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Characterization and Flight Test of a Multi-Antenna Gnss, Multi-Sensor Attitude Determination Algorithm

Nathan Tehrani

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CHARACTERIZATION AND FLIGHT TEST OF A MULTI-ANTENNA GNSS, MULTI-SENSOR ATTITUDE DETERMINATION ALGORITHM

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in partial fulfillment of the requirements for the degree of

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in
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ABSTRACT

Characterization and Flight Test of a Multi-Antenna GNSS, Multi-Sensor Attitude Determination Algorithm

Nathan Tehrani

A multi-antenna Global Navigation Satellite System (GNSS), multi-sensor attitude estimation algorithm is outlined, and its sensitivity to various error sources is assessed. The attitude estimation algorithm first estimates attitude using multiple GNSS antennas, and then fuses a host of other attitude estimation sensors including tri-axial magnetometers, Sun sensors, and inertial sensors. This work is motivated by the attitude determination needs of the Antarctic Impulse Transient Antenna (ANITA) experiment, a high-altitude balloon-suspended science platform. In order to assess performance trade-offs of various algorithm configurations, the attitude estimation performance of various approaches is tested using a simulation that is based on recorded ANITA III flight data. For GNSS errors, attention is focused on multipath, receiver measurement noise, and carrier-phase breaks. For the remaining attitude sensors, different grades of sensor are assessed. Through a Monte-Carlo simulation, it is shown that, under typical conditions, sub-0.1 degree attitude accuracy is available when using multiple antenna GNSS data only, but that this accuracy can degrade to degree-level in some environments warranting the inclusion of additional attitude sensors to maintain the desired level of accuracy. This algorithm was validated in a flight test. A WVU *Phastball* unmanned aerial vehicle was outfitted with GNSS receivers, an IMU, a magnetometer, and a Sun sensor to collect flight data. To determine the wing flex during flight, and correct the body-centric antenna coordinates, a computer vision algorithm was developed to use aircraft-mounted camera data to track markers along the wing surface and estimate the wing deflection.

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Acronyms

ARW	Angular Random Walk
DGPS	Differential GPS
ECEF	Earth Centered Earth Fixed
ECI	Earth Centered Inertial
GLONASS	Global Navigation Satellite System
GPS	Global Positioning System
GNSS	Global Navigation Satellite Systems
IMU	Inertial Measurement Unit
INS	Inertial Navigation System
KF	Kalman Filter
EKF	Extended Kalman Filter
UKF	Unscented Kalman Filter
L_1	1575.42 MHz GPS frequency
L_2	1227.60 MHz GPS frequency
RMS	Root Mean Square

Nomenclature

Symbol	Units	Description
$\mathbf{x}$		State Vector
v	<i>meters/second</i>	Velocity
R	<i>meters</i>	Geometric Range
C		Coordinate Transformation Matrix
ρ	<i>meters</i>	Pseudorange
ϕ	<i>meters</i>	Carrier-Phase
c	<i>meters/second</i>	Speed of light
$\delta\theta$	<i>radians</i>	Delta Angle
δv	<i>meters/second</i>	Delta Velocity
el	<i>radians</i>	User to Satellite Elevation Angle
ϕ	<i>radians</i>	Roll
θ	<i>radians</i>	Pitch
ψ	<i>radians</i>	Yaw
ω_e	<i>radians/second</i>	Earth Rotation Rate

Chapter 1

Introduction

1.1 Motivation

This study is motivated by the requirements of the Antarctic Impulse Transient Antenna (ANITA) experiment. ANITA is an ongoing project that uses a balloon-lofted platform to detect radio impulses from high-energy particle collisions in the ice below. Ultra-high energy neutrinos (UHEN) and ultra-high energy cosmic rays (UHECR) have both been detected by IceCube, a ground-based neutrino observatory which uses detectors embedded in the ice [2]. ANITA, with its high operating altitude, can observe possible particle collisions in a significantly-larger volume of ice [3]. This document outlines the development, simulation, and testing of an attitude determination algorithm. It is motivated by the requirements of the Antarctic Impulse Transient Antenna (ANITA) experiment.

The ANITA I, II, and III flight platforms have made successful radio transient discoveries [3, 4]. ANITA uses several feed-horn antennas with narrow observation beams and a high degree of pointing precision for each antenna. ANITA's angular resolution for radio transients depends on the timing resolution of its radio detectors (utilising phased-array techniques) and the attitude solution of the platform, and that resolution is now limited by the attitude solution, particularly in the pitch and roll axes. For any airborne sensing platform, the pointing accuracy is dependent on the accuracy of the onboard attitude solution [5]. As such, a key to high pointing accuracy is a robust attitude-determination system.

1.2 Review of Literature

1.2.1 GNSS Attitude Determination

Global Navigation Satellite Systems, including the American GPS and Russian GLONASS, are used ubiquitously in navigation and positioning for flight, survey, and automotive applications. In this study, we look at how GNSS observables can be used to determine the *attitude*, or three-dimensional orientation, of an aircraft, in this case an airborne platform. At its most basic level, attitude determination with GNSS is made possible with a *lever arm*, or distance between the antenna and a reference position on the platform body. In the case of GNSS-INS navigation, a single antenna can improve on the attitude-determination ability of an inertial measurement unit alone when combined in a tightly-coupled filter [6].

Using multiple GNSS receivers, without without inertial or other sensors, for attitude determination was first proposed by Clark Cohen in 1991, during graduate study [7]. Cohen proposed using differential GNSS pseudorange and carrier phase observations to solve for the attitude of an aircraft, using at least three receivers [7]. The method was adapted for flight testing on aircraft [8] and also utilized in a post-processing attitude determination exercise using GPS data collected by a space vehicle [9].

The first usage of multi-antenna attitude determination on a scientific platform was conducted by Gang Lu and Elizabeth Cannon, beginning in 1993, for a hydrographic survey [10]. Jennifer Evans, et al made the first use of a combined multi-antenna GNSS and inertial attitude determination system [11].

Multi-antenna GNSS attitude determination has since been tested on ground, waterborne, and flight vehicles [12], and the technology has matured to multiple commercially-available products [13, 14, 15].

Multi-antenna GNSS has been used for remote sensing platforms since shortly after its proposal [16], and it is in use on multiple stratospheric balloon platforms including ANITA [5].

An alternate method, which shows considerable promise, involves solving the integer ambiguity problem for each receiver using the combined attitude solution as a constraint . This method, developed by Gabriele Georgi [17], has been shown to work well in flight applications including remote sensing platforms [18].

This work outlines the design and performance evaluation of a GNSS-based attitude estimator, using conventional carrier-phase differential GNSS baseline observations, that is then augmented with various other attitude sensors to offer a proposed algorithm for the ANITA project, or other

similar balloon-based payloads.

1.2.2 Vector Observations to Attitude

Estimating a rotation matrix between sets of vectors is a well-studied mathematical problem with many engineering applications [19]. Three-axis rotational estimation is particularly common in the field of attitude and pointing determination [19].

The Wahba Problem is a well-known numerical exercise, formulated in 1965 by Grace Wahba, which involves finding the closest rotation matrix between two sets of vectors [20]. Formally, the problem is defined as a cost function to minimize the error in rotation between two sets of vectors. There are a number of methods used to solve the problem, with varying computational efficiency.

This study utilizes the Singular value Decomposition solution to Wahba's problem, as first proposed by Markley in 1988 [21].

Additional nonlinear attitude determination methods exist. The QUEST, or Quaternion Estimation method, seeks the unique quaternion solution for a set of vector measurements and reference vectors [19]. The RE-QUEST, or recursive QUEST, applies the same method but rather than solving for a single epoch, uses a filtering approach to find the time-varying attitude profile [22]. Other filtering techniques include the Multiplicative Extended Kalman Filter (M-EKF) [23] and the Quadratic Extended Kalman Filter (Q-EKF) [24].

1.2.3 Sun sensors and Magnetometers for Aerospace Vehicle Attitude Determination

There has been considerable effort to simulate gyroscope-free attitude determination using 3-axis magnetometers, 2-axis Sun sensors, or both, for spacecraft applications [25]. Highlights include the use of a magnetometer-only Sun-pointing algorithm by Ahn, 2003 [26]. This method did not include filtering and was used to estimate an attitude vector which was being corrected. Magnetometer-derived attitude was within 3° of gyroscope-derived reference attitude for the entire investigated flight. Psiaki (1991) modeled an orbit- and attitude-determination algorithm [27]. Using a 10nT 3-axis magnetometer and a 0.005° Sun sensor, this method showed less than 0.1° error in all axes. Crasidis (1996) created a Sun sensor and magnetometer Kalman filter and showed that a magnetometer-only attitude estimate is markedly improved (error reduced by approximately half) with the inclusion of Sun sensor data [25]. The Balloon-borne Large Aperture Submillimeter Telescope for Polarimetry (BLASTPol) is a similar stratospheric platform that uses Kalman filtering of multi sensor data for

post-flight attitude determination [5].

1.2.4 Kalman Filter and Unscented Kalman Filter

The linear state estimator now known as the Kalman Filter was developed over a period from 1958 to 1961 by Rudolph Kalman, Peter Swerling, and Richard Bucy, in a series of technical papers beginning with satellite-tracking method by Swerling [28] and continuing with two papers on the linear estimator itself by Kalman and Bucy [29, 30].

A limitation of the Kalman Filter is its inability to transform a state space to a measurement space when the transformation is nonlinear. In other words, the transformation must be represented by the observation matrix. There are two Kalman Filter derivatives which allow for nonlinear transformations. The Extended Kalman Filter linearizes the transformation model about its mean and covariance [31]. The Unscented Kalman Filter, developed by Eric Wan and Rudolph Van Der Merwe, expands the state into a Gaussian distribution, then propagates the distribution through the observation equations to obtain a distribution of measurements [32].

1.3 Objectives

This study takes into account the instrumentation setup and attitude performance needs of the ANITA, which are used as a starting point and a set of goals. The objectives of this study are as follows:

- To develop a GNSS-based attitude determination algorithm.
- In support of the first item, to develop a differential GNSS baseline-determination filter.
- To develop a multi-sensor, GNSS and INS attitude-determination filter.

The next set of goals, and the required steps of the study, surround the testing of the algorithm in a simulation environment:

- To adequately model GNSS and other sensor data as needed by the algorithm.
- To create new sensor modelling processes where needed.
- To assess the algorithm's performance in varying states of error sources.

Finally, the algorithm performance is assessed in a flight test:

- To add required instruments to flight vehicle and adapt vehicle firmware as needed.
- To develop a wing-flex determination experiment to resolve the problem of moving wings.
- To adapt the attitude-determination algorithm to support flight test data.

The remainder of this thesis details the design and testing of the attitude-determination algorithm. Chapter 2 outlines the algorithm itself, and its three steps. Chapter 3 provides an overview of the simulation environment used to test the algorithm. Chapter 4 describes the flight test experiment design and data processing. The results are shown in Chapter 5, and future directions for this work are noted in the conclusion, Chapter 6.

Chapter 2

Technical Approach

2.1 Algorithm Overview

Figure 2.1 shows the overall algorithm used. First, a carrier-phase differential GNSS filter, as detailed in Section 2.2.2, estimates the baselines between antennas. Next, this information is used as a measurement update for a GNSS-only multiple antenna attitude estimator as described in Section 2.2.3. Finally, the resulting estimated attitude state is optionally fused with a multi-sensor estimator that also incorporates inertial, magnetometer, and Sun sensor data, as discussed in Section 2.2.4.

Figure 2.1: Block diagram showing the three main estimators: baseline-estimation filter, GNSS-only attitude estimator, and multi-sensor attitude estimator.

2.2 Attitude Determination

2.2.1 Kalman Filter Overview

The Kalman Filter estimator is centered around the state vector $\mathbf{x}$ and its associated state covariance matrix $\mathbf{P}$. A state-transition matrix $\mathbf{F}$ transforms this state and its covariance from one time step k to the next [33]:

$$\mathbf{x}_k = \mathbf{F}_k \mathbf{x}_{k-1}, \quad (2.1)$$

$$\mathbf{P}_k = \mathbf{F}_k \mathbf{P}_{k-1} \mathbf{F}_k^T. \quad (2.2)$$

The measurement residual $\mathbf{y}$ is obtained by subtracting the measurement-containing vector $\mathbf{z}$ from the expected measurements (based on the state), which are computed using the linear observation transform $\mathbf{H}$:

$$\mathbf{y}_k = \mathbf{z}_k - \mathbf{H}_k \mathbf{x}_{k-1}. \quad (2.3)$$

In parallel, the measurement covariance residual $\mathbf{S}$ is also found:

$$\mathbf{S}_k = \mathbf{R}_k + \mathbf{H}_k \mathbf{P}_k \mathbf{H}_k^T. \quad (2.4)$$

The Kalman Gain, a weight applied to each measurement, is calculated using $\mathbf{P}$, $\mathbf{H}$, and $\mathbf{S}$:

$$\mathbf{K}_k = \mathbf{P}_k \mathbf{H}_k^T \mathbf{S}^{-1}. \quad (2.5)$$

Finally, the state and state covariance matrix are updated using the weighted measurements:

$$\mathbf{x}_{k+1} = \mathbf{x}_{k-1} + \mathbf{K}_k \mathbf{y}_k, \quad (2.6)$$

$$\mathbf{P}_{k+1} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_k \quad (2.7)$$

2.2.2 Antenna Baseline Filter

A Kalman filter is used to estimate the relative position between each of the antennas and a single master antenna at each of the 10 Hz measurement epochs. In particular, this Kalman filter uses pseudorange and carrier-phase differential GNSS (PCD-GNSS) measurements to estimate the relative position vectors between the antennas [34]. The state vector, x , for this filter consists of $x_{A,B}$, $y_{A,B}$, and $z_{A,B}$, the relative position vector components between antennas A and B , and a set of double-differenced pseudoranges and carrier-phase biases $N_{A,B}$.

$$x = \begin{bmatrix} x_{A,B} \\ y_{A,B} \\ z_{A,B} \\ N_{A,B}^{1,k} \\ \vdots \\ N_{A,B}^{j,k} \end{bmatrix} \quad (2.8)$$

The measurement models used to model the double-differenced carrier-phase observables follow the same approach outlined in [35], as is discussed next.

The model for an undifferenced GNSS pseudorange, ρ , and carrier-phase measurement, ϕ , (with units of carrier cycles) are given as [34]:

$$\rho = r + I_\rho + T_\rho + c(\delta t_u - \delta t_s) + \epsilon_\rho, \quad (2.9)$$

$$\phi = \lambda^{-1}[r + I_\phi + T_\phi] + \frac{c}{\lambda}(\delta t_u - \delta t_s) + N + \epsilon_\phi; \quad (2.10)$$

where λ is the wavelength corresponding to the frequencies $L1$ and $L2$ and expressed in meters. The geometric range r between the receiver and GNSS satellite is also expressed in meters, as are the ionospheric and tropospheric delays I and T . The speed of light c is expressed in meters per second. The clock biases of the receiver and satellite, δt_u and δt_s , respectively, are expressed in seconds. The un-modeled error sources, which include multipath and thermal noise, are included in ϵ in units of meters.

First, range and phase measurements for the master antenna A (antenna 1) and B (antennas 2,

3, or 4) are differenced to form single-differenced phase measurements:

$$\Delta\rho_{A,B}^j = r_{A,B}^j + c\delta t_{A,B} + \epsilon_{\rho,A,B}^j, \quad (2.11)$$

$$\Delta\phi_{A,B}^j = \lambda^{-1}r_{A,B}^j + \frac{c}{\lambda}\delta t_{A,B} + N_{A,B}^j + \epsilon_{\phi,A,B}^j. \quad (2.12)$$

Within Eq. 2.11 and Eq. 2.12, due to the very short baseline separation between the antennas, the atmospheric delays completely cancel along with the any satellite clock bias and ephemeris errors. Next, the single differenced measurements are then differenced between satellites. For example, between satellite j and a reference satellite k :

$$\nabla\Delta\rho_{fA,B}^{j,k} = r_{A/B_{k|k-1}} + \epsilon_{\rho,A,B}^{j,k}. \quad (2.13)$$

$$\nabla\Delta\phi_{fA,B}^{j,k} = -\lambda^{-1}(\mathbf{1}_A^j - \mathbf{1}_A^k)^T \mathbf{r}_{A/B_{k|k-1}} + N_{A,B}^{j,k} + \epsilon_{\phi,A,B}^{j,k}. \quad (2.14)$$

where the remaining receiver clock bias errors are eliminated, leaving only the unknown phase bias $N_{A,B}^{j,k}$, which is known to be an integer. Because the GPS and GLONASS satellite constellations operate at different frequencies, both a GPS and a GLONASS satellite are used as separate reference satellites [36]. GLONASS satellites operate using Frequency Division Multiple Access (FDMA), and the wavelength varies from satellite to satellite. The resulting inter-channel bias is negligible when using like receivers, as in this model [37]. Because of complications in integer fixing, however, GLONASS phase ambiguities are not fixed in this study.

The observation matrix, H , transforms the state x to predicted measurements. The first three rows of this filter's observation matrix consist of the 3 three-component unit vectors which point

from the reference satellite to the satellite corresponding to each measurement.

$$H = \begin{bmatrix} u_x^1 & u_y^1 & u_z^1 & \lambda_{L1,1} & & & \\ u_x^2 & u_y^2 & u_z^2 & & \lambda_{L1,2} & & \\ \vdots & \vdots & \vdots & & & \ddots & \\ u_x^n & u_y^n & u_z^n & & & & \lambda_{L1,N} \\ u_x^1 & u_y^1 & u_z^1 & \lambda_{L2,1} & & & \\ u_x^2 & u_y^2 & u_z^2 & & \lambda_{L2,2} & & \\ \vdots & \vdots & \vdots & & & \ddots & \\ u_x^n & u_y^n & u_z^n & & & & \lambda_{L2,N} \end{bmatrix}. \quad (2.15)$$

The measurement vector, z , consists of double-differenced pseudorange and phase measurements for each satellite relative to the reference satellite, including measurements for each the $L1$ and $L2$ frequencies:

$$z = \begin{bmatrix} \nabla \Delta \rho_{L1,A,B}^{i\dots n,k} & \nabla \Delta \rho_{L2,A,B}^{i\dots n,k} & \nabla \Delta \phi_{L1,A,B}^{i\dots n,k} & \nabla \Delta \phi_{L2,A,B}^{i\dots n,k} \end{bmatrix}. \quad (2.16)$$

In parallel with this Kalman filter, the floating point estimated phase biases (for GPS satellites only), $N_{A,B}^{j,k}$ and their estimated error-covariance are fed into an integer ambiguity resolution algorithm. In particular, the Least-squares AMBiguity Decorrelation Adjustment (LAMBDA) method [38] is used to determine the integer biases and adjust the estimated relative positions.

2.2.3 Baseline to Attitude

GNSS Attitude Function

Once the antenna relative baselines with respect to a master antenna are estimated using the baseline estimation filter, an Earth-centered, Earth-fixed (ECEF) antenna relative position matrix, L_E is generated at each epoch by vertically concatenating the estimate relative vectors of each of non-master antenna, as adopted from Cohen [7]:

$$L_E = \begin{bmatrix} x_{2,E} & y_{2,E} & z_{2,E} \\ x_{3,E} & y_{3,E} & z_{3,E} \\ x_{4,E} & y_{4,E} & z_{4,E} \end{bmatrix}. \quad (2.17)$$

This matrix is used to estimate the platform attitude given the antenna baseline vectors, in which the state vector x contains the attitude state, expressed in Euler angles, representing the rotation

from the body to navigation-frame:

$$x = \begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix}. \quad (2.18)$$

The body-centric antenna coordinate matrix L_B , with their origin defined as the reference antenna's position, makes up the list of reference vectors. The SVD method, as described in the Eq. 2.20 through Eq. 2.22, is used to find the rotation matrix C_e^b between the *ECEF* and *Body* frames, or the attitude matrix. This is convolved transformed to the local navigation frame using the Earth-to-nav rotation:

$$C_n^b = C_n^e C_e^b. \quad (2.19)$$

This solution, using the Markley SVD method, requires the construction of a matrix $\mathbf{B}$ using the the measured vectors v_i and reference vectors w_i :

$$\mathbf{B} = \sum_{i=1}^n v_i (w_i)^T. \quad (2.20)$$

A singular value decomposition is performed on $\mathbf{B}$, resulting in unitary matrices $\mathbf{U}$ and $\mathbf{V}$. A diagonal, 3×3 matrix $\mathbf{M}$ is constructed using the determinants of $\mathbf{U}$ and $\mathbf{V}$:

$$\mathbf{M} = \begin{bmatrix} 1 & & \\ & 1 & \\ & & |\mathbf{U}||\mathbf{V}| \end{bmatrix}. \quad (2.21)$$

By multiplying $\mathbf{U}, \mathbf{M}$, and $\mathbf{V}$, one can find the rotation matrix R :

$$R = \mathbf{U} \mathbf{M} \mathbf{V}^T, \quad (2.22)$$

The SVD rotation solution also provides a straightforward attitude error covariance matrix which was used as an error covariance matrix in the authors' GNSS-only attitude filter and for the GNSS-attitude measurement covariance estimates in the multisensor Kalman filter [39].

In order to obtain the attitude error covariance matrix, the matrix $\mathbf{B}$ is multiplied by the trans-

pose of the non-transformed rotation matrix C_e^b .

$$\mathbf{D} = \mathbf{B}(C_e^b)^T. \quad (2.23)$$

$\mathbf{D}$ can then be used to find the inverse of the error covariance matrix:

$$P^{-1} = \text{tr}(D) * I(3 \times 3) - D \quad (2.24)$$

The attitude dilution of precision, as proposed by Yoon (2001) is a similar metric which assesses the ability to measure Euler angles [40]. It is defined as [40]:

$$ADOP = \sqrt{\text{tr}[(nI - SS^T)^{-1}]}, \quad (2.25)$$

where n is the number of satellites in view, I is the 3×3 identity matrix, and S is a $3 \times N$ matrix comprising the unit vectors to each satellite, including the reference satellite. [40]. A variable starting location was used to investigate the effect of the lower GDOP and ADOP at high latitudes.

2.2.4 Multi-sensor Unscented Kalman Filter

Finally, a third Kalman filter estimator is used for attitude determination using all sensor data. In this step, an unscented Kalman filter (UKF) was chosen for its ability to handle the nonlinear transformation between platform attitude and solar incidence angles in the Sun sensor measurements. The details of the UKF implementation followed in this study are offered in the tutorial paper by Rhudy and Gu [41] and as such, these details are not discussed in detail herein. In this paper, an outline of the state vector, state prediction $f(x)$, and observation functions $h(x)$ for each measurement update are discussed.

The state vector, x estimated in the Multi-Sensor filter is given as:

$$x = \begin{pmatrix} \phi \\ \theta \\ \psi \\ b_p \\ b_q \\ b_r \end{pmatrix}. \quad (2.26)$$

where ϕ , θ , and ψ are the platform's roll, pitch and yaw, and $b_{p,q,r}$ are the time-varying biases of

Figure 2.2: The roll, pitch, and yaw delta-angles p , q , and r .

the IMU's roll rate, p , pitch rate, q , and yaw rate r gyroscopes.

Within the UKF framework, at each epoch, the the state vector is expanded into a group of $2L + 1$ sigma points, χ , where $L = 6$ is the length of the estimated state vector. For each group of sigma points l , the attitude states are predicted by integrating the IMU gyro data through the attitude kinematic equations [42]:

$$f(\phi, \theta, \psi) : \begin{bmatrix} \phi_i \\ \theta_i \\ \psi_i \end{bmatrix} = \begin{bmatrix} \phi_{i-1} \\ \theta_{i-1} \\ \psi_{i-1} \end{bmatrix} + \begin{bmatrix} 1 & t(\theta_{i-1})s(\phi_{i-1}) & t(\theta_{i-1})c(\phi_{i-1}) \\ 0 & c(\phi_{i-1}) & -s(\phi_{i-1})c(\theta_{i-1}) \\ 0 & \frac{s(\phi)}{c(\theta_{i-1})} & \frac{c(\phi_{i-1})}{c(\theta_{i-1})} \end{bmatrix} \left(\begin{bmatrix} p \\ q \\ r \end{bmatrix} - \begin{bmatrix} b_p \\ b_q \\ b_r \end{bmatrix} \right) \Delta t, \quad (2.27)$$

where $s(\cdot)$ represents sine, $c(\cdot)$ represents cosine, and $t(\cdot)$ represents tangent.

The delta-angles p , q , and r , shown in Figure 2.2, are the measured delta-angles from the gyroscope, corrected for the craft- and Earth-rate rotations which were included in the IMU model.

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} p' \\ q' \\ r' \end{bmatrix} - \left(\begin{bmatrix} p_c \\ q_c \\ r_c \end{bmatrix} + \begin{bmatrix} p_e \\ q_e \\ r_e \end{bmatrix} \right) \quad (2.28)$$

The Earth-rate rotations p_e , q_e , and r_e can be found using the Earth's rotation transformed to the body frame, then converting to Euler angles [43]:

$$R_e = \omega_e \begin{bmatrix} 0 & \sin(L) & 0 \\ -\sin(L) & 0 & -\cos(L) \\ 0 & \cos(L) & 0 \end{bmatrix} C_n^b, \quad (2.29)$$

where ω_{Earth} is Earth's rotation rate and L is the craft's latitude. The craft-rate rotation

component can be found in a similar way [43]:

$$R_e = \begin{bmatrix} 0 & v_{eb,E}^n \tan(L)/(R_E L + h) & v_{eb,N}^n/(R_E L + h) \\ v_{eb,E}^n \tan(L)/(R_E L + h) & 0 & -v_{eb,E}^n/(R_E L + h) \\ v_{eb,E}^n/(R_E L + h) & v_{eb,E}^n/(R_E L + h) & 0 \end{bmatrix} C_n^b, \quad (2.30)$$

where $v_{eb,N}^n$ and $v_{eb,E}^n$ are the north and East components of the craft's velocity, h is the craft's altitude, and R_E is Earth's radius.

Furthermore, ϕ_{i-1} , θ_{i-1} , and ψ_{i-1} are the previous epoch's roll, pitch, and yaw sigma points, are the first three elements of each column of χ , and $b_{p,q,r}$ are the the sigma points corresponding to the IMU bias states, which are predicted as random walk parameters. red

$$f(b_{p,q,r}) : \begin{bmatrix} b_{pi} \\ b_{qi} \\ b_{ri} \end{bmatrix} = \begin{bmatrix} b_{pi-1} \\ b_{qi-1} \\ b_{ri-1} \end{bmatrix} + \begin{bmatrix} w_{b_p} \\ w_{b_q} \\ w_{b_r} \end{bmatrix} \quad (2.31)$$

The measurement-prediction matrix Ψ is populated by the predicted measurement vectors using each set of sigma-points in χ . Because measurements occur at different rates in this filter, it is necessary to have different measurement updates occur at different rates. For epochs coinciding with Sun sensor and GNSS attitude measurements, each column Ψ_i is as follows:

$$\Psi_i = \begin{pmatrix} B_{b,x} \\ B_{b,y} \\ B_{b,z} \\ \angle_X \\ \angle_Y \\ \phi' \\ \theta' \\ \psi' \end{pmatrix}, \quad (2.32)$$

where B_b , $\angle_X$, and $\angle_Y$ are predicted magnetometer and Sun sensor measurements based on the i^{th} sigma point. The observation models, $h(x)$ used to predict the magnetometer and Sen sensor measurements based upon estimate attitude sigma points are identical to those used to generate the data as discussed in Section 3.1, with the exception that no magnetometer biases are estimated

in the filter. That is, the observation equations use $\hat{C}_n^b$, the direction-cosine representation of the predicted attitude states $\hat{\phi}$, $\hat{\theta}$, and $\hat{\psi}$:

$$h_B(\phi, \theta, \psi) : \vec{B}_b = \hat{C}_n^b \vec{B}_n. \quad (2.33)$$

$$V_{Sun,b} = \hat{C}_n^b V_{Sun,n} \quad (2.34)$$

$$h_{\angle_X}(\phi, \theta, \psi) : \angle_X = \pi/2 + atan2(Sun_{z,b}/Sun_{x,b}); \quad (2.35)$$

$$h_{\angle_Y}(\phi, \theta, \psi) : \angle_Y = \pi/2 + atan2(Sun_{z,b}/Sun_{y,b}) \quad (2.36)$$

As GNSS attitude and Sun sensor measurements occur at at 10Hz rate, the remaining (50Hz) measurement updates consist only of magnetometer measurement predictions:

$$\Psi_i = \begin{pmatrix} B_{b,x} \\ B_{b,y} \\ B_{b,z} \end{pmatrix}, \quad (2.37)$$

The measurement update matrix z consists of the simulated sensor measurement at each filter epoch. These are similar in form the the columns of Ψ :

$$z = \begin{bmatrix} B_{b,x} \\ B_{b,y} \\ B_{b,z} \\ \angle_X \\ \angle_Y \\ \phi_{GNSS} \\ \theta_{GNSS} \\ \psi_{GNSS} \end{bmatrix}, \quad (2.38)$$

for filter epochs with GNSS, magnetometer, and Sun sensor measurements, and

$$z = \begin{bmatrix} B_{b,x} \\ B_{b,y} \\ B_{b,z} \end{bmatrix}, \quad (2.39)$$

for epochs with magnetometer measurements only.

2.3 Flight Data Processing

2.3.1 GNSS Baselines

The most significant change to the GNSS-baseline filter to accommodate flight data is a change to $L1$ observations only.

The state vector, which contains the carrier phase ambiguities, becomes:

$$x = \begin{bmatrix} x_{A,B} \\ y_{A,B} \\ z_{A,B} \\ N_{A,B,L1}^{1,k} \\ \vdots \\ N_{A,B,L1}^{j,k} \end{bmatrix}, \quad (2.40)$$

and the measurement vector likewise becomes:

$$z = \begin{bmatrix} \nabla \Delta \rho_{L1 A,B}^{i\dots n,k} & \nabla \Delta \phi_{L1 A,B}^{i\dots n,k} & l_{b,k} \end{bmatrix}, \quad (2.41)$$

with the removal of $L2$ observations. The baseline length at the k^{th} timestep, $l_{b,k}$, is added as a measurement to act as a constraint. H is similarly modified to include the observation transformation

for the baseline length measurement:

$$H = \begin{bmatrix} u_x^1 & u_y^1 & u_z^1 & \lambda_{L1,1} & & \\ u_x^2 & u_y^2 & u_z^2 & & \lambda_{L1,2} & \\ \vdots & \vdots & \vdots & & & \ddots \\ u_x^n & u_y^n & u_z^n & & & \lambda_{L1,N} \\ \frac{x_{b,x}}{|x|} & \frac{x_{b,y}}{|x|} & \frac{x_{b,z}}{|x|} & & & \end{bmatrix}, \quad (2.42)$$

where x_b is the *ECEF* baseline between the antennas, or the first three elements of the state matrix.

For the results in this work, the open-source software RTKlib was used in place of the baseline-determination filter [44].

2.3.2 Wing Flex Determination

As described in experimental setup subsection 4.3, the aircraft wing was marked with evenly-spaced red markers and recorded throughout the flight test with a tail-mounted camera. Data processing, to find the wing flex as a function of time (for each video frame) was accomplished in a two-step process: marker identification and 3D projection.

Marker identification began with each video frame transformed into HSV (hue, saturation, and value) space. In order to isolate red pixels, elements in the hue matrix $\mathbf{H}$ below the red-threshold of 0.9 were set to zero to create a hue-based mask:

$$\mathbf{H}'_{u,v} = \mathbf{H}_{u,v} > 0.9. \quad (2.43)$$

This was then convolved pixel-by-pixel with the value matrix, which represented the brightness of each pixel in the original image, to obtain a mixed hue-brightness matrix $\mathbf{M}$:

$$\mathbf{M}_{u,v} = \mathbf{H}'_{u,v} \mathbf{V}_{u,v}. \quad (2.44)$$

The resulting red-only image matrix $\mathbf{M}$ was converted to a logical matrix $\mathbf{M}'$, with the threshold selected based on the mean pixel value in the value matrix. Saturation information, although often used in segmenation applications [45], was not used, high-glare angles tended to ‘wash out’ the red markers on the wing.

The centroid of each logical-high region was found and cataloged. As it was impossible to remove

all red pixels which were not wing markers, at each time step, a simple form of position-based feature matching was implemented. For each time step, a given marker centroid position was differenced with the positions of each centroid in the previous time step, with the lowest-distance centroid selected as the match. Thus, each marker position was tracked from frame to frame, with non-marker false positives being eliminated from the list of centroids. For the first frame, the position of each marker was selected manually. Figure 2.3 shows the described steps visually.

Figure 2.3: Marker identification in a selected flight video frame: A: Video frame; B: Brightness frame; C: Red-only hue mask; D: Logical matrix containing only red pixels, marker locations noted.

After the first process determined the pixel coordinates of the red markers, their positions in 3D space were needed. The markers, because they were placed directly over the wing spar, are assumed to move only in the vertical plane as the wing flexes up and down (that is, any wing torsion and its effects on the markers is not modeled). The marker coordinates were first projected onto a plane, normal to the camera axis, which intersected the horizontal plane at the wing spar. A second projection mapped these ‘world’ coordinates onto the plane of spar movement.

Each image was first undistorted. The first projection, from pixel coordinates to the camera-normal plane, required prior knowledge of the camera position relative to the wing spar: the identity rotation matrix $\mathbf{R}$ and the translation $\mathbf{T}$ (consisting only of the Z-coordinate l , the distance from the camera to the wing center). In accordance with convention, the last column of $\mathbf{R}$ was set equal

to $\mathbf{T}$, yielding the modified rotation matrix $\mathbf{R}'$ [45]:

$$\mathbf{R}' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & l \end{bmatrix}, \quad (2.45)$$

which represents the full camera pose relative to the wing. This was multiplied by $\mathbf{K}$, the intrinsic camera matrix, to find the homography H [45]:

$$H = \mathbf{R}'\mathbf{K}, \quad (2.46)$$

where $\mathbf{K}$ is constructed using the x and y focal lengths f_x and f_y (in pixels), the pixel center coordinates c_x and c_y , and the skew parameter s :

$$\mathbf{K} = \begin{bmatrix} f_x & s & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}. \quad (2.47)$$

In this study, $\mathbf{K}$ and the camera's spherical distortion were determined in a lab test. To find the 'camera view' plane C coordinates, the homogeneous image coordinates of each marker were multiplied by the homography matrix H .

Finally, these points (mapped onto a plane) were projected onto a second plane, S , defined by the upward motion of the spar and representing the real plane on which the markers were confined to move, as shown in Figure 2.4. A fourth-degree polynomial fit was determined for all twenty markers at each time step, with the polynomial's value at zero set to zero (as there could be no vertical movement of the wing at its center). Figure 2.5 shows the measured wing deflection at a selected time step.

To find the wingtip antenna position on the plane S , a 100-element linearly-spaced array was defined along the x -axis, which were evaluated using the polynomial to find corresponding values on the y -axis. Starting from the wing center, the length of each segment was summed until the half-wingspan was reached. The x and y coordinates corresponding to this point are then defined as the Y and Z body coordinates of the Ublox antennas. These positions were then transformed to the forward-antenna-centric antenna position matrix L_{body} , and used attitude determination in the process explained in Section 2.2.3.

The wing-flex magnitude was calibrated using the time-changing baseline lengths as found using

Figure 2.4: The camera view plane C (blue) and the world plane S (green), defined by the motion of the wing spar and associated markers.

Figure 2.5: Polynomial fit: wing deflection as a function of span, with marker positions.

GNSS, and also found to be in agreement with the expected degree of wing flex as found in a materials modelling exercise by S. D'Urso and I. Pecora.

Chapter 3

Simulation Testing

3.1 Simulation Setup

The simulated flight data used in this study is based upon the recorded flight data of ANITA III. That is, to simulate a balloon flight, the onboard position and attitude solutions were accepted as truth for simulation purposes, and sensor readings with realistic measurement noise were simulated.

3.1.1 Profile Generation

Figure 3.1: Attitude profile used in this work.

Figure 3.1 shows the Euler angle time histories during a two-hour segment of the ANITA III

flight. As indicated in Fig. 3.1, the platform had a small ($< 1^\circ$) oscillation in the roll and pitch axes and a constant rotation about the yaw axis.

3.1.2 Data Simulation

GNSS Observables Simulation

For each simulation run, four GNSS receivers were simulated with baseline separations of one-meter each, such that they are arranged in a square configuration. That is, the antennas were placed according to the following matrix L_b :

$$L_b = \begin{bmatrix} x_{2,b} & y_{2,b} & z_{2,b} \\ x_{3,b} & y_{3,b} & z_{3,b} \\ x_{4,b} & y_{4,b} & z_{4,b} \end{bmatrix}, \quad (3.1)$$

where $x_{i,b}$, $y_{i,b}$, and $z_{i,b}$ are the body-centric coordinates of the i^{th} antenna $i = 1$ denoting the master antenna, as was done by Cohen in the first paper describing multi-antenna attitude determination [7]. GNSS carrier-phase data was simulated for each flight profile at a rate of 10 Hz using the MATLAB SatNav Toolbox [46], which was modified by Watson et al. (2016) [47] to include additional GNSS error sources.

A number of deterministic and non-deterministic error sources are associated with GNSS measurements [34]. Fortunately, for attitude estimation applications, several of the primary GNSS error sources, including satellite and receiver clock biases and atmospheric delays, are canceled through the use of double differenced GNSS observations [34]. However, two important error sources, namely multipath reflections and carrier-phase breaks (AKA cycle-slips) remain present. In particular, when a metallic object reflects a GNSS signal onto the antenna, the multiple paths induce errors [34]. This could be a large problem on balloon-based scientific platforms, as the antennas are spaced closely and in close proximity to science payload. Thermal measurement noise in the receiver is another error source; it is actually amplified by double differencing GNSS data. As such, for this simulation study, multipath, carrier-phase breaks, and receiver thermal errors were assessed with respect to their effect on the attitude estimator's performance using the distributions indicated in Table 3.1.

3.1.3 Inertial Measurement Simulation

In addition to GNSS measurements, inertial measurement unit data was simulated for each flight profile and data at a sampling rate of 200 Hz. In particular, four grades of IMU tri-axial rate

gyroscope and accelerometers were simulated assessed. In this case, ideal gyroscope readings were generated by accepting the truth attitude solution of the ANITA III flight. These ideal measurements were then polluted with both a time-varying bias b_i with in-run stability σ_{inrun} and measurement noise σ_{ARW} :

$$b_i = b_{i-1} + X_1 \sigma_{inrun}, \quad (3.2)$$

$$\Delta\theta'_i = \Delta\ddot{heta}_i + b_i + X_2 \sigma_{ARW}; \quad (3.3)$$

where X_1 and X_2 are normally-distributed random numbers $U[-1, 1]$. The magnitude of these two noise terms were selected based on the grade of the inertial sensors assumed, which were varied as indicated in Table 3.1.

3.1.4 Sun Sensor & Magnetometer Simulation

Two-axis Sun-sensor data and tri-axial magnetometer data were also simulated for each flight based on the measurement models and uncertainties of the sensors current installed on the ANITA IV balloon. In particular, the apparent Sun position and the Earth's magnetic field along the flight profile were calculated and sensor measurements were simulated by polluting these true values with random noise based on the measurement noises quoted by the manufacturers' specification sheets as indicated in Table 3.1.

The magnetometer data consists of magnetic field intensity measurements (B_b) in three orthogonal directions corresponding to the North, N , East, E , and down D axes in the body frame, b . This begins with B_E , a vector constraining the simulated magnetic field intensities in the navigation frame, generated at each location along the flight path:

$$\vec{B}_E = \begin{pmatrix} B_{b,N} \\ B_{b,E} \\ B_{b,D} \end{pmatrix}. \quad (3.4)$$

The magnetic field was generated using the World Magnetic Model (WMM) [48] in an interface developed by J. Hardy [49].

Body-frame magnetic field measurements are generated by multiplying truth attitude (repre-

sented by the direction-cosine matrix C_n^b) by the navigation-frame magnetic field:

$$\vec{B}_b = C_n^b \vec{B}_E. \quad (3.5)$$

With three contributing error sources added: hard and soft iron errors and measurement noise, in a simplified method as described by Gebre-Egziabher et. al. [50]:

$$\hat{B} = A_{si} \vec{B}_b + \vec{B}_{hi}, \quad (3.6)$$

where A_{si} is a 3×3 matrix which describes the soft-iron error effect and $\vec{B}_{hi}$ is a 3×1 vector containing the hard-iron offset, a magnetic field generated by ferromagnetic material on the platform. For this study, nominal values for A_{si} and $\vec{B}_{hi}$ were used, based on the calibrations in the Gebre-Egziabher paper. Simulated measurement noise was then added to $\hat{B}$, corresponding to precision level of the modeled magnetometer.

The simulated Sun sensor data consists of solar incidence angles $\angle_X$ and $\angle_Y$ relative to the two horizontal body-frame axes X_b and Y_b . These were generated using the apparent solar azimuth θ_{Sun} and elevation ϕ_{Sun} calculated for each epoch of the flight duration. First, the solar azimuth and elevation values are transformed into a unit vector representing the Sun's position in the sky with respect to the navigation frame, n :

$$V_{Sun,n} = \begin{pmatrix} Sun_{x,n} \\ Sun_{y,n} \\ Sun_{z,n} \end{pmatrix}. \quad (3.7)$$

This unit-vector is then transformed using the nav-to-body direction cosine matrix, C_n^b :

$$V_{Sun,b} = C_n^b V_{Sun,n} \quad (3.8)$$

and the solar incidence angles $\angle_X$ and $\angle_Y$ are then calculated:

$$\angle_X = \pi/2 + atan2(Sun_{z,b}/Sun_{x,b}), \quad (3.9)$$

$$\angle_Y = \pi/2 + atan2(Sun_{z,b}/Sun_{y,b}); \quad (3.10)$$

where $atan2$ is the four-quadrant tangent inverse.

As with the magnetometer measurements, simulated measurement noise was added to the Sun sensor measurements. However, in the case of a Sun sensor, as measurement noise increases at low solar elevations, the measurement noise was scaled according to solar elevation angle. Sun sensor measurements were simulated at 10Hz intervals.

Error-Source States & Monte Carlo

For this study, a total of 50 one-hour flight profiles were simulated in a Monte-Carlo manner. In particular, the ECEF starting positions, magnitude of GNSS error sources, and quality of IMU, Magnetometer and Sun sensor data were varied as indicate in Table 3.1. Note that by randomly varying the starting location, the GNSS constellation satellite geometry was randomized as well.

Table 3.1: Sensor Error-Source Monte-Carlo Simulation Distribution Parameters

Error-Sources	Model Parameters	Notes
Thermal Noise	$\sigma_\rho = 0.32m$, $\sigma_\phi = 0.16\lambda$	linear scale factor randomly selected between [0,1]
Multipath	1.0 intensity: $\sigma = 0.4m, \tau = 15sec$	linear scale factor randomly selected between [0,2]
Tropospheric Delay	Percent of error assumed handled by broadcast correction	Modified Hopfield with linear scale factor randomly selected between [0.95,1.05]
Ionospheric Delay	First order ionospheric effects mitigated with dual-frequency	linear scale factor randomly selected between [0.7,1]
Carrier phase break	Likelihood set to 1 phase break per 24 minute to 1 phase break per 240 minutes.	
Gyroscope	In-run Bias $\sigma = 9.6e^{-6} \frac{rad}{sec}$, $ARW = 0.2 \frac{deg}{\sqrt{hr}}$	Scaled Honeywell HG1700AG72 SF = $(\frac{1}{50}, \frac{1}{200}, \frac{1}{400})$
Sun Sensor	Zenith measurement noise $\sigma = 0.1$ deg.	Scaled SolarMEMS ISSDX-60 SF = (1, 2, 3, 4)
Magnetometer	Measurement noise $\sigma = 2.67$ nT A_{si} terms scaled between [0.005, 0.01] B_{hi} terms scaled between [25nT, 50nT]	ST LSM9DS0

Chapter 4

Flight Experiment

4.1 PhastBall Zero Background

To verify this algorithm using real flight data, a flight test experiment was developed using the *Phastball Zero* Unmanned Aerial System, one of three operational Phastball UAS vehicles which are cooperatively operated by the WVU Navigation Lab and the Interactive Robotics Laboratory. *Phastball Zero* has been used for several research projects, including Precise-Point Positioning [51], relative navigation [35], and a prior attitude determination project [52].

A suite of sensors were added to *Phastball Zero* to allow for post-flight processing and attitude determination using the study algorithm. Fig. 4.1 shows the flight system as configured.

Figure 4.1: *Phastball Zero*

4.2 Sensor Integration

4.2.1 IMU & Magnetometer

Figure 4.2: *Phastball Zero* avionics data collection system (two Ublox receivers mounted above).

4.2.2 Sun Sensor

One SolarMEMS ISS-DX60 digital solar sensor was installed in the aft fuselage of the experimental flight vehicle. The sun sensor, when illuminated, measures the x and y components of the solar incidence vector, as shown in Fig. 4.3.

The Netburner data collection computer communicates with the sun sensor with the aid of a half-duplex RS485-to-RS232 binary data converter. Command packets are sent, and data returned,

Figure 4.3: Solar incidence angles as measured by the sun sensor.[1]

at 10Hz intervals and saved by the Netburner data collection computer. The solar incidence angles as well as the solar radiation scalar are saved for use in the multisensor filter.

Figure 4.4: Sun sensor as mounted on Phastball UAV

4.2.3 Receivers & Antennas

In addition to the existing NovAtel OEM 618 and antenna mounted at the aircraft's nose, two additional receivers were added to the platform. Two Ublox EVK-M8T combined GNSS receiver/timing servers were mounted internally, above the avionics cluster (Fig. 4.2) with their antennas mounted on the upper surface of each wing. To mitigate any potential multipath, the antennas were mounted atop 10-centimeter squares of aluminum flashing, which were then attached to the wing surface. The layout of the antennas can be seen in Fig. 4.5.

Figure 4.5: *Phastball Zero* general arrangement with GNSS antennas mounted (wingtips and upper nose).

4.3 Wing Flex Experiment

As shown in the antenna offset trial, multi-antenna GNSS attitude determination requires knowledge of the antenna positions on the aircraft body, or at least a rigid body for antenna mounting. Existing bias from antenna misplacement can be nulled in calibration, but any change in the antenna positions relative to the body during collection will result in a changing bias.

The simulation study assumes a rigid body with perfect knowledge of the body-centric antenna positions. On the flight vehicle, the positions can be measured on the ground. However, the aircraft's wings exhibit a large amount (centimeters) of flex during flight, and this flexes depends on the aerodynamic loads the wing is subject to. Because two antennas are mounted at the wingtips, it becomes necessary to measure this flex in real time, so that new body-centric antenna coordinates can be generated for the antennas for each time step. This experiment seeks to measure wing flexed using a camera and a series of markers applied to the wing.

Figure 4.6: Tail camera as mounted.

Figure 4.7: View from tail-mounted camera during flight.

4.3.1 Camera & Marker Setup

The camera, a Sony HDR-AS50 with a fisheye lens, was mounted directly above the vertical stabilizer as shown in Fig. 5.9. The camera mount and positioning are as proposed by J. Strader et al, in a study which estimated flow over the wings with tufts [53].

A row of twenty red, circular, 2cm-diameter decals were applied to the top surface of the wing. To minimize any bias from wing torsion, the decals were applied directly over the wing's spar, such that the markers' movement would be confined to a plane normal to the wing. The markers were applied on a dark felt background to minimize glare during banking turns. Fig. 4.5 shows the arrangement of markers on the wing, and Fig. 4.7 shows a still frame from one collected flight video.

4.3.2 Time Alignment

Time alignment of sensor data is crucial for the algorithm's performance. Luckily, the inertial and magnetometer measurements are logged together at 50Hz intervals during the flight. The Sun sensor

is polled for incidence angle measurements at 10Hz intervals, and these are logged at every fifth data epoch by the avionics data collection system. Because of the 100ms time between signal polling and signal return, each sun sensor measurement is considered for the previous 10Hz timestep. That is, the sun sensor measurement times are shifted back 100ms relative to the inertial and magnetometer data.

Each GNSS receiver logs its own measurement data on separate SD memory cards using separate card data-logging devices. Because of the nature of GNSS measurements, time alignment between receivers is trivial (each measurement epoch is aligned, or can be easily aligned, with GPS signal time).

Time alignment between the GNSS receivers and the Netburner, therefore, is the only ‘missing link’, which is easily solved with the inclusion of a time signal from one GNSS receiver. The left wing-antenna receiver, a Ublox M8T, is equipped to function as a time server. On a dedicated pin, the receiver transmits pulse-per-second (PPS) time signal which is then measured by the NetBurner as an analog input (and logged at the 50Hz rate with the remainder of the data). This signal is time-aligned with the GNSS reference time, greatly easing the time-alignment of the GNSS data with all other data streams. To avoid confusion, before a carrier lock is achieved, the PPS signal is switched to a 3Hz rate. Fig. 4.8 shows the time signal as recorded on the avionics data collection system. Resolution of the integer-second ambiguity between the GNSS and IMU data is achieved by matching the time of the onset of motion.

Figure 4.8: GPS-aligned time signal, as recorded.

4.3.3 Comparison to Loosely-Coupled GNSS-INS

In parallel to the described algorithm, the flight GNSS (NovAtel receiver) and inertial data was processed using a loosely-coupled GPS-INS filter as formulated by Gross, 2012 [52]. This served as a ‘truth,’ attitude solution, to which the study’s algorithm solution was compared. Figure 4.9 shows the flight attitude profile as determined using the alternate filter.

Figure 4.9: GPS-INS-derived Euler angles for flight test.

Chapter 5

Results

5.1 Simulation

5.1.1 GNSS-only

Effect of Including GLONASS Sats

The GNSS-only attitude determination script was run in two modes, the first using GPS data only, and the second adding GLONASS observables. The pitch, roll, and heading error statistics for both filter modes are presented in Tables 5.1. These results include two simulations for which the baseline filter solution failed to converge, presumably due to carrier-phase break.

Table 5.1: GNSS Attitude Performance - Median RMS Error

	Roll (deg.)	Pitch (deg.)	Heading (deg.)
GPS-only			
Median	0.44	0.42	0.16
GPS+GLONASS			
Median	0.28	0.26	0.12

Using GLONASS as well as GPS satellites yielded a median performance improvement of 40 percent lower attitude error. In an Antarctic flight regime, fewer GNSS satellites are observable, and these are seen at lower elevations [54]. This can negatively impact the Geometric Dilution of Precision (GDOP), a metric that describes the geometric diversity of satellite-receiver vectors [34] and also the Attitude Dilution of Precision (ADOP), as defined in Eq. 2.25. Figure 5.1 shows error performance using GPS satellites only and using both GPS and GLONASS satellites, as well as the ADOP calculated in each case, for a high-latitude profile. Figure 5.2 shows the comparison overall

between the GPS-only and GPS and GLONASS attitude solutions.

Figure 5.1: Comparison between GPS-only mode and GPS+GLONASS mode for a polar flight profile.

5.1.2 GNSS Multi-sensor Attitude Filter

Table 5.2 presents overall error statistics for the 50 trials for the GNSS+INS, GNSS+ All sensors, and all sensors without GNSS, respectively.

Figure 5.3 shows the cumulative distribution of the 3D attitude error $= \sqrt{\phi^2 + \theta^2 + \psi^2}$ for the various filter configurations over the 50 simulated flights, and Figure 5.4 shows the corresponding roll, pitch, and yaw errors for the simulated flights.

Table 5.2: Unscented Kalman Filter Error Statistics: Median Attitude Error

Roll (deg.)	Pitch (deg.)	Heading (deg.)
GNSS+INS		
0.59	0.38	0.64
INS+Mag+SS		
0.0041	0.047	0.045
GNSS+INS+Mag+SS		
0.048	0.053	0.047

In these tables, it is clear that that using additional sensors in addition to GNSS can markedly improve performance. For example, Figure 5.5 shows the attitude estimation error for one example

Figure 5.2: Comparison between GPS-only mode and GPS+GLONASS mode for all profiles, latitude shown.

trial, in which the GNSS-only attitude is shown alongside the multi-sensor filters for comparison.

Of great interest is the algorithm’s ability to handle carrier-phase breaks. For example, phase breaks could occur due to radio-frequency interference, such as during a data transmission over the Iridium satellite constellation which operates very close to the GPS L1 frequency [55]. When a carrier-phase break occurs, it can fortunately be detected easily by a data editor [56]. As such, whenever this occurs, the baseline estimation filter re-sets the error-covariance for the impacted carrier-phase ambiguities to a large value. The result is a momentary spike in attitude error, not longer than five filter time steps, but often with multi-degree magnitude. The multi-sensor filter attitude determination performance was lower across the range of phase break likelihoods as shown in Figure 5.6. Notably, the multi-sensor UKF yielded a low error-level attitude solution for the two trials with GNSS-attitude convergence failure.

Also of interest is the filter’s performance with high receiver measurement thermal noise and multipath errors. Figures 5.7 and 5.8 show that the multi-sensor filter yields lower-magnitude errors than the GNSS-only filter across both error scale ranges. Although an increasing level of multipath error did not noticeably affect the result of the GNSS-only filter performance, the multi-sensor filter performed better in nearly all trials.

Sensitivity to the ionospheric and tropospheric error contribution to the GNSS errors was not considered, as the short baseline between antennas led to cancellation of those error sources.

Figure 5.3: Comparison between GNSS-SVD solution and multi-sensor attitude filter in different modes - 3 axis attitude error.

Figure 5.4: Comparison between GNSS-SVD solution and multi-sensor attitude filter in different modes - roll, pitch, and yaw error.

Figure 5.5: Roll, pitch, and heading errors for multi-sensor filter in GNSS+INS mode, GNSS+INS+Mag+SS mode, with GNSS-only result for comparison.

Figure 5.6: RMS attitude vs. phase break likelihood for each trial.

Figure 5.7: RMS attitude vs. thermal error scalar for each trial.

Figure 5.8: RMS attitude vs. multipath error scalar for each trial.

5.2 Flight Test

5.2.1 GNSS-only Attitude Determination

First, the performance of the GNSS-only method was assessed, with both the rigid-body aircraft model as well as the camera-derived flexing-wing model used as body-centric antenna coordinates. Figures 5.9 and 5.10 show the performance with and without the wing flex estimation.

Table 5.3 shows both filter modes' root-mean-square error relative to the loosely-coupled filter results.

Figure 5.9: GNSS-only attitude solution, with wing-flex estimation included.

Table 5.3: GNSS Attitude Filter RMS Error : Flight Test

Roll (deg.)	Pitch (deg.)	Heading (deg.)
Wing-Flex Correction		
51.2	34.8	47.2
No Wing-Flex Correction		
51.7	33.4	47.5

A convenient check of a differential GNSS baseline is to compare the measured baseline length to the expected, or constrained, length. Figure 5.11 shows the baseline length between the two Ublox wingtip antennas as measured by differential GNSS as well as the derived baseline length used by the camera (and calibrated using the antenna distance on the ground with no flex).

Because the computer vision method yielded an expected level of wing flex through the flight,

Figure 5.10: GNSS-only attitude solution, without wing-flex estimation.

Figure 5.11: GNSS-measured and computer vision-measured wingtip baseline.

the large attitude errors of GNSS-only attitude determination for this experiment can be attributed to the low-quality GNSS baseline measurements.

5.2.2 Unscented Kalman Filter Results

The full Unscented Kalman Filter, as modified for flight data, was also run, using the output of the GNSS-only filter as a measurement as in the simulation. Figure 5.12 shows the combined GNSS and inertial Unscented Kalman Filter result. Sun sensor data was not used in measurement updates, as the Sun was obscured by cloud cover during each test flight on the flight test date. Table 5.4 shows the algorithms' root-mean-square error relative to the loosely-coupled filter results. Despite the clearly-degraded performance of the algorithm with flight data, the addition of magnetometer data updates is shown to improve the filter performance.

Table 5.4: Unscented Kalman Filter RMS Error : Flight Test

Roll (deg.)	Pitch (deg.)	Heading (deg.)
GNSS+INS		
18.9	16.2	47.7
INS+Mag		
19.5	16.6	49.4
INS-only		
16.4	18.7	49.4
GNSS+INS+Mag		
30.8	18.6	44.7

Figure 5.12: GNSS-INS filter result compared with that of loosely-coupled filter.

Figure 5.13 shows the combined GNSS and inertial Unscented Kalman Filter result. Figure 5.14 shows the combined GNSS, magnetometer, and inertial Unscented Kalman Filter result.

Much of the attitude error in the flight test experiment can be attributed to poor GNSS antenna baselines. Additionally, the magnetometer was not calibrated for this experiment. Figures 5.16 and 5.15 show the GNSS Euler angle and magnetometer post-fit residuals.

Figure 5.13: GNSS-free filter result compared with that of loosely-coupled filter.

Figure 5.14: Multisensor filter result compared with that of loosely-coupled filter.

Figure 5.15: Magnetometer Measurement Residuals.

Figure 5.16: GNSS Euler-angle Measurement Residuals.

Chapter 6

Conclusions & Future Development

This study outlined the design and testing of a GNSS-based attitude determination algorithm, as well as its augmentation with additional sensor data. GNSS-only attitude solutions are consistently improved when GLONASS satellites are included in addition to GPS, owing to more observables and lower dilution of precision (especially in polar regions). Furthermore, adding inertial measurements, Sun sensor and magnetometer data further improves attitude-determination performance and reliability.

Instrumentation of a flight vehicle for data collection was demonstrated, as was the processing techniques for flight data for incorporation into the attitude-determination algorithm. A wing-flex estimation method was developed and used to refine the body-centric antenna coordinates on the aircraft.

The most immediate follow-up experiment will be a ground or flight test in direct sunlight, incorporating sun sensor measurements to assess their affect on the UKF performance. Further, future testing could incorporate a fourth GNSS receiver, as well as a switch to same-model receivers.

The wing-flex experiment can serve as a real-life test to determine flight wing flex during pre-determined maneuvers for wing-aeroelastic and structural engineering studies.

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