Solar Electric Propulsion Triple-Satellite-Aided Capture With Mars Flyby

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Solar Electric Propulsion Triple-Satellite-Aided Capture With Mars Flyby

Sean Patrick

Thesis submitted
To the Benjamin M. Statler College of Engineering and Mineral Resources
at West Virginia University
in partial fulfillment of the requirements for the degree of
Masters of Science
in
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ABSTRACT

Solar Electric Propulsion Triple-Satellite-Aided Capture With Mars Flyby

Sean Patrick

Triple-Satellite-aided-capture sequences use gravity-assists at three of Jupiter’s four massive Galilean moons to reduce the ΔV required to enter into Jupiter orbit. A triple-satellite-aided capture at Callisto, Ganymede, and Io is proposed to capture a SEP spacecraft into Jupiter orbit from an interplanetary Earth-Jupiter trajectory that employs low-thrust maneuvers. The principal advantage of this method is that it combines the ISP efficiency of ion propulsion with nearly impulsive but propellant-free gravity assists.

For this thesis, two main chapters are devoted to the exploration of low-thrust triple-flyby capture trajectories. Specifically, the design and optimization of these trajectories are explored heavily. The first chapter explores the design of two solar electric propulsion (SEP), low-thrust trajectories developed using the JPL’s MALTO software. The two trajectories combined represent a full Earth to Jupiter capture split into a heliocentric Earth to Jupiter Sphere of Influence (SOI) trajectory and a Jovian centric capture trajectory. The Jovian centric trajectory makes use of gravity assist flybys of Callisto, Ganymede, and Io to capture into Jupiter orbit with a period of 106.3 days.

Following this, in chapter two, three more SEP low-thrust trajectories were developed based upon those in chapter one. These trajectories, devised using the high-fidelity Mystic software, also developed by JPL, improve upon the original trajectories developed in chapter one. Here, the developed trajectories are each three separate, full Earth to Jupiter capture orbits. As in chapter one, a Mars gravity assist is used to augment the heliocentric trajectories. Gravity-assist flybys of Callisto, Ganymede, and Io or Europa are used to capture into Jupiter Orbit. With between 89.6 and 137.2-day periods, the orbits developed in chapters one and two are shorter than most Jupiter capture orbits achieved using low-thrust propulsion techniques.

Finally, chapter 3 presents an original trajectory design for a Very-Long-Baseline Interferometry (VLBI) satellite constellation. The design was created for the 8th Global Trajectory Optimization Competition (GTOC8) in which participants are tasked with creating and optimizing low-thrust trajectories to place a series of three spacecraft into formation to map given radio sources.
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Nomenclature

$V, v =$ velocity
$a =$ semi-major axis
$e =$ eccentricity
$h =$ triangle altitude
$h_p =$ flyby perapsis
$i =$ inclination
$J =$ performance index
$P =$ weighting factor
$r =$ radius
$\alpha =$ right ascension
$\delta =$ declination angle
$\delta_i =$ flyby turning angle
$\mu =$ gravitational parameter
$\nu =$ true anomaly
$\phi =$ VLBI measurement resolution
$\omega =$ argument of periapsis
$\Omega =$ right ascension of the ascending node
$x, y, z =$ Cartesian position components
INTRODUCTION

The use of gravity-assist flybys to reduce overall mission ΔV requirements via gravity assist at Venus\(^1\). First proposed by Longman, the use of has proven useful ever since Mariner 10 succeeded in reaching Mercury satellites to provide a gravity assist around a central planet such as Jupiter have been proposed for missions such as the Europa Orbiter Mission\(^2\)\(^3\). The use of such maneuvers, known as “satellite-aided capture”, allows for significant advantages in mission design as these maneuvers reduce the total ΔV requirements for orbital insertion around the central body. The Galileo mission was able to reduce its Jupiter orbit insertion (JOI) ΔV requirements by 175 m/s using an Io gravity assist\(^4\).

While single gravity-assist trajectories are most commonly proposed\(^5\), double gravity-assists have been proposed for some mission designs\(^3\)\(^6\). Further, the use of three of the Galilean moons for a triple-satellite-aided capture has been explored by Lynam et al. as a means to further reduce the overall ΔV requirements of Jupiter insertion\(^7\)\(^8\). This sequence begins with a hyperbolic arrival trajectory (from a low-thrust Earth-Jupiter trajectory) a few days before the triple-flyby gravity-assist sequence. The triple flyby sequence is designed such that each gravity assist occurs within one day of the previous assist. Following the three gravity-assists, a capture orbit is achieved with a period of one hundred days. The Callisto, Ganymede, and Io sequence is an excellent candidate for triple-satellite-aided capture as they are all much larger than Europa, so they can provide the strongest collective gravity-assist. Lynam and Longuski have shown that deep space
broken-plane maneuvers must be used to target triple-satellite-aided capture sequences since the average orbital plane of the Galilean moons is different from the ecliptic plane. If a chemical engine is used, these maneuvers often range from 40 m/s to 100 m/s. In order to eliminate impulsive maneuvers that use low specific impulse, $I_p$, chemical engines, low-thrust interplanetary trajectories are chosen for the triple-satellite-aided capture trajectories that are used in this paper. Furthermore, low-thrust interplanetary trajectories can have a much lower arrival $V_\infty$ than chemical trajectories; we can remove the need for an impulsive JOI maneuver.

Strange et al. have proposed a SEP design which uses multiple Hall thrusters. The design provided by Strange et al. is a good candidate for a triple-satellite-aided-capture sequence as it allows for optimal low thrust maneuvers at all stages of flight due to the differing thrust profiles of the chosen Hall thrusters. The model used here, specifically in chapter one, assumes a BHT-600 thruster is used upon arrival at Jupiter from an interplanetary trajectory with a power range of 0.49 to 1 kW. For the interplanetary trajectory, two BPT-4000 Hall thrusters are used in a variable configuration. Specific impulse and efficiency for the BHT-600 and BPT-4000 thrusters as given by Kamhawi et al. are used in the modeling. The Current Best Estimate (CBE) wet mass of 1970 kg and the margin mass value of 415 kg given by Strange et al. in the Mass Estimate List for their proposed mission was used as a baseline for the initial mass of the proposed craft. This was derived from the use of the Falcon 9 launch platform, a design choice replicated here.
This thesis expands upon the trajectory design work of Lynam and Longuski to develop multiple low-thrust trajectories. The thesis is broken down into three main chapters each focusing on a separate trajectory design. Chapters one and two explore Jupiter capture trajectories while chapter three details a separate, earth-centered low-thrust trajectory design developed for an industry competition.

Chapter one expands on the work of Lynam by exploring the feasibility of one of the proposed triple-satellite-aided capture sequences, specifically the Callisto-Ganymede-perijove-Io sequence\textsuperscript{11}. This objective is accomplished by modeling the sequence using the low-thrust optimization program MALTO to determine feasibility and to refine the Callisto-Ganymede-perijove-Io sequence as it would be carried out using a SEP design. Further, this paper will expand upon the chemical interplanetary trajectories used by Lynam and Longuski\textsuperscript{9}, by developing a low-thrust interplanetary trajectory using robust low-thrust trajectory optimization software (MALTO).

Section II of this chapter details the methodology and theory of the solution methods employed in this paper, covering the initial design of the proposed Callisto-Ganymede-perijove-Io triple flyby and the interplanetary trajectory. Following this, Section III discusses the resulting optimized trajectories while Section IV discusses and contextualizes these results.

Chapter two expands on Chapter One by performing a high-fidelity optimization of the low-fidelity trajectories devised using MALTO in Chapter One. To perform this optimization, the Mystic software program was employed in the design and modeling of the trajectory\textsuperscript{12}. As in the previous, low-fidelity trajectory, Hall thrusters were employed
using specific impulse and efficiency data as given by Kamhawi et al. for modeling\textsuperscript{10}. Two other trajectories were also found in Mystic that use triple flybys of Callisto, Ganymede, and Europa. Along with this high-fidelity modeling, a comparison is made to the impulsive Earth to Jupiter capture trajectory, featuring a Callisto-Io-perijove-Ganymede triple-flyby capture sequence, as developed by Didion and Lynam\textsuperscript{13}.

Chapter three details an original design for a Very-Long-Baseline Interferometry (VLBI) spacecraft constellation derived for the 8\textsuperscript{th} Global Trajectory Optimization Competition (GTOC8). The competition, organized by a group at the Jet Propulsion Laboratory is based around optimization of low-thrust maneuvers to place three spacecraft into simple formations to map radio sources.

For the stated problem, the three spacecraft begin collocated in a circular orbit with an altitude of 400 km inside the ecliptic plane. The spacecraft each have two propulsion systems, a chemical propulsion system with an impulsive capability of up to 3 km/s and an \textit{I}_\text{sp} of 450 s and a low-thrust system with \textit{I}_\text{sp} of 5000 s and a maximum thrust of 0.1 N. The chemical propulsion system may only be fired once, and it must be fired before the low-thrust system can be used. In addition, the mission must begin between MJD 58849.0 and 58880.0 and must end within three years.

**BACKGROUND**

In the following chapters, much time is taken to detail the design and optimization of various low-thrust trajectories. Here, some background information has been compiled to expand on some of the design elements located in those chapters. It is the author’s intent that this information is to be available to provide a better understanding of certain aspects
used in the proceeding chapters that may not have been explored as thoroughly.

**Propulsion systems**

Cold Gas-based propulsion systems provide thrust via a compressed gas such as molecular nitrogen, $\text{N}_2$, or Helium, He. A typical configuration for this type of system produces this thrust by releasing gas, stored at high pressure, through a feed system. The gas is then released at high velocity through a converging/diverging nozzle. These types of propulsion systems are relatively simple and safe to operate compared to Chemical propulsion systems; however, the maximum change in velocity that can be achieved with cold gas systems is quite low compared to a chemical propulsion system. Because of this, cold gas propulsion is most often used for attitude control and other minor orbital maneuvers.$^{14}$

Chemical propulsion systems are a broad category that covers any system that produces thrust through a chemical reaction of some sort. These types of propulsion systems tend to fall into the Liquid, Solid and Hybrid subcategories. Liquid propulsion systems use stored propellant which is then fed into a combustion chamber. Inside the combustion chamber, energy is released from the fuel via chemical reaction. The energy of the reaction is released as heat from the chamber through a nozzle much the in the Cold Gas system to produce thrust. A liquid propellant system is very attractive for use in spacecraft design as it provides the highest performance of any chemical propulsion system as well as being highly controllable through thrust modulation. Despite this liquid propulsion systems tend to be complex and have a high development cost.$^{14}$

Solid propulsion systems simplify the overall design by combining the feed and
storage systems used. This is done by premixing the propellant before launch and storing the propellant in a solid form inside the combustion chamber. Once activated, the fuel is ignited and it will continue to burn until it is depleted. The heat from the reaction is again funneled through a nozzle to provide thrust. A solid propulsion system is advantageous in that they are very simple to operate and are small due to high propellant packing density. This is offset by difficulties in manufacturing as well as throttling and control. Hybrid systems are similar the bipropellant liquid propulsion system in that separate fuel and oxidizers are used. Here a solid fuel is used in conjunction with a liquid oxidizer. The solid fuel, stored in the combustion chamber, is mixed with the liquid oxidizer and ignited; the resulting exhaust is funneled through a nozzle. These hybrid systems prove to be simpler than comparable bipropellant systems and can have a higher performance than purely solid rocket systems but will have a lower packing density than comparable solid rockets and poorer performance than liquid propulsion systems\textsuperscript{14}.

Nuclear propulsion systems are conceptually similar to liquid chemical propulsion systems in that a propellant is heated an expelled from a converging/diverging nozzle to create thrust. The two systems diverge in the method in which the propellant is heated. In a Nuclear propulsion system, a nuclear fission reaction is created to supply heat to the propellant either through a heat exchanger or directly over the fission material. These systems have high performance and are suitable for non-launch operations with the main limiting factors being high complexity and external opposition due to political factors regarding nuclear technology\textsuperscript{14}.

Finally, electrical propulsion systems use electricity instead of a chemical or nuclear
reactions to excite the propellant. This electrical excitement can be done in two ways. First, a resistive element or an arc discharge is used to heat a propellant in an electrothermal reaction. This method is very similar in design to a liquid chemical propulsion system barring the heat source. The second method is to ionize the propellant in an electrostatic or electromagnetic field. Electrical propulsion systems have a very high efficiency compared to chemical rockets but at the cost of much lower thrust levels making electrical propulsion systems unsuited for launch operations\textsuperscript{14}. Of the four propulsion types, a focus has been placed on electrical propulsion in particular for discussion to better discuss its application in future chapters.

Propulsion systems tend to follow a basic design which can be approximated as a system of six basic elements: propellant, propellant storage, propellant feed system, energy source, energy conversion, and accelerator. The propellant and energy sources have been touched upon previously, but in general the propellant is the primary component for momentum transfer in the rocket. The energy source is the method used for initiating the momentum transfer; as mentioned before, chemical or nuclear reactions are examples of energy sources. The propellant storage is a system that will hold the propellant in the necessary conditions so as to be usable by the propulsion system. For liquid and gas propellants the storage system will keep the contained propellant at the specific temperature and pressure needed to allow the propellant to properly react. Cold Gas systems for example will store the propellant gases under high pressure and low temperature. Solid propellant systems will combine the energy conversion and accelerator systems with the storage eliminating the need for the next component, the feed system\textsuperscript{14}. 
The feed system is designed such that it transfers the propellant from the storage into the accelerator where the energy conversion takes place. The energy conversion will depend on the type of propulsion method used. In chemical based systems, the chemical reaction converts the propellant into heat and pressure, a nuclear propulsion system uses a nuclear reaction to produce thermal energy to convert the propellant. Electrical propulsion systems use an electromagnetic field or thermal energy produced by an electrical discharge. Inside the accelerator following the energy conversion, a thermodynamic accelerator will direct the expansion of the resulting gas of propellant using a nozzle. A second type of accelerator uses electromagnetic fields to direct charged particles to use as thrust\textsuperscript{14}.

Compared to chemical and nuclear systems, Electrical propulsion has much higher specific impulse. This increased specific impulse is due to the fact that electrical energy is being used to generate high speed reaction-mass, and because the added energy to the propellant flow is not limited to the bond strength of matter. In chemical systems the amount of energy added is determined by the chemical reaction involved. Additionally, nuclear-thermal systems are limited by the structural integrity of the rocket at high temperatures. By having electromagnetic forces directly accelerate reaction-mass, the only potential limit of electrical propulsion systems is the amount of power available\textsuperscript{14}.

Electrical propulsion systems can be broken down into three types, based on how reaction-mass is accelerated. Electrothermal propulsion heats the propellant through the use of electrically heated surfaces (resistojet) or via direct heat deposition into the flow (arcjet). If the propulsion system directly accelerates the reaction mass by applying an
Electrostatic force to charged particles it is referred to as Electrostatic propulsion. One example is the ion engine which uses ionized atoms or molecules as the charge particle. Applied electrical forces are not the only method used in electrical propulsion systems. Magnetic fields may be employed. One method of doing so is based on the interactions between electrical currents in the propellant and a magnetic field to produce a Lorentz force used to accelerate the particles. Propulsion systems utilizing magnetic fields may be referred to as plasma propulsion systems due to the fact that plasma is often used as a propellant. However, they fall under the more broad electromagnetic propulsion category which incorporates more than just the use of plasmas\textsuperscript{14}.

**Gravity Assists**

To complement Solar Electric Propulsion (SEP) techniques employed in the following chapters, multiple-gravity-assist maneuvers were implemented. Gravity assist maneuvers, sometimes referred to as a gravitational slingshot, involve a close approach to a planetary body to adjust the velocity of a spacecraft without expending propellant. This velocity adjustment is typically accomplished by placing the spacecraft on a hyperbolic approach to a planet, moon or other massive body. The craft travels along the hyperbolic trajectory and, upon completion of the hyperbolic approach, the craft will have accelerated or decelerated with respect to the central body.
Figure 1: Diagram detailing the path of a velocity increasing gravity assist

To describe this interaction, we can look at conservation of momentum between the planet and spacecraft as shown below.

\[ mV_i + M V_f = mV_{i+} + M V_{f+} \]

Where \( m \) is the mass of the spacecraft, \( M \) is the mass of the planet. Additionally, \( v \) and \( V \) are the craft and planets respective velocities. The equation can be rearranged such that

\[ V_{i+} - V_{i-} = \frac{M}{m}(V_{f+} - V_{f-}) \]

Because the typical mass of a spacecraft is much lower than that of a planetary body, we can assume that the change in velocity of the planetary body is negligible and can this assume \( V_i = V_f = V \).
In addition to momentum conservation, energy conservation must be accounted for between the spacecraft and planetary body. To start, we define the approach velocity of the spacecraft as \( u_i = v_i - V \) relative to the planet. As the craft approaches the planet, \( u_i \) is deflected by the planet’s gravitational pull such that the departure velocity is equal in magnitude, i.e.

\[
|\mathbf{\Phi}| = |\mathbf{\Theta}|
\]

From this, we can derive that, in the space frame, \( v_f = u_f + V \). It is important to note that, the velocity of the spacecraft relative to the planet does not change. However, the velocity relative to the Sun (space frame) does change\(^{15} \). This is due to the fact that the \( v_\infty \) vector rotates, causing a change in the magnitude of the heliocentric velocity vector. A diagram detailing this is shown below in figure 2.

![Diagram showing how the turning angle affects the heliocentric velocity of a spacecraft during a gravity assist maneuver](image)

**Figure 2:** Diagram showing how the turning angle affects the heliocentric velocity of a spacecraft during a gravity assist maneuver
Triple-flybys

As mentioned briefly above, chapters one and two expand upon previous work, primarily by Lynam\textsuperscript{7-9, 11}, on triple-flyby-capture trajectories. These trajectories employ three sequential gravity-assist maneuvers to capture into orbit around a central body.

Historically, single and double flyby sequences have been used in mission design. Lynam and Longuski have explored the use of Triple-flyby sequences for use in Jupiter missions. The triple-flyby sequences explored by Lynam and Longuski make use of the Galilean moons to lower the ΔV requirements needed to capture into Jupiter orbit\textsuperscript{7-9}.

The initial triple-flyby sequences developed by Lynam were refined here to produce complete Earth to Jupiter capture trajectories in chapters one and two.
CHAPTER ONE

Building upon previous work by Lynam, this chapter explores a Callisto-Ganymede-perijove-Io trajectory sequence. The maneuver is used to capture into Jupiter orbit following an interplanetary trajectory from Earth to Jupiter with Mars assist. The use of low-thrust propulsion allowed for the trajectory to be determined using the MALTO software developed at NASA JPL. Additionally, a similar trajectory, which employs a double flyby capture sequence, developed by Strange et al. is used as a point of comparison throughout the chapter. The trajectory presented here makes use of the advantages low-thrust propulsion offers for Jupiter capture such as the lack of Jupiter Orbital Insertion (JOI) maneuver. These advantages are explored along with some of the navigational challenges of a low-thrust triple-flyby-capture.

THEORY & METHODOLOGY

To develop an optimal Triple-flyby trajectory, the MALTO low-thrust optimization software was employed. MALTO was chosen to be used for trajectory optimization as it provides an intuitive interface along with implementing SNOPT optimization. SNOPT implements a sequential quadratic programming method using a reduced-Hessian semidefinite QP solver to solve nonlinear optimization problems. To construct a trajectory, MALTO separates the trajectory into discrete segments around a match point in which the segment is propagated forward from the initial control node and backwards from the later control node. Generally, these propagations are done using the patched-conic method with low-thrust modeled as impulsive maneuvers placed at discrete segments. For low-thrust trajectories, MALTO can model multiple power and propulsion
sources such as NEP and SEP as well as differing launch vehicles\textsuperscript{16}. Of particular interest to Jupiter missions, solar perturbations were recently added to increase the fidelity of MALTO’s patched-conic propagations in the Jupiter System. These factors allow MALTO to model an array of potential missions.

The Callisto-Ganymede-perijove-Io sequence was modeled in MALTO as a single trajectory using an initial Non-body control point (NBCP) to represent the initial approach towards Jupiter from the interplanetary trajectory. Following the flyby sequence, the craft will capture into Jupiter orbit. A second flyby of Ganymede is included. This second flyby represents the start of an extended Jupiter science mission following the initial insertion into Jupiter orbit using the triple flyby sequence. Once set up, the trajectory is optimized using MALTO. The initial conditions for the initial guess of this trajectory are provided below in Table 1. The goal of optimizing the triple-flyby sequence is to achieve a 100 day period capture orbit.

Table 1. Initial guess for Jupiter-centered, ecliptic J2000 parameters for the NBCP used in the Triple Flyby. These values were obtained via GMAT and used as an initial guess for the MALTO optimization.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>3329325.5081</td>
<td>km</td>
</tr>
<tr>
<td>Y</td>
<td>-1043254.1264</td>
<td>km</td>
</tr>
<tr>
<td>Z</td>
<td>-475214.48645</td>
<td>km</td>
</tr>
<tr>
<td>V_X</td>
<td>-7.5286</td>
<td>km/s</td>
</tr>
<tr>
<td>V_Y</td>
<td>4.9635</td>
<td>km/s</td>
</tr>
<tr>
<td>V_Z</td>
<td>2.2582</td>
<td>km/s</td>
</tr>
</tbody>
</table>
The initial conditions chosen were based on previous work by Lynam\textsuperscript{11}. The initial flyby dates for the Callisto-Ganymede-perijove-Io flyby, each one day after the previous, were chosen such that each of the three moons would be in the proper sequence for the triple flyby. The follow up flyby of Ganymede has been chosen to be approximately 100 days after the initial Io flyby following optimization. To account for suboptimalities following initial optimization, the trajectory was re-optimized using forced coast segments. This method ensures a feasible starting trajectory for the force coast and eliminates further potential errors in the optimization.

Once an optimal solution to the Callisto-Ganymede-perijove-Io sequence was found in MALTO, an interplanetary trajectory was developed. The sequence was initialized as an Earth-Mars-Jupiter, low-thrust trajectory. The initial values for the interplanetary trajectory were based of previous work by Lynam\textsuperscript{11} as well as Strange et al.\textsuperscript{6}. The interplanetary trajectory was initially designed as an Earth to Jupiter trajectory with a single gravity-assist of Mars. This initial trajectory was optimized in MALTO and then modified to use an NBC point that was back propagated from the NBCP of triple-flyby trajectory as the final control point instead of Jupiter. The choice of optimizing in two parts, first and Earth-Mars-Jupiter and then Earth-Mars-NBCP, was made to ensure the design of the trajectory was feasible. In addition, the modification of the optimal Earth-Mars-Jupiter trajectory allowed for easier and more accurate optimization of the Earth-Mars-NBCP trajectory as the initial guess was closer to the optimal state.
The NBCP used in the interplanetary trajectory represents the point at which the spacecraft enters Jupiter’s gravitational sphere of influence (SOI). This point was determined by back-propagating the trajectory of the spacecraft from the NBCP of the optimized triple-flyby trajectory. This back propagation was accomplished using GMAT. The GMAT software was chosen for the back-propagation over MALTO due to the nature of the task. Specifically, MALTO and other patched-conic optimizers and propagators have notably low-fidelity in propagating trajectories for long time periods when two gravitating bodies both impart significant acceleration on the spacecraft. During the trajectory region between the Jupiter-centered NBCP and the heliocentric NBCP at Jupiter’s SOI, both Jupiter and the Sun have a significant influence on the spacecraft’s trajectory. Thus, GMAT is used to directly numerically integrate the trajectory with the gravity of Jupiter and the Sun. GMAT can accurately model this regime by using direct numerical integration of the trajectory with the full gravity of Jupiter and the Sun throughout this regime.

Using GMAT, a script was employed to back propagate from the initial Jupiter centered NBC point to Jupiter’s SOI. As mentioned earlier, the numerical integrator for this script used the point mass gravity of both Jupiter and the Sun (For added fidelity, the script also added the point mass gravity of other planets and Pluto). Once the script successfully back propagated the NBCP, the new location was converted to heliocentric coordinates and used in MALTO for the interplanetary trajectory. The back propagation allows for more accurate formulation of the interplanetary trajectory in MALTO. By moving the NBCP to Jupiter's SOI, more accurate heliocentric coordinates could be
determined for use in MALTO. Once the new NBCP is found using GMAT, the interplanetary trajectory is optimized in MALTO based on this new point. The parameters of the new NBCP are given below in Table 2.

Table 2. Initial heliocentric, ecliptic J2000 parameters of the back propagated NBCP at Jupiter’s SOL.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>-662176330.1877</td>
<td>km</td>
</tr>
<tr>
<td>Y</td>
<td>-462209124.5735</td>
<td>km</td>
</tr>
<tr>
<td>Z</td>
<td>16146115.1805</td>
<td>km</td>
</tr>
<tr>
<td>V_x</td>
<td>4.1570</td>
<td>km/s</td>
</tr>
<tr>
<td>V_y</td>
<td>-7.3053</td>
<td>km/s</td>
</tr>
<tr>
<td>V_z</td>
<td>0.0003192</td>
<td>km/s</td>
</tr>
</tbody>
</table>

The work of Strange et al.\textsuperscript{6} was chosen as a basis for the mass and power and propulsion systems for both the triple-flyby and interplanetary trajectories. This choice was made due to the sample mission provided by Strange et al. who provide detailed reasoning for the choices made regarding the power, propulsion as well as the mass allowance of the sample mission.

The power and propulsion system used by Strange et al.\textsuperscript{6} included BPT-4000 and BHT-600 Hall thrusters along with SEP, which provides an optimal set up for triple-flyby sequences. As Strange et al. point out, Hall thrusters provide higher thrust but lower specific impulse than ion engines for the same power consumption. This fact, as well as the fact that Hall thrusters can be magnetically shielded, makes Hall thrusters the more attractive option for use at Jupiter as the solar power supply would be limited at 5.2 AU\textsuperscript{6}.  

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The BHT-600 and BPT-4000 thrusters used in the sample mission were picked specifically based on the efficiency curve of the thrusters. By using two BPT-4000 thrusters for the interplanetary trajectory and a BHT-600 for the Jupiter mission, Strange et al. have designed a propulsion set up that allows for efficient thrust at all points of the mission. To power the Hall thrusters, an Ultraflex Solar Array with a reference power of 30 kW at 1 AU was employed. This array will provide sufficient power to the Hall thrusters for the interplanetary trajectory as well as the triple-flyby trajectory. SEP is an attractive option for long term Jupiter missions compared to NEP due to the fact that, as Strange et al. mention, multiple radioisotope power systems would be required to produce the necessary 1 kW of power the BHT-600 requires at Jupiter\(^6\).

To model the BHT-600 and BPT-4000 Hall thrusters in MALTO, the values given below in Table 3 were used. In addition to these values, the default solar model for MALTO was employed along with the Ultraflex solar array model values included with MALTO. In addition, for thruster modeling of the BPT-4000 a Pmax of 5 and a Pmin of 2.2 were employed.

**Table 3. Coefficients used to model BPT-4000 and BHT-600 hall thrusters. All other coefficients were not utilized and set to zero.**

<table>
<thead>
<tr>
<th>Thruster</th>
<th>Cthrust(2) [N/kW]</th>
<th>Cdot(2) [kg/s/kW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPT-4000</td>
<td>0.066281553843565</td>
<td>4.224273442228 (\times) (10^{-06})</td>
</tr>
<tr>
<td>BHT-600</td>
<td>0.056084391713786</td>
<td>2.8595081762776 (\times) (10^{-06})</td>
</tr>
</tbody>
</table>

For the interplanetary trajectory, the launch mass was chosen to be 2385 kg and the launch velocity was capped at 5.55 km/s. These values were chosen based on the use of the
Falcon 9. For the BPT-4000 thrusters, the power and propulsion system was set such that two thrusters were used until the power dropped below 4.8 kW. Once the power dropped below 4.8 kW one thruster would be shut down. These values correspond to those of the Falcon 9 and of the thruster configuration used by Strange et al. 6.

RESULTS
Using MALTO, the Callisto-Ganymede-perijove-Io trajectory was successfully optimized. The follow up Ganymede trajectory was also successfully optimized with the resulting trajectory including both the triple flyby sequence and the follow up tour displayed in Figures 1 and 2. The triple flyby approach forms an expected hyperbola hitting the three gravity assists in succession. The Callisto and Ganymede flybys, at 282 km and 183 km respectively, are low but not infeasible. The Io flyby distance, even lower than Ganymede at 100km, is still within a safe distance. The follow-up flyby to Ganymede is, at 106.3 days, very near the desired 100 day orbit. This orbit puts the Ganymede arrival position very near that of the original Ganymede gravity-assist.
Figure 3. Close view of Callisto-Ganymede-perijove-Io triple flyby sequence following optimization.

The triple flyby sequence is completed ballistically and does not expend any extra mass. Note however that while the triple flyby sequence is comprised of a smooth conic section, the following section containing the second Ganymede flyby and the capture orbit are not. This is due to the method that segmentation is handled in MALTO. The linear behavior in this region is a result of MALTO’s handling of linear connections between segments rather than making a correct conic section. Figure 2 better illustrates the nature of this.

The successful Ganymede flyby shows that an extended science mission following the triple-flyby sequence is possible. The values for the optimized NBCP called “PreCapture Start” are given in Table 4. While optimized, the trajectory output in MALTO
Table 4. Parameters of the initial Jupiter-centered, ecliptic J2000 NBCP after optimization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>3525495.932</td>
<td>km</td>
</tr>
<tr>
<td>Y</td>
<td>-1913671.4535083</td>
<td>km</td>
</tr>
<tr>
<td>Z</td>
<td>-29262.11</td>
<td>km</td>
</tr>
<tr>
<td>Vx</td>
<td>-6.1082620057458</td>
<td>km/s</td>
</tr>
<tr>
<td>Vy</td>
<td>6.1629269494039</td>
<td>km/s</td>
</tr>
<tr>
<td>Vz</td>
<td>0.17901096148155</td>
<td>km/s</td>
</tr>
</tbody>
</table>

Figure 4. Expanded view of Callisto-Ganymede-perijove-Io flyby following optimization. This figure shows the full follow-up capture orbit.

Following the optimization of the triple-flyby, the interplanetary trajectory was successfully optimized regarding the initial Earth-Mars-Jupiter trajectory and was then subsequently optimized for the Earth-Mars-NBCP trajectory. The NBCP used was first
determined using the back-propagation method described above in Section II. The optimized trajectory is given below in Figure 3.

In Figure 3, the spacecraft launches from Earth on November 16, 2024 on a SpaceX Falcon 9 rocket and begins its interplanetary trajectory with a mass of 2385 kg and a launch $\Delta v$ of 3.6 km/s. The spacecraft coasts until it nearly reaches aphelion.

Near aphelion, the SEP engines provide a low-thrust analog of a $\Delta v$-leveraging maneuver that raises the perihelion of the spacecraft to near Martian orbit (see red arrows in Figure 3). On April 5th, 2026, the spacecraft flies by Mars at its lower bound altitude of 300 km. This Martian gravity assist increases the aphelion of the spacecraft and enables it to reach Jupiter with less thrusting (and therefore less mass expenditure).

The spacecraft optimally coasts for several more months after the Martian gravity assist before thrusting again with its Hall thrusters. These maneuvers are near perihelion and nearly tangent to the spacecraft’s trajectory, so they efficiently raise the aphelion of the spacecraft toward Jupiter’s orbit. The spacecraft coasts again until it reaches a second set of SEP maneuvers. These maneuvers are mostly perpendicular to the spacecraft’s trajectory, so they are not efficient in the sense that they are not consistent with an optimal elliptical spiral and therefore not as efficient as they could be. However, these SEP maneuvers are necessary to ensure that the spacecraft enters Jupiter’s SOI on a trajectory that is consistent with the highly constrained triple flyby sequence in Figure 1. The total mass cost of the SEP trajectory leg from Mars to Jupiter SOI NBCP is 494.3 kg.
Further optimization work in Chapter two improved these maneuvers and enabled a more mass-efficient interplanetary trajectory.

![Earth-Mars-NBCP interplanetary trajectory following optimization](image)

**Figure 5. Earth-Mars-NBCP interplanetary trajectory following optimization**

Below, Table 5 details a complete mission timeline for the interplanetary and triple-flyby trajectories. The GMAT propagation from Jupiter SOI to Jupiter NBCP was ballistic, so it is assumed that no mass would be used in that leg.
Table 5. Timeline detailing the time and mass at each major node for the entire 5.3 year length of the mission.

<table>
<thead>
<tr>
<th>Node</th>
<th>Date</th>
<th>Mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth Launch</td>
<td>11/16/2024</td>
<td>2345</td>
</tr>
<tr>
<td>Mars Gravity-Assist</td>
<td>04/05/2026</td>
<td>2197.8</td>
</tr>
<tr>
<td>Jupiter SOI arrival</td>
<td>08/11/2029</td>
<td>1703.5</td>
</tr>
<tr>
<td>Jupiter NBCP</td>
<td>11/28/2029</td>
<td>1703.5</td>
</tr>
<tr>
<td>Callisto flyby</td>
<td>12/01/2029</td>
<td>1703.3</td>
</tr>
<tr>
<td>Ganymede flyby 1</td>
<td>12/02/2029</td>
<td>1703.2</td>
</tr>
<tr>
<td>Io flyby</td>
<td>12/03/2029</td>
<td>1703.0</td>
</tr>
<tr>
<td>Ganymede flyby 2</td>
<td>03/19/2030</td>
<td>1694.4</td>
</tr>
</tbody>
</table>

To better compare the interplanetary trajectory developed here to that of Strange et al., a second interplanetary trajectory, as shown in Figure 4, was developed in which a Jupiter Flyby was used in place of the back-propagated NBCP. This interplanetary trajectory was designed to be as close as possible to the original NBCP trajectory as possible to allow for a more direct comparison of the effects the final state was on the overall mission mass expenditure. The optimized conditions for the Jupiter flyby were unchanged from the optimal NBCP trajectory prior to optimization of the interplanetary trajectory. Following optimization, the new Earth-Mars-Jupiter trajectory resulted in a final mass of 1939.4 kg as compared to the Earth-Mars-NBCP trajectory which resulted in a 1703.5 kg final mass, a difference of 235.9 kg.
Figure 6. Earth-Mars-Jupiter interplanetary trajectory following optimization

Following optimization, the trajectory design stayed quite similar to the original Earth-Mars-NBCP trajectory as well as Strange et al.’s Earth-Mars-Jupiter trajectory. Due to the use of Jupiter as an end point instead of the GMAT derived NBCP, the initial launch date changed during optimization from 11/16/2024 to 12/19/2024. Despite this the arrival date of the mission to Jupiter is in line with the date of approach to perijove of the triple-flyby at 12/3/2029.

DISCUSSION

Comparison with SEP Double Flyby Capture at Jupiter

Since the modeling choices for this paper were based on those of Strange et al.⁶, the partial mission designed in this paper can be directly compared to the appropriate
portions of the mission designed by Strange et al. The optimized Earth-Mars-NBCP interplanetary trajectory was found to be very comparable to the Earth-Mars-Jupiter interplanetary trajectory depicted in Figure 3 of Strange et al. The first point of comparison is that the shapes of the two interplanetary trajectories are similar. This similarity is due to the similar MALTO design methodology used in the two trajectories. The second point of comparison is that their times of flight are similar. The interplanetary trajectory of Strange et al. had a time of flight from Earth to Callisto flyby of 4.86 years while the time of flight of the interplanetary trajectory in this paper from Earth to Callisto flyby was 4.96 years. Hence, this paper’s trajectory only needed slightly more than 1 month longer to arrive at Jupiter than that of Strange et al.

Although the shape, design methodology, and time from Earth to Jupiter of the two interplanetary trajectories were very similar, Strange et al.’s interplanetary trajectory resulted in a final mass of 1839.4 kg while the trajectory presented here in Figure 3 resulted in a final mass of 1703.5 kg. However, this comparison is somewhat misleading since Strange et al. did not use the GMAT integration used in this paper to patch their interplanetary trajectory to their Jupiter-centered trajectory. If they had, it is possible they would have also obtained a less optimal solution. To illustrate this point, the trajectory as presented in Figure 4 uses a more directly analogous design to that of Strange et al. which resulted in a final mass of 1939.4 kg. By using Jupiter as an end state versus the NBCP and changing no other aspects of the trajectory before optimization, a change of 235.9 kg of mass occurred. This change can be directly attributed to the use of Jupiter as an end
state. The NBCP used in figure 3 requires more constraints on optimization in MALTO to properly optimize and, because of this, the trajectory expends more mass.

While optimal within the modeling constraints imposed in this study, the trajectory in this paper could potentially be improved to further reduce the amount of mass spent. This suboptimality, along with the jagged thrust profile in the triple-flyby, can partially be attributed to artifacts in the low-thrust optimization of MALTO. Additionally, the optimality of the completed trajectory was reduced due to the fact that the triple flyby and the interplanetary trajectory could not be simultaneously optimized in MALTO since MALTO cannot change central bodies within a single optimization run. The use of forced coast arcs in the Triple Flyby trajectory did eliminate much of the excess thrust arrows during Jupiter approach, the constrained nature of the interplanetary trajectory needed to successfully target the NBCP will still cause more mass expenditure over the entire mission. Future design work using higher fidelity optimizer software such as Mystic was employed in Chapter two to further improve upon these designs.

The principal difference between the mission design in this paper and that of Strange et al.⁶ is that this paper uses a triple flyby of Callisto, Ganymede and Io while Strange et al. used a double flyby of Callisto and Ganymede. There is several design tradeoffs associated with using double vs triple flybys to capture into Jupiter orbit. First, the triple flyby is better than the double flyby at quickly reducing Jupiter-centered orbit period of the spacecraft due to its larger number of gravity assists. The capture orbit used in this paper had a period of 106.3 days before the second Ganymede flyby (which would ostensibly reduce the orbit period to a reasonable value if the full tour was modeled).
Strange et al.’s Jupiter-centered mission required a 350.4 day capture orbit, a second Callisto flyby a second, 84.1-day orbit, and a third Callisto flyby before capturing into a reasonable 33.4 day orbit. Despite the shorter capture period, the use of a triple-flyby capture sequence here gives the spacecraft a perijove below that of Io. This low perijove would expose the spacecraft to increased radiation levels.

Combining the interplanetary trajectory time with the time required to obtain a low-period orbit for both approaches gives the following total times. The double flyby approach used by Strange et al. would reach a low-period Jupiter orbit on 9/7/2028, roughly 6 years from the initial 8/18/2022 launch date. Comparatively, this triple flyby approach would achieve low-period Jupiter orbit on 3/19/2030, about 5.3 years following the 11/16/2024 launch date. Hence, the triple flyby approach saves a substantial amount of mission time.

A final comparison of the two approach strategies involves this paper’s use of an Io flyby to aid in capture. As mentioned above, in order to use Io as a gravity-assist body, the spacecraft must have a perijove that is below Io’s orbital radius of 5.9 Jupiter radii. Such a close approach to Jupiter would expose the spacecraft to increased levels of radiation from the “Io torus” that would need to be mitigated. Furthermore, in a more complete mission design, the spacecraft would have to perform more SEP thrusting during the capture orbit to raise its perijove in order to compensate for the effects of solar perturbations and reduce the radiation exposure of the spacecraft in future orbits. This SEP thrusting was modeled by Strange et al.6, but not in this paper due to its focus on the interplanetary trajectory and capture.
Navigation Challenges
This analysis strongly focuses on the mission design of SEP triple flyby capture trajectories and does not directly address the navigational challenges of triple satellite aided capture in general or SEP triple satellite aided capture in particular. Lynam and Longuski\(^7\) performed a preliminary navigation analysis for chemical triple satellite aided capture, so their results will be contextualized to SEP triple satellite aided capture. Lynam and Longuski showed that using only radiometric navigation would require two trajectory correction maneuvers (TCMs) with a total impulsive \(\Delta V\) of about 5-9 m/s to precisely guide a spacecraft through a triple flyby trajectory. Using both radiometric and optical navigation, they showed that it would take 2-4 m/s. For both cases, they assumed an expedited ground processing loop that can determine the spacecraft’s orbit after each flyby, calculate a trajectory correction maneuver (TCM), and command the spacecraft to perform that maneuver within 4.5-8.5 hours after each flyby. Due to the 30-50 minute one-way light time between Earth and Jupiter, achieving these results in an operational mission would be challenging.

Since Lynam and Longuski’s results\(^7\) assumed impulsive maneuvers, it is difficult to predict their applicability to SEP triple satellite aided capture. Strange et al.\(^6\) stated that their BHT-600 Hall thruster has a control authority of 1.6 m/s/day applied continuously rather than impulsively. Since there is about 1 day between flybys and the continuous application of \(\Delta V\) would be less efficient at correcting the flyby errors than an impulsive
ΔV, it is clear that only using radiometric navigation would not be sufficient for guiding an SEP spacecraft through a triple flyby. Using both radiometric and optical navigation to guide the spacecraft through the triple flyby from the ground would be extremely challenging, but not necessarily impossible. However, the spacecraft would probably require an extremely powerful imaging telescope such as Deep Impact’s\textsuperscript{18} High Resolution Imager (HRI) and the use of stereophotoclinometry\textsuperscript{19-21}, which would complicate the payload choices and the operation of the mission. The use of autonomous navigation is another option which would save hours of light time delay, but also would require a robust onboard navigation system and probably still require an HRI-level imager. Another possible strategy would be to use low Isp RCS attitude control thrusters to navigate the flybys, but use the high Isp SEP thrusters for every other maneuver in the mission. Despite the challenge of the TCM maneuvers involved, the SEP propulsion employed would not require an impulsive Jupiter Orbit Insertion (JOI) maneuver (that would add operational difficulty and statistical uncertainty to the capture). A full GNC analysis of SEP triple flybys would be an interesting topic for further research, but is beyond the scope of this chapter.
CHAPTER TWO

Previously, chapter one went into detail on the design and optimization of the application of gravity-assist maneuvers to lower the orbital energy of a spacecraft to allow it to capture into Jupiter orbit. These gravity-assists using a satellite around the capturing body, referred to as satellite-aided capture, made use of three of the Galilean moons, specifically Callisto, Ganymede, and Io. The previous chapter made use of the MALTO low-thrust optimization program developed by NASA JPL to develop two separate trajectories that, when combined, detail a complete, low-thrust Earth to Jupiter capture trajectory using a triple-satellite-aided-capture sequence.

This chapter further improves upon the trajectories developed in Chapter one by using the Mystic optimization software also developed by NASA JPL to improve upon the MALTO trajectories. By exploiting the fact that Mystic allows for the central body of the trajectory to change mid integration, a single trajectory detailing the complete Earth-Jupiter capture complete with triple flyby sequence was able to be developed. This is contrast to the need to develop two separate trajectories, one sun-centric and one Jupiter-centric, in MALTO. Additionally, following optimization of the Earth-Mars-Callisto-Ganymede-perijove-Io-Jupiter trajectory, two additional trajectories employing different triple flyby sequences were developed.

METHODOLOGY

The trajectory design started, as stated above by using previously developed trajectories by Patrick and Lynam as a basis for the current design work. These
trajectories were devised using the MALTO design software developed by NASA Jet Propulsion Laboratory (JPL) as a means of performing low-fidelity trajectory optimization and mission planning. The trajectories derived using MALTO showed promising results, the capture trajectory used a Callisto-Ganymede-perijove-Io capture sequence to obtain direct Jupiter capture with a period of 106.3 days with negligible SEP $\Delta V$ expenditure. The interplanetary trajectory conversely showed a distinct possibility of improvement.

Patrick and Lynam’s interplanetary trajectory had a final mass 135.9 kg lower than the interplanetary trajectory devised by Strange et al. that was used as a basis of comparison. A few possibilities were proposed as to the reasons for this. The most important reason, as far as this paper concerned, is that MALTO does not allow for the changing of central bodies in a single optimization run. Because of this factor the interplanetary and triple flyby trajectories could not be simultaneously optimized in MALTO. To correct this, the high fidelity optimization software Mystic was employed to design a new trajectory incorporating both the interplanetary and triple flyby trajectories into one trajectories allowing for simultaneous optimization.

**Mystic vs MALTO**

Mystic, also developed at JPL, employs a patented “Static/Dynamic Optimal Control (SDC)” algorithm for optimization$^{23}$. The SDC algorithm is a patented, nonlinear optimization method that is designed to optimize both static and dynamic variables at the same time. This, as noted by Whiffen, previously developed optimization schemes do not incorporate both static and dynamic optimization. In the case of Mystic, the application
of the SDC algorithm allows for the use of both static constraints such as Launch dates and the dynamic Low thrust integration. The Mystic software applies the SDC algorithm to computing optimal low-thrust trajectories by way of either maximizing the final net mass of the spacecraft or by minimizing user defined infeasibility by way of the magnitude of a constraint violation.

In addition, Mystic allows for the central body of the trajectory to be changed mid-trajectory, this allowed for the integration of the triple-flyby trajectory with that of the interplanetary trajectory to create a single trajectory starting from Earth launch and ending in Jupiter capture orbit. By creating a single, continuous trajectory, Mystic reduces the overall error in the trajectory by eliminating the need to match end points of trajectories. This fact is relevant to the trajectory devised by Patrick and Lynam as the GMAT program was required to back propagate the start of the MALTO triple flyby trajectory to Jupiter’s sphere of influence (SOI) to allow for matching with the MALTO interplanetary trajectory. By using Mystic here, no back propagation is required.

Beyond the fact Mystic allows for the changing of the central body for the trajectory, Mystic also allows for more control over trajectory end states. MALTO, for example, builds trajectories by defining trajectory segments each ending with an approach of a pre-defined body or control point. Mystic conversely allows for a wider range of intermediate and end states by allowing for simple creation of bodies via ephemeris or orbital element definitions as well as custom orbit constraints as defined around a specific central body.

Especially relevant to the work presented here is how both MALTO and Mystic optimize flybys of massive bodies. As mentioned previously in Chapter one, MALTO
uses a patched conic method for trajectory construction causing flybys of massive bodies
and gravity assist maneuvers to be treated as zero redius, center-of-mass flybys. A post
processing script is used after optimization to derive the flyby altitudes for the trajectory.
Mystic conversely does not use a patched-conic approach to trajectory construction and
will integrate and develop flybys of massive bodies directly based off of the physical
parameters of said bodies as described in the dynamical model.

Despite its disadvantages, MALTO does provide two benefits over Mystic. First,
MALTO is better at finding trajectories without initial guesses than Mystic is. Secondly,
MALTO allows for the definition of custom launch vehicles inside the GUI in a much
more user-friendly manner. Mystic conversely does not allow for custom launch vehicle
definitions in the GUI. Mystic does, however, allow for a greater degree of control over
launch parameters as well as more in-depth launch mass curve manipulation for the
predefined launch vehicles.

Below, Table 6 provides a comparison between some of the key aspects of both
MALTO and Mystic that are relevant to the work presented here.
| **Table 6: Comparison of key features and limitations of MALTO and Mystic software packages** |
|-------------------------------------------------|-----------------|
| **Optimizer**                                   | **Mystic**      |
| MALTO employs the SNOPT SQ optimization algorithm. | Mystic employs a patented “Static/Dynamic” Optimization technique. |
| **Dynamic Model**                               | **Mystic**      |
| Zero Sphere of Influence Patched Conic dynamic Model. | Fully integrated dynamic model |
| **Integrator**                                  | **Mystic**      |
| No integrator included.                         | A Geometric Integrator is included. The integrator includes evaluation criteria allowing for automatic changes in the central body around which the trajectory is optimized |
| **Trajectory Construction**                    | **Mystic**      |
| The trajectory is broken into discrete segments between control nodes representing massive bodies relevant to the mission. | Single, End to End trajectories are constructed. |
| **Gravity Assist**                              | **Mystic**      |
| Due to the use of a patched conic approach flybys of massive bodies are treated as being zero altitude for trajectory determination. Altitude of Gravity assist maneuvers is determined using a post processing script. | Gravity assist maneuvers are integrated during trajectory optimization using the full Dynamic model available in Mystic requiring no post processing |
| **Low-thrust Modeling**                        | **Mystic**      |
| Low-thrust maneuvers are handled individually for each segment and are treated as a single impulsive ΔV maneuver. | Low-thrust maneuvers are integrated fully overtime and are not treated as single impulsive ΔV maneuvers |
| **Other Limitations**                           | **Mystic**      |
| Does not allow for the changing of the central body of the trajectory. This creates a need for multiple trajectories for different stages of mission design to be developed and patched. | With regards to the application presented here, specifically targeting of a triple-flyby sequence, the validity of the initial guess trajectory is crucial. |
Trajectory Design

Much like the previous MALTO trajectory design, the triple flyby trajectory was designed first to ensure feasibility of the capture with the interplanetary trajectory designed afterwards. Once the triple flyby was developed and a feasible interplanetary trajectory was developed, the two were combined to further optimize the full trajectory.

The optimized MALTO trajectory was used to provide reasonable initial guesses for the trajectory design. The triple flyby was designed using the optimized Non-body control point (NBCP) from MALTO as the starting conditions leading into the Callisto-Ganymede-perijove-Lo sequence. One major change to the design is the change in end state for the trajectory. Due to previously discussed limitations in the MALTO software, a second Ganymede flyby representing the Jupiter capture orbit and the start of a subsequent science mission was used as the end state for the trajectory. Here, Mystic allows the end state to be represented by an energy constraint representing achievement of a 100-day Jupiter capture orbit.

Once the triple flyby trajectory was developed and optimized, the interplanetary trajectory was developed using the newly optimized Non-body control point as the initial target location. This optimization was achieved through an iterative design process starting with an Earth to Mars trajectory and working up to an Earth-Mars-Jupiter and finally Earth-Mars-NBCP trajectory. This iterative method was employed due to the highly constrained nature of the trajectory (the constraint requirements can be seen in the previous chapter). By iterating the trajectory piecewise, a greater degree of control over the optimization can be achieved.
The initial design of the trajectory was based on the optimized interplanetary MALTO trajectory with a few changes. The largest change to the trajectory is in the use of an Atlas V with constrained launch $V_\infty$ vs Falcon 9 for the initial earth centered launch. This change was made for optimization simplification. The only parameter of the trajectory that was affected by this change is that of the initial launch mass, by switching to the Atlas V a larger starting mass value can be assumed.

Once the triple-flyby and interplanetary trajectories were developed the combined trajectory was developed by adding 30 days to the Earth-Mars-NBCP interplanetary trajectory. In addition, the Callisto, Ganymede and Io gravity assists were inserted and optimized.

As mentioned previously, each trajectory was based on the previous MALTO trajectories derived by Patrick and Lynam. From these the following initial conditions as displayed in Tables 1 and 2 were developed for the triple flyby and interplanetary trajectories. The combined trajectory’s initial conditions were developed based on those of the triple-flyby and interplanetary trajectories.

Following the optimization of the final combined trajectory, two more trajectories were developed similarly to the combined trajectory. The optimized Earth-Mars trajectory was then used as a base and expanded up to complete Earth-Mars-Jupiter capture orbits with inserted and optimized Callisto, Ganymede and Europa gravity assists. The two additional trajectories differ in the placement of the Europa flyby, which is either before or after perijove.
RESULTS

Following the optimization of the individual triple-flyby and interplanetary trajectories for the first solution, an optimal combined trajectory was found using Mystic. This trajectory demonstrates a complete, Earth to Jupiter capture orbit over the course of 1848.875 days (5.0619 years). The trajectory includes a successful gravity assist of Mars as well as a complete triple-flyby capture using of Callisto, Ganymede, and Io (with each flyby one day apart). Table 1 below gives a complete timeline of this trajectory.

Table 1. Earth-Mars-Callisto-Ganymede-perijove-Io mission timeline

<table>
<thead>
<tr>
<th>Event</th>
<th>Date</th>
<th>Mass [kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth Launch</td>
<td>November 16, 2024 01:43:58</td>
<td>5024.9739</td>
</tr>
<tr>
<td>Mars Gravity Assist</td>
<td>April 5, 2026 21:41:48</td>
<td>4626.9528</td>
</tr>
<tr>
<td>Callisto Gravity Assist</td>
<td>December 1, 2029 14:44:14</td>
<td>3779.7404</td>
</tr>
<tr>
<td>Ganymede Gravity Assist</td>
<td>December 2, 2029 09:43:25</td>
<td>3779.7386</td>
</tr>
<tr>
<td>Perijove</td>
<td>December 3, 2029 02:57:57</td>
<td>3779.7369</td>
</tr>
<tr>
<td>Io Gravity Assist</td>
<td>December 3, 2029 7:05:56</td>
<td>3779.7366</td>
</tr>
<tr>
<td>Jupiter Capture</td>
<td>December 8, 2029 22:42:48</td>
<td>3779.7238</td>
</tr>
</tbody>
</table>

The optimized trajectory results in a spacecraft final mass of 3779.7238 kg out of the launch mass of 5024.9739 kg along with a capture orbit period of 89.8 days. Figure 1 below shows the overall Sun-centered trajectory, and with Figure 2 shows the Jupiter-centered triple-flyby.
Figure 7. Expanded view of complete Earth-Mars-CGPI trajectory following optimization.

Figure 8. Close view of CGPI section of trajectory.
With minimal effort, the combined trajectory was found to be able to model the feasibility of other Jupiter approach sequences involving triple-flyby maneuvers of the Galilean moons. To demonstrate, a Callisto-Ganymede-Europa-perijove sequence was developed as shown in Figure 3 and Figure 4 along with a Callisto-Ganymede-perijove-Europa sequence as shown in Figure 5 and Figure 6.

**Figure 9.** Complete Earth-Mars-CGEP trajectory following optimization
Figure 10. Close view of CGEP section of trajectory

Figure 11. Complete Earth-Mars-CGPE trajectory following optimization
Figure 12. Close view of CGPE section of trajectory

A complete mission timeline of both trajectories has been included below in Table 2 and Table 3. The Earth-Mars-Callisto-Ganymede-Europa-perijove trajectory was successfully optimized to have a final mass of 3503.2 kg and a capture orbit period of
112.8 days. Additionally, the Earth-Mars-Callisto-Ganymede-perijove-Europa trajectory has an optimized final mass of 3451.8 kg and a capture orbit period of 137.2 days.

Table 8. Earth-Mars-Callisto-Ganymede-Europa-perijove mission timeline

<table>
<thead>
<tr>
<th>Event</th>
<th>Date</th>
<th>Mass [kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth Launch</td>
<td>November 12, 2024 14:24:27</td>
<td>5024.9739</td>
</tr>
<tr>
<td>Mars Gravity Assist</td>
<td>March 19, 2026 08:35:10</td>
<td>4581.5853</td>
</tr>
<tr>
<td>Callisto Gravity Assist</td>
<td>January 20, 2030 20:36:01</td>
<td>3503.2441</td>
</tr>
<tr>
<td>Ganymede Gravity Assist</td>
<td>January 21, 2030 18:13:56</td>
<td>3503.2421</td>
</tr>
<tr>
<td>Europa Gravity Assist</td>
<td>January 22, 2030 05:49:00</td>
<td>3503.2410</td>
</tr>
<tr>
<td>Perijove</td>
<td>January 21, 2030 13:13:11</td>
<td>3503.2403</td>
</tr>
<tr>
<td>Jupiter Capture</td>
<td>February 15, 2030 22:08:30</td>
<td>3503.1852</td>
</tr>
</tbody>
</table>

Table 9. Earth-Mars-Callisto-Ganymede-perijove-Europa mission timeline

<table>
<thead>
<tr>
<th>Event</th>
<th>Date</th>
<th>Mass [kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth Launch</td>
<td>December 2, 2024 17:16:25</td>
<td>5024.9739</td>
</tr>
<tr>
<td>Mars Gravity Assist</td>
<td>April 13, 2026 02:57:09</td>
<td>4458.8470</td>
</tr>
<tr>
<td>Callisto Gravity Assist</td>
<td>March 11, 2030 22:53:13</td>
<td>3451.8969</td>
</tr>
<tr>
<td>Ganymede Gravity Assist</td>
<td>March 12, 2030 20:30:46</td>
<td>3451.8949</td>
</tr>
<tr>
<td>Perijove</td>
<td>March 13, 2030 15:00:42</td>
<td>3451.8931</td>
</tr>
<tr>
<td>Europa Gravity Assist</td>
<td>March 13, 2030 22:15:57</td>
<td>3451.8924</td>
</tr>
<tr>
<td>Jupiter Capture</td>
<td>April 7, 2030 02:06:05</td>
<td>3451.8379</td>
</tr>
</tbody>
</table>

DISCUSSION

The optimized Earth-Mars-Callisto-Ganymede-perijove-Io trajectory developed in Mystic shows a marked improvement over the previous separated interplanetary and triple-flyby trajectories designed by Patrick and Lynam using MALTO. Specifically, the combined Mystic trajectory reduces the overall mission SEP ΔV requirements by roughly 1.0 km/s from 6.602 km/s to 5.587 km/s.
Each of the three trajectories found using Mystic have nearly equatorial Jupiter-centered orbits. Because of this, in all three trajectories, the spacecraft will travel through Jupiter’s equatorial region where the radiation environment is the worst. The radiation exposure is most severe below 5 RJ and progressively improves at higher radii. The initial Earth-Mars-Callisto-Ganymede-perijove-Io trajectory has a radius of perijove of 4.225 RJ, exposing the spacecraft to high levels of radiation. The subsequent Earth-Mars-Callisto-Ganymede-perijove-Europa and Earth-Mars-Callisto-Ganymede-Europa-perijove trajectories have perijove radii of 7.834 RJ and 7.639 RJ respectively. These higher radii expose the spacecraft to much less radiation than that of the Earth-Mars-Callisto-Ganymede-perijove-Io trajectory, about 1/3 as much exposure. The lower radiation exposure does come with a trade off in regards to the period of the capture orbits and the SEP ΔV requirements. While exposing the craft to higher radiation, the Earth-Mars-Callisto-Ganymede-perijove-Io (EMCGPI) trajectory has a period of 89.8 days. This is shorter than the Earth-Mars-Callisto-Ganymede-perijove-Europa (EMCGPE) trajectory at 137.2 days and the Earth-Mars-Callisto-Ganymede-Europa-perijove (EMCGEP) trajectory at 112.8 days.

The EMCGPI trajectory requires 5587.3 m/s of SEP ΔV, while the EMCGPE and EMCGEP trajectories require 7367.8 m/s and 7077.8 m/s, respectively. However, this difference is mostly due to the fact that the MALTO interplanetary trajectory that all three were based on was optimized for the EMCGPI trajectory, rather than the other two. Also, the other two are less optimal in terms of their heliocentric flight path angle as they approach Jupiter. Thus, we cannot draw general conclusions from these specific
examples about whether EMCGPI trajectories require more SEP $\Delta V$ than the other two captures. Other EMCGPE trajectories may very well require less SEP $\Delta V$ than other EMCGPI trajectories. The other conclusions about perijoves, the radiation, and capture orbit period are more generalizable because they are based on the fact that Io has a larger mass and a smaller perijove than Europa.

Previously, Chapter one briefly explored some of the navigation challenges related to a SEP triple-flyby trajectory. Specifically, a comparison was made of the applicability of the navigation analysis performed by Lynam and Longuski relating to triple flyby capture trajectories assuming impulsive maneuvers to SEP triple satellite aided capture sequences presented in Chapter one. The main navigation challenge was determined to be that continuous application of thrust from a SEP source would be unable to provide enough $\Delta V$ over the short intervals between flybys. Because of this there is insufficient control authority to guide a SEP craft through the given triple flyby sequences.

To correct for this, Patrick and Lynam proposed low Isp RCS thrusters dedicated to attitude control, that are also capable of performing Trajectory Correction Maneuvers (TCM). Additionally, autonomous navigation was suggested as a solution to save hours of light time delay, but it would require a robust onboard Navigation system.

While unexplored here, Didion and Lynam explored the use of autonomous navigation for an impulsive Callisto-Io-perijove-Ganymede Jupiter capture trajectory and by inserting reasonable, random errors into the propagation model$^{24}$. The focus of this aspect was to determine if autonomous mission navigation of a triple-flyby trajectory was feasible after taking into account trajectory correction. Didion and Lynam determined
that an autonomous navigation routine is feasible when dealing with reasonable error. The maneuvers required for trajectory correction are infeasible for the use in an SEP trajectory as presented here, but they could be performed with the addition of a Hydrazine thruster dedicated to attitude control and TCMs.
CHAPTER 3

This chapter details the methodology used to solve the problem provided for a competition problem. The methodology presented here was designed and implemented by Dr. Alfred Lynam, Alan Didion, and, the author, Sean Patrick. This problem was developed by a team at NASA JPL headed by Anastassios Petropoulos. For the competition, each team was given one month to develop the best possible solution they could. More information about this competition including the original problem statement and the history of the competition can be located at the GTOC8 website*.

PROBLEM DESCRIPTION AND CONSTRAINTS

The problem as given begins with three spacecraft initially placed in a circular orbit of altitude 400 km. This orbit is located in the ecliptic plane with each spacecraft initially located on the x-axis. The spacecraft have two propulsion systems, a chemical propulsion system and a low-thrust SEP system. The chemical propulsion system has a total ΔV capacity of 3 km/s and an Isp of 450 s. The SEP system has a maximum thrust of 0.1 N and an Isp of 5000 s. For each spacecraft, the total mass of the craft cannot exceed 4000 kg, 1890 kg is dry mass. The remaining 2110 kg can be assigned to either propulsion system as the competitors see fit. As a constraint, the chemical propulsion system may only be fired once and is required to be fired before the low-thrust system can be used. Despite this, the complete 3 km/s of ΔV do not need to be used. As an additional constraint, the impulsive ΔV maneuver must occur between MJD 58849.0 and 58880.0. Finally, the mission must be completed within three years.

* http://sophia.estec.esa.int/gtoc_portal/?page_id=560
The competition problem is designed to represent placing the three given spacecraft into a triangular constellation. The normal vector of this constellation represents a VLBI measurement boresight vector. Once the constellation is in place, the measurement vector would sweep the sky. During this time, the geocentric direction of the measurement vector would be compared to a list of 420 given radio sources. A measurement would be counted if the vector would cross within 0.1 degrees of a radio sources. In addition, there is a required 15 minute gap between measurements. This gap represents a cool-down period for the spacecraft, representing slewing and data recording.

Furthermore, the spacecraft orbital radii must remain between 6578.14 km and km. The dynamical model implemented must only include the gravitational influence of the Earth. This neglects the moon and sun. Flybys of the moon are also permitted but are to be treated as instantaneous and are modeled using patched-conics and assumed to occur at the position of the moon’s center.

GIVEN CONDITIONS

In addition to the constraints detailed above, the constraints detailed in Table 9 were provided so as to allow for all submissions to be graded upon the same metric.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravitational Parameter, Earth</td>
<td>( \mu )</td>
<td>398600.4329</td>
<td>km(^3)/s(^2)</td>
</tr>
<tr>
<td>Gravitational Parameter, Moon</td>
<td>( \mu_m )</td>
<td>4902.8006</td>
<td>km(^3)/s(^2)</td>
</tr>
<tr>
<td>Radius, Earth</td>
<td>( R_E )</td>
<td>6378.14</td>
<td>km</td>
</tr>
<tr>
<td>Radius, Moon</td>
<td>( R_M )</td>
<td>1737.5</td>
<td>km</td>
</tr>
<tr>
<td>Gravitational Acceleration</td>
<td>( g )</td>
<td>9.80665</td>
<td>m/s(^2)</td>
</tr>
<tr>
<td>Day</td>
<td>N/A</td>
<td>86400</td>
<td>S</td>
</tr>
<tr>
<td>Year</td>
<td>N/A</td>
<td>365.25</td>
<td>day</td>
</tr>
</tbody>
</table>
SCORING

For each observation, a number of scoring parameters were considered. One of these is the maximum altitude of the VLBI triangle at the time the measurement was taken. Because the constellation is a triangle, the altitude is defined as the distance from a vertex to a perpendicular side. Here, there are three possible altitudes to this triangle each of which corresponds to the three spacecraft, A, B, and C. For a measurement to be considered valid, the maximum altitude of the triangle must be greater than or equal to 10,000 km.

The triangle normal, n, is calculated from the geocentric radius vectors of the three spacecraft as shown below:

\[ \vec{n} = \pm (\vec{r}_A - \vec{r}_B) \times (\vec{r}_A - \vec{r}_C) \]

In addition, the source vectors, s, are calculated from the data for the declination, \( \delta \), and right ascension, \( \alpha \).

\[ \vec{s} = (c \hat{\alpha} \hat{\delta} + c \hat{\nu}) \times (c \hat{\alpha} \hat{\delta} - c \hat{\nu}) \times (c \hat{\alpha} \hat{\delta}) \]

With these equations, s and n can be compared by taking the inverse cosine of the dot product between them as shown below. If the resulting angle is less than the 0.1 constraint mentioned above, the measurement is valid.

\[ \phi = \cos^{-1}(\vec{n} \cdot \vec{s}) \]

The problem statement also provides a reward for repeat observations by including an observation factor, P, in the scoring equation. P follows a series of complex rules as follows:

- The first valid observation of a unique source: \( P=1 \)
• The second observation of a source:
  
  ▪ If \( h \) is at least three times the previous observation of the source: \( P=3 \), otherwise \( P=1 \)

• The third observation of a source
  
  ▪ If \( h \) is at least six times the second observation: \( P=6 \)
  ▪ If \( h \) is at least three times that of the second: \( P=3 \)
  ▪ If \( h \) is neither: \( P=1 \)

• The fourth or greater observation of a source: \( P=0 \)

With \( P \) and \( h \) defined for each observation, the total score for the mission, \( J \), can be determined as:

\[
\sum_{l} Ph(0.2 + c^2 \delta)
\]

In addition to the repeat observation factor \( P \) and the maximum altitude of the VLBI triangle \( h \), \( \delta \) is the declination angle of the source being observed.

**DYNAMIC MODEL**

For the competition, a dynamic model was developed based upon a given set of parameters so as for competitors to develop the necessary simulation environment. The model used represents an Earth-centered inertial (J2000) coordinate frame. Included in the problem description are the equations necessary to allow for astrodynamical propagation. Note however, that patched conics are used for lunar flybys. For the purposes of the competition, the earth is treated as the central body of the model and is fixed. As mentioned in the problem description, the gravitational influence of the Sun and
moon are neglected. The moon is modeled as propagating dynamically using Newtonian mechanics in place of Keplarian elements.

To describe the motion of the moon, a table of Keplarian elements were provided to allow for initialization within the model, this is given in Table 10 below. These elements have an initial epoch of Modified Julian Date (MJD) 58849.0.

<table>
<thead>
<tr>
<th>Orbit Element</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semimajor-Axis</td>
<td>a</td>
<td>383500.0</td>
<td>km</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>e</td>
<td>0.04986</td>
<td>N/A</td>
</tr>
<tr>
<td>Inclination</td>
<td>i</td>
<td>5.2586</td>
<td>deg</td>
</tr>
<tr>
<td>LAN</td>
<td>Ω</td>
<td>98.0954</td>
<td>deg</td>
</tr>
<tr>
<td>Arg. Of Periapsis</td>
<td>ω</td>
<td>69.3903</td>
<td>deg</td>
</tr>
<tr>
<td>Mean Anomaly</td>
<td>M₀</td>
<td>164.35025</td>
<td>deg</td>
</tr>
</tbody>
</table>

Table 11: Initial Keplarian Orbital Elements

In addition to the initial Keplarian elements, a series of equations where provided to represent the two-body acceleration of the moon.

\[
\begin{align*}
\ddot{\phi} &= \mu = 0, \quad \ddot{\theta} = \mu = 0, \quad \ddot{\tau} = \mu = 0 \\
\dot{\phi} &= \sqrt{\dot{\theta}^2 + \dot{\tau}^2} + \dot{\phi} \\
\end{align*}
\]

Another series of equations were provided to model the propagation of the spacecraft.

\[
\begin{align*}
\phi &= \mu \phi, \quad \theta = \mu \theta, \quad \tau = \mu \tau \\
\dot{\phi} &= \phi \\
\dot{\theta} &= \theta \\
\dot{\tau} &= \tau \\
\end{align*}
\]

The equations for the spacecraft are similar to those of the moon; however, the
equations for the spacecraft include terms for the thrust, $T$, and the instantaneous mass, $m$, 
of each spacecraft. The equation is used to decrement the mass of each spacecraft in accordance with the specific impulse of the low-thrust engines on each craft.

In addition to the equations modeling the low-thrust engine of each spacecraft above, the impulsive chemical engine was modeled using the rocket equation to decrement the mass. This relation can be expressed as:

$$\dot{m}(t') = \dot{m}(t)\exp(-\frac{\Delta V}{\dot{m}(t)})$$

Note that $m(t')$ and $m(t)$ represent the mass before and after the maneuver respectively.

For the solution presented here, a lunar flyby was used. The problem statement assumes that any lunar flyby is to occur instantaneously using the patched-conic method. By using the patched-conic method, the trajectory of the craft turns according to a turning angle, $\delta$, which can be found using:

$$\delta = \sin^{-1}\left(\frac{\mu_{M}/(R_{M} + h)}{\mu_{M}/(R_{M} + h) + \mu}/2\right)$$

The $v_{\infty}$ value used here is derived automatically by the simulation. In addition, $h_{p}$, the periapsis radius was treated as a design parameter. For a lunar flyby to be considered valid the $v_{\infty}$ must be conserved throughout the flyby and be higher than 0.25 km/s. Additionally, $h_{p}$ must be greater than 50 km for the flyby.

**METHODOLOGY**

For Lynam et al.’s approach, the author proposed a preliminary design in which the three spacecraft are placed in an equatorial triangular constellation. The spacecraft would be positioned such that spacecraft A is placed in a polar or near polar orbit. Spacecrafts B and C would be left in the ecliptic plane and, using the initial chemical burn and a series
of low-thrust maneuvers, would be placed into orbits with apsides offset by 180 degrees. Once the spacecraft are in their final orbit inclinations and position, the remaining low-thrust fuel would be expended over the course of the mission to increase the maximum altitude of the constellation.

![Diagram](image)

**Figure 13: An example diagram detailing the basic design of the initial proposed constellation**

The decision to place craft A in a polar orbit was made by the author based on the fact that, the path the normal vector of the VLBI triangle makes, is dependent upon the inclination between the three spacecraft. To illustrate this, if craft B and C are placed in opposing orbits in the ecliptic plane the path of the normal vector will change based on the inclination of craft A’s orbit. Because craft B and C are not stationary, the path of the normal vector will have sections of the sky that are not able to be directly “observed” by
the normal vector depending on the inclination of craft A’s orbit. With this in mind, a polar orbit for craft A and crafts B and C being left in the ecliptic plane with apsides offset by 180 degrees would allow the normal vector to sweep out the complete sky over the course of the crafts orbits.

While this initial design is appealing in its thoroughness of mapping radio sources, Lynam et al. noted some flaws in the application of the design. The most glaring issue presented is that, based on the constraints of the chemical and low-thrust propulsions systems, reaching a polar orbit from the ecliptic plane would prove to be difficult to implement. In addition, the consistency of the design would allow for too many repeat measurements of radio sources, limiting potential scoring opportunities.

To solve this, Lynam, et al. modified the initial design proposed by the author. In this new design, one spacecraft (craft A, red in figures) is inserted into an elliptical, high inclination orbit. This insertion is accomplished through the use of a maximum, purely prograde impulsive maneuver followed by continuous velocity direction low-thrust spiral and finally a gravity-assist of the moon. After this flyby, the spacecraft continues to thrust in the normal/antinormal direction as appropriate, to raise the inclination further as the mission continues, switching when passing through the equatorial plane. The other two spacecraft (craft B and craft C, green and blue respectively) are put into elliptical orbits, apsides offset by 180 degrees from each other with each craft also offset 90 degrees from craft A’s line of apsides. This is intended to ensure optimal radio source mapping geometry for this configuration. The maneuvers to put the craft into these orbits are timed such that the new periods have a resonance of an integer plus 0.5 with the starting orbit.
This is a combined V-N (velocity-normal) maneuver, which uses the remaining delta-V to modify the inclination of both orbits up to a possible 10 degrees. The inclination component of the burns for craft B and craft C are opposite so as to place the two craft in the same plane.

Once the spacecraft constellation is constructed, the model is left to propagate until two years have passed. During this time, normal vectors are calculated and measurements are collected to determine source matches. A post-processing sorting algorithm is employed to determine which matches are valid by applying the 15-day interval rule and striking the invalid measurements.

**RESULTS**

A full, three year mission trajectory was successfully developed. This trajectory includes the three spacecraft successfully being placed into the intended constellation. Table 1 below provides a mission time table.
### Table 12. Mission Timeline

<table>
<thead>
<tr>
<th>Event</th>
<th>Time MET [day]</th>
<th>Time MJD [day]</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mission Start</td>
<td>0.000</td>
<td>58849.000</td>
<td>Given Epoch</td>
</tr>
<tr>
<td>SC A Impulse</td>
<td>4.324</td>
<td>58853.324</td>
<td>Maximum V-direction (3 km/s)</td>
</tr>
<tr>
<td>SC A SEP- V-direction</td>
<td>4.324</td>
<td>58853.324</td>
<td>Maximum V-direction (0.1 N)</td>
</tr>
<tr>
<td>SC B Impulse</td>
<td>4.340</td>
<td>58853.340</td>
<td>Combined V-N direction (norm &lt;3 km/s)</td>
</tr>
<tr>
<td>SC C Impulse</td>
<td>4.565</td>
<td>58853.565</td>
<td>Combined V-N direction (norm &lt;3 km/s)</td>
</tr>
<tr>
<td>SC A Targets Moon</td>
<td>181.997</td>
<td>59030.997</td>
<td>Optimizer with full VNC authority (norm &lt; 0.1 N)</td>
</tr>
<tr>
<td>SC A Moon Rendezvous</td>
<td>186.997</td>
<td>59035.997</td>
<td>Centimeter accuracy to lunar center</td>
</tr>
<tr>
<td>Lunar Flyby</td>
<td>186.997</td>
<td>59035.997</td>
<td>Instantaneous 20k km altitude flyby</td>
</tr>
<tr>
<td>SC A long-term inc-change</td>
<td>186.997</td>
<td>59035.997</td>
<td>Begin N-N direction low-thrust</td>
</tr>
<tr>
<td>Mission End</td>
<td>1095.750</td>
<td>59944.750</td>
<td>Mission time limit reached</td>
</tr>
</tbody>
</table>

These specifics are for a mission with a B/C orbit resonance of 3.5 to the period of the initial orbit. The rest of their delta-V is used to change the inclination of their orbits to about 10 degrees.

At mission end, each craft successfully expended all chemical propellant mass and spacecraft B and C had 2026.8 kg of mass remaining. Spacecraft A performed a longer post-flyby maneuver, ending with 1914.7 kg of mass. With a dry mass of 1890.0 kg, this leaves spacecraft A, B and C with 24.7 kg and 136.8 kg of fuel respectively at mission end. A full Mass Fuel budget is given in Table 2.
Over the course of the mission, the spacecraft constellation successfully matched 156 sources. After striking 4th and higher repetitions and applying the 15-day measurement interval rule, 21 valid measurements were retained and submitted.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Impulse</td>
<td>58853.324</td>
<td>N/A</td>
<td>1973.191</td>
<td>2026.809</td>
</tr>
<tr>
<td>Spiral</td>
<td>58853.324</td>
<td>59030.997</td>
<td>31.264</td>
<td>1995.545</td>
</tr>
<tr>
<td>Target Moon</td>
<td>59030.997</td>
<td>59035.997</td>
<td>0.778</td>
<td>1994.767</td>
</tr>
<tr>
<td>End of Mission</td>
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<td>59944.750</td>
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To follow are several mission trajectory plots that detail specific points in the mission design. As defined earlier, the convention is spacecraft A in red, spacecraft B in green, and spacecraft C in blue. The first two plots, Figures 11 and 12 detail geocentric plots during the impulsive maneuvers of the three spacecraft. Figure 11 shows and X-Y view of this while Figure 12 details a Y-Z view. Both figures show the formation of the resonance orbits of crafts B and C in addition to the beginning of craft A’s spiral out maneuver.

The next two plots, Figure 13 provides and expanded view of the constellation so as to show the spiral trajectory of craft A immediately before the Lunar Flyby maneuver. The plot as presented shows the X-Y view of the initial impulsive maneuver and low-thrust spiral, ending at Lunar Rendezvous. Figure 14 switches from the geocentric views of the
previous figures to a selenocentric view of the lunar flyby maneuver craft A undertakes. Finally, Figures 15 and 16 details the trajectories of the three crafts 60 days after the lunar flyby of craft A.

Figure 14: Geocentric X-Y plot showing location of impulsive maneuvers and the resonant orbits of B and C with respect to A and the Earth, to scale.
Figure 15: Geocentric Y-Z plot showing the inclination of the resonant orbits of B and C, after their combined impulsive maneuvers.

Figure 16: Geocentric X-Y plot showing the impulsive maneuver of A followed by low-thrust spiral to lunar rendezvous.
Figure 17: Selenocentric X-Y plot showing lunar rendezvous to within centimeters, moon to scale, axes in geocentric coordinates.
Figure 18: Geocentric X-Y plot of the first 60 days of the post-flyby orbit of spacecraft A.
Figure 19: Geocentric X-Z plot showing the first 60 days of the post-flyby orbit of spacecraft A, also showing the continued inclination change.
CONCLUSION

Triple gravity-assist flybys of the Galilean moons, in association with SEP propulsion, allows for a lower ΔV requirement to enter Jupiter orbit. In Chapter one, an application of the previous work by Lynam and Strange et al. is proposed in which a spacecraft approaches Jupiter orbit using an Earth-Mars-Jupiter low-thrust trajectory into a Callisto-Ganymede-perijove-Io triple flyby. This application shows that the proposed triple-flyby method, in conjunction with SEP, is feasible for insertion into Jupiter orbit while providing ample mass reserves (1703.5 kg) for an extended Science mission once orbit is achieved. In addition to the mass reserves, the proposed trajectory requires less time to achieve low-period Jupiter orbit at only 5.3 years compared to the trajectory proposed by Strange et al. at 6 years.

The optimized trajectory, while comparable to that of Strange et al., left room for improvement. Follow up work in Chapter two expanded upon the interplanetary and triple-flyby trajectories using higher fidelity optimization software (Mystic) to improve optimization results. In addition, the use of triple-flyby capture trajectories would require trajectory correction maneuvers with challenging navigation requirements or the addition of RCS attitude control thrusters to the spacecraft. Along with the difficulty of the TCMs required for the triple-flyby the proposed trajectory requires a perijove lower than Io's orbital radius (5.9 Jupiter radii), exposing the spacecraft to increased radiation that must be accounted for in mission design. Despite these issues, the low-thrust SEP design does
not require an impulsive JOI maneuver lowering the operational difficulty and statistical uncertainty of the mission. In addition to the lack of an impulsive JOI maneuver, the patched Earth-Ganymede trajectory proposed here is faster than the similarly designed mission given by Strange et al.

In chapter one triple gravity-assist flybys of the Galilean moons, in conjunction with SEP propulsion, were shown to allow for a lower propellant mass requirement to enter Jupiter Orbit. In chapter two, an expansion on that work was developed and expanded upon. There, a Jupiter approach was designed based on the previous Earth-Mars-Jupiter low-thrust trajectory with a Callisto-Ganymede-perijove-Io triple-flyby designed in Chapter one. This trajectory was modified and expanded using the Mystic software developed by JPL into a complete Earth-Mars-Callisto-Ganymede-perijove-Io trajectory. Following this, an Earth-Mars-Callisto-Ganymede-Europa-perijove trajectory and an Earth-Mars-Callisto-Ganymede-perijove-Europa trajectory were also developed.

Once optimized, these trajectories show a marked improvement over the original MALTO trajectories developed in chapter one. The fact that Mystic allows for a complete Earth to Jupiter orbit capture in a single high-fidelity trajectory reduces uncertainty and error as compared to the need for separate patched-conic Jupiter-centered and heliocentric trajectories in MALTO.

While the optimized Mystic trajectories do show an improvement over the MALTO trajectories, the navigation challenges presented in Chapter one are still present. In addition, the Mystic EMCGPI trajectory still has the issue of exposing the spacecraft to high levels of radiation due to having a perijove lower than Io’s orbital radius. However,
the EMCGEP and EMCGPE trajectories, with a higher radii of perijove, have about 1/3 as much radiation exposure.

In Chapter three, the author, along with Dr. Alfred Lynam and Alan Didion, worked to design and implement a trajectory model for the 8th Global Trajectory Optimization Competition (GTOC8). The trajectory developed was designed to model a three spacecraft VLBI constellation according to a series of constraints designated by the authors of the competition problem. For this, an initial design was proposed by the author and expanded upon to place one craft into a highly elliptical orbit with the other two craft in elliptical, resonant orbits.

The three-spacecraft constellation designed in chapter three benefits from a few key features. Namely, the resonance of the orbits of spacecraft B and C ensure that they are only ever as close as the sum of their periapsides (at least the diameter of Earth), ensuring a consistent altitude for the measurement triangle. Additionally, the flyby of spacecraft A with the moon allows for low-declination measurements, improving possible scores for the measured sources. By having spacecraft A continue normal/antinormal thrust after the lunar flyby, its inclination continues to change, sweeping lower and lower declinations for a more diverse set of source matches.

This arrangement, however, suffers from a few insufficiencies. Namely, the resonant orbits used in the results delivered here (factor of 3.5) have small, consistent size. Thus, the measurement triangle altitude is arguably too consistent; repeat observations are wasted, rather than accrue score multipliers. Furthermore, spacecraft A takes more than half a year, one sixth of the mission allowance, spiraling to the moon in the equatorial
plane. It is possible that spacecraft A could have been inclined with the initial impulse, but this would have drastically lengthened the time necessary to reach the moon. Finally, multiple lunar flybys would possibly improve the equatorial searching capabilities of the constellation, but this was not examined because the post-flyby normal thrust profile was deemed adequate.
REFERENCES


