Semi Analytical Approach for Binary Mixture Conductivity in Hydraulic Fracturing

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Abstract
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In hydraulic fracturing, a proppant injection schedule practice typically applies a binary proppant mixture (for example: 100 Mesh sand following by 40/70 Mesh sand in well MIP-3H, Marcellus Shale). The former injection agent is finer in size or less resistant to stress, whereas the latter injection agent is coarser in size or more resistant to stress. This practice creates a special region inside the fracture, in which two injected proppant types co-exist and is defined as the mixture zone. This research concentrates on the variability of the mixture zone under impact of different factors, introduces novel semi-analytical modelling approach to better estimate the hydraulic conductivity inside the mixture zone, and further applies this novel approach to prove its efficacy in conductivity estimation and cumulative production prediction.

Variability of the mixture zone is studied by an OAT (One at A Time) sensitivity analysis to examine the percentage of the mixture zone's area over the total propped area under variability of different parameters, including reservoir properties, geo-mechanics, and different design parameters in a proppant injection schedule. The novel semi-analytical model is derived by independent modelling for proppant pack's permeability and width. Permeability model is an improvement from Carman Kozeny equation, in which Internal Specific Area is re-derived to differ a binary mixture from a single proppant type. Fracture width model is derived from Hertzian contact theory under assumptions of parabola distributed stress, elastic behavior and dual-layer schematic. Trust-region method is applied to determine all coefficients in the permeability model, which complies with a non-linear least square problem. Coefficients in the width model are determined by linear approximation from an in-house fracture width database.

Based on satisfied validation results (7.98%-21.82% MRE), trust-region algorithm determines the novel model's coefficients using lab data of only two Weight Concentration Ratios. The novel model is expanded to predict binary mixture's conductivity at arbitrary confining stress and Weight Mixing Ratio values, which avoids misleading experimental outcomes. Proppant particle size distribution between two discrete Mesh numbers, deformation complexity between particles (quadratic form) and proppant crush effect are prospective improvements for better modelling validation results. LWAM is proved to overestimate conductivity compared to the novel model. Overestimation degree, which can exceed 70%, is separated into 3 overestimation zones ($\leq 20\%$, 20-60% and $\geq 60\%$) and examined by the comparison matrices. When contrast in density and Mesh size between proppant types in a mixture is clear (for example: mixture 40/70 sand - 20/40 ceramic), overestimation is extreme, and when internal contrast between proppant types in a mixture is reduced (for example: mixture 20/40 sand - 20/40 ceramic), overestimation is dampened.

The case study for Marcellus Shale applies comparison matrices to predict LWAM's conductivity overestimation and conduct 1-year and 10-year cumulative production analyses. Reduction of confining stress axis in comparison matrices to 5400-6200 psi with a maximum difference of 200 psi between different depths (based on Marcellus Shale minimum horizontal stress data) allows predictive reasoning for conductivity overestimation from weight concentration ratio distribution. Overestimation for cumulative production data is observed to approach 10.73% (3.274 MMSCF in the early production time for a quarter fractured area). This suggests the level of risk caused by application of LWAM in reservoir simulation, depending on the intrinsic contrast between proppant types' Mesh sizes and densities, in the selected proppant injection mixture.

Dedications

This thesis is totally dedicated to my family members. My father, albeit being a tough father when communicating with his children, always support me financially and never complains about my educational expenses to me or my mother. My mother, who is truly my great supporter, life advisor and listener, has been following me closely since I was born. From her, I continuously learn how to be a kind, educated and thoughtful adult. Without their continuous support, I will never achieve my life targets and proceed higher education successfully.

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Nomenclature

$\underset{C}{\operatorname{argmin}} F$: minimization of function F, depending on the variable C (equation (2.2))

a : radius of contact surface (equation (1.11))

a_v : Internal Specific Area

A : contact area of the loaded proppant pack in API conductivity cell

A_{mix} : fractured area in which two proppant types co-exist (i.e. Mixture Zone area)

A_{tot} : fractured area in which there exists injected proppant particles

$A_{surface}$: surface area of solid particles

$ArConc_1$: areal concentration of proppant type 1

$ArConc_2$: areal concentration of proppant type 2

B^{-1} : inverse of matrix B

CFD: Computational Fluid Dynamics

C : standard weight concentration in conductivity lab measurements (typically 2lb/ft²)

C_f : hydraulic conductivity of a fracture

C_{lab} : conductivity estimation based on lab conditions

$C_{reservoir}$: conductivity estimation based on reservoir conditions

$C_{fracture\ simulator}$: conductivity estimation from the fracture propagation simulator

C_{CMG} : conductivity data imported into CMG reservoir simulator

C_{LWAM} : conductivity estimation using LWAM

$C_{semi-analytical\ model}$: conductivity estimation using the novel conductivity model

C_o : initial value for iteration process of variable C

C_k, C_{k-1} : value of variable C (at iterations kth and k-1th)

d : diameter of flow tubes (in Hagen-Poiseuille equation)

$d_{coarser}$: diameter of the coarser particle type

d_{finer} : diameter of the finer particle type

D_i ($i = 1, 2$): mean diameter of particle types in a binary mixture

ΔD : width deformation of a proppant pack

E, ν : Young Modulus and Poisson Ratio

E_r, ν_r : Young Modulus and Poisson Ratio of contact rock surfaces or steel plates

E_p, ν_p : Young Modulus and Poisson Ratio of contact proppant particles
 F_D : multiplication factor for effect of particle size distribution
 $f(C, x_{data})$: general function of the coefficient determination problem (equation (2.1))
 G, E^* : notation of equivalence to quantity $\frac{E}{1-\nu^2}$
 h : total of width deformation and width embedment of a proppant pack
 i ($i = 1, 2$): subscript for two proppant types in a binary mixture
 ISP: Intermediate Strength Proppant
 I, J, K : notations of 3 dimensions in CMG reservoir simulator
 KGD: Kharsitjanovic Geertsma-de Klerk (name of a 2D fracturing model)
 K_o : Kozeny empirical constant/coefficient
 k : permeability
 k_f : fracture permeability
 k_{CMG} : permeability data imported into CMG reservoir simulator
 LWC: Light Weight Ceramic
 Mesh: US Sieve size (i.e. the number of openings in one square inch of a screen)
 MRE: Mean Relative Error
 M : total weight of a proppant pack
 M_i ($i = 1, 2$): weights of particle types in a binary mixture
 n : coefficient in the exponential tortuosity model (equation (1.10))
 nI, nJ, nK : number of grids for 3 dimensions in CMG reservoir simulator
 N_i ($i = 1, 2$): number of particle proppant types in a binary mixture
 N_{mono-i} ($i = 1, 2$): number of particles in a monolayer of proppant types in a binary mixture
 OAT: One at a Time
 PKN: Perkins Kerns Nordgen (name of a 2D fracturing model)
 P : pressure
 P_L : pressure corresponding to one-half of Langmuir volume V_L (equation (5.1))
 P_c : effective confining pressure/stress
 P_o : pressure/stress at initial condition (i.e. atmospheric condition in this thesis) or maximum contact pressure (in Hertzian contact theory)

P_s : average contact pressure on each proppant particle (equation (4.19))

$\mathbf{p}$: norm of vector $\mathbf{p}$ (i.e. length of vector $\mathbf{p}$)

p : coefficient in the natural logarithmic tortuosity model (equation (1.8))

p : step size (equation (2.3))

p_k : step size between iterations k^{th} and $k-1^{\text{th}}$

r : radius of flow tubes (in Hagen-Poiseuille, equation (1.1)) or distance from the contact point to the located proximity where contact pressure is P (equation (1.12))

$\mathbb{R}$: notation for set of all real numbers

R : Weight Ratio, Weight Mixing Ratio, Weight Concentration Ratio, or contact particle's radius

R^2 : residual error between lab data and modelled data

R_a : equivalent radius of the contact object in Hertzian contact theory

RCS: Resin Coated Sand

s : specific surface area

span: set of all possible combinations of two arbitrary vectors (linear algebra, equation (2.7))

TotArConc: total areal concentration

ULW: Ultra-Light Weight

V_{solid} : volume of solid particles

$V_{\text{one particle type } i} (i = 1,2)$: volume of particle types in a binary mixture

V_L : maximum gas volume at infinite pressure (in Langmuir model, equation (5.1))

$\mathbf{x}^T$: transpose of vector $\mathbf{x}$ (same notation for transpose of a matrix)

X : coefficient in the square-root tortuosity model (equation (1.9))

w_{CMG} : default width data in CMG reservoir simulator

w_{ref} : reference width in the API conductivity cell (for proppant pack's width measurements)

w_f : fracture width of a proppant pack under applied effective confining stress

w_r : vertical deformation of the contact particle (in Hertzian contact theory)

$w_{fi} (i = 1,2)$: fracture width of a proppant pack (loaded by a single proppant type) under applied effective confining stress

w_{f0} : fracture width of a proppant pack under zero effective confining stress (i.e. initial fracture width)

$w_{f0i} (i = 1,2)$: initial fracture width of proppant types' sub-layers in the dual-layer unity

y_{data}, y_{lab} : general notation of the measured/lab-computed quantity (equation (2.1), (4.44))
 $\alpha, \alpha_1, \alpha_2$: coefficient to determine the step size (equations (2.6), (2.7))
 β : compressibility factor or the non-Darcy factor (equation (5.2))
 γ : skewness in particle size distribution of an arbitrary proppant pack
 δ : maximum width embedment (in Hertzian contact theory) or coefficient of variation in an arbitrary proppant pack (equation (1.5))
 ρ_i ($i = 1, 2$): densities of particle types in a binary mixture
 σ_H : Minimum horizontal stress (i.e. equivalent to confining stress in this thesis)
 $\emptyset$: porosity
 $\emptyset_o$: initial porosity (i.e. porosity of proppant pack at zero applied effective confining stress)
 τ : hydraulic tortuosity
 τ_o : initial hydraulic tortuosity (i.e. hydraulic tortuosity of proppant pack at zero applied effective confining stress)
 $\partial RD / \partial P_c$: partial derivative of Relative Difference in term of confining pressure
 $\partial RD / \partial R$: partial derivative of Relative Difference in term of weight concentration ratio
 Δ : upper bounded limit for trust-region radius (in trust-region method)
 ∇ : first derivative operator (equation (2.3))
 $\% \text{ Mixture Zone}$: percentage of A_{mix} compared to A_{tot}
 $\% \text{ Mixture Zone}_{base}$: $\% \text{ Mixture Zone}$ computed at the base case
 $\% \text{ Mixture Zone}_{sensitivity}$: $\% \text{ Mixture Zone}$ computed at any sensitivity cases

Introduction

In hydraulic fracturing, proppant injection schedule is planned to maintain high conductive fractures. A proppant injection schedule may apply a binary proppant mixture (100 Mesh sand following by 40/70 Mesh). MIP-3H is an example well in Marcellus Shale which has binary mixture injection practices. The 1st injection proppant type is finer/less resistant to stress (100 Mesh), and the 2nd injection proppant type is coarser/more resistant to stress (40/70 Mesh). Binary mixtures create a region inside the fracture, in which two injected agents co-exist, defined as a mixture zone. Conductivity in this mixture zone is known to be estimated by Linear Weight Average Method (LWAM). However, validation for this model's efficacy has not been conducted. Therefore, this research aims at two purposes. The first purpose is to examine behavior of the mixture zone under changes in reservoir and treatment conditions. The second purpose is to derive a novel conductivity model which is used for mixture zones and expected to quantify conductivity estimation by LWAM.

Behavior of the mixture zone is examined by designing a local and standard One at A Time (OAT) sensitivity method (Morris, 1991). This sensitivity analysis combines a tested model, input and output factor(s). The tested model is an in-house fracture simulator which follows PKN fracture model (Gu, 2013). Input factors are selected from screening process. Output factor is percentage area of mixture zone. To acquire a novel conductivity model for mixture zones, semi-analytical method and independent modelling for permeability and fracture width are directed. Permeability model inherits fundamentals in Carman-Kozeny equation (Kozeny, 1927; Carman, 1939). Internal Specific Area is re-derived for binary mixtures. Fracture width model inherits fundamentals in Hertzian contact (Hertzian, 1882) and elastic behavior concepts. For binary mixtures, an idealized dual-layer schematic is proposed to use these concepts simultaneously.

CHAPTER 1 –UNCONVENTIONAL RESOURCES AND HYDRAULIC FRACTURING

1.1. Unconventional resources

According to Ehterington (2005), shale resources are characterized by their inability of commercial production without any stimulation techniques and/or special recovery processes. These resources, which have been founded, investigated and studied for their decades (King, 2010), have been playing a dominant role in creating production benefits and playing a dominant role in the oil and gas industry. According to US Energy Information Administration (EIA) in 2014, shale resources have been recorded a remarkable amount of technically recoverable 345 billion barrels of oil and 7299 trillion cubic feet of gas. Figure 1 presents the contribution of unconventional resources worldwide.

Figure 1 – Overview of unconventional plays, worldwide (EIA, 2014)

Besides existence of unconventional resources worldwide (China, Norway and European countries), US has significant and reputable unconventional deposits. A list of unconventional

plays in the US include: Bakken in North Dakota, Barnett in Texas, Collingwood-Utica in Michigan, Woodford in Oklahoma, Marcellus in West Virginia and Pennsylvania. Figure 2 presents the contribution of the unconventional plays in the US.

Figure 2 – Overview of unconventional plays, United States (EIA, 2016)

Based on the dominance of the unconventional plays' capability in figures 1 and 2, it is undeniable to state that investment and study in extracting highest possible amount of petroleum from these unconventional resources is essential and beneficial for the industry. However, the definition of unconventional resources also infer the necessity of special recovery techniques specialized for this type of resources. Stimulation and especially hydraulic fracturing are two most well-known and professionally-applied techniques for unconventional resources in the industry.

1.2. Hydraulic fracturing

1.2.1. Theory of fracturing

Early in 1952, hydraulic fracturing concept was introduced by H.K. Van Poolen. Hydraulic fracturing is the application of fluid pressurized injection to a desired section of a formation until

rock failure occurs. Continuous pumping of fluid extends the break, which creates new and larger flow channels. Additional injection of proppants (for example: Ottawa sand, bauxite, Ultra Light Weight –ULW) supports keeping the fracture open after injection pressure is released from the injection process. (H. K. Van Poolen, 1952). Based on this author's study and the description of hydraulic fracturing , the technique involves the following fundamentals:

- Principle of rock failure
- Injection of fracturing fluid and proppants
- Creation of highly conductive channels

Within three fundamentals, principle of rock failure is the base in fracturing theory, and it initiates the understanding of the remaining two principles. In the reservoir, rock is affected by different types of stresses. For a microscopic representation under general scope, figure 3 describes principle stresses which impact a portion of rock's state (H. K. Van Poolen, 1952):

Figure 3 – Principle stresses impacting a portion of rock (Petrowiki)

With an exception of shallow depth, approximately at 1000 ft or shallower (where no formation of interest for petroleum has been commercially recorded), the value of overburden stress is the largest stress applied to the rock. Additionally, there always exists one lower horizontal

stress and one higher horizontal stress value, within two present horizontal stresses in figure 3. The lower horizontal stress value is defined as the Minimum Horizontal Stress, denoted as σ_H . Several studies prove the opening and propagation of a fracture inside the reservoir rock is perpendicular to the direction of the Minimum Horizontal Stress, albeit a few claims of extreme shear should be the main reasoning in replacement of the Minimum Horizontal Stress.

1.2.2 Fracture modeling

Fracture propagation modelling started to receive concerns from 1955, when the first two-dimensional fracturing model was introduced by Howard and Fast. In the following years, other enhancements of two-dimensional fracture modelling were published, which include: PKN (Perkins, and Kern, 1961, later modified by Nordgen, 1972) and KGD (Khristianovitch and Zheltov, 1955; Klerk and Geertsma, 1969). These modelling approaches have been successful until the intrinsic complexity of fracture propagation inside the reservoir rock arised more focus for a better fracturing design and treatments. Because of this complexity, pseudo three-dimensional and fully three-dimensional fracture modelling approach began replacing two-dimensional models and have been being widely practiced in the industry. Examples of these approaches include Settari and Cleary (1986), Gu and Leung (1993), Sounes et.al (1993).

In this section, theoretical reviews for both two-dimensional models (including PKN and KGD) and three-dimensional models are provided. Primary, derivation and numerical solutions relevant to all models are fully described in Ming Gu (2013).

PKN model

PKN model was initially derived by Perkins and Kerns in 1961. There are 4 primary assumptions in the derivation process of PKN, as follows:

- Fracture height is constant and not dependent on fracture length.

- In the vertical cross-sections (perpendicular to the propagation direction), fracture fluid pressure is constant.
- 2D strain deformation occurs only in the vertical plan.
- The cross-section (vertical section) has an elliptical shape and the maximum width is located at the centre of fracture height.

Figure 4 - Schematic for a PKN model (Petrowiki)

Based on these 4 modelling assumptions, the fracture width, maximum fracture width, were acquired. In 1972, concerns for a Newtonian fluid and Laminar flow regime, the pressure gradient along the fracture propagation orientation was derived by Nordgren using the volume balance equation. Nordgren additionally studied the leak-off effect using constant leak-off rate.

KGD model

This 2D model exhibits similarities with PKN regarding constant fracture height and variable width values along the fracture length. However, KGD has its own differences, as follows:

- Horizontal plane has elliptical shape (in replacement of vertical plane in PKN)

- Fluid pressure varies along the propagation orientation (in replacement of constant in PKN)
- Vertical cross-section plane along the fracture height is a rectangular form (in replacement of a elliptical form in PKN)

Figure 5 – Schematic for a KGD model (Petrowiki)

Three-dimensional models

As PKN and KGD are reviewed, both of the 2D models assume constant fracture height as one of their core assumptions prior to the modelling process. However, reasonable interpretation from stress contrast between the interest formation and its adjacent formations state that fracture height is variable, since fractures may propagate through the interested formation and reach the adjacent formations. This drawback from 2D models were addressed by three-dimensional models, which imply variable fracture height in the modelling process.

Figure 6 – Correction for fracture height in pseudo 3D models (Petrowiki)

1.3. Proppants and hydraulic conductivity

Proppants are natural solid particles (for example: Ottawa sand) or artificially-manufactured particles (for example: synthetic bauxite) which have various physical sizes, densities, chemical properties, and mechanical properties. Based on available records of hydraulic fracturing practice in US shale formations, sand and ceramic are two most common categories in a total of more than 50 proppant types available in the industry (Michael J. Economides and Tony Martin, 2007).

Sand

Sand has been undeniably the most common proppant from the time practice of injection solid material into the fractures was initiated (early 1950s). Ottawa sand (i.e. “white” sand, originality is Illinois, US) or Hickory sand (i.e. “brown” sand, originality is Texas, US) are two popular names of nature sand proppants. Sand is primarily made of quartz, chemically known as silicon dioxide (SiO_2), and typically has nominal density 1.631 g/cc. Being advanced over decades

of hydraulic fracturing practice, sand proppant has been artificially manufactured besides natural resources (for example: Resin Coated Sand, or RCS) to adapt to harsher stress conditions inside reservoirs. Albeit the fact that sand is inexpensive and readily available because of resource diversity, brittle failure issue of quartz combined with high level of embedment under contact with rock surfaces at high stress (Michael J. Economides and Tony Martin, 2007) prevent sand from being evaluated as a qualified candidate in all fracturing practices.

Ceramic

Ceramic was addressed as a better candidate compared to sand, and it was invented in the 1970s. The main chemical component of ceramic is aluminium oxide (Al_2O_3) and its density is nominally 3.8 g/cc. Ceramic is known to be typically heavier and better resistant than sand. High resistance proved ceramic's capability in hydraulic fracturing when being applied to deep gas wells. Similar to sand, ceramic has been enhanced to meet the industry's needs. The approach to diversify ceramic is essentially modification in the percentage of aluminium oxide in ceramic proppant types. For example, Intermediate Strength Proppant (ISP) has 70% Al_2O_3 , bauxite has 80% Al_2O_3 and Light Weight Ceramic (LWC) has 50% Al_2O_3 (Michael J. Economides and Tony Martin, 2007).

In hydraulic fracturing, proppants play a critical role in performance of effective fractures. Proppants are purposed to sustain from the impact of reservoir stresses, keep the fracture remain open, and artificially open a highly porous and especially permeable medium. This new medium exists as a highly conductive channel to support flow of petroleum fluid from the reservoir into the wellbore (as the natural porosity and permeability of shale are notoriously low). Therefore, effectiveness of proppant injection is examined by the fluid flow capability of the created conductive channels. Albeit permeability is the petroleum concept to describe the fluid flow capability inside a porous medium, hydraulic conductivity is a more conventional concept to

describe fluid flow capability inside propped fractures. The definition of hydraulic conductivity (shortly abbreviated as conductivity in hydraulic fracturing field) is:

$$C_f = k_f w_f \quad (1.1)$$

Conductivity modelling has been commonly performed by separate modelling for fracture permeability and fracture width. Later in this chapter, review of different modelling approaches for fracture permeability and fracture width are detailed.

1.4. Proppant studies for binary mixtures

Since this thesis focuses on studying binary proppant mixtures, review for previous studies regarding binary mixtures is certainly necessary. Albeit several studies focusing on permeability and/or conductivity for a single proppant type, binary proppant mixtures have not received similar concern degree. Particularly, binary proppant mixtures, which are composed of two proppant types (different in sizes or densities), have been in shortage of experimental data for permeability and/or conductivity. The earliest experimental study for binary proppant mixtures were conducted by Robert R. McDaniel and John R. Willingham (1978). Recently, other comprehensive studies for binary proppant mixtures were conducted by Eric Schultz at University of Texas at Austin (2014) and Ming Fan et.al (2020).

In the study by Robert R. McDaniel and John R. Willingham (1978), experiments were performed for permeability measurements merely. 3 types of materials (Sand, Sinter Bauxite and Glass Beads) and 3 types of Mesh sizes (10-20, 20-40, and 100) were selected. Additionally, 3 weight ratios were prepared for each binary mixture (90%/10%, 75%/25% and 50%/50%), with two subtle changes to 95%/5% (from 90%/10%) and 80%/20% (from 75%/25%) for one binary mixture. Albeit the diversity of options from these materials and Mesh sizes, only 3 binary mixtures

were tested in this study. Experimental settings and measurement procedure closely followed Darcy's Law to obtain permeability.

There are advantages in the design of experiments in Rober R. McDaniel and John R. Willingham (1978) compared to Eric Schultz (2014). All studied binary mixtures by these authors were prepared with 3 weight ratios, which provides more certainty in analysis of coefficients during analytical/semi-analytical modelling process. Furthermore, the binary mixtures of sand (i.e. Hickory Sand - 100 Mesh Sand), which is still a common practice in the industry because of cost efficiency, was studied more carefully compared to similar sand binary mixtures in Eric Schultz (2014). However, an important disadvantage in the study by these authors is shortage of concern for conductivity data for tested binary mixtures, which challenges the modelling validation processes and does not support the primary motivation of this thesis.

In the study by Eric Schultz (2014), experiments exhibited a more systematic degree compared to Rober R. McDaniel and John R. Willingham (1978). Since a portion of the lab data by Eric is selected for modelling and validation in this thesis, his experiment settings and options of binary mixtures should be reviewed in further details.

Regarding the options of binary mixtures, 3 types of different materials (sand, ceramic and Ultra Light Weight – ULW), and 2 range of Mesh sizes (20/40 and 40/70) were selected. Besides, at least one weight ratio was decided to mix the proppant types in a binary mixture (25%/75%, or 50%/50%, or both ratios). In specific, these binary mixtures were measured in Eric's study:

- 20/40 sand and 20/40 ceramic, weight ratio 25%/75%
- 20/40 sand and 20/40 ceramic, weight ratio 50%/50%
- 20/40 sand and 40/70 ceramic, weight ratio 75%/25%
- 20/40 sand and 40/70 ceramic, weight ratio 50%/50%

- 20/40 ceramic and 40/70 sand, weight ratio 70%/25%
- 20/40 ceramic and 40/70 sand, weight ratio 50%/50%
- 40/70 sand and 40/70 ceramic, weight ratio 50%/50%
- 20/40 sand and 40/70 sand, weight ratio 50%/50%
- 20/40 sand and 20/40 ULW, weight ratio 50%/50%

Regarding the experimental settings, an API conductivity cell assembly coupled with a hydraulic pump were used to measure permeability and width of a loaded proppant pack under equivalent confining stress from the hydraulic pump. The confining stress, exerted from hydraulic pressure from the pump, was applied to the two steel plates in the API conductivity assembly. In addition, a back pressure of 350 psi was pre-set in all experiments to ensure fluid flow inside the loaded proppant pack for permeability measurements.

For all binary mixtures listed as above, permeability and proppant pack's width data were recorded for different confining stress (discrete data) between 2000 and 1000 psi (from 5 to 10 data points, depending on the tested binary mixture). Permeability was measured based on application of Darcy's Law. Experimental processes for permeability are examined to have a reasonably low degree of errors in contribution to the overall conductivity lab data.

Proppant pack's width was measured indirectly using a reference width. This reference width (at each applied stress data point) related to the total height of the API conductivity assembly with no proppant pack loaded inside the cell. Using reference width, proppant pack's width (at each applied stress point) was measured by subtraction of the reference width from the measured width of the API conductivity cell with proppant pack loaded. According to the author's claims, a minor degree of variance during reference width measurement (0.02 – 0.08 inches) was recorded. However, this degree of variance may convert to a variance range between 20 and 1300 mD-ft for

conductivity data. As a result, experimental processes for proppant pack's width are examined to have noticeably higher degree of errors in contribution to the overall conductivity lab data. As width measurement is evaluated as unsatisfaction in degree of measurement errors, lab data for proppant pack's width is not selected for fracture width modelling. In order to compensate for this unsatisfaction, lab data for proppant pack's width with higher quality is selected from FRACPRO® database for modelling in this thesis, which is detailed in chapter 4 and in the Appendix.

Besides evaluation of permeability and proppant pack's width data, another evaluation is required for available lab data from Eric's study. Only 3 binary mixtures (within the total of 6) have available lab data for both weight mixing ratios. Since weight mixing ratio is one of the two priorities in the novel conductivity model, the three binary mixtures with lab data for 25%/75%, and 50%/50% ratios are selected to be modelled. In consequence, selected list includes 20/40 sand-20/40 ceramic, 40/70 sand-20/40 ceramic, and 40/70 ceramic-20/40 sand.

1.5. Fracture permeability modelling

As fracture medium contains proppant particles under impact of different stress orientations, fracture permeability modelling has been evolving from modelling for granular particulate bed medium.

Early approaches, for example in Hagen-Poiseuille equation, modelled the granular bed medium as a tube-bundle schematic. Application of the tube-bundle schematic derived the most generic form of permeability:

$$k = \frac{\phi r^2}{8\tau} \quad (1.2)$$

Further concepts as specific surface area $s = \frac{2\phi}{r}$ and closely-resilistic assumption of spherical forms for particles was implied to transform this concept into $s = \frac{6(1-\phi)}{d}$. Under a schematic of identical spherical-formed particles (which can be referred to a single proppant type

under the use of mean diameter as a representative number), equation (1.2) for permeability is revised as:

$$k = \frac{d^2}{72\tau} \cdot \frac{\phi^3}{(1-\phi)^2} \quad (1.3)$$

Carman and Kozeny examined the validity of this permeability model empirically (detailed in Dullien and F.A.L, 1992) because they considered the effect of proppant arrangement, shape factor and pore trajectories. From their studies, an empirical coefficient, which was abbreviated as Carman-Kozeny coefficient, was integrated into the permeability model as:

$$k = \frac{d^2}{72K_o\tau} \cdot \frac{\phi^3}{(1-\phi)^2} \quad (1.4)$$

Where K_o is 2 for a general particulate bed (Kozeny, 1927). Albeit there has been different modelling approaches from equation (1.2) (Dullien and F.A.L, 1992), Carman- Kozeny permeability model is extensively selected for fracture permeability modelling as a starting point, and further narrowed down to analytical or semi-analytical modelling. One notable modification from Carman-Kozeny equation was implication of particle size distribution inside the granular bed (Panda and Lake, 1994). The multiplication factor which models particle size distribution in their study is formulated as:

$$F_D = \frac{(\gamma\delta^3 + 3\delta^2 + 1)^2}{(1 + \delta^2)^2} \quad (1.5)$$

By far, effect of confining pressure has not been mentioned, albeit this does not infer that this effect has not been studied. Simple understanding from the definition of porosity and tortuosity, which are integrated in the Carman-Kozeny equation, is reasonable to state their dependency on confining stress (Jia-Hyun dong et.al, 2010; Ricardo Dias et.al, 2006). Consequently, they are modelled as a function of confining stress, and further integrated into the Carman-Kozeny equation as a solid option for a permeability model. A few empirical modelling equations for porosity and

tortuosity are in this chapter. For porosity, in Jia-jun dong et.al (2010), two most common porosity models were studied for sandstone, limestone and shale:

$$\phi = \phi_o e^{-\beta(P-P_o)} \quad (1.6)$$

$$\phi = \phi_o \frac{P^{-q}}{P_o} \quad (1.7)$$

Equation (1.6) has been widely used in modelling porosity for sandstone and limestone reservoirs with compressibility coefficient β between 10^{-6} and 10^{-4} psi⁻¹ scale. However, in fractured media, β represents fracture coefficient which can approach 10^{-3} scale (Dong chen et.al, 2014). Equation (1.7) is a better option for shale than conventional reservoir rocks. For tortuosity, its models typically integrate porosity as one of the independent variables, and each tortuosity model may limit itself to an empirical porosity range. A critical tortuosity review in Ghanbarian et.al (2013) provides widely-used tortuosity models for different types of granular media:

$$\tau = 1 - p \ln(\phi) \quad (1.8)$$

$$\tau = 1 + X \sqrt{1 - \phi} \quad (1.9)$$

$$\tau = \phi^{-n} \quad (1.10)$$

Equation (1.8) and (1.9) are typically not applied to a proppant pack's porous media since their porosity range for application exceeds 0.4 or their tested media do not resemble a proppant pack's porous media. Equation (1.10) was referred to model tortuosity for binary spherical mixtures in Mota et.al (2001). In this study, coefficient n in equation (1.10) was experimentally stated between 0.4 and 0.5. However, both Mota et.al (2001) and Ghanbarian et.al (2013) concluded the non-universality of n and its dependence on some intrinsic property of the modelled medium. This statement allows limited use of equation (1.10) in modelling tortuosity for binary mixtures in this thesis, underlying the modelling context as a reminder.

1.6. Fracture width modelling

Similar to permeability, fracture width modelling evolves from modelling of width in a granular bed. Prior to review of fracture width modelling, it is fundamental to review physics of a particle pack under impact of applied stress. In most of fracture width modelling studies, there involve two essential physics: particle deformation and particle contact embedment. Particle deformation is referred to deformation behaviors for a material, beginning from elastic, plastic and eventually material rupture. During the series of deformation behaviors, the particle pack width decreases when applied stress increases.

Particle contact embedment is referred to the phenomenon when there exists contact between any surfaces of similar or different materials. In case of a particle pack, this phenomenon occurs at contact within proppant particles of the pack, and contact between proppant particles and the adjacent rock layers' surfaces (modelled as an elastic, flat surface). In 1882, Hertzian studied these contact embedment phenomena, and this study has been named as Hertzian contact theory (fully described and derived in Anthony C. Fiischer-Cripps, 2007). The theory modelled contact zdeformation occurs in the proximity of the compact point.

Contact pressure in Hertzian contact theory is modelled as:

$$P = P_o \left(1 - \frac{r^2}{a^2}\right)^2, r \ll a \quad (1.11)$$

Where P is the contact pressure at a proximity within the circle of radius a, P_o is the maximum contact pressure at the contact point between two objects, and r is the distance from the contact point to the located proximity in which contact pressure is P. Vertical deformation (Anthony C. Fiischer-Cripps, 2007) corresponding to contact pressure in (1.6) is:

$$w_r = \frac{\pi P_o}{4E^*a} (2r^2 - a^2) \quad (1.12)$$

Where w_r is vertical deformation, and $E^* = \frac{E}{1-\nu^2}$. In case the deformation surface is an arc of a circle with radius R_a , Hertzian proposed the quantity w_r as:

$$w_r = w_o - \frac{r^2}{2R_a} \quad (1.13)$$

Where w_o is the maximum of w_r when $r = 0$. Equations (1.7) and (1.8) have to be equal, therefore w_o and R_a are extracted as:

$$w_o = \frac{\pi P_o R_a}{2E^*} \quad (1.14)$$

$$R_a = \frac{\pi a P_o}{2E^*} \quad (1.15)$$

Equations (1.9) and (1.10) are modified to derive a relationship between w_o and R as:

$$a^2 = R_a w_o \quad (1.16)$$

Application of Hertzian contact theory, primarily using equations (1.14), (1.15), and (1.16), has been referenced in several proppant pack's width modelling approaches: analytical, semi-analytical, numerical, or combination of at least two mentioned methods. Jianchun Guo et.al (2012) combined the contact theory with Laplace mathematical transformation to model relationship between creep effect and proppant deformation. Khanna et.al (2012) combined Hertzian contact theory, monolayer assumption, particle neighbouring effect, and Computational Fluid Dynamics (CFD) to model proppant deformation in narrow fractures. Yuaning Gao et.al (2013) modelled proppant deformation analytically by modifying Hertzian contact theory for a single-layer pattern in coal beds and extended to multiple-layer pattern. Recently, Jianchun Guo et.al (2017) proposed an analytical model for proppant deformation and embedment by combination of Hertzian contact theory and sparse effect better proppant clusters. Albeit different modelling approaches have been proposed as being reviewed, the modelled assembly is selected to resemble a proppant pack of a single proppant type (i.e. single-sized spherical particles).

CHAPTER 2: NON – LINEAR LEAST SQUARE PROBLEMS AND TRUST REGION METHOD

Acknowledgement of theoretical backgrounds, which are related to application in this thesis and described in this chapter, mainly refers to these three references: *Numerical Optimization* (Jorge Nocedal and Stephen J. Wright, 1999), *Inverse theory for petroleum reservoir characterization and history matching* (Dean Stuart Oliver, Ning Liu and Albert Reynolds, 2008), and *Optimization Toolbox documentation* (MATLAB®, version 2019b).

Within the development of any physics-based modelling approach, relevance of constant and coefficients contributes an integrated portion in understanding the behavior of the modelled phenomena. Constants and coefficients represent an intrinsic property of the modelled factor under change of the relevantly-determined factors. One example of a constant is Young Modulus, which represents rate of deformation of a material under its elastic behavior range. One example of a coefficient is the pore compressibility, which represents change of porous volume under change of formation pressure.

Albeit constants and coefficients are integrated in a physics-based modelling process, they exhibit differences. Constants are naturally more universal than coefficients, which may depend on various modelled phenomena. Consequently, coefficients have their empirically known range of values (i.e. known as constraints) to fit their purposes. Different techniques have been exploited in order to determine the suitable values for coefficients in specific contexts. In the modelled context of this thesis, supportive lab data allows application of numerical optimization techniques to determine the coefficients. For more details, determination of coefficients is processed by solving the following equation(s):

$$f(C, x_{data}) = y_{data} \quad (2.1)$$

Where C presents the coefficients, x_{data} presents the values of experimentally-controlled factors (i.e. confining pressure, weight mixing percentages), and y_{data} presents the values of measured quantities (i.e. permeability, conductivity). Numerical optimization techniques used to solve the equation(s) are typically linear based solvers or non-linear based solvers (Jorge Nocedal and Stephen J. Wright, 1999). Because of complexity in petroleum problems, the use non-linear based solvers is typical and further discussed in this chapter.

Non-linear based solvers do not directly solve the aimed variables (which are coefficients C in this context). In contrast, they estimate the best possible outcomes of the aimed variables by solving the following consequent problem generated from equations (Jorge Nocedal and Stephen J. Wright, 1999):

$$\underset{C}{\operatorname{argmin}} F = (f(C, x_{data}) - y_{data})^2 \quad (2.2)$$

The function in (2.2) is a representation of a non-linear least-square problem and is defined as the objective function. For a non-linear least square problem, aimed variables are extracted from a sequence of iterations. The sequence starts from a specific C_0 , updates itself after k^{th} iteration as $C_k = C_{k-1} + p_k$, and stops when the global minimum of the function in (2.2) is approached under desired accuracy degree (i.e. sufficient error tolerance) . The scope of solving the original equation(s) in (2.2) now evolves to the scope of solving the objective function. Two categorized directions to solve equation (2.2) are line-search and trust-region methods. Since trust region method exhibits higher flexibility, and better control within a iteration compared to line-search method, it is selected to be used in this thesis. This method and its fundamental backgrounds (Taylor theorem, positive definite matrices) are addressed in this chapter.

2.1. Taylor theorem

Presumably a function $f(x)$ exists its first and second derivatives, its dependence on the independent variable $(x + p)$, where $p \in \mathbb{R}$, is expanded as:

$$f(x + p) = f(x) + \nabla f(x)^T p + \frac{1}{2} p^T \nabla^2 f(x + tp) p, t \in (0,1) \quad (2.3)$$

Taylor theorem provides the estimation of function $f(x)$ under a step size p . In case the n^{th} derivative of $f(x)$ continuously exists, the estimation continues with the term holding n^{th} derivative. Within the context of Taylor theorem's use in this thesis for its non-linear least square problems, estimation is limited until second derivative term.

2.2. Positive definite matrices

A square matrix A (dimensions $n \times n$) is defined as a *positive definite matrix* if the following condition: $x^T A x > 0$ is satisfied for all non-zero vector x with length n .

2.3. Trust-region method

Overview of the method

Trust region method directs toward the global minimum of function (2.2) indirectly by minimizing a *model function* (different from the function (2.2)) within a selected *trust region* (commonly a circle). *Trust region selection and minimization of the model function are performed in all iterations until reaching global minimum of the model function.*

The latter condition infers that the model function has to similarly behave as the function (2.2), within the trust circle. Additionally, function (2.2) is quadratic. As a result, the model function shall be a quadratic estimation of function (2.2) within control by trust circle, and a reasonable selection of which is application of Taylor theorem. In general, selection of the model function is a quadratic function in which the quadratic term may hold a positive definite matrix in replacement of the second derivative as in Taylor theorem. The model function of function F in (2.2) as and the radius of trust circle are notated as M and Δ , respectively. Under step size p

controlled by the trust circle of radius Δ , Model function $M(x + p)$ is generally written for one iteration as follows:

$$M(x + p) = F(x) + \nabla F(x)^T p + \frac{1}{2} p^T B(x) p, \mathbf{p} \leq \Delta \quad (2.4)$$

Where $\mathbf{p}$ is the norm of p and $B(x)$ is the positive definite matrix which replaces the second derivative in the quadratic term. Description of M as in (2.4) satisfies the latter condition of trust-region method.

Analysis of the model function under restriction by trust region

In each iteration, minimization of the model function certainly guides the direction to its global minimum. Furthermore, trust-region method always limits the step size along this direction by the selected trust circle. Based on description of model function M in (2.4), a proper direction and an optimal step size as two requirements, p needs to be determined in each iteration in order to minimize M .

Minimization of a quadratic function as M in case trust region is neglected is known by interpreting its first derivative:

$$M'(p) = \nabla F + Bp, M'(p) = 0 \leftrightarrow p = B^{-1} \nabla F \quad (2.5)$$

In case the trust region is not neglected and $B^{-1}F \leq \Delta$ certainly $p = B^{-1}F$ becomes the solution of p in that specific iteration. Since the condition $B^{-1}F \leq \Delta$ occurs with uncertainty, different techniques have been developed to find the optimal p in case $B^{-1}F \leq \Delta$ is not satisfied. Two examples are Cauchy Point and Dog Leg Method (Jorge Nocedal and Stephen J. Wright, 1999). There is a similarity in these techniques' solutions: the optimal p has the general formula as:

$$p = \alpha \nabla F, \alpha < 0 \quad (2.6)$$

Therefore, using the *span* concept in linear algebra, optimal p in an iteration is generally formulated as:

$$p \in \text{span}(\nabla F, B^{-1}\nabla F) \leftrightarrow p = \alpha_1 \nabla F + \alpha_2 B^{-1}\nabla F, (\alpha_1, \alpha_2) \in \mathbb{R}^2 \quad (2.7)$$

Determination of the optimal p as in (2.7) is the *two-dimensional subspace minimization*, which is the core intergrated algorithm inside the trust-region method for non-linear least square problems in *MATLAB® R2019b Optimization Toolbox*.

2.4. Setting constraints

From the previous dicussions of solver methods in this chapter, involvement of constraints have not been discussed. In fact, an advantage of solvers for non-linear least square problems (both line search and trust region methods) is their indepence on the aimed variables' constraints. However, modelling approaches for physics-based phenomena always include coefficients' constraints. Therefore, coefficents' constraints have to be enforced along with the solver methods. They should be reasoned properly to fit in the modelling contexts and should not be aligned to the outlier sides when referenced to their empirically known ranges. Further discussions are detailed in chapter 4, which describes the modelling approach of this thesis.

CHAPTER 3 – SENSITIVITY STUDY

3.1 Existence of the Mixture Zone inside a propped fracture

According to the abstract of this thesis, a descriptive statement for the Mixture Zone is initiated from injection of a binary proppant mixture. However, clear visualization for this Mixture Zone is necessary to further study its behaviour under considerable factors (i.e. reservoir properties, injection schedule design, geomechanics). In order to perform the mentioned visualization, a fracture propagation simulator is required. For the specific purpose of this thesis, an in-house fracture propagation simulator is built based on MATLAB®. The fracture propagation simulator acquires the foundation from PKN fracture model and is designed explicitly for a binary proppant mixture injection. A review of PKN is in chapter I of this thesis. As a reminder from the review of PKN, solutions for the finite-discretization processes inside the numerical solution of PKN include the fracture width. Based on the primary solutions from finite-discretization processes for PKN, the fracture propagation simulator computes additional three quantities for each discretized grid:

- Areal concentration for proppant type 1, which is injected formerly ($ArConc_1$)
- Areal concentration for proppant type 2, which is injected latterly ($ArConc_2$)
- Total areal concentration for a binary mixture ($ArConc_{Tot} = ArConc_1 + ArConc_2$)

From these additional quantities, the Weight Concentration Ratio (equivalently to the weight ratio in Eric's experimental settings) is defined and computed as follows: $R = \frac{ArConc_1}{ArConc_{Tot}}$.

All the Areal Concentrations and Weight Concentration Ratio are 2D matrices in the fracture propagation simulator, with two dimensions equal the fracture half-length and fracture height. Albeit the fracture propagation simulator is capable of displaying 3D and 2D images for a variety of fracture parameters, the initial most essential result from it is the 2D image for Areal

Conductivity Ratio. An example for the binary mixture 40/70 sand and 20/40 ceramic is presented as below:

Figure 7 – Mixture Zone inside half-fracture dimensions

The image clearly presents the Mixture Zone. A direct interpretation from the definition of Weight Concentration Ratio shows that this parameter is ranged between 0 and 1. The blue zone presents extremely low value of Weight Concentration Ratio (or 0 margin, which infers the dominance of the second injected proppant type) and the red zone presents extremely high value of R (or 1 margin, which infers the dominance of the first injected proppant type). A further look into the image clearly separates the Mixture Zone inside a half-fracture dimension, in which the color gradient shade lies between the blue region and the region (or the value of Areal Concentration Ratio is between 0 and 1, which infers the co-existence of both proppant types).

3.2 Impact of different factors to the Mixture Zone

Previous interpretation from the fracture propagation simulator shows that the Mixture Zone is evident. Besides the visualization, its variability inside the fracture under different

conditions is necessary to be studied. Obviously, the variability of Mixture Zone depends mainly on the injection process during any fracturing practices. However, the intrinsic properties of the reservoir additionally do impact the Mixture Zone, since they directly affect proppant behaviors. Therefore, consideration of Mixture Zone's variability inside the fracture requires an evaluation of the injection process and reservoir properties. All possible interfering factors are listed as below:

- Reservoir's pressure properties (the most concerned is the Minimum Horizontal Stress)
- Formation geo-mechanics (i.e. Young Modulus and Poisson Ratio)
- Injection parameters (i.e. rate and time)
- Pumping schedule
- Proppant selection (i.e. Proppant size and density)

According to this prospective factor list and proved existence of 2 injected agents in the Mixture Zone, a further detailed list of 10 parameters are possible to be included in the sensitivity study. This detailed list consists of reservoir's minimum horizontal stress, formation's Young Modulus, formation's Poisson Ratio, injection rate, proppant densities for both agents, proppant sizes for both agents, injection time design for both agents. Based on the capability of the in-house fracture propagation simulator, Mixture Zone variability analysis can be conducted for the detailed list of 10 factors above. In this chapter, a sensitivity analysis is designed to examine the variability of Mixture Zone inside the fracture based on the impact of all possibly considerable parameters. Since the in-house fracture propagation simulator is a numerical simulator, it is capable to numerate the number of grids with at least two proppant types in existence (A_{mix}) and the total grids with existence of any proppant types, (A_{tot}). According to these definitions, the variability of Mixture Zone is numerated as: $\%_{\text{Mixture Zone}} = \frac{A_{\text{mix}}}{A_{\text{tot}}}$.

Since one parameter's effect at a time is concerned and there should be no bias between studied parameters, variability of Mixture Zone can be observed as sole dependency on a parameter using One At a Time (known as OAT) sensitivity analysis (Morris, 1991). Since a fixed base case is initiated and processed for all parameters, the design of this sensitivity analysis complies with standard and local OAT design. Following the course of all selected factors, computation of Mixture Zone variability and the intended design of study, a procedure for the sensitivity analysis is conducted in 7 steps as follows:

1. Initially, imply a base case in the simulator. The base case contains a base numerical set of the 10 parameters selected in this sensitivity study. All the other settings inside the simulator are kept as default during this sensitivity study.
2. Numerate % Mixture Zone from the simulator after the base case is implied. Record this base % Mixture Zone.
3. Pre-set a range of sensitivity boundaries for the parameters. In this sensitivity study, a 25% upper boundary and a 25% lower boundary from the base case are implied for all parameters.
4. Pre-set finite increment of sensitivity from the sensitivity boundaries. In this sensitivity study, the increment is 5%. Consequently, a sensitivity range for each parameter should consist of 11 numerical values, or 11 sensitivity points (including the base value).
5. From step 1 and step 2, sensitivity ranges are numerated for all the 10 parameters. Record all generated sensitivity ranges.
6. For every studied parameter, all the remaining 9 parameters are fixed at their base values. Under the pre-set sensitivity range for every parameter, 11 numerical values for

the % Mixture Zone are computed at the correspondingly 10 sensitivity points (excluding the base % Mixture Zone).

7. For every studied parameter, deviation for % Mixture Zone (at one sensitivity point) from the base % Mixture Zone (in step 2) are computed using this formula:

$$Deviation = \% MixtureZone_{Sensitivity} - \% MixtureZone_{Base}.$$

Table 1 provides the base case's values for all parameters in this sensitivity study.

Table 1 - Base case's value of parameters in the sensitivity study

Proppant 1	Sand 40/70 (0.000335 m), 2.75 g/cc
Proppant 2	Ceramic 20/40 (0.000631 m), 3.65 g/cc
Confining Pressure	7000 psia
Injection rate	0.02 m ³ /s
G modulus	0.4 GPa
Poisson Ratio	0.3
Injection time design	As matrix (*)

Table 2 provides the sensitivity points for 8 parameters in this sensitivity study, excluding the two injection time parameters.

Table 2 – Sensitivity points for 8 parameters (excluding 2 injection time parameters)

Sensitivity increment	Diameter 1 (m)	Diameter 2 (m)	S.G 1	S.G 2	Injection Rate (m ³ /s)	Confining pressure (psia)	Poisson Ratio	Young Modulus (GPa)
-25 %	0.000251	0.000473	1.9875	2.7375	0.015	5250	0.1875	0.3
-20 %	0.000268	0.000505	2.12	2.92	0.016	5600	0.2	0.32
-15 %	0.000285	0.000536	2.2525	3.1025	0.017	5950	0.2125	0.34
-10 %	0.000302	0.000568	2.385	3.285	0.018	6300	0.225	0.36
-5 %	0.000318	0.000599	2.5175	3.4675	0.019	6650	0.2375	0.38
5 %	0.000352	0.000663	2.7825	3.8325	0.021	7350	0.2625	0.42
10 %	0.000369	0.000694	2.915	4.015	0.022	7700	0.275	0.44
15 %	0.000385	0.000726	3.0475	4.1975	0.023	8050	0.2875	0.46
20 %	0.000402	0.000757	3.18	4.38	0.024	8400	0.3	0.48
25 %	0.000419	0.000789	3.3125	4.5625	0.025	8750	0.3125	0.5

Table 3 provides the sensitivity points for 2 injection time parameters.

Table 3 – Sensitivity points for 2 injection time parameters

Sensitivity increments	-25%	-20%	-15%	-10%	-5%	5%	10%	15%	20%	25%
Former proppant type (4th stage)	0.75	0.8	0.85	0.9	0.95	1.05	1.1	1.15	1.2	1.25
Latter proppant type (5th stage)	0.375	0.4	0.425	0.45	0.475	0.525	0.55	0.575	0.6	0.625

Within the total of 10 studied parameters in this sensitivity analysis, eight of them are imported into the in-house fracture propagation simulator directly. The remaining two are required extra processing prior to be imported into the simulator, which include proppant size and injection time design (specifically for slurry injection steps). For processing for proppant size, the original lab data provides the proppant diameters in a Mesh-size range (for example Mesh 20-40). Because of the fracture propagation simulator input design for proppant diameter as a single numerical value in mm, Mesh sizes are converted into mm using the ISO Sieve Conversion Table and the mean diameter for a Mesh-size range is eventually imported. For processing for injection time design, the in-house fracture propagation simulator designs input format for injection time design as a 2×7 matrix (a form of vectorization in numerical simulation). The injection time design's base case, converted into the simulator's required format, is presented as below:

$$\begin{bmatrix} 0.2 & 0.4 & 0.7 & \mathbf{1.0} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{0.5} & 0.75 & 1.0 \end{bmatrix} (*)$$

The simulator designs pad injection time as a unit-pumping time (in minutes). Therefore, slurry injection time equals the element-wise multiplication operation of the formulated matrix above and the unit-pumping time. The flush injection time is independent of the unit-pumping time.

As a specific design purpose of the matrix for slurry injection steps, the first row of the matrix corresponds to the injection steps of the 1st injected agent and the second row corresponds to the injection steps of the 2nd agent. Since the simulator implies 1st injected agent in the initial 4 steps and the 2nd injected agent in the last 3 steps, the last 3 elements of the first row and the initial 4 elements of the second row are intuitively set to 0 permanently for all simulations.

Based on the matrix's format for injection time design, it is crucial to examine the effect of the intersection between the end of the 1st agent's injection (or its last step) and the beginning of the 2nd agent injection (or its initial step). This effect is concentrated on the difference between the 4th injection time step (last stage to inject the 1st agent, 1.0 in the base case) and the 5th injection time step (first stage to inject the 2nd agent, 0.5 in the base case). Furthermore, both agents' injection time contribute to this difference. As a result, this sensitivity study, regarding the injection time design, both injected agents are examined as follows:

- Initiate the 25% lower and upper boundaries for the 4th injection step (1.0 in the matrix (*)) with a 5% increment to analyze the impact to the Mixture Zone by the 1st injected agent).
- Initiate the 25% lower and upper boundaries for the 5th injection stage (0.5 in the matrix (*)) with a 5% increment to analyze the impact to the Mixture Zone by the 2nd injected agent).

Impact from the injection time design contributes 2 parameters in the total of 10 studied parameters. Except mentioned changes during the sensitivity analysis, the injection steps remain as the base formulated matrix (*).

CHAPTER 4 – SEMI-ANALYTICAL MODELING APPROACH

4.1. Fracture permeability

As the review of studies for fracture permeability, it has been studied extensively in the past for a single proppant type, and there are well-known models to estimate this quantity. Within previous modelling approaches, Carman-Kozeny equation is one of the most applicable. Its formulation is reminded as follows:

$$k_f = \frac{\phi^3}{K_o(1-\phi)^2} \times \frac{1}{\tau a_v^2} \quad (4.1)$$

When a binary proppant mixture is given consideration, the Carman-Kozeny equation is still effective in application, providing all relevant parameters in the equation need revision. Since the Kozeny coefficient K_o are not commonly modelled analytically or semi-analytically, this parameter is determined as an independent variable. In the case of general particulate beds, a numerical value of 2 is empirically reasonable (Kozeny, 1937; Panda and Lake, 1994).

With the Kozeny coefficient is exempted from being modelled, the remaining parameters include porosity, tortuosity and the Specific Internal Area. Albeit the definition of Internal Specific Area includes the term $(1 - \phi)$, this term is re-arranged inside the Carman-Kozeny model for further convenience. Therefore, Specific Internal Area in this study is reduced to the term $\frac{A_{surface}}{V_{solid}}$.

Since an agreement in similarity between a proppant pack and a granular bed medium (under applied stress) is previously stated, capable modelling approaches for both porosity and hydraulic tortuosity can be applied for the case of a binary proppant mixture. A widely used model mentioned in Jian-jun dong et.al (2010) for porosity of a granular bed medium under applied stress is:

$$\phi = \phi_o e^{-\beta(P-P_o)} \quad (4.2)$$

Where: β is the rock/formation compressibility coefficient. For hydraulic tortuosity, a previous study by Mota et.al (2001), of which experiments closely match the experiment design for a binary proppant mixture by Eric Schultz (2014), proposed tortuosity as:

$$\tau = \phi^{-n} \quad (4.3)$$

Where n is an empirical coefficient which was determined between as low as 0.3 to as high as 0.5 in the authors' experiment settings. However, the authors implicitly concluded n to be not universal and dependent on the intrinsic of the tested binary mixture.

The most comprehensive revision focuses on Specific Internal Area. Referred to the ratio $\frac{A_{surface}}{V_{solid}}$, Specific Internal Area is a concept which involves high degree of geometry. For a single proppant type (geometrically modelled as identical spherical-formed particles), this ratio is derived to become $\frac{6}{D}$. The effect of particle size distribution is mitigated by implication of the average particle diameter. Albeit this implication is certainly simplistic, the equation of Internal Specific Area for a single proppant type clearly does not integrate other impacts of proppant properties, besides the proppant size (i.e. average diameter in the ratio).

For a binary proppant mixture, mean diameter is still applied to represent the particle size distribution. However, this intention does not totally mitigate natural complexity in particle size distribution. A kernel density estimator (i.e. KDE) is actually a better supportive property along with mean diameter. Based on the experiment settings in Eric's study, a binary proppant mixture can be modelled as a two-sized particle assembly, coupled with the Weight Concentration Ratio – abbreviated as R . In details, a two-sized particle assembly contains:

- Particle type 1, average diameter D_1 , density ρ_1 , Weight Concentration Ratio R
- Particle type 2, average diameter D_2 , density ρ_2 , Weight Concentration Ratio $1-R$

Define N_1 and N_2 as the number of particles for particle type 1 and particle type 2, respectively. In addition, area and volume of a spherical form are πD^2 and $\frac{1}{6}\pi D^3$, respectively. As a result, $A_{surface}$ and V_{solid} in the Specific Internal Area definition are derived as:

$$A_{surface} = N_1(\pi D_1^2) + N_2(\pi D_2^2) \quad (4.4)$$

$$V_{solid} = N_1\left(\frac{1}{6}\pi D_1^3\right) + N_2\left(\frac{1}{6}\pi D_2^3\right) \quad (4.5)$$

Replace $A_{surface}$ and V_{solid} into Internal Specific Area:

$$a_v = \frac{N_1(\pi D_1^2) + N_2(\pi D_2^2)}{N_1\left(\frac{1}{6}\pi D_1^3\right) + N_2\left(\frac{1}{6}\pi D_2^3\right)} \quad (4.6)$$

Based on the definition of N_1 and N_2 , they are further derived as:

$$N_1 = \frac{M_1}{\rho_1} \times \frac{1}{V_{\text{one particle type 1}}} \quad (4.7)$$

$$N_2 = \frac{M_2}{\rho_2} \times \frac{1}{V_{\text{one particle type 2}}} \quad (4.8)$$

Where M_1 and M_2 are masses of particle type 1 and particle type 2, respectively.

Based on the experiment settings in Eric' study, condition, a specified total areal concentration and the area of proppant contact under confining stresses from the API conductivity assembly are pre-set. Notate them as C (unit in lbs/ft^2) and A (unit in in^2), the total mass of proppant packed in the API conductivity assembly (notated M) is derived as:

$$M = \frac{AC}{144} (\text{lbs}) \quad (4.9)$$

A combination with the Weight Ratio R derives M_1 and M_2 :

$$M_1 = \frac{MR}{\rho_1} \quad (4.10)$$

$$M_2 = \frac{M(1-R)}{\rho_2} \quad (4.11)$$

Replacement of equations for M_1 and M_2 further derives N_1 and N_2 as:

$$N_1 = \frac{MR}{\rho_1(\frac{1}{6}\pi D_1^3)} \quad (4.12)$$

$$N_2 = \frac{M(1-R)}{\rho_2(\frac{1}{6}\pi D_2^3)} \quad (4.13)$$

Replacement of the equations for N_1 and N_2 into the equation for Internal Specific Area further derives this ratio as:

$$a_v = \frac{\frac{MR}{\rho_1(\frac{1}{6}\pi D_1^3)}(\pi D_1^2) + \frac{M(1-R)}{\rho_2(\frac{1}{6}\pi D_2^3)}(\pi D_2^2)}{\frac{MR}{\rho_1(\frac{1}{6}\pi D_1^3)}(\frac{1}{6}\pi D_1^3) + \frac{M(1-R)}{\rho_2(\frac{1}{6}\pi D_2^3)}(\pi D_2^3)} \quad (4.14)$$

Simplification of the equation above provides the final form of the Internal Specific Area for a binary-sized particle assembly, or a binary proppant mixture as follows:

$$a_v = \frac{6}{D_1 D_2} \times \frac{\rho_1 D_1 + (\rho_2 D_2 - \rho_1 D_1)R}{\rho_1 + (\rho_2 - \rho_1)R} \quad (4.15)$$

Eventually, the modified Carman-Kozeny equation for fracture permeability of a binary proppant mixture is finalized as follows:

$$k_f = \frac{\phi^3}{K_o(1-\phi)^2} \times \frac{1}{\tau a_v^2} \quad (4.16)$$

$$\text{Where: } \phi = \phi_o e^{-\beta(P-P_o)}, \tau = \phi^n, \text{ and } a_v = \frac{6}{D_1 D_2} \times \frac{\rho_1 D_1 + (\rho_2 D_2 - \rho_1 D_1)R}{\rho_1 + (\rho_2 - \rho_1)R}.$$

4.2. Fracture width

4.2.1 Modelling approach for fracture width composed of a single proppant type

Based on analytical modelling for a particle embedment under parabola stress distribution, Hertzian's contact theory has been commonly used in deriving fracture width models for a single proppant type (under microscopic scale and combination with material elastic behavior for macroscopic scale as a proppant pack). Primary backgrounds which are relevant to Hertzian's contact theory is readily in chapter 1. Relevant formulas for a single proppant type case are applied in Jianchun Guo et.al (2017) and provided as follows:

$$a = \sqrt{R\delta} \quad (4.17)$$

$$\delta = \left(\frac{3P_c}{E^*}\right)^{\frac{2}{3}} \times R \left(\text{where: } \frac{1}{E^*} = \frac{1-v_p^2}{E_p} + \frac{1-v_r^2}{E_r}\right) \quad (4.18)$$

$$P_s = \frac{4P_c R^2}{\pi a^2} \quad (4.19)$$

$$\Delta D = \frac{P_s}{E_p} w_{f0} \quad (4.20)$$

$$h = 2 \times \delta + \Delta D \quad (4.21)$$

$$w_f = w_{f0} - h \quad (4.22)$$

Where a is the radius of contact, and ΔD are decrease in fracture by embedment and deformation impact, h is the total decrease in fracture width, w_f and w_{f0} are the initial fracture width (i.e. width with zero exposure to applied stress) and final fracture width after exposure to applied stress. In Eric's experimental settings, the contact surfaces inside the API conductivity assembly are flat steel plates and not rock surfaces as in Jianchun Guo et.al (2017). For steel material, the Young modulus and Poisson ratio are 300 GPa and 0.3, respectively. As a result, the term $\frac{1-v_r^2}{E_r}$ in the equation to compute δ is practically 0. The equation of E^* , under elimination of the term $\frac{1-v_r^2}{E_r}$, reduces to $\frac{1}{E^*} = \frac{1-v_p^2}{E_p}$. Substitution of new equation for E^* into the equation for δ , the new equation for δ is derived as:

$$\delta = (R\sqrt[3]{9}) \times \left(\frac{1-v_p^2}{E_p}\right)^{\frac{2}{3}} \times (\sqrt[3]{P_c})^2 \quad (4.23)$$

Substitution of new equation for d into equation for a , the new equation for a is derived as:

$$a^2 = (3P_c \times \frac{1-v_p^2}{E_p})^{\frac{2}{3}} \times R^2 \quad (4.24)$$

Substitution of new equation for a^2 into equation for P_s , the new equation for P_s is derived as:

$$P_s = \frac{4P_c}{\pi \times (3P_c \times \frac{1-v_p^2}{E_p})^{\frac{2}{3}}} \quad (4.25)$$

Substitution of new equation for P_s into equation for ΔD , the new equation for ΔD is derived as:

$$\Delta D = \frac{4w_{f0}}{(\pi^{\frac{3}{2}} \times \sqrt[3]{E_p(1-v_p^2)^2})} \times \sqrt[3]{P_c} \quad (4.26)$$

Substitution of new equation for ΔD into equation for w_f , the new equation for w_f is derived as:

$$w_f = w_{f0} - \frac{4w_{f0}}{(\pi^{\frac{3}{2}} \times \sqrt[3]{E_p(1-v_p^2)^2})} \sqrt[3]{P_c} \quad (4.27)$$

4.2.2 Modelling approach for fracture width composed of a binary proppant mixture

From a practical viewpoint, proppant particles are randomly distributed in the fracture without any specific distribution schematic. However, in order to reasonably concern the fracture width of a binary proppant mixture with integration of the Weight Concentration Ratio, the binary-sized particle assembly coupled with a specific distribution schematic is required. Referred back to the description of a binary-sized particle assembly, in case these two particle types are arranged in two separate sub-layers (each layer is composed of one particle type), the integration of R into the derivation approach is more feasible. A schematic for this arrangement, which is defined in this study as a dual-layer unity, is provided in figure 8. Based on the dual-layer unity, Hertzian's contact theory is applied to model the width of each sub-layer in the provided schematic. When being deformed and embedded under applied stress, these two sub-layers deform and embed simultaneously. The embedment between two sub-layers is neglected.

Figure 8 - Schematic for a dual-layer unity

Direct application of Hertzian contact theory into each sub-layer in the schematic provides the equation for their widths as follows:

$$w_{f1} = w_{f01} - C_1 \sqrt[3]{P_c}, C_1 = \frac{4w_{f01}}{(\pi \sqrt[3]{9}) \times \sqrt[3]{E_{p1}(1-v_{p1}^2)^2}} \quad (4.28)$$

$$w_{f2} = w_{f02} - C_2 \sqrt[3]{P_c}, C_2 = \frac{4w_{f02}}{(\pi \sqrt[3]{9}) \times \sqrt[3]{E_{p2}(1-v_{p2}^2)^2}} \quad (4.29)$$

A summation of the two equations above formulate the total fracture width after deformation and embedment as:

$$w_f = (w_{f01} + w_{f02}) - (C_1 + C_2) \sqrt[3]{P_c} = w_{f0} - (C_1 + C_2) \sqrt[3]{P_c} \quad (4.30)$$

The terms C_1 and C_2 are composed of the sub-term $(\sqrt[3]{E_p(1-v_p^2)^2})$, which is a formulation of geo-mechanics properties of each particle type. Consequently, these terms: $\frac{1}{Ev_1} =$

$\frac{1}{\sqrt[3]{E_{p1}(1-v_{p1}^2)^2}}$ and $\frac{1}{Ev_2} = \frac{1}{\sqrt[3]{E_{p2}(1-v_{p2}^2)^2}}$ do not need to be derived further. From the presumable

layering schematic, w_{f1} and w_{f2} are expected to exhibit certain relationships to the total initial fracture width (w_{f0}) and the Weight Concentration Ratio (R). Further continuing derivations validate this assumption.

According to previous definitions of N_1 and N_2 in permeability modelling, their equations are reminded as follows:

$$N_1 = \frac{MR}{\rho_1 \left(\frac{1}{6} \pi D_1^3 \right)} \quad (4.31)$$

$$N_2 = \frac{M(1-R)}{\rho_2 \left(\frac{1}{6} \pi D_2^3 \right)} \quad (4.32)$$

Based on the dual-layer unity, there are several monolayers in the sub-layer of each particle type. In each of these mono-layer, the particles need to fill the contact area (A). Therefore, the number of particles in a monolayer for two particle types, N_{mono-1} and N_{mono-2} , respectively, are derived as:

$$N_{mono-1} = \frac{A}{\pi D_1^2} \quad (4.33)$$

$$N_{mono-2} = \frac{A}{\pi D_2^2} \quad (4.34)$$

From N_{mono-1} and N_{mono-2} , the number of mono-layers for two particle types, L_1 and L_2 , respectively, are derived as:

$$L_1 = \frac{N_1}{N_{mono-1}} = \frac{6CR}{\rho_1 D_1} \quad (4.35)$$

$$L_2 = \frac{N_2}{N_{mono-2}} = \frac{6C(1-R)}{\rho_2 D_2} \quad (4.36)$$

In order to closely resemble practical packing for particles, a stable packing geometry as center cubic packing is selected for this modelling. Under center cubic packing, the initial width for each layer, w_{f01} and w_{f02} , respectively, are derived as:

$$w_{f01} = \frac{6\sqrt{2}CR}{\rho_1} \quad (4.37)$$

$$w_{f02} = \frac{6\sqrt{2}C(1-R)}{\rho_2} \quad (4.38)$$

Where C is weight concentration at lab conditions. Summation of the two equations for w_{f01} and w_{f02} provides the total initial fracture width (w_{f0}) of a dual-layer unity as:

$$w_{f0} = 6\sqrt{2}C \left[\frac{R}{\rho_1} + \frac{(1-R)}{\rho_2} \right] \quad (4.39)$$

Division of each of the two equations for w_{f01} and w_{f02} to the equation of total initial fracture width (w_{f0}):

$$\frac{w_{f01}}{w_{f0}} = \frac{\frac{R}{\rho_1}}{\frac{R}{\rho_1} + \frac{(1-R)}{\rho_2}} = \frac{R\rho_2}{R\rho_2 + (1-R)\rho_1} = \frac{Z_2}{Z_2 + Z_1} \quad (4.40)$$

$$\frac{w_{f02}}{w_{f0}} = \frac{\frac{(1-R)}{\rho_2}}{\frac{R}{\rho_1} + \frac{(1-R)}{\rho_2}} = \frac{(1-R)\rho_1}{R\rho_2 + (1-R)\rho_1} = \frac{Z_1}{Z_2 + Z_1} \quad (4.41)$$

The ratios $\frac{Z_1}{Z_1 + Z_2}$ and $\frac{Z_2}{Z_1 + Z_2}$ now prove the pre-deterministic viewpoint that w_{f1} and w_{f2} are expected to exhibit certain relationships to the total initial fracture width (w_{f0}) and the Weight Concentration Ratio (R). Investigation of these ratios into the initially proposed equation for w_f provides the final equation of fracture width for a two-sized particle assembly:

$$w_f = w_{f0} \left[1 - \frac{4}{\pi^{\frac{2}{3}} 9} \left(\frac{Z_2}{Z_1 + Z_2} \frac{1}{\sqrt[3]{E_{p1}(1-v_{p1}^2)^2}} + \frac{Z_1}{Z_1 + Z_2} \frac{1}{\sqrt[3]{E_{p2}(1-v_{p2}^2)^2}} \right) \sqrt[3]{P_c} \right] \quad (4.42)$$

$$\text{Where: } Z_2 = R\rho_2, Z_1 = (1-R)\rho_1$$

4.3. Determination of coefficients

From the development of the semi-analytical modelling approach, there are coefficients in the models for both proppant pack's permeability and proppant pack's width. Similar to the development process, set of coefficients in each model is determined independently.

For proppant pack's permeability, before any further determination for the coefficients, the equation (4.16) for the model is necessary to be reminded, as follows:

$$k_f = \frac{\phi^3}{K_o(1-\phi)^2} \times \frac{1}{\tau a_v^2}$$

$$\text{Where: } \phi = \phi_o e^{-\beta(P-P_o)}, \tau = \phi^n, \text{ and } a_v = \frac{6}{D_1 D_2} \times \frac{\rho_1 D_1 + (\rho_2 D_2 - \rho_1 D_1) R}{\rho_1 + (\rho_2 - \rho_1) R}$$

In the permeability modelling, tortuosity is a function of porosity under effect of coefficient n , and porosity is a function of applied confining stress under effect of coefficient β and the initial porosity ϕ_o (porosity of the proppant pack at zero applied confining stress). The value of P_o in the porosity exponential equation is 14.7 psi. Consequently, the term $\frac{\phi^3}{K_o \tau (1-\phi)^2}$ (which has no unit) becomes a function of applied confining stress (at a specific weight mixing ratio), under effect of 3 coefficients: n , β and ϕ_o

The Internal Specific Area, a_v , was derived to be dependent on the weight mixing ratio, proppant average diameters, and proppant densities. Equation (4.15) infers that a_v can be computed for any binary mixtures since average diameters, densities for two proppant types in a mixture and the weight mixing ratio are all provided. Furthermore, a set of measured permeability data for any binary mixtures exhibits a clear relationship to applied confining stress points (pre-set in the experiments, as aforementioned). A re-arrangement for the permeability equation is now performed as below:

$$\frac{\phi^3}{\tau(1-\phi)^2} = k_f a_v^2 K_o \quad (4.43)$$

Since a_v is unique for each tested binary mixture with a specific weight mixing ratio, the above re-arrangement becomes an equation having a general formula as:

$$k_f a_v^2 K_o = \frac{\phi^3}{\tau(1-\phi)^2} = \frac{\phi^{3+n}}{(1-\phi)^2} = \frac{\phi_o e^{-\beta(P_c-P_o)(3+n)}}{(1-\phi_o e^{-\beta(P_c-P_o)})^2} \rightarrow y_{lab} = f(P_c) \quad (4.44)$$

Where $y_{lab} = k_f a_v^2 K_o$, and $f(P_c)$ is a function of applied confining stress generated from the term $\frac{\phi^3}{K_o \tau (1-\phi)^2}$. Additional unit conversion is required for the term $k_f a_v^2$ as this term is expected

to be unit-less, and the appropriate unit conversion factor in this case is determined to be 9.869223×10^{-13} . Based on the complete derivation for the term $\frac{\phi^3}{K_o \tau (1-\phi)^2}$ in (4.44) and the form of the general formula above, 3 coefficients (n , β and ϕ_o) are required to be solved for each lab data set of permeability versus applied confining stress. Solutions for these 3 coefficients in equation (4.44) clearly arises a non-linear least-square problem since equation (4.44) resembles equation (2.2). Chapter 2 readily describes trust-region method as an effective option to solve non-linear least square problems. The remaining aspect for a complete solution is boundary selection.

Boundaries for the coefficients in the converted $f(P_c)$ rely on commonly known ranges of the coefficients, since all of the concerned coefficients in the novel semi-analytical model have been examined and/or validated in previous researches. The use of porosity exponent model has been previously applied in different studies of porous media, for example porosity of sandstone formations. Albeit a general proppant packing may ensemble the environment of a porous media, the two coefficients in the porosity exponent model, including porosity at initial condition (zero applied stress) and compressibility factor are required to be reviewed for suitable boundary selections.

Porosity at initial condition, within the modelling context, is referred to porosity of a two-sized particle assembly. Experimentally, porosity of a two-sized assembly was measured in Anthony Diyokeyugwu and Paul Glover (2018), and variability of this parameter is known to range between 0.3116 and 0.3716 when the mean diameter ratio in the two-sized system is between 1.5 and 3. Compressibility factor, within the modelling context, is expected to be close to compressibility of unconsolidated formations. Variability of unconsolidated formation's compressibility factor is known to range between $5.5 \times 10^{-6} (psi^{-1})$ and $8.5 \times 10^{-3} (psi^{-1})$.

From the above reasoning sequence, the expected lower – upper boundaries for $\emptyset_o$ and β are $0.3116 - 0.3716$ and $5.5 \times 10^{-6} - 8.5 \times 10^{-5} \text{ (psi}^{-1}\text{)}$, respectively.

For tortuosity, its intrinsic definition does not lead to any considerable difference between previous studies for porous media and application in the modelling context in this thesis. The most common range for this parameter is between 4.5 and 5.2 (general granular beds, according to modelling examination by Carman (1937) and Kozeny (1927)). When combined with the proposed exponent model for tortuosity and the expected range for porosity at initial conditions, the coefficient n in the exponent model for tortuosity has expected lower – upper boundaries between 0.3 and 1.2.

Conclusively, initial settings for the trust-region method in MATLAB® Optimization Toolbox (R2019b) to solve the objective equation (4.44) are listed as follows:

- Sub-problem algorithm for the model function: factorization
- Maximum iteration allowance: 500
- Objective function's tolerance: $1.0 \cdot 10^{-7}$
- Initialization of the coefficients in the algorithm: $[0.25, 1.0 \cdot 10^{-5}, 0.8]$

For proppant pack's width, relatively similar coefficient determination procedure is processed. Before any determination for the coefficients, equation (4.42) for the model is reminded, as follows:

$$w_f = w_{f0} \left[1 - \frac{4}{\pi^3 \sqrt[3]{9}} \left(\frac{Z_1}{Z_1 + Z_2} \frac{1}{\sqrt[3]{E_{p1}(1 - v_{p1}^2)^2}} + \frac{Z_2}{Z_1 + Z_2} \frac{1}{\sqrt[3]{E_{p2}(1 - v_{p2}^2)^2}} \right) \sqrt[3]{P_c} \right]$$

Where $Z_2 = R\rho_2, Z_1 = (1 - R)\rho_1$. Equation (4.42) requires the terms w_{f0} , $\sqrt[3]{E_{p1}(1 - v_{p1}^2)^2}$, and $\sqrt[3]{E_{p2}(1 - v_{p2}^2)^2}$ to be determined. However, as mentioned in chapter 1,

available width data in Eric (2014) does not satisfy to be used for the modelling approach in this thesis, and an alternative is required. The alternative starts from the approach to model proppant pack's width itself. In chapter 4, the terms w_{f0} and $\sqrt[3]{E_p(1 - v_p^2)^2}$ exist in equation (4.27) which models width for a single proppant type. A substitution database from FRACPRO® is extracted for all proppant types in the studied binary mixtures. The list of all components contain 20/40 sand, 20/40 ceramic, 40/70 sand and 40/70 ceramic. The database has width data of all proppant types in the list, which is measured under experimental settings similar to Eric's experiments. For each single proppant type in the list, substituted width data allows extraction of the terms w_{f0} and $\sqrt[3]{E_p(1 - v_p^2)^2}$ using equation (4.27). Substitution database and extraction process to obtain the terms w_{f0} and $\sqrt[3]{E_p(1 - v_p^2)^2}$ are in the Appendix.

As a result, the terms $\sqrt[3]{E_{p1}(1 - v_{p1}^2)^2}$ and $\sqrt[3]{E_{p2}(1 - v_{p2}^2)^2}$ in equation (4.42) are extracted using the substitution database. Under the fact that modelling in this thesis is based on the API unit's dimensions in Eric's experiments, initial widths for sole packing of each proppant type in a binary mixture (w_{f0} in (4.27)) and initial width for packing of the corresponding binary mixture (w_{f0} in (4.42)) are closely equal. Equivalently:

$$w_{f1} \approx w_{f2} \approx w_{f0} \quad (4.45)$$

Where w_{f1} and w_{f2} are initial widths for sole packing of the first and second proppant type in a binary mixture, and they are also extracted using substitution dataset and equation (4.27). Combined with (4.45), equations (4.40) and (4.41) are revised as:

$$\frac{w_{f01}}{w_{f1}} = \frac{Z_2}{Z_2 + Z_1} \quad (4.46)$$

$$\frac{w_{f01}}{w_{f2}} = \frac{Z_1}{Z_2+Z_1} \quad (4.47)$$

Equations (4.46), (4.47) and (4.30) are combined to linearly approximated w_{f0} in equation (4.42) as:

$$w_{f0} = \frac{Z_1}{Z_2+Z_1} w_{f2} + \frac{Z_2}{Z_2+Z_1} w_{f1} \quad (4.48)$$

From the coefficient determination procedures for the semi-analytical modelling, the conductivity lab data, albeit being provided, is not facilitated during any coefficient determination procedures. The reason for this omission is that conductivity lab data will be facilitated during the modelling validation (in chapter 6) for better robustness of the derived conductivity model.

CHAPTER 5 – CASE STUDY FOR MARCELLUS SHALE

This case study, which implies reservoir properties, geo-mechanics and fracture treatment design of well MIP-3H (Marcellus Shale), practically better quantifies the application of the developed semi-analytical modelling approach when compared to LWAM. Conductivity estimation and cumulative production prediction (up to 10 years) are two selected criteria. To conduct this case study and acquire the comparison results, a fracture propagation simulator and a numerical reservoir simulator are combined. This combination is proposed to transform the conductivity data by a modelling approach (LWAM or the novel semi-analytical, from the fracture propagation simulator) to permeability data for the fracture. The transformed permeability data is further imported into the numerical reservoir simulator in order to observe the impact on cumulative production data. This chapter describes the two simulators and their facilitation in this case study. Since three binary mixtures are selected to be studied and modelled in this thesis, all of them are implemented as test cases for Marcellus Shale.

5.1. Fracture propagation simulator

Albeit the study in this thesis primarily focuses on modelling development and validation, prospective applications from the novel conductivity modelling require at least a fracture propagation simulation program. Therefore, an in-house fracture propagation simulator was built to support these potential applications.

A general description for the fracture propagation simulator is detailed chapter 3. However, the use of this simulator for conductivity estimation and transformation from conductivity to permeability are detailed in this chapter.

5.1.1. Conductivity estimation

As this thesis studies 3 specific binary mixtures (selected and described in chapter I), corresponding Mesh sizes and densities are recorded to specify the matched binary mixture (i.e 1st injection: 40/70 sand and 2nd injection: 20/40 ceramic). The binary mixture specification (including two proppant types and the order of injection as the previous example) combined with the Weight Concentration Ratio matrix are further inputted into the conductivity estimation procedure. For the value of Weight Concentration Ratio in a digitized grid, conductivity estimations by LWAM and the semi-analytical modelling are performed since they are both functions of confining stress (equivalently minimum horizontal stress encoded in the simulator) and Areal Concentration Ratio. Conductivity estimations by both models have to be linearly scaled to the total areal concentration value of the grid (reservoir condition) from 2 lb/ft² (lab condition), as follows: $C_{reservoir} = C_{lab} \frac{ArConcTot}{2}$. Conductivity estimation for proppant concentration condition inside the reservoir is the final computation process. By sole use of the fracture propagation simulator, conductivity estimation can be satisfied.

5.1.2. Transformation from conductivity to permeability

Transformation from conductivity data (generated by modelling approaches, LWAM or the semi-analytical model) to permeability data (imported into the numerical reservoir simulator) needs two requirements. The former requirement is the default fracture width in CMG (a numerical value), which facilitates the following data conversion: $k_{CMG} = \frac{C_{CMG}}{w_{CMG}} = \frac{C_{fracture\ simulator}}{w_{CMG}}$ (all variables are 2D matrices, except w_{CMG}). The latter requirement is an integration in dimensions with the numerical reservoir simulator. In this thesis, CMG is used as a 3D numerical reservoir simulator. In CMG, 3 dimensions are declared: I, J, and K. The number of grids for these dimensions are denoted as nI, nJ, and nK, respectively. However, permeability data in CMG is

declared as a 2D matrix with dimensions of $(nJ * nK) * nI$. The structure of this 2D matrix is constructed as follows:

- The row $nI = 1$ stores the fracture permeability data
- The rows of other nI values store the formation permeability data

As a result, fracture half-length and fracture width grid dimensions should be designed in the fracture simulator to be identical to the J and K grid dimensions in CMG, respectively. Following the dimension integration as described, all numerical values in the previously computed k_{CMG} are converted into the row $nI = 1$ of a new k_{CMG} , and the remaining rows in the new k_{CMG} hold the permeability value of the formation (i.e. Marcellus Shale in this case study). The new k_{CMG} is imported into the permeability declaration of the CMG input file (i.e. a .DAT format file).

5.2. Numerical reservoir simulator

As aforementioned, CMG software is used as the numerical reservoir simulator in this thesis and it is coupled with the in-house fracture propagation simulator for cumulative production prediction. This CMG model was proposed by a groups of authors (Gu, Kulkarni, Raflee and Ivarrud) in 2014, and was comprehensively detailed by Courtney L. Rubin in 2018 for a parametric study.

Simulation domain

One-fourth of a single fracture's drainage area is selected as the reservoir simulation region. Bi-wing fracture geometry is implied in the CMG model, which directly consolidate with the 2D-PKN in the fracture propagation simulator. According to the simulation domain and the assumed fracture geometry, cumulative production for a horizontal well equals a multiplication of

the cumulative production from the CMG model by $4N_{fractures}$ (the number of one-fourth simulation region in a horizontal well).

Figure 9 - Schematic for the simulation domain (1/4 of a single fracture's drainage area) in the CMG model (Gu, Kulkarni, Raflee and Ivarrud, 2014)

Reservoir homogeneity

Since this thesis focuses on the effect from the hydraulic fracture's permeability, simplified assumption of homogeneity in the reservoir is implied albeit Marcellus Shale is a heterogeneous formation. Based on the assumption of homogeneity, the permeability matrix data k_{CMG} has an identical value for all rows of nI different from 1 (which is the reservoir permeability value). In addition, Logarithmic Local Grid Refinement (LLGR) is implied to reflect transient flow regime between the reservoir's intrinsic permeability zone and the fractured channels' permeability zone.

Gas desorption

Effect of gas desorption is a requirement since Marcellus Shale is a dry gas reservoir. Langmuir isotherm equation (type I), which assumes dynamic equilibrium at constant temperature

and pressure between absorbed and non-absorbed gas, is used to model the desorption effect (Langmuir, 1918). The Langmuir pressure (P_L) and Langmuir volume (V_L) are correlated by the following relationship:

$$v(P) = \frac{V_L P}{P + P_L} \quad (5.1)$$

Where $v(p)$ is the absorption gas volume as pressure P , V_L is the maximum gas volume at infinite pressure, and P_L is the pressure which corresponds to one half of Langmuir volume. The optimal values of Langmuir pressure and Langmuir volume are selected based on experiment results in Yu et.al (2016).

Flow regime

Within the hydraulic fractures as highly conductive channels for fluid flow, turbulence flow (or non-Darcy flow) is implied in replacement of Darcy flow. In this CMG model, Forchheimer flow equation (Gu, 2013) is used. The non-Darcy factor is computed as follows:

$$\beta = \frac{1.485 \cdot 10^9}{\phi k^{1.021}} \quad (5.2)$$

Where β is the non-Darcy factor in 1/ft. Since equation (5.2) has been implied in a variety of porous media with satisfying precision, its implication in this CMG model is reasonable (Courtney Rubin, 2018).

CHAPTER 6 – RESULTS AND DISCUSSIONS

6.1. Sensitivity study

6.1.1. Dependency of mixture zone on studied factors

Figure 10 – Sensitivity results for former injected proppant type’s diameter (Diameter 1)

Figure 11 – Sensitivity results for latter injected proppant type’s diameter (Diameter 2)

Figure 12 – Sensitivity results for former injected proppant type’s density (Density 1)

Figure 13 – Sensitivity results for former injected proppant type’s density (Density 2)

Figure 14 – Sensitivity results for injection rate

Figure 15 – Sensitivity results for confining pressure

Figure 16 – Sensitivity results for Poisson Ratio

Figure 17 – Sensitivity results for Young Modulus

Figure 18 – Sensitivity results for former injected proppant type's (4th time step)

Figure 19 – Sensitivity results for former injected proppant type's (5th time step)

Figures 10-19 present figurative outcomes of the sensitivity study for the 10 parameters listed in chapter 4. The x axis in figures 10-19 present the sensitivity increments (i.e. percentage increments from the base case's value in chapter 3). The y axis presents relative deviation (i.e. % deviation from the base case's value of all sensitivity incremental points in chapter 3). All the numerical outcomes of the sensitivity study which provide the data for plotting figures 9-18 are detailed in the Appendix.

Confining pressure presents the heaviest impact on the mixture zone with a sensitivity in a range of up to 13.17% from the base case. Increasing the confining pressure increases the mixture zone and vice versa. Rate of increase when increasing confining pressure is lower than rate of decrease when decreasing confining pressure. Injection rate also presents a high impact on the mixture zone (9.29% to -4.15%). The dependence of mixture zone on injection is generally inversely proportional, and the rate of decrease is rapid until 10% increase from the base case. However, when the injection rate increases to exceed 10% from the base case, the impact on mixture zone maintains at approximately -4%. Poisson Ratio and Young Modulus both present fluctuations regarding their impact to the mixture zone. Their impact is observed to be not significant compared to confining pressure and injection rate (up to 2.28% from the base case).

Both proppant diameters and densities present high and clear behavior of impact to the mixture zone, and expected degree of impact to the mixture zone for these parameters is equivalent. Increase in diameter and density of the former injected proppant type increases impact on the mixture zone, whereas the reverse behavior is observed for the latter inject proppant type.

Increase in injection time of the former proppant type (focus of sensitivity at the 4th stage) increases the mixture zone and vice versa. Rate of increase when increasing this parameter is sharper than rate of decrease when this parameter decreases. Injection time of the latter proppant

type (focus of sensitivity at the 5th stage) increases the mixture zone when the change of this parameter is within 10% from the base case. When this parameter increases more than 10% from the base case, change to the mixture zone approaches and stabilizes at closely 1.5%. In contrast, when this parameter decreases more than 10% from the base case, the mixture zones slightly fluctuates within the 0.5% from the base case. Injection time of the former proppant type's impact to mixture zone is higher (-2.87% to 2.81%) compared to impact from injection time of the latter proppant type (-1.03% to 1.39%).

6.1.2. Summary

Table 4: Correlation between output and input factors

Input factor	Correlation sign
Diameter (former proppant)	+
Diameter (latter proppant)	-
Density (former proppant)	+
Density (latter proppant)	-
Injection rate	-
Confining pressure	+
Young Modulus	<i>No clear correlation</i>
Poisson Ratio	<i>No clear correlation</i>
Injection time (former proppant)	+
Injection time (latter proppant)	+

Table 4 provides the first summary viewpoint for this sensitivity study, which is conducted through observations of prospective correlations between output factor (% of Mixture Zone area) and studied input factors (equivalently, observations from figures 10-19). A positive sign indicates proportional correlation between output factor and the corresponding input factor. A negative sign indicates inversely proportional correlation between output factor and the corresponding input factor. Description "No clear correlation" does not clearly indicate a positive or a negative sign.

Based on table 4, assumptions in the sensitivity design (i.e. the standard, local OAT sensitivity method) and the tested model (i.e. the in-house fracture propagation simulator), geo-mechanics properties do not present any clear behaviors of their impact to the Mixture Zone, albeit they are widely known to be core influencers which affects fractured area.

The second summary viewpoint for this sensitivity study is presented through the tornado plot in figure 20. Based on the tornado plot, assumptions in the sensitivity design and the tested model, confining stress and the proppant's properties show the biggest effect to the mixture zone. Geo-mechanics properties (i.e. Young Modulus and Poisson Ratio) show insignificant effect to the mixture zone. Former proppant type's density and later proppant type's diameter contribute similar degree of impact to the mixture zone. Similarly, former proppant type's diameter and latter proppant type's density contribute similar degree of impact to the mixture zone. The degree of impact from these proppant parameters is slightly lower compared to the previously mentioned proppant parameters. The last 4 parameters in the tornado chart (i.e. injection time for the former proppant type and below) are insignificant influencers to the Mixture Zone.

Conclusions from the sensitivity study based on table 4 and figure 20 are capable of predictive applications as the following examples. Utica Shale should have a larger mixture zone compared to Marcellus Shale based on the impact from confining pressure, as Utica has a higher confining stress than Marcellus. Provided that the mixture zone for injection of 40/70 sand and 20/40 ceramic is known, an injection of binary mixture 20/40 sand and 20/40 ceramic should expand the mixture zone because of impact from higher size of the former proppant type. Similarly, an injection of binary mixture 40/70 ceramic and 20/40 sand should also expand the mixture zone because of impact from higher density of the former proppant type and lower density of the latter component. Within all factors from the sensitivity study which may contribute to modelling

development in chapter 4 and assumptions in the in-house fracture propagation simulator, closure pressure and proppant's properties provide indication that they need higher weighting factors whereas the geo-mechanics properties provide indication that they need lower weighting factors.

Figure 20 – Summary of the sensitivity study for all parameters

6.1.3. Drawbacks of the sensitivity study's design

Since the sensitivity study is designed to examine local contribution of a factor's impact to the mixture zone and the sensitivity behavior of input factors are small around an expected value, application of standard OAT and finite-difference approximation (Morris, 1991) are accepted. However, the scale of the sensitivity analysis in this chapter can be expanded to a global sensitivity analysis. According to Morris global sensitivity design (Morris, 1991), the base case is generated as each run within analysis for one input factor. Consequently, the computation runs required scale up to factorial of 10 (i.e. 10!), presumably applied to this thesis, and the analysis for variance of

the mixture zone (i.e. the output factor) is obviously more feasible compared to the current design in this chapter.

Another aspect to be considered is dependency within the input factors. In the list of input factors, three are classified as reservoir relevant (reservoir group), which include confining stress, Young Modulus and Poisson Ratio. The remaining seven are classified as treatment relevant (treatment group). While the factors in the reservoir group exhibit certain correlation (i.e. factor interactions), the factors in the treatment group can be claimed as independent. Therefore, application of elementary effect (Morris, 1991) is only reasonable for the treatment group. In addition, the variance in the reservoir group's factors is known from field measurement's analysis, besides their physics-based relationship. Contribution from these concerns can lead to better design and more realistic discussions for impact from the reservoir's group to mixture zone.

6.2. Determination of coefficients

As described in chapter 4, the coefficients for permeability modelling approach requires facilitation of trust-region-reflective algorithm under control of boundary conditions. Table 4 provides numerical values of all coefficients in the permeability modelling approach and the R^2 (i.e. residual error) values for 6 selected lab data sets. Graphical outcomes visualizing the efficacy of the selected algorithm are in the Appendix.

Table 5 - Values of coefficients and R^2 in the permeability model

Binary mixture and weight mixing ratio	Φ_o	β (1/psi)	n	R^2 error
20/40 sand and 20/40 ceramic, 50/50	0.374456	6.75E-05	0.399795	1.34E-05
20/40 sand and 20/40 ceramic, 25/75	0.356449	2.58E-05	0.47776	2.38E-06
40/70 sand and 20/40 ceramic, 50/50	0.358651	6.32E-05	0.401368	3.77E-06
40/70 sand and 20/40 ceramic, 25/75	0.360489	6.35E-05	0.44827	5.86E-06
40/70 ceramic and 20/40 sand, 50/50	0.31414	2.53E-05	0.614715	1.56E-07
40/70 ceramic and 20/40 sand, 25/75	0.326409	3.25E-05	0.588929	1.52E-06

The residual errors in table 5 are acceptably minimal, which indicate high efficacy of the selected algorithm. However, under a modelling development for a physic-based phenomenon, physical interpretation for the coefficients is an obligation before further applications. In the context of this thesis, examination for coefficients are based on two independent criteria: the physical quantity they relate to (i.e porosity, hydraulic tortuosity) and the effects from intrinsic properties of the modelled phenomenon (i.e. the proppant types in the binary mixture and the weight mixing ratio, or Areal Concentration Ratio). The former criteria is implied in the constraints of the coefficients is already described in chapter 4. In this chapter, the latter criteria is discussed in term of coefficient-based.

6.2.1. Initial porosity at zero applied stress (Φ_o)

Φ_o is initially porosity of the proppant pack at zero applied stress. Because of its definition, Φ_o highly depends on the packing of proppant particles and does not depend on confining stress. Reasonably, Φ_o shall exhibit close relationship with volumetric ratio (which is consequently converted to weight ratio, or R as notation defines) between two proppant types in a binary mixture. However, this relationship should be better analytically reasoned based on an experimental study by Anthony Diyokeygwu and Paul Glover (2018). The study focused on correlation between porosity, $\frac{D_{coarser}}{D_{finer}}$ ratio (i.e. diameter ratio), and finer proppant type's volumetric ratio in binary mixtures, which is represented in figure 21.

Limited to the modelling context in this thesis, the diameter ratios of three studied binary mixtures are averagely between 1.12 and 2.1, which leads to a porosity range between 30% and 40%. Overall, porosity values in table 4 satisfy this nominal range. For binary mixture 20/40 sand and 20/40 ceramic, the finer proppant type's volumetric ratios are 0.682 (i.e. equivalent 50% weight of 20/40 sand) and 0.417 (i.e. equivalent 25% weight of 20/40 sand), respectively. Since

0.682 and 0.417 exceed the critical volumetric ratio of finer proppant type (0.25-0.35), Φ_o value in case 20/40 sand weighs 50% of the mixture is higher than Φ_o value in case 20/40 sand weighs 25% of the mixture. For binary mixture 40/70 sand and 20/40 ceramic, the finer proppant type's volumetric ratios are 0.682 (i.e. equivalent 50% weight of 40/70 sand) and 0.417 (i.e. equivalent 25% weight of 40/70 sand), respectively. Since 0.682 and 0.417 exceed the critical volumetric ratio of finer proppant type (0.25-0.35), Φ_o value in case 40/70 sand weighs 50% of the mixture is higher than Φ_o value in case 40/70 sand weighs 25% of the mixture. For binary mixture 40/70 ceramic and 20/40 sand, the finer proppant type's volumetric ratios are 0.318 (i.e. equivalent 50% weight of 40/70 ceramic) and 0.135 (i.e. equivalent 25% weight of 40/70 ceramic), respectively. Compared to the critical volumetric ratio of finer proppant type (0.25-0.35), 0.318 is in the critical range whereas 0.135 is smaller than the critical range. As a result, Φ_o value in case 40/70 ceramic weighs 50% of the mixture is lower than Φ_o value in case 40/70 ceramic weighs 25% of the mixture. All numerical values of term Φ_o from table 4 successfully reflects analytical reasoning of all studied binary mixtures.

Figure 21 – Relationship between porosity and former proppant type's volumetric ratio
(diameter ratio is 1.5)

6.2.2. Beta coefficient in porosity model (β)

β is the compressibility factor in porosity exponent model (equation (4.2)). Overall, β values in tables 4 satisfy the nominal range of rock compressibility, which is between $1.5 \cdot 10^{-6}$ and $8.5 \cdot 10^{-5}$. Because proppant packs in Eric's experiments are compressed by steel plates, they have high compressibility potential. As a result, β values in tables 4 are reasonable to be all in 10^{-5} scale and not 10^{-6} scale. Furthermore, compressibility factor β relates directly to size and strength of the material. In a binary mixture, more weight of the weaker or coarser proppant type leads to expectation of a higher compressibility factor.

For binary mixture 20/40 sand and 20/40 ceramic, more weight of 20/40 sand shall expect higher compression under applied stress. Therefore, compressibility factor in case 50% 20/40 sand weight in the mixture is higher than compressibility factor in case 25% 20/40 sand weight in the mixture. For binary mixture 40/70 sand and 20/40 ceramic, albeit proppant type 20/40 ceramic is stronger, Eric's experiments present its close similarity of compression under applied stress compared to 40/70 sand. Therefore, compressibility factor in case 50% 40/70 sand weight in the mixture is slightly lower than compressibility factor in case 25% 40/70 sand weight in the mixture. For binary mixture 40/70 ceramic and 20/40 sand, more weight of 20/40 sand shall expect higher compression under applied stress. Therefore, compressibility factor in case 50% 40/70 ceramic weight in the mixture is lower than compressibility factor in case 25% 40/70 ceramic weight in the mixture. All numerical values of term β from table 4 successfully reflects analytical reasoning of all studied binary mixtures.

6.2.3. Exponential coefficient in tortuosity model (n)

n is the coefficient in the tortuosity exponential model (equation (4.3)). As reviewed in chapter 1, nominal range of n values is 0.4-0.5. However, complexity caused by packing because

of experimental conditions does not guarantee the nominal range (Ricardo et.al, 2006; Mota et.al, 2001). In Eric's experiments, the proppant is blended uniformly prior to conducting the tests, therefore a highly dense packing is possible. The values of n for binary mixture 40/70 ceramic and 20/40 sand reflects the above reasoning, otherwise other binary mixtures have nominal n values.

Another aspect which requires discussion is partial impact of coefficient n . Because porosity exists in equation (4.3), integration of equation (4.2) into (4.3) revises (4.3) as:

$$\tau = (\phi_o e^{-\beta(P-P_o)})^{-n} = \phi_o^{-n} \cdot e^{\beta n(P-P_o)} = \tau_o e^{\beta n(P-P_o)} \quad (6.1)$$

In equation (6.1), $\tau_o = \phi_o^{-n}$ is the initial tortuosity (at zero applied stress) since ϕ_n is porosity at zero applied stress. The term βn is the increase rate of tortuosity since this term is positive. Equation (6.1) claims that impact of coefficient n to tortuosity should be reasoned through at least one of the two factors, τ_o or βn . Based on Ghanbarian et.al (2013), two porous media having similar porosity may not have similar tortuosity, and two porous media having slightly different porosity may still have closely equal tortuosity. As a result, analytical reasoning for factor τ_o is not feasible, and βn remains as the sole option to examine impact of n .

For binary mixture 20/40 sand and 20/40 ceramic, 20/40 sand is easier to be crushed into significant smaller Mesh sizes compared to 20/40 ceramic because of material's intrinsic property (numerical results in Eric, 2014). Therefore, more concentration of 20/40 sand leads to higher possibility of a higher tortuosity increase rate. From table 4, tortuosity increase rate in two cases, 50% weight of 20/40 sand and 25% weight of 20/40 sand, equals $2.7 \cdot 10^{-5}$ and $1.233 \cdot 10^{-5}$. For binary mixture 40/70 sand and 20/40 ceramic, 20/40 ceramic is, in fact, easier to be crushed into smaller Mesh sizes compared to 40/70 sand because of original size dominance instead of material's intrinsic property (numerical results in Eric, 2014). Therefore, more concentration of 20/40 ceramic leads to higher possibility of a higher tortuosity increase rate. From table 4,

tortuosity increase rate in two cases, 50% weight of 20/40 ceramic and 25% weight of 20/40 ceramic, equals $2.534 \cdot 10^{-5}$ and $2.847 \cdot 10^{-5}$. For binary mixture 40/70 ceramic and 20/40 sand, 20/40 sand is, in clearly easier to be crushed into smaller Mesh sizes compared to 40/70 ceramic because of both original size and material's intrinsic property (numerical results in Eric, 2014). Therefore, more concentration of 40/70 ceramic leads to higher possibility of a lower tortuosity increase rate. From table 4, tortuosity increase rate in two cases, 50% weight of 40/70 ceramic and 25% weight of 40/70 ceramic, equals $1.556 \cdot 10^{-5}$ and $1.914 \cdot 10^{-5}$. All numerical values of term βn from table 4 successfully reflects analytical reasoning of all studied binary mixtures.

6.3. Semi-analytical modeling validation

Figure 22 – Validation results for binary mixture 20/40 sand and 20/40 ceramic

Figure 23 – Validation results for binary mixture 40/70 sand and 20/40 ceramic

Figure 24 – Validation results for binary mixture 40/70 ceramic and 20/40 sand

As stated in chapter 4, conductivity lab data is used to perform modelling validation. Figures 20-22 present graphical outcomes of the modelling validation for all 6 selected lab data sets. The Mean Square Errors (MREs) between the prediction from the semi-analytical model and the lab data for mixture 40/70 sand and 20/40 ceramic (figure 23) range from 0.82-16%, which is considered as acceptable constraints. Two outliers with higher relative difference error are at the areal concentration 0.5/4000 psi and at the areal concentration 0.75/10000 psi. For mixture 20/40 sand - 40/70 ceramic (figure 24), MREs are lower than mixture 20/40 ceramic - 40/70 sand as they range from 0.17-10.7% with two outliers at 0.5/8000 psi and 0.75/10000 psi. For mixture 20/40 ceramic - 20/40 sand (figure 22), it is clear to observe that the MREs are significantly higher than the normality from the two remaining mixtures. Higher MREs for mixture 20/40 ceramic - 20/40 sand are referred to the lab data for the proppant pack's width, which behaves significantly out of the expected normality from proppant physics (deformation and embedment). This profound impact from lab data is contributed by the large deviations of the reference with, w_{ref} , at each measured confining stress points (as explained in the equipment description in this paper), and mixture 20/40 ceramic - 20/40 sand was observed to have highest deviations within all 3 selected mixtures. Ceramic with large diameter size as 20/40 Mesh has permeability of hundreds Darcy at low confining pressure (nearly 350 Darcy at 2000 psi from measured data), which turns a slight deviation of 0.02 inches in width (based on author's claim in Eric Schultz, 2014) to a 583 mD-ft deviation in measured conductivity.

Another factor which prevents the best validation outcomes is the incorporation between all possible proppant physics into the fracture width model, which include proppant deformation, proppant embedment, and proppant crush effect at a certain confining stress. In the derived semi-analytical model, only proppant deformation was taken into consideration under the assumption

of elastic behaviour of material. Proppant embedment, even neglected at the contact between proppant particles and the steel plates, is still considerable between the proppant layers (as shown in the schematic of the layering system), and Hertzian contact theory is further needed for the case of two spherical surfaces at all contact surfaces between the layers. Proppant crush effect, at a certain confining pressure, may completely change the decrease trend of the proppant pack's width behaviour and become harder to be modelled as areal concentration ratio-dependent implementation. In figure 7, an observation from the conductivity lab data for mixture 20/40 sand and 40/70 ceramic indicates proppant crush effect at a certain confining stress larger than 6000 psi. For this mixture, at the areal concentration 0.75 (more weight of 20/40 sand), conductivity drops drastically from approximately 1500 mD-ft to a close of 1000 mD-ft when confining stress changes from 6000 psi to 8000 psi. Since large diameter sand as 20/40 Mesh exhibits clear crush effect, it explains the conductivity drop as aforementioned. Consequently, current implementation of proppant embedment and the crush effect will significantly increase the complexity of the width model, and the semi-analytical model for the proppant pack's width was acquired without their full contributions.

6.4. Comparison between LWAM and the semi-analytical modeling

In chapter 4, the development of the novel semi-analytical modelling is based on dependency on confining pressure, P_c (or minimum horizontal stress) and the weight mixing ratio, R (or Areal Concentration Ratio). To further interpret the efficacy of the novel semi-analytical model and predictable impact of confining stress or Weight Concentration Ratio variability (as they are two core variables for both LWAM and the novel semi-analytical model), a comprehensive comparison between the two models is conducted for each studied binar mixtures. In order to conduct the comparison as mentioned, the following procedure is facilitated:

1. Two discrete ranges for P_c and R are initiated. The ranges for P_c and R are 2000-10000 psi (100 psi increment, 81 discrete values) and 0.01-0.99 (0.01 increment, 99 discrete values), respectively.
2. For each pair of values for P_c and R from the two discrete ranges (171 pairs in total), conductivity estimation for both models is computed. This computation process creates two conductivity matrixes for the two models, both of which have dimensions of 99×81 .
3. Relative difference of the novel semi-analytical model compared to LWAM is computed as follows: $Relative\ Difference = \frac{C_{LWAM} - C_{semi-analytical\ model}}{C_{LWAM}} \times 100$. As being expected, the quantity *Relative Difference* is also a matrix with dimensions of 99×81 . Matrices representing *Relative Difference* are defined as comparison matrices
4. Two-dimensional plot of *Relative Difference* is performed. The two X and Y axes present discrete ranges for quantities P_c and R , and the quantity *Relative Difference* is presented by a color scale.

6.4.1. Overview of the comparison results

Figure 25 – Comparison matrix for binary mixture 20/40 sand and 20/40 ceramic

Figure 26 – Comparison matrix for binary mixture 40/70 sand and 20/40 ceramic

Figure 27 – Comparison matrix for binary mixture 40/70 ceramic and 20/40 sand

Figures 25-27 provide two-dimensional plots of the comparison outcomes for all 3 binary mixtures. An initial observation for these plots clarifies conductivity overestimation by LWAM compared to the novel semi-analytical model for a majority of pairs of P_c and R values. Furthermore, two essential analysis, which include qualitative understanding of LWAM's overestimation for different conditions of P_c and R and prospective improvements for the novel semi-analytical model, can be conducted. Within all studied binary mixtures, increase in P_c and/or R intensifies LWAM's overestimation at difference rates, depending on the intrinsic properties of the binary mixture. According to the added color scale for *Relative Difference* in figures 25-27, LWAM's overestimation can be classified in three distinguishable zones: low overestimation (20% and below), moderate overestimation (20-60%) and high overestimation (60% and above).

For binary mixture 20/40 ceramic and 20/40 sand, figure 25 indicates a central region of high overestimation in which high coordinates of P_c and R are observed at 8000-10000 psi and 0.7-0.99, respectively. The centre of this area has P_c and R coordinates of 0.99-10000 psi. Movement away from the central region in both decreasing directions of P_c and R (i.e. change diagonally from P_c and R coordinates of (0.99-10000 psi) to (0.01-2000 psi)) decreases *Relative Difference* diagonally from high overestimation zone to low overestimation zone. In addition, these zones are separated parabolally.

For binary 20/40 ceramic and 40/70 sand, a similar central region of high *Relative Difference* zone exists in figure 26, albeit the center of this area has P_c and R coordinates of approximately 0.35-10000 psi. Movement away from the central area in the decreasing direction of confining pressure i.e. change horizontally from P_c and R coordinates of 0.35-10000 psi in the decreasing direction of P_c axis) decreases *Relative Difference*

horizontally from high overestimation zone to low overestimation zone along the R axis. Similarly to the binary mixture 20/40 sand and 20/40 ceramic, these zones are aseparated parabolally.

For binary mixture 20/40 sand and 40/70 ceramic, there is no central region in figure 27. In replacement of a central area, the high, moderate and low overestimation zones expand along the P_c axis. Additionally, these zones in figure 25 are separated almost linearly along the decreasing direction of R axis (i.e. approximately 0.55 between the low and moderate zones, and 0.7 between the moderate and high overestimation zones).

6.4.2. Impact of confining pressure and Weight Concentration Ratio variability

Besides the overview of the comparison outcomes, figures 25-27 are capable of supporting predictions regarding the overestimation degree of LWAM, under conditions of confining pressure and Weight Concentration Ratio variation. In addition, partial derivative matrices (in term of confining pressure, $\partial RD/\partial P_c$ and areal concentration ratio, $\partial RD/\partial R$) presented in figures 28-33 can further disclose rate of overestimation under those specific conditions.

Figure 28 – Matrix of $\partial RD/\partial P_c$ for binary mixture 20/40 sand and 20/40 ceramic

Figure 29 – Matrix of $\partial RD / \partial R$ for binary mixture 20/40 sand and 20/40 ceramic

Figure 30 – Matrix of $\partial RD / \partial P_c$ for binary mixture 40/70 sand and 20/40 ceramic

Figure 31 – Matrix of $\partial RD/\partial R$ for binary mixture 40/70 sand and 20/40 ceramic

Figure 32– Matrix of $\partial RD/\partial P_c$ for binary mixture 40/70 ceramic and 20/40 sand

Figure 33 – Matrix of $\partial RD/\partial R$ for binary mixture 40/70 ceramic and 20/40 sand

For mixture 20/40 sand and 20/40 ceramic, the overestimation degree of LWAM monotonically increases when R increases from 0 to 1. Figure 29 further indicates a faster rate of increase when R is between 0.2 and 0.5 and a slower rate of increase to a slight decrease rate when R exceeds 0.6. Lower value of confining pressure (5000 psi and below) indicates a single transition from low overestimation zone to moderate overestimation zone, and the rate of this transition is possibly high. Higher value of confining pressure (6000 and above) indicates two possible transitions from low overestimation zone to moderate overestimation zone, and eventually to high overestimation zone. However, these transitions occur at lower rate when compared to the case of transitioning in a lower value of confining pressure.

For mixture 40/70 sand and 20/40 ceramic, high overestimation zone exists for most of the R and P_c values. Rate of change in overestimation is almost nuance (i.e. around 0) when R increases, with exception at extremely high values of R (0.9-1), of which condition presents a sudden

decrease in the rate of overestimation. At lower value of confining stress (4500 psi and lower), figures 30 and 31 indicate an abrupt change from low/moderate overestimation zone to high overestimation zone when R enters the zone 0.2-0.6 from the zone 0.01-0.2 and another abrupt reverse change from high overestimation zone to moderate/low overestimation zone by LWAM when R exits the zone 0.2-0.6. At higher value of confining stress (4500 psi and higher), high overestimation zone is dominantly present and rate of change in overestimation is nuance.

For mixture 40/70 ceramic and 20/40 sand, the low, moderate and high overestimation zones are uniformly distributed, approximately. Overestimation degree of LWAM monotonically increases when R increases from 0 to 1. However, figure 33 presents that rate of overestimation is almost not considerable when R increases (with exception at a portion of R values below 0.65 and P_c values above 7000 psi). This observation can be equally stated as almost no abrupt change between low/moderate/high estimation zones. Similarly, figure 32 presents that rate of overestimation is almost not considerable when P_c is within the lower range or higher range.

6.4.3. Summary

In binary mixture 20/40 sand and 20/40 ceramic, two proppant types have similar Mesh sizes and contrast in material strength. Comparison matrix indicates clear dependency of Relative Difference on both confining pressure and areal concentration ratio. Relative Difference increases with faster rate when confining stress and Weight Concentration Ratio are low and increases with slower rate when confining stress and Weight Concentration Ratio are higher.

In binary mixture 40/70 sand and 20/40 ceramic, two proppant types have contrast in Mesh sizes and material strength in conventional order (i.e. finer/weaker proppant type is injected first and coarser/stronger proppant type is injected second). Comparison matrix indicates clearer dependency of Relative Difference on confining stress than areal concentration ratio. High

overestimation zone is dominant, and increase in confining pressure rapidly increase the Relative Difference to the high overestimation zone.

In binary mixture 40/70 ceramic and 20/40 sand, two proppant types have contrast in Mesh sizes and material strength in reverse order (i.e. finer/stronger proppant type is injected first and coarser/weaker proppant type is injected second). Comparison matrix indicates stronger dependency of Relative Difference on Weight Concentration Ratio than confining stress. All overestimation zones (low, moderate and high) are uniformly distributed, and increase in Weight Concentration Ratio monotonically increase Relative Difference.

6.4.4. Drawbacks of the novel semi-analytical model

Albeit figures 25-27 prove a sufficient efficacy of the semi-analytical model compared to LWAM in conductivity prediction, the white color regions in figures 25-27 infers that there are conditions (ranges of P_c and R) in which *Relative Difference* falls below 0. For more details, within the area created by the 99×81 discrete values of P_c and R , binary mixture 20/40 sand and 20/40 ceramic returns 26.49% of *Relative Difference* below 0, binary mixture 20/40 ceramic and 40/70 sand returns 2.01% of *Relative Difference* below 0, and binary mixture 20/40 sand and 40/70 ceramic returns 32.13% of *Relative Difference* below 0. This phenomenon indicates that the novel semi-analytical model does not predict as the aimed expectation (i.e. its conductivity prediction should be below LWAM's prediction) for specific ranges of P_c and R , depending on the binary mixture.

For binary mixture binary mixture 20/40 ceramic and 20/40 sand, the semi-analytical model presents moderate degree of pitfall. The sub-zero region of *Relative Difference* reveal the pitfall of the semi-analytical model occurs at low values of P_c (between 2000-2500 psi, regardless of R values) and the low values of R (between 0.01-0.16, regardless of P_c values). For binary

mixture 20/40 ceramic and 40/70 sand, the semi-analytical model presents lowest degree of pitfall in term of both the area and the value of *Relative Difference*. The sub-zero region of *Relative Difference* only occurs at extreme values of R ($>0.9-1$) and low values of P_c (<4000 psi). For binary mixture 20/40 sand and 40/70 ceramic, the semi-analytical model presents highest degree of pitfall. The sub-zero region of *Relative Difference* only occurs at low-end values of R (0.01-0.3, regardless of P_c values).

There are a number of explanations for these sub-zero regions. Interpolation for coefficients as function of Weight Concentration Ratio during composing the comparison matrices is a drawback. However, this interpolation is unavoidable because of shortage in lab data points, limited understanding of the coefficients as a dependency on areal concentration ratio is a consequence. For example, in binary mixture 40/70 sand and 20/40 ceramic, sub-zero region has extremely high areal concentration ratio. At this condition, the binary mixture is close to pure 40/70 sand LWAM is a better estimator. Another drawback is necessity of complicated physics which may have a critical role in the sub-zero regions. For example, in binary mixtures 20/40 sand and 20/40 ceramic, crush effect of 20/40 sand can drastically reduce conductivity of the mixture. At low confining stress, 20/40 ceramic is capable of mitigating crush effect of 20/40 sand. However, as confining stress increases, this mitigation can not maintain, and the sub-zero region becomes larger. Similar explanation regarding crush effect of 20/40 sand can be state in binary 40/70 ceramic and 20/40 sand, especially when confining stress exceeds 5000 psi.

6.5. Case study for Marcellus Shale

6.5.1. Reviews of confining stress data in Marcellus Shale

Typical trait of minimum horizontal stress variation inside a reservoir is a considerable factor when analyzing conductivity estimation from different models. In consequence, reviews of

minimum horizontal stress variation in Marcellus Shale are provided in this section prior to further conductivity estimation analysis and cumulative production analysis. Raw data for minimum horizontal stress in Marcellus Shale, which is used in this section and also in the in-house fracture propagation simulator, is modelled and detailed in Emre D. et.al (2019). This raw data provides discrete values of minimum horizontal stress corresponding to incremental values of depth data between 7450-7548 ft (0.5 ft increment). A portion of the raw data is provided in table 6.

Table 6 - Sample data of confining stress versus depth, 0.5 ft increment (Marcellus Shale)

TVD (ft)	Confining stress (psi)
7450	5519.07
7450.5	5470.42
7451	5448.04
7451.5	5437.37
7452	5462.19

Figure 34 – Histogram of confining stress data in Marcellus Shale

In order to interpret variation of minimum horizontal stress in the data and comply with further analysis in this section, histogram is selected to present the raw data, as in figure 34.

Using basic statistical analysis, the data have arithmetic mean and standard deviation of 5805.52 psi and 233.75 psi, respectively. The histogram in figure 34 implies that outliers start at 6200 psi and above. Most of the data points are ranged between approximately 5400 psi and 6200 psi, which equals 2 standard deviations from the data's mean. Another analysis for the data is to interpret changes of the data points between two consecutive 0.5-ft increment measurements (figure 35). It is clear to observe a normal distribution with arithmetic mean and standard deviation of 0 and 200 psi, respectively.

Figure 35 – Histogram of stress difference (within 0.5 ft height), Marcellus Shale

As a result, it is stated that Marcellus Shale's confining stress is ranged between 5400-6200 psi, and difference in its values between two different depth data (within the formation's height) is at most 200 psi. The above reviews for minimum horizontal stress data in Marcellus Shale coupled with the observations in the comparison section allow predictively reasoning for the conductivity estimation difference between LWAM and the novel model in this case study.

6.5.2. Post-processing for cumulative production data from CMG

As described in chapter 5, 10-year cumulative production data is extracted (Excel format) from CMG for both LWAM and the novel semi-analytical model. However, the simulation algorithms in CMG do not extract similar date records between the models. Consequently, linear interpolation is coded to acquire similar date records between the models. This technique's pseudo code for each conductivity model is as follows:

- The raw date recording and 10-year cumulative production from CMG for a model are denoted as Day_{raw} and $Prod_{raw}$
- Initiate a day range, denoted as Day , starting from 1 and ending at 3654 (10 years equivalent) with 1-day increment. The processed 10-year cumulative production is denoted as $Prod$.
- For all d in Day , extract $d1_{raw}$ and $d2_{raw}$ from Day_{raw} . $d1_{raw}$ is the maximum value in Day_{raw} satisfying $d1_{raw} \leq d$, and $d2_{raw}$ is the minimum value in Day_{raw} satisfying $d2_{raw} \geq d$. Extract $p1_{raw}$ and $p2_{raw}$ (correspond to $d1_{raw}$ and $d2_{raw}$) simultaneously with $d1_{raw}$ and $d2_{raw}$. Quantity p , which is in $Prod$ and corresponds with d , is computed as: $p = d \cdot \frac{p2_{raw} - p1_{raw}}{d2_{raw} - d1_{raw}} + (p1_{raw} - d1_{raw} \cdot \frac{p2_{raw} - p1_{raw}}{d2_{raw} - d1_{raw}})$

Further analysis is possible after the described linear interpolation technique. Cumulative production's magnitude for both models in this case study is within 10^8 - 10^9 scale. Therefore, plotting the interpolated cumulative production data to visualize the efficacy of the novel model is not practical. To mitigate issue from the scaling, difference in production data between two models is decided as a better option to be plotted in this section. For a dry gas reservoir as Marcellus Shale, the critical period of production is typically within the first year of production, in which production

data depends primarily on properties of fractured area. Consequently, additional analysis of difference in production data between two models is concentrated on fluctuation within the first year of production.

6.5.3. Binary mixture 20/40 sand and 20/40 ceramic

Conductivity estimation analysis

Figures 36 and 37 present the Weight Concentration Ratio distribution (along half fracture dimensions) and its histogram, respectively.

Figures 38-42 present the following quantities (along half-fracture dimensions): Conductivity estimation by LWAM, conductivity estimation by the novel semi-analytical model, absolute difference (mD-ft) between the LWAM and the novel model, and relative difference (%) between LWAM and the novel model. Figure 35 presents the histogram of relative difference.

Figure 36 – Weight Concentration Ratio distribution inside half-fracture, mixture 20/40 sand and 20/40 ceramic injection

Figure 37 – Histogram of Weight Concentration Ratio distribution, mxiture 20/40 sand and 20/40 ceramic injection

Figure 38 – Conductivity estimation inside half-fracture by LWAM, mixture 20/40 sand and 20/40 ceramic injection

Figure 39 – Conductivity estimation inside half-fracture by novel semi-analytical model, mixture 20/40 sand and 20/40 ceramic injection

Figure 40 – Absolute difference (mD-ft) between LWAM and the novel semi-analytical model, mixture 20/40 sand and 20/40 ceramic injection

Figure 41 – Relative difference (%) between LWAM and the novel semi-analytical model, , mixture 20/40 sand and 20/40 ceramic injection

Figure 42 – Histogram of Relative difference (%), mixture 20/40 sand and 20/40 ceramic injection

Cumulative production

Figure 43– Absolute difference (SCF) between production data by LWAM and the novel semi-analytical model, mixture 20/40 sand and 20/40 ceramic injection

Figure 44 – Relative difference (%) difference (SCF) between production data by LWAM and the novel semi-analytical model, mixture 20/40 sand and 20/40 ceramic injection

Figures 43 and 44 present absolute difference (SCF) and relative difference (%) in 10-year cumulative production data (interpolated data) between two models. In figures 43 and 44, the x axis presents cumulative production time in days and the y axis presents absolute difference in SCF or relative difference in %.

6.5.4. Binary mixture 40/70 sand and 20/40 ceramic

Conductivity estimation analysis

Figures 45 and 46 present the Weight Concentration Ratiodistribution (along half fracture dimensions) and its histogram, respectively.

Figures 47-51 present the following quantities (along half-fracture dimensions): Conductivity estimation by LWAM, conductivity estimation by the novel semi-analytical model, absolute difference between the LWAM and the novel model, and relative difference between LWAM and the novel model. Figure 42 presents the histogram of relative difference.

Figure 45 – Weight Concentration Ratio distribution inside half-fracture, mixture 40/70 sand and 20/40 ceramic injection

Figure 46 – Histogram of Weight Concentration Ratio distribution, mixture 40/70 sand and 20/40 ceramic injection

Figure 47 – Conductivity estimation inside half-fracture by LWAM, mixture 40/70 sand and 20/40 ceramic injection

Figure 48 – Conductivity estimation inside half-fracture by novel semi-analytical model,
mixture 40/70 sand and 20/40 ceramic injection

Figure 49 – Absolute difference (mD-ft) between LWAM and the novel semi-analytical
model, , mixture 40/70 sand and 20/40 ceramic injection

Figure 50 – Relative difference (%) between LWAM and the novel semi-analytical model, , mixture 40/70 sand and 20/40 ceramic injection

Figure 51 – Histogram of Relative difference (%), mixture 40/70 sand and 20/40 ceramic injection

Cumulative Production

Figure 52 – Absolute difference (SCF) between production data by LWAM and the novel semi-analytical model, mixture 40/70 sand and 20/40 ceramic injection

Figure 53 – Relative difference (%) between production data by LWAM and the novel semi-analytical model, mixture 20/40 sand and 20/40 ceramic injection

Figures 52 and 53 present absolute and relative differences in 10-year cumulative production data (interpolated data) between two models. In figures 52 and 53, the x axis presents cumulative production time in days and the y axis presents absolute difference in SCF or relative difference in %.

6.5.5. Binary mixture 40/70 ceramic and 20/40 sand

Conductivity estimation analysis

Figures 54 and 55 present the Weight Concentration Ratio distribution (along half fracture dimensions) and its histogram, respectively.

Figures 56-60 present the following quantities (along half-fracture dimensions): Conductivity estimation by LWAM, conductivity estimation by the novel semi-analytical model, absolute difference between the LWAM and the novel model, and relative difference between LWAM and the novel model. Figure 49 presents the histogram of relative difference.

Figure 54 – Weight Concentration Ratio distribution inside half-fracture, mixture 40/70 ceramic and 20/40 sand injection

Figure 55 – Histogram of Weight Concentration Ratio distribution, mixture 40/70 ceramic and 20/40 sand injection

Figure 56 – Conductivity estimation inside half-fracture by LWAM, mixture 40/70 ceramic and 20/40 sand injection

Figure 57 – Conductivity estimation inside half-fracture by LWAM, mixture 40/70 ceramic and 20/40 sand injection

Figure 58 – Absolute difference (mD-ft) between LWAM and the novel semi-analytical model, , mixture 40/70 ceramic and 20/40 sand injection

Figure 59 – Relative difference (%) between LWAM and the novel semi-analytical model, mixture 40/70 ceramic and 20/40 sand injection

Figure 60 – Histogram of Relative difference (%) , mixture 40/70 ceramic and 20/40 sand injection

Cumulative production

Figure 61 – Absolute difference (SCF) between production data by LWAM and the novel semi-analytical model, mixture 40/70 ceramic and 20/40 sand injection

Figure 62 – Relative difference (%) between production data by LWAM and the novel semi-analytical model, mixture 40/70 ceramic and 20/40 sand injection

Figures 61 and 62 present absolute and relative differences in 10-year cumulative production data (interpolated data) between two models. In figures 61 and 62, the x axis presents cumulative production time in days and the y axis presents absolute difference in SCF or relative difference in %.

6.5.6. Predictive reasoning for conductivity estimation

Histograms and the kernel density estimation curves for all binary mixtures indicates an observation that the distribution of relative difference is dependent on the comparison matrices in section 4 when being reduced to the interested confining stress ranges (i.e. Marcellus Shale in the case study) and independent on the distribution of areal concentration ratio. In fact, the histograms and kernel density curves of Weight Concentration Ratio exhibit high degree of similarity.

For binary mixture 20/40 sand and 20/40 ceramic, figure 25 is closely concentrated on the region with reduced confining stress axis (between 5400-6200 psi). The comparison matrix presents a thin low overestimation zone (lower than 20%), an increase in height of the moderate overestimation zone (20-60%), and a thickest high overestimation zone (higher than 60%). These observations indicate a kernel density estimation curve which starts at low probability at low overestimation, progressively increases in probability at moderate overestimation and highly excels when entering high overestimation. Validation is readily available in figure 42.

For binary mixture 40/70 sand and 20/40 ceramic, figure 26 is closely concentrated on the region with reduced confining stress axis (between 5400-6200 psi). The comparison matrix presents nearly unnoticeable low overestimation zone (lower than 20%), extremely thin moderate overestimation zone (20%-60%) and dominant high overestimation zone (higher than 60%). These observations indicate a kernel density curve which starts at extremely low probability at low

overestimation, slight increases at moderate overestimation and exhibits a drastic jump when entering high overestimation. Validation is readily available in figure 51.

For binary mixture 40/70 sand and 20/40 ceramic, figure 27 is closely concentrated on the region with reduced confining stress axis (between 5400-6200 psi). The comparison matrix presents the low (lower than 20%) and high (higher than 50%) overestimation zones are almost similar in size, whereas the moderate (20%-50%) overestimation zone is slightly larger in size. These observations indicate a kernel density curve which gradually increases as overestimation moves from low to high zones orderly and has zero significant jump when entering any overestimation zones. Validation is readily available in figure 60.

Limited to this case study for Marcellus Shale and for all studied binary mixtures, maximum difference of 200 psi in confining stress between different depths does not translate to considerable difference in separation of the overestimation zones (i.e. low, moderate and high), which maintains the predictive reasoning of the overestimation's histogram/kernel density estimation curve. However, it is obvious that applications for other formations which has different confining stress data change observations from comparison matrices, and different predictive reasoning outcomes are expected.

Conclusively, analysis of the comparison matrices and reviews of available confining stress data are important in understanding the efficacy of the novel semi-analytical model in an applied case study. It is suggested that modelling for other practical binary mixtures (i.e. 100 sand and 40/70 sand) shall expand the understanding of the novel model by generating more comparison matrices.

6.5.7. Discussion for the cumulative production

Generally, it is expected that as the novel model predicts lower permeability and conductivity compared to LWAM, its cumulative production data is also lower. The magnitude of difference in gas cumulative production between the two models is within 10^5 - 10^6 SCF scale, which yields a small relative difference (up to 10.73% after the first year and 3.274 MMSCF after 10 years) when compared to the magnitude of the gas cumulative production itself (within 10^8 - 10^9 SCF scale). The magnitude of difference in comparison is dependent on the intrinsic property of the injected binary mixture, presumably the other simulation factors are controlled to remain (as in this case study).

Binary mixtures with contrast in Mesh sizes and densities between the two proppant types (i.e. 40/70 sand-20/40 ceramic and 40/70 ceramic-20/40 sand) exhibit a more stable behavior in cumulative production data, yield higher difference (1.035 MMSCF after 10 years) in cumulative production and consequently higher relative difference (maximum 3.731%, after the first year). Especially, the binary mixtures of which the injection order is finer and weaker proppant type, following by coarser and stronger proppant type (i.e. 40/70 sand and 20/40 ceramic) exhibit extremely stable behavior in cumulative production data, yield highest difference (3.274 MMSCF after 10 years) in cumulative production data and consequently highest relative difference (maximum 10.73% after the first year).

Binary mixtures with similarity in Mesh sizes between the two proppant types (i.e. 20/40 sand and 20/40 ceramic) exhibit a more fluctuating behavior in cumulative production data, yield lower difference (0.787 MMSCF after 10 years) in cumulative production and consequently lower relative difference (maximum 2.346% after the first year) compared to binary mixtures with contrast in both Mesh sizes and densities. Based on these discussions, an injection practice for a binary mixture with injection order of finer and weaker proppant type, following by coarser and

stronger proppant type increases expectation by application of LWAM. An injection practice for a binary mixture with similarity in Mesh sizes mitigates expectation by application of LWAM.

6.5.8. Drawbacks in the case study

It is undeniable that modelling for the three selected binary mixtures limit further possible studies for this Marcellus Shale case. Field production data in Marcellus Shale for the studied well MIP-3H is available and its field injection mixtures are binary mixtures of sand. As reviewed in chapter 1, the lab data for binary mixtures of sand is not selected for being modelled because of scarcity in the available data for binary mixtures of sand. As a result, prospective studies as history matching or production sensitivity are decided not feasible to be conducted, which may reduce the solidity and flexibility of the novel model.

CHAPTER 7 – CONCLUSIONS

The novel concept of Weight Concentration Ratio defined as division of areal concentration of the 1st injected proppant type to the total areal concentration visualizes the mixture zone inside a propped fracture. This novel concept is an integrated part in developing the novel semi-analytical model for conductivity of binary mixtures. Modelling process is conducted by separate semi-analytical modelling for permeability and fracture width.

Assumption of a loaded proppant pack as a binary-sized particle assembly allows a derivation sequence for internal specific area. Previous studies of porosity and tortuosity for binary-sized particle assembly and assumptions for porous media as parallel flow tubes further allow use of Carman-Kozeny equation for modelling permeability of binary mixtures. Determination of coefficients in this permeability model is feasible by trust-region method for non-linear least square problems with support of pre-set coefficient boundaries specific binary mixtures' permeability lab data.

Dual-layering schematic and elastic deformation allow modelling for fracture width by Hertzian contact theory. Unsatisfaction in reviews for available proppant pack width's lab data leads to its termination in use for fracture width modelling. Therefore, fracture width model is derived to be independent on coefficients. Besides Weight Concentration Ratio and confining stress, geo-mechanics involve as independent inputs in the model. These properties are known for each proppant type in a binary mixture.

The novel semi-analytical model for conductivity of binary mixtures perform validation with the conductivity lab data with an acceptable degree of accuracy for 2/3 studied binary mixtures: 40/70 sand - 20/40 ceramic (7.98-9.9%) and 40/70 ceramic - 20/40 sand (13-18.13%). Based on validation results, trust-region method proves its effectiveness in determining the

model's coefficients using highly qualified datasets of just two different Weight Concentration Ratios. This achievement partially allows expansion of the novel conductivity model to predict binary mixture's conductivity at arbitrary confining stress and Weight Mixing Ratio values. Undeniably, the novel conductivity model reduces efforts of conducting several API conductivities tests and misleading experimental results caused by measurement errors.

The novel conductivity model proves that LWAM overestimates conductivity of a binary mixture. Degree of overestimation is studied by relative difference between LWAM and the novel model. Comparison matrices with 2 axes of confining stress (2000-10000 psi) and Weight Concentration Ratio (0.01-0.99) are effective in examining relative difference. All comparison matrices have low ($\leq 20\%$), moderate (20-60%) and high ($\geq 60\%$) overestimation zones.

For binary mixture 20/40 sand and 20/40 ceramic, relative difference increases monotonically in both axes to reach high overestimation zone at the 0.99-10000 psi corner and transition rate between overestimation zones decrease moderately. For binary mixture 40/70 sand and 20/40 ceramic, relative different is dominantly in high overestimation zone which is located at 0.5-10000 psi, and transition rate between overestimation zones can be rapid. For binary mixture 40/70 ceramic and 20/40 sand, overestimation zones are uniformly distributed and transition rate between overestimation zones is low.

Case study for Marcellus Shale further quantify efficacy of the models by cumulative production data analysis (both 10-year production data and production data at the early time of production), besides conductivity estimation. Axis reduction of comparison matrices using Marcellus Shale minimum horizontal stress data allows predictive reasoning in conductivity estimation, which is correlated with histograms of relative difference. Cumulative production data analysis quantifies production over-expectation caused by application of LWAM in reservoir

simulation. Injection practices of a binary mixture with high contrast in its proppant types' Mesh size and density significantly increase the expectation. Injection practices of a binary mixture with low contrast in its proppant types' Mesh size and density decreases the expectation.

CHAPTER 8 – RECOMMENDATION FOR FUTURE WORK

I. Lab data

An expansion of the measurements for more binary mixtures is highly supportive to continue this study. In specific, binary mixtures which are currently practiced in the field's hydraulic fracturing operations, for example 100 Mesh sand and 40/70 Mesh sand in Marcellus Shale, should need further focus.

An expansion of the measurements for more weight mixing ratios (i.e. areal concentration ratios) is highly supportive to continue this study. Besides several efforts to understand the dependence of modelling coefficients on the weight mixing ratios, their intrinsic complexities require validation from lab measurements to prove the accuracy of the current understanding described in this study.

A correction or modification in the measurements for the proppant pack's width is necessary, since the current measurement approach prevents the feasible use of the proppant pack's width lab data from reasonable modelling approach(es).

II. The sensitivity study for mixture zone

Scale of the sensitivity analysis can be expanded to a global sensitivity analysis. According to Morris global sensitivity design, the base case is generated as each run within analysis for one input factor. As a result, variance-based analysis for output factor(s) (i.e. mixture zone) is feasible. In addition, variance or relationships between input factors as minimum confining stress and geomechanics are known from field measurements or modelling. Contribution from these aspects can lead to better screening design and more realistic discussions for impact from the prospectively correlated input factors.

III. Semi-analytical modeling improvement

For the novel semi-analytical permeability model, modification from Carman-Kozeny equation does not eliminate the limitations of the this equation. Pore trajectory has been addressed under a certain degree (related to descriptions in Dullien and F.A.L, 1992) in this thesis. However, an full analytical solution for pore trajectory is not totally feasible and out of the aims of the study. Particle size distributions another essential aspect to be more deeply concerned for better permeability modelling. One prospective approach is to derive the modified internal specific area as a function of Weight Concentration Ratio and in-lab-context particle size distribution kernel density estimators for the two proppant types in a binary mixture. However, time constraints do not allow this direction to be modelled successfully.

For the novel fracture width model, the main expansions for better modelling outcomes primarily aligns to a higher degree of complexity in applying Hertzian contact theory and better realization for the proposed dual-layering schematic. Hertzian contact theory can be applied to contact between proppant particles - rock surfaces (which are considered as elastic flat surfaces) and furthermore contact between proppant particles (which are considered as elastic spherical forms). The proppant particles – rock surfaces contact is addressed in this thesis. However, contact between proppant particles has not been addressed. Comprehensive additions in the effect of proppant layers within one proppant type (i.e. within 20/40 sand) and between two proppant proppant types (i.e. between 20/40 sand and 20/40 ceramic) both increase the complexity degree of the final fracture width model extremely. More effectively simplified form of the final fracture width model, under those additions, has not been finalized and therefore is not presented in this thesis.

Albeit dual-layering schematic integrates the Weight Concentration Ratio concept into the fracture width model reasonably, this arrangement is an idealization concept when compared to

realistic distribution of particles. In reality, distribution of particles inside a proppant pack has a tendency to arrange randomly. This fact still allows integration of Weight Concentration Ratio in the prospective model. However, it enforces the modelling scale to be microscopic.(i.e. between two particles), which produces extreme complexity and become out of scope for an analytical or a semi-analytical approach.

IV. Fracture/reservoir simulation

Albeit usefulness of the in-house fracture propagation simulator in understanding of Areal Concentration Ratio, its functionality has been limited in two primary situations: pseudo-3D scaling for simulation and flexible import settings for different formations.

Additional coding efforts have been performed to visualize the Weight Concentration Ratio in pseudo-3D configuration. However, the visualization outcomes have not been satisfied for all studied binary mixtures in this thesis, which is believed to be caused by numerical errors. In addition, the import code module for fracturing parameters has been restricted to Marcellus Shale merely. Although adjustment for this module has been facilitated to import fracturing parameters from other shale formations (specifically Bakken Oil Shale), functionality after additional adjustment within the module has been improper and debugging issues have not been fixed.

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APPENDIX

I. FRACPRO® database

The FRACPRO® database used in this thesis is selected for all available 20/40 sand, 20/40 ceramic, 40/70 sand and 40/70 ceramic from the public FRACPRO® database (located in FRACPRO® commercial software). Selection is based on similarity in measurement conditions compared to Eric's experiment conditions (for example: 2 lb/ft² proppant concentration, applied stress range, temperature, tested fluid)

1. FRACPRO® database for 20/40 sand (table 7)

Arizona						
Effective stress (psi)	Permeability (D)	Perm at Reservoir Temp (D)	Average Width (in)	Average Width after Embedment (in)	Conductivity after Embedment (mD·ft)	Beta Factor (atm·s ² /g)
0	315	315	0.2406	0.23	5,956.30	0.000155
2,000	274.3	274.3	0.235	0.22	5,058.70	0.00018
4,000	153.956	150.021	0.2294	0.22	2,696.70	0.000338
6,000	79.595	68.615	0.2238	0.2101	1,201.40	0.000694
8,000	33.997	26.147	0.2182	0.2045	445.6	0.001755
10,000	9.4	8.145	0.2126	0.1989	135	0.007123
12,000	2.599	2.482	0.207	0.1933	40	0.028915
14,000	0.719	0.687	0.2014	0.1877	10.7	0.117305
16,000	0.199	0.19	0.1958	0.1821	2.9	0.475658
18,000	0.055	0.053	0.1902	0.1765	0.8	1.931716
20,000	0.015	0.014	0.1846	0.1709	0.2	7.959611
Jordan						
Effective stress (psi)	Permeability (D)	Perm at Reservoir Temp (D)	Average Width (in)	Average Width after Embedment (in)	Conductivity after Embedment (mD·ft)	Beta Factor (atm·s ² /g)
0	273.212	273.212	0.2297	0.2185	4,974.00	0.000152
2,000	244.538	244.538	0.2266	0.2154	4,389.80	0.000172
4,000	174.534	170.073	0.2236	0.2124	3,009.80	0.000251
6,000	89.229	76.92	0.2205	0.2093	1,341.70	0.000532
8,000	36.049	27.726	0.2175	0.2063	476.5	0.001463
10,000	14.564	12.62	0.2144	0.2032	213.7	0.004024
12,000	5.884	5.62	0.2114	0.2001	93.7	0.011072

14,000	2.377	2.271	0.2083	0.1971	37.3	0.030464
16,000	0.96	0.917	0.2053	0.194	14.8	0.083816
18,000	0.388	0.371	0.2022	0.191	5.9	0.230604
20,000	0.157	0.15	0.1991	0.1879	2.3	0.634468
Ottawa						
Effective stress (psi)	Permeability (D)	Perm at Reservoir Temp (D)	Average Width (in)	Average Width after Embedment (in)	Conductivity after Embedment (mD·ft)	Beta Factor (atm·s ² /g)
0	273.212	273.212	0.2297	0.2185	4,974.00	0.000152
2,000	244.538	244.538	0.2266	0.2154	4,389.80	0.000172
4,000	174.534	170.073	0.2236	0.2124	3,009.80	0.000251
6,000	89.229	76.92	0.2205	0.2093	1,341.70	0.000532
8,000	36.049	27.726	0.2175	0.2063	476.5	0.001463
10,000	14.564	12.62	0.2144	0.2032	213.7	0.004024
12,000	5.884	5.62	0.2114	0.2001	93.7	0.011072
14,000	2.377	2.271	0.2083	0.1971	37.3	0.030464
16,000	0.96	0.917	0.2053	0.194	14.8	0.083816
18,000	0.388	0.371	0.2022	0.191	5.9	0.230604
20,000	0.157	0.15	0.1991	0.1879	2.3	0.634468
Boss						
Effective stress (psi)	Permeability (D)	Perm at Reservoir Temp (D)	Average Width (in)	Average Width after Embedment (in)	Conductivity after Embedment (mD·ft)	Beta Factor (atm·s ² /g)
0	244.586	244.586	0.2314	0.2201	4,487.10	0.000171
2,000	209.18	209.18	0.2275	0.2162	3,768.50	0.000202
4,000	123.5	120.343	0.2236	0.2122	2,128.30	0.000359
6,000	56.975	49.115	0.2195	0.2083	852.4	0.000834
8,000	26.65	20.497	0.2156	0.2043	349	0.001907
10,000	12.466	10.801	0.2116	0.2003	180.3	0.004363
12,000	5.831	5.569	0.2076	0.1964	91.1	0.00998
14,000	2.727	2.605	0.2037	0.1924	41.8	0.022829
16,000	1.276	1.218	0.1997	0.1885	19.1	0.052221
18,000	0.597	0.57	0.1958	0.1845	8.8	0.119456
20,000	0.279	0.267	0.1918	0.1805	4	0.273254
Brady						
Effective stress (psi)	Permeability (D)	Perm at Reservoir Temp (D)	Average Width (in)	Average Width after Embedment (in)	Conductivity after Embedment (mD·ft)	Beta Factor (atm·s ² /g)
0	311.366	311.366	0.2304	0.2177	5,649.30	0.000139

2,000	255.787	255.787	0.227	0.2144	4,569.10	0.000172
4,000	141.3	137.688	0.2236	0.211	2,420.80	0.000326
6,000	61.687	53.177	0.2203	0.2076	920	0.000795
8,000	31.1	23.919	0.2169	0.2043	407.1	0.001662
10,000	16.13	13.977	0.2135	0.2009	234	0.003372
12,000	8.366	7.991	0.2102	0.1975	131.5	0.006838
14,000	4.339	4.144	0.2068	0.1941	67	0.013869
16,000	2.25	2.149	0.2034	0.1908	34.2	0.028128
18,000	1.167	1.115	0.2001	0.1874	17.4	0.057048
20,000	0.605	0.578	0.1967	0.184	8.9	0.115701
Poland						
Effective stress (psi)	Permeability (D)	Perm at Reservoir Temp (D)	Average Width (in)	Average Width after Embedment (in)	Conductivity after Embedment (mD·ft)	Beta Factor (atm·s ² /g)
0	326.382	326.382	0.2348	0.2234	6,076.60	0.000128
2,000	253.86	253.86	0.2315	0.2201	4,656.60	0.000167
4,000	127.62	124.358	0.2282	0.2168	2,246.90	0.000348
6,000	48.63	41.921	0.2249	0.2135	745.9	0.000968
8,000	22.92	17.628	0.2216	0.2102	308.8	0.002152
10,000	10.803	9.36	0.2183	0.2069	161.4	0.004784
12,000	5.091	4.863	0.215	0.2036	82.5	0.010636
14,000	2.4	2.292	0.2117	0.2003	38.3	0.023642
16,000	1.131	1.08	0.2084	0.197	17.7	0.052556
18,000	0.533	0.509	0.2051	0.1937	8.2	0.116828
20,000	0.251	0.24	0.2018	0.1904	3.8	0.259703
Accupak						
Effective stress (psi)	Permeability (D)	Perm at Reservoir Temp (D)	Average Width (in)	Average Width after Embedment (in)	Conductivity after Embedment (mD·ft)	Beta Factor (atm·s ² /g)
0	402.545	402.545	0.2311	0.2184	7,327.80	0.000113
2,000	368.61	368.61	0.228	0.2154	6,615.10	0.000125
4,000	257.667	251.081	0.225	0.2123	4,441.30	0.000186
6,000	129.437	111.581	0.2219	0.2092	1,945.00	0.000401
8,000	57.122	43.933	0.2188	0.2061	754.5	0.000997
10,000	25.209	21.843	0.2157	0.203	369.5	0.00248
12,000	11.125	10.626	0.2126	0.1999	177	0.006164
14,000	4.91	4.689	0.2095	0.1968	76.9	0.015324
16,000	2.167	2.069	0.2064	0.1937	33.4	0.038096
18,000	0.956	0.913	0.2033	0.1906	14.5	0.094707
20,000	0.422	0.403	0.2002	0.1875	6.3	0.235445

2. FRACPRO® database for 20/40 ceramic (table 8)

Carbolite						
Effective stress (psi)	Permeability (D)	Perm at Reservoir Temp (D)	Average Width (in)	Average Width after Embedment (in)	Conductivity after Embedment (mD·ft)	Beta Factor (atm·s ² /g)
0	540	540	0.2339	0.22	9,871.20	0.000095
2,000	507.6	507.6	0.2338	0.22	9,275.80	0.000102
4,000	406.761	406.761	0.2337	0.22	7,430.60	0.00013
6,000	332.621	332.621	0.2336	0.2191	6,074.20	0.000163
8,000	196.022	196.022	0.2336	0.2191	3,578.50	0.000291
10,000	112.061	112.061	0.2335	0.219	2,045.00	0.00054
12,000	77.546	77.546	0.2334	0.2189	1,414.70	0.00081
14,000	53.662	53.662	0.2333	0.2188	978.6	0.001215
16,000	37.134	37.134	0.2333	0.2188	677	0.001824
18,000	25.697	25.697	0.2332	0.2187	468.3	0.002738
20,000	17.782	17.782	0.2331	0.2186	324	0.004109
Carboprop						
Effective stress (psi)	Permeability (D)	Perm at Reservoir Temp (D)	Average Width (in)	Average Width after Embedment (in)	Conductivity after Embedment (mD·ft)	Beta Factor (atm·s ² /g)
0	443.563	443.563	0.1976	0.1841	6,803.80	0.000104
2,000	416.49	416.49	0.1966	0.183	6,351.50	0.000112
4,000	359.25	359.25	0.1955	0.1819	5,446.70	0.00013
6,000	305.789	305.789	0.1944	0.1809	4,609.00	0.000154
8,000	238.78	238.78	0.1934	0.1798	3,577.80	0.0002
10,000	173.29	173.29	0.1923	0.1787	2,581.10	0.000281
12,000	127.572	127.572	0.1912	0.1777	1,888.80	0.000388
14,000	106.23	106.23	0.1902	0.1766	1,563.40	0.00047
16,000	88.458	88.458	0.1891	0.1755	1,294.00	0.00057
18,000	73.66	73.66	0.188	0.1745	1,071.00	0.000692
20,000	61.337	61.337	0.187	0.1734	886.4	0.000839
Econoprop						
Effective stress (psi)	Permeability (D)	Perm at Reservoir Temp (D)	Average Width (in)	Average Width after Embedment (in)	Conductivity after Embedment (mD·ft)	Beta Factor (atm·s ² /g)
0	441.273	441.273	0.2382	0.2252	8,281.60	0.000104
2,000	396.297	396.297	0.2359	0.2229	7,360.60	0.000118
4,000	302.391	302.391	0.2336	0.2206	5,557.80	0.000158
6,000	200.974	200.974	0.2312	0.2182	3,654.80	0.000245
8,000	120.974	120.974	0.2289	0.2159	2,176.50	0.000422

10,000	69	69	0.2266	0.2136	1,228.00	0.000771
12,000	39.25	39.25	0.2242	0.2112	690.9	0.001412
14,000	22.327	22.327	0.2219	0.2089	388.7	0.002588
16,000	12.7	12.7	0.2196	0.2066	218.6	0.004744
18,000	7.225	7.225	0.2173	0.2043	123	0.008693
20,000	4.11	4.11	0.2149	0.2019	69.2	0.015932
Naplite						
Effective stress (psi)	Permeability (D)	Perm at Reservoir Temp (D)	Average Width (in)	Average Width after Embedment (in)	Conductivity after Embedment (mD·ft)	Beta Factor (atm·s ² /g)
0	540	540	0.2405	0.2262	10,181.00	0.000094
2,000	510.222	510.222	0.2388	0.2245	9,546.20	0.0001
4,000	410.376	410.376	0.2371	0.2228	7,619.00	0.000127
6,000	299.008	299.008	0.2354	0.2211	5,508.40	0.00018
8,000	183.897	183.897	0.2336	0.2193	3,361.30	0.000307
10,000	100.05	100.05	0.2319	0.2176	1,814.40	0.000601
12,000	76.5	76.5	0.2302	0.2159	1,376.30	0.000809
14,000	58.493	58.493	0.2285	0.2142	1,043.90	0.001087
16,000	44.725	44.725	0.2267	0.2124	791.8	0.001461
18,000	34.198	34.198	0.225	0.2107	600.5	0.001964
20,000	26.148	26.148	0.2233	0.209	455.4	0.00264
Sinter Ball Bauxite						
Effective stress (psi)	Permeability (D)	Perm at Reservoir Temp (D)	Average Width (in)	Average Width after Embedment (in)	Conductivity after Embedment (mD·ft)	Beta Factor (atm·s ² /g)
0	511.855	511.855	0.1856	0.1723	7,349.20	0.000094
2,000	491.655	491.655	0.1847	0.1714	7,022.00	0.000098
4,000	403.975	403.975	0.1838	0.1705	5,739.10	0.000119
6,000	344.477	344.477	0.1829	0.1696	4,867.80	0.000141
8,000	291.475	291.475	0.182	0.1687	4,096.70	0.000167
10,000	226.275	226.275	0.1811	0.1678	3,163.20	0.000217
12,000	162.795	162.795	0.1802	0.1668	2,263.50	0.000304
14,000	135.7	135.7	0.1793	0.1659	1,876.50	0.000367
16,000	113.115	113.115	0.1784	0.165	1,555.60	0.000443
18,000	94.288	94.288	0.1775	0.1641	1,289.50	0.000534
20,000	78.595	78.595	0.1766	0.1632	1,069.00	0.000644
CarboHSP						
Effective stress (psi)	Permeability (D)	Perm at Reservoir Temp (D)	Average Width (in)	Average Width after Embedment (in)	Conductivity after Embedment (mD·ft)	Beta Factor (atm·s ² /g)

0	542.261	542.261	0.1892	0.175	7,908.40	0.000094
2,000	502.604	502.604	0.1867	0.1725	7,225.60	0.000101
4,000	436.547	436.547	0.1842	0.17	6,185.10	0.000117
6,000	359.627	359.627	0.1817	0.1675	5,020.50	0.000143
8,000	272.174	272.174	0.1792	0.165	3,743.10	0.00019
10,000	201.534	201.534	0.1767	0.1625	2,729.70	0.000259
12,000	157.279	157.279	0.1742	0.16	2,097.60	0.000334
14,000	158.323	158.323	0.1718	0.1575	2,078.60	0.000331
16,000	159.374	159.374	0.1693	0.155	2,059.20	0.000329
18,000	160.432	160.432	0.1668	0.1526	2,039.60	0.000327
20,000	161.497	161.497	0.1643	0.1501	2,019.50	0.000325

3. FRACPRO® database for 40/70 sand (table 9)

Arizona						
Effective stress (psi)	Permeability (D)	Perm at Reservoir Temp (D)	Average Width (in)	Average Width after Embedment (in)	Conductivity after Embedment (mD·ft)	Beta Factor (atm·s ² /g)
0	62.715	62.715	0.2366	0.23	1,209.20	0.000575
2,000	53.495	53.495	0.2313	0.23	1,007.80	0.000685
4,000	43.165	42.062	0.2261	0.22	773.90	0.000867
6,000	32.328	27.868	0.2208	0.2155	500.50	0.001191
8,000	18.773	14.438	0.2155	0.2102	252.9	0.002166
10,000	10.15	8.795	0.2102	0.2049	150.2	0.004259
12,000	5.488	5.242	0.2049	0.1996	87.2	0.008375
14,000	2.967	2.834	0.1996	0.1944	45.9	0.016472
16,000	1.604	1.532	0.1943	0.1891	24.1	0.032397
18,000	0.867	0.828	0.189	0.1838	12.7	0.06373
20,000	0.469	0.448	0.1837	0.1785	6.7	0.125259
Jordan						
Effective stress (psi)	Permeability (D)	Perm at Resvr Temp (D)	Avg Width (in)	Avg Width after Embedment (in)	Conductivity after Embedment (mD·ft)	Beta Factor (atm·s ² /g)
0	92.182	92.182	0.2318	0.2265	1,740.20	0.000345
2,000	71.502	71.502	0.2289	0.2237	1,332.60	0.000459
4,000	60.731	59.179	0.2261	0.2208	1,088.80	0.000552
6,000	45.884	39.554	0.2232	0.2179	718.20	0.000758
8,000	27.348	21.033	0.2203	0.215	376.9	0.001359
10,000	16.131	13.977	0.2174	0.2121	247.1	0.002466
12,000	10.66	10.182	0.2145	0.2093	177.5	0.003936
14,000	7.073	6.756	0.2116	0.2064	116.2	0.006254
16,000	4.711	4.5	0.2088	0.2035	76.3	0.009893

18,000	3.15	3.009	0.2059	0.2006	50.3	0.015581
20,000	2.114	2.02	0.203	0.1977	33.3	0.024438
Ottawa						
Effective stress (psi)	Permeability (D)	Perm at Reservoir Temp (D)	Average Width (in)	Average Width after Embedment (in)	Conductivity after Embedment (mD·ft)	Beta Factor (atm·s ² /g)
0	65	65	0.24	0.2344	1,269.60	0.000768
2,000	54	54	0.24	0.2344	1,054.70	0.000962
4,000	38	37.029	0.24	0.2344	723.20	0.001478
6,000	23	19.827	0.24	0.2344	387.30	0.002726
8,000	12	9.229	0.24	0.2344	180.3	0.00603
10,000	6.261	5.425	0.24	0.2344	106	0.013336
12,000	3.267	3.12	0.24	0.2344	60.9	0.029493
14,000	1.704	1.628	0.24	0.2344	31.8	0.065227
16,000	0.889	0.849	0.24	0.2344	16.6	0.144257
18,000	0.464	0.443	0.24	0.2344	8.7	0.319039
20,000	0.242	0.231	0.24	0.2344	4.5	0.705586
Badger						
Effective stress (psi)	Permeability (D)	Perm at Reservoir Temp (D)	Average Width (in)	Average Width after Embedment (in)	Conductivity after Embedment (mD·ft)	Beta Factor (atm·s ² /g)
0	62.715	62.715	0.2274	0.2221	1,161.00	0.00056
2,000	53.495	53.495	0.225	0.2197	979.60	0.00067
4,000	43.165	42.062	0.2226	0.2173	761.80	0.000854
6,000	32.328	27.868	0.2202	0.2149	499.1	0.001183
8,000	18.773	14.438	0.2178	0.2125	255.7	0.002187
10,000	10.15	8.795	0.2154	0.2101	154	0.00438
12,000	5.488	5.242	0.213	0.2077	90.7	0.008775
14,000	2.967	2.834	0.2106	0.2053	48.5	0.017578
16,000	1.604	1.532	0.2082	0.2029	25.9	0.035213
18,000	0.867	0.828	0.2058	0.2005	13.8	0.070541
20,000	0.469	0.448	0.2033	0.1981	7.4	0.141311
Brady						
Effective stress (psi)	Permeability (D)	Perm at Reservoir Temp (D)	Average Width (in)	Average Width after Embedment (in)	Conductivity after Embedment (mD·ft)	Beta Factor (atm·s ² /g)
0	83.632	83.632	0.2297	0.2245	1,564.50	0.000363
2,000	71.51	71.51	0.2266	0.2213	1,318.90	0.000432
4,000	50.745	49.448	0.2234	0.2182	899.00	0.000631

6,000	26.042	22.45	0.2203	0.215	402.3	0.001319
8,000	13.575	10.441	0.2171	0.2119	184.3	0.002709
10,000	9.018	7.814	0.214	0.2087	135.9	0.004257
12,000	5.991	5.722	0.2108	0.2056	98	0.00669
14,000	3.98	3.802	0.2077	0.2024	64.1	0.010513
16,000	2.644	2.526	0.2045	0.1993	41.9	0.016521
18,000	1.757	1.678	0.2014	0.1961	27.4	0.025962
20,000	1.167	1.115	0.1982	0.193	17.9	0.040797
Carbo-northern						
Effective stress (psi)	Permeability (D)	Perm at Reservoir Temp (D)	Average Width (in)	Average Width after Embedment (in)	Conductivity after Embedment (mD·ft)	Beta Factor (atm·s ² /g)
0	102.43	102.43	0.2158	0.2093	1,786.70	0.000306
2,000	56	56	0.2108	0.2043	953.50	0.000605
4,000	47	45.799	0.2058	0.1993	760.70	0.000738
6,000	33	28.448	0.2008	0.1943	460.7	0.001099
8,000	19	14.613	0.1958	0.1893	230.5	0.00205
10,000	10	8.665	0.1908	0.1843	133.1	0.00423
12,000	7.527	7.189	0.1858	0.1793	107.4	0.00583
14,000	4.871	4.653	0.1808	0.1743	67.6	0.009527
16,000	3.152	3.011	0.1758	0.1693	42.5	0.01557
18,000	2.04	1.949	0.1708	0.1643	26.7	0.025444
20,000	1.32	1.261	0.1658	0.1593	16.7	0.041579
Frac Sand						
Effective stress (psi)	Permeability (D)	Perm at Reservoir Temp (D)	Average Width (in)	Average Width after Embedment (in)	Conductivity after Embedment (mD·ft)	Beta Factor (atm·s ² /g)
0	216.504	216.504	0.2274	0.2221	4,007.80	0.000138
2,000	50	50	0.225	0.2197	915.60	0.000723
4,000	41	39.952	0.2226	0.2173	723.60	0.000905
6,000	20	17.241	0.2202	0.2149	308.80	0.002036
8,000	8	6.153	0.2178	0.2125	109	0.005732
10,000	1	0.867	0.2154	0.2101	15.2	0.060064
12,000	0.743	0.71	0.213	0.2077	12.3	0.08404
14,000	0.288	0.276	0.2106	0.2053	4.7	0.244665
16,000	0.112	0.107	0.2082	0.2029	1.8	0.712291
18,000	0.044	0.042	0.2058	0.2005	0.7	2.073684
20,000	0.017	0.016	0.2033	0.1981	0.3	6.037087

4. FRACPRO ® database for 40/70 ceramic (table 10)

Interprop						
Effective stress (psi)	Permeability (D)	Perm at Reservoir Temp (D)	Average Width (in)	Average Width after Embedment (in)	Conductivity after Embedment (mD·ft)	Beta Factor (atm·s ² /g)
0	308.016	308.016	0.2109	0.20	5,259.90	0.00006
2,000	292.212	292.212	0.2079	0.20	4,917.30	0.000064
4,000	248.716	248.716	0.2049	0.20	4,123.40	0.000075
6,000	187.932	187.932	0.2019	0.196	3,068.90	0.000101
8,000	139.941	139.941	0.199	0.193	2,250.40	0.000137
10,000	89.288	89.288	0.196	0.19	1,413.60	0.000218
12,000	69.345	69.345	0.193	0.187	1,080.60	0.000283
14,000	43.68	43.68	0.19	0.184	669.8	0.000456
16,000	27.564	27.564	0.187	0.181	415.8	0.000735
18,000	17.426	17.426	0.184	0.178	258.5	0.001182
20,000	11.036	11.036	0.181	0.1751	161	0.001898
Interprop						
Effective stress (psi)	Permeability (D)	Perm at Reservoir Temp (D)	Average Width (in)	Average Width after Embedment (in)	Conductivity after Embedment (mD·ft)	Beta Factor (atm·s ² /g)
0	141	141	0.2	0.1939	2,278.00	0.001441
2,000	141	141	0.2	0.1939	2,278.00	0.001441
4,000	114	114	0.2	0.1939	1,841.80	0.001879
6,000	97	97	0.2	0.1939	1,567.10	0.002299
8,000	85	85	0.2	0.1939	1,373.20	0.002712
10,000	75	75	0.2	0.1939	1,211.70	0.003172
12,000	55	55	0.2	0.1939	888.60	0.004674
14,000	40.333	40.333	0.2	0.1939	651.60	0.006887
16,000	29.578	29.578	0.2	0.1939	477.90	0.010148
18,000	21.69	21.69	0.2	0.1939	350.40	0.014954
20,000	15.906	15.906	0.2	0.1939	257	0.022036
Carbolite						
Effective stress (psi)	Permeability (D)	Perm at Reservoir Temp (D)	Average Width (in)	Average Width after Embedment (in)	Conductivity after Embedment (mD·ft)	Beta Factor (atm·s ² /g)
0	203.111	203.111	0.1964	0.1898	3,212.80	0.00026
2,000	135	135	0.1924	0.1858	2,090.40	0.000389
4,000	100	100	0.1884	0.1818	1,515.10	0.000524
6,000	80	80	0.1844	0.1778	1,185.40	0.000653
8,000	60	60	0.1804	0.1738	869.10	0.000868
10,000	35	35	0.1764	0.1698	495.30	0.00148

12,000	25	25	0.1724	0.1658	345.4	0.002065
14,000	18.91	18.91	0.1684	0.1618	255	0.002723
16,000	13.47	13.47	0.1644	0.1578	177.1	0.00381
18,000	9.599	9.599	0.1604	0.1538	123	0.005328
20,000	6.838	6.838	0.1564	0.1498	85.4	0.007454
Carboprop						
Effective stress (psi)	Permeability (D)	Perm at Reservoir Temp (D)	Average Width (in)	Average Width after Embedment (in)	Conductivity after Embedment (mD·ft)	Beta Factor (atm·s ² /g)
0	178.322	178.322	0.2061	0.2001	2,974.10	0.000258
2,000	140	140	0.2037	0.1977	2,306.80	0.000349
4,000	110	110	0.2013	0.1953	1,790.40	0.000471
6,000	80	80	0.1989	0.1929	1,286.00	0.000702
8,000	65	65	0.1965	0.1905	1,031.80	0.000911
10,000	50	50	0.1941	0.1881	783.70	0.001265
12,000	40	40	0.1917	0.1857	618.90	0.001672
14,000	30.454	30.454	0.1892	0.1833	465.10	0.002352
16,000	23.659	23.659	0.1868	0.1809	356.6	0.003225
18,000	18.38	18.38	0.1844	0.1784	273.3	0.004423
20,000	14.279	14.279	0.182	0.176	209.5	0.006067

II. Coefficient extraction in fracture width model

Effective stress and average width after embedment are selected from tables 6-9. Since embedment is modelled in this thesis, average width after embedment is selected in replacement of average width data in the tables. An initial scatter plot between 3rd square root of effective stress and average width after embedment indicates linear relationship, which is described in notations as:

$$w_f = I - S\sqrt[3]{P_c}$$

Where I and S are intercept and slope for a linear equation. In addition, fracture width model for a single proppant type is derived in chapter 4, equation (4.27) as:

$$w_f = w_{f0} - \frac{4w_{f0}}{(\pi^3\sqrt[3]{9}) \times \sqrt[3]{E_p(1-\nu_p^2)^2}} \sqrt[3]{P_c}$$

As a result, I and S are equivalent as follows:

$$I = w_{f0}, S = \frac{4w_{f0}}{(\pi^3\sqrt{9}) \times \sqrt[3]{E_p(1 - v_p^2)^2}}$$

Eventually, initial fracture width and geo-mechanics term, two coefficients required to be extracted, are computed as follows:

$$w_{f0} = I, \sqrt[3]{E_p(1 - v_p^2)^2} = \frac{4I}{S\pi^3\sqrt{9}}$$

For a single proppant type (i.e. 20/40 sand, 20/40 ceramic, 40/70 sand, 40/70 ceramic), there are 5-7 different data tables. Therefore, each single proppant type has 5-7 pairs of coefficients. Within each single proppant type, observations for the samples do not raise any drastic differences between the samples, and average values of all available samples are computed as representative coefficients.

III. Sensitivity study

Table 11 provides initial numerical results (mixture zone area percentage) of all sensitivity incremental points for 8 studied parameters (exception of injection time parameters):

Table 11 – Initial numerical results of all sensitivity incremental points for 8 studied parameters (exception of injection time parameters)

Increments	Diameter 1	Diameter 2	Density 1	Density 2	Injection rate	Confining Stress	Poisson Ratio	Young Modulus
-25 %	51.8293	60.0671	51.1765	58.6667	61.0442	48.494	55.2901	56.9395
-20 %	52.6316	60.2694	51.3514	59.1973	58.2707	48.494	56.2712	55.74913
-15 %	53.4591	59.0604	52.4691	58.194	58.6716	51.7134	56.2712	54.57627
-10 %	53.481	58.0537	53.125	56.8562	56.0284	54.1935	56.4189	55.40541
-5 %	54.2484	57.4324	54.3408	56.7568	57.4394	54.0453	55.4455	56.27119
BASE CASE	55.8528	55.8528	55.8528	55.8528	55.8528	55.8528	55.8528	55.8528
5 %	56.2712	54.9669	55.7432	55.8528	55.5556	57.1429	54.9342	56.81063

10 %	54.485	53.9735	56.8027	54.9153	53.6278	57.2881	55.0489	55.62701
15 %	55	53.3557	58.1882	53.6913	53.5168	58.9474	56.6225	56.02606
20 %	57.0934	52.5253	58.7814	53.4884	53.7538	58.1315	56.9536	56.02606
25 %	58.7189	52.0134	59.707	53.0201	53.5294	60.1449	56.9079	55.30547

Mixture zone area percentage for the base case is 55.8528 in table 10. Table 11 provides relative deviations (i.e. percentage) from the base case value of all sensitivity incremental points for 8 studied parameters (exception of injection time parameters):

Table 12 - Relative deviations from the base case value of all sensitivity incremental points for 8 studied parameters (exception of injection time parameters)

Increments	Diameter 1	Diameter 2	Density 1	Density 2	Injection rate	Confining Stress	Poisson Ratio	Young Modulus
-25 %	-7.2038	7.5454	-8.3725	5.0381	9.2948	-13.1753	-1.0075	1.9457
-20 %	-5.7673	7.9076	-8.0594	5.9881	4.3291	-13.1753	0.7491	-0.1856
-15 %	-4.2857	5.7430	-6.0582	4.1917	5.0468	-7.4113	0.7491	-2.2855
-10 %	-4.2465	3.9405	-4.8839	1.7965	0.3144	-2.9708	1.0136	-0.8010
-5 %	-2.8726	2.8281	-2.7071	1.6185	2.8407	-3.2362	-0.7292	0.7491
BASE CASE	0	0	0	0	0	0	0	0
5 %	0.7491	-1.5861	-0.1962	0.0000	-0.5321	2.3098	-1.6447	1.7149
10 %	-2.4489	-3.3647	1.7007	-1.6785	-3.9837	2.5698	-1.4393	-0.4043
15 %	-1.5269	-4.4709	4.1813	-3.8700	-4.1824	5.5406	1.3781	0.3102
20 %	2.2212	-5.9576	5.2434	-4.2333	-3.7581	4.0798	1.9709	0.3102
25 %	5.1315	-6.8741	6.9006	-5.0717	-4.1599	7.6847	1.8891	-0.9800

Table 13 provides initial numerical results and relative deviations (i.e. percentage) from the base case of all sensitivity incremental points for 2 studied injection time parameters:

Table 13 - initial numerical results and relative deviations from the base case of all sensitivity incremental points for 2 studied injection time parameters

	Former proppant type's time injection		Latter proppant type's time injection	
Increments	% Mixture zone	% Deviation	% Mixture zone	% Deviation
-25%	52.9801	-2.8727	55.4455	-0.4073
-20%	53.1353	-2.7175	56.0403	0.1875
-15%	53.4426	-2.4102	55.7756	-0.0772

-10%	53.5948	-2.258	55.6667	-0.1861
-5%	55.2288	-0.624	54.8173	-1.0355
0%	55.8528	0	55.8528	0
5%	55.082	-0.7708	56.3758	0.523
10%	55.814	-0.0388	56.9024	1.0496
15%	56.8106	0.9578	57.1906	1.3378
20%	58.6667	2.8139	57.2391	1.3863
25%	56.2914	0.4386	57.2391	1.3863

Table 14 summarizes absolute values of lowest and highest deviations from the base case value for all 10 studied parameters:

Table 14 - absolute values of lowest and highest deviations from the base case value for all 10 studied parameters

Studied Parameters	Lowest	Highest	Total of absolute values
Injection time (latter)	-1.0355	1.3863	2.4218
Poisson Ratio	-1.01356	1.970895	2.984452
Young Modulus	-2.28552	1.714921	4.000444
Injection Time (former)	-2.8727	2.8139	5.6866
Proppant Density 2	-5.98806	5.071724	11.05979
Proppant Diameter 1	-7.20376	5.131524	12.33528
Injection Rate	-9.29479	4.182422	13.47721
Proppant Diameter 2	-7.90757	6.874141	14.78171
Proppant Density 1	-8.37254	6.900639	15.27318
Confining Pressure	-13.1753	7.684664	20.86001

IV. Efficacy of trust-region method

Figures 63 to 68 present efficacy of the trust-region method (the method's fundamental is described chapter 2) applied for coefficient extraction from the novel permeability model in this thesis (chapter 4). In figures 62-68, the black hollow dots are values of the objective function (equations (4.43) and (4.44) in chapter 4). The lines are figurative outcomes from trust-region algorithm through implication of *MATLAB® R2019b Optimization Toolbox*.

Figure 63 - Initial result of trust-region method, mixture 20/40 sand and 20/40 ceramic
(50%/50%)

Figure 64– Initial result of trust-region method, mixture 20/40 sand and 20/40 ceramic
(25%/75%)

Figure 65 – Initial result of trust-region method, mixture 40/70 sand and 20/40 ceramic
(50%/50%)

Figure 66 – Initial result of trust-region method, mixture 40/70 sand and 20/40 ceramic
(25%/75%)

Figure 67 – Initial result of trust-region method, mixture 20/40 sand and 40/70 ceramic
(50%/50%)

Figure 68 – Initial result of trust-region method, mixture 20/40 sand and 40/70 ceramic
(50%/50%)