Calibrating LaModel for Subsidence

Jian Yang

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Calibrating LaModel for Subsidence

Jian Yang

Thesis submitted
to the Benjamin M. Statler College of Engineering and Mineral Resources
at West Virginia University

in partial fulfillment of the requirements for the degree of

Master of Science in
Mining Engineering

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Morgantown, West Virginia
2016

Keywords: LaModel, CISPM, Calibration, Surface Subsidence Prediction

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ABSTRACT

Calibrating LaModel for Subsidence

Jian Yang

LaModel uses a laminated overburden boundary-element model and can not only calculate seam-level stresses and displacements but also surface subsidence for thin tabular deposit such as coal seams. Up to this point, the material property wizards in LaModel were primarily designed for calculating accurate stress redistribution in single and multiple-seam situations and for investigating and optimizing pillar sizes and layouts in relation to overburden, abutment and multiple-seam stresses. However, the critical input parameters which will give the most accurate seam-level stress distribution do not necessarily produce the best surface subsidence prediction. The objective of this research is to develop a methodology for calibrating the critical input parameters in LaModel to produce the most accurate surface subsidence prediction.

For optimum surface subsidence prediction, it was found that the overburden stiffness as defined by the laminations thickness and the gob convergence as defined by the final gob modulus were the two most critical parameters that needed to be calibrated. Using the WVU (Comprehensive and Integrated Subsidence Prediction Model) (CISPM) program as the best empirical subsidence curve, numerous LaModel runs were performed in order to find the values of lamination thickness and final gob modulus which minimized the least-square error between the CISPM and the LaModel subsidence curves. This subsidence matching process was performed for panels with an assumed offset at the edge of the panel (as typically done with empirical subsidence prediction models) and for panels without an assumed offset. Through this curve fitting process, it was determined that the final gob modulus is best determined as a function of the subsidence factor and the lamination thickness is best determined as a function of overburden depth and/or the panel width-to-depth ratio. Ultimately, three different empirical formulas relating the lamination thickness to the overburden depth and/or the panel width-to-depth ratio were determine for the four cases of: subcritical or supercritical panels, with and without offsets. Further, if the user has measured data for subsidence factor and angle-of-draw, the optimum final gob modulus and lamination thickness can be determined from the measured data. These new subsidence prediction formulas have been implemented into new material wizards in LaModel.
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LIST OF SYMBOLS AND ABBREVIATIONS

List of Symbols

* = project title
\partial = partial differential operator
\int = integration sign
A = computing area
a = (true) subsidence factor
a’ = apparent subsidence factor
d = offset distance of inflection point
d’ = panel offset distance
D = single empirical abutment load extent
D’ = boundary pillar width
e = natural constant, 2.71828...
E = elastic modulus of rock mass
E_f = final gob modulus
E_i = initial modulus
E_j = elastic modulus of j-th layer
F = influence function for 3-D vertical stress
f_s = influence function
H = overburden depth
h_c = caved zone height
i = slope
\( K \) = curvature
\( k_b \) = bulking factor of the immediate roof strata

\( L \) = panel length

\( m \) = seam or mining height

\( n \) = gob height number

\( O \) = origin of global coordinate system

\( O' \) = origin of local coordinate system

\( O'_X \) = X coordinate of grid origin in off-seam level

\( O'_Y \) = Y coordinate of grid origin in off-seam level

\( O_X \) = X coordinate of grid origin in seam level

\( O_Y \) = Y coordinate of grid origin in seam level

\( p \) = pressure or limited percent of displacement volume

\( R \) = radius of major influence

\( S \) = final subsidence

\( s \) = seam convergence

\( S_{\text{max}} \) = maximum possible subsidence

\( S_o \) = maximum subsidence

\( t \) = layer or lamination thickness

\( t_j \) = lamination thickness of j-th layer

\( U \) = displacement

\( W \) = panel width or influence function for 3-D vertical displacement

\( W_c \) = critical width

\( W_e \) = effective width
X = abscissa of global coordinate system
X' = abscissa of local coordinate system
x', y' = distance between the extracted element and the surface point where final subsidence to be calculated along X-axis direction
Y = ordinate of global coordinate system
Y' = ordinate of local coordinate system
Z = vertical axis or vertical distance between points
β = angle of major influence
γ = overburden density
Δ = interval between two subsidence points
δ₀ = angle of draw
ε = gob strain or horizontal stress
η = percent of hardrock
λ = property of laminated overburden defined in equation 2.14
ν = Possion’s Ratio
νₖ = Possion’s Ratio of j-th layer
π = natural constant, 3.1415926...
σᵢ = coal or seam stress
σᵢ = induced stress
σₘ = multiple-seam or inter-seam stress
σₜ = overburden or primitive stress
σₛ = surface-effect stress
σᵤ = ultimate stress
### List ofAbbreviations

<table>
<thead>
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<th>Abbreviation</th>
<th>Description</th>
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<tr>
<td>CISPM</td>
<td>Comprehensive and Integrated Subsidence Prediction Model</td>
</tr>
<tr>
<td>ENX</td>
<td>Numbers of Element in X Axis</td>
</tr>
<tr>
<td>ENY</td>
<td>Numbers of Element in Y Axis</td>
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<tr>
<td>EW</td>
<td>Element Width</td>
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<td>GBS</td>
<td>Grid Block Size</td>
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<td>GNY</td>
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<td>SMCRA</td>
<td>Surface Mining Control and Reclamation Act</td>
</tr>
<tr>
<td>SOR</td>
<td>Successive Over-Relaxation</td>
</tr>
<tr>
<td>TML</td>
<td>Total Model Length</td>
</tr>
<tr>
<td>TMW</td>
<td>Total Model Width</td>
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Chapter 1. Introduction

1.1 Surface Subsidence Background

Underground coal mining in the United States began in the early 18th century (Peng & Cheng, 1980), but major research on surface subsidence induced by underground mining in the United States did not begin until the late 1970s (Peng, 2008). There are three main reasons for this delay in subsidence research.

First, in the old days, the majority of coal mines were located in remote area far away from surface buildings and infrastructure such as residential structures, railroads, highways, pipelines, etc. In these non-urban, if not remote, areas, surface subsidence and environmental damages were scarcely detected and reported. However, with the depletion of coal resources and growth of the population, more and more coal mining operations were conducted closer to suburban, even urban, areas where buildings and infrastructure were concentrated. It was in these populated areas that the subsidence damage of buildings and infrastructure raised the public's concern about the control and prediction of surface subsidence due to underground coal mining (Peng & Cheng, 1980).

Second, the traditional room-and-pillar mining method without pillar extraction employed in the U.S., had minimal effect on the overburden and was designed not to cause immediate surface subsidence. However, with the increased need for highly productive mining techniques, full-extraction mining methods, namely caving methods, were increasingly employed in the U.S. coal mining industry. These total extraction mining methods, which include longwall mining and room-and-pillar retreat mining, normally cause immediate roof caving and the associated surface subsidence, which can cause damage to surface structures and negatively impact on the surface environment.
Finally, with more and more attention paid to environmental and public safety conditions, regulations from federal and state agencies for surface subsidence control were increasingly tightened. The first public law on surface subsidence in the U.S. was in the late 1950s when the state of Pennsylvania enacted the pillar support plan required to protect surface structures (State of Pennsylvania, 1957). In 1977, the U.S. Congress established the Surface Mining Control and Reclamation Act (SMCRA) which required that remediation of surface subsidence became a part of routine mining operations in the U.S. coal mining industry. In response to these subsidence control laws requirement, many subsidence research programs were initiated (Qiu, 2013).

When total extraction is used, it produces a large void in the coal seam and disturbs the equilibrium conditions of the surrounding rock strata. When the excavated area expands to a sufficient size, the roof strata will cave and the ground movements and deformations develop upwards from the excavation level, through the overburden strata, to the surface. Surface subsidence, unlike other issues, such as ground control problems that affect underground miners, involves the general public. Therefore, subsidence is not only a technical but also a public relations issue. In order to deal with the public relations issue safely and efficiently, the most important thing is to predict surface movement and deformation accurately. Further, experience shows that accurate prediction of mine subsidence and its effects are the key to designing and implementing effective mitigation measures in the effort to reduce the severity of the subsidence disturbance and the subsequent consequences (Peng, 2008). Damages to surface environmental conditions induced by overburden strata movements and surface subsidence due to underground coal mining, had prompted the need for an efficient and reliable tool to predict the surface subsidence for mine operators, government agencies and scientific researchers (Luo & Peng, 1989).

In the past three decades, a large number of surface subsidence prediction theories and
mathematics models have been developed all over the world. Most of them were developed for predicting final subsidence only and for full-extraction mining. Based on the underlying mathematics, these surface subsidence methods can be classified into the following four categories: 1) the profile function method, 2) the influence function method, 3) the physical modeling method, and 4) the numerical modeling method (Luo, 1989). In this thesis, two popular programs, Comprehensive and Integrated Subsidence Prediction Model (CISPM) and LaModel, are going to be discussed.

The CISPM computer program is one of the most popular and accurate subsidence prediction programs, which was developed and introduced by Y. Luo in 1989. The program is based on the principles of the influence function method and it uses a number of mathematical models, and empirical formulae for optimizing the input subsidence parameters (Peng, 2008). The subsidence prediction program, CISPM, has been well received and proven to be accurate through numerous applications in the U.S. coal mining industry and in a number of major coal producing countries (Peng, 2008).

Another computer program, LaModel, was initially developed by Heasley in 1996 (Heasley, 2008). LaModel uses a laminated overburden model and was primarily designed to calculate seam-level stresses and displacements for thin tabular deposit such as coal seams, but it can also calculate surface subsidence. The program uses a displacement-discontinuity variation of the boundary-element method and a Successive Over-Relaxation (SOR) iterative technique for solving the elastic equations of equilibrium around the mine openings. If surface subsidence is desired, then an influence function based on the laminated overburden model is used to calculate the resulting surface subsidence due to the previously determined underground seam convergence (Heasley, 1998).
1.2 Statement of Problem

As mentioned above, LaModel does have the capability to calculate seam-level stresses and displacements and surface subsidence. However, up to this point in time, the material property wizards in LaModel have primarily been designed for calculating accurate stress distributions in single and multiple-seam situations, and for investigating and optimizing pillar sizes and layouts in relation to overburden, abutment and multiple-seam stresses. Originally, it was anticipated that LaModel would produce good seam-level stresses and displacements, and good surface subsidence predictions by using same input parameters (Heasley, 1998). However, as experienced was gained using the program, it was found that LaModel can calculate good underground stresses based on the calibrated input parameters for stresses redistribution, but those input parameters do not necessarily produce good surface subsidence prediction. Similarly, the critical input parameters which will give the most accurate surface subsidence do not necessarily produce the best stress distribution results. Therefore, the parameters can be quite different for calculating good stresses and good surface subsidence. Essentially, to get good surface subsidence prediction, very flexible overburden has to be used by inputting thin laminations, low overburden modulus which gives very high, short abutment stresses. For the parameters that give you good stresses by using stiffer overburden, the subsidence values predicted are quite low (Heasley, 2016a).

For a numerical modeling program, the accuracy of the results depend entirely on the quality of the input parameters chose by the user. Over the past twenty years, stress calculation and pillar design have been the primary focus of the LaModel program. Therefore, wizards for calibrating the critical input parameters to give accurate seam-level stress distribution have been thoroughly developed. However, recently there has been increased interest in using LaModel for calculating surface subsidence due to its ability to readily calculate subsidence associated with failing pillars,
multiple-seam mining and/or irregular geometries. (Also, the fact that it is free draws many users to LaModel for subsidence prediction.) Therefore, it seemed timely to now develop wizards for calibrating the critical input parameters for LaModel to produce accurate surface subsidence.

1.3 Research Objectives, Methodology and Scope

The objective of this thesis is to develop a methodology for calibrating the critical input parameters in LaModel to optimize surface subsidence prediction and thereby improve mine safety and health.

In this research, three scenarios were used to calibrate LaModel for subsidence. They are: “with an edge offset” calibration method, “without an edge offset” calibration method, and “using measured data” calibration method. Most empirical subsidence prediction programs are based on the influence-function method (Luo and Peng, 1989; VPI&SU, 1987), and it has been found that the inflection point of the influence function needs to be offset a certain distance in from the edge of the extraction panel in order to get the accurate subsidence prediction. This distance is known as the “offset” distance. Therefore, when I calibrate LaModel for subsidence, the “with an edge offset” calibration method was first performed. However, the “with an edge offset” calibration method is not natural for LaModel. Essentially, assuming an offset distance implies that the seam convergence is zero within this distance from the edge of the extraction. Therefore, the second calibration method, “without an edge offset”, was performed. Further, if a mining company or engineer has measured subsidence with a subsidence factor and angle of draw for a specific site, they may want to use these measured data to back calculate the subsidence, the third calibration method was conducted for this scenario.

In order to calibrate LaModel for subsidence prediction, the actual field measured subsidence resulting from the extraction of panels with a wide range of panel widths, depths, extraction
thicknesses, geology, etc. was required. However, such a database does not exist. In lieu of this information, the CISPM program was chosen to be a substitute for measured subsidence data. This program has a long history of calibration with numerous subsidence case histories and of successful subsidence prediction (Luo & Peng, 1989; Peng & Yuo, 1992; Luo, Peng et al., 2008; Luo & Qiu, 2012); and represents some of the best empirically predicted subsidence curves. To calibrate the critical input parameters, numerous LaModel runs were performed in order to find the values of the critical input parameters: final gob modulus, lamination thickness, and offset distance which minimized the least-square error between the CISPM output subsidence and the LaModel predicted subsidence curves. Finally, the determined optimum values of lamination thickness, final gob modulus and panel offset distance were used to back fit three sets of empirical formulas in relation to the three calibration methods.

The three sets of empirical formulas were implemented into a new lamination thickness wizard in LaModel for subsidence prediction. Based on the calibration method selected, the new wizard helps the user to calculate the critical input parameters for subsidence prediction. The new lamination thickness wizard is the final output of this research.
Chapter 2. Literature Review

2.1 Introduction to Surface Subsidence

When total extraction of an opening of sufficient size is reached in an underground horizontal coal seam, the roof strata in the overburden deform to reach a new equilibrium condition. The severity of deformation decreases upward from coal seam toward the surface. As the downward sagging of the strata propagate and reach the surface, there will be a depression zone on the surface directly above, but typically extending beyond the edges of, the underground opening. This depression zone is called the surface subsidence basin or surface subsidence trough (Peng, 1992). The term, “final surface subsidence trough”, normally refers to a surface depression zone formed over a mined area long after the extraction has occurred; and therefore the depth and shape of the depression is assumed to be constant and will no longer change with time.

2.1.1 Overburden Movement

The study of surface subsidence begins with understanding the overburden movement and associated surface subsidence which is in response to the underground coal mining operations. When total extraction mining is used, it produces a large void in the coal seam and disturbs the equilibrium stress condition of the surrounding rock strata. The roof strata is going to bend or fail downward. When the excavated area (or gob) expands to a sufficient size, the roof strata will cave, bulk and ultimately fill the void space. As part of this caving and bulking process, the overlying strata continue to bend and break until the piles of the fallen rock fragments are sufficiently high and stable to support the overlying strata. At this point, the overlying strata will no longer fail, but simply bend and rest on the underlying strata or gob piles. Bending of the overlying strata develops upward until reaching the surface and thereby forming a subsidence trough. The gob, overburden
strata and the surface subsidence trough will also go through a period of re-compaction and gradually become stabilized (Peng, 1992).

When an underground coal mine employs total extraction, the overburden strata above the coal seam are subjected to various degrees of movement and deformation as the strata is further removed from the seam. On the basis of the strata movement and deformation characteristics, the subsided overburden can be divided into four zones as illustrated in Figure 2.1 (Peng, 1992).

![Figure 2.1 Four zones of strata movement above a longwall panel (after Peng & Chiang, 1984)](image)

**Figure 2.1 Four zones of strata movement above a longwall panel (after Peng & Chiang, 1984)**

Closest to the seam, the “caving zone” is normally formed by the immediate roof failing and filling the void space. The strata in the caving zone not only lose their horizontal continuity, they also tumble and lose their beddings orientations. From empirical experience, the caving zone height is normally 2 to 8 times of the seam height, depending on the bulking factor of the immediate roof and the depth and stiffness of the overlying roof (Peng, 1992). The following equation can be used to calculate the caved zone height (h_c) as a function of the mining height (m):
and bulking factor \( (k_b) \) of the gob material.

\[
h_c = \frac{m}{k_b - 1}
\]  

(2.1)

The “fractured zone” is located immediately above the caving zone. After the immediate roof caves into void formed by extracting the coal seam, the strata above the caved zone start to bend downward. As a result of excessive bending, the strata will break and form fractures in both the horizontal and vertical directions and the strata will lose their horizontal continuity; however, the beddings orientations will generally remain. According to empirical experience, the combined height of the fractured zone and the caved zone normally ranges from 20 to 30 times the mining height \((m)\). Other things being equal, the height of the fractured zone for hard and strong strata is larger than that for soft and weak strata. In the fractured zone, the severity of strata breakage decreases from the bottom to the top, and correspondingly, the porosity and permeability of the strata increase from the top to the bottom. Generally, the shape of the fractured zone is related to the size of opening. When the panel is subcritical, the shape of the fractured zone has a dome-like shape. As the panel width expands to a sufficient size, the panel becomes supercritical resulting in a flat top for the fractured zone in the center of the panel. When the fractured zone is observed along a longitudinal cross section of the panel, its shape will be a flat arch.

The third zone above the full extraction area is the “bending zone”. It is between the fractured zone and the soil zone and strata within this zone continually bend downward toward the mine without significant fractures. The stratified beddings and horizontal continuity remain in the original condition. There may be some open fissures in the tension zone of the strata, but those open fissures do not destroy the strata continuity and the layers in this zone serve as an aquiclude.

The highest zone, called the “soil zone”, is the surface layer above the continuous deformation zone. It consists of soil and weathered rocks and the depth of the soil zone depends on the location.
Cracks could develop in this soil zone. Depending on the physical properties of the soils, cracks developed over and near the panel edges tend to remain open permanently but the crack walls collapse easily to fill up the cracks. The cracks developed in and around the central part of the panel open a short distance ahead of the moving longwall face and they close a short distance behind the longwall face. Depending on the depth, seam thickness and other factors, the cracks vary from barely visible to 3-4 ft wide and from less than 1 ft deep to as deep as the soil zone (Peng, 1992; Luo, 2016a).

2.1.2 Surface Subsidence Characterization

The trough subsidence events are normally associated with longwall mining and room-and-pillar retreat mining operations (see Figure 2.2). Along with the mining operations, the downward sagging of the strata propagates and reaches the surface. As a general rule, in order for the strata deformation to reach the ground surface, the width of the opening should be greater than 0.3 to 0.4 times of the overburden depth. The minimum width of the mined opening for the induced strata movement to reach ground surface is called the effective width ($W_e$). When the width of the opening is larger than the effective width, a subsidence trough will be formed on the surface over the mined opening. It takes time to form a subsidence trough. The final subsidence trough is the one that forms long after the mining has been completed (Luo, 2016a).
Based on the previous research on surface subsidence, several fundamental concepts of subsidence theory are going to be introduced. First of all, the “Subsidence (S)” is the vertical component of surface movement at a surface point. The “maximum subsidence (S₀)”, which is the maximum amount of subsidence measurable in a subsidence trough, increases with panel width (in two-dimension across the faceline) or gob dimensions (in three-dimension). When the panel width exceeds a critical value, the maximum subsidence reaches its maximum possible value (Sₘₐₓ). The panel width at this time, when Sₘₐₓ starts to occur, is called the critical panel width (Wₑ). In general, the value of Wₑ is assumed as 1.2H. The “Displacement (U)” is the horizontal component of surface movement at a surface point within the 360° horizontal plane. The “Slope (i)” is the differential subsidence over a horizontal distance of a unit length and is the first derivative of the subsidence. The “Curvature (K)” is the differential slope over a horizontal distance of a unit length and is the second derivative of the subsistence. The “Horizontal Strain (ε)” is the differential horizontal displacement over a horizontal distance of a unit length and is the first
Next, several subsidence terminologies are going to be defined. The “Subsidence Factor (a)” is defined as the ratio of the maximum possible subsidence ($S_{max}$) to the mining height (m) of the coal seam. The “Inflection Point” is the point dividing the convex and concave portions of the major cross section of the subsidence profile. In general, the subsidence at the inflection point is considered to be half of maximum subsidence. The “Angle-of-Draw ($\delta_o$)” is the outward angle between the normal to the seam at the panel edge and a line connecting the panel edge and the point on the surface where the observed subsidence is zero. The “Internal Angle-of-Draw ($\delta_o'$)” is the internal angle between the normal to the seam at the panel edge and a line connecting the panel edge and the point on the surface interior to the panel where the observed subsidence is no longer affected by the panel edge. The internal angle-of-draw is generally considered to be equal to the angle of draw (Luo, 2016a; Heasley, 2016b).

Figure 2.3 Relationship among subsidence parameters and subsidence trough (after Luo, 2016a).
The term “Subcritical Panel” refers to a panel where the width is so narrow or the depth is so small that the lines denoting the internal angle-of-draw cross before they reach the surface and therefore, maximum possible surface subsidence is never attained (see Figure 2.4). Mathematically, the term, subcritical, means that the width of the panel (\(W\)) is less than twice the product of the tangent of the internal angle-of-draw (\(\delta_o'\)) and overburden depth (\(H\)) (Heasley, 1988).

\[
W < 2 \tan(\delta_o')H
\]  

(2.2)

Figure 2.4 A schematic showing the concept of a subcritical panel and its associated subsidence, displacement, strain, and slope and curvature (after Heasley, 1988).

Using the similar definition, a “Supercritical Panel” is one that is so wide or so shallow that the lines denoting the internal angle-of-draw do not cross or meet before they reach the surface and the maximum possible subsidence is attained at all the points between the surface intersection of the internal angle-of-draw lines (see Figure 2.5). Mathematically, the width of a supercritical panel (\(W\)) is greater than twice the product of the tangent of the internal angle-of-draw (\(\delta_o'\)) and
depth of cover (H) (Heasley, 1988).

\[ W > 2 \tan(\delta) H \] (2.3)

Figure 2.5 A schematic showing the concept of a supercritical panel and its associated subsidence, displacement, strain, and slope and curvature (after Heasley, 1988).

2.1.3 Methods for Predicting Final Subsidence Trough

Since the 1960’s, dozens of models have been developed for predicting subsidence (Voight & Pariseau, 1970; Brauner, 1973a; Kratzsch, 1983; Wittaker & Reddish, 1989). These models can be classified into four general types of methods: 1) the profile function method, 2) the influence function method, 3) the physical modeling method, and 4) the numerical modeling method. However, in this thesis the influence function method used in CISPM and the influence function method which is part of the numerical method used in LaModel will be discussed.

The influence function method employs an approach to subsidence prediction which assumes that the extraction of an elemental area (in plan view) of an underground coal seam will cause the surface to subside in a particular manner (see Figure 2.6). Generally, the surface point located directly above the extracted element receives the most amount of subsidence. The farther the
surface point is away from the extracted element, the less amount of subsidence occurs at the surface point. The mathematical function selected to represent the distribution of the subsidence influence caused by the extraction of the element is called the influence function.

Figure 2.6 A schematic of the influence function method (after Heasley, 1988)

The final subsidence at a given surface point is the result of all influences received at this point from all of the extracted “elements” on the coal seam. Mathematically, the final subsidence at a surface point is expressed as the integral of the influence function from each element within the “mined-area” (see Figure 2.7).

Figure 2.7 A schematic of applying the influence function method in subsidence prediction (after Luo, 2016b)
2.2 Introduction of CISPM

2.2.1 Influence Functions

In the previous research on surface subsidence prediction for underground coal mining, there are a number of different influence functions selected by subsidence researchers to build mathematical models for subsidence prediction (Bals, 1931/1932; Beyer, 1945; Knothe, 1957). Among those selected influence functions, most of them were for the vertical component of the surface movement vector only.

Among these influence functions, one of the most popular and versatile influence functions for surface subsidence prediction was proposed by Knothe in 1957. And one of the most popular and successful subsidence prediction programs in the U.S., CISPM, is based on Knothe’s influence function, and was developed by Luo and Peng (1989). The principle of Knothe’s influence function is that the distribution of the subsidence caused by the extraction of one extraction element can be expressed by a modified normal probability distribution function (Knothe, 1957). In three-dimensional case, the Knothe influence function for subsidence is:

\[
f_s(x', y') = \frac{S_{max}}{R^2} e^{-\left(\frac{x'^2 + y'^2}{R^2}\right)}
\] (2.4)

Where \( S_{max} \) is the maximum possible subsidence; \( R = H / \tan \beta \) is the radius of major influence; \( x' \) and \( y' \) are the distances between the extraction element and the surface point where final subsidence is to be determined along the X and Y axes, respectively; \( H \) is the seam depth; and \( \beta \) is the angle between the horizontal coal seam and the line connecting the point of interest and the limit of influence function (see Figure 2.8). In Equation 2.4, the subsidence increases with an increase in maximum possible subsidence \( (S_{max}) \) and the distance \( (x'^2 + y'^2) \) between the extraction element and the surface point where final subsidence is to be determined, and with a decrease in
radius of major influence (R). The three-dimensional equation is developed for predicting the three-dimensional subsidence trough.

\[
S(x, y) = \frac{S_{\text{max}}}{R^2} \int_{A} e^{-\left(\frac{x^2+y^2}{R^2}\right)} \, dA
\]  

(2.5)

Figure 2.8 A schematic of Influence function for subsidence (after Peng, 2008)

Therefore, the final subsidence at the prediction point is obtained by integrating the influence function for subsidence over the computing area (A) (see Figure 2.9), which is defined by pulling a distance equivalent to the offset of inflection point (d) (see Figure 2.3) back from the actual boundary of the mine gob. In order to make the method flexible, different d values are assumed along the four edges of the rectangular mine gob. The mathematical expression for the final subsidence at the prediction point is shown in Equation 2.5.
2.2.2 Final Subsidence Parameters

Based on the mathematical model used in the CISPM program, the final subsidence parameters play an important role for subsidence prediction. The accuracy of subsidence prediction method totally depends on the accuracy of the parameters selected to use in the model. Depend on the Knothe’s influence function, there are three final subsidence parameters extremely essential as follows: 1) subsidence factor, a; 2) offset distance of inflection point, d; 3) radius of major influence, R, or angle of major influence, $\beta = \arctan(R/H)$.

First, the maximum possible “true” subsidence factor (a) is defined as the ratio of the maximum possible subsidence ($S_{\text{max}}$) to the mining height (m) of the coal seam. The maximum “apparent” subsidence factor ($a'$) is defined as the ratio of the maximum subsidence ($S_0$) to the mining height (m). The subsidence factor determines the depth of the final subsidence trough. When the values of $d$ and $R$ are the same in two panels, the larger the subsidence factor, the deeper the final subsidence trough (see Figure 2.10). The overburden mechanical properties the seam depth and seam thickness affects the magnitudes of subsidence factor. In general, the subsidence factor is
inversely proportional to percent of hard rock in the overburden strata and the seam depth.

\[ a_1 < a_2 \]

\[ a = 1.9381(H + 23.4185)^{-0.1884} \]  \hspace{1cm} (2.6)

Figure 2.10 Influence of different subsidence factors on final subsidence trough (after Luo, 2016a)

During the surface subsidence research over several decades, a large amount of longwall subsidence data has been collected by Peng et al. (1995). By analyzing these field data, an empirical equation was derived to calculate subsidence factor for most of the U.S. coal fields (Luo, Peng et al., 2008). In this empirical equation, the subsidence factor (a) was only correlated to the overburden depth (H) (see Equation 2.6).

However, in the central Appalachian coal fields of the U.S., the overburden strata for underground coal mines normally contain much higher percentage of hard rock (i.e., sandstone and limestone) strata than other areas, often higher than 60%, and many of the strong rock strata are also in thick layers (Luo, 2016b). The high percent of hard rock in overburden strata results in significantly different characteristics of the overburden movements and deformations from areas with a low percent of hard rock. Therefore, the Equation 2.6 does not give an accurate subsidence factor for central Appalachian coal fields. Based on subsidence data collected in central
Appalachian coal fields, Karmis et al. (1984) developed an empirical equation (see Equation 2.7) for apparent subsidence factor which includes the overburden depth, panel width and a parameter for the percent of hard rock to calculate the subsidence factor. (It should be noticed that 35% hard rock should be input as 35 for η in this equation.) Figure 2.11 shows the plotting of Equation 2.7. Look at Figure 2.11, it can be seen that the value of a’ increases with an increase in the value of W/H and keep a relative constant value when the value of W/H reaches 1.4; besides, the value of a’ decreases with an increase in the hard rock percent under the same value of W/H.

\[
a' = \left[ 1.017 - \frac{0.03}{\frac{W}{H} - 0.43} \right] \times \left[ 0.12 + 0.66e^{-0.00034\eta} \right]
\]  

(2.7)

**Figure 2.11 Determination of the Apparent Subsidence Factor from the Width-to-Depth Ratio and the Percent of Hard rock (Karmis, 1984)**

Next, the offset distance of the inflection point (d) is the horizontal distance between the inflection point and the closest edge of the underground opening. The value of d is another critical parameter for determining the final subsidence trough. The inflection point determines the location
of the subsidence trough in relation to the edge of the mined panel. Under the same values of a and R, the larger the offset distance of inflection point, the more the subsidence trough moves in toward the panel center (see Figure 2.11). In generally, harder overburden strata will hang at the edge of the panel and produce a larger offset distance of the inflection point.

\[
d_1 < d_2
\]

![Figure 2.12 Influence of different offset distances of the inflection point on the final subsidence trough (after Luo, 2016a).](image)

Based on the analysis of the collected longwall subsidence cases, Peng et al. (1995) found that the offset distance of the inflection point (d) is a function of the overburden depth (H) and is applicable to most U.S. coal fields. Looking at Figure 2.13, it can be seen that the value of d increases first and then decreases with an increase of overburden depth.

\[
d = \left(0.382075 \times 0.999253^H\right) H \tag{2.8}
\]
Finally, the radius of major influence (R) is defined as the horizontal distance between the inflection point and the edge (or the "zero" subsidence point) of the subsidence trough. It determines the shape of the subsidence trough. Under the same values of a and d in two panels, the larger the radius of major influence, the less steep the subsidence trough wall (see Figure 2.14). The range of the major influence zone increases with an increase in the radius of major influence. Generally speaking, the harder/stronger overburden strata results in a larger radius of major influence.
The angle of major influence, $\beta$, is defined as the angle between the horizontal line at the mining level and the line connecting the edge of the subsidence trough and the vertically projected point of the inflection point on the coal seam. It is also used to define the radius of major influence as shown in Figure 2.3. The relationship between the angle of major influence and the radius of major influence can be defined as:

$$ R = \frac{H}{\tan \beta} \quad (2.9) $$

Based on the analysis of the collected longwall subsidence cases, Peng et al. (1995) found that using 3.0 for $\tan \beta$ or 71.6° for $\beta$ is fairly good for most of the U.S. coal fields.

2.3 Introduction of LaModel

2.3.1 Laminated Overburden Model

LaModel, program for modeling coal seam displacements and stresses, was initially developed by Heasley in 1994 (Heasley & Barton, 1998). The program uses a displacement-discontinuity variation of the boundary-element method with a simplified laminated overburden model which consists of a stack of strata laminations where the interfaces between beds, including the ground
surface, are all horizontal, and free of shear stresses and cohesion (see Figure 2.15). In general, LaModel was assumed that the overburden stratification properties of each layer have the identical elastic modulus (E), Poisson's Ratio (ν) and thickness (t). This "homogeneous stratifications" formulation does not require specific material properties for each individual layer and yet it still provides a realistic suppleness of the laminated overburden model as compared to the previous homogeneous isotropic elastic overburden (Heasley, 1998).

Displacement Discontinuity Approximation

Figure 2.15 Schematic of laminated overburden (after Heasley, 1998).

The LaModel program is primarily designed to be utilized by mining engineers or researchers for investigating and optimizing pillar sizes and layouts in relation to overburden, abutment and multiple-seam stresses, but the laminated overburden model can also be used to derive an influence function that can be used to calculate the surface subsidence induced by the in-seam convergence (Heasley, 1998).

From this homogeneous, frictionless lamination conceptual model, the fundamental second-
order, partial-differential equation which mathematically represents the laminated overburden can be derived (Heasley, 1998):

\[
\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} = \frac{4\sqrt{3(1-v^2)}}{Et} \sigma_i,
\]

(2.10)

This equation mathematically defines the exact relationship between the curvature (second derivative of the convergence, S) of the roof and floor of the seam, the mechanical properties (elastic modulus (E), Poisson’s Ratio (v), and lamination thickness (t)) of the overburden and the stresses (\(\sigma_i\)) applied to the seam.

In general, the induced stress (\(\sigma_i\)) is the sum of the overburden stress (\(\sigma_q\)), the seam material stress (\(\sigma_c(s)\)), the surface-effect stress (\(\sigma_s(s)\)) and the multi-seam stresses (\(\sigma_m(s)\)) as shown:

\[
\sigma_i(s) = -\sigma_q + \sigma_c(s) - \sigma_s(s) - \sigma_m(s)
\]

(2.11)

From the fundamental equation of the laminated overburden model, a three-dimensional displacement influence function can be derived which calculates the vertical displacement (W) at a remote point anywhere in the overburden as a result of a unit convergence over a unit area of the seam that is a given distance away (x’, y’, z) from the point (Yang, 1992).

\[
W(x, y) = \frac{1}{8\pi \lambda z} e^{-\frac{x'^2 + y'^2}{4\lambda z}}
\]

(2.12)

the two-dimensional version of this equation can be derived (Salamon, 1962, 1989a) and Yang (1992):

\[
W(x) = \frac{1}{8\sqrt{\pi \lambda |z|}} e^{-\frac{x'^2}{4\pi |z|}}
\]

(2.13)

Here the parameter, \(\lambda\), which encapsulates a number of constants from the overburden, has been introduced to help simplify the equation:
This displacement influence function is used in LaModel to calculate the surface subsidence, or the subsidence on an overlying seam, due to the calculated convergence on the modelled seam.

2.3.2 Critical Parameters

The two primary factors which influence the shape and magnitude of the seam convergence and hence the associated subsidence in LaModel are the gob compaction stiffness and the overburden flexural stiffness. Therefore, the primary parameters which are adjusted in LaModel for fitting subsidence are the final gob modulus \((E_f)\) which is used to control the gob stiffness, and the lamination thickness \((t)\) which is used to control the overburden stiffness (Heasley & Barton, 1998). From the fundamental equation of the laminated overburden model (see Equation 2.10), it can be learned the lamination thickness and the overburden modulus affect the overburden behavior in essentially the same manner; therefore, in order to simplify the calibration process, the overburden modulus was held constant while only the lamination thickness was varied. The primary gob material used in the LaModel program utilizes the “Strain-Hardening” material model. The strain-hardening material model uses an exponential stress-strain curve (see Figure 2.16) and this material is intended to approximate the strain-hardening behavior of gob or backfill. The fundamental basis of this gob model is the assumption that the tangent elastic modulus of the material increases linearly with stress.
The mathematical derivation for the “Strain-Hardening” gob material was provided by Zipf (1992a, 1992b), where he found that the gob material stress ($\sigma$) is related to the material strain ($\varepsilon$) by the following equation:

$$
\sigma = \left[ \frac{E_i \sigma_u}{E_f - E_i} \right] \left[ e^{\left( \frac{E_f - E_i}{n \sigma_u} \right) \varepsilon} - 1 \right]
$$

(2.15)

Where $E_i$ is the initial tangent modulus at zero stress, $E_f$ is the final tangent modulus at the ultimate stress ($\sigma_u$) and $n$ is the gob height factor.

For a supercritical panel, the gob strain ($\varepsilon$) occurring in the gob directly under the maximum subsidence in the flat middle of the surface subsidence trough is equal to the true subsidence factor (a) (see Equation 2.16), and the corresponding gob stress ($\sigma$) is essentially equal to the overburden stress ($\sigma_q$), which is a function of the overburden density ($\gamma$) and the depth ($H$) (see Equation 2.17):

$$
\varepsilon = a = \frac{S_{\text{max}}}{m}
$$

(2.16)

$$
\sigma = \sigma_q = \gamma H
$$

(2.17)
In order to determine the final modulus for gob which generates the observed surface subsidence, Equation 2.15 can be rewritten as:

\[
\sigma_q - \left[ \frac{E_i}{E_f - E_i} \right] \left[ e^{\left( \frac{E_f - E_i}{n \sigma_u} \right) a} - 1 \right] = 0 \tag{2.18}
\]

For specifying the final gob modulus (\(E_f\)) in Equation 2.18, the required parameters are the initial tangent modulus (\(E_i\)), the ultimate stress (\(\sigma_u\)), the gob height factor (\(n\)), the subsidence factor (\(a\)) and the overburden stress (\(\sigma_q\)).

As stated above, the lamination thickness (\(t\)) which is used to control the overburden stiffness was a critical parameter of the overburden. Increasing the lamination thickness will increase the stiffness of the overburden. With a stiffer overburden, the convergence values over the gob areas or in the entries will decrease, and the subsidence trough will tend to be shallower and wider. Conversely, a softer overburden will cause a deep and more abrupt subsidence trough.

To calculate the remote displacement for a shallow depth seam, the domain should be considered to be infinite half-space. In this case, the stress-free ground surface should be taken into account in calculating remote displacement. In order to create a traction-free plane at the ground surface, the technique of a “mirror-image” seam is used (Salamon, 1989b; Yang, 1992). First, the actual seam is assumed to be in an infinite medium with seam displacements occurred. Next, a fictitious “mirror image” seam is placed above the ground surface at a distance equal to the actual seam depth (see Figure 2.17). The fictitious seam is also considered to be in an infinite medium; however, the calculated convergence in the actual seam is exactly mirrored as divergence in the mirror-image seam. Thus, the distributions of convergence and divergence are identical in magnitude but opposite in sign. Consequently, the sum of the propagated displacements and stresses from the two seams is zero at a plane midway between the two seams, at the ground.
surface. Thus, the union of the two infinite media solutions corresponds to the effect of the actual seam at a finite depth (Heasley, 1998).

When predicting surface subsidence over an extraction panel in LaModel, the free-surface (“mirror-image”) effects essentially double the calculated surface displacements from the seam in an infinite media; therefore this factor of 2 can be used to approximate the effect of the free-surface, without needing the extensive calculations associated with the mirror-image seam (Heasley, 1998).

![Figure 2.17 Schematic of mirror-image and multiple-seam stress calculation (Heasley, 1998).](image)

**2.4 Summary and Conclusion**

The empirical method used in CISPM is specifically designed for surface subsidence prediction. This empirical prediction method is easy to use and is supported by extensive field experience and is therefore more extensively calibrated. However, it should be noted that the influence function in the empirical method has little or no connection to the actual mechanics of the subsidence, and the influence function parameters are back calculated from a large amount of
longwall subsidence data (Heasley & Barton, 1998). Without a mechanistic basis, establishing the exact site-specific parameters to use in cases outside of the present database has been problematic and requires significant additional empirical data. Therefore, the empirical formulas do not naturally handle subsidence prediction for unusual situations such as: random pillar designs, multiple-seams, pillar failure, etc.

For the numerical method used in LaModel for surface subsidence prediction, the influence function is derived from the mechanical properties of the overburden and interestingly has the same mathematical form as the influence function used in the empirical methods. The parameters in the numerical method come from the properties of the overburden and first-principals of mechanics. However, subsidence prediction with LaModel is only as accurate as the laminated overburden model that it uses, and in regard to displacements and surface subsidence, the laminated model has some limitations. Also, the numerical method is a bit more cumbersome to use, because instead of a simple subsidence factor and an angle-of-draw, gob properties and lamination thickness needed to be calibrated. Also, it takes much longer to run because it does much more calculation (Heasley, 2016a). Because of the limitations with the laminated model, to get the most accurate subsidence with LaModel, it needs to be calibrated against the more accurate empirical methods. However, once calibrated, it can easily be applied to the unusual situations such as random pillar designs, multiple-seams, pillar failure, etc.
Chapter 3. Calibrating LaModel for Subsidence

In order to give an accurate surface subsidence prediction, the LaModel program needs to be calibrated carefully. In this chapter, three calibration methods for determining critical input parameters will be conducted for panels. First of all, the LaModel program for predicting subsidence over panels with an edge offset will be calibrated. In CISPM, in order to get the optimum subsidence prediction, the integration area, namely computing area, for calculating subsidence for a surface point is not the same as the actual mined area, but rather move an offset distance of inflection point inward from each side of the panel boundary (see Figure 2.9) (Luo, 2016a). Essentially, assuming an offset distance implies that the seam convergence is zero within this distance from the edge of the extraction. Therefore, the computing area in a panel used for calibrating LaModel for subsidence in the first calibration method is obtained by indenting a certain distance from the actual boundary of the mine gob. The characteristics of overburden movements and deformations resulting in surface subsidence in supercritical panels are significantly different from that in subcritical panels, therefore, for calibrating the optimum input parameters for calculating subsidence with LaModel, supercritical and subcritical panels are separated into two different groups and calibrated individually.

Secondly, the LaModel program for predicting subsidence over panels without an edge offset will be calibrated. Without an edge offset is referred to the computing area is the same as the actual panel gob area. Obviously, it is unrealistic to offset a certain distance in from the edge of the extraction panel when calculating subsidence with LaModel. In order to get the optimum subsidence prediction for actual mined out panel, the LaModel program will be calibrated using panels without an edge offset. Also, the supercritical and subcritical panels will be studied
Finally, the LaModel program for subsidence prediction using field data will be calibrated. For a subsidence problem, if the mining engineers or researchers have measured field data in a previous subsidence case at the mine, LaModel can be used to back-analyze the subsidence problem using the field data.

### 3.1 Calibrating LaModel for Subsidence with an Edge Offset

When calibrating LaModel for surface subsidence prediction, the two primary parameters need to be calibrated are final gob modulus and lamination thickness. The final gob modulus is used to control the gob compaction stiffness and the lamination thickness in overburden determines the overburden flexural stiffness. However, for the models in the first calibration method, the actual mined out panels need to be moved inward for certain distance to create computing areas, the offset distance will also need to be determined.

#### 3.1.1 Supercritical Panels

A supercritical panel is referred to as its panel length and panel width are all greater than critical dimension. In this thesis, there is an assumption that the panel length is always greater than the critical width and is determined as sevenfold of its panel width in any model. Therefore, the panel width is the only factor determining whether a panel is supercritical or not. In all of the calibration models, the range of panel widths is from 300 ft to 1500 ft and the range of seam depths is from 300 ft to 2500 ft.

The LaModel program consists of three modules: LamPre, LaModel and LamPlt. The LamPre module is primarily designed for generating input file and includes the necessary subroutines to input the default data, calibrate the critical parameters in laminated overburden model and build mine layout in grid. The function of the LaModel module is to read the input file, solve the
laminated overburden model numerically and produce the output files. The LamPlt module reads
the output files and automatically extract the output data to give plots according to user interest
(Zhang, 2014).

In order to completely automate the model building for subsidence prediction, several new
algorithms and mathematical formulae needed to be developed and implemented into the LamPre
module. In this chapter, algorithms for lamination thickness calculation, for final gob modulus
calculation, for offset distance calculation need to be developed. The major procedures for
automatically generating input file, for solving the laminated overburden model and for analysis
of the output data are listed below and further detailed in the following sections:

The LamPre Module

• **Default data input:** Input the default data which are used to solve the laminated
overburden model.

• **Boundary pillar sizing:** Based on the overburden properties, determine the radius of
influence, and thereby define the required size of the boundary pillars (D’) around the edges of the
panel.

• **Mine model sizing:** Based on the input panel dimensions and the sizes of boundary pillars,
determine the total model width (TMW) and length (TML).

• **Element sizing:** Based on the overall model dimensions and the lamination thickness of
overburden, determine the reasonable element width (EW).

• **Grid generation:** Insert the opening, coal and gob materials into the mine grid based on
the pillar and gob locations.

• **Yield zone application:** Apply the yield zones to the pillars based on the Mark-Bieniawski
stress gradient.
• **Input file generation**: Synthesize the default data, coal and gob materials properties and mine grid etc. together into the input file and save as *.INP File.* represents the project title.

**The LaModel Module**

• **Run the model**: Read the input file and solve the model using the LaModel module and generate the output files.

**The LamPlt module**

• **Graphical representation**: Read the output files and use plots to display the data according to user’s selection.

### 3.1.1.1 Model development

In order to show the standard procedures in detail in building models, an example model is preformed to calibrate LaModel for subsidence. In this example model, the information of the example panel is listed in Table 3.1.

#### Table 3.1 Information of the example panel

<table>
<thead>
<tr>
<th>Items</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Overburden Depth (H)</td>
<td>1,000</td>
<td>ft</td>
</tr>
<tr>
<td>Longwall Panel Width (W)</td>
<td>1,500</td>
<td>ft</td>
</tr>
<tr>
<td>Longwall Panel Length (L)</td>
<td>10,500</td>
<td>ft</td>
</tr>
<tr>
<td>Average Mining Height (m)</td>
<td>5</td>
<td>ft</td>
</tr>
<tr>
<td>Percent of Hardrock (η)</td>
<td>0</td>
<td>%</td>
</tr>
</tbody>
</table>

Firstly, the CISPM program is used to calculate subsidence for the example model. In the ‘Data Input Screen’ of CISPM, 1000 ft of seam depth and 0 of hardrock percent are input for overburden strata information. Also, 5 ft of mining height and 1,500 ft of panel width are used to present panel information. In any model, it is assumed that the panel length is 7 times of the panel width in order to eliminate the dimension effect along longitudinal direction. Therefore, the panel here is set as
10500 ft long. After inputting the information of example panel into CISPM, the final subsidence parameters (see Figure 3.1), the transverse major cross-section subsidence data (see Table 3.2) and the plot for transverse major cross-section subsidence profile (see Figure 3.2) can be got.

![Data Input Screen]

**Figure 3.1 ‘Data Input Screen’ of CISPM for the example model**

Table 3.2 Transverse major cross-section subsidence data for the example model

<table>
<thead>
<tr>
<th>Information About Overburden Strata</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Overburden Depth:</td>
<td>1000 ft</td>
<td></td>
</tr>
<tr>
<td>Percent of Hardrock (Limestone and sandstone):</td>
<td>0%</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Information About the Longwall Panel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Mining Height:</td>
</tr>
<tr>
<td>Longwall Panel Length:</td>
</tr>
<tr>
<td>Longwall Panel Width:</td>
</tr>
<tr>
<td>Location of Prediction Point from Left Edge:</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Default Final Subsidence Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subsidence Factor, a:</td>
</tr>
<tr>
<td>Radius of Major Influence, R:</td>
</tr>
<tr>
<td>Offset of Inflection Point, d:</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Output for the Specified Prediction Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subsidence:</td>
</tr>
<tr>
<td>Horizontal Displacement:</td>
</tr>
<tr>
<td>Slope:</td>
</tr>
<tr>
<td>Strain:</td>
</tr>
<tr>
<td>Curvature:</td>
</tr>
</tbody>
</table>
### Figure 3.2 Transverse major cross-section subsidence profile for the example model

Next, the LaModel program is employed to calculate subsidence for the example model.

First of all, the LamPre module is used to create a laminated overburden model for subsidence prediction over the example panel (see Figure 3.3).
**Figure 3.3 The LamPre Module**

**Default data input**

For calibrating LaModel for subsidence, except the three critical parameters need to be calibrated, the default values for other parameters which are the same with that used for stress distribution prediction, and the mine geometry were used to create in input file for LaModel.
The first form ‘General Model Information’ for building example model is shown in Figure 3.4.

Using this form, the user can enter a project title and the general model parameters of number of seams, number of in-seam materials, and number of steps. Also, the user can select a couple of options: an off-seam plane or a fault plane. And the units system for the current model is located at the lower part of the form. Let’s begin by naming the example model. In order to distinguish this example model from others, the name of this example is defined as “seam depth & panel width”, namely 1000&1500. In the ‘General Parameters’ section, one seam, ten different in-seam materials and one step should be entered. A step is any change in the mining plan, large or small. For instance, you could pull an entire pillar in one step or you could just take a single cut. Next, going down the form, the ‘Off-Seam Plane’ box for calculating displacements in the off-seam plane should be checked. Finally, the units system that will be used for the input parameters in the
upcoming forms need to be defined. For any model, the American Standard Units of feet and pounds per square inch should be selected.

Click the next form button, the second form ‘seam geometry boundary conditions’ will be opened (see Figure 3.5).

![Seam Geometry Boundary Conditions Form](image)

**Figure 3.5 ‘Seam geometry boundary conditions’ form**

In this form, there are three sections: ‘Seam Geometry’, ‘Seam Location’, and ‘Seam boundary Conditions’. In the ‘Seam Geometry’ section, the element width will be determined in the following “Element Sizing” part. In order to determine the number of elements in X and Y axes, the overall dimensions of mine model need to be determined. The number of elements in X and Y axes are equal to the ratios of total model width and length to element width, respectively. The total model width and length are equal to two times of boundary pillar width plus panel width and panel length, respectively. The relationship between the total model dimensions and the panel dimensions is shown in Figure 3.6.

In the ‘Seam Location’ section, the ‘X coordinate of grid origin’ ($O_X$) and ‘Y coordinate of
grid origin’ (Oy) are all kept default values, 0. That means the origin of model is coincident with the origin of the coordinate system. In the example model, 1000 ft of overburden depth and 5 ft of seam thickness are input for calculation.

In the ‘Seam Boundary Conditions’ section, there are two types seam boundary conditions, Rigid and Symmetric, for each seam edge. The seam boundary conditions determine the nature of the seam convergence at the edge of the model. When the boundary is ‘Rigid’, in this case the boundary seam material will have zero convergence and will effectively support the roof around the grid’s edge. However, when the ‘Symmetric’ boundary condition is applied, the convergence of the material outside of the grid is a mirror image of the convergence inside the grid. The convergence of the materials outside and inside of the grid have the same value. In order to obtain large seam convergence value, the ‘Symmetric’ boundary condition is applied to all four panel edges through all models in this thesis.
For the third form ‘Overburden/RockMass Parameters’ (see Figure 3.8), the parameters control the property of the laminated overburden model. The default values for all parameters in this form will be unchanged except for the value of lamination thickness. The optimum lamination thickness employed to predict subsidence for a model will be calibrated in ‘Calibrating rock mass stiffness’ later. On the right of the form, there is a button for opening the ‘Lamination Thickness Wizard’. This wizard helps calibrate a lamination thickness to match a desired abutment extent. Since the objective of this thesis is calibrating LaModel for subsidence, the wizard is beyond the scope of...
For the fourth form ‘Wizard for Defining In-Seam Material Models’, there are three wizards for developing properties for ‘Elastic-Plastic’ coal, ‘Strain-Softening’ coal, or ‘Strain-Hardening’ gob. The ‘Elastic-Plastic’ wizard defines the coal properties as one ‘Linear-Elastic’ material model (A) for in-situ coal and another ‘Elastic-Plastic’ material model (B-I) for yield zone coal (see Figure 3.9). The ‘Strain-Hardening’ wizard defines the gob property as ‘Strain-Harding’ material model (J) (see Figure 3.10). The optimum final gob modulus used to predict subsidence for a mine model will be calibrated in ‘Calibrating gob stiffness’ later.
The fifth form is ‘Program Control Parameters’ (see Figure 3.11). In the ‘control options’ section, the four parameters generally do not need to be changed and use their default values. However, for a model, when the lamination thickness is less than or equal to the element width, the ‘Over-Relaxation Factor’ needs to be reduced in order to make the seam displacement converge. In the ‘solution options’ section, in order to include a “mirror-image” seam in the model which essentially double the calculated surface displacements, the ‘include free surface effects’ box should be checked.
The last form is ‘Off-Seam Plane Characteristics’ (see Figure 3.12). In the ‘Off-Seam Grid Geometry’ section, there are three parameters need to be input. In order to compare the subsidence outputs from LaModel and CISPM over the same surface points, the first point and the interval between two adjacent points in LaModel should be the same with that in CISPM. The algorithm for calculating the interval in CISPM can be established as:

$$\Delta = \frac{0.4H + W}{50} \quad (3.1)$$

The start point is set as 0.2H far away from the left panel edge outward the panel.

In the example model, substitute 1000 ft of overburden depth and 1500 ft of panel width into Equation 3.1 and the interval can be determined as 38 ft, equivalent to grid block size (GBS).
In a transverse major cross-section subsidence profile in CISPM, 51 surface points were used to divide the subsidence profile evenly. Therefore, the ‘Number of Grid Points in X Axis’ (GNX) should be input as 51, the same with that in CISPM. In any model the panel length is equal to 7 times of the panel width, a reasonable value of 301 is selected for ‘Number of Grid Points in Y Axis’ (GNY).

In the ‘Off-Seam Grid Location’ section, the origin of off-seam coordinate system need to be determined based on the seam coordinate system. Two algorithms are developed to determine the origin of the off-seam grid coordinate system. The ‘X coordinate of Grid Origin’ (O’X) and ‘Y coordinate of Grid Origin’ (O’Y) can be calculated as:

\[
O'_X = O_X + \frac{TMW - GBS \times GNX}{2}
\]

\[
O'_Y = O_Y + \frac{TML - GBS \times GNY}{2}
\]

\[\text{Figure 3.12 ‘Off-Seam Plane Characteristics’ form}\]
**Boundary pillar sizing:**

For LaModel, as a boundary element program, the boundary effect is critical to the calculation accuracy of displacement and stress of mine model. If the grid boundary pillar size is not sufficient, stress and displacement effects from the boundary of the mine grid can cause errors in the numerical calculations at the center of the grid. In order to help eliminate the boundary effect on the critical areas of the model, a sufficient boundary size is needed around the edges of the model. For the boundary zone in the model, it could be assumed to a solid coal pillar. In LaModel analysis, it should be noted that too narrow a boundary pillar may not effectively eliminate the boundary effect; while an overly boundary pillar may eliminate the boundary effect but will require a much longer running time without improving the model’s accuracy. Therefore, to balance the numerical calculation accuracy and program running time, an optimum boundary pillar size need to be determined (Zhang, 2014).

According to field observations, the width of the side abutment load is related to the overburden thickness. The empirical relationship can be expressed as follows (Peng & Chiang, 1984):

\[ D = 9.3\sqrt{H} \]  \hspace{1cm} (3.4)

In the LaModel calibration process for the width of abutment load, it also gives the overburden stiffness a similar influence zone. In order to be a bit conservative, the minimum boundary pillar width is set as two times the single empirical abutment load extent (Zhang, 2014):

\[ D' = 2D = 18.6\sqrt{H} \]  \hspace{1cm} (3.5)

In this example model, the boundary pillar width can be determined as:

\[ D' = 18.6\sqrt{H} = 18.6 \times \sqrt{1000} = 588 \text{ ft} \]  \hspace{1cm} (3.6)
**Mine Model Sizing:**

As shown in Figure 3.7, the total model dimensions can be determined by the boundary pillar width and the panel dimensions. The total model width (TMW) and length (TML) can be calculated as two times boundary pillar width plus the panel width and length, respectively:

\[
\text{TMW} = 2D' + W \\
\text{TML} = 2D' + L
\]  

(3.7)  

(3.8)

In this example model, the mine model dimensions can be determined as:

\[
\text{TMW} = 2D' + W = 2 \times 588 + 1500 = 2676 \text{ ft} \\
\text{TML} = 2D' + L = 2 \times 588 + 10500 = 11676 \text{ ft}
\]  

(3.9)  

(3.10)

**Element sizing:**

For a model, according to the total model dimensions and the lamination thickness, a reasonable element width will be determined. For calibrating LaModel for subsidence, in order to obtain reasonable subsidence data, relative thin lamination thicknesses compared to the lamination thicknesses used for stress distribution calculation should be used to soften the overburden stiffness. When a very thin lamination thickness (less than 10 ft) is determined, the element width cannot be too large (greater than 10 ft). The combination of a large element width and a thin lamination thickness in a model results in seam displacement diverging. There is an assumption that when a lamination thickness is less than or equal to 10 ft in a model, the element width of 5 ft will be adopted, however, when a lamination thickness is greater than 10 ft, the element width of 10 ft will be employed.

In the example model, since the TMW and TML are greater and optimum lamination thickness cannot be determined, 10 ft of element width will be first selected to use. In Figure 3.5, the values for ‘Number of Elements in X axis’ and ‘Number of Elements in Y axis’, which are equivalent to
the ratios of the TMW and TML to the element width, respectively, have to be integer multiples of 10, so the TMW and TML calculated before need to be modified. The modified TMW and TML should be equal to or greater than the calculated ones before. Therefore, the TMW and TML should be modified as 2700 ft and 11700 ft, respectively.

**Grid generation:**

Based on the previously calculated total model dimensions and determined element width in this model, the numbers of element in X axis (ENX) and Y axis (ENY) can be determined as:

\[
\text{ENX} = \frac{\text{TMW}}{\text{EW}} = \frac{2700}{10} = 270
\]

\[
\text{ENY} = \frac{\text{TML}}{\text{EW}} = \frac{11700}{10} = 1170
\]

For calibrating LaModel for subsidence with an edge offset, the optimum offset distance will be determined later in ‘Calibrating panel offset distance’.

Apply the defined in-seam material models into the grid. Specifically, input the boundary pillars as solid coal material and define the panel area as gob material (see Figure 3.7).

**Yield zone depth:**

In a mine model, an appropriate set of material properties for a yield zone will be applied to the coal pillars. This yield zone provides a stress gradient on the pillar consistent with the Bieniawski pillar strength formula (Mark & Chase, 1997). This algorithm will automatically assign the coal elements with associated in-seam coal properties based on its location in the pillar in relation to the nearest opening. The Mark formula implies a stress gradient within the pillar, such that the vertical stress at a point ‘x’ distance inside the pillar is a function of the in-situ coal strength (\(S_i\)) and the pillar height (\(h_p\)) of the pillar shown as:
\[
\sigma_v = S \left( 0.64 + 2.16 \frac{x}{h_p} \right)
\] (3.13)

In this thesis, a default value (40 ft) for yield zone depth which should be more than adequate for all models will be used.

**Input File Generation:**

With all of the pre-processing steps completed and mine grid developed, click the ‘Save’ button to produce the input file for the LaModel module. The input file is named as “*.INP” where the “*” represents the input file base name for the project.

**The LaModel Module:**

The LaModel module (see Figure 3.13) solves the model for in-seam displacements and stresses, and surface subsidence. Four LaModel output files are produced. In the “*.F1” file, the first column lists seam convergence for each seam element which will be used for surface subsidence calculation. Another output file, “*.OF” file, includes the calculated vertical subsidence, slope and strain for each surface block. In the “*.CHR” file, it shows the seam layout same as the grid code created in ‘Edit-Grids’ form. The final “*.OUT” file which is generated for the convenience of debugging and further analysis includes all input information and numerical calculation information.
The LamPlt Module:

Among the output files, the “*.OF” file contains the subsidence data and is fully compatible with the LamPlt module. The LamPlt can read the data and show them on plots (see Figure 3.14). However, the “*.OF” file can also be manually analyzed. The “Stability Mapping” program is employed to extract the calculated subsidence data from the output file and calculate the least-square error between CISPM and LaModel subsidence data.
3.1.1.2 Critical parameters calibration

3.1.1.2.1 Calibrating gob stiffness

In a LaModel analysis with gob areas, an accurate stiffness for the gob (in relation to the stiffness of the roof) is critical to accurately calculating pillar stresses and safety factors. However, its stiffness accuracy is also critical to calculate seam convergence and surface subsidence accurately. The relative stiffness of the gob determines how much overburden weight is carried by the gob; and therefore, not carried by the surrounding pillars. This means that a stiffer gob which carries more load and the surrounding pillars carry less gives less seam convergence, while a softer gob which carries less load and the surrounding pillars carry more produces gives more seam convergence. In a LaModel analysis for accurately calculating surface subsidence, it is imperative that the gob stiffness be calibrated with the best available information and using good engineering judgment (Heasley, 2008).

In past laboratory tests, it was determined that gob materials generally follow an exponentially hardening stress-strain curve (see Figure 2.16). This type of material curve is implemented in LaModel using the “Strain-Hardening” material model, and this material model is highly recommended for modeling gob material in LaModel. The stiffness of the gob is primarily determined by adjusting the “Final Gob Modulus” of the “Strain-Hardening” gob model. A higher final modulus gives a stiffer gob and a lower modulus value produces a softer gob material (Heasley, 2008).

The fundamental basis of this gob model is the assumption that the tangent elastic modulus of the material increases linearly with stress. The mathematical derivation of this material model is provided by Zipf (1992a, 1992b), where he found that the material stress ($\sigma$) is related to the material strain ($\varepsilon$) by the following equation:
In order to calculate the final modulus for gob material, Equation 3.14 should be converted to the following form:

$$\sigma = \left[ \frac{E_i \sigma_u}{E_f - E_i} \right] \left[ e^{\left( \frac{E_f - E_i}{n \sigma_u} \right) \varepsilon} - 1 \right]$$  \hspace{1cm} (3.14)

For specifying the final gob modulus ($E_f$) in Equation 3.15, the required parameters are the initial tangent modulus ($E_i$), the ultimate stress ($\sigma_u$), the gob height factor ($n$), the gob strain ($\varepsilon$) and the gob stress ($\sigma$) at the given gob strain ($\varepsilon$). In LaModel, the default values for the initial tangent modulus, the ultimate stress and the gob height factor are 100psi, 4000psi and 1, respectively (Heasley & Barton, 1998). So as long as the values of the gob strain and the corresponding gob stress are known, the final gob modulus can be calculated.

For a supercritical panel, the gob strain ($\varepsilon$) occurring in the gob directly under the maximum possible subsidence in the flat middle of the surface subsidence trough is equal to the maximum possible, or “true” subsidence factor ($a$) and the corresponding gob stress ($\sigma$) is essentially equal to the overburden stress ($q$), which is a function of the overburden density ($\gamma$) and the depth ($H$).

First of all, a true subsidence factor should be determined for a model. In CISPM, an empirical formula which has been proven to be good for most of the U.S. coal fields for calculating the true subsidence factor was derived by Luo (Luo, Peng et al., 2008) (see Equation 2.6). Another empirical formula for calculating apparent subsidence factor for central Appalachian coal fields which contain much higher percent of hard rock strata has been developed by Karmis, Goodman
et al (1984) (see Equation 2.7). It includes a variable of percent of hard rock which controls the stiffness of overburden strata and the calculated apparent subsidence factor needs to be converted to its corresponding true subsidence factor by Equation 3.16 or just simply input the ratio of panel width to overburden depth as 1.4 into Equation 2.7 regardless of the actual ratio of panel width to overburden depth.

\[ a = a' \sqrt{\frac{W}{W_c} \frac{L}{W_c}} \]  \hspace{1cm} (3.16)

If the ratio of panel width to critical width or the ratio of panel length to critical width is greater than 1, set it to 1.

In the example model, since there is not hard rock in the overburden strata, the empirical formula developed by Luo should be used to determine the true subsidence factor. Substitute 1000 ft of overburden depth into Equation 2.6, the true subsidence factor can be calculated as:

\[ a = 1.9381(H + 23.4185)^{-0.1884} = 1.9381 \times (1000 + 23.4185)^{-0.1884} = 0.53 \]  \hspace{1cm} (3.17)

The overburden stress can be calculated as:

\[ \sigma_q = \gamma H = 1.125 \times 1000 = 1125 \text{ psi} \]  \hspace{1cm} (3.18)

Since the gob strain (\( \varepsilon \)) is equal to true subsidence factor (\( a \)) and the corresponding gob stress (\( \sigma \)) is essentially equal to the overburden stress (\( \sigma_q \)), Equation 3.15 can be rewritten as:

\[ \sigma_q - \left[ \frac{E_i \sigma_u}{E_f - E_i} \right] \left[ e^{\left( \frac{E_f - E_i}{n\sigma_u} \right) a} - 1 \right] = 0 \]  \hspace{1cm} (3.19)

In the example model, substitute 1125 psi of overburden stress, 0.53 of true subsidence factor, 100 psi of initial tangent modulus, 4000 psi of the ultimate stress and 1 of the gob height factor into Equation 3.19, the final gob modulus can be calculated as 35160 psi and input into the final
3.1.1.2.2 Calibrating rock mass stiffness

To best understand how the rock mass properties affect the LaModel result, the fundamental differential equation (see Equation 3.20) of the laminated overburden model needs to be analyzed:

\[
\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} = \frac{4\sqrt{3(1-v^2)}}{Et} \sigma_i \tag{3.20}
\]

In this equation, the stiffness of the rock mass in LaModel is primarily determined by two parameters, the rock mass modulus (E) and the rock mass lamination thickness (t). Increasing the modulus or increasing the lamination thickness of the rock mass will increase the stiffness of the overburden. With a stiffer overburden, the extent of the abutment stresses will increase, the convergence and stress over the gob areas will decrease and the multiple seam stress concentrations will be smoothed over a larger area. Since changes in either the modulus or lamination thickness will cause a similar response in the model, the rock mass modulus is typically held constant while the lamination thickness is varied to calibrate the model (Heasley, 2008).

When determining an optimum lamination thickness to use in a model, the interactive trial and error process should be employed during the calibration process. The lamination thickness was initially estimated. The LaModel was run to calculate the surface subsidence, and calculate the least-square error between subsidence data from CISPM and LaModel. Then the lamination thickness was adjusted to improve the fit, the program was run again, etc. This cycle continues until the calculated least-square error reaching the minimum value.

In the example model, the first estimated value of 15 ft was selected for the lamination thickness. The optimum panel offset distance should be used in the example model will be calibrated later. In the calibration process for lamination thickness, the panel offset distance (d’) is
hold constant as 60% of the offset distance of inflection point.

\[ d' = 0.6d = 0.6 \times (0.382075 \times 0.999253^{1000}) \times 1000 = 108.6 \text{ ft} \]  

(3.21)

Since the panel has to be offset by changing gob element to in-situ coal element along each side of the panel, the panel offset distance should be integer multiples of element width. Therefore, 110 ft of panel offset distance should be used in the example model and the actual ratio of the panel offset distance to the corresponding offset distance of inflection point is 60.8%.

After running the first model, calculate the least-square error. Then, adjust the lamination thickness to 18 ft, 21 ft and 24 ft and run the models again. Summarize the above results, the following table can be obtained.

**Table 3.3 Least-square errors with different lamination thicknesses**

<table>
<thead>
<tr>
<th>( t )</th>
<th>15</th>
<th>18</th>
<th>21</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error ( \frac{p}{p_0} )</td>
<td>0.361</td>
<td>0.125</td>
<td>0.055</td>
<td>0.103</td>
</tr>
<tr>
<td>60.8% (110)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For the example model, the lamination thickness was calibrated from 15 ft to 24 ft increasing with a constant interval of 3 ft. The sum of the squares of the errors initially decreases, but turns to increase after reaching a minimum at a certain lamination thickness.

3.1.1.2.3 Calibrating panel offset distance

The panel offset distance affects the location of the subsidence trough in relation to the edge of the mined gob. The greater the panel offset distance is, the further the subsidence trough wall moves inward panel center. In order to fit well to the subsidence profile in CISPM, the optimum panel offset distance for the example model need to be determined in LaModel. The example model with the input parameters combinations of the panel offset distance which is 110 ft (60.8%) and a set of lamination thicknesses has been calculated previously. Next, the combinations of 130
ft (71.8%) and 150 ft (82.9%) of panel offset distance and the same set of lamination thicknesses for the example model need to be input for subsidence calculation. The results are shown in Table 3.4.

**Table 3.4 Least-square errors with different lamination thicknesses and panel offset distances**

<table>
<thead>
<tr>
<th>Error (p)</th>
<th>t (x1)</th>
<th>15</th>
<th>18</th>
<th>21</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>60.8% (110)</td>
<td>0.361</td>
<td>0.125</td>
<td>0.055</td>
<td>0.103</td>
<td></td>
</tr>
<tr>
<td>71.8% (130)</td>
<td>0.114</td>
<td>0.029</td>
<td>0.089</td>
<td>0.249</td>
<td></td>
</tr>
<tr>
<td>82.9% (150)</td>
<td>0.378</td>
<td>0.472</td>
<td>0.682</td>
<td>0.973</td>
<td></td>
</tr>
</tbody>
</table>

Looking at Table 3.4, the minimum value of least-square error is 0.029 with given 18 ft of lamination thickness and 130 ft of panel offset distance. Convert the minimum least-square error and its surrounding eight values with their corresponding panel offset distances and lamination thicknesses into the following table.

**Table 3.5 Converted dataset for Table 3.4**

<table>
<thead>
<tr>
<th>t (x1)</th>
<th>p (x2)</th>
<th>Error (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.608</td>
<td>0.361</td>
</tr>
<tr>
<td>15</td>
<td>0.718</td>
<td>0.114</td>
</tr>
<tr>
<td>15</td>
<td>0.829</td>
<td>0.378</td>
</tr>
<tr>
<td>18</td>
<td>0.608</td>
<td>0.125</td>
</tr>
<tr>
<td>18</td>
<td>0.718</td>
<td>0.029</td>
</tr>
<tr>
<td>18</td>
<td>0.829</td>
<td>0.472</td>
</tr>
<tr>
<td>21</td>
<td>0.608</td>
<td>0.055</td>
</tr>
<tr>
<td>21</td>
<td>0.718</td>
<td>0.089</td>
</tr>
<tr>
<td>21</td>
<td>0.829</td>
<td>0.682</td>
</tr>
</tbody>
</table>

Input the above data into MatLab program and use the ‘Curve Fitting Tool’ to create a full quadratic regression equation (see Equation 3.22). The lamination thickness and the percent are
the two independent variables and the least-square error is the corresponding dependent variable.

\[ y = a + bx_1 + cx_2 + dx_1^2 + ex_2^2 + fx_1x_2 \]  \hspace{1cm} (3.22)

Where the coefficients \(a, b, c, d, e\) and \(f\) are equal to 18.7961, -0.6164, -38.2515, 0.007898, 21.8952 and 0.4602, respectively.

Use the ‘fminsearch’ function in MatLab to find the minimum value of the least-square error and its corresponding lamination thickness and percent. After analyzing the above equation, the optimum lamination thickness and the percent for the example model can be determined as 19.6 ft and 66.8%, respectively.

Plot the transverse major cross-section subsidence profiles calculated by LaModel with calibrated critical parameters and CISPM; and it can be seen that the calibrated LaModel can give a good surface subsidence prediction for the example model compared with CISPM result.

![Subsidence profiles for the example model from LaModel and CISPM](Figure 3.15)

**Figure 3.15** Subsidence profiles for the example model from LaModel and CISPM

So, for a range of panel depths (see Table 3.6), a number of supercritical width (1.5H) and extended length (7W) panels were modeled using the same calibrating procedures stated before. With these models, the optimum combination of panel offset distance and lamination thickness
which minimized the least-square error between the CISPM predicted subsidence and the LaModel calculated subsidence was determined through an interactive trial-and-error process. Since both the offset distance and the lamination thickness greatly affect the subsidence at the edge of the panel, these two parameters need to be simultaneously optimized. These optimum values of the offset distance and lamination thickness were the plotted against depth and a line was fitted through the data to be used in design.

Table 3.6 Optimum $t$ and $p$ for supercritical models with an edge offset

<table>
<thead>
<tr>
<th>Panel Model</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overburden Depth (ft)</td>
<td>300</td>
<td>500</td>
<td>700</td>
<td>900</td>
<td>1100</td>
<td>1300</td>
</tr>
<tr>
<td>Panel Width (ft)</td>
<td>450</td>
<td>750</td>
<td>1050</td>
<td>1350</td>
<td>1650</td>
<td>1950</td>
</tr>
<tr>
<td>Optimum Percent</td>
<td>62.4</td>
<td>61.5</td>
<td>62.9</td>
<td>64.8</td>
<td>66.7</td>
<td>68.7</td>
</tr>
<tr>
<td>Optimum LamThk (ft)</td>
<td>1.03</td>
<td>4.43</td>
<td>9.8</td>
<td>15.99</td>
<td>22.86</td>
<td>29.31</td>
</tr>
</tbody>
</table>

The equation for the optimum lamination thickness ($t$) that provides the best fit to the subsidence for supercritical panels with an edge offset is shown below as Equation 3.23, and graphed in Figure 3.16.

$$t_{\text{super-w}} = 106.9 - 110.3e^{-5.07 \times 10^{-6} H^{1.35}} \quad (3.23)$$
The equation for the percent ($p$) of the CISPM edge effect that pairs with the optimum lamination thickness for supercritical panels is shown below as Equation 3.24 and graphed in Figure 3.17.

$$\eta = 108.3 - 47.3e^{-2.5 \times 10^{-7} H^{47}}$$  \hspace{1cm} (3.24)
3.1.1.3 Summary and Conclusion

In this section, a set of supercritical panels were modeled to calibrate the LaModel for subsidence prediction and the laminated overburden model with calibrated properties was demonstrated to have the potential to provide fairly accurate subsidence predictions for supercritical panels with an edge offset. For the supercritical panels, the range of overburden depth is from 300 ft to 2500 ft; the panel width is 1.5 times of its overburden depth and the panel length is 7 times of its panel width. The seam thickness is assumed as 5 ft and the hardrock percent in overburden strata is assumed as 0.

For all supercritical panels with an edge offset, three critical parameters have been calibrated. Firstly, after inputting a true subsidence factor, a corresponding overburden stress and the default values for the initial modulus, the ultimate vertical stress and the gob height factor, Equation 3.19 can calculate a reasonable final gob modulus for subsidence prediction. This final gob modulus for subsidence calculation is always less than that for stress distribution calculation.
with every things being equal. The less final gob modulus gives soft gob stiffness which is critical to get more seam convergence and surface subsidence.

Secondly, looking at Figure 3.16, the observation that lamination thickness is almost linearly increasing with overburden depth can be made. Also, a fitting equation (see Equation 3.23) is determined to describe the relationship between lamination thickness and overburden depth.

Finally, looking at Figure 3.17, the optimum percent almost increases linearly with overburden depth. A fitted equation (see Equation 3.24) is also determined.

3.1.2 Subcritical Panels

3.1.2.1 Model development and critical parameters calibration

In this section, calibrating LaModel for subsidence prediction over subcritical panels with an edge offset is going to be studied. In general, a subcritical panel is referred as its panel length and panel width both less than the critical width. However, there is an assumption that in any model the panel length is equal to 7H and the panel length is always greater than the critical width. Therefore, a panel is considered as subcritical only when its width is less than the critical width. The critical width is the minimum width of a square underground opening that the surface movements and deformations above which can be fully developed. In general, the range of the critical width is around from 1.1 to 1.4 times overburden depth. In CISPM, the default value of critical width was found to be 1.2H for the Pittsburgh coal seam and most U.S. coal fields (Luo, 2016a). In order to consistent with the empirical value for critical width used in CISPM, the default value of 1.2H is also utilized for calibrating LaModel for subsidence.

For calculating subsidence with LaModel for subcritical panels, the final gob modulus, offset distance and lamination thickness parameters are all interdependent, and determining a unique combination of these parameters that provides the optimum subsidence fit with CISPM is nearly
impossible without constraining the parameters in some respect. Therefore, the final gob modulus and associated maximum possible subsidence factor for subcritical panels are assumed to be the same as for a supercritical panel at that depth and then the final gob modulus can be calculated with Equation 3.19. Similarly, edge offset distance for a subcritical panel is assumed to be the same as for a supercritical panel at that depth, and then the offset distance can be calculated with Equations 3.24. However, the offset distances for a deep and narrow panel can be larger than the panel width; therefore, the calculation of the subsidence for subcritical panels with offset is only valid for panel widths greater than 0.5H (and less than 1.2H).

Once the final gob modulus and the offset distance for a subcritical panel had been determined, the optimum lamination thickness was then determined using the same type of interactive trial-and-error process that was used for the supercritical panels.

For this subcritical lamination thickness optimization, a suite of models was developed with overburden depths ranging from 300 to 2500 ft, panel widths ranging from 300 to 1500 ft, seam thickness being 5 ft and hardrock percent being 0. Also, the procedures for generating input files, solving input files and calculating the least-square errors for subcritical panels are the same with that for supercritical panels. With these models, the lamination thickness was varied until the subsidence predicted by LaModel minimized the least-square error with the CISPM subsidence. These optimum values of lamination thickness (Table 3.7) were listed below.

For the subcritical panels with an edge offset, the optimum lamination thickness was found to be essentially the same as that found for the supercritical panels with an edge offset; therefore, Equation (3.23) can be used for both the subcritical and supercritical panels with an edge offset.
Table 3.7 Optimum lamination thicknesses for subcritical models with an edge offset

<table>
<thead>
<tr>
<th>Overburden Depth (ft)</th>
<th>Subcritical Panel Width (ft)/Lamination Thickness (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>300/0.96, 360/0.98</td>
</tr>
<tr>
<td>500</td>
<td>300/4.58, 500/3.91, 600/4.43</td>
</tr>
<tr>
<td>700</td>
<td>500/8.53, 700/9.18, 840/9.58</td>
</tr>
<tr>
<td>900</td>
<td>500/14.15, 700/15.19, 900/15.82, 1080/16.17</td>
</tr>
<tr>
<td>1100</td>
<td>700/21.54, 900/21.96, 1100/22.13, 1300/22.23, 1320/22.50</td>
</tr>
<tr>
<td>1300</td>
<td>700/28.18, 900/28.74, 1100/29.19, 1300/29.17, 1500/29.25</td>
</tr>
<tr>
<td>1500</td>
<td>900/35.99, 1100/35.89, 1300/36.02, 1500/36.13, 1700/36.14</td>
</tr>
<tr>
<td>1700</td>
<td>900/42.42, 1100/42.28, 1300/42.23, 1500/42.25, 1700/42.29</td>
</tr>
<tr>
<td>1900</td>
<td>1100/48.19, 1300/48.04, 1500/47.97, 1700/47.97, 1900/47.95</td>
</tr>
<tr>
<td>2100</td>
<td>1100/53.60, 1300/53.53, 1500/53.43, 1700/53.35, 1900/53.35</td>
</tr>
<tr>
<td>2300</td>
<td>1300/60.57, 1500/60.30, 1700/60.13, 1900/60.10, 2100/60.09</td>
</tr>
<tr>
<td>2500</td>
<td>1300/65.47, 1500/65.23, 1700/65.01, 1900/64.87, 2100/64.96</td>
</tr>
</tbody>
</table>

3.1.2.2 Summary and Conclusion

As stated above, a suite of subcritical panels have been modeled to calibrate LaModel for subsidence prediction. For those subcritical panels, the range of overburden depth is from 300 ft to 2500 ft; the panel width is ranging from 300 ft to 1.2H and the panel length is defined as 7W. The seam thickness is assumed as 5 ft and the hardrock percent in overburden strata is set as 0. It should be noted that only subcritical panels which panel widths are greater than 0.5H (and less than 1.2H) were calibrated since the offset distance for a deep and narrow panel can be larger than the panel width.

During the calibration process for a subcritical panel, the final gob modulus and associated maximum possible subsidence factor are assumed to be the same as for a supercritical panels at that depth and then the final gob modulus can be calculated with Equation 3.19. Similarly, the edge offset distance is assumed to be the same as for a supercritical panel at that depth, and then the offset distance can be calculated with Equations 3.24. After using the same calibration method found out the optimum lamination thickness for subcritical panels, those optimum lamination
thickness was found to be essentially the same as that found for the supercritical panels with an edge offset. Therefore, Equation 3.23 can be used for both the subcritical and supercritical panels with an edge offset.

### 3.2 Calibrating LaModel for Subsidence without an Edge Offset

In the previous part, using an optimized offset distance is able to provide a more accurate prediction of the surface subsidence with LaModel; however, this offset distance distorts the calculation of displacement and stresses at the seam level. In practice, these seam level displacements and stresses may be important to the user; therefore, in order to allow the user to use without an edge offset subsidence prediction method, calibrating LaModel for subsidence will be conducted over panels without an edge offset here. Without an edge offset means the panels will use the actual mined out area as the computing area. In order to give a good subsidence prediction for an actual mined out panel, the optimum gob stiffness and rock mass stiffness need to be found which minimized the least-square error between the CISPM and the LaModel predicted subsidence curves.

#### 3.2.1 Calibrating Gob Stiffness.

In this part, the panels are not going to be separated to supercritical and subcritical when calibrating gob stiffness. The method for calibrating gob stiffness is the same as that stated before; therefore, Equation 3.19 is used to calculate the final gob modulus for panels without an edge offset, both supercritical and subcritical.

#### 3.2.2 Calibrating Rock Mass Stiffness.

For calibrating rock mass stiffness, supercritical and subcritical panels are separated into two different groups and calibrated individually. First of all, the rock mass stiffness is going to be calibrated for supercritical panels. The information of supercritical panels is the same as the
supercritical panels with an edge offset. The range of overburden depth is from 300 ft to 2500 ft; the panel width is assumed as 1.5H and the panel length is set as 7W. The seam thickness is assumed as 5 ft and the hardrock percent in overburden strata is set as 0.

Repeat the same calibration method to find the optimum lamination thickness for all supercritical panels.

With these models, the optimum lamination thickness (see Table 3.8) which minimized the least-square error between the CISPM predicted subsidence and the LaModel calculated subsidence was determined through an interactive trial-and-error process.

**Table 3.8 Optimum lamination thicknesses for supercritical models without an edge offset**

<table>
<thead>
<tr>
<th>Panel Model</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overburden Depth (ft)</td>
<td>300</td>
<td>500</td>
<td>700</td>
<td>900</td>
<td>1100</td>
<td>1300</td>
</tr>
<tr>
<td>Panel Width (ft)</td>
<td>450</td>
<td>750</td>
<td>1050</td>
<td>1350</td>
<td>1650</td>
<td>1950</td>
</tr>
<tr>
<td>Optimum LamThk (ft)</td>
<td>4.98</td>
<td>18.01</td>
<td>34.64</td>
<td>51.58</td>
<td>66.16</td>
<td>77.01</td>
</tr>
</tbody>
</table>

The equation for the optimum lamination thickness \( t \) that provides the best fit to the subsidence for supercritical panels with an edge offset is shown below as Equation 3.25, and graphed in Figure 3.18.

\[
t_{\text{super-w/o}} = 91.6 - 94.2e^{-4.8 \times 10^{-7} H^{1.12}}
\]  

(3.25)
Figure 3.18 Optimum lamination thickness for supercritical panels without an edge offset

Next, the rock mass stiffness is going to be calibrated for subcritical panels. The information of subcritical panels is the same as subcritical panels with an edge offset. The range of overburden depth is from 300 ft to 2500 ft; the panel width is ranging from 300 ft to 1.2H and the panel length is defined as 7W. The seam thickness is assumed as 5 ft and the hardrock percent in overburden strata is set as 0. Also, the calculation of the subsidence for subcritical panels with an edge offset is only valid for panel widths greater than 0.5H (and less than 1.2H).

The final gob modulus for subcritical panels are assumed to be the same as for a supercritical panel at that depth and then the final gob modulus can be calculated with Equation 3.19. Once the final gob modulus for a subcritical panel had been determined, the optimum lamination thickness was then calculated using the same type of interactive trial-and-error process that was used for the supercritical panels (see Table 3.9).
Table 3.9 Optimum lamination thicknesses for subcritical panels without an edge offset

<table>
<thead>
<tr>
<th>Overburden Depth (ft)</th>
<th>Subcritical Panel Width (ft) / Lamination Thickness (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>300/3.99 360/3.92</td>
</tr>
<tr>
<td>500</td>
<td>300/27.90 500/14.46 600/14.76</td>
</tr>
<tr>
<td>700</td>
<td>500/32.59 700/30.71 840/32.22</td>
</tr>
<tr>
<td>900</td>
<td>500/59.86 700/49.75 900/48.35 1080/49.93</td>
</tr>
<tr>
<td>1100</td>
<td>700/65.77 900/63.18 1100/62.53 1300/64.44 1320/66.67</td>
</tr>
<tr>
<td>1300</td>
<td>700/94.90 900/77.05 1100/73.31 1300/74.45 1500/76.01</td>
</tr>
<tr>
<td>1500</td>
<td>900/90.04 1100/81.52 1300/81.20 1500/82.61 1700/83.59</td>
</tr>
<tr>
<td>1700</td>
<td>900/100.45 1100/88.03 1300/85.54 1500/86.34 1700/87.28</td>
</tr>
<tr>
<td>1900</td>
<td>1100/93.42 1300/89.10 1500/88.76 1700/89.37 1900/90.61</td>
</tr>
<tr>
<td>2100</td>
<td>1100/98.33 1300/92.27 1500/90.62 1700/90.67 1900/92.67</td>
</tr>
<tr>
<td>2300</td>
<td>1300/94.95 1500/92.42 1700/91.83 1900/93.61 2100/93.69</td>
</tr>
<tr>
<td>2500</td>
<td>1300/97.84 1500/93.90 1700/92.60 1900/94.49 2100/94.69</td>
</tr>
</tbody>
</table>

The equation for the optimum lamination thickness that provides the best fit to the subsidence for subcritical panels without an edge offset is shown below as Equation 3.26 and graphed in Figure 3.19. In Equation 3.26, the first part represents the effects of the ratio of panel width to seam depth on lamination thickness while the second part is the equation for calculating lamination thickness for supercritical models with panel offset.

\[
t_{\text{sub--w/o}} = \left( 128.5 - 127.4 \left( \frac{W}{H} \right)^{5.0 \times 10^{-4}} \right) t_{\text{super--w/o}} \quad (3.26)
\]
3.2.3 Summary and Conclusion

As stated above, numerous models have been performed to calibrate LaModel for subsidence prediction over panels without an edge offset. In all panels, the range of overburden depth is from 300 ft to 2500 ft. The seam thickness is assumed as 5 ft and the hard rock percent in overburden strata is set as 0. In supercritical models, the panel width is 1.5H and the panel length is 7W. However, in subcritical models, the panel width is ranging from 300 ft to 1.2H and the panel length is 7W. It should be noted that the subcritical panels which the panel widths are less than 0.5H were neglected to calibrate since the panels are too narrow compared to their seam depths.

During the calibration process for models without an edge offset, the calibration methods for determining the optimum final gob modulus and lamination thickness are the same as that for models with an edge offset.

After determining the optimum lamination thickness for each panel model, two fitting
equations for the optimum lamination thickness that gives the best fit to the subsidence for supercritical and subcritical panels are shown below as Equation 3.27 and 3.28, respectively.

\[
t_{\text{super-w/o}} = 91.6 - 94.2e^{-4.8 \times 10^{-7} H^{1.12}}
\]  
(3.27)

\[
t_{\text{sub-w/o}} = \left(128.5 - 127.4 \left(\frac{W}{H}\right)^{5 \times 10^{-3}}\right) t_{\text{super-w/o}}
\]  
(3.28)

3.3 Calibrating LaModel for Subsidence Using Measured Data

3.3.1 Calibrating Gob Stiffness Using a Measured Subsidence Factor

For a scenario where the user has a measured subsidence factor for a specific site, Equation 2.6 and 2.7 are not needed, and the measured subsidence factor can be used to determine the appropriate final gob modulus. First, it needs to be determined if the measured subsidence factor is the maximum possible subsidence factor (true subsidence factor for a supercritical panel) or the maximum observed subsidence factor (apparent subsidence factor for a subcritical panel). So the question is whether the panel is subcritical or supercritical. To determine the condition of the panel, the width, length and depth of the panel which induced the surface subsidence need to be known.

To determine whether a panel is supercritical dimension or not, the critical width \(W_c\) of the panel needs to be determined. By empirical formula, the critical width of a panel is assumed as 1.2H. When both the panel width and length are greater than the critical width, the panel is supercritical. However, when either the panel width or length is less than the critical width, the panel is subcritical.

If the panel is supercritical with both the width and length greater than the critical width, then the measured subsidence factor is a true subsidence factor \((a)\). In this case, the measured subsidence factor can be directly used in Equation 3.19 to calculate the final gob modulus.
However, if either the panel width or length is less than critical width, then the panel is subcritical and the measured subsidence is only presenting an apparent subsidence factor (a’). The apparent subsidence factor cannot be directly used to calculate final gob modulus because the corresponding peak gob load has not yet been developed due to incompletely developed caving. In order to determine the true subsidence factor for a subcritical panel, the observed apparent subsidence factor can be converted to the true subsidence factor using the Equation 3.16 (Luo, 2016a):

If the ratio of panel width to critical width or the ratio of panel length to critical width is greater than 1, set it to 1. For the subcritical panel, the maximum possible subsidence factor determined using Equation 3.16 can be used in Equation 3.19 to calculate the final gob modulus.

3.3.2 Calibrating Rock Mass Stiffness Using a Measured Angle-of-Draw

There have been an equation (Zhang & Heasley, 2013) which shows the relationships among lamination thickness, angle-of-draw, vertical distance between two interest of points and limited percent of displacement volume to calculate lamination thickness as shown:

\[
t = -\frac{\tan^2 \beta |Z| \sqrt{3(1 - v^2)}}{2\ln(1 - p)}
\]  

(3.29)

Currently, the optimum value of p is assumed to be 99.9999% in LaModel (Zhang, 2012) primarily based on the distance of stress influence. However, by using this value of p and measured angle-of-draw, the above equation does not give a reasonable lamination thickness to use to predict surface subsidence. Because the reported typical values of angle-of-draw in the surface subsidence trough ranges from 4°~45° (Peng, 1992), which is much less than the values of angle-of-draw in LaModel (Zhang, 2012) as shown in Table 3.10 that were used to get optimum overburden loading. Therefore, the percent of included displacement volume for the surface subsidence cannot be as high as 99.9999%. For this calculation the value of p which gives good subsidence angles-of-draw
as measured in the field needs to be determined.

**Table 3.10 Angles-of-draw in LaModel (Zhang, 2012)**

<table>
<thead>
<tr>
<th>Case</th>
<th>Lamination thickness, t, ft</th>
<th>Percent of displacement volume, p, %</th>
<th>Vertical distance, z, ft</th>
<th>Radius of influence, R, ft</th>
<th>Distance factor</th>
<th>Angle-of-draw, β, °</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>50</td>
<td>99.9999</td>
<td>100</td>
<td>287.02</td>
<td>2.87</td>
<td>70.79</td>
</tr>
<tr>
<td>B</td>
<td>300</td>
<td>99.9999</td>
<td>100</td>
<td>759.38</td>
<td>7.59</td>
<td>82.5</td>
</tr>
<tr>
<td>C</td>
<td>500</td>
<td>99.9999</td>
<td>100</td>
<td>907.63</td>
<td>9.08</td>
<td>83.71</td>
</tr>
</tbody>
</table>

In CISPM and LaModel, the influence function method (Knothe, 1957) is used to calculate surface subsidence. In CISPM, the influence function is based on Knothe’s theory, which is the distribution of the subsidence caused by the extraction of one unit area can be expressed by a modified normal probability distribution function. The influence function proposed by Knothe for subsidence prediction is

\[
f_s(x', y') = \frac{1}{R^2} e^{-\pi(x'^2 + y'^2)/R^2}
\]  

(3.30)

The maximum possible subsidence \( S_{\text{max}} \) is determined as mining height \( m \) times the true subsidence factor \( a \) as shown in

\[
S_{\text{max}} = ma
\]  

(3.31)

The radius of major influence \( R \) can be calculated by seam depth \( H \) times the tangent of angle-of-draw \( \beta \).

\[
R = H \tan \beta
\]  

(3.32)

However, the influence function in LaModel was derived from the frictionless laminated
overburden model proposed by Salamon (Salamon, 1962, 1989a). In this case, because the influence function is for surface subsidence prediction and the calculation is done in an infinite media, the surface subsidence, \( W \), from a unit point seam convergence can be presented as:

\[
W(x', y') = \frac{1}{8 \pi \lambda H} e^{-\left(\frac{x'^2 + y'^2}{4\lambda H}\right)}
\]  

(3.33)

The value of \( \lambda \) can be calculated as:

\[
\lambda = \frac{t}{2\sqrt{3(1 - \nu^2)}}
\]  

(3.34)

To calculate the remote displacement in an infinite half-space, LaModel uses a “mirror-image” seam which essentially double the calculated surface displacements (Heasley, 1998). After implementing “mirror-image” seam into the mathematical model, the influence function can be converted to:

\[
W(x', y') = \frac{1}{4 \pi \lambda H} e^{-\left(\frac{x'^2 + y'^2}{4\pi \lambda H}\right)}
\]  

(3.35)

Compared Equation 3.35 with Equation 3.30, it can be see that the square of radius of major influence \( R^2 \) in Knothe’s influence function is equivalent to \( 4\pi \lambda H \) in the LaModel influence function.

\[
R^2 = 4 \pi \lambda H
\]  

(3.36)

The measured angle-of-draw can be used to calculate the radius of major influence as:

\[
R = H \tan \beta
\]  

(3.37)

Substituting Equation 3.37 back into Equation 3.36, simplifying and rearranging gives:

\[
H^2 \tan^2 \beta = 4 \pi \lambda H
\]  

(3.38)

Making a substitution of:
\[\lambda = \frac{t}{2\sqrt{3(1 - \nu^2)}} \quad (3.39)\]

After rearranging and simplifying, the equation which relates the lamination thickness to the observed angle-of-draw is:

\[t = \frac{\tan^2 \beta H \sqrt{3(1 - \nu^2)}}{2\pi} \quad (3.40)\]

When the lamination thickness is calibrated for surface subsidence prediction, the vertical distance between points in Equation 3.29 is equivalent to seam depth. Then the Equation 3.29 can be converted to:

\[t = \frac{\tan^2 \beta H \sqrt{3(1 - \nu^2)}}{-2\ln(1 - p)} \quad (3.41)\]

Compared Equation 3.41 with Equation 3.40, the only difference happens in the denominators of the right side of the equations. In order to determine the value of \(p\) to calculate lamination thickness for calculating subsidence, let Equation 3.40 and Equation 3.41 equal, then simplifying gives:

\[-\ln(1 - p) = \pi \quad (3.42)\]

So:

\[p = 1 - e^{-\pi} = 0.9568 \quad (3.43)\]

That means when 0.9568 of \(p\) value be used to calculate lamination thickness in Equation 3.41, it gives the same result by using Equation 3.40.

### 3.3.3 Summary and Conclusion

In this section, calibrating final gob modulus and lamination thickness was conducted based on the measured subsidence factor and angle-of-draw.
The Equation 3.19 is still used to determining final gob modulus but Equation 2.6 and 2.7 are not needed to calculate the subsidence factor, rather the measured subsidence factor will be used to determine the appropriate final gob modulus.

For the equation determining lamination thickness, it was derived through comparing the influence functions used in LaModel and CISPM. Those two influence functions have the same form; therefore, by letting them equal, the equation for calculating lamination thickness can be determined as:

\[ t = \frac{\tan^2 \beta H \sqrt{3(1 - \nu^2)}}{2\pi} \]  

(3.44)
Chapter 4. Computer Implementation and Case Study

4.1 Computer Implementation

In the previous chapter, calibrating LaModel for surface subsidence prediction has been conducted. Three sets of empirical formulae for calculating critical input parameters were derived for three different prediction methods. In this chapter, the subroutine codes for defining the critical input parameters for surface subsidence prediction is going to be implemented into the LamPre module. This subroutine will be validated using a case study.

The overall organization of the subroutine is fairly simple. The subroutine’s primary function is calculating critical parameters for subsidence prediction. After starting the calibrating process, the users are required to choose the method they want to use to calculate those parameters. Then, the subroutine will automatically calculate the critical parameters based on the input longwall panel information, and the calculation method selected by users. Finally, the calculated values for critical parameters will be shown in each text box. The flowchart for the subroutine is shown in Figure 4.1 with an explanation of the flowchart symbol in Figure 4.2.
Figure 4.1 Flowchart of subroutine for defining subsidence parameters
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Start/End]</td>
<td>Start/End</td>
<td>Indicates the beginning or end of a program flow in a flowchart.</td>
</tr>
<tr>
<td>![Process]</td>
<td>Process</td>
<td>Indicates and processing function. This is the most common symbol in process flowcharts.</td>
</tr>
<tr>
<td>![Decision]</td>
<td>Decision</td>
<td>Indicates a decision point between two or more paths in a flowchart.</td>
</tr>
<tr>
<td>![Stored data]</td>
<td>Stored data</td>
<td>Indicates any type of stored data.</td>
</tr>
<tr>
<td>![Import/Output data]</td>
<td>Import/Output data</td>
<td>The Data flowchart shape indicates inputs to and outputs from a process. As such, the shape is more often referred to as an I/O shape than a Data shape.</td>
</tr>
</tbody>
</table>

Figure 4.2 Explanation of flowchart symbols.

4.1.1 Details of Subsidence Calculation with an Edge Offset

When the users determine to calculate surface subsidence using the “with an edge offset” method, the procedures for determining critical parameters are stated as below. First, Equation 3.23, 3.19 and 3.24 will be employed automatically to calculate the lamination thickness, the final gob modulus and the percent of edge offset in CISPM at that depth, respectively.

4.1.2 Details of Subsidence Calculation without an Edge Offset

However, if the users choose the “without an edge offset” method to predict surface subsidence, the procedures for calculating the critical parameters are listed as below. First, the subroutine will automatically calculate the value of 1.2H and compare it with the panel width. If the panel width is greater than 1.2H and the panel is supercritical, Equation 3.25 will be employed to calculate the lamination thickness; otherwise, the panel is subcritical and Equation 3.26 is going to be used. Next, Equation 3.19 will be adopted to calculate the final gob modulus regardless if the
panel is supercritical or subcritical.

4.1.3 Details of Subsidence Calculation Using Measured Data

If the method using measured data to calculate surface subsidence is selected, the procedures for calculating the critical parameters are listed as below. First, the subroutine will also automatically calculate the value of 1.2H and compare it with the panel width. If the panel width is greater than 1.2H, the panel is supercritical and the measured subsidence factor is a maximum possible (true) subsidence factor; otherwise, the measured subsidence factor is an apparent subsidence factor. The subroutine will automatically convert the apparent subsidence factor to the true subsidence factor at that depth using Equation 3.16. After determining the true subsidence factor, Equation 3.19 is going to be used to calculate the final gob modulus. Next, Equation 3.44 will be employed to calculate the lamination thickness using the measured angle-of-draw.

4.2 Case Study

In order to validate the equations presented above and analyze the accuracy and utility of the new subsidence prediction capabilities in LaModel, a case study was performed.

The location for this subsidence case study is a longwall mine in Barbour County, West Virginia. This mine started production in 1975 with continuous miners in room-and-pillar sections. In 1982, the first longwall was installed and by the time of the final subsidence monitoring in this study, the mine had successfully completed 5 longwall panels (Heasley & Barton, 1998).

The first panel at which the subsidence was investigated using the new LaModel subsidence prediction calculations is called V-1 panel, and it is actually the fifth longwall panel to be extracted at the mine. The panel is 935 ft wide and 2100 ft long with an extraction thickness of 5.9 ft and an average overburden depth of 394 ft. At the mine, there were one longitudinal line and two transverse lines of subsidence monitoring stations over the latter half of the panel (see Figure 4.3)
(Heasley & Barton, 1998). Since the panel is located in Northern Appalachia coal fields, the hardrock percent in overburden strata was assumed as 0.

Figure 4.3 Map of the V-1 panel (after Heasley, 1998)
Summarize the above information of the V-1 panel, the following table can be obtained:

### Table 4.1 Information of the V-1 panel

<table>
<thead>
<tr>
<th>Items</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Overburden Depth (H)</td>
<td>394</td>
<td>ft</td>
</tr>
<tr>
<td>Longwall Panel Width (W)</td>
<td>935</td>
<td>ft</td>
</tr>
<tr>
<td>Longwall Panel Length (L)</td>
<td>2100</td>
<td>ft</td>
</tr>
<tr>
<td>Average Mining Height (m)</td>
<td>5.9</td>
<td>ft</td>
</tr>
<tr>
<td>Percent of Hardrock (P)</td>
<td>0</td>
<td>%</td>
</tr>
</tbody>
</table>

4.2.1 Using “with an Edge Offset” Method to Calculate Subsidence over V-1 Panel

Initially, for optimum accuracy of the predicted subsidence, an offset distance was assumed; therefore, Equations 3.19, 3.23 and 3.24 were employed to calculate the final gob modulus (20,630 psi), lamination thickness (2.34 ft) and offset distance (69.3 ft) for the V-1 panel. These critical input parameters along with default values for other parameters and the mine geometry were used to create an input file for LaModel. The LaModel program then solved the model for in-seam displacements and stresses, and surface subsidence. Next, the “Stability Mapping” program was employed to extract the calculated subsidence data from the output files. Finally, the calculated (and measured), transverse and longitudinal subsidence results were plotted in Figure 4.4 and 4.5.
Figure 4.4 Transverse subsidence curves for the V-1 panel using “with an edge offset” method

Figure 4.5 Longitudinal subsidence curves for the V-1 panel using “with an edge offset” method
Examining Figure 4.4, it can be seen that the calculated subsidence curves fit the measured data fairly well. In particular, the default subsidence factor matches the mid-panel subsidence for Line #2 fairly well, but is a little low for Line #1. The difference in subsidence factor between the measured data and the default value is also seen in the longitudinal subsidence in Figure 4.5. Also, in Figure 4.4 and 4.5, it can be seen that the default angle-of-draw at the edges of the panel is a bit lower than what was observed in the fields.

4.2.2 Using “without an Edge Offset” Method to Calculate Subsidence

In order to highlight the difference in the subsidence prediction between using an offset distance from the edge of the panel and not, the subsidence over the V-1 panel was also calculated without using an offset distance. The subsidence prediction approach is very similar to the previous approach. Equation 3.19 is still used to calculate the final gob modulus (20,630 psi), but now the offset distance is set to 0 and Equation 3.25 is used to calculate the appropriate lamination thickness (10.7 ft) to use without an edge offset. The calculated (and measured), transverse and longitudinal, subsidence results without using an offset distance were plotted in Figure 4.6 and 4.7.
Figure 4.6 Transverse subsidence curves for the V-1 panel using “without an edge offset” method

Figure 4.7 Longitudinal subsidence curves for the V-1 panel using “without an edge offset” method
Examining Figure 4.6 and 4.7, it can still be seen that the calculated maximum subsidence is smaller than that measured for Line #1. In addition, it can now be seen that the calculated subsidence curves at the edge of the subsidence trough show the actual subsidence starting some 60 to 150 ft wider than the measured subsidence, essentially the offset distance. This problem, that the calculated subsidence using an influence function method tends to be wider than the measured subsidence, has been found throughout the history of subsidence prediction and is the reason the offset distance was originally introduced (Heasley & Barton, 1998).

4.2.3 Using “Measured Data” Method to Calculate Subsidence

For the V-1 panel, the measured subsidence data shows that the subsidence factor for Line #1 is 0.72 (comparing with the default subsidence factor of 0.62) and that the angle-of-draw is 11.8°. As previously mentioned, the maximum possible subsidence factor is the key parameter to calibrate the final gob modulus, and since the V-1 panel is supercritical, the measured subsidence factor at the middle of the panel is the maximum possible subsidence factor. Therefore, Equation 3.19 is still employed to calculate the final gob modulus (16,510 psi, comparing with the final gob modulus of 20,630 psi matching the default subsidence factor of 0.62) to match the measured maximum subsidence factor for Line #1. Then, Equation 3.24 is used to calculate the default value for the offset distance (69.3 ft). Next, Equation 3.44 is used to calculate the optimum lamination thickness (4.6 ft) to match the measured angle-of-draw. The subsidence output from the calibrated process is shown in Figure 4.8 and 4.9.
Figure 4.8 Transverse subsidence curves for the V-1 panel using measured data and “with an edge offset” method

Figure 4.9 Longitudinal subsidence curves for the V-1 panel using measured data and “with an edge offset” method
Examining Figures 4.8 and 4.9, it can be seen that the new specific calculated subsidence curves fit the measured subsidence curves fairly well. In particular, the measured subsidence factor matches the mid-panel subsidence for Line #1 pretty well, but is a little high for Line #2. For the longitudinal subsidence curves, the measured subsidence factor and angle-of-draw match the calculated subsidence curve pretty well both at the mid and panel edges with the default offset distance.
Chapter 5. Conclusion and Recommendation

5.1 Summary and Conclusion

In this thesis, the LaModel program for surface subsidence prediction has been calibrated for single panel in single seam. In lieu of a large subsidence database, the WVU Comprehensive and Integrated Subsidence Prediction Model (CISPM) program was used as the best empirical subsidence curve. Then, numerous LaModel runs were performed in order to find the values of final gob modulus, edge offset distance and lamination thickness which minimized the least-square error between the CISPM predicted subsidence and the LaModel calculated subsidence curves. This subsidence matching process was performed for panels with an assumed offset at the edge of the panel (as typically done with empirical subsidence prediction models) and for panels without an assumed offset. Also, a separate optimization process was done for supercritical panels and subcritical panels. Further, if the user has measured data for subsidence factor and angle-of-draw, the optimum final gob modulus and lamination thickness can be determined from the measured data. These new subsidence prediction formulas are being implemented into new material wizards in LaModel. Through this curve fitting process, a number of equations for optimizing the values of the critical input parameters for predicting subsidence are developed.

1) In the “with an edge offset” calibration method, it was determined that the final gob modulus is best determined as a function of the maximum possible subsidence factor and the depth, and the optimum offset distance is solely a function of the depth. Also, an empirical formula relating the lamination thickness to the overburden depth was determined regardless if the panels are supercritical or subcritical.
2) In the “without an edge offset” calibration method, the calculation for final gob modulus is the same as the function in the “with an edge offset” calibration method. Also, two different empirical formulas relating the lamination thickness to the overburden depth and/or panel width-to-depth ratio were determined for the cases of: supercritical panels and subcritical panels.

3) In the “measured data” calibrating method, if measured subsidence data is available for the subsidence factor and/or angle-of-draw, it is found that the optimum final gob modulus is a direct function of the measured subsidence factor and the optimum lamination thickness is a direct function of the overburden depth and the measured angle-of-draw.

For the case study, it was found that using the optimum offset distance to predict the subsidence gives a much better surface subsidence prediction than not using and offset distance. Predicting subsidence without using an offset distance does not produce a good fit at the edge of the panel. Also, if site-specific data on the subsidence factor and angle-of-draw is known, then the subsidence prediction/fitting can quickly be improved using the calculations for a site-specific gob modulus and laminations thickness.

5.2 Future Research Recommendation

Based on the investigations conducted in this thesis, the following work need to be further studied:

1) In this research, investigations focused on calibrating LaModel for surface subsidence prediction over single longwall panel. Further investigations should put on the surface subsidence prediction over two and more adjacent longwall panels and chain pillars.
2) In this research, models were developed for single seam. However, for the multiple seam extraction operation, the LaModel program for surface subsidence prediction should be conducted in the future research.

3) A lack of awareness of abandoned mines combined with human activities area expansion has caused surface buildings and infrastructure to be built over these old mines. Subsidence over abandoned coal mines is a potential hazard for these buildings and infrastructure. Therefore, the subsidence prediction over abandoned mines with LaModel also need to be studied in the future research.
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