Phased Array Feed Beamforming and its Application in Mapping Extended
Sources

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ABSTRACT

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Ever since the detection of radio signals, technology has strived to improve the performance of the hardware associated with detecting and observing faint radio sources by improving signal processing and detection techniques. Phased array feeds (PAFs) for radio telescopes can yield in an increase in the survey speed and field of view for reflector antennas. Beamforming, calibration, and imaging are some of the challenges that need to be addressed in order to further the development of PAFs. There is a trade-off between achieving high sensitivity and maintaining a stable clean beam pattern when it comes to beamforming design strategies. A new beamforming strategy is discussed in this paper which is sub optimal but can be improved with time.

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1Introduction

For years the technology for high sensitivity radio astronomy has been based on very large individual spherical or parabolic dishes focusing energy to a single horn feed as seen in Figure 1¹. This increases the sensitivity but forming images becomes challenging. A single dish has one pixel so it can only record the total power captured within its primary beam. To make an image, the single beam must be pointed in different directions and the measurements combined to form a map (Mangum et al. 2007). If the primary beam shape is known, it is possible to mosaic over a given field with near-uniform sensitivity. With a single dish, overall resolution is set by the diffraction limit.

The Parkes 20 cm multi-beam in Australia is a 13-element horn feed (Staveley-Smith et al. 1996). Parkes multi-beam was so successful, a similar instrument was commissioned by Cornell University for the Arecibo Observatory. Arecibo is the world's largest single dish radio telescope. The Arecibo L-band Feed Array (ALFA) is smaller than the Parkes multi-beam system and is a seven feed system instead of thirteen. It allows large-scale surveys of the sky to be conducted with unprecedented sensitivity using the 305-m Arecibo telescope in Puerto Rico. ALFA, operating near 1.4 GHz (Giovanelli et al. 2005), consists of a cluster of seven cooled dual-polarization feeds, a fiber-optical transmission system, and digital back-end signal processors. The idea is each of these feeds or antennas observe the dish from an offset angle in the focal point. So at one pointing, we can see more points in the sky.

This whole idea has been elaborated and improved by some major groups in the world like ASTRON (The Netherlands), DRAO (Canada), CSIRO (Australia), FLAG (National Radio Astronomy Observatory (NRAO), Green Bank Observatory (GBO), Brigham Young University

¹<https://images.app.goo.gl/3vGFm7bTfyN1AqF7A>

(BYU), West Virginia University (WVU); U.S.). These groups look at densely packed array feeds (smaller antennas) rather than separate high gain horns. Their main focus is to form an arbitrary number of beams on the sky by electronically phase shifting instead of forming discrete, physically separate beams on the sky. This provides a much more continuous look at the distribution of sources. Such systems are called phased array feeds (PAFs)

One main objective of PAFs is to expand our field of view and observe a number of spots on the sky at any given point of time. Any telescope captures a plane wave incident on an aperture or reflecting surface. Mirror-based telescopes focus the plane wave in free space using the geometry of the reflecting surface to provide directional gain. Phased arrays record an incident plane wave in several locations and focus or align the signals using electronics. Signals added in phase interfere constructively.

Single dish applications are not the only possible use of PAFs. Such an array feed, like the Apertif on the Westerbork Synthesis Radio Telescope (WSRT) operated by ASTRON in the Netherlands could be placed on the Very Large Array (VLA) to expand the field of view dramatically. So instead of looking at one small patch on the sky we can observe a much larger region with a single observation. Figure 2² is a pictorial representation of this concept.

A primary driver of the PAF is an increased field of view. As discussed earlier with a PAF we can steer simultaneous beams in as many directions as we want within some limited field of view. The beams are formed electronically after the signal is digitized. This is done by multiplying in our computer or in the beam former by a set of complex weights that shifts the pointing direction of the beam. As long as we have computational resources we can do this as many times as we want. PAFs help in survey operations by reducing the time required

²<https://images.app.goo.gl/5cX9hsTmvacpEsgX8>

Figure 1: The 64-meter radio telescope at Parkes Observatory as seen in 1969.

Figure 2: Increased field of view using a PAF.

to collect the same data on the sky since by processing the same set of array data, we observe a deep space object with multiple high gained pencil beams all at once (Ivashina & Klooster 2002). Another exciting opportunity is that, PAFs can do radio camera imaging directly. The big dish acts as the camera lens. Instead of doing synthesis images as one would do with the VLA we can take a look at the sky and make a direct image of a small patch of the sky. With PAFs we can form beams as arbitrarily close to each other as we want so that we can get a smooth distribution of measurements of source intensity on the sky.

Some of the fringe benefits of this is the ability to do interference cancellation and increased sensitivity. In the presence of a man made source of interference like a satellite or a ground based transmitter or some unknown interference source, we can use the beam former to form a spatial null in the direction of the interference and that ways we can remove or reduce the interfering signal and continue on with our observation (Landon et al. 2010). We can remove it before we get to the spectral processing. This can be done in the spatial domain rather than a frequency or time domain. Increased sensitivity is attained if

we have a fairly large PAF. By manipulating the illumination pattern on the dish, we can use a higher percentage of the total dish surface area without increasing noise from the spill over regions, like the warm

temperature of the ground (Warnick & Jeffs 2006). One of the problem that we face with horn feeds is that every time we move the feed off axis in the focal point of the parabolic dish, it causes some distortion in the beam pattern. The side lobe level goes up in the beam pattern. With a fully controlled PAF we should be able to correct some of those distortions. Thus, PAFs increase the field of view of a radio telescope and grant some degree of control over our beams (Warnick & Jeffs 2007).

Of course, there are some technical challenges of PAFs, the most significant one being the increase in the system noise temperature. As the antennas are placed close to each other, they couple and noise propagates between them (Warnick & Jensen 2005). As the system noise temperature goes up, we are less sensitive and as a result we cannot see some of the weak sources.

2Focal L-band Array for the Green Bank Telescope (FLAG)

In Radio Astronomy high sensitivity and large-scale surveys are of paramount importance. While large apertures improves sensitivity they tend to limit the FoV, when equipped with a single feed. The survey speed of a telescope is proportional to its FoV and the square of the signal-to-noise (SNR) ratio required for the survey. Roshi et al. (2018) describes a new 1.4 GHz (L-band) PAF system, built for the Robert C. Byrd Green Bank Telescope as part of FLAG project, which can enhance the FoV of the telescope by a factor of seven and has sensitivity comparable to the existing single feed 1.4 GHz receiver.

FLAG was first commissioned in the summer of 2016. Single pulses from PSR B1933 + 16 and bright giant pulses from PSR B1937 + 21 were detected in the summer of 2017 demonstrating the capability of

the instrument to detect pulsars (Rajwade et al. 2018). The HI mapping results are summarized in Pingel (2017).

Galaxies are accreting gas in order to sustain the star formation in them. With the current known gas content of galaxies, the star formation rate can be sustained for a few billion years but there is evidence that the gas content in galaxies have remained constant for the past 10 billion years. This could only mean that there is some intergalactic medium that feeds the galaxies to sustain their star formation rate (Prochaska, Herbert-Fort & Wolfe 2005). Current theories suggest that there are two modes by which galaxies accrete gas. A hot mode where the gas is heated to up to 10^6 K and a cold mode where the gas remains below 10^5 K (Kereš et al. 2005; 2009; Sancisi et al. 2008). The cold mode accretion is a dominant form of accretion of gas for low mass galaxies in low density environments. Such accretions are detected via observations of neutral hydrogen at 21 cm (HI) at column densities of $N(\text{HI}) \sim 10^{18} \text{ cm}^{-2}$ (Popping et al. 2009). This requires sensitive single-dish observations and requires enormous amount of survey time since the expected line strengths are weak (Chynoweth et al. 2008; Mihos et al. 2012; Wolfe et al. 2013; Pisano 2014; deBlok et al. 2014; Wolfe, Lockman & Pisano 2016). The detection of Fast Radio Bursts (FRBs), which is a transient radio pulse has sparked interest among many in unravelling the radio sky (Lorimer et al. 2007; Katz 2016). In order to reduce the observation time it is needed to study all these phenomena essential to increase the FoV of the telescope with high sensitivity multi-beam systems.

The FoV of a radio telescope when equipped with a single feed is limited to the full width at half maximum (FWHM) of the primary beam, which is $\sim \lambda/D$, where λ is the observing wavelength and D is the aperture diameter of the reflector. The feeds are usually optimized to receive maximum radiation from the reflector while attenuate the ground spillover. This optimization process determines the physical size of the feed. In a focal plane array (FPA) the FoV of telescopes

are increased by placing multiple optimized feeds in the focal plane. The size of the feeds determine the separation between the feeds in a FPA. Non-overlapping beams on the sky are formed if the physical separation of the feeds are large. Further, the off-axis beams suffer efficiency degradation since the feed optimization is usually done for the central beam. There is no improvement in the mapping efficiency on account of these effects. In the last decade, a set of smaller focal plane radiating elements referred to as PAFs have been employed which has gained wide interest among the radio astronomy community (Fisher & Bradley 2000; Hay & Bird 2015; Warnick et al. 2016). In such PAF receivers, each element is electrically small, and thus does not optimally illuminate the reflector. However an optimal illumination with low spillover is obtained by the weighted sum of the amplified signal voltages from multiple elements. By adding signals with different sets of complex weights, multiple beams can be formed. The beams formed in this way can be made to overlap, thus increasing the mapping efficiency. Mutual coupling effects between elements can complicate the design and optimization of PAFs (Diao & Warnick 2017). The mutual coupling distorts the element beam pattern, and introduces channel-to-channel noise coupling between neighboring low-noise-amplifiers (LNAs) which follow these elements. Thus, accurate methods for electromagnetic modeling and beamforming are needed in order to achieve efficient performance. All of these results have led to the development of FLAG.

FLAG is a collaborative project between the National Radio Astronomy Observatory (NRAO), the Green Bank Observatory (GBO), Brigham Young University (BYU), and West Virginia University (WVU). The frequency at which the FLAG receiver operates ranges between 1.2 - 1.5 GHz. As a part of the FLAG project, a 19-element dual-polarization cryogenic PAF receiver has been developed for the Green Bank Telescope (GBT). The receiver is reported to have the lowest beamforming system temperature making it the most sensitive

PAF in the world and comparable to the existing receiver on the GBT. The system temperature is normalized by the aperture efficiency (η) and the measured T_{sys}/η is 25.4 ~~2.5~~ K (Roshi et al.2018). We can image a FoV with a diameter of 30° . FLAG is capable of forming seven Nyquist-sampled beam producing a survey speed larger by a factor of

2.1 to 7 to a single beam forming system (Roshi et al. 2018). The survey speed depends on the observing applications. The measured performance of the PAF both in frequency and offset from boresight, qualitatively agree with predictions from a rigorous electromagnetic model.

Figure 3: (a) A 19 element dual-polarized dipole array. (b) The dipole and custom low-loss, low thermal conductivity transition to the low noise amplifier (Figure 1. in Roshi et al. (2018)).

2.1 Instrumentation

FLAG consists of 19-element dual polarization dipoles connected to 19 pairs of cryogenic LNAs located in vacuum dewar. For achieving maximum sensitivity over the antenna FoV of angular diameter of 20° , and across a bandwidth of 300 MHz, the geometric parameters of the

Figure 4: Block diagram of a phased array receiver system (Figure 2. in Roshi et al (2018)).

dipoles need to be adjusted in order to optimize the design. This has been developed by Warnick et al. (2011). The optimal design depends on the noise parameter of the LNA and uses maximum SNR beamforming as a parameter. The dipole array is shown in Figure 3(a). The dipoles are fabricated using brass, copper and gold to minimize ohmic loss. In order to reduce dielectric loss an air filled coaxial conductor transmission line is used. These vertical transmission lines help maintain the correct separation between the radiating elements and the ground plane. The inner conductor is centred using two teflon beads located at either end of the transmission line. A low-loss coaxial assembly serves as a thermal transition and a vacuum barrier between the LNAs and the dipoles as shown in Figure 3(b). SiGe transistors which are known to have a low-noise performance at cryogenic temperatures are used by the LNAs (Weinreb et al. 2009). All the remaining functions of the receiver are encompassed by highly integrated, 40-channel electronics assembly which includes calibration signal injection, warm post-amplification, power leveling, local oscillator distribution, down conversion, and serial data transmission through optical fiber, as shown in Figure 4. The PAF and the receiver box is placed at the prime focus of the GBT. GBT is shown in Figure 5³ where FLAG is located.

3 Theory of Beamforming

In this section, we discuss beamforming which is a signal processing technique using a PAF instrument. Beamforming is mostly used in sensor arrays for directional transmission or reception of signals. This is achieved by linearly combining signals from an array of sensors in a phased array in such a way that signals at particular angles experience constructive interference while others experience destructive interference (Ellingson 2003). Beamforming has been widely used in radio

³<https://images.app.goo.gl/jC2e5jb4gBVkVaWr8>

Figure 5: The Green Bank Telescope where FLAG is located.

astronomy since the first detection of radio waves. Analog beamforming was initially used in radio astronomy to increase the directive gain of antenna arrays. More recently, beamforming is used for agile steering of antenna arrays and adaptive control of telescope performance during observations.

Another form of beamforming is digital beamforming which enables a PAF instrument to form multiple simultaneous beams thus increasing the field of view (FoV) for the telescope. Figure 6 illustrates the different ways in which digital beamforming technique is used.

3.1 Signal Model

Landon et al. (2010) made an assumption that the array output signals are processed in narrow sub bands such that $B \ll D/c$, where B is the bandwidth, D is the PAF diameter, and c is the speed of light. This is in contrast to an interferometric array, like the VLA. An M element PAF for n time samples produces a complex base band data vector of length $M \times 1$ given by:

$$\mathbf{x}[n] = \mathbf{a}s[n] + \mathbf{n}[n] \quad (1)$$

where $\mathbf{s}[n]$ is the signal of interest (SOI) and $\mathbf{n}[n]$ is the total system noise vector. If any interference is present then we also include components due to man made detrimental signals $d[n]$ in the total system noise vector. This is shown in Figure 6. $\mathbf{a}$ is the normalized array response vector to a unit amplitude point source in the far-field direction corresponding to the SOI. Assuming that there is statistical independence between the SOI and noise, the array covariance matrix is defined as:

$$\mathbf{R} = E[\mathbf{x}[n]\mathbf{x}^H[n]] = \mathbf{R}_s + \mathbf{R}_n \quad (2)$$

$$\mathbf{R}_s = \sigma_s^2 \mathbf{a}\mathbf{a}^H$$

$$\mathbf{R}_n = \mathbf{R}_{rec} + \mathbf{R}_{sp} + \mathbf{R}_{sky} + \mathbf{R}_{loss} + \mathbf{R}_{int} \quad (3)$$

We have assumed that the mean wide-sense stationary signals are zero and there is no dependence between distinct signals and noise sources. σ_s^2 is the SOI signal, and $\mathbf{R}_{rec}$, $\mathbf{R}_{sp}$, $\mathbf{R}_{sky}$, $\mathbf{R}_{loss}$, $\mathbf{R}_{int}$ are receiver, spillover, sky, loss due to antenna resistance, and interference noise covariance matrices, respectively. $\mathbf{R}_{sky}$ is the sky noise contribution from sources such as the atmospheric, cosmic microwave background (CMB), and galactic background (GB) radiation.

$\mathbf{R}$ is estimated for the j th short term integration (STI) window with sample covariance matrix from observed data samples as

$$\hat{\mathbf{R}}^{(j)} = \frac{1}{L} \sum_{n=jL}^{(j+1)L-1} \mathbf{x}[n] \mathbf{x}^H[n] = \frac{1}{L} \mathbf{X}_j \mathbf{X}_j^H \quad (4)$$

$$\mathbf{X}_j = [\mathbf{x}[jL], \mathbf{x}[jL + 1], \dots, \mathbf{x}[(j + 1)L - 1]]$$

where $\mathbf{X}_j$ is the L sample long STI data window. The length of L depends on the signal scenario and desired operational mode. L is short if adaptive cancellation is used to suppress moving interference. Beamformer weights (computed from $\hat{\mathbf{R}}^{(j)}$) must be updated rapidly with L short enough so that $\mathbf{R}_{int}$ does not change over the STI window. The length of L is thus large in case of slower update rate. A longer L is used to adapt to spillover and sky noise. When the signal conditions are stable, L can be very long (on the order of minutes or hours) to reduce sample estimation error and yield accurate calibration or high sensitivity beamformer solutions.

3.2 Calibration

As discussed earlier, PAF can generate multiple simultaneous beams as shown in Figure 6. This requires us to calibrate the signal array

Figure 6: Multiple simultaneous beams increases the field of view. A spatial null is used to cancel any man made interference (Figure 4. in Landon et al. (2010)).

response vector $\mathbf{a}_i$ for each direction Ω_i corresponding to the boresight direction of each beam. Due to the strict beam pattern stability requirements, re-calibration may be necessary from time-to-time. This provides a correction for differential electronic phase and gain drift and accounts for the change in receiver noise temperature. The calibration algorithm is obtained from Landon et al. (2010) and is as follows:

1. Noise covariance: $\check{\mathbf{R}}_n$

Steer the dish to a relatively empty patch of sky so $\mathbf{x}[n] \perp \mathbf{n}[n]$, and collect a long term (large L , e.g., 10 minutes) sample covariance estimate for the noise field $\check{\mathbf{R}}_n$ using Equation 4.

2. Signal-plus-noise covariances: $\check{\mathbf{R}}_i$.

While tracking the brightest available calibration point source, steer the dish to calibration angle Ω_i (relative to this source). The observed signal model is $\mathbf{x}[n] = \mathbf{a}_i s[n] + \mathbf{n}[n]$, where $\mathbf{a}_i$ is the desired calibration vector at direction ω_i . Calculate $\check{\mathbf{R}}_i$ using Equation 4 and the same L as in step 1.

3. Array response: $\check{\mathbf{a}}_i$.

Compute $\check{\mathbf{a}}_i = \check{\mathbf{R}}_n \mathbf{u}_{max}$, where $\mathbf{u}_{max}$ is the dominant solution to the generalized eigen equation $\check{\mathbf{R}}_i \mathbf{u}_{max} = \lambda_{max} \check{\mathbf{R}}_n \mathbf{u}_{max}$. This noise-whitening method produces a lower variance estimate with less bias arising from correlated noise in $\mathbf{R}_n$ (caused by inter-element mutual coupling) than does the method of Jeffs et al. (2008b).

4. SOI covariance: $\check{\mathbf{R}}_{s,i}$

Compute $\check{\mathbf{R}}_{s,i} = \lambda_{max} \check{\mathbf{a}}_i \check{\mathbf{a}}_i^H$.

5. Form calibration grid

Repeat steps 2–4 in a grid pattern corresponding to the desired distribution of beam centers and constraint points, e.g., for spherical angles $\Omega_i / 1 \leq i \leq K$.

3.3 Beamformer Design

Producing beams with a PAF model requires a design procedure to obtain a set of beam former weight vectors $\mathbf{w}_i^{(j)}$. The output for the i th beam is computed as

$$y_i[n] = \mathbf{w}_i^{(j)H} \mathbf{x}[n], \quad 0 \leq i \leq k, \quad j = \frac{n}{L} \quad (5)$$

Each main lobe steering angle Ω_i is associated with a distinct beam former weight. The j th index in Equation 5 can be dropped if the weights are kept constant during each STI. A rectangular or hexagonal beam pattern is usually preferred to steer the beams with cross over points at the -1 to -3 dB levels. Depending on the beam former weights $\mathbf{w}_i$, the main beam can be steered in various directions. One can also control the shape of the beam, side lobe levels, optimize sensitivity, and place the nulls directly towards interference. An introduction to beamformer design and array signal processing can be found in Van Veen & Buckley (1988) and Van Trees (2002).

4 Max-SNR Beamformer

The maximum sensitivity beamformer is defined as (Landon et al. 2010)

$$\mathbf{w}_{mSNR} = \underset{\mathbf{w}}{\operatorname{argmax}} \frac{\mathbf{w}^H \mathbf{R}_s \mathbf{w}}{\mathbf{w}^H \mathbf{R}_n \mathbf{w}} \quad (6)$$

where $\mathbf{w}_{mSNR}$ is the optimum weight vector that maximizes the $SNR = \frac{\mathbf{w}^H \mathbf{R}_s \mathbf{w}}{\mathbf{w}^H \mathbf{R}_n \mathbf{w}}$ in Equation 6. The optimization of Equation 6 is defined as follows:

$$\begin{aligned} & \max \mathbf{w}^H \mathbf{R}_s \mathbf{w} \\ & \text{subject to } \mathbf{w}^H \mathbf{R}_n \mathbf{w} = 1 \end{aligned}$$

The Lagrangian for the above equation is defined as $\mathcal{L} = \mathbf{w}^H \mathbf{R}_s \mathbf{w} - \lambda(\mathbf{w}^H \mathbf{R}_n \mathbf{w} - 1)$ where λ is the Lagrangian multiplier. Equating the derivative of the Lagrangian to zero gives us:

$$\mathbf{R}_s \mathbf{w} = \lambda \mathbf{R}_n \mathbf{w} \quad (7)$$

which is the generalized eigenvalue problem. We solve this equation for the dominant eigen vector ($\mathbf{w}_{mSNR}$) corresponding to the eigen value of maximum magnitude (λ_{max}). The solution to Equation 7 gives us the Max-SNR beamformer. All existing PAF systems in operation today use the maximum sensitivity beamformer.

The results obtained from the simulation code written in MATLAB by Karl.F. Warnick for the Max-SNR beamformer which assumes a perfect calibration is as follows:

Figure 7 represents the 19-element PAF layout.

Figure 8(a) represents the response pattern for one formed beam with sensitivity measured in logarithmic scale. This allows us to see how the side lobes behave. Since an ideal beam should follow a Gaussian distribution, we fit the central beam to a Gaussian as shown in Figure 8(b). This provides us with some knowledge about the behaviour of the central beam. The better the beam fits to the Gaussian, better the beam shape. The goodness of fit which is given by the root mean square error (RMSE) of the residuals obtained from fitting the Gaussian to Figure 8(b) is found to be 1.5288. A RMSE value of zero represents a perfect fit. Thus it is evident from the figures that the Gaussian fits well to the main beam in Figure 8(b) but the presence of side lobes as seen in Figure 8(a) could be problematic when trying to achieve a refined beam shape.

The Max-SNR beamformer has its own downsides as it does not guarantee pattern stability and can not be recomputed to optimize over variations in the noise field and offers little direct control of the beam pattern shape, since it naturally responds to the noise covariance structure. Changes in array response vectors due to spillover noise and electronic drift in the receiver causes variation in the beam pattern stability making them unpredictable and may cause undesired signals from a nearby bright source. However, with a Max-SNR beamformer, the PAF sensitivity is maximized in a desired direction. It provides efficient steering of beams as it only requires a single pair of on and off-source measurements and a single calibration vector. We see this clearly in Figure 8.

The simple phased array feed algorithm using 1 dimensional physical optics integration for the reflector model is as follows (The original code is written by Karl F. Warnick):

1. Define the feed, receiver, and reflector system
 - a. Set the operating frequency, design frequency, and the bandwidth. The design frequency is used to convert all parameters in wavelengths to meters, such as element length, element spacing, and ground plane standoff distance. The operating frequency is the frequency at which the simulation is run. The bandwidth is measured in Hz.
 - b. Define the phased array feed geometry (Array layout type, number of hexagonal rings, array element spacing) and ground plane (ground plane=1, 0=no ground plane)
 - c. Define the array element (Type= Hertzian, length)
 - d. Define the reflector (Dish focal length/diameter, Dish diameter, Sky temperature)
 - e. Define the receiver and low noise amplifier (LNA minimum noise temperature and LNA noise resistance parameter)
2. Get the system noise response
 - a. Calculate the array element pattern overlap integral matrix
 - b. Calculate the spillover noise correlation matrix, isotropic noise

correlation matrix, sky noise correlation matrix, Array element loss noise correlation matrix, external and loss thermal noise correlation matrix, array input impedance matrix using resonant approximation ($x=0$), receiver noise.

- c. Transform all open circuit voltages to loaded array output voltages.
- d. Calculate the total system noise correlation matrix.
3. Get the system signal response.
 - a. Get the embedded array element radiation field patterns.
 - b. Convert the element patterns to open circuit voltages.
 - c. Transform from open circuit voltages to loaded voltages.
4. Apply a beamformer and compute figures of merit for boresight beam.
 - a. Compute the beamformer weight vectors.
 - b. Get stable maximum sensitivity beamformer calibration.
 - c. Fit a Gaussian to the sensitivity and compute the root mean square error (RMSE) of the residuals.
 - d. Compute the sensitivity map (peak sensitivity for all steered beams).

The results obtained from running the MATLAB code is as follows:

Aperture efficiency: 0.80003

Spillover efficiency: 0.97674

Tsys: 47.4072 K

Tlna: 33.0925 K

Tspill: 6.4235 K

Tloss: 3.9456 K

Sensitivity: 130.7385 m²/K

Tsys/efficiency 60.074 K

5 Stochastically Optimized Beamformer

Various beamforming design methods have been implemented to achieve maximum sensitivity with a stable beam pattern spatial structure. Stability and sensitivity are the two most significant parameters while forming beams. Noise and gain stability is needed for weak source detection whereas, high sensitivity is required to detect faint radio signals from distant celestial sources. Thus a balance between the demand for high sensitivity with a known stable beam pattern is desirable for a PAF beamformer model. There is a trade-off between sensitivity and stability since PAFs achieve their optimal SNR performance by adapting the beam pattern to the noise field and array response parameters.

There are several beamforming methods that have been suggested for PAF such as Max-SNR Beamformer, Equiripple Beamformer, Hybrid Beamformer, Transforming Modeled Beamformers. Most of these beamforming methods involve obtaining the beamformer through numerical optimization. As a part of my master's thesis I have developed a beamforming technique called Stochastically Optimized Beamforming by perturbing the beamforming weights from the Max-SNR Beamformer and generating a new set of weights optimized for alternative constraints. We are testing this new algorithm using the existing code to examine the central formed beams. The response pattern for one formed beam is obtained by plotting the sensitivity derived from the beamforming algorithm vs. the offset angle. This is shown in Figure 9. By normalizing the sensitivity we can see the behaviour of the side lobes as shown in Figure 10. In an attempt to understand how sensitivity relates to beam shape we have conducted a study where different set of weights are generated from our model to form one main beam. A Gaussian is then fitted to the main beam and a RMSE of the residuals of the fit is calculated for each formed beam. It can be seen from Figures 12 through 17 that as the RMSE decrease, the sen-

sitivity gradually declines which is indicative of the fact that there is a trade-off between sensitivity and the beam shape.

The algorithm for the Stochastically Optimized Beamformer is as follows:

1. Repeat all the steps from the Max-SNR Beamformer algorithm.
2. Count iterations from 1 to 100000.
3. Perturb the weights that gives maximum sensitivity for the Max-SNR beamformer.
 - a. Multiply each complex weight of the optimum beamformer weight vector by a randomly generated number between 0 and 1 (excluding 0 and 1) in such a way that the i th element of our beamformer weight vector is defined as follows:
 $w_{new,i} = \text{real}(w_i) \times \text{rand}(0, 1) + i \times \text{imag}(w_i) \times \text{rand}(0, 1)$, where $\text{rand}(0,1)$ is a randomly generated number between 0 and 1. $\text{real}(w_i)$ and $\text{imag}(w_i)$ are the real and the imaginary parts of the component of the i th weight of the Max-SNR beamformer weight vector.
4. Form a beam from the newly generated weights w_{new} and fit a Gaussian to it.
5. Calculate the residuals from the fit and find the RMSE of the residuals $RMSE_{new}$.
6. Compare $RMSE_{new}$ to the $RMSE$ from Max-SNR beamformer model ($RMSE$).
 - a. If $RMSE_{new} < RMSE$.
 - a(i). Set $RMSE = RMSE_{new}$, $w = w_{new}$
 - a(ii). Increase the count on the iteration and calculate the next $RMSE_{new}$ and compare this with the $RMSE$ from step a(i).
 - a(iii). Run this for all iterations until we get the smallest $RMSE_{new}$.
 - b. If $RMSE_{new} > RMSE$, increase the count on the iteration and repeat steps 3 to 6 until $RMSE_{new} < RMSE$.
7. Plot for one formed beam obtained from the weights that give us the smallest $RMSE_{new}$ and fit a Gaussian to it.

8. Run this code any number of times to generate beams with $RMSE_{new} < RMSE$

The results obtained from running this code are shown in Figure 9, Figure10, Figure11, Figure12, Figure13, Figure14, Figure15, Figure16, and Figure17

6Conclusion

The maximum sensitivities obtained from the perturbed model as shown in Figures12through17are much less than the Max-SNR sensitivity of $130.7385 \text{ m}^2/\text{K}$. Thus sensitivity along boresight is low for our generated weights. Also, by perturbing the weights we lower the sensitivity for our model which in turn lowers the RMSE. As a result, even though we achieve RMSEs which are less than the Max-SNR RMSE of 1.5288, the normalized sensitivity plot of one formed beam as shown in Figure10and the plots from Figures12through17depicts the presence of some prominent side lobes in these beams. Thus, even though we were able to reduce the RMSE for our model from that of the Max-SNR beamformer model, we still cannot accept this approach unless we try to improve our model by altering the constraints so that we simultaneously lower the residuals and maintain a sensitivity closer to the Max-SNR results.

Figure 7

Figure 8: (a) The resulting beam shape obtained from plotting sensitivity of the Max-SNR beamformer in the logarithmic scale vs. the angle from boresight θ . This allows us to see how the sidelobes behave. (b) The resulting beam shape obtained from plotting sensitivity of the Max-SNR beamformer vs. the angle from boresight θ . A Gaussian is fit to the central beam to show that it follows a Gaussian distribution.

Figure 9: Response pattern for one formed beam for the Stochastically Optimized Beamformer model.

Figure 10: Response pattern for one formed beam with normalized sensitivity for the Stochastically Optimized Beamformer.

Figure 11: Response pattern for one formed beam for the Stochastically Optimized Beamformer model with normalized sensitivity. This allows us to see how the sidelobes behave. A Gaussian fit to the central beam allows us to see that the main beam shape follows a Gaussian distribution.

Figure 12: RMSEs from fitting a Gaussian to the main beam obtained from the Stochastically Optimized Beamformer.

Figure 13: RMSE from fitting a Gaussian to the main beam obtained from the Stochastically Optimized Beamformer.

Figure 14: RMSE from fitting a Gaussian to the main beam obtained from the Stochastically Optimized Beamformer.

Figure 15: RMSE from fitting a Gaussian to the main beam obtained from the Stochastically Optimized Beamformer.

Figure 16: RMSE from fitting a Gaussian to the main beam obtained from the Stochastically Optimized Beamformer.

Figure 17: RMSE from fitting a Gaussian to the main beam obtained from the Stochastically Optimized Beamformer.

References

- [1]J. G. Mangum, D. T. Emerson, and E. W. Greisen. The on the fly imaging technique. *A&A*, 474:679—687, 2007.
- [2]L. Staveley-Smith, W. E. Wilson, T. S. Bird, M. J. Disney, R. D. Ekers, K. C. Freeman, R. F. Haynes, M. W. Sinclair, R. L. Webster R. A. Vaile, and A. E. Wright. The parkes 21 cm multibeam receiver. *Astron. Soc. Aust.*, 13:243—248, 1996.
- [3]R. Giovanelli et al. The arecibo legacy fast alfa survey. i. science goals, survey design, and strategy. *AJ*, 130:2598–2612, 2005.
- [4]M. Ivashina and C. G. M van’t Klooster. Focal fields in reflector antennas and associated array feed synthesis for high efficiency multibeam performances. *Proceedings of the ESTEC Antenna Workshop, Noordwijk, The Netherlands*, 2002.
- [5]J. Landon, M. Elmer, J. Waldron, D. Jones, A. Stemmons, B. D. Jeffs, and K. F. Warnick. Phased array feed calibration, beam-forming and imaging. *AJ*, 139, 2010.
- [6]K. F. Warnick and B. D. Jeffs. Gain and aperture efficiency for a reflector antenna with an array feed. *IEEE Antennas and Wireless Propagation Letters*, 5:499–502, 2006.
- [7]K. F. Warnick and B. D. Jeffs. Efficiencies and system temperature for a beamforming array. *IEEE Antennas and Wireless Propagation Letters*, 2008.
- [8]K. F. Warnick and M. A. Jensen. Effects of mutual coupling on interference mitigation with a focal plane array. *IEEE Trans. Ant. Propag.*, 53:2490–2498, 2005.

-
- [9]K. M. Rajwade, N. M. Pingel, R. A. Black, and M. Burnett
M. Ruzindana, B. Jeffs, K. Warnick, D. J. Pisano, D. R. Lorimer,
R. M. Prestage, L. Hawkins, J. Ray, P. Marganian, T. Chamber-
lin, J. Ford, W. Shillue, and D. A. Rosh. Commissioning of flag:
A phased array feed for the gbt. *Proceedings of the International
Astronomical Union*, 13:398–399, 2018.
- [10]N. M. Pingel. Deep extragalactic neutral hydrogen surveys with
the green bank telescope. *Graduate Theses, Dissertations, and
Problem Reports*, 2017.
- [11]D. A. Rosh, B. Simon W. Shillue, K. F. Warnick, B. Jeffs, D. J.
Pisano, R. Prestage, S. White, J. R. Fisher, M. Morgan, R. Black,
M. Burnett, J. Diao, M. Ruzindana, V. van Tonder, L. Hawkins,
P. Marganian, T. Chamberlin, J. Ray, N. M. Pingel, D. R. Lorimer
K. Rajwade, and, A. Rane, J. Castro¹, W. Groves, L. Jensen,
J. D. Nelson, T. Boyd, and A. J. Beasley. Performance of a highly
sensitive, 19-element, dual-polarization, cryogenic l-band phased-
array feed on the green bank telescope. *ApJ*, 155, 2018.
- [12]J. X. Prochaska, H. F. Stéphane, and A. M. Wolfe. The sdss
damped ly survey: Data release 3. *ApJ*, 635:123–142, 2005.
- [13]D. Kereš, N. Katz, D. H. Weinberg, and R. Davé. How do galaxies
get their gas? *MNRAS*, 363:2–28, 2005.
- [14]D. Kereš, N. Katz, R. Davé, M. Fardal, and D. H. Weinberg.
Galaxies in a simulated cdm universe – ii. observable properties
and constraints on feedback. *MNRAS*, 396:2332–2344, 2009.
- [15]R. Sancisi, F. Fraternali, T. Oosterloo, and J. M. van der Hulst.
Cold gas accretion in galaxies. *NewAR*, 48:1013–1027, 2004.

-
- [16]K. M. Chynoweth, G. I. Langston, M. S. Yun, F. J. Lockman, K. H. R. Rubin, and S. A. Scoles. Neutral hydrogen clouds in the m81/m82 group. *AJ*, 153:1983–1992, 2008.
- [17]J. C. Mihos, P. Harding, C. E. Spengler, C. S. Rudick, and J. J. Feldmeier. The extended optical disk of m101. *ApJ*, 762:82, 2012.
- [18]D.J. Pisano. Green bank telescope observations of low column density hi around ngc 2997 and ngc 6946. *AJ*, 147:48, 2014.
- [19]W.J.G. de Blok, K. M. Keating, D.J. Pisano, F. Fraternali, F. Walter, T. Oosterloo, E. Brinks, F. Bigiel, and A. Leroy. A low hi column density filament in ngc 2403: signature of interaction or accretion. *A&A*, 569:11, 2014.
- [20]S. A. Wolfe, F. J. Lockman, and D. J. Pisano. Sensitive 21 cm observations of neutral hydrogen in the local group near m31. *ApJ*, 816:81, 2016.
- [21]D. R. Lorimer, M. Bailes, M. A. McLaughlin, D. J. Narkevic, and F. Crawford. A bright millisecond radio burst of extragalactic origin. *Science*, 318:777–780, 2007.
- [22]J. R. Fisher and R. F. Bradley. Full-sampling array feeds for radio telescopes. *Proc. SPIE 4015, Radio Telescopes*, 2000.
- [23]S. G. Hay, T. S. Bird, M. A. McLaughlin, D. J. Narkevic, and F. Crawford. Handbook of antenna technologies. *Springer*, 2015.
- [24]K. F. Warnick, R. Maaskant, M.V. Ivashina, and et al. D. B. Davidson. Phased arrays for radio astronomy, remote sensing, and satellite communications. 2016.
- [25]M. Elmer and B. D. Jeffs. Beamformer design for radio astronomical phased array feeds. *Proc ICASSP*, pages 2790–2793, 2010.

-
- [26]J. Diao and K. F. Warnick. Practical superdirectivity with resonant screened apertures motivated by a poynting streamlines analysis. *IEEE Transactions on Antennas and Propagation*, 66:432–437, 2018.
- [27]S. Weinreb, J. Bardin, H. Mani, and G. Jones. Matched wideband low-noise amplifiers for radio astronomy. *IEEE Transactions on Antennas and Propagation*, 80, 2009.
- [28]S. W. Ellington. Beamforming and interference cancellation with very large wideband arrays. *IEEE Transactions on Antennas and Propagation*, 51:1338–1346, 2003.
- [29]B. Jeffs, K. Warnick, J. Landon, M. Elmer, J. Fisher, and R. Norrod. Signal processing for phased array feeds in radio astronomical telescopes. *IEEE Journal of Selected Topics in Signal Processing*, 2, 2008a.
- [30]H. Van Trees. Detection, estimation, and modulation theory. *John Wiley and Sons*, 2002.
- [31]B. Van Veen and K. Buckley. Beamforming: A versatile approach to spatial filtering. *ASSP Magazine, IEEE*, 5,4, 1988.