1}{T_G}
\end{bmatrix} \begin{bmatrix}
\Delta T_m \\
\Delta T_v
\end{bmatrix} + \begin{bmatrix}
0 & 0 \\
\frac{1}{RT_G} & \frac{1}{T_G}
\end{bmatrix} \begin{bmatrix}
\Delta \omega \\
\omega_b
\end{bmatrix}
\]

\[
\begin{bmatrix}
\Delta T_m \\
\Delta T_v
\end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix}
\Delta T_m \\
\Delta T_v
\end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix}
\Delta \omega \\
\omega_b
\end{bmatrix}
\]

B.5 Static Load

\[
V_o = \sqrt{V_{qo}^2 + V_{do}^2}
\]

\[
I_{qL_o} = V_{qo} P_L - V_{do} Q_L
\]

\[
I_{dL_o} = V_{do} P_L + V_{qo} Q_L
\]

\[
h_1 = \frac{A_p P_L V_{qo}}{V_o} - I_{qL_o}
\]

\[
h_2 = \frac{A_p P_L V_{do}}{V_o} - I_{dL_o}
\]

\[
h_3 = \frac{A_q Q_L V_{qo}}{V_o} - I_{qL_o}
\]

\[
h_4 = \frac{A_q Q_L V_{do}}{V_o} + I_{qL_o}
\]

\[
d_{11} = \frac{V_{qo} \times h_1 - V_{do} \times h_3}{V_o^2}
\]

\[
d_{12} = \frac{V_{qo} \times h_2 - V_{do} \times h_4}{V_o^2}
\]

\[
d_{21} = \frac{V_{do} \times h_1 + V_{qo} \times h_3}{V_o^2}
\]
### Appendix B. Linear Modeling of Components

\[ d_{22} = \frac{V_{d0}^2 \times h_2 + V_{q0}^e \times h_4}{V_0^2} \]

\[
\begin{bmatrix}
\Delta I_{qL} \\
\Delta I_{dL}
\end{bmatrix} =
\begin{bmatrix}
d_{11} & d_{12} \\
d_{21} & d_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta V_{qL} \\
\Delta V_{dL}
\end{bmatrix}
\]

#### B.6 Rectifier Model

\( PBR = 0.1103 \)

\[ E_m = \sqrt{V_{q0}^2 + V_{do}^2} \]

\[ \delta_o = \cos^{-1}\left(\frac{V_{dm}}{E_m}\right) \]

\[ \phi_o = \cos^{-1}\left(\frac{\pi V_{dc}}{\sqrt{108 E_m T}}\right) \]

\[ R_4 = \sqrt{108T^2 E_m^2 - \pi^2 V_{dc}^2} \]

\[ R_5 = \frac{\pi}{E_m R_3} \]

\[ R_6 = 4\sqrt{3} \pi \]

\[ R_7 = \cos(\phi_o + \delta_o) \]

\[ R_8 = \sin(\phi_o + \delta_o) \]

\[ R_9 = R_6 R_7 P_{BR} \]

\[ R_{10} = R_6 R_8 P_{BR} \]

\[ R_{11} = -R_9 I_{dc} \]

\[ R_{12} = R_{10} I_{dc} \]

\[ R_{13} = (R_5 V_{dc} \frac{V_{q0}}{E_m}) - \frac{V_{do}}{E_m^2} \]

\[ R_{14} = (R_5 V_{dc} \frac{V_{q0}}{E_m}) + \frac{V_{do}}{E_m^2} \]

\[
\begin{bmatrix}
\Delta e_c \\
\Delta I_{dc} \\
\Delta V_t
\end{bmatrix} =
\begin{bmatrix}
0 & -\frac{G_k}{0.05} & 0 \\
\frac{\sqrt{108}}{\pi} \frac{\omega_b}{X_{sr}} & -\frac{6 X_t}{\pi} \frac{\omega_b}{X_{sr}} - \frac{\omega_b}{X_{sr}} & \Delta I_{dc} \\
0 & 0 & \Delta V_t
\end{bmatrix}
\begin{bmatrix}
\Delta e_c \\
\Delta I_{dc} \\
\Delta V_t
\end{bmatrix} +
\begin{bmatrix}
0 & 0 & G_k \frac{0.05}{0.05} & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & \omega_b X_c & \Delta I_{ref}
\end{bmatrix}
\begin{bmatrix}
\Delta V_{q1} \\
\Delta V_{d1} \\
\Delta I_{ref}
\end{bmatrix}
\]
Appendix B. Linear Modeling of Components

\[
\begin{bmatrix}
\Delta I_{q1} \\
\Delta I_{d1} \\
\Delta I_{dc} \\
\Delta V_t
\end{bmatrix} =
\begin{bmatrix}
-1.5R_6E_mR_{11}R_5 & 1.5R_5R_{11}E_mX_l + R_9 & 0 \\
-1.5R_6E_mR_{12}R_5 & 1.5R_5R_{12}E_mX_l + R_{10} & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\Delta e_c \\
\Delta I_{dc} \\
\Delta V_t
\end{bmatrix}
+ \begin{bmatrix}
R_{11}R_{13} & R_{11}R_{14} & 0 & 0 \\
R_{12}R_{13} & R_{12}R_{14} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta V_{q1} \\
\Delta V_{d1} \\
\Delta I_{ref} \\
\Delta I_{cl}
\end{bmatrix}
\]

B.7 Inverter Model

\[E_m = \sqrt{V_{q0}^2 + V_{do}^2}\]
\[\delta_o = \cos^{-1}\left(\frac{V_{q0}}{E_m}\right)\]
\[\phi_o = \cos^{-1}\left(\frac{\pi V_{dc}}{\sqrt{108}E_mT}\right)\]
\[I_4 = \sqrt{108T^2E_m^2 - \pi^2V_{dc}^2}\]
\[I_5 = \frac{T}{E_m}\]
\[I_6 = 4\sqrt{3}\frac{T}{\pi}\]
\[I_7 = \cos(\phi_o + \delta_o)\]
\[I_8 = \sin(\phi_o + \delta_o)\]
\[I_9 = I_6I_7P_{BR}\]
\[I_{10} = I_6I_8P_{BR}\]
\[I_{11} = -I_{10}I_{dc}\]
\[I_{12} = I_9I_{dc}\]
\[I_{13} = -\frac{\sqrt{108}}{\pi E_m} V_{q0}\cos(\gamma_o)\]
\[I_{14} = -\frac{\sqrt{108}}{\pi E_m} V_{do}\cos(\gamma_o)\]
\[I_{15} = I_5\left(V_{q0}\frac{V_{dc}}{E_m}\right) - E_mI_{13}\]
\[I_{16} = I_5\left(V_{do}\frac{V_{dc}}{E_m}\right) - E_mI_{14}\]
\[I_{17} = I_{11}I_{15}\]
\[I_{18} = I_{11}I_{16}\]
\[I_{19} = I_{12}I_{15}\]
\[I_{20} = I_{12}I_{16}\]
\[I_{21} = I_9 - \frac{I_8I_{11}+E_{m_1}X_1}{2}\]
\[I_{22} = I_{10} - \frac{I_8I_{12}+E_{m_1}X_1}{2}\]

\[
\begin{bmatrix}
\Delta I_{dc} \\
\Delta V_t
\end{bmatrix}
= \omega_b \times \begin{bmatrix}
\frac{6X_l}{X_{sr}\pi} & 0 \\
0 & \frac{X_{sr}}{X_{sr}^2 - X_{dc}}
\end{bmatrix}
\begin{bmatrix}
\Delta I_{dc} \\
\Delta V_t
\end{bmatrix}
+ \omega_b \times \begin{bmatrix}
I_{13} & I_{14} & 0 \\
I_{13} & I_{14} & 0 \\
0 & 0 & X_c
\end{bmatrix}
\begin{bmatrix}
\Delta V_{q1} \\
\Delta V_{d1} \\
\Delta I_{cl}
\end{bmatrix}
\]

Appendix B. Linear Modeling of Components

\[
\begin{bmatrix}
\Delta I_q^e \\
\Delta I_d^e \\
\Delta I_{dc} \\
\Delta V_t
\end{bmatrix} =
\begin{bmatrix}
I_{21} & 0 \\
I_{22} & 0 \\
1 & 0 \\
0 & 1
\end{bmatrix}\begin{bmatrix}
\Delta I_{dc} \\
\Delta V_t
\end{bmatrix} +
\begin{bmatrix}
I_{17} & I_{18} & 0 \\
I_{19} & I_{20} & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}\begin{bmatrix}
\Delta V_{q1}^e \\
\Delta V_{d1}^e \\
\Delta I_{c2}
\end{bmatrix}
\]

B.8 Static Var Compensator Model

\[V_{q_0}^e = 1.308\]
\[V_{d_0}^e = -0.2153\]
\[\alpha_s = 140^o\]
\[T_\alpha = 0.04\]
\[K_\alpha = 20\]

\[X_s = 1\]
\[V_o = \sqrt{V_{q_0}^e V_{d_0}^e} = V_{q_0}^e \frac{2V_{d_0}^e (1 - \cos(2\alpha_s))}{X_s \pi} \]
\[c_{11} = \frac{V_{q_0}^e c_{11}}{V_{d_0}^e} \]
\[c_{21} = -\frac{V_{q_0}^e c_{11}}{V_{d_0}^e} \]
\[d_{12} = \frac{1}{X_s \pi} (2\pi - 2\alpha_s + \sin(2\alpha_s)) \]
\[d_{21} = -d_{12} \]

\[
\begin{bmatrix}
\Delta \alpha_s
\end{bmatrix} = -\frac{1}{T_\alpha} \begin{bmatrix}
\Delta \alpha_s
\end{bmatrix} + K_\alpha \begin{bmatrix}
-\frac{V_{q_0}^e}{V_o} & -\frac{V_{d_0}^e}{V_o}
\end{bmatrix} \begin{bmatrix}
\Delta V_q^e \\
\Delta V_d^e \\
\Delta V_{ref}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\Delta I_{qs} \\
\Delta I_{ds} \\
\Delta \alpha_s
\end{bmatrix} =
\begin{bmatrix}
c_{11} & d_{12} & 0 \\
c_{21} & 0 & 0 \\
1 & 0 & 0
\end{bmatrix}\begin{bmatrix}
\Delta \alpha_s \\
\Delta V_q^e \\
\Delta V_d^e \\
\Delta V_{ref}
\end{bmatrix}
\]
Appendix C

Component Connection Modeling

C.1 Component Connection for the DC System

Table C.1: Component Connection Table for the DC system

<table>
<thead>
<tr>
<th>No.</th>
<th>$\Delta a$</th>
<th>$\Delta b$</th>
<th>$\Delta x$</th>
<th>$\Delta y$</th>
<th>$\Delta u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\Delta V_{eq}^c$</td>
<td>$\Delta I_{eq}^c$</td>
<td>$\Delta I_{dc}$</td>
<td>$\Delta I_{eq}^c$</td>
<td>$\Delta I_{ref}$</td>
</tr>
<tr>
<td>2</td>
<td>$\Delta V_{dq}^e$</td>
<td>$\Delta I_{dq}^e$</td>
<td>$\Delta I_{dc}$</td>
<td>$\Delta I_{dq}^e$</td>
<td>$\Delta V_{eq}^c$</td>
</tr>
<tr>
<td>3</td>
<td>$\Delta I_{ref}$</td>
<td>$\Delta I_{dc}$</td>
<td>$\Delta V_{t1}$</td>
<td>$\Delta V_{d2}^e$</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$\Delta I_{c1}$</td>
<td>$\Delta V_{t1}$</td>
<td>$\Delta I_{dc}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$\Delta V_{eq}^c$</td>
<td>$\Delta I_{eq}^c$</td>
<td>$\Delta V_{t2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$\Delta V_{dq}^e$</td>
<td>$\Delta I_{dq}^e$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$\Delta I_{c2}$</td>
<td>$\Delta I_{dc}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>$\Delta V_{t1}$</td>
<td>$\Delta V_{t2}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>$\Delta V_{t2}$</td>
<td>$\Delta I_{t1}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>$\Delta I_{t2}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For rectifier: $I_{dc1} = I_{c1} + I_{t1}$

For inverter: $I_{dc2} + I_{c2} = -I_{t2}$
Table C.2: Connection Values for the DC system (Rectifier + DC Line + Inverter)

<table>
<thead>
<tr>
<th>Connection Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{11}(4, 3)$</td>
<td>1</td>
</tr>
<tr>
<td>$L_{11}(4, 9)$</td>
<td>-1</td>
</tr>
<tr>
<td>$L_{11}(7, 7)$</td>
<td>-1</td>
</tr>
<tr>
<td>$L_{11}(7, 10)$</td>
<td>-1</td>
</tr>
<tr>
<td>$L_{11}(8, 4)$</td>
<td>1</td>
</tr>
<tr>
<td>$L_{11}(9, 8)$</td>
<td>1</td>
</tr>
<tr>
<td>$L_{12}(3, 1)$</td>
<td>1</td>
</tr>
<tr>
<td>$L_{12}(5, 2)$</td>
<td>1</td>
</tr>
<tr>
<td>$L_{12}(6, 3)$</td>
<td>1</td>
</tr>
<tr>
<td>$L_{21}(1, 5)$</td>
<td>1</td>
</tr>
<tr>
<td>$L_{21}(2, 6)$</td>
<td>1</td>
</tr>
</tbody>
</table>
## C.2 Component Connection for Synchronous Generator

Table C.3: Connection Table for the Synchronous Generator (Fifth order)

<table>
<thead>
<tr>
<th>No.</th>
<th>$\Delta a$</th>
<th>$\Delta b$</th>
<th>$\Delta x$</th>
<th>$\Delta y$</th>
<th>$\Delta u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\Delta i_q^e$</td>
<td>$\Delta \Psi_q^e$</td>
<td>$\Delta \Psi_{kq}^e$</td>
<td>$\Delta i_q^e$</td>
<td>$\Delta V_{q_2}^e$</td>
</tr>
<tr>
<td>2</td>
<td>$\Delta i_d^e$</td>
<td>$\Delta \Psi_d^e$</td>
<td>$\Delta \Psi_{kd}^e$</td>
<td>$\Delta i_d^e$</td>
<td>$\Delta V_{d_2}^e$</td>
</tr>
<tr>
<td>3</td>
<td>$\Delta V_{q}^e$</td>
<td>$\Delta \Psi_{kq}^r$</td>
<td>$\Delta \Psi_{fd}^r$</td>
<td>$\Delta \omega_b$</td>
<td>$\Delta T_m$</td>
</tr>
<tr>
<td>4</td>
<td>$\Delta V_{d}^e$</td>
<td>$\Delta \Psi_{kd}^r$</td>
<td>$\Delta \omega_b$</td>
<td>$\Delta \delta$</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$\Delta \Psi_{mq}^r$</td>
<td>$\Delta \Psi_{fd}^r$</td>
<td>$\Delta \delta$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$\Delta \Psi_{md}^r$</td>
<td>$\Delta I_{d_2}^r$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$\Delta E_x$</td>
<td>$\Delta \omega$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>$\Delta T_c$</td>
<td>$\Delta \delta$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>$\Delta T_m$</td>
<td>$\Delta \Psi_{q}^r$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>$\Delta \Psi_{q}^e$</td>
<td>$\Delta \Psi_{d}^r$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>$\Delta \Psi_{d}^e$</td>
<td>$\Delta \Psi_{mq}^r$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>$\Delta \delta$</td>
<td>$\Delta \Psi_{md}^r$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>$\Delta \Psi_{q}^r$</td>
<td>$\Delta i_{q}^r$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>$\Delta \Psi_{d}^r$</td>
<td>$\Delta i_{d}^r$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>$\Delta \Psi_{kd}^r$</td>
<td>$\Delta i_{q}^e$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>$\Delta \Psi_{kq}^e$</td>
<td>$\Delta i_{d}^e$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>$\Delta \Psi_{fd}^r$</td>
<td>$\Delta T_c$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>$\Delta \Psi_{mq}^r$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>$\Delta \Psi_{md}^r$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>$\Delta \Psi_{q}^r$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>$\Delta \Psi_{d}^r$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>$\Delta i_{q}^r$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>$\Delta i_{d}^r$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>$\Delta \delta$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>$\Delta i_{q}^r$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>$\Delta i_{d}^r$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>$\Delta \Psi_{d}^r$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>$\Delta \Psi_{d}^r$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table C.4: Connection values for the AC synchronous Generator (Fifth order)

<table>
<thead>
<tr>
<th>$L_{11}(1, 14) = 1$</th>
<th>$L_{21}(1, 14) = 1$</th>
<th>$L_{12}(3, 1) = 1$</th>
<th>$L_{11}(2, 15) = 1$</th>
<th>$L_{21}(2, 15) = 1$</th>
<th>$L_{12}(4, 2) = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{11}(5, 10) = 1$</td>
<td>$L_{21}(3, 6) = 1$</td>
<td>$L_{12}(7, 4) = 1$</td>
<td>$L_{11}(6, 11) = 1$</td>
<td>$L_{21}(4, 7) = 1$</td>
<td>$L_{12}(9, 3) = 1$</td>
</tr>
<tr>
<td>$L_{11}(8, 16) = 1$</td>
<td></td>
<td></td>
<td>$L_{11}(10, 1) = 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{11}(11, 2) = 1$</td>
<td></td>
<td></td>
<td>$L_{11}(12, 7) = 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{11}(13, 8) = 1$</td>
<td></td>
<td></td>
<td>$L_{11}(14, 9) = 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{11}(15, 3) = 1$</td>
<td></td>
<td></td>
<td>$L_{11}(16, 4) = 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{11}(17, 5) = 1$</td>
<td></td>
<td></td>
<td>$L_{11}(18, 10) = 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{11}(19, 11) = 1$</td>
<td></td>
<td></td>
<td>$L_{11}(20, 8) = 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{11}(21, 9) = 1$</td>
<td></td>
<td></td>
<td>$L_{11}(22, 12) = 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{11}(23, 13) = 1$</td>
<td></td>
<td></td>
<td>$L_{11}(24, 7) = 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{11}(25, 12) = 1$</td>
<td></td>
<td></td>
<td>$L_{11}(26, 13) = 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{11}(27, 8) = 1$</td>
<td></td>
<td></td>
<td>$L_{11}(28, 9) = 1$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
C.3 Component Connection Model for the Overall System

### Table C.5: Connection Table for the Overall System

<table>
<thead>
<tr>
<th>No.</th>
<th>( \Delta a )</th>
<th>( \Delta b )</th>
<th>( \Delta x )</th>
<th>( \Delta y )</th>
<th>( \Delta u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( \Delta V_{q2}^e )</td>
<td>( \Delta I_{qm}^e )</td>
<td>( \psi_{kq} )</td>
<td>( \Delta \frac{\phi_k}{\omega} )</td>
<td>( \Delta V_{ref\text{exciter}} )</td>
</tr>
<tr>
<td>2.</td>
<td>( \Delta V_{d2}^e )</td>
<td>( \Delta I_{dm}^e )</td>
<td>( \psi_{kd} )</td>
<td>( \Delta \delta )</td>
<td>( \Delta V_{ps} )</td>
</tr>
<tr>
<td>3.</td>
<td>( \Delta T_m )</td>
<td>( \Delta \frac{\omega}{\omega} )</td>
<td>( \psi_{f_d} )</td>
<td>( \Delta \alpha_s )</td>
<td>( \Delta P_c )</td>
</tr>
<tr>
<td>4.</td>
<td>( \Delta E_{fd} )</td>
<td>( \Delta \delta )</td>
<td>( \Delta \frac{\omega}{\omega} )</td>
<td>( \Delta E_{fd} )</td>
<td>( \Delta I_{ref} )</td>
</tr>
<tr>
<td>5.</td>
<td>( \Delta V_{q2}^e )</td>
<td>( \Delta E_{fd} )</td>
<td>( \Delta \delta )</td>
<td>( \Delta V_{q2}^e )</td>
<td>( \Delta V_{ref\text{svc}} )</td>
</tr>
<tr>
<td>6.</td>
<td>( \Delta V_{d2}^e )</td>
<td>( \Delta T_m^e )</td>
<td>( \Delta E_{fd} )</td>
<td>( \Delta V_{d2}^e )</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>( \Delta V_{ref} )</td>
<td>( \Delta V_{q2}^e )</td>
<td>( \Delta T_m )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>( \Delta V_{ps} )</td>
<td>( \Delta V_{d2}^e )</td>
<td>( \Delta T_v )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>( \Delta \frac{\omega}{\omega} )</td>
<td>( \Delta I_{qL}^e )</td>
<td>( \Delta e_c )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>( \Delta P_c )</td>
<td>( \Delta I_{dL}^e )</td>
<td>( \Delta I_{dc1} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>( \Delta I_{q2}^e )</td>
<td>( \Delta I_{qL}^e )</td>
<td>( \Delta V_t )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.</td>
<td>( \Delta I_{d2}^e )</td>
<td>( \Delta I_{dqL}^e )</td>
<td>( \Delta I_{dc2} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13.</td>
<td>( \Delta V_{q2}^e )</td>
<td>( \Delta I_{qSV}^e )</td>
<td>( \Delta V_t )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14.</td>
<td>( \Delta V_{d2}^e )</td>
<td>( \Delta I_{dSV}^e )</td>
<td>( \Delta \alpha_s )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15.</td>
<td>( \Delta V_{q2}^e )</td>
<td>( \Delta \alpha_s )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.</td>
<td>( \Delta V_{d2}^e )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17.</td>
<td>( \Delta I_{ref}^e )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18.</td>
<td>( \Delta V_{q2}^e )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19.</td>
<td>( \Delta V_{d2}^e )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20.</td>
<td>( \Delta V_{ref\text{svc}}^e )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Applying KCL: \( \Delta I_{qd_m}^e + \Delta I_{qd_1} + \Delta I_{qdc} = \Delta I_{qd_L}^e + \Delta I_{qdc} \)
## C.4 Connection Matrix

Table C.6: Connection Values for the overall system

| \( L_{11}(1,7) \) = 1 | \( L_{12}(7,1) \) = 1 | \( L_{21}(1,3) \) = 1 |
| \( L_{11}(2,8) \) = 1 | \( L_{12}(8,2) \) = 1 | \( L_{21}(2,4) \) = 1 |
| \( L_{11}(3,6) \) = 1 | \( L_{12}(10,3) \) = 1 | \( L_{21}(3,15) \) = 1 |
| \( L_{11}(4,5) \) = 1 | \( L_{12}(17,4) \) = 1 | \( L_{21}(4,5) \) = 1 |
| \( L_{11}(5,7) \) = 1 | \( L_{12}(20,5) \) = 1 | \( L_{21}(5,7) \) = 1 |
| \( L_{11}(6,8) \) = 1 | | \( L_{21}(6,8) \) = 1 |
| \( L_{11}(9,3) \) = 1 | | |
| \( L_{11}(11,1) \) = 1 | | |
| \( L_{11}(12,2) \) = 1 | | |
| \( L_{11}(11,9) \) = -1 | | |
| \( L_{11}(12,10) \) = -1 | | |
| \( L_{11}(11,11) \) = 1 | | |
| \( L_{11}(12,12) \) = 1 | | |
| \( L_{11}(11,13) \) = -1 | | |
| \( L_{11}(12,14) \) = -1 | | |
| \( L_{11}(13,7) \) = 1 | | |
| \( L_{11}(14,8) \) = 1 | | |
| \( L_{11}(15,7) \) = 1 | | |
| \( L_{11}(16,8) \) = 1 | | |
| \( L_{11}(18,7) \) = 1 | | |
| \( L_{11}(19,8) \) = 1 | | |
Appendix D

System Data

D.1 Synchronous Generator (Full Order)

Table D.1: Data for Synchronous Machine (Full Order)

| \( f = 60 \) | \( E_{fd} = 2.7165 \) |
| \( \omega_b = 2\pi f \) | \( \delta_o = 38.58^\circ \) |
| \( r_{fd} = 0.000742 \) | \( V_{qo}^e = 1.308 \) |
| \( r_{kq} = 0.054 \) | \( V_{do}^r = -0.2153 \) |
| \( r_{kd} = 0.0131 \) | \( V_{qo}^r = 1.16 \) |
| \( r_s = 0.001096 \) | \( V_{do}^r = 0.625 \) |
| \( H = 4 \) | \( D = 0 \) |
| \( x_q = 1.64 \) | \( i_{qo}^r = 0.383 \) |
| \( x_d = 1.7 \) | \( i_{do}^r = 0.7203 \) |
| \( x_{ls} = 0.15 \) | \( i_{qo}^e = 0.7434 \) |
| \( x_{kq} = 1.526 \) | \( i_{do}^e = 0.3364 \) |
| \( x_{kd} = 1.605 \) |
| \( x_{fd} = 1.65 \) |
D.2  Synchronous Generator (Classical Model)

Table D.2: Data for Synchronous Machine (Third Order)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_o = 20.7$</td>
<td>$E'_{q_o} = 1.06$</td>
</tr>
<tr>
<td>$r = 0.00031$</td>
<td>$V_{q_o} = 0.94$</td>
</tr>
<tr>
<td>$X_d = 1.2$</td>
<td>$V_{d_o} = 0.36$</td>
</tr>
<tr>
<td>$X'_d = 0.3$</td>
<td>$V_{q_o}^{r} = 1.16$</td>
</tr>
<tr>
<td>$H = 4$</td>
<td>$V_{d_o}^{r} = 0.625$</td>
</tr>
<tr>
<td>$X_q = 1.2$</td>
<td>$\tau_d = 6$</td>
</tr>
</tbody>
</table>

D.3  Static Load

$P_L = 1.3$
$Q_L = 0.6$

D.4  Static Var Compensator

$K_\alpha = 30$
$T_\alpha = 0.3$

D.5  Data for DC Converter

Table D.3: Data for DC Converter

<table>
<thead>
<tr>
<th>Rectifier</th>
<th>Inverter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{q_s} = 1.0$</td>
<td>$V_{q_s} = 1.308$</td>
</tr>
<tr>
<td>$V_{d_s} = 0.0$</td>
<td>$V_{d_s} = -0.2153$</td>
</tr>
<tr>
<td>$I_{dc} = 0.9832$</td>
<td>$V_{dc} = -0.8932$</td>
</tr>
<tr>
<td>$X_c = 10$</td>
<td>$I_{dc} = 0.9832$</td>
</tr>
<tr>
<td>$X_{ar} = 0.6$</td>
<td>$X_L = 0.06$</td>
</tr>
<tr>
<td>$X_L = 0.1$</td>
<td>$T_a = 0.9548$</td>
</tr>
<tr>
<td>$T_{a} = 1.0276$</td>
<td>$\gamma = 15^\circ$</td>
</tr>
</tbody>
</table>
Appendix D. *System Data*

**DC Link:** $P_{\text{DC}} = 0.2, X_{\text{DC}} = 1.3, G_{\text{DC}} = 21.645$

### D.6 Synergetic Controller

#### D.6.1 For Classical Model

$K = 1.0$

$\lambda = 0.1$

#### D.6.2 For Full Order Model

$K = 0.5$

$K_1 = 1.0$
Bibliography


systems interconnected by h.v.d.c. links. *Generation, Transmission and Distribu-
tion, IEE Proceedings C*, 127(1):15–19, January 1980. ISSN 0143-7046. doi:

9781400841042.

[33] Seraji H. Pole assignment using dynamic compensators with pre-specified poles.

[34] M.A Laughton. Matrix analysis of dynamic stability in synchronous multimachine

[35] M.A Choudhry and D.P. Carroll. Coordinated active and reactive power modulation
318487.

affected by excitation control. *Power Apparatus and Systems, IEEE Transactions
292452.

[37] S. Lefebvre. Tuning of stabilizers in multimachine power systems. *Power En-
gineering Review, IEEE*, PER-3(2):22–23, Feb 1983. ISSN 0272-1724. doi:
10.1109/MPER.1983.5519619.

[38] Hammad AE. and El-Sadek M. Application of a thyristor controlled var compen-
sator for damping subsynchronous oscillations in power systems. *Power Apparatus
and Systems, IEEE Transactions on*, PAS-103(1):198–212, Jan 1984. ISSN 0018-
9510. doi: 10.1109/TPAS.1984.318608.

for transient stability studies using a simplified dc system representation. *Power Apparatus
ISSN 0018-9510. doi: 10.1109/TPAS.1985.319246.

with multiterminal hvdc links and static compensators. *Power Engineering Review,
5520974.


