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Dynamic Response in Transient Stress-Field Behavior Induced by Hydraulic Fracturing

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Dynamic Response in Transient Stress-Field Behavior Induced by Hydraulic Fracturing

Andrew Jenkins

Thesis Submitted
to the Benjamin M. Statler College of Engineering and Mineral Resources
at West Virginia University

in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE in
Petroleum and Natural Gas Engineering

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2015

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ABSTRACT

Dynamic Response in Transient Stress-Field Behavior Induced by Hydraulic Fracturing

Andrew T. Jenkins

Hydraulic fracturing is a technique which is used to exploit geologic features and subsurface properties in an effort to increase production in low-permeability formations. The process of hydraulic fracturing provides a greater surface contact area between the producing formation and the wellbore and thus increases the amount of recoverable hydrocarbons from within the reservoir. The use of this stimulation technique has brought on massive applause from the industry due to its widespread success and effectiveness, however the dynamic processes that take part in the development of hydraulic fractures is a relatively new area of research with respect to the massive scale operations that are seen today.

The process of hydraulic fracturing relies upon understanding and exploiting the in-situ stress distribution throughout the area of study. These in-situ stress conditions are responsible for directing fracture orientation and propagation paths throughout the period of injection. The relative magnitude of these principle stresses is key in developing a successful stimulation plan. In horizontal well plan development the interpretation of stress within the reservoir is required for determining the azimuth of the horizontal well path. These horizontal laterals are typically oriented in a manner such that the well path lies parallel to the minimum horizontal stress. This allows for vertical fractures to develop transversely to the wellbore, or normal to the least principle stress without the theoretical possibility of fractures overlapping, creating the most efficient use of the fluid energy during injection.

The orientation and magnitude of these in-situ stress fields however can be dynamic, controlled by the subsequent fracture propagation and redistribution of the surrounding stresses. That is, that as the fracture propagates throughout the reservoir, the relative stress fields surrounding the fractures may see a shift and deviate from their original direction or magnitude. These types of shifts are of great concern because they can impact subsequent fracture development causing non-uniform fracture propagation and the potential overlapping of fracture paths as they extend from the wellbore at the point of injection.

The dynamics of stress variation that occur with respect to hydraulic fracturing is a somewhat new area of study. In order to accomplish the goals of this thesis and continue future research in this area a new transient model has been developed in order to assess these dynamic systems and determine their influence on fracture behavior. This applies the use of a fully coupled finite element method in 2-D using linear elastic fracture mechanics which is then expanded using displacement discontinuity to a cohesive zone model in 3-D. A static boundary element model was also used to determine stress fields surrounding static, predetermined fracture geometries. These models have been verified against analytical solutions for simple cases and are now being applied to more detailed case studies and analysis. These models have been briefly discussed throughout this thesis in order to give insight on their current capabilities and application as well as their future potential within this area of research.

The majority of this work introduces transient stress field prediction to cases of single and multiple hydraulic fractures. The static assessment of these stresses is determined for verification of results to those found in publication which leads into these transient stress field variations. A new method has been developed and applied to the stress state prediction for the first time in a transient fracture model which is partly based upon a critical distance theory. These dynamic interactions can provide useful insight to pertinent issues within the petroleum and natural gas industry such as those to hydraulic fracturing fluid loss and induced seismic events, as well as to applications of efficiency and optimization of the stimulation treatment plan.

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DEDICATION

This work is dedicated to:

My mother and father, Sue and Herb Jenkins for their truly unconditional love and support, doing far more than they ever had to in order to ensure that I succeed. My loving wife Sierra for her sacrifice and support over the past few years and to the United States military personnel who have made the greatest sacrifice so that I may have the opportunity to pursue my education.

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CHAPTER 1 - INTRODUCTION

1.1. BACKGROUND

The focus of this research has been primarily upon unconventional shale gas reservoirs and their response to the induced stresses which occur throughout the hydraulic fracturing process. Shale reservoirs differ entirely from the traditional reservoir development techniques which consist primarily of exploiting naturally permeable beds and formation traps. Unlike these traditional reservoirs, such as sandstone or carbonates, shale reservoirs serve as both the trap and source rock for hydrocarbons, whereas conventional reservoirs serve only for the accumulation of hydrocarbons from another organic source. Conventional reservoir development allows for the profitable production of hydrocarbons without the requirement for complex stimulation techniques, such as hydraulic fracturing, as the formations permeability and porosity are high enough to allow sufficient volumes of fluid migration throughout the field. With the advancement of horizontal drilling and stimulation techniques, it has allowed for unconventional shale reservoirs to become a new staple of the petroleum and natural gas industry, as these were previously deemed to be unprofitable for development by using past techniques.

The inherent problems with shale deposits lies in their unique properties which associate some of their mechanical behavior more with that of coal bed formations than that of other more conventional reservoirs, or traps. Shale reservoirs allow for adsorption to occur through the bonding of entrained hydrocarbons throughout the rock matrix. This allows for additional hydrocarbon storage beyond that of the associated pore space volume available within the shale rock. The surrounding lithological stresses also play an important role in the physical behaviors related to shale reservoirs as it impacts the total compressibility. This is along the lines of shrinkage and swelling that is seen in coal bed formations which is dependent upon the in-situ and applied stress distributions (Mansoori, 1996). The permeability and porosity of the shale rock matrix varies directly with these applied stresses due to the compressibility and higher elastic limit, relative to other types of geological formations in conventional reservoir development. These characteristics affect the trans-missive ability of shale formations, influencing the fluid migration potential through a reduction in inherent permeability and natural fracture closure throughout the formation.

Hydraulic fracturing is a process that is used in order to stimulate both conventional and unconventional reservoirs in an effort to make them more economically feasible to develop, produce or remediate; both new and old fields alike. The process of hydraulic fracturing allows for access to vast amounts of previously inaccessible hydrocarbon deposits throughout a reservoir. This is achieved by generating a greater contact area between the producing formation and the wellbore. As a fracture propagates through the formation, reservoir to wellbore conductivity is increased by the drastic change to permeability throughout the stimulated zone. These fractures increase the fluid migration potential by significantly increasing permeability and therefore, the amount of production and recoverable hydrocarbons from the reservoir is drastically increased throughout these zones (Cleary, 1958).

The process of hydraulic fracturing uses the injection of water or gel and a propping agent (i.e. sand) at high pressure into low permeability formations that do not allow sufficient fluid migration. This process first utilizes perforating guns to puncture the production tubing, cement and steel casing to initiate the location of fracture. These locations are then sealed off on an individual basis by the use of packers which allow for an increase of applied pressure that is isolated only to that specific zone of the wellbore and surrounding formation. The fracturing fluid, which is typically 99.5 percent water and sand, is then pumped down the wellbore to the isolated perforations. As fluid pressures are suddenly increased at the injection site by the use of high pressure pumps on the surface, the surrounding stress regimes are affected. Once the applied fluid pressures meet or exceed the fracture initiation pressure of the formation then the crack, or fracture begins to develop. These stress fields

continue to be altered throughout the immediate and surrounding area as the fracture(s) propagate throughout the formation.

Fracture initiation and propagation is controlled by the relationship of applied pore pressure to the rocks tensile strength and overburden pressure. The propagation of the fracture is further controlled and directed by the orientation of the principal stresses at the immediate fracture tip. The direction of fracture propagation will continue perpendicular to the path of least resistance through the formation (i.e. the least principal normal stress). This implies that a fracture will propagate parallel to the maximum horizontal stress, such that closure and fracture width is related to the minimum horizontal stress.

Proppant within the fracturing fluid stabilizes the developing fracture as it propagates throughout the formation. The distribution of proppant within this developing fracture aids in preventing full aperture closure, providing support to the compressive forces trying to close the fracture. The use of proppant aids in extending both the length of fractures as well as stabilizing the newly created void space to the formations closure pressure. This allows the induced fracture network to maintain maximum conductivity from the producing formation to the wellbore once the treatment process has been completed, while also increasing the volume or surface area of augmented permeability. The types of proppant to be used vary depending upon the specific treatment plan that has been developed, however these are typically a mixture of multiple grades (or sizes) of sand or plastic beads that settle through the developing fracture. These methods of transport are dependent upon the injection pressure gradient and fluid-mass velocity through the fracture which generally cause larger particles to settle first and smaller particles to settle last.

As wellbore zones or stages are fractured throughout a reservoir, the in-situ stress fields are modified. The redistribution of these stress fields relies upon the in-situ conditions of the initial geological strata, rock matrix and induced hydraulic gradients through the propagating fracture network. This redistribution of these stress fields generates the possibility of reduced stability throughout the modified stress regime. This is suggesting that the natural faults and fractures may potentially reactivate within the affected treatment zone and/or surrounding reservoir area that has seen these stress influences. These new stress regimes can lead to a number of possible complications if instability occurs, most notably as felt seismic anomalies leading to fluid loss and reservoir or aquifer damage.

The seismicity induced by hydraulic fracturing is inherent to the process and has been a principal study of the process. These induced seismic events are typically used to map the generated fracture network, aiding in the optimization of the well completion and stage treatment design. As the fracture propagates through the rock matrix, energy is released in the form of vibrations. Monitoring these vibrations indicates the path, geometry and distribution of fractures within the induced fracture network. These induced seismic events are typically on a micro-seismic scale which indicates that these events do not transmit enough energy to the surface to be harmful or felt. This is because the applied fluid energy is dissipated over a very large area. The occurrence of these micro-seismic events would therefore be expected to last throughout the duration of the treatment period, or until fracture development has subsided below the threshold of the seismic monitoring equipment.

In many instances however, field cases have revealed that the release of seismic energy continues well beyond the length of the injection period. At times, these secondary events are of a higher magnitude than those induced during staged treatments. These types of seismic events can be classified as seismic anomalies, as they are not directly attributed to the fluid injection and fracture propagation but instead to the redistributed stress fields that are developed by these events (Jonny Rutqvist, 2013).

Induced seismic events and related anomalies that fall outside the expected fracture network give an indication to areas of possible fault slippage or reactivation. This is expected to occur along zones or areas of discontinuity, as the modified in-situ stress fields relax to a new equilibrium. These events, can at times help further increase conductivity to the wellbore along the fault faces and interconnecting fractures, increasing hydrocarbon production. In other cases, these seismic anomalies can be detrimental to the well as they are often times associated with hydraulic fracture fluid loss, reservoir or aquifer cap-rock damage and in extreme cases has the potential to damage surface infrastructure(s) through felt seismicity and associated subsidence.

There have been several cases of felt seismicity which possess the capability and have been shown to damage the reservoir or surrounding aquifers given the proper conditions. Although these cases are extremely rare, the possibility is an unacceptable consequence that requires a better understanding of the hydraulic fracturing process in order to diminish the probability of these related seismic anomalies. Typical seismic activity throughout the stimulation process tends to fall below a 1.0 magnitude; however in very rare cases the seismicity associated with hydraulic fracturing has been shown to reach magnitudes in excess of 3.0 (Ellsworth, 2013). In addition to the overall magnitude of the induced seismic anomalies, the frequency of these events occurring is also an extremely important factor. These seismic anomalies have been shown to directly correlate in areas that have been subjected to, or sustained multiple well and fracture treatments.

These types of seismic events can not only further discredit the fracturing process and industry as a whole from the public viewpoint, but it also affects the economic feasibility and recoverable hydrocarbons of the reservoir. In retrospect, these events can also further increase wellbore conductivity and thus surface production. Therefore, understanding these occurrences is an important objective of the industry and this research in order to diminish, predict and better understand the nature of these post-treatment seismic anomalies.

In comparison to the seismicity generated during fracture treatment, the additional or continuation of micro-seismic anomalies post-injection is evident from microseismic monitoring during the treatment process. The location of these seismic events places them a distance beyond that of their respective fracture half-lengths and localized seismicity that can be associated with the induced fracture. This offset distance of seismic activity is suggestive that natural faults, fractures or areas of discontinuity are being stimulated in addition to the intended pay zone where areas of localized seismicity can be seen. This confirms that stress fields are being modified far beyond that of the immediate area surrounding the induced fracture network and outside the period of injection. The extent of these stress modifications depends upon the specific interaction of geological, thermal and fluid properties in the producing and surrounding formations as well as that of injection fluid (A. Dhari Talegani, 2013). This often becomes an extremely difficult problem to not only measure, but to correlate to analytical and numerical methods of prediction and simulation due to the complicated coupling that occurs through these processes.

The most notable controlling factor in cases of fault reactivation had been shown to be the frictional coefficient that is present along the pre-existing fault and fracture plane(s). As the in-situ stress fields are modified through the induced fracture network and surrounding area, the friction between previously deactivated faults that lie in these altered stress zones may be affected. This can cause material failure or general instability throughout the area, leading to a displacement along the relative fault surfaces. This geological process is most often described by the Mohr-Coulomb Failure Criterion, which describes the principal stress relationship to a materials inherent properties in a straightforward manner by relating the shear and normal stresses along the acting plane (Labuz, 2012). In their natural state, deactivated fault zones do not pose an issue. The confining pressure and past geological events have caused a resistance to sliding or slippage throughout these zones due to the current in-situ stress regimes, as they have been brought to equilibrium over time. With the introduction of fracture(s,

discontinuity and pore pressure is increased throughout the induced fracture network as it propagates throughout the reservoir. The related stress changes can lead to a loss of friction as increases in the principal stresses or alterations to the entirety of these in-situ stress fields along these deactivated fault planes.

The process of hydraulic fracturing has become an increasingly complex procedure as it has become more widely used throughout the petroleum and natural gas industries. This has led to the development of new application methods due to industry advancements in horizontal drilling and stimulation techniques, as well as an increased understanding of the key principals involved with the hydraulic fracturing process. The impacts of hydraulic fracturing have been studied throughout its use; however their effects have only recently become readily apparent as it is being used with such an increased frequency. This influx of data has led to the realization that in some cases, what was once merely seen as a coincidental occurrence may be in direct relation to the stimulation techniques used for fracture treatments. The relative natures of these impacts that may be associated with hydraulic fracturing are becoming apparent due to statistical probability of their recent, widespread use.

The delicate nature of this subject, with relation to both the energy industry and environmental concerns has caused it to become a trending sociopolitical topic throughout not only the United States, but throughout the globe. The increased scrutiny that has been brought about from both the public viewpoint and government agencies alike has caused the industry to take notice in an effort to better understand the process of hydraulic fracturing and its related effects. The expansion of research related to this topic will ultimately benefit all parties involved as it is a necessary step in sustainable energy development both here in the United States and abroad.

The study of hydraulic fracturing has been done through a number of differing techniques in order to better understand the consequent alterations of in-situ stress properties and their relation to the associated interactions throughout porous media. Many of the studies within this field have not yet addressed several of the more complex issues involved with the hydraulic fracturing process. In order to study these complex interactions involved with stress induced fault reactivation, the progression between pre and post-fracturing must be studied in great detail. The complexities surrounding cases of fault reactivation require consideration of the influential geological parameters, rock mechanics, fluid properties and dynamic stress interactions in an effort to understand, classify and predict the proper in-situ stress response and it's relating effects.

1.2. OBJECTIVES

The underlying goal of this project and the associated research is intended to investigate advanced methods to predict cases of fault reactivation and therefore improve the effectiveness of hydraulic fracturing treatment plans in horizontal shale gas wells. The research area will focus on the Marcellus shale formation and associated geology throughout the Appalachian Basin region.

The previous work done in this area has not fully related or addressed the complex interactions within the fracture network. Therefore these methods do not offer the desired capability or performance of predicting the pre and post fracturing effects, primarily relating to these modified stress fields. This is beneficial in preventing the reactivation of natural faults lying within these modified stress regimes as well as optimizing the overall design of fracturing stage treatments. Understanding these relationships can provide further insight to creating more effective stimulation plans. In addition, this research may be further applied to similar types of geological deformation problems and fault study investigations due to the close relationship(s) in methodology.

In order to better determine the interactions involved when fracturing a reservoir, components of previous work will be examined and altered in order to extend or expand the capabilities of current modeling software. This will consider and reference many of the related topics to this problem. In more recent years, the amount of available

material in relation to this research has increased due to advancements made through both analytical and numerical methodology allowing for multiple approaches to this complex problem.

The entirety of the research project will introduce the following topics and requirements for completion, although specific attention for the material presented in this thesis will focus upon the last bulleted point of stress state characterization.

- Assessing the reactivation potentials of faults by identifying the in-situ stress conditions of faults nearby fracture treatment wells.
 - Acquisition of data for horizontal shale gas reservoirs
 - Characterization of critically stressed faults
- Development of a 3-D propagation model for multiple transverse fractures created simultaneously in one fracturing stage.
 - Fracture deformation model
 - Fluid flow and leak-off model
 - Fracture tip advance criterion
 - Mass conservation equation
 - Discretization of fracture propagation model with triangle element
- Further extending the 2-D variation model of stress field distribution coupled with the 3-D fracturing propagation model in order to predict fracture geometry and varied stress throughout the stage.
 - Green's functions and displacement continuity method
 - Balance of stress and internal fluid pressure on fracture surfaces
 - Coupled 3-D model (stress variation versus fracture propagation)
 - Discretization of the coupled 3-D model
- Generate code for a 3-D numerical model so that different cases may be studied. These will be validated with synthetic examples to ensure proper modeling.
 - Programming a coupled 3-D model
 - Validation and modification of the coded model
 - Generation of a 3-D simulator for fracturing design
- Characterization of the stress state changes of a fault and near-fault zones, evaluate the potential of fault slippage due to hydraulic fracturing and optimization of fracture design.
 - Reveal changes of stress state of fault and/or near-fault zones during a fracture treatment.
 - Evaluate the potential of fault slippage
 - Optimization of the fracture design by use of the 3-D simulator

1.3. RESEARCH PROJECT SCOPE

The specific focus of this research has been to study the influence of hydraulic fracturing on the geomechanical properties and response of a reservoir throughout the fracturing process. This information is useful to determine the effect, or potential of fault reactivation caused by hydraulic fracture and the related stress field modifications that occur throughout a reservoir and its surrounding geology. In relation to this subject, there are numerous commercially available software models which possess the capability to model hydraulic fracturing treatment plans or stages. The downfall in many of these programs or techniques however is that they do not address many of the more complex rock and fluid mechanics occurring throughout the reservoir or fracturing stage. In addition much of the previous study in these areas has focused primarily upon stresses surrounding the fracture for the determination of propagation criteria. In addition many techniques to date do not adopt a fully coupled solution and thus assume a pressure boundary solution or one way coupling that ignores rock deformation or displacement effect upon the fracture pressure.

The assumptions made through the previous examinations of the topic, raise question to the overall accuracy of these assumptions and thus their associated results. The primary focus of this research is to evaluate the more complex reservoir interactions that occur throughout the fracturing process, including many of these previously neglected parameters and mechanics.

The overall goal of addressing these complexities is to further enhance the development of a three-dimensional model to analyze reservoir stress interactions. This will aid in increasing the result quality related to the prediction of fault reactivation tendency. The previous models used in relation to this objective have addressed the basic mechanical interactions that occur within a reservoir. These solutions allows for a more general determination of stress distributions, both pre and post fracturing. In many cases, this is a satisfactory in creating an effective treatment plane but it does not provide the capability required in relation to specific stability issues.

In conjunction with the Mohr-Coulomb Failure Criterion, the slip tendency seen across the fault plane may be predicted by the results. The preliminary model referenced for this research addresses the reservoir as an isotropic, homogeneous elastic medium which disregards the poroelasticity and leak-off related effects. These assumptions present a number of possible adaptations that can be made in order to account for a larger amount of the effective variations on geological stresses. The proposed methods and theory relating to the material presented in this paper will aid in the development of an effective three-dimensional model, relating reservoir stress interactions throughout the time of the injection period.

A primary focus here is the inclusion of poroelastic effects into the three-dimensional model, also addressing the inclusion of a proper leak-off model. The underlying theory of the model is based upon the Finite Element Method. This will be used to account for each aspect of fracture propagation and their interaction with in-situ stresses, both within and surrounding the reservoir through the coupling of the associated governing equations. These aspects will require that the fluid flow within the fracture be modeled simultaneously with propagation in order to account for stress interactions that occur throughout the time that a stage or fracture is being stimulated. This dependency will require an adaptation in domain discretization by the use of a dynamic mesh structure. This allows the address of time dependent boundary conditions that are associated with a propagating fracture through porous media. The coupling of these parameters will allow for a more precise and in turn, realistic modeling of interactions throughout the induced fracture network. This will ideally create a greater correlation to the responses often seen in field related data.

The inclusion of these parameters will allow for better assumptions to be made through the fracture stage development. The increasing scrutiny in regard to hydraulic fracturing has created the need of a more thorough understanding of the process in an effort to reduce the possible negative impacts surrounding the use of hydraulic fracturing.

Features	Model		Remarks for Proposed Model
	(previous)	(proposed)	
Model			
Geometry	3-D	2-D and 3-D	
Properties	Homogenous	Heterogeneous	PETREL Data Extraction
Layering	Single Layer	Multiple Layer	
Input	Manual	Autonomous	
Solution			
Type	Static	Dynamic	Iterative + Direct
Method	FEM	FEM	
Fluid Flow	Fluid Viscosity + Leak-Off		

TABLE 1 SUMMARY OF FEATURES OF PROPOSED AND PREVIOUS NUMERICAL MODELS

1.4. METHODOLOGY

- 1) Conduct a literature review of the previous research done involving this topic.
 - a. Geomechanical interactions and redistribution of stresses due to hydraulic fracturing.
 - b. Mechanics of hydraulic fracturing and Mohr-Coulomb failure criterion.
 - c. Poroelastic properties of the reservoir and involved during the fracture propagation.
 - d. Fluid and leak-off interaction as a fracture stage develops through the reservoir.
- 2) Comprise a fundamental background of the theory and methodology relating to this research.
- 3) Generate a set of governing equations for the poroelastic effects and fluid flow interactions involved in hydraulic fracturing, throughout the reservoir and fracture network.
- 4) Coupling of the fluid flow equations and the geomechanical problem that includes both the fluid flow and leak-off model.
- 5) Incorporate the (fully or partial-) coupled equations into the geomechanical problem.
- 6) Generate code in order to support the aforementioned principles by the use of a dynamic mesh structure, allowing a time dependency to model real time stress variation.
- 7) Evaluate the generated results to ensure consistency and accuracy within the model.
- 8) Apply sensitivity analysis to the input parameters in order to study the key principals involved and their influences on fault reactivation tendency.
- 9) Identify patterns of in-situ stress and their correlated reactivation tendency that allows the ability to classify or categorize key zones of, or expected of failure.
- 10) Write and complete thesis and documentation.

Task	2013			2014			2015
	Spr.	Sum.	Fall	Spr.	Sum.	Fall	Spr.
Literature Review							
Previous Model							
Generate Governing Equations							
Implement New Code							
Evaluate Results of New Model							
Apply Sensitivity Analysis							
Identify Stress Patterns							
Complete Thesis Work							

TABLE 2 TIMETABLE OF THESIS RESEARCH

1.5. STRUCTURE OF THE THESIS

This thesis includes seven chapters which outline the methodology, approach and results that support much of this research. The current chapter provides a general description of the research problem along with its objectives, scope and methodology. A literature review was conducted on the various related topics to fault reactivation and reservoir stress change interactions in Chapter 2. The fundamental concepts supporting a majority of material presented in this work are provided throughout Chapter 3. This encompasses most of the basic information which the more detailed assumptions and observations are based later in this work. Chapter 4 outlines the preliminary results which were determined using the initially developed, static finite element model. Chapter 5 covers the specific analytical methods required to solve this problem, this includes the new transient model description and more specific methodology supporting this work. The results and conclusion of this work are summarized by Chapter 6 which identify any relationships that are seen throughout the various models and initial conditions that have been used. Chapter 7 includes the appendix, covering material that was not put directly into the body of this thesis but offers reference as indicated to material that was deemed important but too large or specific to put into the body.

CHAPTER 2 - LITERATURE REVIEW

2.1. INTRODUCTION

Since the advent of hydraulic fracturing it has become a key process that is crucial to the success of the petroleum and natural gas industry through its application to unconventional shale and tight gas plays. The process of hydraulic fracturing with respect to this thesis refers specifically to the injection of a water or gel mixture into extremely low-permeability (i.e. unconventional) formations or reservoirs. This process has been studied through numerous analytical, numerical and computational approaches in an effort to better understand the induced stress relationships occurring through fracture initiation, propagation and termination along with its transient effects on reservoir fluid migration and storage (Fitzgerald, 1998). These stress relationships not only explain but dictate the propagating fracture geometry. In many of these cases however, the explicit relationships of subsurface properties have been neglected for reasons of simplicity which therefore limits not only their application, but also their validity.

Early methods of hydraulic fracturing date back to the late 1940s where this process involved similar, albeit crude methods in today's terms. The massive scale multi-fracturing operations seen today were not extensively used until the turn of the 21st century. Through this time there has been significant progress made to optimizing these injection methods with the continuous refinement of the fracturing process, its associated technology and supporting methodology. This has created an understanding of the principle relationships involved with hydraulic fracturing although the extent of their role in fracture efficiency and stress modification has not been explicitly correlated or validated in many cases.

The overall efficiency of a stimulation treatment plan today is typically inferred by methods of micro-seismic monitoring and mapping in combination with fracture fluid flow back and fluid loss. In a simpler sense, these resultant effects may also be seen by analyzing the production data from newly stimulated wellbores and formations. These experimental studies however do not yield all the necessary data in many cases for increasing the efficiency or accuracy of their associated numerical and analytical solution methods. In order to understand and correlate the principle physics of two seemingly dissimilar problems they must be broken down into their fundamental components and analyzed.

The addition of new or more complex parameters in determining a solution is typically respective to specific case studies or regions and therefore does not offer the same benefits as would a universal solution, in terms of its application. The application of a universal solution to a specific problem typically requires one of two approaches. One approach is to simply disregard these complex interactions within the physical problem, which in this case are those that occur sub-surface, as these complexities are typically referenced on a case by case basis. This is often due to the exponential increase in difficulty of determining a solution as the number of unknowns surrounding the problem is increased. Limiting the problem to previously identified relationships and key mechanical interactions can provide a general solution by introducing key assumptions that attempt to reduce the number of possible unknowns. The other approach to developing a universal solution is to address complexities surrounding the problem by a method of coupling. In this method the dependencies can be reduced through various mathematical techniques and determining a solution of these complexities simultaneously.

These simplifications to complex problems have been deemed acceptable by industry standards and their requirements for most cases, as the correlation to the available field data is typically within an acceptable range of error. There is however the concern that ignoring certain parameters can potentially and/or drastically impact the final solution. This identifies one of the key underlying goals of this research. It is ultimately important to identify specific stress modification relationships through a general solution approach so that the application is not limited

to a specific problem, but rather is capable for a broad range of problems and complexities. This robustness can be achieved through solution methods such as full or partial coupling of the governing constitutive relationships.

Previous study in the concern of in-situ stress modification and fault reactivation or slip tendency has been done primarily in the areas of CO₂ sequestration, reservoir depletion and deep-well injection where the expansion or contraction of large geologic volumes over time generates noticeable patterns of altered stress. These relationships have become an extremely important topic to the industry due to the numerous environmental and related concerns that surround the extraction or production of hydrocarbons. The area of study with regard to reservoir expansion or contraction generally follows similar large scale geomechanical principles as those affected from the hydraulic fracturing process. As pore pressures are modified throughout the reservoir with the advent of fluid injection, the in-situ stress fields are modified by a deviatoric stress relationship.

In order to study the afflicting factors caused by hydraulic fracturing, a geomechanical model must be incorporated with a hydraulic fracture and fluid flow model. This will allow the study and identification of the effects relating to these in-situ stress modifications, both throughout and surrounding the reservoir. There is a great difficulty associated with the modeling and simulation in cases of fracture however which is due to the highly complex nature in which such cases of fatigue and failure must be addressed.

Various case studies were used in order to correlate and test the validity of previous research throughout this area. These case studies cover the possibility of fault or seismic reactivation involved with the injection or depletion that occurs through the expansion or contraction of a reservoir due to these relative stress field modifications. These numerical solutions typically assume that the reservoir is a simple homogenous elastic medium, ignoring many of the associated fluid, geological and temperature dependent effects. These assumptions raise numerous concerns in regards to the complex reservoir interactions that occur in the study of fault reactivation and stress reorientation that occurs due to hydraulic fracturing.

This has been a relatively new area of research over the past two decades. Over that time it has benefited the industry through new analytical and numerical approaches or techniques that have not only aided in the direct study of these stress relationships, but also had an impact in many other aspects of this and other related industries. These major revelations that have occurred during this time have been summarized throughout this chapter in order to provide a brief background on the related research to this topic, as well as to provide support for some of the major assumptions and methodology that are used throughout this work.

2.2. IN-SITU STRESS FIELD MODIFICATION

The in-situ stress refers to the initial state of stress throughout the domain. These measurements are an extremely important aspect of geotechnical or geomechanical analysis. A particular difficulty in exploring this subject is in relation to the location of interest which typically lies a few thousand feet or more below the earth's surface. This often requires an indirect method of evaluating these sub-surface properties as it becomes extremely difficult to take an account of the in-situ reservoir properties and related stresses by means of direct measurement. In order to visualize and quantify these relationships they are typically modeled using derived analytical relationships so that they replicate, or mimic these real observations and more importantly their trends by an indirect means. In order to infer these parameters directly, it would require access to these subsurface geologic areas, which in many instances is not a feasible, practical or even achievable task.

Early methods of studying geological stress fields applied the theory of strain nuclei. This was originally designed to solve for general field equations in an elastic medium for point loading conditions or singularities. As methods of mathematical integration were developed, these types of solutions were expanded to include cases of

distributed loading (Love, 1944). The strain nuclei theory was applied in solving for cases of a homogenous, poroelastic medium. In these cases there is no differentiation between reservoir rock properties and the surrounding rock properties that would typically encapsulate the reservoir. This limits its overall use and reliability with respect to cases of complex or stratified geology. The entirety of this model representation is that it is assumed to act as an isotropic homogenous medium that has no property variation throughout the domain.

The concept of strain nuclei in poroelastic media was used to calculate ground surface subsidence above axis-symmetric, disc-shaped reservoirs that were pressure depleted through fluid production by (Geertsma, 1973). The concept was then later applied to determining poroelastic strain and stress fields surrounding a depleted reservoir and also used to explain induced seismic events associated with these field changes (Segall, 1985). The concept of strain nuclei in combination with a discrete element approach was used to calculate the subsidence and compaction effects of pressure (i.e. stress) changes within a reservoir. These theories were then used to model synthetic examples of reservoirs that had 1 to 4 producing wells (Du, 2001).

A semi-analytical approach was taken by (Hamidreza Soltanzadeh, 2008) in order to explore the physical interactions within a reservoir and its constraining cap-rock formation(s). This was performed in order to study and prevent reservoir damage that occurs by cap-rock discontinuity caused by fault slippage or reactivation. This process used the theory of inclusions and inhomogeneities to provide a geochemical analysis of the reservoir. This study however did not address the fluid or gas migration through faults and fractures throughout the reservoir. This assumption allowed for semi-analytical and closed-form solutions of induced stress and pore-pressure change throughout the producing reservoir area(s).

The theory of inclusions and inhomogeneities was used with success in stress field modeling throughout a reservoir by (Rudnicki J. , 1999). An inclusion can be identified as a homogenous isotropic elastic medium that would undergo arbitrary strain if it was unbounded, however due to constraints imposed by the matrix that surrounds it; the strain field within it is modified (Eshelby, 1961). This arbitrary strain is known as an eigenstrain. Eigenstrains are a generic reference to the compilation of internal strains that would be caused by various complex mechanisms (poroelastic, plastic, and thermal changes). Eigenstrains are referred to by Eshelby as stress-free transformation strains, while Eigenstress is a self-equilibrated internal stress caused by several of these influencing eigenstrains.

The assumption of an infinitely extending material limits the complexity of the mathematical solution to practical nature. This was suggested for use in ellipsoidal inclusion in full-space and is based on the Green's Function method. These were used to find stress fields both within and surrounding horizontal and dipping reservoirs having elliptical and rectangular cross-sectional geometries in a plane strain half-space. The basic nature of Eshelby's Theory is that strain variations (therefore stress) are uniformly distributed throughout the entirety of the inclusion. The equivalent inclusion method allows for inhomogeneity to be simulated with the use of these eigenstress within a homogenous material. This allows for an incorporation of complex stress effects without increasing the overall complexity of the problem solution.

These methods require that the reservoir is treated with the assumption that no pressure or temperature change occurs in the surrounding rocks, so there is no hydraulically driven flow, no heat transfer and fully-drained loading occurs (Hamidreza Soltanzadeh, 2008). These assumptions make it difficult to explain many of the more complex reactions that can be identified but the direct relationship causing this response is not often a specific answer. These assumptions become particularly questionable in solutions involving a time-dependency as there is a large variation in these characteristics throughout this time.

2.3. STRESS DEFORMATION THROUGH INDUCED FRACTURE NETWORKING

The characterization of natural and induced fracture networks is an extremely important aspect of fluid induced stresses. The shape, geometry, size, distribution, roughness, connectivity, etc. of the fracture network all influence the stress response under hydraulically induced pressure(s). This is an extremely complex process that has seen multiple approaches to fracture analysis and propagation. The elastic deformation behavior of rock fracture seen through hydraulic fracturing requires intensive refining techniques and complex numerical techniques in order to properly define the boundaries of the discontinuous zones that occur throughout the process (B.J. Carter, 2000).

There have been various methods developed to deal with these discontinuous zones throughout the solid media as the fluid front propagates in their proximity or through their solid boundary. This is most commonly approached through the Boundary Element Method (BEM) and displacement discontinuity method (DDM) that include linear and non-linear fracture mechanics (FM). Many other techniques have been developed as well, each with their own unique advantages and disadvantages. In the similar field of study, concrete dam failure coincides well with hydraulic fracture modeling and thus there is a cross-over between these applicable techniques.

There have been many techniques to numerical modeling of crack propagation and crack analysis. Early methods of analysis assume strength-based models in which propagation is determined via the tensile strength of the material and the calculated tensile stresses at the crack tip. In these cases there is a lack of mesh objectivity in which the FE mesh discretization has a significant influence upon the results. A newer approach and widely used is that of linear elastic fracture mechanics (LEFM) in which a stress intensity factor is used in combination with the materials fracture toughness. The application of non-linear fracture mechanics (NLFM) have become more widely used in recent times for analyzing the fracture process zone (FPZ) at the front of the crack tip.

The deformation induced by these applied hydraulic stresses requires the coupling of fluid and mechanical principles so that there is mathematical relationship established to a failure criterion. The principal theory in describing rock fracture fluid flow is traditionally the cubic law, which assumes that there are two smooth parallel plates at a specified separation. This assumption drastically affects fluid flow at the outer boundary of the fluid-rock transition because real fracture walls are most often not a smooth surface and contain a given amount of roughness or imperfection across the surface. The roughness along these given boundaries influences the ease of fluid movement through these pathways, altering their induced stresses. In order to account for these variations in fracture properties the problem is typically simplified to incorporate an average roughness value that would yield an equivalent water conducting capacity as the real rock fractures (Zhou, 2011). In more simplified approaches the fracture surface is assumed a perfectly smooth surface in contact with an incompressible fluid and therefore reduces the requirements of the solution drastically (Sudipta Sarkar, 2004).

As one would presume, as the number of hydraulic fractures increases so does the associated complexity of the modified stress fields. The effect of multiple hydraulic fractures can cause an effect known as stress shadowing. The influence of propagating fractures through a media influence each other based upon numerous parameters such as: the geometry of propagating fracture, the speed and direction of propagation, the magnitude and direction of in-situ properties, the distance of fracture spacing as well as the associated fluid and proppant properties used for treatment. This ideally separates the interaction of static and dynamic stress fields, where in addition to the static stress changes throughout an in-situ stress field the associated dynamic or transient stress field modifications also play a vital role in the study of induced seismicity. (Harris, 2000)

2.4. FAULT REACTIVATION DUE TO RESERVOIR PRESSURE FLUCTUATIONS

Seismic activity is controlled primarily by friction between the coinciding fault planes and fractures within the reservoir. Changes in hydrostatic pressure influence the potential for micro-seismic or seismic events to occur due to changes in stress distributions within the reservoir (Segall P, 1998). Whether a fault is tectonically active will affect the ease at which a fault plane will undergo slippage in addition to numerous fluid and geological related properties (Huppert, 2000). The fault is further susceptible to stress fluctuations if the plane lies in a critically stressed state (Gao, 2012). Fault planes which are in a critically stressed state require that only small fluctuations in stresses surrounding these discontinuities may cause slippage, as their equilibrium state is sensitive to such fluctuations (Rudnicki J. W., 2006).

Semi-analytical analysis of fault reactivation tendency done in previous research has typically focused on the use of the Coulomb or Hoek-Brown Failure Criterion as an accepted theoretical means of computation. The stress-strain relationships evaluated using the Coulomb Failure Theory determine if a fault zone would increase or decrease in the potential to undergo slippage based upon the stress magnitude parameter and principle stress orientations. This method is a suggestive means of fault zone classification, but it does not explicitly indicate failure. The classical Hoek-Brown failure theory is based upon brittle failure of intact rock along with model studies of jointed rock mass behavior. A modified criterion was developed in order to distinguish intact rock mass from those that have been disturbed as their associated strengths are modified greatly through the introduction of discontinuity.

Based upon these failure analysis results, the slip potential can be mapped relative to the induced fracture and stress change versus that of the in-situ stress distributions. This method was developed in order to easily apply stress changes to more general scenarios (faults of varying dip located both within and outside of the reservoir) and reservoirs which maintained a close relationship to numerical results.

Coulomb failure theory was used in most research regarding to fault reactivation due to induced stress changes by (Hillis, 2004; Rudnicki J. , 1999; Segall, 1985; Segall P, 1998). The broad application of the Coulomb Failure Criterion allows for application of altering case and study conditions easily while maintaining acceptable tolerances within the accepted spectrum Coulomb Failure envelope. This has been established through experimental analysis of confined rock samples that appear to explain many field observations (Jaeger & Cook).

The typical or classical approach to this failure approach uses the Mohr Circle in order to map the magnitudes of shear and normal stress acting on a given fault plane. This theory was further expanded by Gephart in order to also incorporate shear stress direction perpendicular to the Mohr Circle normal stress axis. The result of this application is a Mohr Sphere which is useful in considering the relation between stress and fault slip data (Allmendinger, Gephart, & Marrett, 1989).

The reservoir pressure and stress interaction determines the relation of potential failure throughout the zones of discontinuity. The previous approach through related reactivation studies did not capture the significant stress interactions that occur by means of hydraulic fracturing as they instead focused primarily on pressure fluctuation alone, regarding to injection or depletion of the reservoir versus in-situ stress magnitude and orientation (Çiftçi, 2012). This requires that a new methodology be developed for cases of induced fracture due to the significant difference in the time and magnitude of stress variation surrounding the fracture. In addition, the interconnection and spacing of fracture through the rock mass adds further complexity to these stress interactions.

2.5. INDUCED SEISMICITY AND EARTHQUAKE MECHANICS

Throughout the United States there has become an increased awareness to the environment and the effects of human related activity. One of the most prominent issues related to this topic is that of global greenhouse gasses. These gasses, such as carbon dioxide have been attributed to various causes and environmental effects throughout the world. These emissions go hand in hand with the modernization of the world and the growing use of fossil fuels in both developing and established areas.

The United States has been a leading contributor to global greenhouse gas emissions and has therefore established numerous regulations to control these emissions in an effort to lead by example. A conservationist type approach is required by contributing industries in order to avoid costly federal fines and sanctions. The energy industry is especially affected by these changes as heavy pressure from outside sources within the government and public sector from special interest groups aim to alleviate the use of deplete able resources altogether. The feasibility of this viewpoint is simply not acceptable as technological and economical limitations cannot provide what is required for these changes. This has created a separation in the energy industry between the use of what is seen as cleaner types of fossil fuels and those that have been the staple of the past, such as coal fired power plants.

One of the largest contributors to the greenhouse gas effects is that of carbon dioxide. In an effort of reducing both global and national emissions, various ways of dealing with this gas have been developed. For example, storage programs have been developed that seek to inject carbon dioxide far beneath the earth's surface by using depleted reservoirs or aquifers. This methodology is quite similar to that of typical natural gas and petroleum exploration and extraction as reservoir and aquifer integrity is essentially to trapping these gasses beneath the surface so that they cannot be released to the atmosphere.

The classical theory of Newton's Third Law in which for every action, there is an equal and opposite reaction can be used to outline the pros and cons of nearly all physical problems. The balance in these cases is made by weighing the pros and cons of these cause and effects in order to classify them as being successful or beneficial. The method of reducing emissions by underground storage techniques offers a beneficial means of their disposal however it does not come without a cost. These storage methods have recently been shown to be linked to an increase in the number and size or magnitude of earthquakes throughout the neighboring area. This was found by studying the cumulative number of earthquakes throughout the United States above a certain threshold magnitude. Using a predefined magnitude of seismic events greater than three, it was found that these events have been increasing at a higher rate after the turn of the century. This appears to show a correlation of these seismic events with the increasing use of these underground storage techniques based upon their location and frequency. A steady earthquake rate that once averaged around 21 events per year has seen a drastic increase in the past few decades to a peak of 188 events in 2011 (Ellsworth, 2013).

(Richard Davies, 2013) Investigated these associations by studying the change in reservoir stress distributions in relation to earthquakes by mining, oil and gas depletion, water injection, reservoir impoundment, waste disposal, academic research of boreholes, solution mining, geothermal operations, and hydraulic fracturing for oil and gas recovery. There are numerous observations available from around the world which has related the production or injection of reservoirs with occurrences of seismic activity.

The case studies identified by Davies covered the use hydraulic fracturing in which a felt seismicity had occurred; that is, seismicity which is capable of being felt at the earth's surface due to the fracturing process. Three of these cases were located in the Eola Field, US; Etsho and Kiwiganaola, Canada; and Lancashire, UK. Of these three cases the largest seismicity generated was in the Horn River Basin, Canada with a felt seismic magnitude of 3.8. His

study outlined the general process to predicting fault reactivation and analyzing the selected data sets. This general process was used as the methods of fault reactivation, outlined below:

1. Fracturing fluid or displaced pore fluid could enter the fault.
2. Fluid pressure pulse could be transmitted to the fault and due to poroelasticity; deformation or 'inflation' of the rock due to injection could increase fluid pressure in the fault or in fractures connected to the fault.

The cases of felt seismicity, such as those exemplified in the case studies above by Davies depict the upper range of induced seismicity magnitudes with relation to induced reservoir stress change. These are a relatively rare occurrence at this magnitude; however the supporting evidence of fault reactivation exists through numerous case studies involving smaller scale seismic anomalies.

Much like the injection of greenhouse gasses for underground storage, other harmful substances are often buried beyond that of the public viewpoint. This includes nuclear waste from radioactive material and more widespread, the landfills that are spread throughout the country. In relation to the petroleum industry, these methods are also often used in the disposal of wastewater that is not suitable for treatment and release. This type of wastewater injection induces similar, often larger and more noticeable stress changes throughout the injection area compared to those incurred by greenhouse gas disposal. Injection-induced earthquakes can have much higher stress differentials prior to failure as the pore pressure changes and lubrication that occurs around the critically stressed faults or discontinuity.

2.6. MODELING RESERVOIR STRESS DISTRIBUTIONS

(Hamidreza Soltanzadeh, 2008) Used the theory of inclusions to present semi-analytical solutions for the modeling of horizontal and dipping reservoirs of finite depth with rectangular or elliptical cross-sections, under plane strain conditions. These criteria were assessed by using dimensionless parameters in order to allow their application to a broad range of reservoir geometries and depths. Much of this work was based upon that done earlier by Rudnicki (1999) whose theory was based upon the theory of inclusions and inhomogeneities which was used to analyze ellipsoidal reservoir(s) in full space to assess rock failure within deep reservoirs.

Research done by (HuiHong et al, 2012) took into consideration the complex and realistic interactions occurring in heterogeneous reservoir modeling including fault zone, high-resolution topography, impounding and drainage of the reservoir. The focus of this study was in the region of the Xinfengjiang Reservoir (China) in relation to the induced earthquakes throughout the region thought to be linked with the reservoir. The developed model incorporates use of the finite element method (FEM) analysis in combination with poroelastic effects, whereas the reservoir is considered a continuous porous media.

In order to obtain realistic and accurate solutions, the boundary conditions of the coupled poroelastic model must be elaborately considered. Models were compared based upon the criteria of altering diffusion coefficients. Pore pressure is initially set as the hydrostatic pressure in a water-saturated state of equilibrium, while lateral and bottom pore pressure boundaries are set by a flowing pressure gradient. Solution involves first the equation of elastic deformation of a solid medium and secondly solved with the seepage diffusion equation until a convergence is met at each time step of the analysis window.

The fundamental principles considered by (Gao, 2012) investigated the stress relationships within the reservoir using finite element method analysis (FEM) to determine a 3D model of distributed stress elements. Failure criteria were consistent with the Anderson theory of faulting which applies the Coulomb Failure Criterion to

study fault reactivation and slippage. Fault activity is reliant upon the in-situ stress distributions at depth below earth's surface and can have the potential to reactivate when hydraulic fracturing alters these initial states of stress throughout the fault plane. The two fundamental principles effecting these redistributions are an increase in minimum stress due to the poroelastic effect and the stress increase due to the opening of the fracture(s).

CHAPTER 3 - FUNDAMENTAL CONCEPTS

3.1. GEOMECHANICS

3.1.1. INTRODUCTION

The principles of geomechanics conceptualizes the behavior of rock and soil masses. The study of geomechanics divides the subject into the two main disciplines of soil and rock mechanics, of which the latter will be discussed with its relation to reservoir fluid flow, deformation and fracture. This field of study encompasses the classification and property evaluation of differing types of rock and their key physical relationships to internal and external forces under applied or in-situ conditions. These properties are typically obtained or derived from laboratory tests of available field samples in order to establish a correlation to compressive strength, tensile strength, shear strength, bulk modulus, modulus of elasticity and other important parameters for any given state which may be of interest.

Understanding the fundamentals relating to rock mechanics allows for the hydraulic fracturing and related processes to be studied with application in mind, as more practical parameters and scenarios can be developed or referenced. The foregoing principles outline the general concept and theory that is involved in determining the expected response of subsurface materials. This will help provide an insight with engineering application and design in mind, much as it has in the Civil and Mining engineering fields.

The introduction of any alterations in or around a rock masses physical environment provide for a great variability in its associated properties. These variability's are one of the key concerns of geological engineers that are particularly interested in understanding subsurface physical and chemical interactions. The askew nature associated with these heterogeneous properties however causes great difficulty in explaining the exact relationships that exist throughout these sub-surface systems by simple means. This is an important aspect of rock mechanics in which its state and condition have the ability to drastically affect the associated parameters between two nearly similar samples of the same rock mass. This again raises another key distinction in the differences of solid rock mass versus that of disturbed rock mass and property. A solid rock mass is considered homogenous with uniform, isotropic properties while a disturbed rock mass may have variation or discontinuity of these properties.

Faults, fracture and other discontinuities often form from the introduction of stress changes which alter the in-situ stress distributions around the reservoir and surrounding rock matrix. These stress changes can be induced by natural or man-made means; such as tectonic plate movement or hydraulic fracturing, respectively. The response of the rock matrix relies heavily on the ratios of in-situ stress to induced or applied stresses. This is otherwise known as a stress differential. These stress differentials control the overall response of a material based upon their inherent properties and related behavior. The magnitude of the stress differential is a key parameter in regards to the study of deformation and failure, or more specifically for this research that of fault reactivation as slight fluctuations to the surrounding stress fields may alter stability and thus create a potential for reactivation. In other cases it may take a much larger stress magnitude to reactivate these same areas, however this depends upon their relative state of stress and surrounding environmental conditions.

The strength of a given rock matrix is indicative to its composition (or mineralogy) and surrounding boundary conditions which control both the stress responses and solid-fluid interactions. If discontinuities are ignored throughout an area of study then the expected responses are thought to be uniform throughout that zone. In areas which contain faults or discontinuities however the physical response of the rock matrix is altered by these discontinuous material interfaces. This allows for stress concentrations to change variably along these planes of altered contact causing them to react unlike they would in a uniform homogenous case. The variability and

distribution of stress along these zones is an indicator of where and when slippage is most likely to occur along these discontinuities.

The in-situ rock properties are of great importance to the stick-slip phenomenon which is commonly used to describe the frictional sliding and failure along fault planes. This stick-slip scenario is caused by the variability that exists along the surface contact of the discontinuity due to related material properties, such as the surface roughness and fluid interactions within these fluctuating stress fields. Accurate prediction of fault reactivation is often an incredibly difficult task as it requires sophisticated methods such as 3-D finite element modeling along with a detailed representation of sub-surface systems.

The inclusion of fracture or discontinuity whether it be natural or induced creates a secondary fracture permeability that depends upon the understanding of mechanically driven fracture and pore pressure interactions throughout the geological area of study. This is a complicated process that requires extensive geologic knowledge and data to support the assumptions being made. There is no guaranteed way to represent these interactions directly and therefore certain assumptions must be made in order to simplify the process. These fracture interactions occur at random within a specified parameter window throughout a formation, therefore modeling is typically represented as such, random assignments.

The induced fractures produced throughout a rock mass lend opportunity to stimulate production of hydrocarbon deposits in porous media. This includes both naturally occurring fractures as well as those induced through the hydraulic fracturing process. These fracture networks are explicitly required for reservoirs having extremely low permeability, which are traditionally termed as being unconventional. Unconventional reservoir characterization is typically inclusive of shale, coal bed and tight sands that have permeability's in the nano-scale range. In terms of shale gas reservoirs, such as those in the Appalachian region, this applies to the terminology of porosity; as permeability is affected by the size of pores within the shale rock matrix.

These principles will further the understanding of associated reservoir interactions as well as the methodology supporting many of these key aspects. Due to the complicated processes involved with geological formations, the inclusion of this material is intended to ease both their introduction and support their formulation.

FIGURE 1 EXTENT OF THE DEVONIAN MARCELLUS PLAY FROM (GREGORY R. WRIGHTSTONE, 2011)

3.1.2. GEOLOGICAL INFLUENCES

The Appalachian Basin is comprised of an area generally described as lying within eight states, totaling an aerial extent of roughly 54,000 square miles. Hydrocarbon deposits throughout the Appalachian region are comprised of varying organic rich formations in the form of shale and coal bed deposits lying at various depths throughout this region. A primary target and one of the most well-known formations within this region for natural gas exploration is the Devonian Marcellus which lies beneath the Allegheny Plateau Region of North America; stretching primarily through Ohio, West Virginia, Pennsylvania and New York with smaller outcrops located through Kentucky, Maryland and Virginia which can be seen from **Figure 1** above.

With the advancement of drilling and stimulation techniques, these areas have now recently allowed for profitable development of these so called “unconventional” deposits. This has created a large influx of drilling and production operations throughout the North East region of the United States after initial exploration of these formations in the early 2000s displayed both intriguing and profitable results.

The past development and knowledge of this region is generally in reference to the well-known mining operations that have occurred throughout the organic rich coal beds that lie in this area. The continuing development of the mining industry has been a great influence to the economy, industry and residents associated with these areas. The types of deposits used for mining operations and coal bed methane extraction generally lie above the interval of the shale deposits mentioned throughout this research as they are typically within a 2,000 foot interval of the surface within the Appalachian region. The access to deeper organic type formations was, and has not been fully explored for mining operations mainly due to technological restraints. Additionally, the availability of vast coal resources through the more proven bed deposits in the Dunkard, Monongahela, Conemaugh, Allegheny and Pottsville Coal Groups did not require the progression to more formidable zones.

The increase of federal regulation surrounding coal extraction and its uses in the past few decades to present time has created an obstacle for energy production, which once relied so heavily on coal. The interest in these other types of organic formations grew throughout the turn of the century when some of these energy companies began to expand their exploration of these new low-permeability shale formations to the Appalachian Basin due to their success in other more well-known shale basins throughout the country, such as the Eagle Ford, Haynesville, Barnett and Bakken. Previous to natural gas exploration and production in this region, the presence of gaseous hydrocarbons throughout coal seams was found and seen as a nuisance which posed a severe safety hazard to mining operations. These gases are highly explosive and therefore it is advantageous to remove them prior to mining operations in order to reduce the risk of causing an inadvertent explosion within the mine.

The Marcellus is located in the Middle Devonian and is a subset of the Hamilton Group as can be seen in **Figure 2**. This figure illustrates the stratigraphy throughout the Appalachian region which has been categorized by its location and geologic period throughout and surrounding Pennsylvania, which lies at the center of the Marcellus shale play. The Middle Devonian epoch is as the name states a subsidiary classification to the Devonian period, referring to its (the formations) specific age group in geological time. The Devonian period occurred roughly between 420 – 360 million years ago. Of this there are three subsidiary groups which include the early Devonian that lasted 420 to 395 million years ago, the Middle Devonian which lasted from 395 to 387 million years ago and the Late Devonian period which occurred approximately 387 to 360 million years ago.

During this period the development of the Acadian mountains for what now is known as Eastern North America provided the depositional environment of the Appalachian Basin. A large amount of sediments were produced by the erosion of the Acadian mountains, accumulating into this basin. This depositional period occurred prior to and during the development of the continents as they are seen today.

FIGURE 2 STRATIGRAPHY OF THE MIDDLE DEVONIAN IN THE APPALACHIAN REGION FROM (KRISTIN M. CARTER, 2011)

The Devonian Marcellus shale, like all shale, is a sedimentary material that is rich in organic content. These types of materials are formed over millions of years with the layering, compaction and consolidation of deposited materials and marine organics. Stratification is an easily visible trait of these shale formations which is developed subsequently through time with the addition of temperature and pressure, creating a fine grained laminated structure.

The generation of hydrocarbons is later caused due to the thermal transformation of kerogen, a naturally occurring solid organic material. This chemical reaction is influenced by the temperature, pressure, organics and length of time that these interactions are allowed to occur, as the kerogen content evolves from reactive carbon to dead carbon. This progressive process of hydrocarbon transformation occurs through a series of three stages known as diagenesis, catagenesis and metagenesis as hydrocarbons progress from their immature state, to oil and then from oil to gas.

The surrounding bedding planes to the source formation encapsulate and direct movement of these developed hydrocarbons to a point or a zone of accumulation, just as with conventional reservoirs. The entrainment of these hydrocarbons throughout the rock matrix is controlled by the pore space movement while accumulation is due to the surrounding interconnections of the formation that is available by the natural fracture network. The terminology of shale gas is in reference to shale formations which have not expelled hydrocarbons entrained or adsorbed tightly within their matrix. This is what differentiates shale from other types of hydrocarbon deposits, as shale serves as both the source rock and the reservoir for developed hydrocarbons. Understanding the specific reservoir rock properties and their associated characteristics is crucial in developing and progressing this type of research.

The study and description of sedimentary rocks is typically defined by four key components that are outlined by **Figure 3**. These parameters are extremely useful in the explanation of many important characteristics required for reservoir exploration and production. Although these parameters do not specifically address the mechanical behavior of a rock sample, they aid in forecasting or explaining these mechanical behaviors by examining its structuring.

These fundamental structural components as seen in **Figure 3** include:

1. **Porosity** the space with no mineral matter, commonly filled with water or other fluids.
2. **Grains** the sand grains and fossils, loose structure of the matrix.
3. **Cement** the bonding agents of grains and matrix, chemically precipitated minerals.
4. **Matrix** the fine grained clay sized sediment throughout the granular structure.

FIGURE 3 FUNDAMENTAL COMPONENTS OF SEDIMENTARY ROCKS FROM (RAILSBACK, 2002)

The accumulation of hydrocarbons occurs through a process described by permeability which is controlled by the formation(s) inherent properties and pressure gradient. The inter-granular pore space of a given rock matrix controls to what degree and speed at which fluid movement or migration may occur without regard to discontinuity. In unconventional reservoirs it is often necessary to make a distinction of the fluid movement between these two zones and in some cases take even greater distinction is required. The primary permeability of a formation describes the homogenous rock matrix response with minimal to zero discontinuities included. In areas which have undergone higher degrees or longer periods of stress differential, discontinuities develop through the natural process of plate tectonics which is typically the dominant sub-surface force.

These natural forces lead to the development of what is termed the secondary permeability that is much higher than that of the primary permeability, as discontinuities develop through these naturally occurring stress variations. This secondary permeability allows for fluid migration to occur at a much higher rate than possible through the intact (shale) formation. Secondary permeability is controlled primarily by the available interconnections throughout a formation caused by fault, fracture or other discontinuities. Therefore, pay zone accumulation and integrity is primarily affected by secondary permeability on a large scale basis, while production relies more heavily on primary permeability depending on the extent of the natural or induced discontinuities and fracture network.

Throughout the research of shale reservoirs it has also been useful to use even further distinction between the primary and secondary properties by the application of the micro and macro scales, as well as a differentiation between the organic and inorganic content distribution within the sample. In specific applications the use of a triple permeability, quad porosity model has been determined to best represent these associated characteristics and responses.

Porosity and Permeability go hand in hand with reservoir production and profitability which is especially true in unconventional plays such as the Marcellus Shale. Shale formations, until recently were a particularly difficult formation to produce as they require stimulation or treatment in order to be productive. Unlike the permeability of traditional reservoir formations such as sandstone or carbonate, a shale formation has an extremely low matrix permeability which lies in the nano-darcy range. In addition, this relative permeability is further influenced by the in-situ stresses at confined conditions due to the differences in the mechanical properties and behavior of shale and coal beds versus that of more conventional reservoirs. This relationship creates a high degree of nonlinearity between the two types of permeability, making it incredibly important to correlate these parameters to their downhole conditions.

The porosity of the formation is defined as the available inter-granular or pore space volume within the rock matrix that allows for the measurement of possible fluid storage. This property is typically referenced as a ratio (i.e. percentage) of open space volume to solid rock volume. The permeability of the rock sample is characteristic to fluid flow and movement throughout the sample and often yields a reliable insight to production expectations. In high-porosity, high-permeability formations these properties are often assigned as a constant value, however this is not a valid assumption for unconventional reservoirs where fluid flow through the matrix is affected on a molecular scale (Soeder, 1988). In these cases it is often necessary to pay special care to these properties due to not only their dynamic nature but their wide range of value.

These respective properties play a vital role in the movement of entrained fluid throughout the available pore space volume. These terms are most often associated with the reservoir characterization and provide insight to reservoir production and profitability in a quick and understandable manner. **Table 3** below illustrates the permeability and pore size ranges for reservoir characterization.

Formation Type	Pore Size	Permeability
Conventional Oil & Gas	$\geq 1 \mu\text{m}$	$\geq 1 \text{ mD}$
Shale Gas	$10^{-1} - 10^{-2} \mu\text{m}$	$1 \text{ mD} - 1 \mu\text{D}$
Tight Gas	$1 - 10^{-3} \mu\text{m}$	$1 \mu\text{D} - 10^{-3} \mu\text{D}$

TABLE 3 TYPICAL POROSITY AND PERMEABILITY RANGES BY RESERVOIR TYPE

In unconventional reservoirs the understanding of these principles on a macro scale requires that their micro scale influence be recognized specifically. The nano-darcy permeability's of these formations can be largely influenced with changes to distributed stress regimes and the expansion or contraction of pore space and available interconnections between these spaces.

Figure 4 identifies a general micro and macro scale characterization of the available pore space volume. On the left, a micro scale look of the rock matrix identifies a homogenous state with little dependency while on the right is a macro scale look of the rock matrix yielding insight to the more complicated interactions of natural faults, fracture and available pore space volume. This micro and macro scale structure is pertinent to fluid flow throughout the structure in addition to influencing the expansion or contraction of these void areas with the application of stresses.

In deformable porous media the structure of this matrix plays an integral role in the deformation of the matrix. These void spaces and discontinuity reduce the homogenous rock responses and has the possibility to enhance the elastic or brittle behavior, depending on its (the deformable media's) particular structure.

FIGURE 4 MICRO AND MACRO SCALE PORE VOLUME CHARACTERIZATION

The relation of these properties with respect to reservoir fluid flow can be easily inferred from the simplest form of Darcy's law, explaining the interactions related to reservoir fluid production for a steady-state radial reservoir. This equation highlights the particular importance of permeability and skin factor to stimulation engineers for production purposes. This establishes the importance of in-situ rock properties and the variance of them throughout the formation.

$$q = \frac{kh(p_e - p_{wf})}{\alpha_r B \mu \left(\ln \frac{r_e}{r_w} + s \right)}$$

Where q is the production rate, k is the permeability, h is the formation height, B is the formation volume factor, μ is the viscosity, α_r is the , p_e is the original reservoir pressure, p_{wf} is the flowing BHP, r_e is the external drainage radius, r_w is the wellbore radius and s is the skin factor.

In tight formations which are of focus throughout this research, the typical approach to flow within the reservoir is through that of a dimensionless time relationship. The assumption of an infinitely acting reservoir as shown through the equation above is usually not feasible, as it would take an undesirable amount of time to reach a semi-log straight line relationship. The effects of permeability throughout these formations are an effect of both primary and secondary permeability contributions which is the introduction that both the matrix and fracture permeability must be accounted in order to produce a reliable reservoir representation. That is due to the magnitude of difference in fluid migration through the rock matrix versus that of the natural and induced fracture network.

3.1.3. STRESS EFFECTS AND RHEOLOGY

The effects of applied stresses upon a rock sample are reliant upon its properties and composition or distribution of them. The strength of the individual grains comprising a sample and their bonding typically defined by cementation present the characteristics which resist applied stresses to the sample. These properties are further influenced by the porosity of the sample along with the entrained fluid within these void spaces.

The in-situ stress distributions throughout a reservoir area directly affect the techniques involved with a successful stimulation and treatment plan. The orientation of the natural state of stress directs induced fractures in the path

of least resistance. The path of least resistance can ideally be described as the least resistive path between two points through a given formation. In most cases this pathway lies perpendicular with that of the least principle stress axis. Since any point within a reservoir can be described by the relative stress and strain acting upon it and its neighboring points the fracture path can be predicted with relative ease. These in-situ forces remain in equilibrium unless forces are to act upon them, causing a stress differential between neighboring points. If the differential is great enough, then deformation occurs, eventually leading to failure. The state of stress in a given rock mass is dependent upon its properties, geologic history and surrounding boundary or confining conditions.

The geologic stress forces acting upon subsurface formations due to the overlying formations (overburden/confining pressure) are typically depicted through a positive convention. This stress within the crust has been found to be directly dependent upon its relative depth, varying with sub-surface distance. This has been found to be a relatively constant value throughout the world. The lithostatic stress acting upon a formation is a constant static force distributed upon the reservoir matrix. The overburden pressure is typically calculated as uniform hydrostatic stress acting from the uppermost reservoir or formation boundary, continuing in a linearly increasing fashion that is based on the applicable pressure gradient. This is associated by a linear pore pressure gradient for in-situ stress approximation. In a state of hydrostatic stress, the three principle stresses are assumed in or near an equilibrium state, where the geological area is under a constant force from surrounding media. Variations in the principal stresses are generally seen over long periods of geologic time due to the mobility of the earth's crustal forces influencing the respective force(s) and orientation(s).

It has been found that typically beyond depths of 2,000 feet that the controlling maximum principle stress becomes that of the vertical stress component. This is an indication that in most general cases for hydraulic fracturing that the minimum principle stress will be one of the horizontal stress components, as formations such as the Marcellus typically lies below this depth in reference to instances of where fracturing is used. The Marcellus shale in drilling exploration and production typically falls within the range of 2,000 to 10,000 feet below the earth's surface, supporting this assumption. It has been found from locations throughout the world that the vertical stress component through most of the crustal region increases in a linear fashion prior to the brittle-ductile transition zone regardless of geology. This was determined from numerous mining and civil engineering experiments which found this correlation shown in the figure below.

Geologic history or naturally induced stresses may be idealized as creep and it's relating effects. This "creeping" motion references the slow periodic forces which occur naturally through the geological and tectonic cycles. These forces are responsible for natural seismic events or anomalies which occur throughout the world. The motion of creep however, varies greatly from the stress-strain relationship experienced in response to cases of hydraulic fracturing. The process of creep occurs through an extended period of time, this may be thousands and more often millions of years of stress deformation. The differential stress caused throughout this time is allowed to build to a point which meets or exceeds the formations resistive forces, at which point permanent deformation, failure or slippage occurs. These differential stress variations are controlled by the inherent rock matrix properties.

The process of naturally occurring failure and slippage is typically described by a cyclical stick-slip phenomenon in which a larger stress differential may form relative to the other points along the failure plane. The moment that slippage starts to occur, the built up energy begins to diminish until the resistive forces of the failure plane are again capable of stopping movement. The balance of these forces has been extensively studied in reference to natural earthquake and tectonic events; however the extent of their interactions throughout complex case studies is far from being well understood, leaving much open to interpretation.

FIGURE 5 VERTICAL STRESS APPROXIMATION (AFTER HOEK & BROWN)

Differential stress plays a vital role in the deformation of any media. The magnitude of stress differential in combination with the inherent properties of the material will produce varying response effects. In geologic formations such as shale, the stress response is typically susceptible to larger scaled deformations versus those of other rock formations. This deformation can be broken into two categories, set by the elastic limit of the material and the magnitude and/or rate of stress being applied.

The differential stress applied to a media and surroundings will alter the in-situ stress fields. This stress is therefore the principal determination in not only post-primary and secondary stress fields, but also the prediction of fault reactivation cases. The magnitude of differential stress will directly influence stability predictions in addition to the ratio of in-situ stresses to the formations resistive stress capacity. In cases of high in-situ stresses, the ratio of resistive stress becomes particularly important as they may place the area in a critical state of stress. The critical stress state is identified by a failure criterion such as the Mohr–Coulomb, Tresca or Von-Mises.

FIGURE 6 PRINCIPLE STRESS DEFORMATION AND ORIENTATION

The strain experienced by these stress variations can be described in terms of tension, compression or shearing effects as shown in **Figure 6**. This is typical of underground stress conditions, where geological materials have been repeatedly deformed to their failure limit by folding and faulting over time through large variations in temperature and confining conditions. The characterization of these forces is relative to the sample volume or area in question, as material properties can differentiate these stress-strain effects. In mathematical explanation, the force(s) surrounding any point within a media can be identified by three principal stresses: the minimum, intermediate, and maximum stress tensor. The principal stresses act normal to the respective planes of a sample cube, parallel to its central axis. These principal stresses are translated from the stress contributions of all neighboring points and forces.

The amount of strain experienced correlates to the stress-strain relationship of that specific rock matrix at any given point. The complicated nature of these interactions however, introduces a new stress-strain curve each time the material experiences a cycle of loading/unloading. This occurs as micro-fractures develop throughout the matrix each time stresses are applied, slightly altering the response curve.

Geological rheology becomes an important discussion in terms of this research, describing the reaction of a rock matrix to these outside (external) forces. The strength of a rock describes its ability to resist outside forces (stress) prior to failure or breaking. The stress applied to the rock matrix causes strain to develop, again the particular reaction is dependent upon the rheology of the material properties. In the case of Devonian Marcellus shale these properties become increasingly difficult to understand due to the inherent nature of shale materials.

The capacity of a given rock specimen to resist deformation is measured by Young's Modulus. This "stiffness" factor plays an essential role in the hydraulic fracturing process, directly affecting the fracture geometry as it governs the width that a fracture will open, when subjected to a given pressure. The relationship of horizontal stress governing the likely fracture propagation path is influenced by related Poisson effects. These horizontal stresses are affected by the confining pressures as the rock matrix is compressed vertically, it will expand laterally. The deformed geometry of the material correlates to Poisson's ratio. The relations between moduli of the linear elastic law may be seen below.

$$G = \frac{E}{2(1 + \nu)}$$

$$K = \frac{E}{3(1 - 2\nu)}$$

Where E is the Elastic Modulus (a.k.a. Young's Modulus), ν is Poisson's Ratio and G is the Shear Modulus (a.k.a. Modulus of Rigidity).

Due to the stratification of organic materials through shale formations; additional parameters are required to explain subsurface stress-strain relationships versus that of hard-brittle formations such as granite which may undergo a larger stress magnitude with little deformation prior to failure. The compressibility of a shale formation however, causes them to experience a much higher elastic limit which allows for a greater probability of deformity through the bedding planes. In addition, the stress variation seen throughout their respective bedding planes is typically on a higher scale than the surrounding formations. The effects of compressibility and fracture toughness around the propagating fracture will be discussed in further detail in subsequent chapters, explaining the initiation and propagation of rock fracture through the matrix.

The response of any given rock matrix can be generally categorized by its deformation modulus. This is often not a well-known or easily measurable parameter, but it is required for many types of numerical analysis. The deformation modulus (Young's Modulus) defines the elastic, brittle or ductile behavior of a given material matrix based upon applied stresses and strains. Under any loading, the material matrix, in this case a given rock volume begins to deform at the microscopic level which is defined by the mineralogy and natural fractures present in the given sample. Due to the interactions of micro-fractures during the initial stages of loading, the modulus value increases slightly with the increase in applied pressure.

FIGURE 7 ROCK MASS DEFORMATION MODULUS

FIGURE 8 SIMPLISTIC STRESS-STRAIN RELATIONSHIP

The early period of deformation experienced here is not defined by Young's Modulus and is instead classified as the crack closure deformation. This identifies the early period of axial deformation as the natural cracks and discontinuity throughout the formation begin to close throughout the matrix. As the applied axial stress increases, the deformation transitions to a linear relationship of stress which is identified by Young's Modulus. As compaction occurs the linear stress-strain relationship continues until reaching a peak of elastic behavior. This is not the peak of the stress-strain relationship but the peak of linear behavior as the onset of inelastic behavior occurs. At this point the material will display a higher degree of deformation for each successive unit of applied stress. At this point, if loading is continued then it will eventually lead to brittle failure. This process can be seen from **Figure 7**. These detailed interactions are often summarized by a simplified stress-strain relationship such as that in **Figure 8** in which a defined material elastic limit is established. Prior to this limit, elastic deformation is controlled by the linear relationship of Young's modulus and therefore ignoring the initial crack closure deformation differences.

The later stages of deformation by either incremental or sudden loading can be further explained by the toughness of the material. This parameter is required to determine failure. The common geophysical properties for various types of rock are highlighted in **Table 4**. The large range of values clearly indicates the need for testing of site specific samples due to the large variation that is seen throughout this data. These values depend upon the locations, depth and quality of the rock sample in addition to the porosity and entrained fluids.

Rock Classification	Poisson's Ratio	Density (g/cm ³)	Young's Modulus (GPa)	Bulk Modulus (GPa)
Shale	0.20 – 0.40	2.00 – 2.67	1 – 70	5 – 25
Granite	0.15 – 0.24	2.53 – 2.62	2 – 50	25 – 42
Sandstone	0.17 – 0.40	1.91 – 2.58	10 – 20	1 – 30
Marble	0.25 – 0.38	2.51 – 2.86	2 – 20	40 – 44
Limestone	0.16 – 0.31	2.44, 2.67 – 2.72	3-27, 70	23 – 54
Dolomite	0.28 – 0.30	2.876	7 – 15	34 – 62

TABLE 4 COMMON GEOPHYSICAL ROCK PROPERTIES

In-Situ stress distributions are determined through a combination of the afflicting stresses mentioned throughout this chapter. The vertical stress value is typically available through density log integration, however if this is not an option, it is typically assumed that the vertical gradient is 1.0 to 1.1 psi/ft as a general approximation. Typically

this causes vertical stress to be the controlling maximum stress at depth as it is inferred from the relationship described below.

$$\sigma_v = \int_0^H \rho(h)g dh$$

Where σ_v is the resulting vertical stress due to overlying rock layers at a depth, H , and ρ is the rock mass density and g is a gravitational constant.

Vertical stress measurements are typical of civil and mining applications where they have been accumulated from throughout the world. Brown and Hoek (1978) compiled this data and determined a relatively linear relationship, regardless of depth. Therefore, in most applications the equation above is typically referenced as a simple estimation, from its form below.

$$\sigma_v = \gamma z$$

Where σ_v is the resulting vertical stress due to overlying rock layers at a depth, z and γ , the unit weight of the overlying rock.

The typical minimum horizontal stress component for transverse isotropic shale plays is calculated as follows:

$$\sigma_{hmin} = \left(\frac{E_h}{E_v} \right) \left(\frac{V_v}{1 - V_h} \right) (\sigma_v - \alpha P_p) + \alpha P_p + \left(\frac{E_h}{1 - V_h^2} \right) \varepsilon_{hmin} + \left(\frac{E_h V_h}{1 - V_h^2} \right) \varepsilon_{hmax}$$

Where σ_{hmin} is the minimum horizontal stress, E_h and E_v are the isotropic elasticity properties, ε_{hmin} and ε_{hmax} are the respective horizontal tectonic strains.

A specific focus of reservoir rock mechanics involves the pore pressure and effective stress throughout the rock matrix in order to determine the total in-situ stress magnitude. When subjected to applied forces the total stress across the rock matrix will be reduced by the entrainment of fluids in the open pore space volumes. These fluids throughout the pore spacing aid in supporting the total applied stress to the formation. This affects the response of reservoir rock dynamically with changes to fluid characteristics and pore-pressure diffusion. This introduces an important material state in rock mechanics, differentiating between drained and un-drained properties.

In the case of a porous elastic media (i.e. reservoir rock matrix) these states reflect the stress-strain behavior of the material significantly. In un-drained porous media the reaction will be in a stiff manner versus those in drained cases it will react softer, as the un-drained cases assume an instantaneous pressure response. This was addressed by Terzaghi in the introduction of the effective stress concept which offset the total stress directly by the pore pressure. This was later modified by Biot which introduced a scaling factor to the pore pressure coefficient. This reduced the contribution of pore pressure counteracting the applied load due to the effects of cementation between the neighboring matrix grains. This correction factor is applied as an efficiency of the granular cementation varying between 0 and 1.

$$\sigma' = \sigma - \alpha p$$

Where σ' the resulting effective stress of the total stress is applied and adjusted pore pressure coefficient. The poroelastic constant α varies between 0 and 1 reducing overall pore pressure contribution. This relationship governs deformation of porous media.

The determination of horizontal stress values requires a slightly different approach as it is influenced by numerous geologic factors. This determination of horizontal stress orientation is the single most important determination or approximation with respect to hydraulic fracturing. The minimum and maximum horizontal stress will control fracture propagation and therefore must be determined effectively. This is typically achieved by one of two categories of applicable methods: direct, where horizontal stress is measured directly and indirect. Normally, the average vertical to horizontal effective stress relationship is determined through the use of Poisson's ratio (ν).

$$\sigma'_h = k\sigma'_v = k\gamma z$$

Where σ'_h is the resulting effective horizontal stress based upon the predetermined stress ratio, k .

$$k = \frac{\nu}{(1 - \nu)}$$

Where ν is Poisson's ratio and k is the stress ratio.

Just as stress has been shown to cause deformity throughout a rock mass, additional effects also contribute to the fundamental geomechanical behavior. This includes the effects of stress shadowing in which patterns of altered stress can influence the successive stress developments throughout the field. This can potentially affect both successive fracturing stages, natural fractures and fault kinematics throughout the affected zone.

In three-dimensional analysis the representation of stresses may be readily accomplished through a matrix notation. In this case, there are 9 components comprising stress in three dimensions throughout the x, y and z axis. This can be inferred by the common 3x3 stress matrix shown below.

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

It can be seen from this notation that the matrix diagonally dominated by normal stresses. This introduces the theory of shear stress reciprocity, describing the symmetry of the three-dimensional stresses. This relationship minimizes the stress matrix to six independent components.

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} \rightarrow \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ & \sigma_{yy} & \tau_{yz} \\ \text{symm} & & \sigma_{zz} \end{bmatrix}$$

Where $\tau_{yx} = \tau_{xy}$, $\tau_{zx} = \tau_{xz}$, and $\tau_{zy} = \tau_{yz}$. The conventional notation for stress is followed here in which for τ_{xy} , stress is acting on the x-plane in the y-direction.

The transformation of the full stress tensor shown above in its distinct coordinate system can be transferred and analyzed in any other coordinate system through stress tensor transformation. When oriented in principal stress the shear stresses vanish and the x, y and z axis become aligned with the first, second and third principle stresses. These stresses are defined by the following criteria $\sigma_1 > \sigma_2 > \sigma_3$.

$$\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

Where σ_1 is the first or maximum principle stress, σ_2 is the second or intermediate principle stress and σ_3 is the third or minimum principle stress.

The common geomechanical properties relating to stress and strain, including failure relationships are outlined in **Table 5**. These properties are summarized from various scientific applications of rock mechanics from differing fields. It is apparent from these values that there is a large fluctuation in the respective properties. This can be attributed to the defects and imperfections which occur throughout natural samples. This can add another dimension of uncertainty which can severely impact the matching numerical models and simulation to real world events.

Rock Classification	UC Strength	Tensile Strength	Elastic Modulus	Poisson's Ratio	Strain at Failure	Fracture Mode I Toughness
Granite	100 – 300	7 – 25	30 – 70	0.17	0.25	0.11 – 0.41
Basalt	100 – 350	10 – 30	40 – 80	0.1 – 0.2	0.35	> 0.41
Quartzite	150 – 300	5 – 20	50 – 90	0.17	0.20	> 0.41
Marble	50 – 200	7 – 20	30 – 70	0.15 – 0.30	0.40	0.11 – 0.41
Sandstone	20 – 170	4 – 25	15 – 50	0.14	0.20	0.027 – 0.041
Limestone	30 – 250	6 – 25	20 – 70	0.30		0.027 – 0.041
Shale	5 – 100	2 – 10	5 – 30	0.10		0.027 – 0.041

TABLE 5 COMMON GEOMECHANICAL ROCK PROPERTIES

In a two dimensional system the principal stresses can be recovered using the associated stresses from the stress matrix of σ_x , σ_y and σ_{xy} . The maximum principal stress is then defined as follows:

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2} \right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

The minimum principal stress is calculated as follows:

$$\sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2} \right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

The subsurface stress throughout regional geological strata generally follows a similar pattern or trend which can be used to estimate incurred stresses without the need of drilling exploratory wells. This regional data has been compiled from borehole breakouts, induced fracture networks, geological indicators and other similar methods by the World Stress Map Project. This compiled data is shown in **Figure 9** which encompasses the Appalachian Region of North America where the Marcellus Shale is located. It can be inferred from this figure that the primary horizontal stress runs in a North-Eastern like pattern in a parallel trend with the Appalachian Mountain topography.

FIGURE 9 STRESS MAP FOR APPALACHIAN REGION (HEIDBACH, 2008) FROM THE WORLD STRESS MAP PROJECT

3.1.4. JOINTS, FAULTS AND ROCK DISCONTINUITY

A fault is a rock or formation fracture in which two sides displace relative to each other and their respective failure plane. Natural faults and fractures are controlled primarily by the orientation of surrounding stress field and the resistive capacity of the lithology or formation in question. In addition, other complex factors such as the cementing of inhomogeneous material that precipitates through the aperture of the fault plane and the surface roughness of the adjacent fault face. These parameters are often unknown which increases the difficulty in understanding the complex nature of fault mechanics, although generalized theories have proved to approximate these interactions.

Faults and fracture give opportunity for seismic events to occur, providing areas of discontinuity where stress variations may build along the plane of the fault. Seismic events generated in this fashion typically operate in a cyclical stick-slip mode of failure. In this type of failure, seismicity is created during periods of slip, while natural resistive forces cause stick or periods of no movement. An estimate of generated seismicity lies with the increase in lithostatic stresses with depth, therefore as depth increases it can be said that seismic potential is also expected to increase due to the relative stress magnitudes which may be generated over periods of no movement. In deeper faults the possibility for sudden large seismic events is increased as the energy released is over a short period of time prior to becoming dormant. Therefore it can be predicted that with an increase in depth the duration of slippage decreases but the probability of higher magnitude seismic events increases.

The generalized theory of fault kinematics is often explained by Anderson's Criteria of Faulting which encompasses all modes of faulting by using a simplified classification method. Anderson's Theory follows the principle that faulting is a mode of brittle fracture in accordance with the Coulomb criterion. This states that failure or fracture will occur in a planar fashion in which the fault plane slips with a uniform friction release across the entire contacting surface. This categorizes the available faulting regimes to the following three cases of Normal, Strike-Slip and Reverse type faulting that is based upon the magnitude and orientation of the applied stress distributions surrounding the faulting environment, this can be seen from **Figure 10** below.

FIGURE 10 ANDERSON'S THEORY OF FAULTING

Table 6 below highlights the interchangeability of principal stress between the horizontal and vertical stresses.

Tectonic Regime	σ_1	σ_2	σ_3
Normal Faulting	S_v	S_{Hmax}	S_{Hmin}
Reverse Faulting	S_{Hmax}	S_{Hmin}	S_v
Strike-Slip Faulting	S_{Hmax}	S_v	S_{Hmin}

TABLE 6 TECTONIC REGIMES BY PRINCIPAL STRESS RELATIONSHIP

It is particularly important to mention that in regards to this research, the faulting regimes presented by Anderson only apply to the stress state at the time that the fault is formed. This is relatively similar to the presentation of the in-situ stress fields in which the current state of stress does not dictate which faults are present, only which

faulting regime may be dominant at any given time. Furthermore faults are found to rotate away from the principle stress direction σ_1 as they accrue slippage (Scholz).

FIGURE 11 FAULT TYPE CLASSIFICATION

The determination of a specific faulting regime is therefore dependent upon the orientation at which the principal stress magnitudes lie. The given horizontal stress distributions for a specific depth allow a prediction of the faulting regime that is most likely to be present as displayed in **Figure 11**. The dynamic nature of these subsurface stresses however increases the difficulty of predicting these regimes in many instances. This is because the horizontal stress will shift throughout the field depending upon stress conditions and surrounding discontinuities. That is, that the faulting regime may change relative to the static stress distribution at any specified time, so that there may be multiple types of faults that have formed through time but only one type of regime will be active at any given time. This can introduce an endless array of complexity as the controlling fault type may be further influenced by the proximity of these other fault planes or discontinuities and their associated state(s) of stress. This then presents one of the general ideas that the principle stress surrounding a particular fault regime is relative to the time in which these stresses were applied.

Using this methodology described above, it can then be expanded for analysis purposes by limiting the complexity of then these dynamic fault regimes. Reducing the complexity of the problem can then be achieved by controlling the application of the principle stress distributions surrounding these single fault regimes and addressing them on a case by case basis.

Figure 11 describes the static stress relationship in a horizontal stress regime that changes relative to depth. This can lead insight to the current active faults for that particular stress pattern, however it does not indicate whether previous stress patterns have caused other types of faulting in the past, nor whether they are currently present in the formation.

This generalization provided by Andersonian theory is typically used to categorize types of faulting where more complex cases would need greater attention to detail and further investigation to properly classify. These types of faulting environments may transition through multiple types of lithology and geology where in-situ rock properties

and stress vary greatly between rock layer types. These specific types of interaction highlight the deformation variance through changes in lithology, such as brittle and ductile rock layers and the shifting of principle stress through geologic time. Ductile rock layers are more prone to high stress variance as their elastic limit provides some flexibility that is necessary to deform without full failure, while brittle rock layers will shear creating a fault plane, allowing these areas to shift relative to each other. This is an approximation that is not fully accounted through Anderson's Theory as the discontinuity does not typically follow the assumed planar relationship.

The theory of overthrust faulting was proposed by Hubbert-Rubey in order to explain a class of faults not readily explained by that of Anderson's theory of faulting. These types of faults are low-angle overthrusts in which thin sheets of rock have been transported some distance horizontally. In relation to the specific problem of fault reactivation related to stimulation of petroleum reservoirs, this is a specifically important aspect to cover. Hubbert-Rubey hypothesized that under some conditions; the pore pressure may greatly exceed the hydrostatic head and even approach those of the applied lithostatic load. This theory that is limited to sedimentary basins has been supported by petroleum related data (Scholz).

$$F = \sigma_h = (\sigma_z - p)I\mu$$

The characterization of fault environments is a much simpler process than the characterization of the natural fracture and joint network(s). These systems are extremely dynamic and may vary greatly within even the same formation. The difficulty incurred by replicating these systems can become a daunting task which is why these systems are most often assigned to be representative of the general pattern or trend seen throughout the formation rock mass. The particular assignment of discontinuity is relative to the geological formation and supporting data from the lab and field, however this is still often just a generalization of the actual in-situ conditions.

FIGURE 12 GENERAL BEHAVIORS OF J1 AND J2 JOINT SETS OCCURRING ACROSS PENNSLVANIA THROUGHOUT THE MARCELLUS SHALE AFTER (ENGELDER, 2008) FROM (KRISTIN M. CARTER, 2011)

The natural occurrence of stress or strain causes fracturing throughout the rock matrix known as jointing (Cliffs Minerals, Inc., September 1982). This joint network throughout the Marcellus shale matrix is well established and has been attributed to means of natural hydraulic fracturing by fluid pressures during hydrocarbon generation. The stresses throughout shale plays such as the Marcellus develop a specific orientation of jointing that can be further used to interpret in-situ stress distributions as was discussed with **Figure 9**. The natural distribution of joints occurs such that they appear perpendicular to each other in nature, and in the Marcellus appear to be aligning parallel with the minimum and maximum horizontal stresses, respectively. These joint mechanisms are often referred to as J_1 and J_2 joints, where J_1 is approximately perpendicular to J_2 as can be seen in **Figure 12** (Terry Engelder, 2009).

3.1.5. REACTIVATION FACTORS AND SLIP TENDENCY

The initiation of induced stresses on a formation require that in-situ stress field be balanced to the pre and post fracturing stages in order to mitigate or control areas of slippage. The determination of reactivation is based primarily upon the friction between the fault plane surfaces. The surface friction controls the rate and extent of sliding, or slippage between two coinciding fault planes. This frictional relationship requires that the properties of the material and entrained fluids be known in order to correctly determine a stress failure envelope. The stress fields surrounding fault and fractures is also a key insight to determining their interactions.

In similar fashion to the interaction of faults themselves, the interaction of fault and fracture can be viewed in a similar manner. This is because faults are often observed to occur and grow in systems rather than as isolated features influencing their behavior. These areas of altered stress surrounding induced fractures in the vicinity of fault systems are then expected to influence their behaviors in a similar fashion. Such that in areas of stress enhancement propagation and failure occur more readily while in areas of stress drop, propagation and failure may be reduced, implying an increase in stability.

The stress failure envelope is most often described by the maximum and minimum principal stresses and their orientations. The criteria used for failure has been proposed through many different types of stress relationships. This includes the methods of the: maximum tensile stress, Tresca criterion, maximum octahedral shear stress, Von Mises stress and the Coulomb-Navier criterion. The Mohr Criterion is a more general approximation of failure which can be observed experimentally. The Coulomb Criterion identifies a strength envelope in which slippage is most likely to occur based upon its state of stress. This provides a window of allowable stress concentration that is easily determinable, providing a wide range of application to both soil and rock mechanics.

The Coulomb Failure Criterion is defined as a function of stress resistance. Therefore the potential of reactivation is most largely affected by the differential in stress across these deactivated fault zones. This can be inferred below based upon the equation to determine Coulomb based modes of failure.

$$\Delta CFS = \Delta\tau - \mu_s \Delta\sigma'_n$$

Where $\Delta\tau$ and $\Delta\sigma'_n$, respectively are the induced changes of shear and effective stresses on the fault plane. The sign of $\Delta\tau$ is positive when it points in the same direction as the initial shear stress on the plane. The sign of $\Delta\sigma'_n$ is positive for an increase in compressive stress. Therefore ΔCFS indicates a tendency towards fault reactivation.

A visual representation is shown displaying the Mohr-Coulomb stress failure relationships in **Figure 13**. This is an intrinsic curve criterion in which a linear relationship between shear and normal stress on a failure plane is expressed. The shear failure envelope is determined at the point in which shear failure of the material is expected

to occur. The Mohr circle is constructed based upon the magnitude and angle of stress throughout the formation. The prediction of failure through this method is altered by the inclusion of poroelastic effects which can cause shifting of the Mohr circle.

FIGURE 13 MOHR COLOUMB FAILURE ENVELOPE WITH RELATION TO STATE OF STRESS

The optimal conditions for failure and lock-up along a fault plane are often derived from laboratory tests which have been further validated with case studies. In many areas of research regarding rock mechanics and fault kinematics it is often assumed that a critical friction coefficient of 0.6 is used as it has been shown to agree well in most cases (Scholz). This correlation was determined by using multiple types of rock at various depths which tend to display a failure to occur at an average value of 0.6.

The dimensionality of the problem reflects the interactions of vertical and horizontal stress components in relation to fault kinematics. The relative magnitude of these stresses controls the delineation of strike-slip or dip-slip faulting which alters the predicted Coulomb stress change (Geoffrey C.P. King, 1994).

FIGURE 14 THE GENERALIZED FAILURE CRITERION ENVELOPES BY METHOD

3.1.6. FAILURE CRITERION METHODS

COULOMB FAILURE CRITERION

The coulomb failure criterion was used in order to describe the slip tendency of the given fault plane or stress state. These stresses, taken from the finite element stress tensors evaluate their ratio to a generalized failure criterion. The general approach of Coulomb Failure Criterion is shown below.

$$\tau > \tau_f = S_0 + \mu \sigma_n$$

Where τ_f , a failure criterion is set by a maximum shear stress limit and τ is the measured or calculated shear stress. Therefore, σ_n is the stress normal to the plane and μ is the resistive or frictional coefficient.

Applying the general Coulomb Failure Criterion in terms of principle stresses the following relationship can be determined based upon the equation shown below.

$$\frac{\sigma_1}{\sigma_3} = \frac{S_1 - P_p}{S_3 - P_p} \geq \left[(\mu^2 + 1)^{\frac{1}{2}} + \mu \right]^2$$

Where σ_1 and σ_3 are the maximum and minimum principle stresses, respectively.

To determine the initial optimal angle of faulting, with angle θ to σ_1 .

$$\theta = \frac{1}{2} \tan^{-1} \left(\frac{1}{\mu} \right)$$

The effects of shear and normal stresses across the fault plane are respective to the orientation of the fault.

$$\sigma_{n(\beta)} = \frac{1}{2}(\sigma_1 + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3) \cos 2\beta$$

$$\tau_{n(\beta)} = -\frac{1}{2}(\sigma_1 - \sigma_3) \sin 2\beta$$

The Anderson fault theory is used to determine the principal stress relationships, representative of each type of faulting (i.e. possible reactivation).

<i>Normal Faulting</i>	$\frac{\sigma_1}{\sigma_3} = \frac{S_v - P_p}{S_{h \min} - P_p} \leq \left[(\mu^2 + 1)^{\frac{1}{2}} + \mu \right]^2$
<i>Strike – Slip Faulting</i>	$\frac{\sigma_1}{\sigma_3} = \frac{S_{h \max} - P_p}{S_{h \min} - P_p} \leq \left[(\mu^2 + 1)^{\frac{1}{2}} + \mu \right]^2$
<i>Reverse Faulting</i>	$\frac{\sigma_1}{\sigma_3} = \frac{S_{h \max} - P_p}{S_v - P_p} \leq \left[(\mu^2 + 1)^{\frac{1}{2}} + \mu \right]^2$

Effective overburden pressure at a given depth is given below.

$$\sigma_v = \rho g z (1 - \lambda)$$

This is used to determine the limiting conditions in regards to the three types of faulting methods.

VON MISES FAILURE CRITERION

The Von Mises failure criterion is also known as the distortion energy theory as it is a coalescence of stress in any direction, only caring for magnitude and not direction. This is why it is often used as a yield criterion throughout many applications.

In a two-dimensional study the Von Mises stress can be determined using the relationship below in terms of general stress components, where $\sigma_{zz} = \tau_{xz} = \tau_{yz} = 0$.

$$\sigma_{VM} = \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 - \sigma_{xx}\sigma_{yy} + 3\tau_{xy}^2}$$

For three-dimensional study the Von Mises can be determined as follows using the principle stress relationships.

$$\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] \leq T^2$$

This can also be written in terms of the principle stress magnitudes in a two-dimensional form, where the following is valid for a two-dimensional plane stress state, $\sigma_3 = 0$ and $\sigma_3 = \nu(\sigma_1 + \sigma_2)$ for a plane strain state.

$$\sigma_{VM} = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$$
$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

TRESCA FAILURE CRITERION

The Tresca criterion is also known as the maximum shear stress theory. This simply states that the material will yield when the shear stress at a point exceeds the critical shear stress of the material.

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} \geq \tau_Y = \frac{\sigma_Y}{2}$$

Where σ_{max} and σ_{min} are the maximum and minimum principal stresses, σ_1 and σ_3 .

The Tresca failure envelope is very similar to that of the Von Mises as can be seen in Figure 15 where the Tresca is slightly more conservative in its predictions leading up to failure.

RANKINE FAILURE CRITERION

The Rankine principle stress theory simply states that for a brittle material, rupture occurs when the maximum principal stress in the specimen reaches some sort of limiting value of the material.

$$\sigma_1 \geq \sigma_U$$

This can be written conditionally as follows for all principle stresses.

$$f = \max(|\sigma_1|, |\sigma_2|, |\sigma_3|) = Y$$

FIGURE 15 FAILURE ENVELOPE OF STRESS STATE CONDITIONS

3.1.7. SEISMIC RESPONSE

The seismic response of a geologic system is the acoustic representation of energy release per unit volume per unit time that occurs from rock failure. This is a conservation of energy encompassing the sudden release of energy from the fluid volume, fracture propagation and frictional slippage throughout the fracture or fault system. This is the most often used terminology in the description of faulting systems to describe the relative magnitude of energy release. The potential of the energy magnitude increases with the size of the fault and the strength of the adjacent rock masses, in addition to the depth at which the fault plane is located. This is due to the increase in potential energy that can be stored in the system which is directly dependent upon its depth, as the overburden pressures are typically increasing in this fashion.

In hydraulic fracturing operations the treatment well may be monitored through an offset observation well through applied earthquake seismology, using microseismic monitoring arrays. The travel times of P-wave and S-wave response generates a fracture map that displays the generated fracture relationships. This relates the interpretation of complexity and stimulated rock volume which displays that not all microseismic events are related to pressure changes, which can be classified as wet events. The fracture complexity index (FCI) defines the relationship of the microseismic cloud as a function of width to length; this is helpful in establishing a mathematical relationship of hydraulically induced seismicity and those generated by associated far field stress modification.

$$FCI = \frac{\text{Total Width}}{\text{Total Length}}$$

Where FCI is the fracture complexity index used to assess the degree of network development. The range of values is delineated as follows, where high complexity > 0.25, moderate complexity is 0.15-0.25 and low complexity < 0.15.

Microseismic events are approximated based upon the magnitude of plastic slip along a plane, known as the microseismic moment.

$$M_0 = G \times l_{slip} \times A_{slip}$$

$$M_0 = DA\mu$$

Where D is the average displacement over the entire fault surface, A is the area of the fault surface and μ is the average shear rigidity of the faulted rocks.

The microseismic cloud is generated by the calculated moment magnitude of these microseismic events.

$$M_s = \frac{2}{3} \log(M_0)$$

The proximity of fracturing stages and discontinuity significantly affects the characteristics of this parameter.

$$M_w = \frac{2}{3} \log_{10}(M_0) - 6.0$$

3.2. MECHANICS OF HYDRAULIC FRACTURING

3.2.1. INTRODUCTION

The mechanics involved in simulation of a hydraulic fracture require a coupling of a fluid-driven crack, propagating within an elastic, permeable medium. This analysis is based upon a set of governing fluid flow equations that relate the hydraulic part of the problem (fluid mass balance and fluid flow within the fracture) to those of the mechanical problem. These equations are based upon a toughness factor introduced as a dependent parameter of fracture size, since the process zone (PZ), in general depends on the fracture size as well as the associated boundary conditions. Green's function technique for linear elasticity is typically used to reduce the system of integro-differentials to a system of non-linear algebraic equations which may be solved through the use of finite difference or finite element techniques.

For a given stress state caused by hydraulic pressure and a set of boundary conditions for the governing fluid-solid relationship, the displacements (fracture geometry) can be calculated. The additional requirements for solution of propagation are provided through the modes of fissure opening and advancement. This implies that extension of the fracture is proportional to the speed of the fluid, in which the direction of propagation is a function of mode I and II stress intensity factors (SIF).

The modes of loading that can be defined for a fracture in a three-dimensional homogenous body were defined mathematically by Irwin through the following types of displacements:

FIGURE 16 IRWIN'S KINEMATIC FRACTURE MODES

Figure 16 identifies the distinct criteria for crack propagation within each of these modes. In mode I, or opening mode, the crack surfaces are pulled apart by tensile forces creating symmetric deformation on the plane parallel to the tip of fracture advancement or propagation. In mode II, or sliding mode, the crack surfaces are subjected to shear forces parallel to the fracture surface. In mode III, or tearing mode, the crack surface are subjected to shear

forces parallel to the fracture front. These idealized opening modes often work in combination of each other creating a mixed mode situation. This requires that the stress superposition to be used in order to determine the individual components and their respective contribution.

$$\sigma_{ij} = \frac{1}{\sqrt{2\pi r}} [K_I f_{ij}^I(\theta) + K_{II} f_{ij}^{II}(\theta) + K_{III} f_{ij}^{III}(\theta)] + \vartheta(r^{1/2})$$

Where K_I , K_{II} , and K_{III} are the fracture mode stress intensity factors corresponding to Mode I, II or III.

The stresses induced by hydraulic fracturing dynamically change the in-situ stress distributions through a time dependent, or transient relationship. It is therefore highly important to capture these interactions during the injection period when trying to model the dynamic stress regimes during the injection process.

This correlates to the stress intensity factor governing the fracture opening mode.

$$K_I = \frac{1}{\sqrt{\pi a}} \int_{-a}^a p_R(t) \sqrt{\frac{a+t}{a-t}} dt$$

This equation is reduced to the following when subject to a uniform stress field, however this is not the case for heterogenous anisotropic materials.

Through this the fracture toughness is defined in terms of the critical stress intensity factor.

$$\Delta\sigma_c = \frac{K_{Ic}}{\sqrt{\pi a}}$$

Where $\Delta\sigma_c$ is the defining critical stress differential, K_{Ic} is the fracture intensity parameter.

These time-dependent relationships require a complex association between hydraulic and mechanical behaviours. The pressure induced by fluid injection surrounding the borehole controls the opening of new fractures, dilation of pre-existing fractures and their associated propagation. The transient pressure relationship during injection can be referenced from the figures below.

The injection pressure increases until surpassing the fracture initiation or breakdown pressure which is controlled by the material matrix surrounding the point of injection. The pressure required to start initiation is greater than that of the pressure required to propagate the fracture and therefore the pressure declines slightly after fracture initiation throughout the period of propagation. As the fracture propagates the pressure plateaus at what is known as the closure pressure until reaching the transient reservoir behavior. This process can be visualized in **Figure 17** below.

FIGURE 17 HYDRAULIC FRACTURE INJECTION PRESSURE WITH TIME

3.2.2. STRESS CONCENTRATIONS AND FRACTURE INITIATION

One of the most important parameters controlling the propagation or initiation of fractures is the fracture toughness parameter derived from the concept of the stress intensity factor. It is assumed through fracture mechanics that preexisting defects will always pervade a continuum. These inclusions provide areas of high stress concentration with respect to the surrounding intact media. The initiation and propagation of fracture is predicted to occur in these areas of localized stress concentration. The inclusion of these defects requires complicated stress interaction to be accounted for either directly or indirectly in analytical and numerical methods.

The stress that is required to initiate a fracture usually differs from the amount of stress required to propagate the fracture. The difference in breakdown to extensional pressure relies in the rock specific properties relating to deformation.

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) + \dots$$

On a microscopic scale the nucleation of fracture is controlled at the fracture tip by the strength of the atomic bonds throughout the rock matrix. The primary stress component for the crack tip region was summarized by Irwin with the following relation to the three displacement modes of failure.

$$\sigma_y = \frac{K_I}{(2\pi r)^{\frac{1}{2}}} f(\theta)$$

$$\sigma_{xy} = \frac{K_{II}}{(2\pi r)^{\frac{1}{2}}} f(\theta)$$

$$\sigma_{yz} = \frac{K_{III}}{(2\pi r)^{\frac{1}{2}}} f(\theta)$$

The stress intensity factors are related to the strain energy release rate with the following.

$$G_I = \frac{(k+1)}{8\mu} K_I^2$$

$$G_{II} = \frac{(k+1)}{8\mu} K_{II}^2$$

$$G_{III} = \frac{(k+1)}{8\mu} K_{III}^2$$

Secondary permeability lies with the efficiency of stimulated fracture treatments as it is typically inferred as the resistance to driving force between two parallel plates.

$$-\mu(hw) \frac{\partial^2 \mu}{\partial w^2} = \frac{\partial p}{\partial x} (hw)$$

3.2.3. FRACTURE PROPAGATION AND GEOMETRY

The real geometry of fracture is an extremely dynamic process controlled by heterogeneity and anisotropy developed over millions of years. In order to simplify the fracture process, certain assumptions are made to limit the uncertainty of the solution while maintaining a relatively close relationship to field data. The classical method

of fault geometry in a homogenous, isotropic medium considers a symmetric wing shaped pattern to the development of fractures from the point of initiation, or the wellbore in this case. This follows the assumption that the fracture initiates from a maximum height at the point of injection and declines as the distance from the injection point increases, to the extents of the fracture or “half-length” (SPE Reprint Series, 1990).

There are multiple theories that follow this approach using slightly differing cross-sectional geometries. These geometries are typically described by the use of three common modeling schemes:

- (1) PKN fracture geometry
- (2) KGD fracture geometry
- (3) Radial fracture geometry

These types of fracture propagation are governed by three sets of equations that require coupling through each of the models. This is in reference to the continuity, elasticity or linear elastic fracture mechanics (LEFM) and fluid-flow (momentum) equations.

KHRISTIANOVIC-GEERTSMA DE KLERK FRACTURE MODEL

The Khristianovic-Geertsma de Klerk (KGD) model follows the assumption that the fracture height is fixed and that fracture width is constant in the vertical direction as it does not depend on the height. The rock stiffness is calculated in the horizontal plane only, or 2D plane strain deformation in this plane. The fluid pressure gradient is determined from the equation below.

$$w(0, t) = \frac{2(1 - \nu)L(p_f - \sigma_h)}{G}$$

$$p(0, t) - p(x, t) = \frac{12\mu q_i}{h_f} \int_0^x \frac{dx}{w^3(x, t)}$$

FIGURE 18 KGD FRACTURE GEOMETRY, SCHEMATIC DIAGRAM

PERKINS-KERN NORDGREN FRACTURE MODEL

The Perkins and Kern (PKN) model geometry is defined by a relationship of width to excess pressure where the fracture length as width are determined based upon a constant fracture height. Nordgren later altered this model in order to adapt for the inclusion of leak-off within the model.

$$w(x, t) = \frac{2\Delta p_{wi}(x, t)}{E'} h$$

$$w(x, t) = \frac{(1 - \nu)h_f(p - \sigma_h)}{G}$$

$$\frac{\partial(p - \sigma_h)}{\partial x} = -\frac{64}{\pi} \frac{q\mu}{w^3 h_f}$$

The PKN geometry has an elliptical cross-section in both horizontal and vertical directions which can be seen from **Figure 19**. This model makes the assumption that although fracture height is fixed that the length is much greater than the height such that $L \gg h$. Due to this and plane strain assumptions (in the vertical direction) the solution can be reduced to a two dimensional problem.

FIGURE 19 PKN FRACTURE GEOMETRY, SCHEMATIC DIAGRAM

The PKN model only assumes the loss of energy due to the fluid flow along the fracture and does not incorporate the energy consumption required to induce fracture propagation (overcome fracture “toughness”). This type of modeling then simplifies to channel flow in an elliptical fracture.

RADIAL FRACTURE GEOMETRY

The radial fracture geometry is the third most commonly used fracture model and is often referred to as the “penny crack”. This type of fracture is seen in shallow formations where the minimum stress is equal to the overburden stress. This geometry is assumed to be symmetric with respect to the injection point. This also assumes that the injection rate and pressure within the fracture remain constant.

FIGURE 20 RADIAL FRACTURE GEOMETRY, SCHEMATIC DIAGRAM

The principle stresses again maintain a critical relationship to fracture geometry by the magnitude and ratio of these three values. In cases of hydraulic fracturing, such as that in the Marcellus shale it has been determined that the maximum stress where this formation lies is that of the overburden or overlying formation due to the vertical pressure gradient. If this is true, then the fracture geometry will grow in a vertical fashion from the horizontal wellbore and extend outwards. The two main stresses which control the fracture geometry are then associated with that of the minimum and maximum horizontal or the minimum and intermediate principle stresses. Beyond this, the fracture width and height is related to the in-situ properties and stresses by the relationships stated above for the commonly used modeling schemes. The stress relationships can be seen in **Figure 21**.

FIGURE 21 PRINCIPLE STRESS ORIENTATION TO THE WELLBORE

Figure 21 exemplifies the orientation of the three principle stress states in three dimensions which surround a given point below the surface. These principle stresses are oriented along the primary axes in which all stresses can be defined. With regard to the problem at hand, the reservoir is anticipated to lie at some depth below the earth's surface, at which the vertical stress becomes the dominant subsurface force. These principle stresses play an integral role in hydraulic fracture geometry. The state of these stresses does not only direct these fractures in a vertical manner but their growth in the transverse or longitudinal direction(s) from the horizontal wellbore.

FIGURE 22 TRANSVERSE VERTICAL FRACTURE

FIGURE 23 LONGITUDINAL VERTICAL FRACTURE

3.2.4. STRESS FIELD REORIENTATION

The role of the in situ stresses has been described to play a vital role in the initial geometry and direction of the fracture. These stresses are often used as the principle method whether for orienting the wellbore and predicting fracture growth, whether applied to determining real treatment design or to simulation design parameters.

This is often observed due to the lack of information and testing regarding transient stress field behaviors throughout the injection and fracture growth. In some cases the in situ stresses remain in relative harmony throughout the injection period and do not supersede one another, this is the standard case for hydraulic fracture propagation. On the other hand when given the proper conditions these in-situ stresses can change, which may drastically impact each other, shifting the direction of S_{hmin} and S_{Hmax} with which the fracture(s) attempt to realign themselves with during the injection period.

The reorientation of the primary stress fields may cause fractures to terminate growth as the energy from the injection becomes unevenly distributed. This may cause fractures within the same stage or subsequent stages to negatively impact the efficiency of the treatment.

FIGURE 24 FRACTURE PROPAGATION IN A STATIC STRESS FIELD

FIGURE 25 FRACTURE PROPAGATION IN A DYNAMIC STRESS FIELD

3.3. MECHANICS OF FRACTURE FLUID FLOW

Fluids are considered an isotropic material which undergoes continuous deformation through external forces; this is the driving force of fracture throughout the rock mass matrix. The typical theories involving fluid flow through rock fractures has been described by the cubic law which assumes that the fracture is treated as two parallel plates and as such is considered to be one-dimensional flow (J. D. Anderson, 2009; Nadai, 1963; Nolte, 1988).

Fluid aperture flow between parallel or nearly parallel crack faces of an incompressible Newtonian fluid with negligible surface roughness:

$$q = -\frac{w^3}{12\mu} Ga^3 \Delta h$$

This is also referred to as Poiseuille's law in which one-dimensional flow is modeled using the lubrication theory:

$$q = -\frac{w^3}{12\mu} \nabla p_f$$

This is used as the relationship of fluid flow and fracture aperture based upon the cubic law. The cubic law follows that transmissivity is proportional to the cube of the aperture size and that the flow rate is directly proportional to the pressure gradient.

3.3.1. LAMINAR FLOW

The Navier-Stokes equation is the differential form of the momentum equation for Newtonian fluids. In relation to incompressible fluid flow in a rectangular coordinate system, the Navier-Stokes equation can be defined as:

$$\begin{aligned} \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} &= g_x - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right) \\ \frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} &= g_y - \frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2} \right) \\ \frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} &= g_z - \frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right) \end{aligned}$$

Also, from the continuity equation, the motion of an incompressible fluid in a 3D continuum can be associated with the Navier-Stokes Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Where,

$$\begin{aligned} \frac{\partial u_x}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= -\frac{\partial P}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \\ \frac{\partial u_x}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= -\frac{\partial P}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \end{aligned}$$

$$\frac{\partial u_x}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{\partial P}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)$$

Where u , v and w are the respective x , y and z components of the velocity.

The Neumann equation expresses the first law of thermodynamics for a linearly viscous fluid. In this presentation the heat capacity c and the thermal conductivity k have been assumed constant.

$$\rho c \frac{dT}{dt} + p \operatorname{div} \vec{v} = k \Delta T + \phi$$

The expansion of this equation is shown in rectangular coordinates below.

$$\begin{aligned} \rho c \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) + p \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) \\ = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + (2\mu + k) \left[\left(\frac{\partial v_x}{\partial x} \right)^2 + \left(\frac{\partial v_y}{\partial y} \right)^2 + \left(\frac{\partial v_z}{\partial z} \right)^2 \right] \\ + \mu \left[\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right)^2 + \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right)^2 + \left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} \right)^2 \right] \end{aligned}$$

Under steady-state flow conditions a ‘no-slip’ condition is assumed by the Navier-Stokes solution in which the components of velocity (both normal and shear) disappear at the fluid-solid interface. This requires that at fluid inlet(s) and outlet(s) the pressure and/or velocity must be specified.

3.3.2. PROPPANT DISTRIBUTION

An important aspect of hydraulic fracturing is the flow of the slurry mixture and particle migration that occurs through the flow regime. This outlines the relation of proppant transport and distribution throughout the aperture. This distribution is often assumed to be constant however both field and simulated data have displayed that the distribution of proppant within the fracture does not have this linear correlation.

The distribution of proppant throughout the induced fracture relates to the effective permeability changes experienced throughout the stimulated fracture network or treatment area. This is a principle relationship of fracture efficiency which is greatly impacted due to the settling and packing of proppant during fluid transport. This process occurs due to the injection rate properties, type and size of proppant injected along with fracture geometry influences. This causes a nonlinear distribution of proppant which does not extend the full length of the induced fracture. The applied fluid stress through the fracture helps further propagation, however at the end of treatment these fracture will start to close due to the closure stress of the formation. The underlying goal of fracturing is to enhance the conductivity to the wellbore by induced fractures which make the proppant distribution an important aspect of hydraulic fracturing theory.

3.3.3. POROELASTIC EFFECTS

The presence of fluids throughout pore space in the rock matrix modifies its mechanical response to applied stresses. This pore pressure effect is known as poroelasticity. This becomes particularly important as it incorporates time-dependency through fluid diffusion. This creates an expected response that the material behaves stiffer as the rate of loading increases. The influence of induced fluids and fluid pressure is typically influential to failure mechanisms such as Terzaghi effective stress, which is of particular importance to the nature of this work.

As discussed earlier, the deformation of a fluid saturated rock is not proportional to the total compressive stress (overburden) but instead to the Biot modified effective stress. That is that the bearing capacity of the formation to resist the compressive forces applied is reduced by entrained fluids. This is described by the relationship below.

$$\text{Biot Effective Stress} = \Delta P - \alpha \Delta p$$

$$\alpha = 1 - \frac{K}{K_s}$$

The determination of entrained fluid reduction is found through the drained and undrained response of the media. This represents the dry strength of the rock matrix without entrained fluids and the entrained fluid sample to an applied load. The difference of these two samples represents the introduction of Biot's contribution.

$$K_u = K + K_f \frac{(K_s - K)^2}{K_f(K_s - K) + \phi K_s(K_s - K_f)}$$

Where K_f is the fluid bulk modulus and ϕ is the porosity.

The relationship of pore pressure to compressive stress is expressed by the Skempton pore pressure coefficient and undrained Poisson ratio.

$$\Delta p = B \Delta P$$

$$\Delta p = \frac{2B(1 + \nu_u)}{3} \Delta P$$

$$\Delta p = \frac{2B(1 + \nu_u)}{3(1 - \nu_u)} \Delta P$$

Respectively for three-dimensional, plane-strain and one-dimensional problems.

The induced pore pressure stress that is generated under certain conditions is expressed by the relationship below.

$$\sigma_{xx} + \sigma_{yy} + \sigma_{zz} = -4\eta p$$

$$\sigma_{xx} + \sigma_{yy} = -2\eta p$$

$$\sigma_{xx} = -2\eta p$$

Respectively for three-dimensional, plane-strain and one-dimensional problems.

In hydraulic fracturing the poroelastic effects are generated in the form of a poroelastic stress (back-stress) from the injection and a pressure dependent leak-off.

3.3.4. LEAK-OFF

The effects of leak-off are caused by fluid pressure and flow through the porous media. This can be directly related to the determinations of fracture efficiency which relates the volume of fluid injected to the volume of the generated fracture. Due to the effects of leak-off, the fracturing efficiency will always be less than one. This is caused by the loss of fluid to reservoir porosity during the hydraulic fracturing treatment.

$$\eta = \frac{V_f}{V_i}$$

Where η , is the fracture efficiency and V_f , is the volume of fracture and V_i , is the volume of injected fluid.

The most widespread leak-off model is typically that using Carter's equation for fluid leak-off.

$$v_l = \frac{C_L}{\sqrt{t - \tau}}$$

Where v_l is the fluid loss velocity normal to the fracture face and τ is the time at which filtration starts.

In a semi-analytical form, Carter's leak-off model is used as the relationship described below:

$$g(x, t) = \frac{2C_L}{\sqrt{t - t_0(x)}}$$

Where C_L is the leak-off coefficient and t_0 is the time at which the fracture tip arrives.

3.4. RESERVOIR STRESS MODELING

Various computational methods have been developed in order to solve for approximate numerical solutions. It is rare that analytical solutions can be found for many rock mechanics problems in a practical concern. These types of numerical solutions require specified boundary locations in order to extrapolate their relationship to the domain of the problem. In order to solve for these types of boundary value problems there are two typical numerical methods used which are those that require approximation throughout the entire problem domain and those that require approximation to be made only along the specified boundary conditions. These are known as differential methods and integral methods, respectively.

The first characterization of the reservoir model requires that the continuum be allowed to account for discontinuities throughout the material. These continuum theories account for the various properties throughout the reservoir rock matrix model. This is the main differentiation in the classification of fundamental rock mass mechanics. In reference to porous media, this requires that the model accounts for these variable inclusions and associated properties.

In an integral method solution the overlying problem boundary encloses the problem domain so that it is referred to as an interior problem. In the area of rock mechanics however, that of the exterior problem is of more concern in which the problem domain is an infinite or semi-infinite region outside the boundary of concern. Differential continuum methods are of the most often used throughout engineering related mechanics problems which include the finite difference and the finite element methods.

Two typical approaches are typically used in the solution of a coupled hydro-mechanical problem such as this. The first solution is a balance equation approach in which the derived partial differential equations for fluid flow in the fracture are input to the solution of the surrounding elastic deformation problem. This provides a staggered solution technique, as the input-output of the independent solutions are exchanged for each successive solution. In a poroelastic approach the partial differential equations from Biot's theory are used in a fully coupled solution scheme in which unknowns are solved simultaneously. This type of solution is more computationally expensive, requiring a higher degree of spatial discretization surrounding the thing geometry of the crack. The accuracy of the solution is greater than uncoupled techniques however this relies heavily on the accuracy of the special discretization.

In general the specific solution method determined for the problem is chose based upon both the requirements of the solution as well as the requirements of the computational analysis. Regardless of the solution method sought, the desired accuracy of the solution can be defined easily as the least number of iterations possible to convergence. This encompasses all applications without regard to dimensionality or solution requirements. In many cases this requires a trade-off between accuracy of the solution and the expenditure required to reach the solution.

The modeling strategy used throughout this work encompass a variety of different fracture simulation software and in-house code which was developed specifically for this research. These models have been verified for numerical accuracy using simplified cases which has allowed for further research and development to commence. These strategies encompass three main sets of modeling code.

1. Preliminary Static Model	three-dimensional stress analysis with uniform pressure
2. Transient Model	two-dimensional stress analysis with non-uniform pressure
3. Transient Cohesive Zone Model	three-dimensional propagation model with non-uniform pressure

These codes will be discussed in further detail throughout the coming chapters in order to give a brief description of the methodology behind the techniques applied. The preliminary model was developed for simple stress problems and does not offer a transient solution. The static preliminary model uses an applied, constant hydraulic pressure that is uniformly distributed across all of the fluid elements. There is no leak off considered into the solid elements and therefore fractures do not have any complex boundary issues and a loading can be applied at the areas considered to be fractured. The transient model trades some of the stress density seen in the static model for the ability to perform time integrated solutions. This allows for the problem to be determined at specified time steps according to the constraint conditions and thus dynamic stress patterns and interactions can be observed. In addition the transient model observes the fluid and solid interactions that are occurring, including leak-off and distributed pressure within the fracture. The last model described is the cohesive zone model which applies many of the techniques seen in the previous transient model, however it adds a dimension for full three-dimensional coupled solutions.

3.4.1 FINITE ELEMENT METHOD ANALYSIS

The finite element method is used in various engineering applications in order to find relative, or approximate solutions for complex scenarios. This method allows for analytical solutions to be found at any point body within the contained structure. In more complex scenarios, unknown quantities are made by generalizations in order to make approximate, yet acceptable solutions (Zienkiewicz, 1971; Lewis & Schrefler, 1987; Smith & Griffiths, 1988).

The basis of the finite element method relies in its ability to simplify the complexities of a problem to their constitutive relationships, to their simplest representative geometry and conditions. This is done by breaking down the original complex body into an equivalent number of smaller, simpler elements. These simplifications allow for solutions to be found over small areas which are then extrapolated over the problem domain. This is done by discretizing the problem domain to any number of (typically) similar, simple geometric shapes or bodies. The simplified geometric representations are then defined by the density of points throughout each element. By breaking down the element into a certain number of discrete points, an approximate solution may be found over each elements domain by extrapolating the known point values over the element. The process of selecting only points within a media body is known specifically as the discretization. The interconnecting nodes of these adjoining elements are governed by their constitutive equations representing the problem or solution in question. This method allows an exact solution to a differential approximation of the problem.

The assemblage of discrete subdivisions is known as finite elements because each one occupies a small but finite sub-domain of the original domain, while each one of these elements is connected at known locations or nodal points. The solution of these types of problems is expressed through a series of governing equations and associated boundary conditions.

The finite element procedure can be broken down into six generalized steps (Abel, 1972).

1. Discretization of the continuum
2. Selection of the field variable models
3. Derivation of the finite element equations
4. Assembly of the algebraic equations for the overall discretized continuum
5. Solution for the variable field nodal vector
6. Computation of the element resultants from the nodal field variable

The finite element procedure is typically introduced by the following three-step process in **Figure 26**.

FIGURE 26 BASIC FINITE ELEMENT FLOWCHART

These process steps are required by any finite element software in order to determine an appropriate solution. In many commercial solutions of the finite element method, these three steps will be determined by a single software package. In areas of specified interest such as academic research however, these processes are often handled by multiple types of software in order to allow specific problem requirements to be addressed.

The pre-processing area of FEA defines the problem domain and its associated geometry. The basic idea of the pre-processing section is to assign the problem domain geometry, its defining material data and associated boundary conditions. In order to accomplish this, the domain must be described in finite detail, through a process known as discretization. The defining elements that comprise the problem geometry are themselves connected through a set of discrete points. These point locations are then assigned their associated data from the breakdown of the original domain. Throughout the course of FEA the pre-processing domain is referenced in one of two ways. This is through the element, or nodal connectivity and the location of the node or element in their special coordinates. In other words for a discretized problem domain there is a distinct number of elements, these elements are surrounded by interconnecting elements which are known. The same applies for points, where each point is surrounded and interconnected by a set of known points at known locations.

The processing section is the heart or core of finite element analysis as it describes the constitutive, governing equations of the problem at each finite location throughout the domain. The nodal and element information provided by the pre-processing section, such as material data and location are then used to generate a system of equations. This system of equations may then be solved by any number of available solution techniques depending upon the dimensionality of the problem and the solutions application requirements. These solutions are often found through iterative solution techniques which determine an approximate solution to a specified degree of uncertainty.

The post-processing section relies heavily on the solution requirements and intended use of the analysis data. This section describes the visualization of the nodal and element solutions over the problem domain. This approach is

similar to that of reversing the pre-processing section as the solutions from each node and element are assigned to the larger problem domain. This solution data is dependent upon the given problem but includes such data as displacement, velocities, temperature and so on.

The finite element method is better suited for non-linear material behavior than that of integral methods. The largest limitation of this method is the need to define the outer boundary of the problem domain arbitrarily. The main concern here is that far-field stress conditions may not be satisfied completely. Although the computation time is increased through the use of finite element method of those of similar finite difference theories, the advantages provide the best possible solution due to problem constraints.

A simultaneous set of equations covers the interaction of elements via the element nodes and boundary conditions of the problem. The set of equations is generally represented as follows.

$$[K]\{u\} = \{F\}$$

Where $[K]$ is a definitive material property (identified as the stiffness matrix), $\{u\}$ is a specified governing material behavior (such as displacement) and $\{F\}$ is a resulting action of these material properties and behavior (such as the vector of nodal loads).

The known and unknown data is then segregated, which identifies the known inverse matrix $[K]^{-1}$.

$$\{u\} = [K]^{-1}\{F\}$$

Common solutions of various FEM problems are outlined in **Table 7** below. The highlights a very simple explanation to the applications of FEA and how they relate to the solution of various physics problems.

	Property $[K]$	Behavior $\{u\}$	Action $\{F\}$
Elastic	Stiffness	Displacement	Force
Thermal	Conductivity	Temperature	Heat Source
Fluid	Viscosity	Velocity	Body Force
Electrostatic	Dialectri Permittivity	Electric Potential	Charge

TABLE 7 SIMULTANEOUS EQUATION SOLUTION OF VARIOUS PROBLEMS

The finite element method is used to determine the stress distribution analysis pre and post fracturing. The finite element method divides the continuum (reservoir) into a series of elements which are connected at a finite number of points (nodal points). These interconnecting points create a web over the element volume which requires an iterative solution, interpolated by a piecewise polynomial. This mesh and nodal structure is outlined in **Figure 27** for a two-dimensional continuum.

FIGURE 27 GENERAL FINITE ELEMENT MESH OF A 2D CONTINUUM

The types of elements which typically may be used for numerical analysis of discretized problems are identified in **Figure 28**. This chart is broken down by dimension, progressively increasing from left to right. On the left are 1-dimensional beam elements. In the middle are 2-dimensional triangular or quadrilateral elements. On the right are 3-dimensional elements, tetrahedron, hexahedron and pentahedrons. The difference of these types of elements, aside from their shape is the number of nodal points which each element is divided. The more nodal points in the element discretization, the more precise the relative stress and strain values however this typically creates an exponential increase in the length of time in order to reach a stable or steady solution.

FIGURE 28 GENERAL ELEMENT TYPE BY DIMENSIONAL CONSTRAINT

The governing equilibrium equation through the finite element method may be seen from the following relationship. This describes the total potential energy which may be obtained by minimizing the potential energy of the system. This identifies the typical solution algorithm for finite element analysis, not the generalized problem differential equations.

$$\pi = \frac{1}{2} \int_V \{\sigma\}^T \{\varepsilon\} dV - \{\delta\}^T \{p\} dV - \{\delta\}^T \{q\} dS$$

Where π is the total potential energy, σ and ε are the stress and strain vectors respectively, δ the displacements at any point, p the body forces per unit volume and q the applied surface tractions. Integrations are taken over the volume V of the continuum and loaded surface area, S .

The process of using the displacement method for finite element analysis assumes that unknown values are only located at the nodal points, in which variation within any element is described in terms of these nodal values by means of interpolation functions.

$$\{\delta\} = [N]\{\delta^e\}$$

Where N is the set of interpolation functions termed the shape of functions and δ^e is the vector of nodal displacements of the element. For differing types of elements the shape of function and vector of nodal displacements are different.

The strains within each element are expressed in terms of the element nodal displacement as follows.

$$\{\delta\} = [B]\{\delta^e\}$$

Where B is the strain matrix generally composed of derivatives of the shape functions. Again, for differing types of elements the strain matrix and vector of nodal displacements are different.

Lastly is the stresses related to the strains by use of an elasticity matrix D .

$$\{\sigma\} = [D]\{\varepsilon\}$$

The total potential energy of the continuum is defined as the sum of the energy contributions of the individual elements. This can be inferred from the following summation of all element contributions.

$$\pi = \sum_e \pi_e$$

Where π is the total potential energy and π_e is the energy contribution of individual elements.

The total potential energy of each individual element is described by the following relationship.

$$\pi_e = \frac{1}{2} \int_{V_e} [\delta^e]^T [B]^T D B \delta^e dV - \int_{V_e} [\delta^e]^T [N]^T p dV - \int_{S_e} [\delta^e]^T [N]^T q dS$$

Where V_e is the element volume and S_e is the loaded element surface area.

Performance of the minimization for element e with respect to the nodal displacements δ^e for the element results in the following.

$$\frac{\partial \pi_e}{\partial \delta^e} = \int_{V_e} ([B]^T DB) \delta^e dV - \int_{V_e} [N]^T p dV - \int_{S_e} [N]^T q dS = K^e \delta^e - F^e$$

When simplifying the energy contribution for each element, the equivalent nodal forces are as follows.

$$F^e = \int_{V_e} [N]^T p dV + \int_{S_e} [N]^T q dS$$

The remaining term of the minimization is known as the element stiffness matrix.

$$K^e = \int_{V_e} [B]^T DB dV$$

The final results consisting of stress and strain are located in a global coordinate system. Each element within the three-dimensional model has six contributing components of strain $\{\varepsilon\}$ and stress $\{\sigma\}$ which can be seen from each vector format.

$$\begin{aligned} \sigma &= \{\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{23}, \sigma_{31}\}^T & \text{or} & \quad \sigma = \{\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx}\}^T \\ \varepsilon &= \{\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \varepsilon_{12}, \varepsilon_{23}, \varepsilon_{31}\}^T & \text{or} & \quad \varepsilon = \{\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{zx}\}^T \end{aligned}$$

The calculations at each of the nodal points are the most accurate solution to the problem. The field variables are extrapolated to element nodes after evaluated at integration points throughout the predefined matrix. In order to accurately determine fault stability, these shear and normal stresses are correlated using the Coulomb Failure Criterion once the solution for fracture propagation at the current time step has been found.

Strain displacement gradients given in Cartesian coordinates for small strains and rotations.

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} & \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \varepsilon_y &= \frac{\partial v}{\partial y} & \gamma_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\ \varepsilon_z &= \frac{\partial w}{\partial z} & \gamma_{zx} &= \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \end{aligned}$$

Where $u = u(x, y, z)$, $v = v(x, y, z)$ and $w = w(x, y, z)$ are displacement components in an arbitrary material in the x , y and z directions.

The principal stresses are in essence stress invariants acting perpendicular to each other. These invariants are used in determining the mean stress, deviator stress and angular stress invariant of principle stress orientation.

$$\begin{aligned} p &= -\frac{1}{3}(\sigma_x + \sigma_y + \sigma_z) \\ q^2 &= \sigma_x(\sigma_x - \sigma_y) + \sigma_y(\sigma_y - \sigma_z) + \sigma_z(\sigma_z - \sigma_x) + 3(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2) \end{aligned}$$

$$\theta = \frac{1}{2} \sin^{-1} \left[-\frac{27J_3}{2q^3} \right]$$

Where J_3 is of the following:

$$J_3 = \begin{vmatrix} \sigma_x + p & \tau_{xy} & \tau_{zx} \\ \tau_{xy} & \sigma_y + p & \tau_{yz} \\ \tau_{zx} & \tau_{yz} & \sigma_z + p \end{vmatrix}$$

With the third stress invariant θ then the principle stress relationships can be established as follows.

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{pmatrix} = \begin{pmatrix} p \\ p \\ p \end{pmatrix} - \frac{2}{3} q \begin{pmatrix} \sin \left(\theta_0 - \frac{2}{3} \pi \right) \\ \sin \theta_0 \\ \sin \left(\theta_0 + \frac{2}{3} \pi \right) \end{pmatrix}$$

3.4.2. COUPLED INTEGRATION

A requirement for any type of modeling is describing the interactions occurring through a variety of processes relating to the changes of state and transfer of energy. These interactions have dependencies relating to material properties and state, in addition to surrounding conditions. The influence of internal-external forces and properties which govern a materials response are correlated through fundamental relationships. There are three basic algorithms which are used for multiphysics simulation which are full coupling, loose coupling and one-way coupling. These coupling techniques are described below for reference.

FULLY COUPLED

In a fully coupled simulator, there is a single set of constitutive equations describing all the relative physics of the problem. In most cases this is referring to multiphysics where there are multiple processes occurring, such as fluid flow, heat transfer and stress deformation. These types of solutions typically offer the most exact solution, as all of the relating physics are determined simultaneously. As it would be expected these types of nonlinear, inelastic solutions are often extremely difficult to derive.

ONE-WAY COUPLED

In order to simplify the solution, fully coupled models often disregard multiphase flow and nonlinear material behavior. The opposite approach is taken through one-way coupling in which the constitutive equations of various interactions are solved independently, so that the solution from one equation is used as the input for the other, however that value or solution is not returned to the original governing equation.

LOOSE COUPLED

A loose-coupling technique lies anywhere between the other two methods. Specifically, loose coupling uses separate constitutive equations, as a one-way system, however information is exchanged in both directions at designated time intervals. This type of approach allows for a similar physical relationship to that of fully coupled solutions at considerably less cost.

In the basic case of hydraulic fracturing this process involves the coupling of the governing fluid flow and geomechanical interactions. The basis of this coupling is through the use of partial derivatives that exemplify the mathematical physics of the related parameters. In simple terms the theory of coupling can be viewed as an interpreter between two different languages where fully coupled solutions would be representative of someone fluent in speaking or writing both languages. One way coupling would be like someone who could understand

another language but is not fluent enough to speak it. Loose coupling would be like someone who could understand the language but also had a foreign language book with common terms in order to formulate a response.

CHAPTER 4 - ANALYTICAL APPROACH

4.1. INTRODUCTION

The previous approach to this problem of fault kinematics through hydraulically induced stress does not reference the problem as it occurs naturally. The main causes for reworking the model involve the inclusion of a time dimensionality, allowing for the determination of transient poroelastic and leak-off effects. In addition, the numerical solution technique is also addressed in order to create a more robust and capable program that more closely resembles the times found through industry standard modeling software.

This preliminary approach views the injection process as a snapshot, using uniformly distributed pressure and geometry; therefore it is incapable of displaying the developing stress patterns throughout the reservoir. The mathematical approach to this new solution requires a fluid-mechanical coupling that allows for both sets of constitutive equations to be addressed simultaneously.

Injection is further controlled by specific rate, instead of a specified pressure which allows for easier integration between real-world and theoretical results.

4.2. STATIC METHODOLOGY

4.2.1. DISCRETIZATION

In order to account for variations throughout the in-situ stress fields the mechanical deformation needs to be correlated to their associated stress fields, either naturally occurring or induced by the hydraulic fracturing process. This requires that the constitutive equations be determined for the descriptive physics of the mechanical problem to be solved.

Nonlinearity is often observed in structures due to stress-strain relations, large deformations, conditions of support, conditions of failure, or a combination of these causes. Nonlinear behavior is most often associated with cases of material behavior or conditions of support involving inelasticity (Nathan M. Newmark, 1971). In an effort to determine the trends in developing stress regimes, it is required that the problem be expanded to allow for time integration so that these dynamic relationships may be further understood and accounted for.

The 20-node brick element is also known as a quadratic (hexahedron) element, shown in **Figure 29**. The quadratic element is referenced from that the interpolation along each edge of the element is a quadratic function. This allows them to simulate curved boundaries by a piecewise-quadratic approximation. The largest disadvantage of this type of element is during the mesh generation process as it requires more experience to properly design in order to reduce numerical difficulties, such as hour-glassing. It is noted however that if one of the faces of the 20-node brick element is collapsed then the element can simulate the singular strain and stress fields used in applications such as those of linear elastic fracture mechanics from (Dhondt, 2004).

FIGURE 29 SCHEMATIC OF THE 20 NODE BRICK ELEMENT

At each vertex of the brick element a node exists such that the shape functions are as follows:

$$\varphi_i(\xi_j, \eta_j, \zeta_j) = \delta_{ij}$$

This satisfies the discrete position and shape function relationship using the isoperimetric (i.e. the same) formulation defined below:

$$U(X) = \sum_{i=1}^N \varphi_i(\xi, \eta, \zeta) U(X_i)$$

$$X = \sum_{i=1}^N \varphi_i(\xi, \eta, \zeta) X_i$$

The shape functions of the 20 node element are as follows for each of the individual nodal points throughout the element. In the case of the 20 node hexahedron these nodes are located at each of the corner points and mid points of a three dimensional cube as shown in the figure above. These nodal shape functions define the exact solutions whereas the intermediate areas of the solution are approximated from the known solution at these points.

FIGURE 30 MAPPING A SURFACE ELEMENT IN LOCAL COORDINATES ONTO GLOBAL COORDINATES

The coordinates of the nodes with respect to a central local and global system are defined in Error! Reference source not found..

	X	Y	Z
N1	-1	-1	-1
N2	1	-1	-1
N3	1	1	-1
N4	-1	1	-1
N5	-1	-1	1
N6	1	-1	1
N7	1	1	1
N8	-1	1	1
N9	0	-1	-1
N10	1	0	-1
N11	0	1	-1
N12	-1	0	-1
N13	-1	-1	0
N14	1	-1	0
N15	1	1	0
N16	-1	1	0
N17	0	-1	1
N18	1	0	1
N19	0	1	1
N20	-1	0	1

TABLE 8 NODAL COORDINATE LOCATIONS OF A 20-NODE HEXAHEDRON ELEMENT

$$[N] = [N_1 \ N_2 \ N_3 \ N_4 \ N_5 \ N_6 \ N_7 \ N_8 \ N_9 \ N_{10} \ N_{11} \ N_{12} \ N_{13} \ N_{14} \ N_{15} \ N_{16} \ N_{17} \ N_{18} \ N_{19} \ N_{20}]$$

The first eight equations here represent the shape functions at the corners of the hexahedral, while the remaining twelve equations are representative of the associated midpoints on the cube. The three dimensional notation of [x,y,z] is represented by $[\xi, \eta, \zeta]$ respectively.

$$\varphi_1 = -(1 - \xi)(1 - \eta)(1 - \zeta)(2 + \xi + \eta + \zeta)/8$$

$$\varphi_2 = -(1 + \xi)(1 - \eta)(1 - \zeta)(2 - \xi + \eta + \zeta)/8$$

$$\varphi_3 = -(1 + \xi)(1 + \eta)(1 - \zeta)(2 - \xi - \eta + \zeta)/8$$

$$\varphi_4 = -(1 - \xi)(1 + \eta)(1 - \zeta)(2 + \xi - \eta + \zeta)/8$$

$$\varphi_5 = -(1 - \xi)(1 - \eta)(1 + \zeta)(2 + \xi + \eta - \zeta)/8$$

$$\varphi_6 = -(1 + \xi)(1 - \eta)(1 + \zeta)(2 - \xi + \eta - \zeta)/8$$

$$\varphi_7 = -(1 + \xi)(1 + \eta)(1 + \zeta)(2 - \xi - \eta - \zeta)/8$$

$$\varphi_8 = -(1 - \xi)(1 + \eta)(1 + \zeta)(2 + \xi - \eta - \zeta)/8$$

$$\varphi_9 = (1 - \xi)(1 + \xi)(1 - \eta)(1 - \zeta)/4$$

$$\varphi_{10} = (1 + \xi)(1 - \eta)(1 + \eta)(1 - \zeta)/4$$

$$\varphi_{11} = (1 - \xi)(1 + \eta)(1 + \zeta)(1 - \zeta)/4$$

$$\varphi_{12} = (1 - \xi)(1 - \eta)(1 + \zeta)(1 - \zeta)/4$$

$$\varphi_{13} = (1 - \xi)(1 + \eta)(1 - \zeta)(1 + \zeta)/4$$

$$\varphi_{14} = (1 + \xi)(1 - \eta)(1 + \zeta)(1 + \zeta)/4$$

$$\varphi_{15} = (1 - \xi)(1 + \eta)(1 + \zeta)(1 + \zeta)/4$$

$$\varphi_{16} = (1 - \xi)(1 - \eta)(1 + \zeta)(1 + \zeta)/4$$

$$\varphi_{17} = (1 - \xi)(1 - \eta)(1 - \zeta)(1 + \zeta)/4$$

$$\varphi_{18} = (1 + \xi)(1 - \eta)(1 - \zeta)(1 + \zeta)/4$$

$$\varphi_{19} = (1 + \xi)(1 + \eta)(1 - \zeta)(1 + \zeta)/4$$

$$\varphi_{20} = (1 - \xi)(1 + \eta)(1 - \zeta)(1 + \zeta)/4$$

The relationship of nodal points and elements is controlled through the constitutive equations of the defining problem. These are established through a series of independent, partial or fully coupled differential equations and associated boundary conditions. These differential equations establish the related physical relationship to the laws of conservation which control the behavior of many types of problems, such as: elastic, thermal and fluid flow.

Defining an element vector containing all displacements belonging to the element, such as:

$$\{U\}_e = \begin{bmatrix} U_{11} \\ U_{12} \\ U_{13} \\ U_{21} \\ \dots \\ U_{N1} \\ U_{N2} \\ U_{N3} \end{bmatrix}_e$$

So that in three dimensions the element stiffness matrices' properties are derived and represented as follows:

$$[k_m] = \int \int \int [B]^T [D] [B] dx dy dz$$

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}$$

The following 3 x 3 x 3 integration scheme **Figure 31** represents the full-integration of the 20-node hexahedral element:

FIGURE 31 THE 3 X 3 X 3 INTEGRATION POINT SCHEME ON HEXAHEDRAL ELEMENTS

In a reduced integration scheme, the next smallest available integration scheme is chosen. In this case, the reduced integration scheme for a 20-node hexahedral element is the same as that of a fully integrated 8-node element shown in **Figure 32** below. This is represented by a 2 x 2 x 2 integration scheme. This provides the vertex nodal values in which the middle values of the element are obtained by taking the mean of the neighboring vertex nodal values. The process of reduced integration allows for a reduction in the number of unknowns which decreases the size of the linear system in order to increase the efficiency of the solution.

FIGURE 32 THE 2 X 2 X 2 INTEGRATION POINT SCHEME ON HEXAHEDRAL ELEMENTS

Scheme	Location	Number	Weight
1 x 1 x 1	(0,0,0)	1	8
2 x 2 x 2	$\left(\pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}\right)$		
3 x 3 x 3	$\left(\pm \frac{3}{\sqrt{5}}, \pm \frac{3}{\sqrt{5}}, \pm \frac{3}{\sqrt{5}}\right)$	8	$\left(\frac{5}{9}\right)^3$
	$\left(0, \pm \frac{3}{\sqrt{5}}, \pm \frac{3}{\sqrt{5}}\right)$	12	$\left(\frac{8}{9}\right)\left(\frac{5}{9}\right)^2$
	$\left(0, 0, \pm \frac{3}{\sqrt{5}}\right)$	6	$\left(\frac{8}{9}\right)^2\left(\frac{5}{9}\right)$
	(0,0,0)	1	$\left(\frac{8}{9}\right)^3$

TABLE 9 LOCATION OF INTEGRATION POINTS IN HEXAHEDRAL ELEMENTS

4.2.2. GOVERNING EQUATIONS

The solution of the governing finite element equations will yield the displacements at each node. This may then be used to calculate the stresses and strains throughout the element and nodal matrices. Since the accuracy of the solution is greatest at the given integration points, the field variables are typically evaluated at these locations and then extrapolated to the surrounding nodes. This extrapolation is done on an element basis so that for any given node one obtains as many values as the number of elements it belongs to. This usually means that the values are discontinuous at the element border, however this is remedied by calculating the mean value over all elements that the node belongs to. **Table 9** highlights the common integration point techniques used in hexahedral type elements.

$$\sigma_{xxj} = \sum_{i=1}^8 \varphi_i(\xi_j, \eta_j, \zeta_j) \sigma_{xxi}, \quad j = 1, \dots, 8$$

Where i are the nodes and j are the integration points for a reduced integration 20-node hexahedral.

This can be presented in matrix form as follows:

$$\{\sigma_{xx}\}_{integration\ points} = [A]\{\sigma_{xx}\}_{nodes}$$

From this the nodal values can be determined by inverting the previous equation.

$$\{\sigma_{xx}\}_{nodes} = [A]^{-1}\{\sigma_{xx}\}_{integration\ points}$$

The size of $[A]$ in this case (20-node reduced integration hexahedron) is an 8 x 8 matrix which can be evaluated explicitly as both the shape functions and integration point location are known. The explicit definition of the inverse matrix $[A]^{-1}$ is as follows:

$$[A]^{-1} = \begin{bmatrix} +2.549 & -0.683 & -0.683 & +0.183 & -0.683 & +0.183 & +0.183 & -0.049 \\ -0.683 & +2.549 & +0.183 & -0.683 & +0.183 & -0.683 & -0.049 & +0.183 \\ +0.183 & -0.683 & -0.683 & +2.549 & -0.049 & +0.183 & +0.183 & -0.683 \\ -0.683 & +0.183 & +2.549 & -0.683 & +0.183 & -0.049 & -0.683 & +0.183 \\ -0.683 & +0.183 & +0.183 & -0.049 & +2.549 & -0.683 & -0.683 & +0.183 \\ 0.183 & -0.683 & -0.049 & +0.183 & -0.683 & +2.549 & +0.183 & -0.683 \\ -0.049 & +0.183 & +0.183 & -0.6683 & +0.183 & -0.683 & -0.683 & +2.549 \\ +0.183 & -0.049 & -0.683 & +0.183 & -0.683 & +0.183 & +2.549 & -0.683 \end{bmatrix}$$

In order to evaluate the fully integrated case of a 20-node hexahedral then the system becomes over determined as there becomes a system of 27 equations and only 20 unknowns, seen from the element shape function below:

$$\sigma_{xxj} = \sum_{i=1}^{20} \varphi_i(\xi_j, \eta_j, \zeta_j) \sigma_{xxi}, \quad j = 1, \dots, 27$$

In order to solve these types of systems a solution scheme must be chosen. A standard procedure in the solution of an over determined system such as this is the least squares method where the fully integrated equation shown above becomes:

$$b_j = \sum_{i=1}^{20} a_{ji} x_i, \quad j = 1, \dots, 27$$

$$I := \sum_{j=1}^{20} \left(\sum_{i=1}^{27} a_{ji} x_i - b_i \right)^2$$

The least squares approximation allows the solution to be found by the use of typical differentiation techniques:

$$\frac{\partial I}{\partial x_k} = 2 \sum_{j=1}^{27} \left[\left(\sum_{i=1}^{20} a_{ji} x_i - b_i \right) a_{jk} \right] = 0, \quad k = 1, \dots, 20$$

$$\sum_{i=1}^{20} \left[\left(\sum_{j=1}^{27} a_{ji} a_{jk} \right) x_i \right] = \sum_{j=1}^{27} a_{jk} b_j, \quad k = 1, \dots, 20$$

Or can also be defined by the following condensed form instead of that shown above:

$$\sum_{i=1}^{20} c_{ki} x_i = d_k, \quad k = 1, \dots, 20$$

Where the expansion of c_{ki} and d_k is:

$$c_{ki} = \sum_{j=1}^{27} a_{jk} a_{ji}, \quad k = 1, \dots, 20$$

$$d_k = \sum_{j=1}^{27} a_{jk} b_j, \quad k = 1, \dots, 20$$

This produces a solvable system of 20 equations and 20 unknowns. If $\{b_1\}$ is a unit vector with a unit value located in the first row then the solution $\{x_1\}$ contains the nodal values for a unit value in the first integration point and zero in all of the other integration points. This process is repeated for all other integration points throughout the element which yields a 20 x 27 matrix $[B]$ that appears in the equations below:

$$\{\sigma_{xx}\}_{nodes} = [B] \{\sigma_{xx}\}_{integration\ points}$$

Which takes the following form for all of the associated integration points as noted above.

$$[B] = [\{x_1\}\{x_2\} \dots \{x_{27}\}]$$

The specific application of the finite element method to discontinuity modeling is a particularly difficult task. Much of the difficulty in determining these solutions lies in the discretization of the continuum during stages of propagation as the elements are breached by discontinuity. This causes a special case of interpolating integration point values where discontinuity arises.

FIGURE 33 ANALYTICAL SOLUTION OF STRESS DISTRIBUTION SURROUNDING A PRESSURIZED FRACTURE

The determination of slip tendency was performed using a projection of stresses found from the FEM analysis in accordance with **Figure 33** above (Qian Gao, 2013).

$$\frac{1}{2}(\Delta\sigma_y + \Delta\sigma_x) = -p_{net} \left(\frac{r}{\sqrt{r_1 r_2}} \cos(\theta - 0.5\theta_1 - 0.5\theta_2) - 1 \right)$$

$$\frac{1}{2}(\Delta\sigma_y + \Delta\sigma_x) = p_{net} \left(\frac{2r \sin \theta}{H} \right) \left(\frac{H^2}{4r_1 r_2} \right)^{\frac{3}{2}} \sin \left(\frac{3}{2}(\theta_1 + \theta_2) \right)$$

$$\Delta\tau_{xy} = -p_{net} \left(\frac{2r \sin \theta}{H} \right) \left(\frac{H^2}{4r_1 r_2} \right)^{\frac{3}{2}} \cos \left(\frac{3}{2}(\theta_1 + \theta_2) \right)$$

$$\Delta\sigma_z = \nu(\Delta\sigma_y + \Delta\sigma_x)$$

The coulomb failure criterion was used in order to describe the slip tendency of the given fault plane or stress state. These stresses, taken from the finite element stress tensors evaluate their ratio to a generalized failure criterion.

The general approach of Coulomb Failure Criterion is shown below.

$$\tau > \tau_f = S_0 + \mu \sigma_n$$

Where τ_f , a failure criterion is set by a maximum shear stress limit and τ is the measured or calculated shear stress. Therefore, σ_n is the stress normal to the plane and μ is the resistive or frictional coefficient.

Applying the general Coulomb Failure Criterion in terms of principle stresses the following relationship can be determined based upon the equation shown below.

$$\frac{\sigma_1}{\sigma_3} = \frac{S_1 - P_p}{S_3 - P_p} \geq \left[(\mu^2 + 1)^{\frac{1}{2}} + \mu \right]^2$$

Where σ_1 and σ_3 are the maximum and minimum principle stresses, respectively.

To determine the initial optimal angle of faulting, with angle θ to σ_1 .

$$\theta = \frac{1}{2} \tan^{-1} \left(\frac{1}{\mu} \right)$$

The effects of shear and normal stresses across the fault plane are respective to the orientation of the fault.

$$\sigma_{n(\beta)} = \frac{1}{2}(\sigma_1 + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3) \cos 2\beta$$

$$\tau_{n(\beta)} = -\frac{1}{2}(\sigma_1 - \sigma_3) \sin 2\beta$$

The Anderson fault theory is used to determine the principal stress relationships, representative of each type of faulting (i.e. possible reactivation).

$$\text{Normal Faulting} \quad \frac{\sigma_1}{\sigma_3} = \frac{S_v - P_p}{S_{h \min} - P_p} \leq \left[(\mu^2 + 1)^{\frac{1}{2}} + \mu \right]^2$$

$$\text{Strike - Slip Faulting} \quad \frac{\sigma_1}{\sigma_3} = \frac{S_{h \max} - P_p}{S_{h \min} - P_p} \leq \left[(\mu^2 + 1)^{\frac{1}{2}} + \mu \right]^2$$

$$\text{Reverse Faulting} \quad \frac{\sigma_1}{\sigma_3} = \frac{S_{h \max} - P_p}{S_v - P_p} \leq \left[(\mu^2 + 1)^{\frac{1}{2}} + \mu \right]^2$$

Effective overburden pressure at a given depth is given below.

$$\sigma_v = \rho g z (1 - \lambda)$$

This is used to determine the limiting conditions in regards to the three types of faulting methods.

4.3. TRANSIENT METHODOLOGY

4.3.1. GOVERNING EQUATIONS

There are many difficulties that arise when dealing with geomechanical fracture mechanics, especially those introduced through hydraulic means. Aside from the numerical discretization of the domain, the propagation of a fracture fluid front is difficult to track as it propagates through the domain and causes a discontinuity between once adjoining nodes and elements. In order to overcome this an M integral method is used for fracture propagation.

Application of the finite element method to time-dependent problems modifies the original formulation of the form:

$$-\nabla^2 u = f$$

To the following:

$$\frac{\partial u}{\partial t} - \nabla^2 u = f$$

The equations governing the response of fluids and solids alike obey the principles of conservation of mass, momentum, and energy. In fluid flow when considering an arbitrarily small volume the following relationships hold true. In order to correctly model the interactions that occur throughout the fracturing process a hydro-mechanical coupling must be made in order to distinguish and relate these principle relationships to each other.

The mathematical model responsible for describing incompressible or Newtonian fluids is the system of Navier-Stokes equations. This system of equations incorporates the conservation of mass, momentum and energy.

$$\partial_t \rho + \nabla \cdot [\rho v] = 0$$

$$\partial_t(\rho v) + \rho v \cdot \nabla v - \nabla \cdot \left[\mu \nabla v + \frac{1}{3} \nabla \cdot v I \right] + \lambda p_{tot} = \rho f$$

$$\partial_t(c_p \rho T) + c_p \rho v \cdot \nabla T - \nabla \cdot [\lambda \nabla T] = h$$

Where v is the velocity, ρ is the density, p_{tot} is the (total) pressure, and T is the temperature of the fluid occupying the two- or three-dimensional region Ω .

LINEAR ELASTIC FRACTURE MECHANICS

The concept of linear elastic fracture mechanics (LEFM) follows a point of view using that of the minimum potential energy established by Griffith. This theory identifies two necessary conditions for crack growth:

1. The bonds located at the tip of the crack must be stressed to the point of failure. The failure determination is a function of the stress concentration factor relating the radius of curvature to its length.
2. For crack extension to occur over an increment then the amount of energy that is released must be greater than or equal to the required surface energy of the two new crack faces.

These principles established by Griffith identify an energy balance relationship while the determination of crack extension was further identified by Irwin in which the stress intensity factor quantifies the magnitude of the effects from the stress singularities at the crack tip.

CONTINUITY EQUATION

The continuity equation, otherwise known as the conservation of mass relates the sum of all incoming and outgoing masses.

Rate of Increase of q in CV	=	Rate at which q enters through the surface of	-	Rate at which q leaves through the surface of CV	+	Net sum of sources and sinks of q within CV
-----------------------------	---	---	---	--	---	---

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0$$

In the case of an incompressible fluid within the fracture, then the fluid mass conservation can be written as follows:

$$\frac{\partial w}{\partial t} + \frac{\partial q}{\partial x} + g = 0$$

Where q is the fluid flux and g is the leak-off term.

Integrating this equation over the domain with respect to x yields the following equation:

$$\frac{\partial}{\partial t} \int_x^l w dx + \int_x^l g dx = q$$

Then taking the second time integral in order to develop the global fluid continuity equation over the entire domain.

$$2 \int_0^l w dx + 2 \int_0^t \int_0^{l(t')} g dx dt' = Q_o t$$

MOMENTUM EQUATION

The momentum equation, much like that of the continuity equation relates the sum of all incoming and outgoing rates.

Rate of Increase of x-component momentum of fluid within CV	-	Rate of flux of x-component momentum into CV	+	Rate of flux of x-component momentum out of CV	=	Sum of x-components of forces applied to CV
---	---	--	---	--	---	---

$$\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u^2}{\partial x} + \frac{\partial \rho uv}{\partial y} + \frac{\partial \rho uw}{\partial z} = -\frac{\partial p}{\partial x} + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + \rho f_x$$

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = 0$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \frac{\partial \sigma_{xx}}{\partial x} + \frac{1}{\rho} \frac{\partial \sigma_{xy}}{\partial y} + \frac{1}{\rho} \frac{\partial \sigma_{xz}}{\partial z} + f_x$$

One of the three Navier-Stokes equations is as follows.

The flow stiffness matrix is then calculated by the following:

$$k_{f-f} = \frac{\partial F_f}{\partial x_f}$$

The maximum tangential stress criterion (MTS) is used in the determination of the fracture propagation angle by following the assumption that the crack will propagate in the direction of the maximum tangential stress. This tangential stress is calculated in throughout a sufficiently small radius surrounding the crack tip. The following equation was determined for the angle of crack propagation θ as follows:

$$\tan \frac{\theta}{2} = \frac{-2K_{II}}{K_I + \sqrt{(K_I)^2 + 8(K_{II})^2}}$$

Laplace's Equation (the potential equation) which arises in incompressible fluid flow (in which case ϕ is the velocity potential), gravitational potential, electrostatics, magnetostatics, steady-state heat conduction with no sources (in which case ϕ is the temperature) and torsion of bars in elasticity (in which case $\phi(x, y)$ is the warping function). Functions which satisfy Laplace's equation are also referred to as harmonic functions.

$$\nabla^2 \phi = 0$$

$$\nabla = e_x \frac{\partial}{\partial x} + e_y \frac{\partial}{\partial y} + e_z \frac{\partial}{\partial z}$$

$$\nabla^2 = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

$$\nabla^2 \phi = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \phi}{\partial r} \right) + \frac{1}{r^2}$$

4.3.2. COUPLED INTEGRATION

The transient approach to fracture propagation requires an entirely different approach to solution due to the added processes occurring which govern fracture propagation and nodal discontinuity throughout the FEM mesh. This can be idealized to four main physical processes which govern fracture propagation.

1. Fracture Opening* 2. Fluid Flow Within the Fracture* 3. Fluid Leak-Off into the Formation 4. Fracture Propagation	Dependent on the net fluid pressure within the fracture Dependent upon the fracture width and fluid pressure

A coupling procedure is used for processes 1 and 2 in the following manner:

$$F_1(w, p_f) = 0$$

$$F_2(w, p_f) = 0$$

The solid media, made up of elements representing the reservoir follow the governing equations described below for equilibrium, constitutive law and geometry. This controls the behavior of the solid elements.

$$\text{Equilibrium: } \nabla \cdot \sigma + b = 0$$

$$\text{Constitutive Law: } \sigma(x) = D: \varepsilon(x)$$

$$\text{Geometry: } \varepsilon(x) = [\nabla(u) + (u)\nabla 0]/2$$

Where ∇ is the divergence operator, Δ is the gradient operator and $(:)$ is the double dot product operator, σ is the stress, b is gravity, D is the elastic stiffness tensor, ε is the strain and u the displacement.

The determination of fracture opening depends upon the interaction of the fluid elements and the solid elements, along with their associated interface interaction. The fracture opening is controlled via the following relationship:

$$w(x, t) = [u(x^+, t) - u(x^-, t)] \cdot n$$

Where n is the unit normal of the fracture and $x^+ = x^-$ before the fracture reaches $(x, 0)$.

The boundary interactions occurring at the material interface are controlled via the following relationship:

$$\sigma \cdot n = p_f(x, t), x \in S_f$$

Where $u_x = 0$ at $x = 0$ and $\sigma = \sigma_0$, and σ_0 is the confining stress.

FIGURE 34 MATHEMATICAL MODEL OF SOLID ELEMENT FRACTURE OPENING

The combined effects for determining fracture opening are concluded with the following equation governing the solid elements opening.

$$w(x, t) = \int_{l_t} F(x_1; x) (p_f(x_1, t) - \sigma_0) dx_1$$

For a plane strain, uniform and isotropic media then the integral F is defined by the following.

$$F(x_1; x) = \frac{4}{\pi E'} \ln \left| \frac{\sqrt{l_t^2 - x^2} + \sqrt{l_t^2 - x_1^2}}{\sqrt{l_t^2 - x^2} - \sqrt{l_t^2 - x_1^2}} \right|$$

Where $F(x_1; x)$ is the non-local kernel function, l_t is the half length at time t and E' is the plane modulus.

Fracture propagation is determined using the mode I failure that was discussed earlier (see **Figure 16**) which defines tensile fracture. This determination of fracture propagation is done through a stress intensity factor and corresponding fracture toughness.

This failure criterion is defined as follows:

$$K_I > K_{IC}$$

Where K_I is the stress intensity factor (SIF), $K_I = K_I(p)$ and K_{IC} is the fracture toughness.

The mathematical fluid model is governed via the following set of equations for mass conservation and fluid flow in accordance with Carter's leak-off model. The key assumptions of the mathematical fluid model are that of incompressibility, laminar flow and a Newtonian fluid. **Figure 34** displays the mathematical model of the solid elements governing fracture opening.

$$\text{Mass Conservation: } \frac{\partial w}{\partial t} + \nabla \cdot q + g = 0$$

$$\text{Fluid Flow: } q = -\frac{w^3}{12\mu} \nabla p_f$$

$$\text{Leak Off: } g(x, t) = \frac{2C_l}{\sqrt{t - t_0(x)}}$$

Where μ is the fluid viscosity, C_l is the leak-off coefficient and t_0 is the fracture tip arrival time.

The development of a coupled model then requires that the fluid and solid element interactions be combined to account for their interactions throughout the solution. The solid coupling over the boundary is known as the elastic response.

The coupling equations for the elastic response are defined below:

$$w(x, t) = \int_{l_t} F(x_1; x) (p_f(x_1, t) - \sigma_0) dx_1$$

$$w_i = \sum_j T_{ij} (p_f - \sigma_0)_j$$

$$\mathbf{F}_1(\mathbf{w}, \mathbf{p}) = \mathbf{w} - \mathbf{T}\mathbf{p} + \mathbf{C}_0 = \mathbf{0}$$

Where T_{ij} is the fracture width at point i that is induced by unit pressure applied on points j .

FIGURE 35 COUPLED SOLID AND HYDRAULIC INTERACTION, ELASTIC RESPONSE

$$K_u U = F_j$$

$$T_{ij} = (u^{i+} - u^{i-}) \cdot n$$

The coupled equations for fluid flow are defined as follows using the boundary conditions defined by the mathematical fluid model. This interaction is set forth in **Figure 35** above. In combination with FEM the following relationships are established:

$$K_w(W)P + L\dot{W} + H = 0$$

Where W is a vector formed by the widths of the nodes on the fracture surface, Where $K_w(W)$ is the assembly of the flux stiffness of the fluid elements, L is the assembly of the length stiffness of the fluid elements and H includes the contributions of the fluid leak-off and the fluid injection.

Integrating the equation seen above with time yields the following expression.

$$\int_{t_n}^{t_{n+1}} [K_w(W) + L\dot{W} + H] dt = 0$$

$$K_w(W_{n+1})(P_{n+1})\Delta t + L(W_{n+1} - W_n) + H\Delta t = 0$$

Where w_{n+1} and P_{n+1} are the unknown fracture widths and net fluid pressures at the $(n + 1)$ th step, respectively, W_n is the known fracture width at the n th step and Δt is the time step between the n th step and the $(n + 1)$ step.

$$W_{n+1} = W_n + \Delta W, \quad P_{n+1} = P_n + \Delta P$$

Where ΔW is a vector formed by the width increments of the nodes on the fracture surface.

At any moment then we have the following:

$$\Delta w(x) = w \Delta u_y(x), \quad 0 \leq x \leq l_t$$

Where Δu_y the displacement increment in the y direction on the fracture is surface in the equivalent quarter model and Δw is the fracture width increment. Therefore the following generalization can be made as shown below.

$$K_w(U_n + \Delta U)(P_n + \Delta P) - L' \Delta U + H \Delta t = 0$$

Where L' determines the contribution of the node displacement increments to the fracture width increments.

$$F_2(w_{n+1}, p_{n+1}; w_n, p_n) = 0$$

The solution of the coupled system is then found using a Newton-Raphson method. The first term in this solution F_1 is linear and the second term F_2 is non-linear. The Newton-Raphson method is used in order to solve the nonlinear coupled equations for the width and pressure at the new time level as follows:

$$F_1(w, p) = w - Tp + C_0 = 0$$

$$F_2(w_{n+1}, p_{n+1}; w_n, p_n) = 0$$

$$w_{n+1} \quad p_{n+1}$$

In each case then the initial guess of ΔW and ΔP are set to be zero for the Newton-Raphson iteration algorithm. The following criteria is used for convergence:

$$\|\Delta W^{(m+1)} - \Delta W^{(m)}\| / \|\Delta W^{(m+1)}\| \leq \varepsilon_{tol}$$

Where $\| \cdot \|$ is the two-norm operator, $\Delta W^{(m+1)}$ and $\Delta W^{(m)}$ are the results of the fracture width increments after the $m+1$ th iteration and the m th iteration, respectively, and ε_{tol} is the specified tolerance which equals $1.0e - 5$.

FIGURE 36 NUMERICAL ASPECTS OF FRACTURE PROPAGATION WITHOUT REMESHING

The fracture propagation length is determined by the element size along the fracture tip which is defined in **Figure 36**. This strategy allows for fracture propagation to be determined without the requirement of remeshing throughout the domain as the fracture propagates. The use of a local refinement method such as the one use here aids in increasing the efficiency and reducing the time that is spent on remeshing by reducing CPU costs by more than 60% in some cases (Mohammadi, 2008).

This solution strategy is only conditionally stable and therefore the time step Δt is selected such that the following criteria is met.

$$K_I \leq (1.0 + \varepsilon_{tol}^S) K_{IC}$$

Where ε_{tol}^S is the allowable tolerance for the SIF.

The calculation of the stress intensity factor (SIF) is done through the use of the M-Integral method. According to the M-integral method, the approximation of the SIF in accordance with the plane strain model is as follows:

$$K_I = \frac{2E}{1-\nu} \left\{ \int_A \left[\sigma_{ij} \frac{\partial u_i^c}{\partial x_1} + \sigma_{ij}^c \frac{\partial u_i}{\partial x_1} - \sigma_{mn}^c \varepsilon_{mn} \delta_{1j} \right] \frac{\partial \chi}{\partial x_j} dS - \int_{S_e} \chi p \frac{\partial u_i^c}{\partial x_1} dL \right\}$$

Where E is the elastic modulus, ν is the Poisson's ratio, the domain A of the integral surrounding the fracture tip is shown in the figure below, S_e is a set of edges of the finite elements in set A and these edges coincide with the fracture surface, σ_{ij} is the stress, u_i is the local displacement, x_j is the local coordinate, δ_{ij} is the Kronecker delta, χ is a scalar field, ε_{mn} is the strain, and σ_{ij}^c and u_i^c are the auxiliary stress and displacement, respectively.

The characteristic radius surrounding the fracture tip is defined with the relationship below within the domain A .

$$r_c = \sqrt{A_{tip}}$$

FIGURE 37 DOMAIN OF THE INTERACTION INTEGRAL AT THE FRACTURE TIP

As mentioned previously, the technique applied for this solution does not require a remeshing of the domain. This node-split technique allows for the fracture to propagate as depicted in **Figure 37** above and if fracture or failure conditions are met over that element then the original node is split into two separate nodes if the fracture propagates. If the fracture is determined to propagate then the fracture tip is pushed forward to the next node in front of the recent split node pair such that the node ahead of the fracture tip is always the potential fracture tip.

4.4. COHESIVE ZONE MODEL

The cohesive zone model is an approach to idealize the Mode I type of fracture opening for three dimensional analysis, neglecting in-plane and out-of-plane shear. This approach identifies a prospective fracture path prior to injection, along which the fracture is expected to propagate. This area is subject to local mesh or grid refinement in order to correctly map the large fluctuations that occur near the area of injection and surrounding the fracture tip. The propagation and further geometry of the fracture is then controlled by the geological properties, resistance to fracture and the hydraulic injection properties.

The cohesive zone model in three dimensions essentially states an initial 2-D fracture plane within the 3-D model that can be seen in **Figure 38**. This is known as a zero thickness element which can be implemented to the classic finite element framework. This cohesive zone is prescribed as having zero width between the surrounding solid elements comprising the mesh. As fluid pressure is applied to the injection point then this 2-D plane will begin to separate, thus expanding to 3-D like a balloon if the conditions for fracture advancement are met. This process is continued for each solution time step until either a prescribed time or the end of propagation by encroaching upon the edge of the domain boundary.

FIGURE 38 COHESIVE ELEMENT FRACTURE PLANE

The structure of the 2-D finite element code remains relatively intact with the use of the cohesive zone model. The main difference is in the element discretization and thus the shape functions, using an 8 node solid element for reservoir domain discretization and a 4 node hydraulic element for fluid flow and fracture propagation. For the case of a single fracture the 2-D fracture plane is “sandwiched” between the solid elements as depicted in **Figure 39**.

FIGURE 39 THREE DIMENSIONAL COHESIVE ZONE MODEL DISCRETIZATION FOR A SINGLE FRACTURE

The same technique is applied for multi-fracture cases. For these simultaneous fracture cases the fracture plane is stipulated at a pre-determined interval, between solid reservoir elements. Figure 40 below depicts the case of three fractures where the injection rate is set at a constant rate divided between the three fracture planes. The expansion of these cohesive zone elements will be discussed in further detail in **Chapter 6.2.3**. Transient Changes in 3-D Fracture Profile

FIGURE 40 THREE DIMENSIONAL COHESIVE ZONE MODEL DISCRETIZATION FOR MULTIPLE FRACTURES

CHAPTER 5 - PRELIMINARY INVESTIGATION

5.1. INTRODUCTION

The preliminary results reflect the modifications of stress field distributions throughout a reservoir based upon in-situ stress distributions described using a critical stress assumption. As hydraulic fracturing techniques are applied to the reservoir they alter the in-situ stress fields within the reservoir and in turn, this can affect the associated stability surrounding discontinuity in these areas of altered stress.

The fundamental principles of mechanical deformation were used to investigate the stress relationships within the reservoir using finite element analysis in order to generate a three-dimensional model of distributed stress elements. These modified stress fields were compared using failure criteria that are consistent with the Anderson theory of faulting and applied Coulomb Failure Criterion. Fault activity is reliant upon the in-situ stress distributions occurring at a depth below the earth's surface and can have the potential to reactivate when hydraulic fracturing alters these initial stress regimes. The two fundamental principles affecting these redistributions are the increase in minimum stresses, due to the poroelastic effect and the stress increase due to the opening and interconnecting of induced and natural fracture(s).

Application of these theories provides simplified approach to reservoir mechanics in order to provide a generalized solution that is not specific to certain cases. The generality of these assumptions provides a versatile solution which does not require complex parameters that are often difficult to determine and correlate accurately.

5.2. MODEL REPRESENTATION

The preliminary model representation was done using the finite element method to investigate stress distributions throughout the reservoir and their relation to faults, fracture and slip tendency. The modes of failure are indicated based upon modifications to the in-situ stress fields surrounding areas of specified discontinuity, which in this case is a static predetermined hydraulic fracture. Stress field modification creates unbalanced stress states which can lead to increased slip tendency. If a critically stressed state is assumed throughout the body then these stress zones can be determined based upon the inherent changes that occur under different loading conditions. This critically stressed state theory is typically assumed in the study of geomechanical cases. The reservoir and fracture properties are defined in **Table 10** and **Table 11**, respectively.

The solution follows a specified preconditioning (SSOR-PCG) method in order to expedite the analysis procedure which is done using a conjugate gradient algorithm. The preconditioned conjugate gradient method does not require the assembly of a global stiffness matrix and instead solutions are found on an element-by-element basis. This is particularly useful when solving for nonlinear or transient solutions as the material constants and mesh requirements can change without the requirements of re-assembling the global coefficient matrix.

Reservoir Properties		
Geometry	4000 x 4000 x 800	Length/Width/Height, Ft.
Depth of Top	5794.15	Ft.
Young's Modulus	4.02E+6	PSI
Poisson's Ratio	0.209	
Bulk Density	156	Lb/Ft ³
Vertical Stress	6277	PSI
Maximum Horizontal Stress	8673	PSI
Minimum Horizontal Stress	4616	PSI
Reservoir Pore Pressure	0.433	PSI/Ft.

TABLE 10 PRELIMINARY RESERVOIR PROPERTIES

Fracture Properties		
Geometry	200 x 200	Length/Height, <i>Ft.</i>
Fracture Spacing	100	<i>Ft.</i>
Number of Fractures	1 or 3	
Net Hydraulic Pressure	1000	<i>PSI</i>

TABLE 11 PRELIMINARY FRACTURE PROPERTIES

5.3. NUMERICAL VERIFICATION

The first preliminary evaluation was done with a simple fracture model allowing stress modifications induced by a single fracture presented into the reservoir. This solution is based upon a static equilibrium in which there is no fracture prior to the induced hydraulic pressure. Again, the final analysis is calculated post fracturing for a pre-determined fracture geometry. This ignores the dynamic stress regimes occurring throughout the induced fracture stage. The results that are provided below are based upon previous work presented through the published paper and compared against the most recent renderings of the same software in order to ensure correlation between the results.

The stress diagrams for typical Andersonian faulting environments have been determined for a single fracture model. In presenting previous results, a slip tendency of 0.6 was used in order to represent the coefficient of sliding friction. This was established based upon research done by (Byerlee 1978) which established the coefficient of friction for a wide variety of rocks was within a small range: $0.6 \leq \mu \leq 1.0$. The influence of surface roughness, rock type and normal stress were found to have little to no effect on the effect of friction and thus sliding tendency based upon the aforementioned research.

The preliminary data determined that the distribution of shear and normal stresses around a single fracture in a normal faulting environment tends to decrease along the distance perpendicular to the fracture face, indicating that these zones become more stable. However, parallel to the fracture face the slip-tendency tends to increase, indicating that they become more unstable and thus increase the potential to slip in this region. This observation was said to be applicable to both normal and strike-slip faulting environments (Gao, 2012). These results are verified in below in the following two sections.

5.3.1 SINGLE FRACTURE MODEL

NORMAL FAULTING ENVIRONMENT

Figure 41 (a) normalized shear stress, normal to the z-axis and **(b)** normalized shear stress, normal to the y-axis below represents shear stress distributions in a normal faulting environment in a homogenous environment surrounding a single fracture. It can be seen from the distribution on the diagrams that the shear stress magnitude (dark blue) have decreased slip tendency. The magnitude of shear stress is seen to be amplified along the tips of the fracture while a reduction in shear stress is seen across the body of the fracture.

FIGURE 41 RATIO OF SHEAR STRESS BY NET PRESSURE SURROUNDING A SINGLE FRACTURE IN A NORMAL FAULTING ENVIRONMENT

The normal stress surrounding the fracture is seen below in **Figure 42 (a)** normalized normal stress, normal to the z-axis and **(b)** normalized normal stress, normal to the y-axis. This indicates that the stresses are highest around the body of the fracture in this environment. The skewed nature of these stresses is representative of the ratios of σ_{hmin} and σ_{Hmax} . This normal stress shown in orange/red have an increased slip tendency.

FIGURE 42 RATIO OF NORMAL STRESS BY NET PRESSURE SURROUNDING A SINGLE FRACTURE IN A NORMAL FAULTING ENVIRONMENT

STRIKE-SLIP FAULTING ENVIRONMENT

The following results present the stresses determined for a strike-slip faulting environment where σ_{hmin} and σ_{Hmax} are switched from the results shown above for a normal faulting environment. **Figure 43 (a)** normalized shear stress, normal to the z-axis and **(b)** normalized shear stress, normal to the y-axis displays the effects of shear stresses upon the fracture. This also indicates the role that the principle stresses play in the stress surrounding the fracture in comparison with **Figure 41**.

FIGURE 43 RATIO OF SHEAR STRESS BY NET PRESSURE SURROUNDING A SINGLE FRACTURE IN A STRIKE-SLIP FAULTING ENVIRONMENT

Figure 44 (a) the normal stress, normal to the z-axis and **(b)** the normal stress, normal to the y-axis displays the effects of the normal stress surrounding the fracture. This displays a similar pattern to that seen in the shear stresses above due to the way in which the principal stresses are oriented. This again displays significantly different results than the normal faulting environment.

FIGURE 44 RATIO OF SHEAR STRESS BY NET PRESSURE SURROUNDING A SINGLE FRACTURE IN A STRIKE-SLIP FAULTING ENVIRONMENT

FAULT REACTIVATION / SLIP-TENDENCY

The following Figure 45 **(a)** slip tendency, normal to the z-axis and **(b)** slip tendency, normal to the y-axis display the distributions of slip-tendency in a normal faulting environment. The contour lines on both the 2D models represent a 0.6 slip-tendency (reactivation factor), as discussed earlier. The diagrams below indicate that in the regions surrounding the fracture tips the zones can be said to decrease in stability while the zones lying adjacent to the fracture tend to increase in stability.

FIGURE 45 SLIP-TENDENCY FOR A NORMAL FAULTING ENVIRONMENT SURROUNDING A SINGLE FRACTURE, TS=0.6

The pattern described above becomes more visible with the aid of the 3D diagram shown below in **Figure 46**. This is a representation of the reservoir volume where the zones highlighted in red are indicative of a particular reactivation factor in a critically stressed state.

FIGURE 46 3-D SLIP-TENDENCY FOR A NORMAL FAULTING ENVIRONMENT AND SINGLE FRACTURE, TS=0.6

The distributions of slip-tendency in Figure 47 **(a)** slip tendency, normal to the z-axis and **(b)** slip tendency, normal to the y-axis for a strike-slip faulting environment. The contour lines on both 2D models represent the critically stressed conditions surrounding a single fracture in a homogenous reservoir domain.

FIGURE 47 SLIP-TENDENCY FOR A STRIKE-SLIP FAULTING ENVIRONMENT AND SINGLE FRACTURE, TS=0.6

The pattern for a strike-slip environment is shown below in **Figure 48** using a contour plot of the slip tendency factor in a critically stressed state. The strike-slip environment is shown to extend its instability towards the outer extents of the reservoir domain

FIGURE 48 3-D SLIP-TENDENCY FOR A STRIKE-SLIP FAULTING ENVIRONMENT AND SINGLE FRACTURE, TS=0.6

5.3.2 THREE FRACTURE MODEL

NORMAL FAULTING ENVIRONMENT

The identical simulation was carried out for a set of three fractures as was in the base of a single fracture in the previous section. The shear and normal stress are shown below for the case of a normal faulting environment in **Figure 49 (a)** shear stress surrounding three fractures, normal to the z-axis and **(b)** shear stress surrounding three fractures, normal to the y-axis. For a normal faulting environment the stress is distributed non-uniformly for both shear and normal stresses, however the normal stress depicts an increase in slip tendency while the shear stress shows a decrease in slip tendency across the normal faulting environment.

FIGURE 49 RATIO OF SHEAR STRESS BY NET PRESSURE SURROUNDING MULTIPLE FRACTURES

Figure 50 (a) normal stress surrounding three fractures, normal to the z-axis and **(b)** normal stress surrounding three fractures, normal to the y-axis displays the normal stresses surrounding three congruent fractures in a normal faulting environment. This displays a similar trend as those depicted in the single fracture case. This static analysis does not lend insight to many of the interactions that will be portrayed later in this thesis regarding these transient stress dynamic.

FIGURE 50 RATIO OF SHEAR STRESS BY NET PRESSURE SURROUNDING MULTIPLE FRACTURES

STRIKE-SLIP FAULTING ENVIRONMENT

The shear and normal stress are shown below for a strike-slip faulting environment. **Figure 51 (a)** shear stress surrounding three fractures, normal to the z-axis and **(b)** shear stress surrounding three fractures, normal to the y-axis depict these results. The stresses are distributed uniformly across all three fractures for both normal and shear stresses. Again, the normal stress depicts an increase in the slip tendency while the shear stress depicts a decrease in slip tendency across the strike-slip faulting environment.

FIGURE 51 RATIO OF SHEAR STRESS BY NET PRESSURE SURROUNDING MULTIPLE FRACTURES

Figure 52 (a) shear stress surrounding multiple fractures, normal to the z-axis and **(b)** shear stress surrounding multiple fractures, normal to the y-axis represent the shear stress distributions for a strike slip faulting environment. These results indicate a similar behavior to that seen in the single fracture case with a uniform distribution spread across all three fractures.

FIGURE 52 RATIO OF SHEAR STRESS BY NET PRESSURE SURROUNDING MULTIPLE FRACTURES

FAULT REACTIVATION / SLIP-TENDENCY

The distribution of slip tendency is shown below in Figure 53 (a) slip tendency surrounding three fractures, normal to the z-axis and (b) slip tendency surrounding three fractures, normal to the y-axis for a normal faulting environment. Just as in the single fracture case a symmetrical slip tendency profile is seen with respect to the fracture tips.

FIGURE 53 SLIP-TENDENCY FOR A NORMAL FAULTING ENVIRONMENT SURROUNDING MULTIPLE FRACTURES, TS=0.6

Figure 54 below depicts the three dimensional slip tendency profile in the case of three fractures in a critically stressed state. This displays an expansion of the slip profile, however it is observed this is due to the additional fractures and not to the interaction of stresses as the profile does not change.

FIGURE 54 3-D SLIP-TENDENCY FOR A NORMAL FAULTING ENVIRONMENT AND MULTIPLE FRACTURES, TS=0.6

Slip tendency for a strike-slip faulting environment is depicted in **Figure 55 (a)** slip tendency surrounding three fractures, normal to the z-axis and **(b)** slip tendency surrounding three fractures, normal to the y-axis. These slip tendency profile are again reflective of the same observations shown in the single fracture case.

FIGURE 55 SLIP-TENDENCY FOR A STRIKE-SLIP FAULTING ENVIRONMENT SURROUNDING MULTIPLE FRACTURES, TS=0.6

A three-dimensional slip tendency plot is shown in **Figure 56** below for a three fracture case in a strike-slip faulting environment. This reflects the same results as shown in the single fracture case in **Figure 48**. There is a slight expansion of this profile but this is attributed to the extra 100 feet to each side of the center fracture due to the fracture spacing.

FIGURE 56 3-D SLIP-TENDENCY FOR A STRIKE-SLIP FAULTING ENVIRONMENT AND MULTIPLE FRACTURES, TS=0.6

5.4. MULTIPLE LAYER MODEL

The previous investigation of fault reactivation was used in determining the slip tendency surrounding fractures in a homogenous single layer domain. This was expanded to apply this stress and slip tendency calculation to cases of multiple layers to investigate these effects. The assumption in the static model of a constant height and length does not introduce any complexities to the solution which would concern real world physical interactions. In the following case the reservoir domain was split into three regions so that the effects of layering could be seen as prescribed in **Figure 57** below.

FIGURE 57 DIMENSIONING LAYERING GEOMETRY OF PRELIMINARY MODEL

Formation Type	Elastic Modulus	Poisson's Ratio	Bulk Modulus
Shale	27.717 E+09	0.209	0.2449 E+05
Limestone	55.020 E+09	0.300	0.2653 E+05

TABLE 12 STATIC MODEL LAYERING PARAMETERS

This discretization of the reservoir domain was done such that it encapsulates the fracture in the shale formation, as would occur between two cap-rocks which in this case were chosen to be a limestone material. This idea ties a basic similarity to the confines of the Marcellus shale between the Tully and Onondaga limestone's. These material properties of the model layering are defined in **Table 12** below.

Reservoir Properties		
Geometry	4000 x 4000 x 800	Length/Width/Height, <i>Ft.</i>
Depth of Top	5794.15	<i>Ft.</i>
Young's Modulus	See Table 12	
Poisson's Ratio	See Table 12	
Bulk Density	156	<i>Lb/Ft³</i>
Vertical Stress	6277	<i>PSI</i>
Maximum Horizontal Stress	8673	<i>PSI</i>
Minimum Horizontal Stress	4616	<i>PSI</i>
Reservoir Pore Pressure	0.433	<i>PSI/Ft.</i>

TABLE 13 GENERAL RESERVOIR PROPERTIES OF THE STATIC LAYERED MODEL

The individual layer properties of the three dimensional static model are defined by **Table 12** below. These properties were determined using simplified stratigraphy of traditional shale basins.

The properties of the static fracture geometry are the same as those prescribed in the previous section. The general fracture geometry is defined in **Table 14** below.

Fracture Properties		
Geometry	200 x 200	Length/Height, <i>Ft.</i>
Number of Fractures	1	
Net Hydraulic Pressure	1000	<i>PSI</i>

TABLE 14 STATIC LAYERED MODEL FRACTURE PROPERTIES

The application of slip tendency or stability analysis is an interesting topic and is used throughout many different fields and for different applications. In this case the prescribed model is interesting with its effects to laying and the integrity of the cap formations whether they are protruded with any faults or discontinuity throughout their stratigraphy. The pay zones in shale formations are typically constrained with these types of boundaries as those displayed in the model above and therefore become integral to incorporating these effects on slip tendency analysis and stress redistributions.

The effects of the layering become apparent in **Figure 58 (a)** cross-sectional stress magnitude normal to the x-axis at the center of fracture and **(b)** cross-sectional stress magnitude normal to the z-axis. These diagrams of the stress magnitude clearly indicate the interactions that occur surrounding the fracture as well as an indication to the effects of layering.

The stress magnitude is expanded within the confined zone specified by the weaker material which will be said to be shale. This expansion of the stress shows that the dissipation of stresses created by the fracture show that a greater impact area is caused.

FIGURE 58 CROSS-SECTIONAL STRESS MAGNITUDE SURROUNDING A VERTICAL FRACTURE IN LAYERED MEDIA

Figure 59 below indicates the stress magnitudes in comparison with a single layered homogenous material. This case depicts the stress magnitude in the same scale as that shown in **Figure 58**. This displays a drastic increase in the stresses surrounding the fracture by the inclusion of the material boundaries. The stress differences surrounding this fracture are seen to display an increase in stress in the range of 6 times that of the homogenous case (21,000 psi versus 3,500 psi).

FIGURE 59 CROSS-SECTIONAL STRESS MAGNITUDE SURROUNDING A VERTICAL FRACTURE IN HOMOGENOUS MEDIA

Figure 60 displays the stresses corresponding to the stress magnitudes within the single fracture case only. Here the distribution of stress is clearly evident when scaled relative to its maximum observed stresses.

FIGURE 60 CROSS-SECTIONAL STRESS MAGNITUDE SURROUNDING A VERTICAL FRACTURE IN A HOMOGENOUS MEDIA, RELATIVE MAGNITUDE

NORMAL FAULTING ENVIRONMENT

In the limestone formation the slip tendency is seen to contract across the x-axis whereas the shale formation this is seen to expand. This can be inferred that the stronger or stiffer formation is able to direct these higher stress loads across a smaller area prior to an expected failure. Since the determination of slip tendency is based upon a ratio shear stress to relative normal stress then this is to be expected as the stronger material can withstand a higher limit however it is the same ratio of 0.6 which determines instability.

The **Figure 61** below identifies the slip tendency surrounding the confined layering case in which the sale is surrounded by limestone. In comparison with the single layer case in **Figure 62** it is apparent that the slip tendency is exaggerated by the confinement between layers. The tendency to experience failure is much higher in the outer boundaries of the confining layers that it is within a single layer.

FIGURE 61 CROSS-SECTIONAL SLIP TENDENCY FOR A NORMAL FAULTING ENVIRONMENT IN LAYERED MEDIA

FIGURE 62 CROSS-SECTIONAL SLIP TENDENCY FOR A NORMAL FAULTING ENVIRONMENT IN HOMOGENOUS MEDIA

In order to visualize the slip tendency profile throughout the reservoir a 3-D view is used with a contour plot at the point of failure for a critically stressed zone using a coefficient of 0.6. These profiles in Figure 63 (a) and (b) clearly indicate the change in layers throughout the slip tendency contour plot. In the shale layer, the failure zone is expanded whereas in the limestone layers it is closer to the fracture profile.

FIGURE 63 3-D SLIP-TENDENCY FOR A NORMAL FAULTING ENVIRONMENT AND MULTIPLE LAYERS

Figure 64 and **Figure 65** display an assembly type view of the fracture for stress magnitude and slip tendency, respectively. The profile of slip tendency is displayed to the left of the cross-section normal to the x-axis. This displays that the slip tendency is not necessarily confined to the areas of high stresses as the slip tendency profile extends beyond the range of the highest stress magnitudes.

FIGURE 64 ASSEMBLY VIEW OF STRESS SURROUNDING A SINGLE FRACTURE IN A NORMAL FAULTING ENVIRONMENT

FIGURE 65 ASSEMBLY VIEW OF SLIP TENDENCY SURROUNDING A SINGLE FRACTURE IN A NORMAL FAULTING ENVIRONMENT

STRIKE SLIP FAULTING ENVIRONMENT

In the limestone formation the slip tendency is again seen to contract across the x-axis whereas the shale formation this is seen to expand. The same conclusions can be drawn for this behavior as was done in the normal faulting case. The same relationship where the slip tendency seems to be exxagerated, **Figure 66** for a confined layering case versus a single layer case, **Figure 67** as was in the normal faulting case.

FIGURE 66 CROSS-SECTIONAL SLIP TENDENCY FOR A STRIKE SLIP FAULTING ENVIRONMENT IN LAYERED MEDIA

FIGURE 67 CROSS-SECTIONAL SLIP TENDENCY FOR A STRIKE SLIP FAULTING ENVIRONMENT IN HOMOGENOUS MEDIA

A three-dimensional contour plot was again used to display the profile of the slip tendency at a critically stresses state using a value of 0.6 in **Figure 68**. The strike-slip environment displays a similar behavior with the expansion of the shale slip tendency from the x-axis however this is much less prevalent than that in the normal faulting case.

FIGURE 68 3-D SLIP TENDENCY FOR A STRIKE-SLIP FAULTING ENVIRONMENT AND MULTIPLE LAYERS

Figure 69 and **Figure 70** display an assembly type view of the fracture for stress magnitude and slip tendency, respectively. The profile of slip tendency is displayed to the left of the cross-section normal to the x-axis. Again, just as in the normal faulting environment, this displays that the slip tendency is not confined to the areas of high stresses, as the slip tendency profile extends beyond the range of the highest stress magnitudes.

FIGURE 69 ASSEMBLY VIEW OF STRESS SURROUNDING A SINGLE FRACTURE IN A STRIKE-SLIP FAULTING ENVIRONMENT

FIGURE 70 ASSEMBLY VIEW OF SLIP TENDENCY SURROUNDING A SINGLE FRACTURE IN A STRIKE-SLIP FAULTING ENVIRONMENT

CHAPTER 6 – ANALYSIS AND RESULTS

6.1. INTRODUCTION

The study of stress change surrounding fracture(s) is extremely important in regards to ensuring longevity and integrity of the reservoir. Furthering the understanding of how these stresses influence the surrounding reservoir and how parameters affect these stress relationships will allow for a better understanding and prediction of the reservoir and surrounding rock behavior. This is accomplished through the use of a new hydraulic fracture model which couples the governing processes controlling stress interactions throughout the fracture propagation process.

Unlike the analysis that have been performed for similar study these methods allow for a fully coupled solution to the governing hydro-mechanical model. This offers a faster and more precise solution technique to be applied which in turn allows for the incorporation of heterogeneous reservoir properties and dynamic injection of hydraulic fracturing fluid. These methods and techniques which have been described in the preceding chapters introduce many of the aspects which are vital to this problem and its solution.

The simulator identifies the initial path of fracture based upon a predetermined fracture path, however width in two-dimensions is controlled by the surrounding geology and injection while width and height in three-dimensions. When the initial stress state of the reservoir is considered to be at zero, then the resulting stress surrounding the fracture may be identified as the stress increase or difference between in-situ and final stress states. These relative changes offer an insight to the transient stress effects that occur surrounding the fracture and its propagation.

The figures below identify the initial state of the generalized 2-D mesh structure for the fracture propagation model. The initial location of fracture propagation is stated at a nodal location that is indicated within the red box. This indicates the injection location for a point source. The surrounding gray area identifies the location of local mesh refinement where the fracture patch is expected to propagate. Therefore, although the fracture path is expected to propagate along the predefined areas of mesh refinement, the fracture path itself is not predefined and is freely available to deviate from this predefined area.

FIGURE 71 EXPECTED FRACTURE PATH FOR SINGLE FRACTURE

FIGURE 72 EXPECTED FRACTURE PATH FOR MULTIPLE FRACTURES

Figure 73 below identifies the injection state of fracture propagation. Using a steady rate of injection at the initially opened cohesive nodes. This is representative of the fracture propagation at some time, t after injection starts.

FIGURE 73 MAGNIFIED VIEW OF FRACTURE DISCRETIZATION IN SOLID MEDIUM

The injection point of a bi-wing fracture at which the left hand most side is that of the injection is identified in **Figure 74**. Due to symmetry the fracture is displayed via its half-length where w , is the width of the fracture and p_f is the fluid pressure at their associated nodal locations. The blue arrows indicate the leak-off into the solid media surrounding the fracture.

FIGURE 74 SKETCH OF FLUID DRIVEN FRACTURE

6.2. TWO-DIMENSIONAL CASE

A single fracture is used in order to establish a baseline for study of congruent fracture stress relation. The relationship of stress surrounding a single fracture will identify to its corresponding properties of injection rate and surrounding geomechanical properties. Therefore in future scenarios, under identical conditions, as the number of fractures is increase their effect upon the relative stress variation can be seen. An outline of the sensitivity cases can be seen in **Table 15**.

Single Fracture at 0.00005 Injection				
Cases	Fracture Spacing	Injection Rate	Modulus of Elasticity	Poisson's Ratio
1	N/A, single	0.00005	150	0.2
2	N/A, single	0.00005	170	0.2
3	N/A, single	0.00005	190	0.2
4	N/A, single	0.00005	170	0.1
5	N/A, single	0.00005	170	0.2
6	N/A, single	0.00005	170	0.3

TABLE 15 SINGLE FRACTURE SENSITIVITY

The figures below identify the effects of the major geomechanical properties on fracture length and width. These properties include the elastic modulus of the reservoir, corresponding to its ability to resist deformation and Poisson's ratio. There were a total of 6 cases run in order to establish their respective relationships.

FIGURE 75 ELASTIC MODULUS EFFECT ON FRACTURE LENGTH

FIGURE 76 ELASTIC MODULUS EFFECT ON FRACTURE WIDTH

FIGURE 77 POISSON RATIO EFFECT ON FRACTURE LENGTH

FIGURE 78 POISSON RATIO EFFECT ON FRACTURE WIDTH

The figures above indicate the sensitivity of associated geomechanical properties on fracture width and length through time. In this comparison the results were determined over equal time periods for each solution which was predefined at 100 seconds of injection at an injection rate of $0.00005 \text{ m}^2/\text{s}$. These results indicate that the fracture geometry, as expected is much more sensitive to variations in the materials elastic modulus than that of Poisson's ratio.

FRACTURE LENGTH

The maximum rate of change seen in fracture length was 9.71% with an average of 7.78% as the elastic modulus was decreased from 170k to 150k. The maximum rate seen with an increase in elastic modulus from 170k to 190k was 8.85% with an average of 7.10%. In comparison, the maximum rate of change seen when increasing Poisson's ratio from 0.2 to 0.3 was 4.2% with an average of 3.35%. The maximum rate of change seen when decreasing Poisson's ratio from 0.2 to 0.1 was 2.51% with an average of 1.82%.

FRACTURE WIDTH

The maximum rate of change seen in fracture width was 4.00% with an average of 3.38% as the elastic modulus was decreased from 170k to 150k. The maximum rate seen with an increase in elastic modulus from 170k to 190k was 3.48% with an average of 2.95%. In comparison, the maximum rate of change seen when increasing Poisson's ratio from 0.2 to 0.3 was 1.73% with an average of 1.43%. Decreasing Poisson's ratio from 0.2 to 0.1 saw a maximum rate of change of 1.00% with an average of 0.84%.

These results clearly indicate that the modulus of elasticity has the greatest impact upon both fracture length as well as width. Furthermore, the largest rate change in both length and width with respect to Poisson's ratio were seen as it was increased. On the other hand, the largest rate change for both length and width with respect to elastic modulus were seen as it was decreased. This depicts a positive correlation between rate changes for length and width but their response is inversed between changes in Poisson's ratio and modulus of elasticity. This correlation was similarly observed using an injection rate 10 fold of that depicted here however the fracture width was much less influenced due to the high rate of injection.

The figures below depict the illustration of fractures in the reservoir for a single fracture case and for a multiple, or three fracture case. In the case of two parallel fractures, the center fracture is removed from the three fracture illustration and they will lie equidistant from the centerline of the reservoir.

FIGURE 79 ILLUSTRATION OF EXPECTED SINGLE FRACTURE

FIGURE 80 ILLUSTRATION OF EXPECTED MULTIPLE FRACTURES

Reservoir Properties

- Poisson's Ratio = 0.20
- Young's Modulus = 17,000,000
- Injection Rate = 0.0005
- Injection Rate Division = 0.3333
- Leak-Off Rate = 1.47×10^{-5}
- Fracture Spacing = 5 m

In the case of three congruent fractures using an equivalent spacing of 5 meters and a uniform volume injection rate of 0.0005 m/s^2 divided between the three fractures. These figures to the left below displays the horizontal stress distribution along the x-axis at a snapshot taken at 45 seconds. This displays a concentration of stress at the fracture tips of the offset fractures. The centric fracture displays an interaction with the offset fractures in both direction and magnitude of stresses along with an alteration of fracture geometry. The figure below to the right displays the horizontal stress distribution along the y-axis. This displays a stress concentration at the fracture tips that is mirrored around the central fracture where a maximum is observed, again both magnitude and direction of stresses are influenced by this central fracture.

FIGURE 81 STRESS CHANGE IN THE X-DIRECTION

FIGURE 82 STRESS CHANGE IN THE Y-DIRECTION

The fracture geometry is affected by the spacing of the offset fractures and the relative stress magnitudes that are seen around the developing fracture. Under equidistant spacing and injection rates the fracture geometry is mirrored around the central fracture location. Assuming that each fracture is undergoing injection at the same time then the central fracture begins to trade width for length in comparison to the two offset fractures.

6.2.1. TRANSIENT CHANGES TO STRESS FIELDS

The most effective way of analyzing stress distributions for any system is through the principle stress relationships. This is merely a stress transformation which orients the X and Y stress fields, whose vectors may point in any given direction and magnitude along the principle axis. Therefore the shear stress is diminished to zero, as the principle stresses are aligned along the corresponding axis by minimum, maximum and intermediate values in three dimensional analyses. In two dimensional analyses the third principle stress is dependent upon the analytical solution of plane strain or stress. The Von Mises stress is an important criteria selected for this research as it exemplifies the entire stress field with only a single consolidated value.

The changes in horizontal stress can be interpreted from the resultant values of the simulation through the solution time. In order to see the effects of these stress relationships, the stress values are interpolated over a cross-sectional and a profile cross-section of the reservoir domain at specified locations, as seen below in **Figure 65**. These locations are plotted at the intersection of the injection point as well as at an offset location that is determined by the number of fractures and the fracture spacing, if applicable. In the case of multiple fractures, the profile will always be plotted against the uppermost fracture. In order to see cross-sectional and profile views intersecting at the center of the reservoir then please refer to **Appendix 8.4 Figures**.

FIGURE 83 ILLUSTRATION OF LINE LOCATIONS FOR STRESS-CHANGE PLOTS

Figure 65 represents the line locations at which stresses will be plotted. The lines A – A' and B – B' run North to South and represent the cross-sectional plots running perpendicular to the fracture at the point of injection and at a specified distance 50 meters from the point of injection. The lines C – C' and D – D' which run East to West represent the profile plots of the stress and stress relationships which are running parallel to the fracture. In order to visualize these stress transformations, the stresses will be displayed at single time steps as well as for the rate of change in order to determine the general static and dynamic response which surrounds fracture propagation and the redistribution of stress.

SINGLE FRACTURE CASE

The following three figures of **Figure 84** (a) σ_x , (b) σ_y , and (c) τ_{xy} represent the stress distributions at the injection point and at distance of 50 meters from the injection point. This is for a single fracture that has been injected at a constant injection rate of 0.0005 m/s. These figures represent the cross-sectional view of the reservoir at lines A – A' (green) and B – B' (blue) shown in **Figure 83**. As one would expect, the stress change seen at the site of injection is much higher than that seen a distance (50 m) from the fracture tip.

FIGURE 84 THE STATIC COMPONENT STRESS DISTRIBUTION SURROUNDING A SINGLE FRACTURE, T=60S

These figures of **Figure 85** (a) σ_1 , (b) σ_2 , and (c) σ_{vm} represent the consolidated component stress distributions surrounding the fracture at the injection site and at a distance 50 meters from the injection site. The first two **Figure 85** (a) and (b) represent the principle stresses in two-dimensions, that is the maximum and minimum. The third **Figure 85** (c) is representative of the Von Mises stress distribution. These represent the stress transformation of the component stress tensors shown independently in **Figure 84**. This information is useful in deciphering the interactions in comparison with the component stresses. Their appearances seem to simply be rearranged between the diagrams but they provide useful insight to other applications. The Von Mises is reflective of all the stress contributions and therefore lends insight to the dominant stresses. A different relationship can clearly be seen in the peak of the Von Mises stress which displays the transition between the dominant stress components.

FIGURE 85 THE STATIC PRINCIPLE STRESS DISTRIBUTION SURROUNDING A SINGLE FRACTURE, T=60S

Figure 86 represents the profile of the fracture with respect to the component stresses. These figures (a) σ_x , (b) σ_y , and (c) τ_{xy} are shown with respect to the profile reservoir cross-sections along lines C – C' (green) and D – D' (blue). These same line location profile plots are shown in **Figure 87** (a) σ_1 , (b) σ_3 , and (c) σ_{vm}

FIGURE 86 THE STATIC COMPONENT STRESS PROFILE SURROUNDING A SINGLE FRACTURE, T=60S

FIGURE 87 THE STATIC PRINCIPLE STRESS PROFILE SURROUNDING A SINGLE FRACTURE, T=60S

The previous figures defined a static snapshot of the stress distribution at two different locations taken at the same instant in time. This is useful for comparison to other similar studies however the transient response is of concern with relation to this investigation. The time dependent behavior of the stress fields are described by **Figure 88 (a)** maximum component stress field variation, **(b)** minimum component stress field variation and **(c)** the maximum principal and Von Mises stress variations. These plots display the maximum or minimum value of stress versus time without regard for distance to or from the fracture or injection point. This identifies the general behavior that is to be seen throughout the period of injection.

FIGURE 88 THE GENERAL TRANSIENT BEHAVIOR OF STRESS SURROUNDING A SINGLE FRACTURE, T=0 TO 60S

These **Figure 88 (a), (b)** and **(c)** depict a similar behavior for the component stress distributions through time. The principle and Von Mises stress are also shown to depict a predictable behavior. These consolidated stresses are

shown in order to introduce their behavior and the influence of all the contributing component stresses in order to verify their use in further discussion.

In addition to their predictable behavior(s), a relationship can be seen between the principle stress σ_1 and the Von Mises stress σ_{VM} in **Figure 88 (c)**. This relationship is key to defining the Von Mises stress as an applicable consolidated stress value to this type of problem.

TWO FRACTURE CASE

The following three figures represent the stress distributions at a distance of 50 meters from the injection point for a system of two equally injected fractures at a distance of 5 meters spacing. Unlike the single fracture case described in the previous section, the case of two fractures introduces an additional complexity to the stress distributions, as would be expected. These stresses portray themselves in a variety of ways which can be seen below in **Figure 89** where (a), (b) and (c) are σ_x , σ_y , and τ_{xy} respectively after 60 seconds of injection.

FIGURE 89 THE STATIC COMPONENT STRESS BEHAVIOR SURROUNDING DUAL FRACTURES AT 5 METERS SPACING, T=60S

The static stress distributions are an important step to the classification of these stress interactions as it provides verification of the trend that is seen in similar static case studies but also provides the ability to compare states when adding additional complexities, such as by increasing the number of fractures. The static principle and Von Mises stress distributions are depicted below by **Figure 90**. These figures represent a cross-sectional view of the reservoir and fractures at the lines A – A' (green) and B – B' (blue) after an injection time of 60 seconds. These two cross-sections are again taken at the point of injection and at a distance of 50 meters of the injection site as depicted in **Figure 83**.

FIGURE 90 THE STATIC PRINCIPLE STRESS DISTRIBUTION SURROUNDING DUAL FRACTURES AT 5 METERS SPACING, T=60S

The following **Figure 91** represents a profile view of the stress distribution along the outermost fracture where **(a)** is σ_x , **(b)** is σ_y , and **(c)** τ_{xy} at a time of 60 seconds after injection. This is to say that this profile is 2.5 meters from the reservoir centerline and 5 meters from the second fracture. These lines represent that of C – C' and D – D' as described in **Figure 83**. This same process is repeated for the corresponding principle stresses of σ_1 , σ_2 , and σ_{VM} in **Figure 92 (a), (b), and (c)**.

FIGURE 91 THE STATIC COMPONENT STRESS PROFILE ALONG THE OUTERMOST FRACTURE IN A DUAL FRACTURE SYSTEM AT 5 METERS SPACING, T=60S

FIGURE 92 THE STATIC PRINCIPLE STRESS PROFILE ALONG THE OUTERMOST FRACTURE IN A DUAL FRACTURE SYSTEM AT 5 METERS SPACING, T=60S

Figure 93 is representative of the profile at the reservoir centerline. It is not along the profile of a fracture but simply taken at the center of the reservoir, therefore it lies 2.5 meters from both fractures. These figures **(a), (b)** and **(c)** are of σ_x , σ_y , and τ_{xy} respectively.

FIGURE 93 THE STATIC COMPONENT STRESS PROFILE ALONG THE RESERVOIR CENTERLINE BETWEEN DUAL FRACTURES AT 5 METERS SPACING, T=60S

FIGURE 94 THE STATIC PRINCIPLE STRESS PROFILE ALONG THE RESERVOIR CENTERLINE BETWEEN DUAL FRACTURES AT 5 METERS SPACING, T=60S

The transient response of the stresses is of the most concern with respect to this type of research as these have rarely been fully explored. These stresses are often viewed for static cases and do not analyze the representative patterns as they develop. In the case of two fractures the transient stress response can be seen in **Figure 95 (a)** the maximum component stresses, **(b)** the minimum component stresses, and **(c)** the maximum principle and Von Mises stresses where the development of these profiles can be used to infer some of these interactions.

FIGURE 95 THE GENERAL TRANSIENT BEHAVIOR OF STRESS SURROUNDING DUAL FRACTURES AT 5 METERS SPACING, T=0 TO 60S

THREE FRACTURE CASE

The following three figures represent the stress distributions at a distance of 50 meters from the injection point for a system of three equally injected fractures at a distance of 5 meters spacing. Unlike the single fracture case described in **Figure 84** the case of three fractures brings another layer of complexity to the stress distributions. The following **Figure 96 (a), (b), and (c)** represent σ_x , σ_y , and τ_{xy} respectively which are taken at the cross-sectional lines A – A' (green) and B – B' (blue) at the injection point and at a distance 50 meters from the injection point.

FIGURE 96 THE STATIC STRESS DISTRIBUTION SURROUNDING A SYSTEM OF THREE FRACTURES AT 5 METER SPACING, T=40S

Again, unlike the single fracture cases where the surrounding stress fields are only representative of the lone fracture(s) interaction with the surrounding media, multiple fractures are further influenced of the surrounding stress magnitude by the introduction of additional fractures. Typically the stresses that are incurred surrounding in the x-direction off of the fracture tips generate the larger stress magnitude. At this spacing of 5 meters it can be seen that the maximum stress magnitude is dominated by the y-direction stress due to the proximity of the fractures interacting causing a stress magnification.

Figure 97 below illustrates the principle stress distribution of the component stress shown in **Figure 96** above. These are again plotted over the cross-sectional lines of A – A' (green) and B – B' (blue) at the site of injection and at a distance of 50 meters from the site of injection.

FIGURE 97 THE STATIC PRINCIPLE STRESS DISTRIBUTION SURROUNDING THREE FRACTURES AT 5 METER SPACING, T=40S

The next figures are representative of the profile stress distributions along the length of the fracture(s) and beyond. **Figure 98** illustrates the static stress profile along the outermost fracture in a three fracture system where (a), (b) and (c) are representative of σ_x , σ_y , and τ_{xy} respectively along the profile lines C – C' (green) and D – D' (blue) at a time of 40 seconds.

FIGURE 98 THE STATIC COMPONENT STRESS PROFILE ALONG THE OUTERMOST FRACTURE IN A THREE FRACTURE SYSTEM AT 5 METERS SPACING, T=40S

The next figures are representative of the profile stress distributions along the length of the fracture(s) and beyond. **Figure 99** illustrates the static stress profile along the outermost fracture in a three fracture system where (a), (b) and (c) are representative of σ_1 , σ_2 , and σ_{VM} respectively along the profile lines C – C' (green) and D – D' (blue) at a time of 40 seconds.

FIGURE 99 THE STATIC PRINCIPLE STRESS PROFILE ALONG THE OUTERMOST FRACTURE IN A THREE FRACTURE SYSTEM AT 5 METERS SPACING, T=40S

The following displays the component stress profile at the central fracture location; therefore it lies at the center of the reservoir and at the center of the two outlying fractures, both 5 meters away. **Figure 100** (a), (b), and (c) correspond to the component stresses are σ_x , σ_y , and τ_{xy} at a time of 40 seconds. At the same locations the corresponding principle and Von Mises stresses are depicted in **Figure 101** at the same solution time.

FIGURE 100 THE STATIC COMPONENT STRESS PROFILE ALONG THE CENTER FRACTURE IN A THREE FRACTURE SYSTEM AT 5 METERS SPACING, T=40S

FIGURE 101 THE STATIC PRINCIPLE STRESS PROFILE ALONG THE CENTER FRACTURE IN A THREE FRACTURE SYSTEM AT 5 METERS SPACING, T=40S

The change in the principal stresses is displayed in **Figure 102 (a)** dynamic profile of maximum principle stress and **(b)** dynamic profile of minimum principle stress, in order to observe the changes that occur throughout fracture propagation and verify whether the assumptions that are made in static models can be justified which were found in (Cheng, 2012). Although both of these profiles are similar, their relationship is seen to change dramatically through time in relative and overall stress magnitude. This indicates that the assumption of static models cannot be trusted as it incorrectly skews the depiction of stress towards early stage behavior, at the initial time of injection.

FIGURE 102 THE DYNAMIC BEHAVIOR OF THE PRINCIPLE STRESSES, T=0 TO 60S

The development of the transient response in the three fracture system which is shown below in **Figure 103 (a)** the maximum component stress, **(b)** the minimum component stress, and **(c)** the maximum principle and Von Mises stress. Keep in mind that the length of the simulation is shorter for this cases than that of one and two fractures so the decline in maximum stresses cannot be seen, however they have reached their peak and is expected to follow the same trend as displayed in the previous two examples.

FIGURE 103 THE GENERAL TRANSIENT BEHAVIOR OF STRESS SURROUNDING THE THREE FRACTURE SYSTEM AT 5 METERS SPACING, T=0 TO 400

The Von Mises stress clearly illustrates its behavior with respect to the other stress magnitudes in all three cases. When the stresses are at their maximum value this correlation of the Von Mises to the maximum principle stress displays its greatest difference. During the initial periods of injection and after the fracture interactions have occurred they are much more resembling of each other.

The previous figures throughout this section were used to illustrate the component contributions to the consolidated stresses which in this case are the principle and Von Mises stress. These behaviors offer an insight to the dominating stress influence at any particular time and distance which is critical in establishing dynamic stress relationships. **Figure 104 (a)** indicates the maximum and **(b)** the minimum Von Mises stress with respect to the distance from the fracture tip. This data is representative of a single fracture after 60 seconds of injection at 0.0005 m/s^2 .

FIGURE 104 VON MISES STRESS WITH RELATION TO THE FRACTURE LENGTH AND DISTANT FROM THE FRACTURE TIP AT T=60S FOR A SINGLE FRACTURE

The Von Mises stress is a culmination of the surrounding stress fields such that one value can be used to compare given states of stress. This is particularly useful when you are not so concerned with the direction of the force but the magnitude. This is often why Von Mises is used as a yield criterion. The distance from the fracture tip to the maximum Von Mises stress occurring on each cross-section from the point of injection is plotted in the figure above. Here, the zero is identifying the fracture tip such that the left of zero includes the fracture to the point of injection.

The Von Mises stress is used to identify two distinct patterns that are seen surrounding fracture propagation and associated stresses. There are two distinct sets of behavior for the area trailing the fracture tip and then for the area leading the fracture tip that can be seen in **Figure 105**. At the point of injection the maximum Von Mises stress decline throughout the length of the fracture. As the fracture tip is approached, the maximum Von Mises stress begin to increase sharply to a maximum value which is observed at the tip of the fracture. After the tip of the fracture a secondary behavior is observed leading the fracture tip. In this behavior the maximum Von Mises stress declines rapidly in an exponential manner.

FIGURE 105 MAXIMUM VON MISES STRESS BEHAVIOR SURROUNDING A SINGLE FRACTURE

This behavior head and tail behavior of the fracture surrounding the fracture tip can be used in order to quantify the data that is observed throughout the stress interactions of the Von Mises stress distributions. The behavior at the head of the fracture for a single fracture is least impacted by the fracture propagation due to the homogenous reservoir properties and constant fluid injection properties. With these values held constant, then the dynamics of the fracture propagation can be limited to the surrounding stress interactions. As one would expect the head of the fracture is least impacted and therefore displays a trend like behavior which can be predicted. This is due to the limited interactions that are occurring ahead of the fracture tip throughout the undisturbed media since the only stress fields are those due to the observed fracture propagation.

The tail behavior of the fracture is more reliant upon the injection parameters and the distance at which the fracture tip extends from the point of injection. This identifies two specific zones within the head and tail of the fracture known as the zones of influence. In the tail of the fracture there is a zone of influence surrounding the point of injection which affects the surrounding stresses, much like the zone of influence surrounding the fracture tip in the head of the fracture. This can be seen in **Figure 87** below.

FIGURE 106 ZONES OF INFLUENCE SURROUNDING A SINGLE FRACTURE

The critical distance in question can be observed by evaluating the solution for a single fracture throughout its transient behavior. This identifies the change in distance that is observed which can then be used to establish a critical distance between the injection point and fracture tip with which stress influences are reduced. A single fracture at a single time step is used in order to establish the main points of interest on the Von Mises stress plot.

FIGURE 107 POINTS OF CONCERN FOR STATIC VON MISES STRESS DISTRIBUTION

Point Data for Von Mises Stress in a Single Fracture Case								
Time	Point 1	Von Mises	Point 2	Von Mises	Point 3	Von Mises	Point 4	Von Mises
10	6.49	431.2	2.49	349.2	0	668.7	100	1.67
30	14.24	309.4	5.24	248.2	0	602	100	4.16
45	18.64	273.5	6.64	215.8	0	655.7	100	5.58
60	22.89	248.6	7.89	195.4	0	534	100	6.94
90	30.57	216.8	10.57	169.3	0	369.8	100	9.20
300	68.66	144.9	24.66	108.3	0	231.7	100	16.96
900	125.1	108.8	40.12	77.14	0	247.1	73*	31.43*
1500	172.2	99.35	48.16	67.1	0	254.2	26*	89.22*

TABLE 16 POINT DATA FOR TRANSIENT MAXIMUM VON MISES STRESS FOR A SINGLE FRACTURE CASE

Table 16 illustrates the position and magnitude of Von Mises stress along the fracture and surrounding reservoir area at the head of the fracture. The data for points 1 and 2 lie to the left of zero and therefore their absolute value has been shown in the table above. Data point 3 is a zero that is representative of the fracture tip and is shown for reference but has become obsolete now that the absolute values of previous points are used. The data at point 4 is representative of the Von Mises stress at a distance of 100 meters from the head of the fracture tip.

Figure 108 below displays the critical distance that was observed in the interactions of the Von Mises stress. This corresponds to the tail of the fracture behavior where the local minimum was seen versus the length of the propagating fracture. Plotting these distances shows a linear relationship between the distance of the local minimum and the fracture length as the fracture propagates through time. This behavior demonstrates that the Von Mises can be useful in predicting fracture and stress behavior.

FIGURE 108 CRITICAL DISTANCE OF THE VON MISES STRESS FROM THE POINT OF INJECTION

This linear trend is fitted with a trend line to determine a describing equation and corresponding fit value which can be seen below.

$$y = 1.3974x + 2.6701$$

$$R^2 = 0.9979$$

As the fracture propagates and begins to increase the distance at which the tip is located from the point of injection a dynamic behavior is observed in the maximum Von Mises stress. This behavior is simplified in the following **Figure 109** in order to understand the relationships being identified. This will be carried into further discussion with a detailed view in **Figure 110** representing the solution data observed with a single fracture case.

FIGURE 109 TRANSIENT GROWTH OF THE VON MISES INFLUENCE IN THE FRACTURE TAIL

This behavior can be idealized in such a way that the maximum Von Mises stress creates an angle with the propagating fracture plane or fracture tip. The maximum Von Mises emanates from the fracture tip creating a critical angle. As the distance from the fracture tip is increased by moving outwards into the reservoir domain the critical angle remains relatively constant until a plateau is reached at which point the stresses stabilize and the critical angle returns to zero. This critical angle can be used to justify the stability of the reservoir or in regard to stress reorientations that are seen around propagating fractures. These reorientations can affect the geometry and relative stress distributions surrounding adjacent fracture propagation by either easing or impeding the ability of the fracture to propagate.

FIGURE 110 CRITICAL ANGLE OF THE MAXIMUM VON MISES STRESS OBSERVED FROM THE FRACTURE TIP

The maximum Von Mises stress is depicted in **Figure 111** which displays a behavior similar to that seen in the traditional Mohr Coulomb or Von Mises failure criteria as depicted in **Figure 14**. This trend is seen at the head of the fracture regardless of the length of simulation or the number of fractures. This displays an ability to predict the dynamic stress behavior surrounding fractures throughout propagation. This transient behavior can be used to determine dynamic behaviors due to its predictability.

FIGURE 111 MAXIMUM VON MISES STRESS BEHAVIOR AT THE HEAD OF A SINGLE FRACTURE

Using the approach that was outlined for the determination of a critical angle, a linear trend line is generated for the set of data above. This data is mirrored over the x-axis creating a symmetrical v shape pattern. Calculating the critical angle for this case it was found to be roughly 39.4° above the x-axis. The tail of the fracture displays a similar behavior which extends in length, however remains at the same angle throughout time.

The following **Figure 112** describes the maximum Von Mises stress from the fracture tips located at the head of the fracture(s). This depicts a similar pattern to that seen above in which the maximum stress value follows a predictable path.

FIGURE 112 THE MAXIMUM VON MISES STRESS BEHAVIOR AT THE HEAD OF DUAL FRACTURES AT 5 METERS SPACING

The last figure in the series, **Figure 113** represents the maximum Von Mises stress for a three fracture system at 5 meters spacing. This behavior is again displaying a predictable behavior at the head of the fracture tips and beyond into the reservoir.

FIGURE 113 MAXIMUM VON MISES STRESS BEHAVIOR AT THE HEAD OF A THREE FRACTURE SYSTEM AT 5 METERS SPACING

The noticeable difference in the occurrence of the maximum stress is that these stress fall into a tighter pattern within a specific area of influence away from the fracture tips. This is clearly evident for the two and three fracture cases where the stress interactions magnify this effect. This is seen as a jump between the linear behaviors prior to reaching the extents of the reservoir domain.

The fracture displays a constant profile in both the head of the fracture and the tail of the fracture. The tail profile is seen to correlate linearly with the distance from the injection point while the head of the fracture there is a

constant behavior, as would be expected with the previous investigation. The interesting find is that these predictions remain constant through time with respect to their aforementioned zones such that their dynamic behavior can be predicted. **Figure 114 (a)** single fracture case, **(b)** dual fracture case, and **(c)** three fracture case depict the maximum Von Mises stress profile with relation to a best fit curve to that seen in the single fracture case.

These figures show the interaction that occurs in the maximum Von Mises stress with the addition of one or more fractures. The influenced zone is constant with respect to time so its dynamic response can be predicted independently depending upon the number of fractures that are developed.

FIGURE 114 A COMPARISON OF THE MAXIMUM VON MISES STRESS PROFILE SURROUNDING ONE, TWO AND THREE FRACTURES

UNIFORM INJECTION

The terminology of uniform injection is used in this sense as the injection of two congruent fractures which are equal in both injection rate and volume. This allows for an ease in the comparison between fractures and stress fields as there is a congruency between these. A uniform rate of injection stipulates the rate at which each fracture is injected based upon an initial total mass rate of injection for all fractures and an allotted percentage determined by the number of fractures.

FIGURE 115 UNIFORM INJECTION OF TWO FRACTURES, $Q = 0.0005 \text{ M/S}^2$

FIGURE 116 STRESS CHANGE IN X-DIRECTION 50/50

FIGURE 117 STRESS CHANGE IN Y-DIRECTION 50/50

NON-UNIFORM INJECTION

The terminology of non-uniform injection refers to an unequal distribution between the mass rate injection distributions between congruent fractures. This can be seen from the figure below which represents the first fracture receiving an injection rate that is 40% of the total specified while the second fracture receives 60% of the specified mass rate of injection. This would be representative of the head or pressure changes between two fractures that are spaced by some distance Y .

The application of differing mass injection rates can lend insight to the interactions that occur between the two concurrent stress fields surrounding the individual fractures and developing as a whole. The representation of the model is symmetric with respect to location and properties, therefore by creating non-symmetry in the injection rate then the interactions of these stresses may be observed.

FIGURE 118 NON-UNIFORM INJECTION OF 60/40 FOR TWO FRACTURES, $Q=0.0005 \text{ M/S}^2$

FIGURE 119 STRESS CHANGE IN X-DIRECTION 60/40

FIGURE 120 STRESS CHANGE Y-DIRECTION 60/40

The stress along the Y-direction becomes condensed upon the smaller fracture. The X-direction stresses display that they are dominant and impact the larger fracture stresses.

6.2.2. TRANSIENT CHANGES IN 2-D FRACTURE PROFILE

One of the most observable impacts of these stresses surrounding the fracture is that upon the fractures geometry. This can be seen with propagation in systems with more than one concurrent fracture as the stresses from one or more fractures influences their length and width. The impacts of these stress behaviors and fracture width are depicted below for a system of three concurrent fractures in order to visualize these interactions. The fracture displacements have been magnified 2000 times in order to clearly see the interaction of these behaviors.

This series of figures in **Figure 121** displays a three fracture case at different moments in time. At **(a)** the initial fracture propagation the central fracture is smaller and longer as it is compressed by the large stresses that are exchanged with the onset of injection. As the fracture begins to propagate the stresses begin to interact as they being to equalize in **(b)**. The profile at **(c)** again displays an expansion of width where the impacts of the stresses from the central fracture begin to become apparent. The last figure **(d)** is taken at a time of 60 seconds where the effects of the stress interactions are clearly evident with the central fracture appearing symmetrical but the offset fractures appear oblique and the outer edge begins to point inwards, towards the central fracture.

FIGURE 121 THE DEVELOPMENT OF THE FRACTURE PROFILE IN THREE CONCURRENT FRACTURES AT 5 METERS SPACING

The resultant effects of these stress interactions are clearly evident, however the development of this transient behavior is difficult to recognize from these images alone. In order to see these stress interactions the Von Mises stress has been displayed in **Figure 122** throughout the same time period as shown above for the fracture profile.

FIGURE 122 THE DEVELOPMENT OF THE VON MISES STRESS SURROUNDING THREE CONCURRENT FRACTURES AT 5 METERS SPACING

6.2.3. TRANSIENT CHANGES IN 3-D FRACTURE PROFILE

The three dimensional model using the cohesive zone method was used to compare results throughout the two dimensional simulations. The verification of these previous assumptions and conclusions should hold true when expanding to the three dimensional model for validity if these conclusions are indeed true. Figure 39 and Figure 40 identify the discretization of the 3-D model used in this study.

These interactions are further investigated using the three-dimensional model where the fracture interaction can be visualized. The fracture width profiles are displayed below in **Figure 124** and **Figure 126** for the offset and center fracture in a three fracture system, respectively. Being a homogenous system at this point, the two outlying fractures are identical in their geometry and internal pressure distribution as they influence the center fracture equivalently. This interaction can be seen through the development of the center fracture through time as both its length and width are seen to be influenced in both rate of change and magnitude versus that of the outlying fracture(s).

FIGURE 123 CENTRAL FRACTURE PRESSURE PROFILE OF 3D SOLUTION AT 5 METERS SPACING

FIGURE 124 CENTRAL FRACTURE WIDTH PROFILE OF 3D SOLUTION AT 5 METERS SPACING

Although there is an impact seen throughout the development of the center fracture, it appears counterintuitively as the center fracture growth becomes arrested to the outer fracture development. In order to verify this observation this is being investigate in more detail in future work surrounding this topic.

FIGURE 125 OFFSET FRACTURE PRESSURE PROFILE OF 3D SOLUTION AT 5 METERS SPACING

FIGURE 126 OFFSET FRACTURE WIDTH PROFILE OF 3D SOLUTION AT 5 METERS SPACING

The visualization of the fracture profile in 3-D is displayed below for same fracture in **Figure 127** and **Figure 128**. This highlights the opening of the 2-D cohesive zone layer which was discussed earlier. The development of the fracture occurs as the conditions are met for the entire element and therefore these diagrams do not depict a smoothly distributed fracture geometry.

FIGURE 127 THREE DIMENSIONAL FRACTURE GEOMETRY AT T=30S

FIGURE 128 THREE DIMENSIONAL VIEW OF FRACTURE GEOMETRY AT T=60S

The effects of injection pressure and fracture width have been studied using a relative relationship. Since the reservoir domain is homogenous and symmetrical, then the outlying fractures during simultaneous injection are equivalent. This generalization was used for comparison of the fracture width and pressure development throughout the injection period using the central fracture as a point of reference. In addition this generalization is made to the case of a single fracture at the center of the domain in order to determine the impacts of the simultaneous fracture development. These multi-fracture effects can be seen from the figures below.

Figure 129 depicts the pressure and width ratios of the central fracture versus the outlying or offset fractures in a three fracture system. This was to determine the impact of these outlying fractures on the central fracture development. The ratio of fracture pressure versus fracture width was used in order to determine these impacts. This investigation found that a linear relationship occurred, regardless of the injection rate which can be further developed in future study to create a direct correlation between these properties. It is predicted that this linear relationship will continue until the distance between fractures is reduced to a specific distance between the offset fractures.

FIGURE 129 PRESSURE AND WIDTH OF A CENTRAL FRACTURE VERSUS OFFSET FRACTURES

The baseline for the fracture geometry was made using the central fracture and a single fracture case to see the impact of the offset fractures in **Figure 130** below. This information will be used in addition to that found above in future study but it is apparent that the increase in the number of fractures impedes the growth of the fracture width. This is determined due to the influence of stresses surrounding the development of the fracture throughout its propagation which was discussed throughout the 2-D fracture simulation results.

FIGURE 130 SINGLE FRACTURE DEVELOPMENT VERSUS SIMULTANEOUS FRACTURE DEVELOPMENT

CHAPTER 7 - SUMMARY AND CONCLUSIONS

7.1. SUMMARY

Many of the semi-analytical approaches which have been studied through previous work(s) have been neglecting of the fluid interaction which occurs at the boundary of the producing fracture and do not hold valid for a variety of cases. Fracture propagation plays a key role in the stress redistribution that occurs throughout a reservoir as it migrates throughout the area and by ignoring the fluid and proppant properties this reduces the accuracy and quality of the results as well the validity of our assumptions that are made from these types of simulations. Previous works have dealt with the theory of inclusions or finite element method and the Mohr Coulomb Failure criterion to assess fault stability and near fault stress distribution within a homogenous media. This is typically assessed on a case by case basis however that has little application to practical design or study and rarely captures the transient behavior of these stress and stability distributions.

The investigations presented throughout this work have shown to display a congruency with similar study in this area along with available field data, presenting an initial platform to build from. This data will be used in conjunction with fluid flow theories in order to add more diversity to modeling and simulation capabilities in an effort to further enhance the predication of dynamic stress behavior and its impact on surrounding stability. Furthering this research will focus on the finite element method and general stress state failure criterions in order to further enhance the predictability and accuracy of failure. This has been done by coupling additional fluid flow equations with the existing model.

The application of finite element analysis has been an excellent method to determining the solution of multiphysics problems. In the preliminary approach the primary focus of the solution was upon the stress fields surrounding fractures. This applied the use of high density elements with static case analysis for comparison against analytical solutions and case studies. The downfalls to this approach are high computational times due to the element and nodal structuring and the lack of a dynamic behavior which is required to properly model transient fracture and reservoir behaviors. The static solution does however provide a general insight into surrounding fracture stress fields although it negates many important aspects within the fracture and its propagation.

The secondary approach that has been used exclusively throughout the results of this thesis uses a more complicated numerical technique in combination with a simpler mesh structure. The reduction of nodal and element complexity along with the efficiency of the propagation method aids in reducing the computational time significantly, even when processing the transient solutions of time dependent fracturing cases. Various solution techniques were applied and studied in order to enable this efficient, robust solution that features fast computational times and accurate solutions to a fully coupled hydro-mechanical problem.

These simulation techniques have aided in recognizing critical stress behaviors that occur throughout the dynamics of fracture propagation. The novelty of this study is in its infancy and enables many different types of future research to be conducted throughout the material presented. Many of the ideas and theories discussed are currently awaiting publication or are in the process of. The most beneficial result of this work will be the application of this theory to practical cases which can be analyzed using field data so that it can be of more value to both the industry and to furthering our understanding of these complex interactions.

7.2. CONCLUSION

The stress distributions surrounding a fracture have shown to depict a dynamic behavior that differentiates entirely from the standard, static stress case. This becomes significantly important to the study of fracture mechanics in both porous media as well as having application to many other fields of engineering research and design. These dynamic relationships relate specifically to the topic of hydraulic fracturing as it can cause complications with regard to fracture interaction, efficiency and surrounding stability prediction in addition to general fracture predictions.

The behaviors of stresses due to processes such as hydraulic fracturing are often assumed to be constant throughout the injection period however the determination of these stresses is a dynamic process with differing stress characteristics in comparison to static models. The static distribution of stress has been shown to favor the early period of injection just as hydraulic forces are applied to the fracture boundaries. This information also moves to introduce a failure criterion or area of weakened state which is based upon the Von Mises stress, without the requirement of interpreting multiple stress parameters. This will define a critical distance and angle with relation to the fracture(s) dynamic propagation path. This is anticipated to provide insight to fracture interaction and reservoir stability as these results clearly indicate that the Von Mises stress is capable of predicting these behaviors of not only the static interaction surrounding the fracture, but also this dynamic stress behavior which is similar to that of the commonly used principle stress relationships.

The relevancy of these stress fields is not only regarding the surrounding stability but also their direct correlation to fracture propagation of both current and subsequent fractures. The in-situ stress distributions control the immediate response of the fracture propagation, however the dynamic stress fields of these fracture interactions can impede or enhance the subsequent fracture paths. The stresses projected off of the fracture tip display a high increase in the immediate stress fields to the point which they will impact these subsequent fractures and further fracture development. This has been determined both in this thesis and throughout other similar bodies of work however at the time this was the first application of such methodology to the transient behavior of these stresses. The proposed use of the Von Mises stresses is the ability to predict near and far field stress patterns using single consolidated value from which the most efficient stimulation plan may be developed.

The use of varying injection rates gives an insight to the interaction of trailing fracture stresses of the leading fracture impacting the path, direction and magnitude of stresses surrounding subsequent fractures throughout the same injection period. Therefore creating stimulation plans which either mitigates the energy loss occurring from interactions or causing interactions which enhance fracture tip stresses to aid in propagation is an interesting proposition. This is directed at pulsed injection or delayed staging techniques to enhance the quality and efficiency of the treatment zone and in turn increase production from unconventional reservoir formations by thoroughly understanding these dynamic stress variations and using them to our advantage.

The continuation of this type of research is important to furthering our knowledge of geomechanics as well as to increasing the understanding of hydraulic fracture and unconventional reservoir development. Hydraulic fracturing has been crucial to the advancement in the search for energy independence in the United States by aiding in the profitable production of shale formations which dominate most of North America. There is still a great need however to further the effectiveness and understanding of hydraulic fracture treatment so that we can increase production and profitability while decreasing the risks commonly associated with public health and the environment.

7.3. DIRECTION OF FUTURE WORK

The direction of this work leaves many directions for future advancement by integrating the modeling techniques seen through the static model in the preliminary stress relationship study to the later approach seen in the transient model. Both of these methods have been described throughout this thesis. The static model utilizes a 20 node hexahedron element using reduced integration. These elements can be utilized between the prescribed cohesive zones of the current transient model.

Since the three-dimensional transient model utilizes the cohesive zone theory for fracture propagation then a mixed element mesh can be used to describe the reservoir domain, essentially sandwiching a layer of cohesive nodes between the solid elements at each injection point. This can allow for the transient stress patterns to be mapped surrounding the injection points, as well as those stress patterns in the surrounding solid elements representing the reservoir domain. Essentially it is being said that the transient fracture propagation technique will be taken from the new 3-D model and the stress calculation will be taken from the preliminary 3-D model.

In addition to fracture propagation and element stress determination the inclusion of more complicated fracture parameters can be useful in further study. Similar to the changes seen between the static and dynamics stress field due to non-uniform pressure distributions, the fracture proppant distribution also has a significant impact upon these areas. As the fracture is filled with proppant the available fracture aperture is diminished which can affect the pressure distribution throughout the fracture. These proppant distributions can also be useful in the determination of post-injection related stress changes. This can be attributed to events during the relaxation of the reservoir as fluid flow back occurs and the proppant begins to redistribute the stress fields to an equilibrium between the fracture surfaces. This is suspected to provide a great insight to the redistribution of these transient stress fields.

The overall direction of future work is to incorporate these techniques and methodologies to a fracture propagation model which is capable of simulating fracture fluid and proppant distribution during propagation to a heterogeneous, pre-stressed porous media. It is believed that the most interesting methods for revisiting this type of discussion are based upon similar theory to that which is applied in pre-stressed concrete models where the magnitude of the residual stress differentials can further alter fracture geometry prior to the arrival of the fluid front.

CHAPTER 8 - APPENDICES

8.1. NOMENCLATURE

c	diffusivity
e	void ratio
e_o	initial void ratio
g	gravitational acceleration
h	height
k	permeability
k	k-value, ratio of mean horizontal stress to vertical stress
k_f	hydraulic conductivity
m_f	fluid mass content per unit volume
m_i	Hoek-Brown parameter
p	mean stress
q	fluid volume per unit area and time
r	distance
t	time
w	width, breadth or thickness
g	gravitational constant
h	general thickness or height
u_a	average pressure stress in other fluid
u_w	average pressure stress in wetting fluid
x_i	rectangular coordinates
B	Skempton's coefficient
C	cohesion
E	Young's modulus
G	shear modulus
H	reciprocal of poroelastic expansion coefficient
L	distance or length
R	gas constant
N_{Re}	Reynolds number
z	gas compressibility factor, deviation factor
K	permeability

K_d	drained bulk modulus
K_f	fluid bulk modulus
K_g	grain bulk modulus
K_u	undrained bulk modulus
P	pressure
Q	external source, fluid volume per time
R	reciprocal of specific storage coefficient at constant stress
T	temperature
t	time
V_{fi}	injection rate, fluid volume per time
μ	<i>coefficient of internal friction</i>
Δ	change in a variable, used as prefix
δ	angle of internal friction
ε	strain, normal and general
γ	specific weight
θ	angle
α	Biot-Willis coefficient
λ	mobility
λ_u	undrained bulk modulus
η	diffusivity
σ	stress, normal
τ	stress, shear
ρ	density
ϕ	porosity
ϕ_c	crack porosity
Φ	dip, azimuth of
Θ	angle of dip
ϑ	angle, angular coordinate
ν	Poisson's ratio
ψ	flux or flow rate, per unit area

8.2. UNIT CONVERSIONS

Base Unit	Converting To	Multiply By
acre	meter ² (m ²)	4046.85642
atm	MPa	0.101325
bar	mega-pascal (MPa)	0.1
btu	joules (J)	1055.05585
centipoises (cP)	pascal second (Pa s)	0.001
degree	radian (rad)	0.0174532925
foot	meter	0.3048
square foot	meter ²	0.092903
ft ² /hr	meter ² per second (m ² /s)	2.58064x10 ⁻⁵
ft ⁴	meter ⁴ (m ⁴)	0.00863097
ft/hr	meter per second (m/s)	8.46667x10 ⁻⁵
ft/min	meter per second (m/s)	0.00508
ft/s	meter per second (m/s)	0.3048
mile/hr	meter per second (m/s)	0.44704
ft/s ²	meter per second ² (m/s ²)	0.3048
ft lbf	joule (J)	1.3558
cm/s ²	meter per second ² (m/s ²)	0.01
gallon	meter ³ (m ³)	0.00378541
gram	kilogram (kg)	0.001
g/cm ³	kilogram per meter ³ (kg/m ³)	1000
hour	second (s)	60
inch	meter (m)	0.0254
cubic foot	meter ³ (m ³)	0.0283168
sq/mile	acre	640
sq/mile	meter ² (m ²)	2589988.11
pound force (lbf)	newton (N)	4.44822
ton (US)	pound force (lbf)	2000

8.4. FIGURES

Using 0.0005 injection of one single fracture over time a distance of 0m from the point of injection.

Using 0.0005 injection of one single fracture over time a distance of 50m from the point of injection.

Using 0.0005 injection of one single fracture over time a distance of 100m from the point of injection.

Using 0.0005 injection of one single fracture over time a distance of 225m from the point of injection.

Results from (Cheng, 2012) referenced in comparison to **Figure 102 (a) and (b)**.

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