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Optimizing Indoor Location Based Tracking through Proper Filter Selection and Wireless Sensor Network Design

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Abstract

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Indoor positioning system (IPS) is a topic that is coming up more and more for various reasons, such as allowing companies to track important objects using radio frequency identification (RFID) and employees with Bluetooth devices inside a facility. Geofencing is one of the biggest topics with IPS and is meant to limit access to a network in specified areas. Devices that incorporate indoor tracking are not initially precise when objects and employees are on the move. This movement requires devices to have a reliable filter for noise and package lose. For this paper, the comparison between extended Kalman filters and unscented Kalman filter in a controlled environment will help indicate which is ideal for IPS tracking. Both filters will be applied and compared on location accuracy metrics. The proper design of the wireless network is also crucial for having an effective IPS method. This will show the difference in wireless networks and how the initial design will lead to greater chance of success for IPS.

Dedication

I would like to dedicate this problem report to my family, my fiancé and friends. My family has supported me every step of the way in graduate school and I wouldn't be where I am without them. My fiancé for being patient with me and being the anchor in keeping me sane throughout my college career. My friends for making school and projects something to enjoy and being there for me when I needed it.

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Chapter 1: Introduction

1.1: Reasons for Research

The advancement in the wireless networking field of indoor positioning systems (IPS) has led to great leaps inside industry and the medical field. There have been numerous attempts to implement tracking of specific objects through the use of wireless signals. In light of the Internet of Things (IoT) topic, IPS has become a major aspect being researched. The purpose of an accurate IPS has been and will always be a valuable topic since the introduction of radio frequency identification (RFID). RFID patented by Charles Walton in 1983 was initially used for 'Portable radio frequency emitting identifier' and has grown substantially to be one of the most commonly used components in IPS. In recent years IPS moved to location tracking for more reasons, such as employee tracking and Geofencing, in hopes to increase productivity and traceability.

Indoor positioning systems can be defined as any wireless system designed to locate or track a specified object inside a building using different types of sensory information. This can be done with radio frequency (RF), magnetic fields, and many others. Another definition that has been used for IPS is a system which can infer the position of a target inside the physical space where the detection system is installed, within a maximum time delay or in real time [1]. Both definitions are vague manly because of all the options out for indoor tracking.

There are numerous ways of implementing these IPS but choosing the correct system topologies for the user is as equally important. The four different system topologies are defined as: remote systems, self–positioning systems, indirect remote positioning systems, indirect self-positioning systems [1]. These have even been researched to aid visually impaired. There have been multiple techniques that have come out over the years, but each technique has its own downfalls that is why different positioning techniques can be combined to compensate the limitations of a single method [1]. The focus of this paper is to identify what filters, design, and hardware would make the most efficient indoor location tracking system.

Advancements in the medical field call for a need of accurate IPS to help track specialists, such as doctors, to where the most efficient path can direct them to the area of need, where to find the specialist, and who else is nearby. This can be provided to the specialist through different paths such as Bluetooth, ZigBee, and Wi-Fi. There are plenty of issues that arise from medical facilities and current tracking systems. The most common form for reaching the specialist is through the pager system. For instance, if the called party does not reply, the controller has no idea whether they are in an area where the signal does not penetrate, have been completely out of the area for some time, have been too busy to reply or have misheard the call-back number [2]. Since the architecture of each location can be drastically different, the proper wireless design would be necessary to avoid running into the issue of areas where the communication signal cannot penetrate or is not normally covered.

One of the biggest challenges with picking hardware and software for the IPS is tailoring it toward the parameters of interest, whether it be toward the medical field previously discussed or if it were to be toward industry where warehouse tracking is necessary. There are many different systems out there and they all vary widely in many parameters including accuracy, cost, size, configurability, security, and reliability [3]. Taking the warehouse example, the user could run into issues such as multipath phenomenon (packets being received 180 degrees out of phase causing nullification). These are issues that cannot be solved using Global Navigation Satellite Systems (GNSS) such as Global Positioning System (GPS). There are many factors with buildings that affect GPS for indoor tracking. One of the greatest issues that comes up is from multipath noise which affects the localization.. This is one of the main motivations for an effective indoor location tracking system.

The filter selection process associated with the indoor positioning systems provides several capable filters, especially for the purpose of tracking. This is the reason why two types of Kalman filter will be explored and tested. The first to be explored in depth will be the Extended Kalman filter which in general uses the same application of the Kalman filter but is able to linearize within a nonlinear system which is most commonly found out in the real world. The next filter will be a new variation of the Extended Kalman filter called the Unscented Kalman filter, which is an extension of the Extended Kalman filter that uses the probability density of the system to estimate points in a form that is closer to Gaussian.

Issues that arise from indoor location positioning devices and algorithms stem from performance of hardware and software versus the cost of the product. This is what makes Kalman filters valuable. Kalman filters provide an efficient way of estimating location and reducing noise at a low cost [4]. This is the reason for testing two different Kalman filters in the same environment to see how they perform and whether or not the Unscented Kalman filter should be used in devices that have location tracking.

Another problem encountered is how to define the system the Kalman filters are used in. Having a linear system is not practical because that is not how environments behave, which is why the system has to be seen as non-linear system. The Kalman filters are able to take this into account with little alterations which will be explained later in this paper. The information required when calculating location with Kalman filters includes the covariance matrix and the mean value. Because we are using linear filters for a nonlinear system, some approximations will have to be made. The reason behind why the mean and covariance are valuable information to the Kalman filter is because they are capable of finding estimated parameter by involving iterative equations expressed in terms of the previous value of the estimated parameters and updated mean and error covariance matrix. In respect to probability density, points are represented as moments. The zeroth moment is always the total probability, the first moment is the mean and this is the reason for why it is important.

One more filter was initially considered as a potential tracker. We researched the particle filter because of its many advantages such as: target presence and absence are explicitly modeled by the probability function, the method can track targets moving randomly in the field of deployment, non-Gaussian noise in sensor readings can be incorporated into the filter by estimating the distribution function of this noise. This incorporates the noise due to calibration errors in sensors in addition to the environmental noise, it permits us to detect targets with variable levels of intensity [5]. The main downfall and reason for not investigating the particle filter further comes from its computationally costly implement. It is very complex.

Aside from having the proper software in place, the design of the WSN and the hardware to be used needs to be identified. There are many different types of products out there that are capable of indoor location tracking. One of the most common products used for tracking is RFID chips, but there are multiple issues that are associated with them. The first problem is that none of the currently available RFID products provides the signal strength of tags directly. Instead, the reader reports “detectable” of “not detectable” in a given range. The second problem is the long latency between a tracking tags being physically placed to its location and being computed by the location server. The third problem is the variation of the behavior of tags [6]. There are other products such as ZigBee that has offered promising results for tracking but could still be more accurate. ZigBee nodes can communicate with each other within a range of nearly 100m outdoors, in free space, but indoors it is usually 5 to 20 m [1]. These all come with their own pros but face significant losses when applied indoors. Investigating a new tracking system which directly applies the acceleration sensor information to the system model based on movement equations and trying field experiments using a software with the proposed algorithm [7] would be ideal for the indoor tracking problem.

Due to the advancements in signal processing and wireless systems, the IoT craze offers an amplitude of innovative ideas that will change the way internet works. The IoT makes every physical object a potential part of a distributed network in which heterogeneous devices autonomously and spontaneously abstract and share context information received from the real world [1]. This is the reason for the focus on WSN and selection process of filters. In IoT environments, position information covers a primal role because it provides useful context knowledge to be associated with other monitored parameters [1]. Once the filter selection and wireless design are implemented to the correct specifications, then an effective indoor positioning system can be ensured.

1.2: Related Work and Standards

Understanding what makes an efficient IPS is an important step for when making the decision of hardware and software. A number of wireless technologies have been used for indoor location sensing, such as Infrared, IEEE 802.11 Wi-Fi, Ultrasonic, RFID, and Ultra-wideband [6]. There has also been numerous techniques that have come up over the past few years that are unique compared to the last. One of the most popular implementations includes calculation of location based on received signal strength indication (RSSI) and variations of time of arrival (ToA). Initially these techniques were tested based on having line of sight (LOS), but as mentioned earlier no building is designed the same, nor is it designed with having LOS from every possible location. This is why there is an emphasis on the proper implementation of a WSN to avoid holes in coverage. If only anchor-anchor synchronization is maintained, the time difference of arrival (TDOA) data can be exploited for localization [8]. Due to the plethora of options out there, a substantial amount of time was required for selecting the optimum technique to implement.

RSSI and ToA as mentioned above are some of the most common techniques for location detection. A clear definition of RSSI is the measurement of the received power level by a sensor. The reason RSSI is used so much is because it follows the inverse-square law, which in the case of location detection with radio waves is approximating the distance of an object by the signal strength information from the transmitter and receiver. This is ideal if everything in the building is LOS, but are not a significant issue if the WSN is implemented correctly and the proper filter is used to reduce the amount of noise in the environment. Equation [1] shows how the inverse-square law relates RSSI (i.e. intensity) and distance (location from sensor).

$$Intensity \propto \frac{1}{distance^2} \qquad [1]$$

Using the RSSI as the indicator for detection allows for a relatively accurate estimation of distance depending on the hardware used. To estimate the unknown position of mobile nodes, at least three anchor nodes must be able to detect and measure mobile node’s signal strength. Each anchor node stores its position coordinates and value of RSSI received from the anchor node and each anchor node is calculated from the measured RSSI shown in Equation [2]

$$D = 10^{\left(\frac{A-RSSI}{10n}\right)} \qquad [2]$$

The cost-effective implementation and the availability of the received signal strength (RSSI) data in all systems makes RSSI-based localization a common choice. However, RSSI-based methods suffer from relatively lower accuracy [9]. With plenty of options for calculating the location of an object and to track it, exploiting a hybrid of angle and range information can reduce the number of required line of sight (LOS) nodes for localization [8]. This is something discussed in great detail in [8] which has a focus on a WSN that has no line of sight (NLOS) between sensor and object. This is a case that is extremely common in WSN and can be unstable under most circumstances [1]. That brings up the point of having the WSN cover the area of desire to the standard of voice over internet protocol (VoIP) which works to -65 decibel-milliwatts (dBm) specifications. The KF preprocessing cannot mitigate the effect of NLOS effectively, especially if the nodes are initially in a severe NLOS situation. Second, the online variance calculation of the range measurement of a moving target might not be done with good accuracy for NLOS links [8]. For this reason, there is a huge focus on how the WSN is designed and implemented.

Some methodologies that can be incorporated with indoor location tracking that will need to be expanded upon are angle of arrival (AoA), fingerprinting, and Cell of Origin (CoO). These techniques are primarily done with triangulation following the mapping of the indoor facility. The triangulation location-sensing technique uses the geometric properties of triangles to compute object locations. Triangulation is divisible into the sub-categories of lateration, using distance measurements, and angulation, using primarily angle or bearing measurements [10]. With the use of the WSN reducing the NLOS areas, the tracking technique can be used to accurately portray the location of the object. This is done with the use of multiple base stations and central server to do computations. Multiple base stations provide signal strength measurements mapping to an approximate distance. A central server then aggregates the values to triangulate the precise position of the tagged object. Finally, the computed object positions are published to client applications [3]. This shows how the deployment of the WSN can affect the accuracy of the location tracking.

There has been plenty of research into what provides the most accurate reading using RSSI. Two common techniques to exploit RSSI for localization are based on fingerprinting signal strengths and conversion of signal strength to distance [1]. This exploitation is seen throughout “Hybrid WSN and RFID indoor positioning and tracking system” with an interest in Ultra-Wide Band technology (UWB). UWB uses low energy devices, such as Bluetooth low energy (BLE), and a high bandwidth to communicate. UWB uses a short-range and high-bandwidth communication technology, with strong multipath resistance and building penetrability [1]. The theoretical accuracy for UWB is supposedly as small as a few centimeters from the object source. This comes up as the ideal choice for technology, but with this high accuracy there is a high cost associated. An issue of UWB regards the expensive cost of a single node which makes the technology unsuitable for extensive deployments [1]. Being that an implementation of IPS would be on such a large scale, this factor weighs in on why UWB is not a viable option.

When trying to calculate the location of an object, most technology will use a lateration or angulation technique to assist in the calculation. There are three general approaches to measuring the distances required by the lateration technique: Direct, Time-of-flight, Attenuation. Angulation is similar to lateration except, instead of distances, angles are used for determining the position of an object [10]. The reason behind these techniques come from the fact that these are based on using proximity sensors to determine position. A proximity location-sensing technique entails determining when an object is “near” a known location [6]. These lead into the use of different technologies, such as RFID, Bluetooth, and Near Field Communication.

Looking more into the hardware necessary for indoor location tracking, passive and active RFID tags are used on a case to case basis. Some key specifications that a consumer would like when having their WSN and indoor location tracking implemented is the system is easy to set up, requires few base stations, and uses the same infrastructure that provides general wireless networking in the building [6]. Identifying the hardware and software will only work as well as the design of the WSN. With that being said, the variety of hardware to choose from can be a cumbersome task, especially considering how unpredictable the medical field can be. Using RFID tags, the network is able to provide a valuable option for the consumer. RFID tags are categorized as either passive or active.

RFID is a means of storing and retrieving data through electromagnetic transmission to an RF compatible integrated circuit and is now being seen as a radical means of enhancing data handling processes [6]. Passive RFID tags operate without a battery and reflect the RF signal transmitted to them from a reader and add information by modulating the reflected signal. Passive RFID technology uses choke point and grid concepts which are meant to use a dense network of low-range receivers in a grid pattern to locate tagged objects. Active tags are ideally suited for the identification of high-unit-value products moving through a tough assembly process [6]. This technology has had promising results for tracking products throughout industry and their speed. In some cases, RF tags can be read in less than 100 milliseconds. The other advantages are their promising transmission range and cost-effectiveness [6]. Active tags suffer from several drawbacks, such as they scale poorly due to the limited range of IR, incurs significant installation and maintenance costs, and performs poorly in the presence of direct sunlight, which is likely to be a problem in rooms with windows [11]. Due to these facts, RFID tags are an extremely viable option for indoor location tracking and could be used in systems where the person or product is constantly tracked.

Depending on the situation of the environment that is being worked in, Wireless Fidelity (Wi-Fi) might seem like a more suitable option. There is an inconsistency between buildings and hospitals which will be discussed later when covering the environment RF works in. RADAR, which is a RF based system, was discovered to have a signal strength at a given location to vary quite significantly (by up to 5 dBm) depending on the user's orientation, i.e. the direction he/she is facing [11]. The Wi-Fi based positioning system is meant to be for applications where an accurate RSSI can be produced such that it can be used in conjunction with fingerprinting to determine position.

One technique that was implemented and analyzed was the iBeacon from Apple. The performance was analyzed in [12] which covered the advantage of a beacon based system. Beacons are Bluetooth Low Energy (BLE) based short-distance communication technologies. Their advantages include low power consumption, miniaturization, wide signal range, and low cost [12]. This beacon system proves that using a BLE system has numerous advantages at a cost. Although the BLE-based iBeacon has many advantages, its performance is poor in terms of the indoor positioning accuracy of a smartphone, and it is difficult to estimate the distance accurately using only the strength of the signal transmitted from the iBeacon transmitter to the smartphone [12]. Perhaps in other research having a hybrid implementation with the iBeacon system would be advantageous to explore.

Now that most of the common technology has been investigated, the selection of the appropriate hardware can be done. Table 1 shows the accuracy of said technologies established in [1].

Table 1: Accuracy and Methods of Current Technologies

Technology	Accuracy	Positioning Methods
WLAN	Meters	Fingerprinting and CoO
Bluetooth	Decimeters to Meters	Fingerprinting and CoO
ZigBee	Meters	RSSI
RFID	Decimeters to Meters	Fingerprinting and CoO
UWB	Centimeters to Decimeters	ToA

The implementation these technologies no matter the positioning method would all perform the same functions as the Kalman Filter tracker. These general functions include: (1) Receive the WSN packets, (2) Extract the sensor ID (node ID), location, RFID tag value (person ID), and time from WSN packet, (3) Calculate velocity & direction of movement of the person using two successive WSN packets having same RFID, (4) Determine the expected location of the person using Kalman filter system model, and (5) Determine the sensor nodes IDs in the expected location with an appropriate Gate size [13].

Everything thus far are dependent on what antennas are used with the transmitter. The solution to location determination using triangulation with Omni directional antennas is not applicable to directional antennas, because the received power depends on angle of transmission and reception as well as distance [9]. This is why the design will be a hybrid implementation. Larger areas, such as warehouses, would cover better using directional antennas, whereas hospital hallways, waiting areas, and emergency rooms would be better suited with Omni antennas. These are typically designed to make the handoff from access point to access point to be smoother.

Some equipment that has been used in recent years include Cisco and Aruba, which uses Wi-Fi services. Cisco's uses Connected Mobile Experiences (CMX) for onsite, online and social analytics for its location-based mobile services. Aruba's hardware uses wireless access points with built-in BLE for asset tracking through any facility. One IPS that has been used more recently is Mist, which uses their own access points to create virtual beacons to detect objects and use machine learning to improve its estimates. Vocera and Stanley Healthcare are devices with their own software that are currently used in hospitals all over the country. These all have issues of requiring a solid network structure.

The only standard thus far for indoor positioning and tracking is ISO/IEC 18305:2016. This standard covers different applications, such as emergency response, government, navigation, social networking, asset tracking in warehouses, and many more. The reasons for creating a standard for IPS is that applications for location tracking inside, for instance warehouses and hospitals, are growing at a tremendous rate and the need for a standard will ensure requirements are set to meet user satisfaction, define metrics for which indoor location tracking can be evaluated, and set jurisdictions on specific products such that there is not a rise in manufacturing costs.

1.3: Practical Uses for IPS

There are many uses for having an indoor location positioning system that push for advancements in technology. Some of the reasons include Geofencing, object tracking and indoor mapping systems. Estimating the position of mobile entities is an important problem for several emerging applications in areas such as advanced manufacturing, Internet of Things, and healthcare systems [14]. Looking into the healthcare system as an example, tracking down specialist in a big hospital can seem difficult most times, but most of the issue comes from having networks with holes in them or that are not designed to handle the tracking devices implemented. Location determination is essential to many sensor networks because most sensor information can only be interpreted by knowing the locations of the source nodes [9]. This is the reasoning for a focus on the initial wireless network design which will ensures the user is getting the best results from their system. One of the main issues that arise when designing a WSN is considering attenuation. GPS is ineffective for indoor tracking because of attenuation, but this is also a possible problem for WLAN.

One of the uses of location tracking that has a lot of interest in the wireless networking field is the topic of Geofencing. Geofencing is essentially using the wireless network, whether it be GPS or indoor, to create a virtual perimeter that restricts access to specific users. This Geofencing could be used in healthcare systems and schools to manage network access to specific areas, such as break rooms and libraries.

Another purpose for the indoor positioning systems that have been used for a couple of year now is using RFID chips and the WSN to help identify objects in a warehouse or storage location. A signal would go out from a transmitter and the RFID receiver would be received and output a sound or flash a light to help a work find the specific object. With the implementation of more IPS there will be more areas that will want to adopt it, such as mapping buildings for museums or malls, buildings that need to be more accessible for the visually impaired, or even public transportation. Indoor location tracking can play a role in emergency services, such as tracking first responders to determine whether or not they are in good condition. With the help of IPS, first responders would be able to determine if more help is required. This is of interest to the government primarily for the purpose of safety and efficiency.

Following everything mentioned thus far, a proper wireless design and accurate indoor location tracking will help industry and the medical field. Tracking with low cost Wireless Sensor Network (WSN), presents its own challenges, namely real time decision making, high frequency sampling, multi-modal sensing, complex signal processing, and data fusion [5]. This is why the implementation of a WSN with the correct hardware and filter will provide an IPS that can be efficient and at a reasonable cost.

Chapter 2: Filter, Hardware and Environment

2.1: Filter Definition and Applications

Before getting into the filter selection between extended Kalman filters and unscented Kalman filter, the definition of what a filter is, what it is used for, and where it came from must be established. A filter is used in signal processing to remove noise that is associated with the environment of application. In this research it is used to remove noise from the signal to enhance the accuracy of the location tracking. Ideally the accuracy of the location tracking is less than 1 meter, but this is no easy task.

The channel model and noise model for this research exclusively deal with an Additive White Gaussian Noise system and a discrete channel where the received signal, R[n], includes the measured , S[n],signal added with the AWGN, W[n], which is shown below in Equation [3]

$$r[n] = S[n] + \mathrm{W[n]} \qquad [3]$$

One of the first implementations of the filter that was used to reduce the amount of noise received is the Bayes filter, also known as the Recursive Bayesian estimation. The Bayes filter is an estimation to determine an unknown probability density function (PDF). The advantages of the Bayes filter is that it has the ability to use multiple sensors to develop weight estimates which provides a higher level of confidence in the estimation. Some of the applications of the Bayes filter are tracking and fusion, and military radar tracking.

The Bayes filter is a proper filter to do state estimation primarily for calculating and determining the orientation and position of an object. The Bayes filter uses the Markov assumption which is essentially using the present state of a process to determine the future state, but not the past events. The Markov assumption is based on the fact that you need to know the state of the current position and do not need to know anything else. With all of this you can use the Bayes filter in two steps: 1) Prediction Step and 2) Correction step. These steps are done by creating a prediction and update step that is also seen in the Kalman filter section. The prediction step is determined by Equation [4].

$$p(x_k|z_{1:k-1}) = \int p(x_k|x_{k-1})p(x_{k-1}|z_{1:k-1})dx_{k-1} \qquad [4]$$

This demonstrates how the prediction step is the sum of the products of the transition between the previous time step (k-1) to the current time step (k) and probability distribution of the last time step. After completing this, the update step can be performed which is proportional to the product of the measured likelihood and previously predicted state shown in Equation [5].

$$p(x_k|z_{1:k}) = \frac{p(z_k|x_k)p(x_k|z_{1:k-1})}{p(z_k|z_{1:k-1})} \propto p(x_k|x_{k-1})p(x_{k-1}|z_{1:k-1}) \qquad [5]$$

The denominator in the update step listed above is a constant to the position of the state, x. The result is what is left in the numerator, which is calculated and normalized.

The perk of the Bayes filter is that there are different realizations of it and the Bayes filter is simply the framework for recursive state estimation. There are different properties to each realization that can be applied to specific applications. The reason these Bayes filters are a popular filter for object tracking is its ability to work in linear and Non-Linear models for motion and observation.

2.2: Kalman Filter

The Kalman filter, which is a form of the Recursive Bayes Filter, is a linear quadratic estimator and an effective and recursive estimator for discrete time linear filtering problem [1]. One of the most applications of the Kalman Filter were used for the navigation and control of vehicles. This easily applies to signal processing in estimating the position of a moving object, but is limited to linear systems. The noise that the environment provides can be minimized with the proper implementation of the Kalman filter when specification of said environment are determined. Calculations for this are made using weighted averages when the confidence of specified points are higher.

For the Kalman Filter, the filtering method consists of three steps: time update, prediction step, and measurement update [15]. When including the design of the wireless sensor network and its implementation, the method with the Kalman filter will be longer. This method is done in five steps: Creation of sensor deployment scenario, Generation of Target(s) movements, Finding the possible paths for the target movement (in regular deployment-in door application), Event generation (detection of target(s) movement by sensor node(s)), Predicting the future position(s) of the target(s) [13]. This is how the prediction step is completed and leads into the time update step.

The KF tracker will perform the following time update stage and measurement step. The update step is to provide a reliable estimate for the next time step of the algorithm as well as the compare the predictions to the actual measurements. The process for the update stage is as follows: Get the next packet having the same RFID, Extract the source ID (sensor ID), Check whether this source ID is contained in the already predicted sensor ID's Gate, if not, determine the velocity and direction from the present and next successive packet [13]. With the prediction and update step being complete, the measurement will be completed. The measurement stage is done by the following process to complete the algorithm: Extract the sensor ID and its location information (obtain through GPS/Localization algorithm) from WSN packet, Compare this location information with the already stored WSN deployment location information, available in the database, Calculate the measurement error and the Kalman gain, and finally to perform measurement updates to minimize the error [13]. This will complete the algorithm process of the standard Kalman filter algorithm.

Below Figure 1 recaps how the Kalman filter performs its process using the prediction, update, and measurement steps mentioned above.

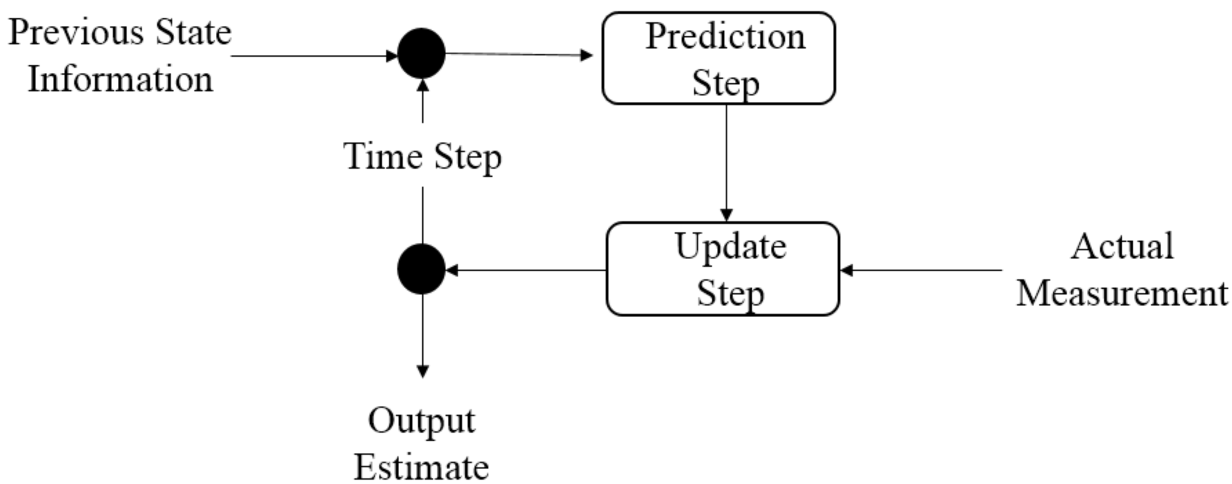

Figure 1: Kalman Filter process

The common route the Kalman filter follows when doing the above approximation is by using the nodes (Wi-Fi access points, Bluetooth receivers, etc.) to receive the information, whether it be the RSSI, ToA, AoA, etc., and sends it to the base station for processing. Kalman filtering then outputs a path estimate that can be used for the desired application shown below in Figure 2 [16].

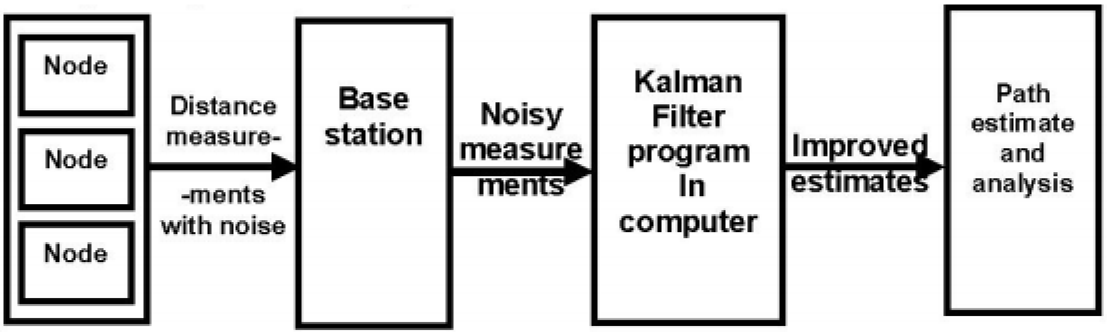

Figure 2: Kalman filter route from sensor to estimation

Understanding the prediction and update step of the Kalman filter need to be explored before diving into the extended Kalman filter and unscented Kalman filter. The prediction stage of the Kalman filter is determined using Equations [6] and [7] where m_k^- and P_k^- are the predicted mean and covariance of the state, respectively, before seeing the measurement:

Prediction

$$m_k^- = A_{k-1} m_{k-1} \quad [6]$$

$$P_k^- = A_{k-1} P_{k-1} A_{k-1}^T + W \quad [7]$$

The update step follows and is determined using Equations [8] through [12] where m_k and P_k are the mean and covariance estimates, respectively, and v_k, S_k, and K_k are the measurement residual, measurement prediction, and Kalman filter gain, respectively.

Update:

$$v_k = y_k - H_k m_k^- \quad [8]$$

$$S_k = H_k P_k^- H_k^T + R_k \quad [9]$$

$$K_k = P_k^- H_k^T S_k^{-1} \quad [10]$$

$$m_k = m_k^- + K_k v_k \quad [11]$$

$$P_k = P_k^- - K_k S_k K_k^T \quad [12]$$

Now that a basis for the extended Kalman filter and unscented Kalman filter have been established, the transition to the two should be apparent.

2.3: Extended Kalman Filter

One of the most common application of Kalman filter to nonlinear systems is the extended Kalman filter. The application for the extended Kalman filter was to extend the Kalman filter to nonlinear problems by creating a Gaussian approximation using Taylor series. The key to nonlinear Kalman filtering is to expand the nonlinear terms of the system equation is a Taylor series expansion around a nominal point [17]. The idea behind the EKF is to linearize a nonlinear function. The way to do this linearization is for the EKF to do local linearization of the using the state transformation function.

The general set up for how variables are fed into the extended Kalman filter are shown below in Figure 3 where **f** is the input and the state transformation, w is the noise from the environment, r_k is the measurement noise, y_k is the output from the nonlinear system, and x_k is the state at which the object is estimated to be.

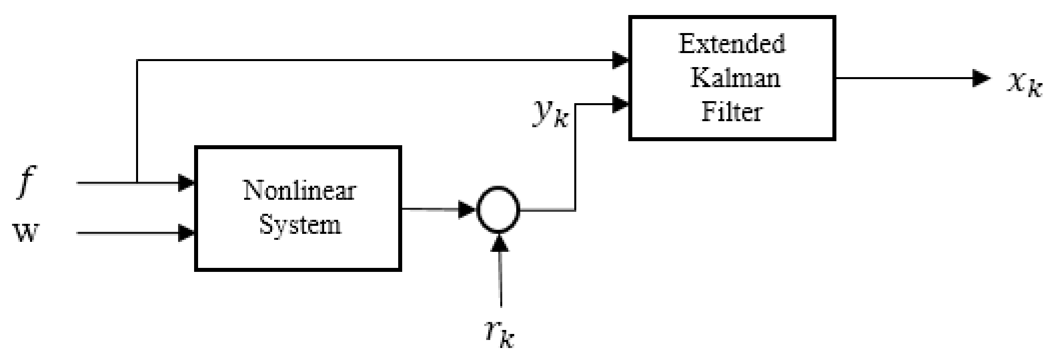

Figure 3: Block diagram of Extended Kalman filter

Predictions are approximated as simply the function of the prior mean value for estimates without any exceptions. The covariance is determined by linearizing the dynamic equations, and the determining the posterior covariance matrices analytically for the linear system [18]. As such, the EKF can be viewed as providing “first-order” approximations to the optimal terms. These approximations, however, can introduce large errors in the true posterior mean and covariance of the transformed Gaussian random variable (GRV), which may lead to suboptimal performance and sometimes divergence of the filter [18]. The EKF provides accurate measurements when using the first order of the Taylor series. Higher orders can be used but are not necessary for this application. In the EKF, the state distribution is approximated by a GRV, which is then propagated analytically through the first-order linearization of the non-linear system [18]. The EKF is able to take the state transformation function and typically output a Gaussian distribution shown below in Figure 4 [19].

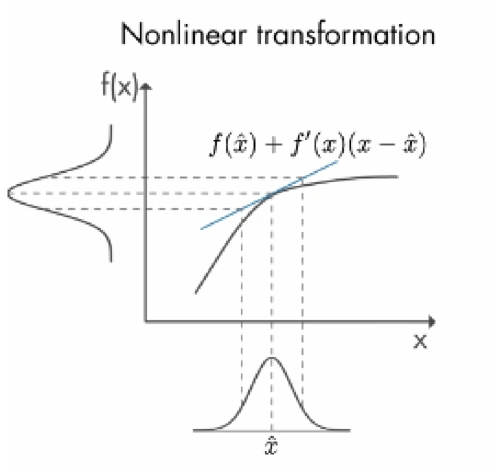

Figure 4: Nonlinear Transformation of Extended Kalman Filter

The model used for developing the EKF is shown below in Equation [13] and [14]

$$x_k = f(x_{k-1}, k-1) + w \tag{13}$$

$$y_k = h(x_k, k) + r_k \tag{14}$$

To reiterate, x_k is the state of the object, y_k is the measurement from the nonlinear system, **f** is the state transformation function, **h** is the measurement model, w is the environment noise, and r_k is the noise associated with the measurement.

The EKF follows the same procedure as the KF with having a prediction and update step which apply to both orders of the Taylor series. For the first order the prediction and update steps are as followed in Equations [15] through [21]

Prediction:

$$m_k^- = f(m_{k-1}, k-1) \quad [15]$$

$$P_k^- = F_x(m_{k-1}, k-1)P_{k-1}F_x^T(m_{k-1}, k-1)P_{k-1} + W \quad [16]$$

Update:

$$v_k = y_k - h(m_k^-, k) \quad [17]$$

$$S_k = H_x(m_k^-, k)P_k^- H_x^T(m_k^-, k) + R_k \quad [18]$$

$$K_k = P_k^- H_x^T(m_k^-, k)S_k^{-1} \quad [19]$$

$$m_k = m_k^- + K_k v_k \quad [20]$$

$$P_k = P_k^- - K_k S_k K_k^T \quad [21]$$

This is where the Jacobian is seen in the EKF, which is one key that differentiates it from the UKF. $F_x(m, k-1)$ and $H_x(m, k)$ are the Jacobians for the state transformation and measurement model which are defined as the following partial derivatives in Equations [22] and [23]

$$[F_x(m, k-1)]_{j,j'} = \frac{\partial f_j(x, k-1)}{\partial x_{j'}} \text{ for x = m} \quad [22]$$

$$[H_x(m, k)]_{j,j'} = \frac{\partial h_j(x, k)}{\partial x_{j'}} \text{ for x = m} \quad [23]$$

One limitation associated with the EKF is that the error propagation plays a big role in whether or not the linear and quadratic transformations are reliable. If the error propagation cannot be approximated then the filter does not perform well. Another limitation of the EKF comes from using the Jacobian matrix. If the Jacobian matrix does not exist then there is a chance of the transformation not working. The EKF has applications that are clearly applicable to the use of indoor location tracking and handling nonlinear systems which are seen everywhere in real world applications.

2.4: Unscented Kalman Filter

The Unscented Kalman Filter is an extension of the EKF that does not use the Jacobian matrices. Initially, the UKF was created with the intuition that it is easier to approximate a probability distribution than it is to approximate an arbitrary nonlinear function or transformation [4]. The reason why this fits into the indoor location tracking problem is because it is meant for nonlinear systems, whereas the EKF is meant for locally linear systems.

The UKF has a lot of similarities to another filter called the particle filter (PF), but there are some key differences that help advocate the use of a UKF. One difference is how the UKF uses sigma points, which are the location of the estimates [15]. Another key difference is the computational cost of the UKF versus the PF since the UKF requires significantly less points to accurately estimate the state of a trackable object. There a few key features that need to be understood to use in any application. First, sigma points are not drawn at random; they are deterministically chosen so that so that they exhibit certain specific properties (i.e. mean and covariance). Second, sigma points can be weighted in ways that are inconsistent with the distribution interpretation of sample points in a particle filter [4]. With that being said, the UKF can be demonstrated the same way as the EKF in Figure 5 below

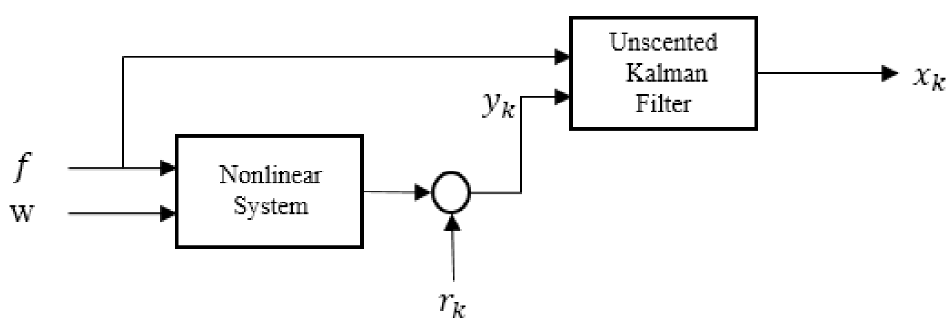

Figure 5: Block Diagram for Unscented Kalman Filter

Since the EKF is approximated by a GRV, which is then propagated analytically through the first-order linearization of at the nonlinear system. This can introduce large errors in the true posterior mean and covariance of the transformed GRV, which may lead to sub-optimal performance and sometimes divergence of the filter. The UKF addresses this problem by using a deterministic sampling approach. The state distribution is again approximated by a GRV, but is now represented using a minimal set of carefully chosen sample points (i.e. Sigma points) [18]. The UKF will provide accurate estimations for the state estimation problem, but comes at a cost. The computational cost of the UKF is equal if not higher than the EKF depending on how nonlinear the system is behaving. In the most extreme case, the UKF can give a delta spike predictive distribution. We call this sigma point collapse [15]. Ideally, the UKF will be able to take the input function and output a Gaussian distribution through the nonlinear system shown in Figure 6 [19].

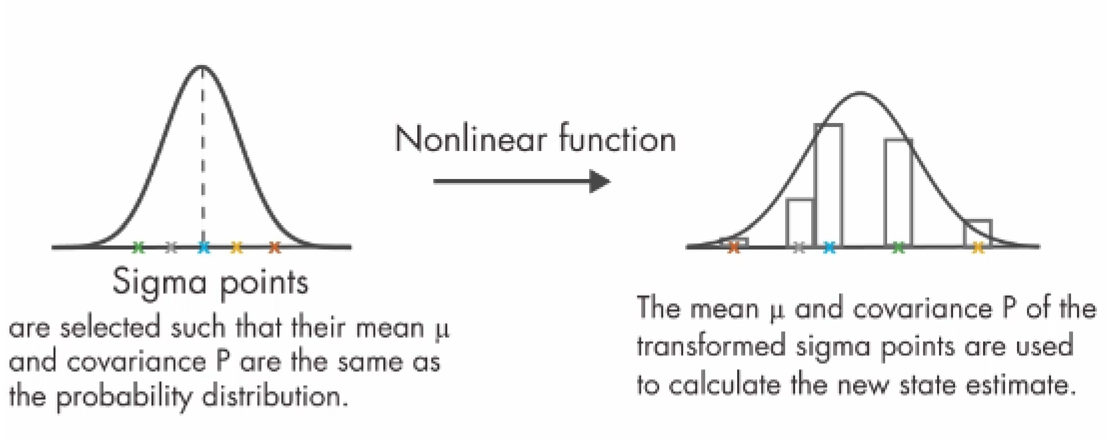

Figure 6: Example of Nonlinear transformation with Unscented Kalman Filter

The UKF represents the extra uncertainty on a linearized function due to linearization errors by the covariance of the deviations between the nonlinear and the linearized function in the regression points [20]. The model for the UKF are defined in Equations [24] and [25]

$$x_k = f(x_{k-1}, k-1) + w \quad [24]$$

$$y_k = h(x_k, k) + r_k \quad [25]$$

Where x_k is the state of the object, y_k is the measurement from the nonlinear system, **f** is the state transformation function, **h** is the measurement model, w is the environment noise, and r_k is the noise associated with the measurement. This leads into performing the prediction and update steps shown below in Equations [26] through [34]. These equations will use the predicted state mean m_k^-, and the predicted covariance P_k^- to determine state of x and output prediction $\hat{x}_k$

Prediction:

$$x_{k-1} = [\, m_{k-1} \;\cdots\; m_{k-1}] + \sqrt{c}\,[0 \quad \sqrt{P_{k-1}} \quad -\sqrt{P_{k-1}}] \qquad [26]$$

$$\hat{x}_k = f(x_{k-1}, k-1) \qquad [27]$$

$$m_k^- = \hat{x}_k\, G_m \qquad [28]$$

$$P_k^- = \hat{x}_k G[\hat{x}_k]^T + w \qquad [29]$$

The Update step of the Unscented Kalman filter computes the predicted mean μ_k and covariance S_k, along with the cross-covariance or both the state X_k^- and the measurement Y_k^-

Update:

$$X_k^- = [\, m_k^- \;\cdots\; m_k^- \,] + \sqrt{c}\,[0 \quad \sqrt{P_k^-} \quad -\sqrt{P_k^-}] \qquad [30]$$

$$Y_k^- = h(X_k^-, k) \qquad [31]$$

$$\mu_k = Y_k^- g_m \qquad [32]$$

$$S_k = Y_k^- G[Y_k^-]^T + R_k \qquad [33]$$

$$C_k = X_k^- G[Y_k^-]^T \qquad [34]$$

Using the above equation, the Kalman filter gain,K_k and mean and covariance of the updated state, m_k and P_k respectively. These are done using Equations [35] through [37]

$$K_k = C_k S_k^{-1} \qquad [35]$$

$$m_k = m_k^- - K_k[y_k - \mu_k] \qquad [36]$$

$$P_k = P_k^- - K_k S_k K_k^T \qquad [37]$$

The Kalman gain follows Figure 7 [21] for the calculation of listed above.

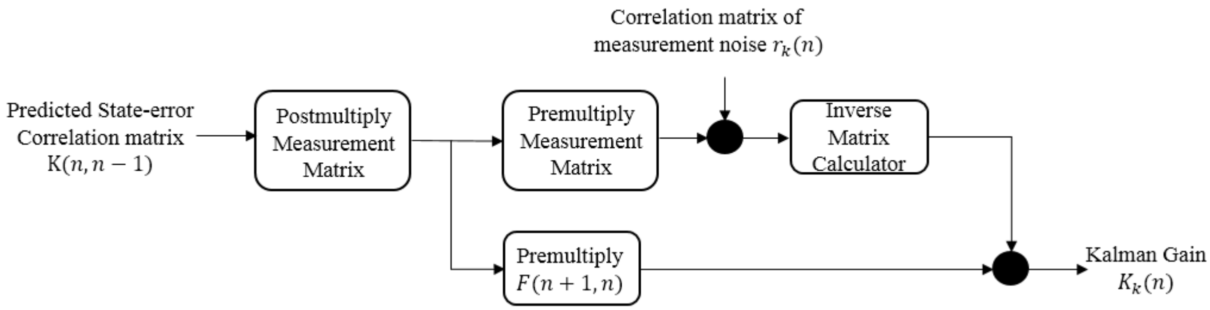

Figure 7: Kalman Gain flow diagram

The main limitation associated with the UKF is the performance time.

2.5: Environment Definition

After establishing the different types of filters that are going to be compared for an optimized indoor location tracking, looking into what affects the environment has on a wireless network and why the initial design is crucial will be established. The characteristics of the environment being worked in are far from Gaussian and require precision placement of wireless access points to ensure proper coverage.

The focus of this report is to create an indoor location tracking system capable of handling nonlinear systems and heavy user density. In Figure 8, the environment is the medium for which the tracking message is sent from the transmitter to the receiver and shows all of the sampling, quantizing, encoding, and modulation required for sending a message. This is where the noise will be added for the filters to manage.

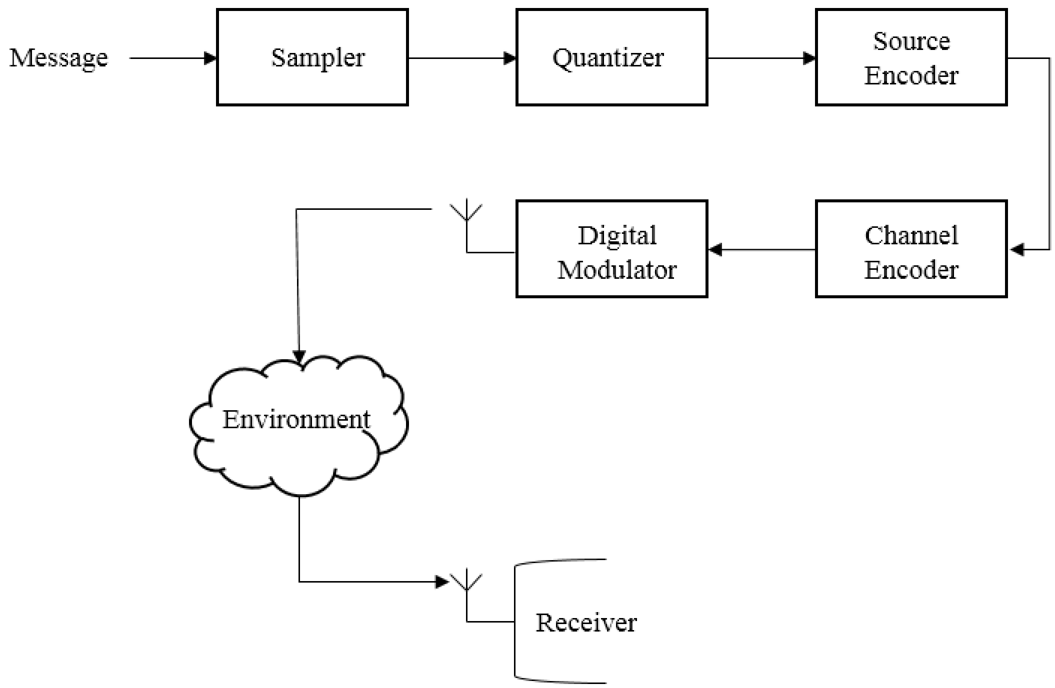

Figure 8: Signal path from transmitter to receiver

The construction of the indoor facility that needs to be covered can vary in material from location to location. Everything from steel to wood will affect how the radio frequency coverage performs. This can be seen with types of walls, material, and even the product inside certain warehouses. The phenomena that will need to be covered are reflection, refraction, diffraction, scattering and absorption. This is where the multipath phenomenon comes into play that was mentioned previously in the first section. There is also the consideration of what antennas are to be used, whether it be directional or Omni directional antennas. Specific access points are meant to operate to certain specifications like height and density of users. This briefly shows the magnitude of attention is required to design a WSN that can support an indoor positioning system.

Reflection, refraction, diffraction, scattering and absorption are different types of annulations that are experienced within wireless networks and designs. These being the reason for packet loss and other WSN design issues. Attenuation is a reduction in the amplitude of the wireless signal that is being sent and will deteriorate the accuracy of the positioning accuracy. Looking at the types of material being used in the area of interest you will see how radio frequency will attenuate. Reflection is when a signal bounces off of reflective surfaces, similar to how light works.

Refraction is the bending of signal that causes issues between sender and receiver. Diffraction being when there is an obstruction in between sender and receiver. Scattering, which is similar to refraction but is caused by dust, humidity and other unevenness formed in nature. Lastly, Absorption, which is when material receives the signal and turns the energy into heat. These attenuations will have negative effects on the performance of the Wi-Fi, for instance the users will most likely experience low data rates and interference.

Another consideration for the design of the wireless network in which indoor location tracking is required is knowing the number of clients that will be communicating with each access point, what types of devices will be used, does the facility allow others to bring their own device on to the site, and the amount of bandwidth required. This all ties back into how the types of hardware being used for indoor location tracking plays a huge role into how the radio frequency will work.

The RF considerations and hardware to do indoor tracking are primarily based around long and short range sensor concepts. Long range sensor concepts deal primarily with turning the power of the device up to cover the entirety of the desired area. This is not ideal for bigger areas such as warehouses, but sufficiently for smaller medical office buildings. For wireless designs, a short range method is used to cover the desired space with sufficient overlap, typically 20% overlap. This is also to ensure that if there was a failure in a node or access point, that the system would still cover the area of the failed access point sufficiently. If the wireless design is done with the consideration of building materials, client requirements, and the type of devices being used then the indoor positioning system can be implemented without the worry of insufficient coverage.

2.6: Pre-Comparison of Filters and Wireless Sensor Network design

Looking into what the consumer desires from their WSN and indoor location tracking filters, a cost conscious network that has accurate equipment and low upkeep costs is the ideal solution. The computational cost of the filters is one way to compare the filters and is important for the consumer depending on their application. The EKF would cost less computationally and perform faster typically, but is not effective on severely nonlinear systems and has the complexity of calculating the Jacobian matrices. The UKF would cost more because of the process of calculating the sigma points, but would most likely have a greater accuracy especially in the sever system the EKF would have trouble in.

The next area that the consumer will be conscious of for their indoor location tracking system is the cost of the equipment and how complex it would be to implement them into a pre-existing network. Depending on the condition of the wireless network, the cost and implementation can come with the task of a complete redesign of the network which ultimately effects productivity. With the implementation comes the upkeep cost which can determine the type of hardware required.

With that being said, both filters will should perform well in the environment laid out. The hardware that would be best for an indoor positioning system is some combination of RFID and Bluetooth that can easily utilize Wi-Fi networks.

Chapter 3: Results

3.1: Filter Performance

The filters were tested using Matlab 2018a to simulate and calculate their performance tracking an object moving through the halls of a hospital and in a warehouse scenario. The multipath phenomenon was ignored in this scenario. The tracking in the hospital was done by starting with the object in the top corner of Figure 9 and having it move around into different rooms, walking through hallways, and going back to a big office area without stopping, similar to how hospital employees have to work.

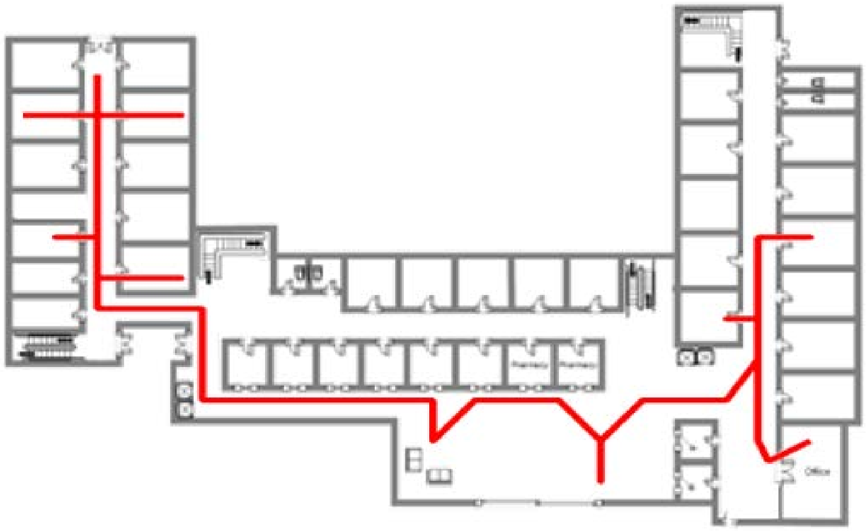

Figure 9: Hospital layout and Path

In Figure 10, a package would be tracked from being brought into a warehouse and then followed until it is sent out.

Figure 10: Warehouse layout and path

After creating a model for the actual path of the object, the Kalman filter, Extended Kalman Filter, and Unscented Kalman filter were used to filter out noise associated with the environment. The path that each of the filters for the Hospital problem are shown below in Figure 11 and Figure 12

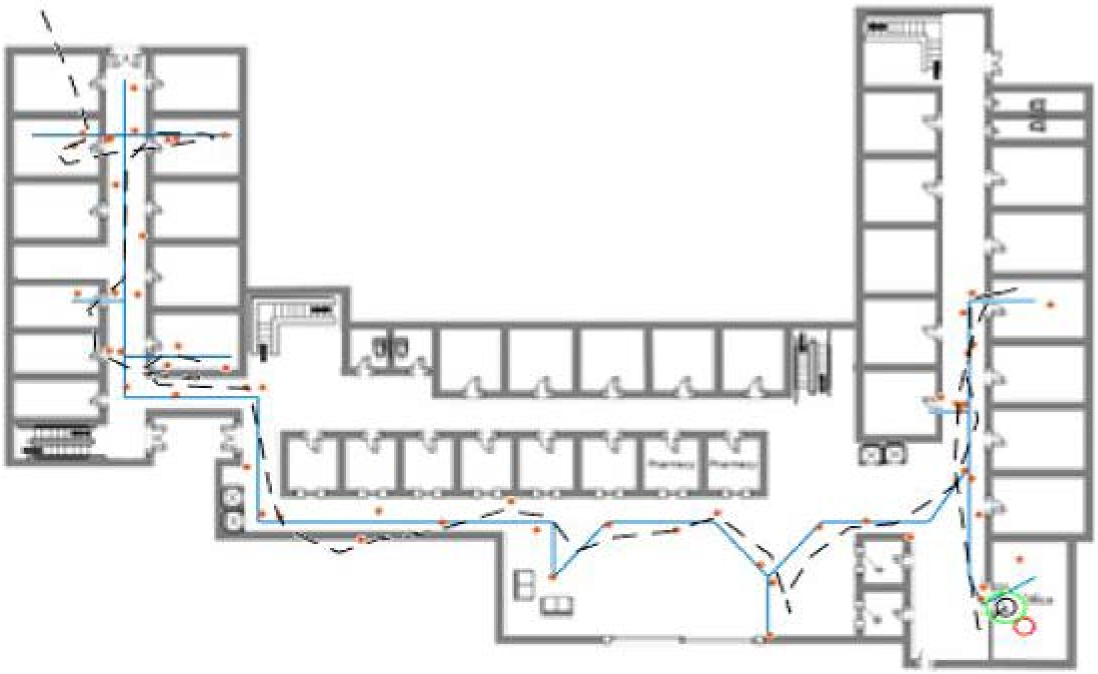

Figure 11: Extended Kalman Filter path for Hospital tracking

Figure 12: Unscented Kalman Filter path for Hospital tracking

The average mean square errors of the KF, EKF, and UKF are provided below in Table 2.

Table 2: MSE of Filters for Hospital tracking after 50 simulations

Method	MSE
EKF	1.70
UKF	1.67

The Signal to Noise (SNR) was calculated to ensure that the filters would eliminate enough noise to work properly shown in Table 3.

Table 3: SNR associated with filters in Hospital tracking

Method	SNR (dB)
EKF	47.5
UKF	48.4

The warehouse problem was performed similar to the hospital tracking problem, but with more data points and the object is stationary at a few locations. The path for the warehouse problem are shown in Figure 13 and Figure 14.

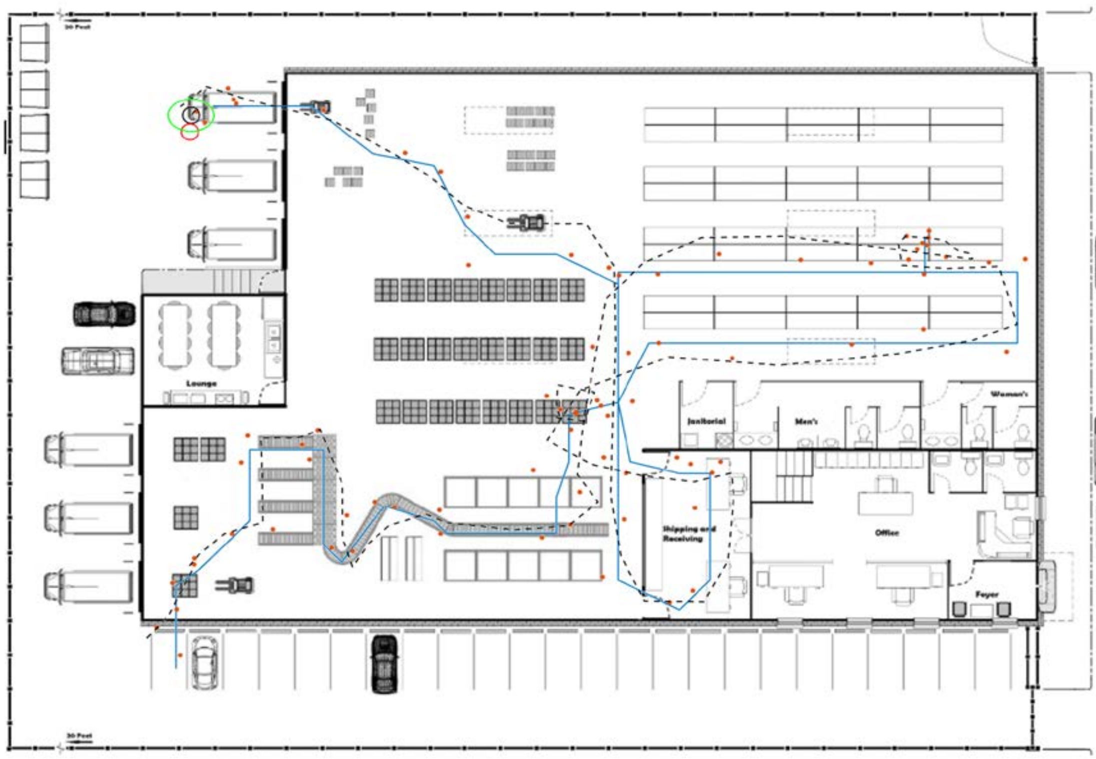

Figure 13: Extended Kalman Filter path for Warehouse tracking

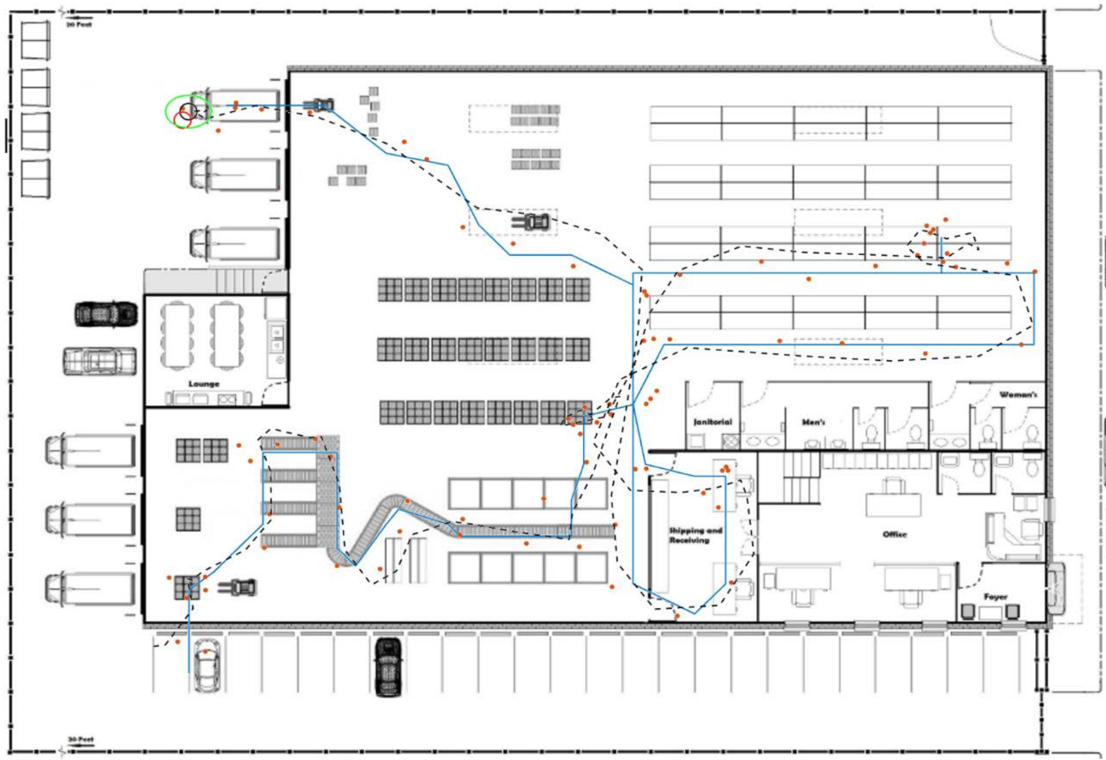

Figure 14: Unscented Kalman Filter path for Warehouse tracking

The average mean square errors of the KF, EKF, and UKF are provided below in Table 4.

Table 4: MSE of Filters for Warehouse tracking after 50 simulations

Method	MSE
EKF	1.086
UKF	1.092

The SNR for the warehouse tracking is shown below in Table 5.

Table 5: SNR associated with filters in Warehouse tracking

Method	SNR (dB)
EKF	31.2
UKF	31

3.2: In-depth Comparison

After analyzing how the filters performed in the simulation, the filters performed well in tracking the object moving throughout the hospital floor and provided close errors. With the two filters performing relatively close, factors like cost and accuracy are used to determine the best filter for any indoor tracking application. Due to the fact that one facility to another can be drastically different in materials and density of users, the EKF would be ideal for its low computational cost. The UKF would have a far higher computational cost and a slightly higher accuracy. The time difference the filters took to perform the same nonlinear tracking problem was substantial, shown below in Table 6.

Table 6: Computation time of filters for Hospital problem

Method	Time (ms)
EKF	3.73
UKF	17.33

The UKF took approximately 4.6 times longer to have a 1.76% increase than the EKF. This is backed up with the warehouse example where the UKF was 6.8 times longer than the EKF and had a 0.5% decrease from the EKF. The times for the warehouse problem are shown below in Table 7.

Table 7: Computation time of filters for Warehouse problem

Method	Time (ms)
EKF	2.59
UKF	17.77

The error that is associated to each of the problems would have a difference of a couple of feet. For the hospital tracking problem, the EKF would have an MSE of 5.1 ft and the UKF would have an MSE of 5 ft. For the warehouse problem, the EKF would have an MSE of approximately 3.25 ft and the UKF would have an MSE of 3.27 ft. These would be increased with the mapping of the facilities being implemented because the tracking would be able to determine the objects possible routes through the facility. The Signal to Noise Ratio for both tracking scenarios were relatively close for both filters and point to the EKF being the optimal choice.

Chapter 4: Conclusion

4.1: Conclusion

This report provides a start for the development of indoor positioning systems and the filters associated with the hardware that perform the tracking. All of the current hardware that is being use by Cisco, Ekahau, and Mist were covered and are already being use throughout industry but still require improvement. With new hardware and algorithms coming out regarding indoor tracking, the need for a well-defined standard will be required.

The wireless sensor network design was briefly covered to keep some emphasis on the initial design of the network having an effect on wireless applications like indoor location tracking. Design of the network is the basis for all of the indoor tracking applications, such as Geofencing. The implementation of Geofencing should be capable of granting or limiting access to as close as a few feet of the specified border depending on how the wireless network is designed, the user maps out the facilities for the indoor location tracking, and the filters ability to manage noise from the environment.

With hardware improving every year, the proper filter selection was explored and concluded on implementing the Extended Kalman filter because of its ability to work to create locally linear systems that simulate the typical path someone or an object would take throughout the indoors of a building. Due to the cost in the two filters being relatively equivalent in the complex nonlinear system case and the error that the filters provide, the EKF would only have the advantage of being less computationally in less severe scenarios and being able to perform close in the extreme cases.

4.2: Future Works

Moving on from this work, I would like to see this being applied on hardware, whether it be RFID, Bluetooth, or even creating a sensor to use. I would also suggest looking into using some sort of learning system since it requires no modeling. Another idea that should be explored is using active tags in an ad hoc network style tracking. These will all prove to be challenging and novel tasks that will move indoor location tracking quicker into industry and the medical field.

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