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Metaheuristics for the Generalized Quadratic Assignment Problem

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Metaheuristics for the Generalized Quadratic Assignment Problem

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**Thesis submitted to the
College of Engineering and Mineral Resources
at West Virginia University
in partial fulfillment of the requirements
for the degree of**

**Master of Science
in
Industrial Engineering**

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Simulated Annealing**

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ABSTRACT

Metaheuristics for the Generalized Quadratic Assignment Problem

Roseline Mostafa

The generalized quadratic assignment problem (GQAP) is the task of assigning a set of facilities to a set of locations such that the sum of the assignment and transportation costs is minimized. The facilities may have different space requirements, and the locations may have varying space capacities. Also, multiple facilities may be assigned to each location such that space capacity is not exceeded. In this research, an application of the GQAP is presented for assigning a set of machines to a set of locations on the plant floor. Two meta-heuristics are proposed for solving the GQAP: tabu search (TS) and simulated annealing (SA). In addition, two types of neighborhood structures are considered for each meta-heuristic. A set of 21 test problems, available in the literature, is used to evaluate the performances of the meta-heuristics using one or two neighborhood structures. Computational experiments show that the proposed SA heuristics performed better than the proposed TS heuristics. The SA heuristics obtained results better than those presented in the literature for three of the test problems. On the other hand, the TS heuristics did not perform well for the problems with high space capacity utilization.

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CHAPTER 1

INTRODUCTION

1.1 The Facility Layout Problem

The facility layout problem (FLP) is defined as the placement of facilities in a plant area, with the aim of determining the most effective arrangement in accordance with some criteria or objectives under certain constraints, such as shape, size, orientation, and pick-up/drop-off point of the facilities.

The facility layout problems can be of different types. Different types of layouts are shown in *Figure 1.1*. *Figure 1.1(a)* shows a quadratic assignment problem (QAP) where all the departments are equal and only one machine is assigned to each location. *Figure 1.1(b)* shows an unequal area facility layout problem (UFLP) where we have rectangular grids and each machine is assigned a number of grids based on their area requirement.

M15	M5	M11	M2
M6	M1	M4	M13
M14	M8	M12	M7
M10	M3	M16	M9

(a) QAP (Discrete)

	M4			M2	M5
			M1		M3

(b) UAFLP (Discrete)

M1,5,6		M4
M2	M3,7	

(c) GQAP (Discrete)

D1		D4
D2	D5	D3

(d) Continuous

Figure 1.1: Different types of facility layouts

Figure 1.1(c) shows the generalized quadratic assignment problem (GQAP) where we have unequal area locations and more than one machine can be assigned to one location. Note, the first

one (*Figure 1.1(a)*) deals with equal area location and the next two (*Figures 1.1(b) & (c)*) deals with unequal area locations. All of these are discrete layout problems. However, *Figure 1.1(d)* shows a continuous UFLP where several departments with different shapes and areas are located on a rectangular floor.

A simple classification of facility layout problems can be shown by the figure below.

Figure 1.2: Simple classification of facility layout problems

In this research, the focus is on the GQAP (shown in *Figure 1.1(c)*) which is a discrete UFLP.

1.2 The Importance of the Facility Layout Problem

Facility layout dramatically influences the efficiency of material handling within a manufacturing system. Therefore, the facility layout is of great concern for manufacturers. An efficient facility

layout will improve profit and productivity. Moreover, it has been estimated that materials handling cost is between 20 to 50% of the total operating cost, and effective facility layout planning can reduce the material handling costs by 10 to 30% (Tompkins *et al.*, 2003).

Since customer demand is constantly changing, the material handling paths and layout of machines (or departments) are varied constantly. In other words, in order to ensure optimal performance of a facility, the layout should reflect changes to the system that may occur over time. Therefore, the FLP exists when either new plants are built or older plants are modified. Francis *et al.* (1992) proposed some reasons that may cause the modification of the layout of a facility:

- a) Change of the product design.
- b) The addition or deletion of a product from the product line.
- c) Significant increase or decrease in the demand for a product.
- d) Changes on the design of the process.
- e) The replacement of equipment.
- f) The adoption of new safety standards.
- g) Bottlenecks in production.
- h) Unexplainable delays and idle time.
- i) Excessive temporary storage space.

Therefore, the FLP often occurs and exists for many different reasons. As a result, the facility layout may need to be modified constantly. The decision of the facility layout is made at the strategic level and has a long-lasting effect on the manufacturing system. Once the decision is made, changing the layout of the facility can be very costly. Some of the costs associated with the re-layout of a facility are cost of rearranging the machines, cost of purchasing or leasing equipment for rearranging the machines/department, and the cost associated with the loss of production. Therefore, the importance of the FLP is obvious.

1.3 The Generalized Quadratic Assignment Problem

The generalized quadratic assignment problem (GQAP) is the task of assigning a set of facilities to a set of locations such that the sum of the assignment and transportation costs is minimized. The facilities may have different space requirements, and the locations may have varying space

capacities. Also, multiple facilities may be assigned to each location such that space capacity is not exceeded.

The material handling cost depends on the material flows between departments and the distances between their locations. The most commonly used distance measures for the FLP are rectilinear and Euclidean. The rectilinear distance between two points (x_1, y_1) and (x_2, y_2) is defined as $|x_1 - x_2| + |y_1 - y_2|$, and the Euclidean distance between the two points is defined as $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. Other distance measures can also be used to determine the distances between two departments. Nevertheless, in this research the rectilinear distance measure is used to determine the distance between two departments.

See *Figure 1.3* for an example of a layout configuration with 4 locations. Notice the centroid of each location is given. For example, the centroid of location 4 is (30, 5). Using the rectilinear distance measure discussed above, the distance between locations 1 and 3 is 17.5 distance units and between locations 1 and 4 is 27.5 distance units.

Figure 1.3: Example of a layout configuration

1.4 Objective of the Thesis

The objectives of this research are given as follows:

1. To develop a tabu search (TS) heuristic for the GQAP.
2. To develop a simulated annealing (SA) heuristic for the GQAP.
3. To test the performance of the heuristics by solving test problems taken from the literature.
4. To compare the performances of the two heuristics.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

A good number of papers have been published on solving FLPs. But unlike QAP, GQAP literature is very limited. Exact methods cannot achieve optimal solutions for large scale problems. Heuristics are used to obtain good quality solutions in short computation times. However, the optimality of the solutions are not guaranteed by heuristic approaches. The literature review can be divided in two sections. The first one reviews exact methods and heuristic methods for GQAPs and their applications. And the second one is a brief review for tabu search (TS) and simulated annealing (SA) metaheuristics used in different types of FLPs.

2.2 Generalized Quadratic Assignment Problem (GQAP)

2.2.1 Exact Methods

Lee and Ma (2004) presented the first formulation for the GQAP. The authors presented three methods for the linearization of the formulation and a branch and bound algorithm to optimally solve the GQAP. Hahn *et al.* (2008) presented a new algorithm to optimally solve the GQAP. Pessoa *et al.* (2010) presented two exact algorithms for the GQAP which combine a previously proposed branch and bound scheme with a new Lagrangean relaxation procedure over a known RLT formulation.

2.2.2 Heuristic Methods

Cordeau *et al.* (2006) presented a linearization of the GQAP formulation as well as a memetic heuristic for the GQAP, which combines genetic algorithms (Holland,1975) and tabu search (Glover, 1986). Mateus *et al.* (2011) proposed several GRASP (greedy randomized adaptive search procedures) with path-relinking heuristics for the GQAP using different construction, local search, and path-relinking procedures (GRASP-PR). McKendall and Li (2017) presented an application of the GQAP for assigning a set of machines to a set of locations on the plant floor. Also, the authors developed a TS heuristic using different construction algorithms. McKendall (2008) presented three tabu search heuristics for a dynamic space allocation problem (DSAP) where each of the proposed heuristics outperformed the heuristics available in the literature for the DSAP. The DSAP is a layout-type problem that assigns items (project activity resources) to locations (either

work or storage spaces) during a multi-period planning horizon such that the cost of rearranging the items is minimized.

2.2.3 Applications

Apart from facility layouts we have multiple application of the GQAP. Cordeau *et al.* (2006) presented an application which uses the GQAP to manage containers in a storage yard (assign M container groups to N storage areas). Cordeau *et al.* (2007) considered the service allocation problem as a GQAP with side constraints. Unal and Uysal (2014) used the GQAP to design a curriculum at a university. GQAP can also be used to assign maintenance activities and their required resources to workspaces and their idle resources to storage locations inside the reactor containment building during planned outages at a nuclear power plant (McKendall and Jaramillo, 2006).

2.3 Metaheuristics for the FLP

2.3.1 Tabu Search

In different types of FLPs, the TS has been widely used in optimizing material handling cost, utilization of space, minimization of re-layout cost for both single- and multi-objective problems (Abdinnour-Helm and Hadley, 2000; McKendall and Hakobyan, 2010; Scholz *et al.*, 2009; Seo *et al.*, 2006; Yang *et al.*, 1999). TS is also deployed for optimizing single row fixed layout design problem (a special class of FLP) by Samarghandi and Eshghi (2010). Liang and Chao (2008) proposed a special different intensification and diversification strategies, which shows better convergence of searching to get proper arrangement of facilities. Improved TS algorithm with intensification, reconstruction and solution acceptance operation was proposed by Singh (2009), which gives comparative result with respect to some benchmark problems found in QAP-based FLP literature.

2.3.2 Simulated Annealing

SA is being used by the researchers in the field, starting from manufacturing cell design to multi-objective optimization of dynamic and static behaviour of FLP irrespective of whether there are equal or unequal sized facilities (Castillo and Peters, 2003; Dong *et al.*, 2009; Ioannou, 2007; Sahin and Türkbey, 2009; Sahin *et al.*, 2010; Souilah, 1995). Hybrid method of SA with TS (Sugiyono, 2006), SA and GA (Matsuzaki *et al.*, 1999) helps to avoid high computational cost (Mir and Imam,

2001) and improves the solution. McKendall *et al.*, (2006) developed two SA algorithms for dynamic facility layout problems. A basic SA heuristic for the DFLP, and an SA heuristic with a look-ahead/look-back strategy. Gunawan *et al.* (2014) solved QAP using GRASP and improved the solution using a hybrid SA-TS algorithm. Privault *et al.* (1998) and Hamam *et al.* (2000) solved GAP using simulated annealing metahuristics.

2.4 Proposed Tabu Search (TS) Heuristic

As mentioned above in the GQAP literature review, a TS heuristic has been presented for the GQAP. In the TS heuristic presented in McKendall and Li (2017), the best admissible move is selected randomly among the top solutions at each iteration, to add randomness to diversify the search. This is a type of probabilistic TS heuristic and is not considered in this thesis. In other words, in the proposed TS heuristic the best admissible move is selected at each iteration. Also, McKendall and Li (2017) added even more diversification to their TS heuristic by generating a set of diverse initial solutions. In the proposed TS heuristic, only one initial solution is generated. For these reasons the proposed TS heuristic is more restricted. That is, the proposed TS heuristic presented in this thesis is deterministic.

2.5 Contributions of the Thesis to the GQAP Literature

The GQAP literature is very limited and there is no SA algorithm developed for the GQAP. Therefore, the contributions of this thesis, to the GQAP literature, is given as follows.

1. Development of a SA heuristic for the GQAP.
2. Improvement of the best found solutions for 3 test problems taken from the literature.

CHAPTER 3

PROBLEM DEFINITION

3.1 Problem Definition

There are M machines to be installed in N locations ($M \geq N$) in a department. The machines have different space requirements and the locations have varying space capacities. Multiple machines can be assigned to each location such that space capacity is not exceeded. The problem is to assign the machines to the locations such that the total cost (i.e., the sum of installation cost and material handling costs) is minimized.

3.2 Problem Assumptions

The assumptions for the GQAP are as follows:

1. The layout configuration is given.
2. Machines and locations may be of unequal sizes.
3. The locations have rectangular areas.
4. Material flows between machines are given.
5. The distances between locations are known.
6. The area requirements for the machines are known.
7. The layout representation is discrete.
8. The objective is to minimize the sum of the installation and material handling costs.

3.3 Mathematical Formulation

The formulation of the GQAP from McKendall and Li (2017) is given below and is an adaptation of the model presented by Lee and Ma (2004).

$$\text{Minimize } z = \sum_{i=1}^M \sum_{k=1}^N a_{ik} x_{ik} + \sum_{i=1}^M \sum_{\substack{j=1 \\ j \neq i}}^M \sum_{k=1}^N \sum_{\substack{l=1 \\ l \neq k}}^N c_{ijkl} f_{ij} d_{kl} x_{ik} x_{jl} \quad (1)$$

$$\text{Subject to } \sum_{k=1}^N x_{ik} = 1, \text{ for } i=1, \dots, M \quad (2)$$

$$\sum_{i=1}^M r_i x_{ik} \leq C_k, \text{ for } k=1, \dots, N \quad (3)$$

$$x_{ik} = 0 \text{ or } 1, \text{ for } i=1, \dots, M \text{ and } k=1, \dots, N \quad (4)$$

where,

$x_{ik} = 1$ if machine i is assigned to location k , and zero otherwise

M = number of machines

N = number of locations

a_{ik} = cost of assigning machine i to location k

f_{ij} = units of materials transported from machine i to machine j

d_{kl} = distance from location k to location l

c_{ijkl} = unit cost per distance unit of moving materials from machine i , at location k , to machine j , at location l

r_i = the space requirement of machine i

C_k = the amount of space capacity available at location k

Objective function (1) minimizes the sum of the assignment (or installation) and material handling costs. Constraint set (2) ensures that each machine is assigned to only one location. Constraint set (3) ensures that the space capacity of each location is not exceeded, and the restrictions on the decision variables are given in (4).

The term in objective function (1) used to obtain material handling cost has a quadratic term (i.e. product of two variables). As a result, the mathematical formulation (1)–(4) is nonlinear and is called a nonlinear programming model. The model is linearized by substituting w_{ikjl} for $x_{ik}x_{jl}$. Then, replace objective function (1) with

$$\text{Minimize } z = \sum_{i=1}^M \sum_{k=1}^N a_{ik} x_{ik} + \sum_{i=1}^M \sum_{\substack{j=1 \\ j \neq i}}^M \sum_{k=1}^N \sum_{\substack{l=1 \\ l \neq k}}^N c_{ijkl} f_{ij} d_{kl} w_{ikjl} \quad (1a)$$

and add the following constraints,

$$x_{ik} + x_{jl} - 1 \leq w_{ikjl}, \text{ for } i, j = 1, \dots, M \text{ where } j \neq i \text{ and } k, l = 1, \dots, N \text{ where } l \neq k \quad (5)$$

$$w_{ikjl} \geq 0, \text{ for } i, j = 1, \dots, M \text{ where } j \neq i \text{ and } k, l = 1, \dots, N \text{ where } l \neq k \quad (6)$$

As a result, the linearized model (i.e. a mixed integer linear programming model) for the GQAP consists of objective function (1a) subject to constraints (2)-(6).

For example, we can consider a GQAP instance where six machines are assigned to four locations on the plant floor (see *Figure 1.3*) and use the following data from McKendall and Li (2017).

$$a_{ik} = \begin{bmatrix} 700 & 1600 & 1900 & 1400 \\ 1300 & 1800 & 1700 & 800 \\ 800 & 1400 & 3000 & 1100 \\ 3000 & 800 & 700 & 1500 \\ 1200 & 1500 & 1300 & 1800 \\ 1700 & 800 & 1200 & 1100 \end{bmatrix}$$

$$f_{ij} = \begin{bmatrix} 0 & 33 & 41 & 9 & 16 & 56 \\ 0 & 0 & 49 & 91 & 78 & 23 \\ 0 & 0 & 0 & 41 & 38 & 44 \\ 0 & 0 & 0 & 0 & 6 & 17 \\ 0 & 0 & 0 & 0 & 0 & 68 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$d_{kl} = \begin{bmatrix} 0 & 22.5 & 17.5 & 27.5 \\ 22.5 & 0 & 15 & 15 \\ 17.5 & 15 & 0 & 10 \\ 27.5 & 15 & 10 & 0 \end{bmatrix}$$

$$r_i = \begin{bmatrix} 90 \\ 120 \\ 100 \\ 110 \\ 110 \\ 70 \end{bmatrix}$$

$$C_k = [300 \ 200 \ 100 \ 100] \text{ and } c_{ijkl} = 1 \text{ for all } i, j, k, l$$

Using the linearized mathematical model given above for the GQAP and the MPL/CPLEX software, the optimal solution is obtained for this problem: $x_{13} = x_{21} = x_{34} = x_{42} = x_{51} = x_{61} = 1$, $z^* = \$17,165$.

Interpretation of the solution is:

Machines 2, 5, and 6 are assigned to location 1.

Machines 4 is assigned to location 2.

Machines 1 is assigned to location 3.

Machine 3 is assigned to location 4.

The solution of the GQAP instance is shown in *Figure 3.1*.

Figure 3.1: Solution of the GQAP Instance

The sum of total installation cost and total material handling cost is $z^* = \$17,165$ where

$$\text{total installation cost} = \sum_{i=1}^6 \sum_{k=1}^4 a_{ik} x_{ik} = \$8,000$$

$$\text{and total material handling cost} = \sum_{i=1}^6 \sum_{j \neq i}^6 \sum_{k=1}^4 \sum_{l \neq k}^4 c_{ijkl} f_{ij} d_{kl} w_{ikjl} = \$9,165.$$

3.4 Combinatorial Optimization Problem (COP) Model

COPs are problems in which all or some of the decision variables are discrete. For the GQAP the COP model is given below:

$$\text{Minimize } TC(S) = \sum_{i=1}^M a_{is(i)} + \sum_{i=1}^M \sum_{j \neq i}^M c_{ijs(i)s(j)} f_{ij} d_{s(i)s(j)} \quad (7)$$

$$\text{Subject to } \sum_{i \text{ s.t. } s(i)=k} r_i \leq C_k \text{ for } k = 1, 2, \dots, N \quad (8)$$

where,

$\mathbf{J}$ = set of machines $|\mathbf{J}| = M$ = number of machines

$\mathbf{K}$ = set of locations $|\mathbf{K}| = N$ = number of locations

The solution representation is: $S = \{S(1), S(2), \dots, S(M)\}$ where $S(i) = k$, i.e. machine i is assigned to location k .

Objective function (7) minimizes the total cost of the solution S . Constraint set (8) ensures that the capacity of each location is not violated.

The solution obtained in *Section 3.3* for the GQAP instance can be represented as $S = (3, 1, 4, 2, 1, 1)$ which indicates machines 2, 5, and 6 are assigned to location 1, machines 4 is assigned to location 2, machines 1 is assigned to location 3, and machine 3 is assigned to location 4. Also, total cost of solution is $TC(S) = \$17,165$.

CHAPTER 4

METHODOLOGY

4.1 Introduction

Metaheuristics can improve the performance of many other heuristics. Tabu search (Glover, 1986) is an aggressive metaheuristic that guides a local search out of local optimal while simulated annealing (Metropolis *et al.*, 1953) uses a probabilistic approach to obtain the same end. In this thesis, a tabu search (TS) and a simulated annealing (SA) heuristic are developed to solve the COP model presented in the previous section to solve the GQAP.

4.2 Solution Representation for the GQAP

Recall, the solution representation for the GQAP is defined as follows:

$$S = \{S(1), S(2), \dots, S(M)\}$$

where

S = solution of the GQAP

M = number of machines

$S(i) = k$, i.e. machine i is assigned to location k .

Next, a construction algorithm is presented for the GQAP.

4.3 Construction Algorithm to Generate Initial Solution

An initial solution for the GQAP is generated by a construction algorithm described in McKendall and Li (2017). It is given as follows.

Step 1: Initialize the capacities of the locations (i.e. $C = \{C(1), C(2), \dots, C(N)\}$). Initialize the space requirements of the machines (i.e. $r = \{r(1), r(2), \dots, r(M)\}$).

Step 2: Sort machines in set r in descending order with respect to $r(i)$. Break ties based on smallest index (i.e. lowest machine number). Then put the sorted machine numbers in the eligible machine set (EMS). For example, if $r = \{5, 3, 5\}$, then $r(1) = r(3) = 5 > 3 = r(2)$. Since index $1 < 3$, EMS = $\{1, 3, 2\}$.

Step 3a: Set $k = 1$ where k = location number.

Step 3b: If $k > N$ (N = number of locations), then go to step 5b. Else, go to position 1 of EMS (i.e. set $p = 1$).

Step 4: Set i = the machine in position p of set EMS.

Step 5a: If $r(i) \leq C(k)$, then

1. Assign machine i to location k (i.e. set $s(i) = k$), and set $C(k) = C(k) - r(i)$;
2. Remove machine i from EMS. If EMS is empty, then go to step 5b;
3. If $C(k) < r(i)$ for Last (i) in set EMS, then set $k = k + 1$, and go to step 3b; Else, go to step 4. Else, set $p = p + 1$, and go to step 4.

Step 5b: Terminate algorithm. If EMS is empty, then display feasible solution S . Else, display “No feasible solution!”

For the GQAP instance mentioned before, the solution generated using this construction algorithm is $S_0 = (2, 1, 3, 1, 2, 1)$ where total cost of this solution is $TC(S_0) = \$21,255$. Recall, the optimal solution obtained using MPL/CPLEX and the mathematical formulation (objective function (1a) subject to constraints (2)-(6)) presented in Chapter 3 is $S = (3, 1, 4, 2, 1, 1)$ where $TC(S) = \$17,165$.

4.4 Neighborhood Structures

A neighborhood of a solution S_0 is a set of solutions that can be obtained from S_0 by a simple operation or move. In our proposed metaheuristics we used two types of neighborhood structures as defined in McKendall and Li (2017): drop/add and pairwise exchange.

4.4.1 Drop/Add Neighborhood Structure

Drop/add neighborhood considers a simply removal of an element and addition of an element to the set of solution variables. Using the notation taken from McKendall and Li (2017), the drop/add operation $(u, v: v')$ represents exchanging (dropping) location v assigned to machine u with (adding) location v' . For example, if $S_0 = \{2, 1, 3, 1, 2, 1\}$, as obtained from using the construction algorithm presented above to solve the 6 machines 4 locations GQAP instance, then the drop/add operation $(1, 2; 3)$ produces the solution $\{\underline{3}, 1, 3, 1, 2, 1\}$. In other words, machine 1 assigned to location 2 is reassigned to location 3. Since there are $M = 6$ machines and $N = 4$ locations, and

each machine is already assigned to one location, there are $N-1 = 3$ possible drop/add operations for each machine. Therefore, there will be $M(N-1) = 6(3) = 18$ possible drop/add operations. See *Table 4.1* (similar to Table 6 in McKendall and Li, 2017) for all possible drop/add operations on solution $S_0 = (2, 1, 3, 1, 2, 1)$.

Table 4.1 Solutions obtained from drop/add operations on initial solution

#	Move	Solution	Cost	#	Move	Solution	Cost
1	(1, 2; 1)	(<u>1</u> , 1, 3, 1, 2, 1)	infeasible	10	(4, 1; 2)	(2, 1, 3, <u>2</u> , 2, 1)	infeasible
2	(1, 2; 3)	(<u>3</u> , 1, 3, 1, 2, 1)	infeasible	11	(4, 1; 3)	(2, 1, 3, <u>3</u> , 2, 1)	infeasible
3	(1, 2; 4)	(<u>4</u> , 1, 3, 1, 2, 1)	\$21,580	12	(4, 1; 4)	(2, 1, 3, <u>4</u> , 2, 1)	infeasible
4	(2, 1; 2)	(2, <u>2</u> , 3, 1, 2, 1)	infeasible	13	(5, 2; 1)	(2, 1, 3, 1, <u>1</u> , 1)	infeasible
5	(2, 1; 3)	(2, <u>3</u> , 3, 1, 2, 1)	infeasible	14	(5, 2; 3)	(2, 1, 3, 1, <u>3</u> , 1)	infeasible
6	(2, 1; 4)	(2, <u>4</u> , 3, 1, 2, 1)	infeasible	15	(5, 2; 4)	(2, 1, 3, 1, <u>4</u> , 1)	infeasible
7	(3, 3; 1)	(2, 1, <u>1</u> , 1, 2, 1)	infeasible	16	(6, 1; 2)	(2, 1, 3, 1, 2, <u>2</u>)	infeasible
8	(3, 3; 2)	(2, 1, <u>2</u> , 1, 2, 1)	infeasible	17	(6, 1; 3)	(2, 1, 3, 1, 2, <u>3</u>)	infeasible
9	(3, 3; 4)	(2, 1, <u>4</u> , 1, 2, 1)	\$20,695	18	(6, 1; 4)	(2, 1, 3, 1, 2, <u>4</u>)	\$20,495

4.4.2 Pairwise Exchange Neighborhood Structure

Pairwise exchange considers interchanging or swapping two elements in the current solution. Using the notation taken from McKendall and Li (2017), the pairwise exchange operation $(u, v; u', v')$ represents exchanging location v assigned to machine u with location v' assigned to machine u' . If $S_0 = \{2, 1, 3, 1, 2, 1\}$, then the pairwise exchange operation $(1, 2; 4, 1)$ produces the solution $\{1, 1, 3, 2, 2, 1\}$. In other words, machine 1 assigned to location 2 exchanges locations with machine 4 assigned to location 1. Since two machines are swapping locations and there are $M = 6$ machines, there are a combination of $M = 6$ pick two i.e., $M(M-1)/2 = 15$ possible pairwise exchange operations. If $v = v'$, the solution does not change. Therefore, there are always less than $M(M-1)/2$ pairwise exchange operations when a location has more than one machine assigned to it. See *Table 4.2* (similar to Table 7 in McKendall and Li, 2017) for all possible pairwise exchange operations on solution $S_0 = (2, 1, 3, 1, 2, 1)$.

Table 4.2 Solutions obtained from pairwise exchange operations on initial solution

#	Move	Solution	Cost	#	Move	Solution	Cost
1	(1, 2; 2, 1)	(<u>1</u> , <u>2</u> , 3, 1, 2, 1)	infeasible	9	(2, 1; 6, 1)	(2, <u>1</u> , 3, 1, 2, <u>1</u>)	same as S_0
2	(1, 2; 3, 3)	(<u>3</u> , 1, <u>2</u> , 1, 2, 1)	infeasible	10	(3, 3; 4, 1)	(2, 1, <u>1</u> , <u>3</u> , 2, 1)	infeasible
3	(1, 2; 4, 1)	(<u>1</u> , 1, 3, <u>2</u> , 2, 1)	infeasible	11	(3, 3; 5, 2)	(2, 1, <u>2</u> , 1, <u>3</u> , 1)	infeasible
4	(1, 2; 5, 2)	(<u>2</u> , 1, 3, 1, <u>2</u> , 1)	same as S_0	12	(3, 3; 6, 1)	(2, 1, <u>1</u> , 1, 2, <u>3</u>)	infeasible
5	(1, 2; 6, 1)	(<u>1</u> , 1, 3, 1, 2, <u>2</u>)	infeasible	13	(4, 1; 5, 2)	(2, 1, 3, <u>2</u>, <u>1</u>, 1)	\$18,050
6	(2, 1; 3, 3)	(2, <u>3</u> , <u>1</u> , 1, 2, 1)	infeasible	14	(4, 1; 6, 1)	(2, 1, 3, <u>1</u> , 2, <u>1</u>)	same as S_0
7	(2, 1; 4, 1)	(2, <u>1</u> , 3, <u>1</u> , 2, 1)	same as S_0	15	(5, 2; 6, 1)	(2, 1, 3, 1, <u>1</u> , <u>2</u>)	infeasible
8	(2, 1; 5, 2)	(2, <u>2</u> , 3, 1, <u>1</u> , 1)	infeasible				

4.5 Determining Cost of the GQAP Solution

4.5.1 Calculating the Total Cost

The total cost of the GQAP solution is the sum of assignment and material handling costs. The total assignment cost ($A(S)$) is calculated as follows:

$$A(S) = \sum_{i=1}^M a_{is(i)}$$

where a_{ik} = cost of assigning machine i to location k

and the material handling cost ($MH(S)$) is calculated as follows:

$$MH(S) = \sum_{i=1}^M \sum_{\substack{j=1 \\ j \neq i}}^M c_{ijs(i)s(j)} f_{ij} d_{s(i)s(j)}$$

where

f_{ij} = units of materials transported from machine i to machine j

d_{kl} = distance from location k to location l

c_{ijkl} = unit cost per distance unit of moving materials from machine i (at location k) to machine j (at location l)

So, the total cost of the GQAP solution is calculated as follows:

$$\begin{aligned}
TC(S) &= A(S) + MH(S) = \sum_{i=1}^M a_{is(i)} + \sum_{i=1}^M \sum_{\substack{j=1 \\ j>i}}^M c_{ijs(i)s(j)} f_{ij} d_{s(i)s(j)} \\
&= \sum_{i=1}^M a_{is(i)} + \sum_{i=1}^M \sum_{j=i+1}^M c_{ijs(i)s(j)} w_{ij} d_{s(i)s(j)}
\end{aligned}$$

where $w_{ij} = f_{ij} + f_{ji}$

4.5.2 Calculating the Change in Total Cost, $\Delta TC(S)$

Once the objective function value (OFV) is obtained for the initial solution, $TC(S_0)$, the OFV of the neighboring solutions, $TC(S)$, can be obtained efficiently by calculating the change in total cost. The change in total cost is the sum of the change in assignment and material handling costs.

4.5.2.1 $\Delta TC(S)$ in Drop/Add Neighborhood

If a solution S is generated in the drop/add neighborhood of S_0 by performing the move $(u, v: v')$, the change in assignment cost $\Delta A_{uv}(S)$ is calculated as follows:

$$\Delta A_{uv}(S) = a_{uv} - a_{uv'}$$

And the change in material handling cost $\Delta MH_{uv}(S)$ is calculated as follows:

$$\begin{aligned}
\Delta MH_{uv}(S) &= \sum_{i=1}^M c_{ius(i)v} w_{iu} d_{S(i)v} - \sum_{i=1}^M c_{ius(i)v'} w_{iu} d_{S(i)v'} \\
&= \sum_{i=1}^M c_{ius(i)v} w_{iu} [d_{S(i)v} - d_{S(i)v'}]
\end{aligned}$$

So, the total change in cost after the drop/operation is:

$$\begin{aligned}
\Delta TC &= TC(S_0) - TC(S) = \Delta A_{uv}(S) + \Delta MH_{uv}(S) \\
&= (a_{uv} - a_{uv'}) + \sum_{i=1}^M c_{ius(i)v} w_{iu} [d_{S(i)v} - d_{S(i)v'}]
\end{aligned}$$

4.5.2.2 $\Delta TC(S)$ in Pairwise Exchange Neighborhood

If a solution S is generated in the pairwise exchange neighborhood of S_0 by performing the move $(u, v: u', v')$, the change in assignment cost $\Delta A'_{uv}(S)$ is calculated as follows:

$$\Delta A'_{uv}(S) = (a_{uv} + a_{u'v'}) - (a_{uv'} + a_{uv})$$

And the change in material handling cost $\Delta MH'_{uv}(S)$ is calculated as follows:

$$\begin{aligned}
\Delta MH'_{uv}(S) &= \left[\sum_{i=1}^M c_{ius(i)v} w_{iu} d_{S(i)v} + \sum_{i=1}^M c_{iu's(i)v'} w_{iu'} d_{S(i)v'} \right] - \left[\sum_{i=1}^M c_{ius(i)v'} w_{iu} d_{S(i)v'} + \sum_{i=1}^M c_{iu's(i)v} w_{iu'} d_{S(i)v} \right]
\end{aligned}$$

$$= \sum_{i=1}^M c_{ius(i)v} [w_{iu} (d_{S(i)v} - d_{S(i)v'}) + w_{iu'} (d_{S(i)v'} - d_{S(i)v})]$$

So, the total change in cost after the pairwise exchange operation is:

$$\Delta TC = TC(S_0) - TC(S) = \Delta A'_{uv}(S) + \Delta MH'_{uv}(S)$$

$$= [(a_{uv} + a_{u'v'}) - (a_{uv'} + a_{u'v})] + \sum_{i=1}^M c_{ius(i)v} [w_{iu} (d_{S(i)v} - d_{S(i)v'}) + w_{iu'} (d_{S(i)v'} - d_{S(i)v})]$$

4.6 Steepest Descent Local Search Heuristic

The local neighborhood search technique, called the steepest descent local search heuristic, is used in the proposed metaheuristics to get to the local optima. The steepest descent local search heuristic for the GQAP is given as follows.

Step 1: Construct a solution, $S_0 = (S(1), S(2), \dots, S(M))$, using the construction algorithm presented earlier and obtain its cost, $TC(S_0)$.

Step 2: Evaluate all feasible solutions obtained from all possible drop/add operations on S_0 and all possible pairwise exchange operations on S_0 .

Step 3: Pick best operation/move which gives best feasible neighboring solution, S , with respect to total cost, $TC(S)$. If $TC(S) < TC(S_0)$, set $S_0 = S$, $TC(S_0) = TC(S)$, and go to step 2. Else, terminate heuristic and display local optimum S_0 .

Next the SA heuristic is presented for the GQAP. Since SA is a probabilistic heuristic, it does not always converge to a local optimum solution. As a result, the solutions obtained from the proposed SA heuristic are either improved and/or verified as a local optimum using the steepest descent local search heuristic presented above.

4.7 Simulated Annealing (SA)

4.7.1 Basic Simulated Annealing

Simulated annealing (SA) algorithms model the process of annealing in solids to optimize complex functions or systems. In the physical world, annealing is accomplished by heating a solid to an elevated temperature and then allowing it to cool slowly enough so that the thermal equilibrium is maintained. Atoms in the material then assume a globally minimum energy state. SA algorithms have been successfully applied to a variety of problems. The algorithm starts with an initial

solution and generates a new solution by changing one or more of the solution variables. The objective function is then evaluated for the new solution. While a better solution is always accepted there is a possibility that a worse solution may be accepted based on a probability function (Metropolis *et al.*, 1953).

The change in energy is expressed as the change in objective function value (OFV), while the temperature is a control parameter that sets the probability of selection. In the general method, the temperature is held constant for a prescribed number of iterations to allow the system to gain “thermal equilibrium” and is then decreased in accordance with a cooling curve. As the temperature decreases, so does the probability that a worse solution will be accepted. This forces the algorithm to converge to an optimal, or near optimal, solution.

4.7.2 Outline of SA for the GQAP

SA heuristics starts with an initial solution S_0 and its cost $TC(S_0)$. Generates a neighboring solution S randomly. The cost of the neighboring solution $TC(S)$ is obtained and compared to the cost of the initial solution $TC(S_0)$. If the cost of the neighboring solution is better (i.e., $TC(S) < TC(S_0)$) this solution becomes the best solution, and it is used as the starting solution at the next iteration (i.e., $S_0 = S$). Otherwise, the initial solution S_0 is used as the starting solution at the next iteration with respect to some probability. This process continues until a stopping criterion is reached. However, strategies, which use a local search technique that allowed accepting non-improving neighboring solutions, were considered and performed well. This is the idea behind the SA heuristic and other meta-heuristics. In other words, the idea of annealing is used to accept non-improving moves (or neighboring solutions S) to avoid getting trapped at a poor local optimum. In SA, the probability of accepting non-improving moves initially is high, but as the search proceeds (and the temperature is reduced), the probability of accepting non-improving moves reduces. An outline of a basic SA heuristic (McKendall *et al.*, 2006) for combinatorial optimization problems is as follows:

1. Select settings for the heuristic parameters and generate an initial solution S_0 and its cost $TC(S_0)$. This solution S_0 is defined as the current solution.
2. Obtain a neighboring solution S of the current solution using a move or operation (e.g., drop add or pairwise exchange move).

3. Obtain the cost of the neighboring solution $TC(S)$ and compare it to the cost of the current solution $TC(S_0)$.

(a) If the cost is better (i.e., $TC(S) < TC(S_0)$), it is accepted as the current solution (i.e., $S_0 = S$).

(b) If the cost is worst or same (i.e., $TC(S) \geq TC(S_0)$), then it is accepted as the current solution (i.e., $S_0 = S$) with some probability. Otherwise, keep the current solution S_0 .

4. Update counters and parameters, and repeat steps 2–4, until an appropriate stopping criterion is reached.

4.7.3 Components of SA Heuristic for the GQAP

4.7.3.1 Initial Temperature $T(0)$

Initial Temperature controls the probability that a non-improving move will be accepted. The initial temperature is determined such that the probability of accepting a neighboring solution S with a cost of $x\%$ above the cost of the initial solution S_0 is $P(\Delta TC)$ where $\Delta TC = TC(S_0) - TC(S)$. The probability $P(\Delta TC) = \exp(-\Delta TC / T(t))$. For iteration 1, $T(t) = T(0)$, which gives the equation:

$$T(0) = -\Delta TC / \ln(P(\Delta TC)) = \frac{-\left(\frac{x}{100}\right)TC(S_0)}{\ln(P(\Delta TC))}.$$

4.7.3.2 Number of iterations at each temperature $N(t)$

This is the number of iterations at each temperature which can be static or dynamic.

4.7.3.3 Cooling Ratio α

Cooling ratio specifies the fraction of the current temperature $T(t)$ that will be used as the next temperature once we have attempted a number of iterations equal to $N(t)$. The cooling schedule is $T(t) = \alpha T(t-1)$ where $0 < \alpha < 1$.

4.7.3.4 Stopping Criterion

The stopping criterion for the proposed SA heuristic is the minimum allowable temperature. The heuristic terminates when the temperature reaches a value close to zero. Other commonly used stopping criteria are computational time, maximum number of iterations performed and maximum number of consecutive iterations without an improvement. The detailed settings of the heuristic parameters are explained in *Chapter 5*.

4.7.4 Pseudo-code for the SA Heuristic for the GQAP

The pseudo-code for the SA heuristic with the different neighborhoods (i.e., drop/add, pairwise exchange neighborhoods) for the GQAP is given below.

Step 1: Obtain initial solution $S_0 \in X$ where X is the solution space (set of all feasible solutions). Set $Best_sol = S_0$ and $Best_cost = TC(S_0)$.

Step 2: Select initial temperature $T(0)$, $T > 0$, the cooling ratio α , attempted number of moves at each temperature, and the minimum allowable temperature.

Step 3: Set temperature counter $t = 0$, iteration counter at current temperature $n = 0$.

Step 4: Randomly choose a neighborhood structure between drop/add and pairwise exchange.

Step 5: Randomly generate a neighbor of S_0 . For drop/add neighborhood, randomly select a machine u and a location v' , then assign machine u to location v' . For pairwise neighborhood, randomly select two machines u and $v \neq u$ and exchange their locations.

Step 6: Obtain the change in the total cost, $\Delta TC = TC(S_0) - TC(S)$.

Step 7: If $\Delta TC > 0$, then $S_0 = S$. If $TC(S_0) < Best_cost$, update $Best_sol$ and $Best_cost$ such that $Best_sol = S_0$ and $Best_cost = TC(S_0)$.

Else if $\Delta TC < 0$ and $\exp(\Delta TC/T) > \text{rand}(0,1)$, then $S_0 = S$. Else keep S_0 .

Set $k = k + 1$ and $n = n + 1$. Repeat steps 4-7 until $n = N(t)$ or until reaching stopping criterion.

If $n = N(t)$ go to step 8. If stopping criterion is reached go to step 9.

Step 8: Set $t = t + 1$, $T = T(t) = \alpha T(t-1)$, $n = 0$. Go to step 4.

Step 9: Use steepest descent local search heuristic on the best solution obtained.

Note: For SA heuristic with only drop/add neighborhood, insteps 4 and 5, consider and generate a neighbor of S_0 using a drop/add move only.

4.8 Tabu Search (TS)

4.8.1 Basic Tabu Search

Tabu search (TS) has been shown to be effective on a wide variety of classical optimization problems, such as graph colouring and travelling salesman problems, and has also been applied to

practical problems such as scheduling and electronic circuit design. The method uses aspiration levels and tabu restrictions which will be discussed below. The search method in the proposed implementation is a steepest descent algorithm. It explores the set of feasible solutions by moving from a solution to its best neighbor (the solution with the best objective function value in a neighborhood), even if this results in a worst objective function value. The short-term memory contains representations of certain numbers of recent moves (formed as a tabu list). These moves are forbidden or declared as tabu restricted for a number of iterations which is known as the tabu duration. As the search progresses the list is updated on a first in-last out basis so that the list remains a fixed size. It is the short-term memory and the notion of tabu restriction that provide the capability to escape local optima. The algorithm is run for a prescribed number of iterations or a prescribed duration of time and then the best solution generated so far is chosen. In this research, a TS heuristic is presented similar to the TS used in McKendall and Li (2017). The different between two will be discussed below.

4.8.2 Components of TS Heuristic for the GQAP

4.8.2.1 Tabu List

After performing one iteration of the TS heuristic, the best admissible move (discussed below) is defined as tabu (tabu restricted) for a certain number of iterations (called tabu length or duration). The tabu list is used to keep track of the tabu moves. The best admissible move is either a non-tabu move that generates the lowest cost feasible solution out of all the possible non-tabu moves at the current iteration or a tabu move that produces the lowest cost feasible solution found thus far, considering all the iterations performed.

If the best feasible solution in the neighborhood of the current solution S_0 is S , and S is obtained by the drop/add move $(u, v; v')$, then this move is tabu restricted for a certain number of iterations, called tabu list size (tabusize). The tabu status and tabu list size of each move are maintained in the tabu list structure $tabu[u][v]$. For example, we can consider the results obtained in the first iteration (see *Table 4.1* and *Table 4.2*). In iteration 1, the best move is the exchange move $(4, 1; 5, 2)$, which yields the best feasible solution $S_0 = (2, 1, 3, 2, 1, 1)$ (with lowest OFV of \$18,050) in the neighborhood of the current solution S_0 . Therefore, the move $(4, 1; 5, 2)$ is tabu until $iter = tabusize + iter$ and update tabu list: $tabu[4][1] = tabu[5][2] = tabusize + iter$. That is, the move, exchanging location 1 assigned to machine 4 with location 2 assigned to machine 5, can be

performed again when $iter > tabusize + iter$ i.e. $iter > tabusize + 1$ since current iteration is iteration 1 ($iter = 1$). If we consider a tabu size of 6, after iteration 1, the tabu list will look like Table 4.3.

Table 4.3 Tabu list after iteration 1

	1	2	3	4	5	6
1				6+1=7		
2					6+1=7	
3						
4						

Table 4.4 Drop/add neighborhood in iteration 2

#	Move	Solution	Cost	#	Move	Solution	Cost
1	(1, 2; 1)	(<u>1</u> , 1, 3, 2, 1, 1)	infeasible	10	(4, 2; 1)	(2, 1, 3, <u>1</u> , 1, 1)	infeasible
2	(1, 2; 3)	(<u>3</u> , 1, 3, 2, 1, 1)	infeasible	11	(4, 2; 3)	(2, 1, 3, <u>3</u> , 1, 1)	infeasible
3	(1, 2; 4)	(<u>4</u> , 1, 3, 2, 1, 1)	\$18,305	12	(4, 2; 4)	(2, 1, 3, <u>4</u> , 1, 1)	infeasible
4	(2, 1; 2)	(2, <u>2</u> , 3, 2, 1, 1)	infeasible	13	(5, 1; 2)	(2, 1, 3, 2, <u>2</u> , 1)	infeasible
5	(2, 1; 3)	(2, <u>3</u> , 3, 2, 1, 1)	infeasible	14	(5, 1; 3)	(2, 1, 3, 2, <u>3</u> , 1)	infeasible
6	(2, 1; 4)	(2, <u>4</u> , 3, 2, 1, 1)	infeasible	15	(5, 1; 4)	(2, 1, 3, 2, <u>4</u> , 1)	infeasible
7	(3, 3; 1)	(2, 1, <u>1</u> , 2, 1, 1)	infeasible	16	(6, 1; 2)	(2, 1, 3, 2, 1, <u>2</u>)	infeasible
8	(3, 3; 2)	(2, 1, <u>2</u> , 2, 1, 1)	infeasible	17	(6, 1; 3)	(2, 1, 3, 2, 1, <u>3</u>)	infeasible
9	(3, 3; 4)	(2, 1, <u>4</u> , 2, 1, 1)	\$17,460	18	(6, 1; 4)	(2, 1, 3, 2, 1, <u>4</u>)	\$19,075

Table 4.5 Pairwise exchange neighborhood in iteration 2

#	Move	Solution	Cost	#	Move	Solution	Cost
1	(1, 2; 2, 1)	(<u>1</u> , <u>2</u> , 3, 2, 1, 1)	infeasible	9	(2, 1; 6, 1)	(2, <u>1</u> , 3, 2, 1, <u>1</u>)	same as S_0
2	(1, 2; 3, 3)	(<u>3</u> , 1, <u>2</u> , 2, 1, 1)	infeasible	10	(3, 3; 4, 2)	(2, 1, <u>2</u> , <u>3</u> , 1, 1)	infeasible
3	(1, 2; 4, 2)	(<u>2</u> , 1, 3, <u>2</u> , 1, 1)	same as S_0	11	(3, 3; 5, 1)	(2, 1, <u>1</u> , 2, <u>3</u> , 1)	infeasible
4	(1, 2; 5, 1)	(<u>1</u> , 1, 3, 2, <u>2</u> , 1)	infeasible	12	(3, 3; 6, 1)	(2, 1, <u>1</u> , 2, 1, <u>3</u>)	infeasible
5	(1, 2; 6, 1)	(<u>1</u> , 1, 3, 2, 1, <u>2</u>)	infeasible	13	(4, 2; 5, 1)	(2, 1, 3, <u>1</u> , <u>2</u> , 1)	\$21,255
6	(2, 1; 3, 3)	(2, <u>3</u> , <u>1</u> , 2, 1, 1)	infeasible	14	(4, 2; 6, 1)	(2, 1, 3, <u>1</u> , 1, <u>2</u>)	infeasible
7	(2, 1; 4, 2)	(2, <u>2</u> , 3, <u>1</u> , 1, 1)	infeasible	15	(5, 1; 6, 1)	(2, 1, 3, 2, <u>1</u> , <u>1</u>)	same as S_0
8	(2, 1; 5, 1)	(2, <u>1</u> , 3, 2, <u>1</u> , 1)	same as S_0				

Similarly, in iteration 2 (see *Tables 4.4* and *4.5* for all possible drop/add and pairwise exchange moves for current solution $S_0 = (2, 1, 3, 2, 1, 1)$), since the best admissible move is the drop/add move $(3, 3; 4)$, $tabu[3][3] = tabusize + iter = tabusize + 2$. So the tabu list will be updated accordingly (see *Table 4.6*).

Table 4.6 Tabu list after iteration 2

	1	2	3	4	5	6
1				6+1=7		
2					6+1=7	
3			6+2=8			
4						

4.8.2.2 Aspiration Criterion

The aspiration criterion is a short-term memory strategy used in the tabu search heuristic to override the tabu restriction. If a move is tabu in the tabu list and produces a solution with a better objective function value compared to that of the best found solution so far, the move is admissible. In other words, the tabu restriction is overridden even though the move is tabu.

4.8.2.3 Stopping Criterion

The stopping criterion for the proposed TS heuristic is computational time. This stopping criterion is used to compare the performances of the proposed TS heuristics with the proposed SA heuristics. Other commonly used stopping criteria are maximum number of iterations performed and maximum number of consecutive iterations without an improvement. The detailed settings of the heuristic parameters are explained in *Chapter 5*.

4.8.3 Pseudo-code for the TS Heuristic for the GQAP

The TS heuristic proposed here is similar to the TS heuristic proposed in McKendall and Li (2017). The proposed TS heuristics consider the drop/add neighborhood structure by itself, and both the drop/add and pairwise exchange neighborhood structures together. However, in McKendall and Li (2017) the authors consider the drop/add and pairwise exchange neighborhood structures together. The pseudo-code for the TS heuristic with the different neighborhoods (i.e., drop/add, pairwise exchange neighborhoods) for the GQAP is given below.

The pseudo-code for the TS heuristic with both drop/add and pairwise exchange neighborhood for the GQAP is given below.

Step 1: Initialize parameters and counters: Set $tabu[u][v] = 0$, for all $u = 1, \dots, M$ and $v = 1, \dots, N$. Set $tabusize$, where $tabusize$ = tabu tenure length. Set $MaxTime$, where $MaxTime$ = maximum computational time for the problem before terminating heuristic. Set $iter = 0$, where $iter$ = iteration number.

Step 2: Obtain an initial solution $S_0 = (S(1), S(2), \dots, S(M))$ using the construction algorithm presented in *Section 4.3*. Obtain $TC(S_0)$ using objective function.

Step 3: Set $S^* = S_0$, where S^* is the best solution found thus far, and set $TC(S^*) = TC(S_0)$;

Step 4: Set $iter = iter + 1$; Find the best admissible move, which gives best feasible solution (S) in neighborhood S_0 . Find best feasible solution (S) by considering all possible drop/add and pairwise exchange operations on S_0 .

Step 5: Set $S_0 = S$ and $TC(S_0) = TC(S)$; If $TC(S_0) < TC(S^*)$, set $S^* = S_0$ and $TC(S^*) = TC(S_0)$.

Step 6: Update tabu list as either $tabu[u][v] = iter + tabusize$, if drop/add operation ($u, v; v'$) gives best admissible move or $tabu[u][v] = tabu[u'][v'] = iter + tabusize$, if pairwise exchange operation ($u, v; u', v'$) gives best admissible move.

Step 7: Stopping criterion: If $iter < MaxTime$, go to Step 4. Else, terminate the heuristic and return solution S^* and total cost of solution $TC(S^*)$.

Note: For the TS heuristic with only drop/add neighborhood, consider all possible drop/add moves only in step 4.

4.9 Comparison of Proposed Meta-heuristics

In TS heuristic the tabu list provides a short-term memory and it is a very deterministic heuristic. That is, the best admissible move is selected at each iteration. To diversify the search, the best admissible move in, say, the top 10-25% can be selected at each iteration to add randomness to the heuristic. This is called the probabilistic TS heuristic. Although this heuristic exists in the literature, it is not considered in this thesis. On the other hand, SA is very stochastic in nature (i.e.,

at each iteration, a move is randomly selected). As a result, an extremely deterministic heuristic (i.e., TS) and a very stochastic heuristic (i.e., SA) is developed for the proposed GQAP.

After solving problems of different types and sizes, the results obtained from the two diverse heuristics are compared. To compare the performance of the heuristics, the objective function value (total cost) as well as time taken to solve the problems should be considered. The comparisons are discussed in *Section 5.4*.

CHAPTER 5

COMPUTATIONAL RESULTS

5.1 Introduction

The proposed simulated annealing (SA) and tabu search (TS) heuristics were tested on a set of 21 test problems given by Cordeau *et. al.* (2006). This problem set was also used in other literature for the GQAP (Mateus *et. al.*, 2011 and McKendall and Li, 2017). The set of test problems contains problem instances with 20, 30, 35, 40, and 50 machines with locations ranging between 6 and 20 such that the percentages of location area required (i.e. tightness of the capacity constraint) ranges between 35 and 95. As mentioned previously, the proposed SA and TS heuristics used one of the construction algorithms presented by McKendall and Li (2017) to generate the initial solutions (see *Section 4.3*).

5.2 Computational Environment

The proposed SA and TS heuristics were programmed using MATLAB and executed on a 3.30 GHz Intel Core i5 CPU with 16 GB of RAM under the Microsoft Windows Operating System.

5.3 Setting Parameters

5.3.1 Parameters in Simulated Annealing (SA) Heuristics

Preliminary analyses are performed to determine the SA heuristic parameter settings.

1. Initial Temperature, $T(0)$

McKendall *et. al.* (2006) accepted a non-improving neighboring solution (with 25% probability) with a cost of 10% above the cost of the initial solution to set the initial temperature for a SA heuristic for a dynamic facility layout problem. Privault *et. al.* (1998) accepted 80% of the candidate solutions (i.e., non-improving feasible or infeasible neighboring solutions) for a type of assignment problem. Note, in this research, only feasible solutions are candidate solutions.

In a preliminary experiment, three problems from the data set with different problem sizes and different tightness of the capacity constraint are chosen to evaluate the performance of the SA heuristics with drop/add neighborhood structure. The probability of accepting non-improving candidate solutions are set at 80% and 90%. The other parameters were fixed for both cases. That is, the number of iterations at each temperature $N(t)$ is defined to be 10% of the size of the

neighborhood, and the cooling parameter α is set to 0.99. See the results of this preliminary study in Table 5.1 below. The 90% case (i.e., 90% probability of accepting non-improving candidate solutions) gave better results. Therefore, this is used in the later experiments.

Table 5.1: Setting Initial Temperature for SA Heuristics

Probability of accepting candidate solutions	Problem Instances	Best solution in literature	Trial 1	Trial 2	Trial 3	Average		Best	
						Cost	% Deviation	Cost	% Deviation
80%	20-15-35	1471896	1718733	1781154	1614116	1704668	15.81%	1614116	9.66%
	35-15-55	4639128	4913912	5486242	5456673	5285609	13.94%	4913912	5.92%
	50-10-95	12845598	18171288	20514591	19860331	19515403	51.92%	18171288	41.46%
90%	20-15-35	1471896	1521604	1506076	1535928	1521203	3.35%	1506076	2.32%
	35-15-55	4639128	4819067	4944205	4818008	4860427	4.77%	4818008	3.86%
	50-10-95	12845598	18052809	17049092	17380906	17494269	36.19%	17049092	32.72%

2. Minimum Allowable Temperature, T_{min}

Minimum allowable temperature is used as the stopping criterion in the proposed SA heuristics. The minimum allowable temperature, T_{min} is set to 0.01, as in McKendall *et. al*, 2006.

3. Cooling Ratio, α

In the SA heuristics literature, the value of the cooling ratio α is set between 0.9-0.998. In the experiments below, this value is set to 0.99.

4. Number of iterations at each temperature, $N(t)$

Using the parameter settings mentioned above, a preliminary experiment was performed to see how the value of $N(t)$ affects the results. The values of $N(t)$ used are 10%, 20%, 30%, 40% and 50% of the size of the neighborhood. The results are shown in *Tables 5.2-5.6*. The results equal to or better than the best found solutions in literature are given in bold font.

Table 5.2: Results with $N(t) = 10\%$ of the size of the neighborhood

Problem Instances	Best solution in literature	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Best Results	% Deviation
20-15-35	1471896	1562334	1666277	1654817	1683947	1633818	1506076	1506076	2.32%
20-15-55	1723638	1774373	1760779	1746373	1765301	1768575	1775879	1746373	1.32%
20-15-75	1967315	2164399	2082325	2197493	2271365	2110196	2097599	2082325	5.85%
30-06-95	5160920	7343965	7396391	7282153	7231103	7261463	7338332	7231103	40.11%
30-07-75	4383923	4866487	4925550	5261271	5162753	5043916	4876711	4866487	11.01%
30-08-55	3501695	3604426	3583662	3640803	3687505	3501695	3859019	3501695	0.00%
30-10-65	3620959	4061961	4015158	3983075	4046989	4317204	4030877	3983075	10.00%
30-20-35	3633644	3470425	3668551	3786653	3712153	3806562	3824489	3470425	-4.49%
30-20-55	3599602	3769255	3811693	3867163	3852868	3626272	3847910	3626272	0.74%
30-20-75	4050938	4186411	4428979	4275888	4050938	4342809	4403353	4050938	0.00%
30-20-95	5726530	7980938	7780961	8055889	7942314	7909580	7780961	7780961	35.88%
35-15-35	4456670	4550406	4560935	4637488	4602290	4756974	4630814	4550406	2.10%
35-15-55	4639128	4935291	4780621	5148577	5005799	4969027	5049512	4780621	3.05%
35-15-75	6301723	6676525	6623197	6543257	6571093	6589578	6606436	6543257	3.83%
35-15-95	6670264	11073897	10502375	10888223	10527428	11600152	11534541	10502375	57.45%
40-07-75	7405793	7853993	7533579	7481964	7675760	7476115	7570414	7476115	0.95%
40-09-95	7667719	9824366	10769720	10357320	10763954	10556437	11343446	9824366	28.13%
40-10-65	7265559	7312813	7265559	7737908	7265559	7390261	7325468	7265559	0.00%
50-10-65	10513029	10716593	10768262	10641717	10624654	10718070	10942884	10624654	1.06%
50-10-75	11217503	12175256	12398118	11902391	12195717	12164193	11939694	11902391	6.11%
50-10-95	12845598	17769538	19658267	15751197	18582569	18149669	17617411	15751197	22.62%

Table 5.3: Results with $N(t) = 20\%$ of the size of the neighborhood

Problem Instances	Best solution in literature	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Best Results	% Deviation
20-15-35	1471896	1588987	1548769	1562145	1601312	1525718	1521604	1521604	3.38%
20-15-55	1723638	1774373	1764158	1752972	1817353	1782379	1723638	1723638	0.00%
20-15-75	1967315	2041918	2126257	2115180	2057468	2213512	2120431	2041918	3.79%
30-06-95	5160920	7247483	7160004	7412682	7475173	7395715	7497761	7160004	38.74%
30-07-75	4383923	4677043	5347809	5079456	4678065	5166234	4594686	4594686	4.81%
30-08-55	3501695	3532566	3501695	3584958	3532566	3599746	3638030	3501695	0.00%
30-10-65	3620959	3943414	3941497	4006612	3777807	3880017	3976571	3777807	4.33%
30-20-35	3633644	3460504	3731187	3813073	3830739	3742375	3724273	3460504	-4.76%
30-20-55	3599602	3726465	3819853	3622258	3763267	3770997	3746221	3622258	0.63%
30-20-75	4050938	4239695	4233113	4301731	4452933	4210272	4440242	4210272	3.93%
30-20-95	5726530	8034139	7845030	7919961	7606477	8080334	8010218	7606477	32.83%
35-15-35	4456670	4620810	4604970	4538589	4576827	4694835	4544283	4538589	1.84%
35-15-55	4639128	4819067	4876326	4818008	5016659	4954007	4944205	4818008	3.86%
35-15-75	6301723	6585733	6442490	6445856	6392312	6413066	6451959	6392312	1.44%
35-15-95	6670264	10626593	10627451	11146052	10525688	10968883	10937121	10525688	57.80%
40-07-75	7405793	7504465	7664909	7595950	7570121	7559801	7709946	7504465	1.33%
40-09-95	7667719	11306285	9685784	10502930	10540251	10942780	10807231	9685784	26.32%
40-10-65	7265559	7321819	7298103	7385528	7288263	7302026	7360432	7288263	0.31%
50-10-65	10513029	10779442	10637068	10749210	10704071	11054040	10596584	10596584	0.79%
50-10-75	11217503	12012667	12284677	11821395	11752506	12055603	12605241	11752506	4.77%
50-10-95	12845598	17439024	17059700	17530023	18013567	17043366	17355431	17043366	32.68%

Table 5.4: Results with $N(t) = 30\%$ of the size of the neighborhood

Problem Instances	Best solution in literature	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Best Results	% Deviation
20-15-35	1471896	1559407	1535928	1584946	1519070	1556821	1511402	1511402	2.68%
20-15-55	1723638	1738832	1738832	1761175	1839191	1775879	1751669	1738832	0.88%
20-15-75	1967315	2318448	2391599	2153335	2015529	2058889	2179943	2015529	2.45%
30-06-95	5160920	7409304	7178809	7340597	7191420	7250003	7104924	7104924	37.67%
30-07-75	4383923	5283280	4570820	5047180	4732311	4402834	4718853	4402834	0.43%
30-08-55	3501695	3532566	3501695	3501695	3585428	3622115	3501695	3501695	0.00%
30-10-65	3620959	4126158	4028574	3959570	3693834	3785624	3847868	3693834	2.01%
30-20-35	3633644	3710782	3489490	3724920	3845483	3380252	3791180	3380252	-6.97%
30-20-55	3599602	3679311	3717663	3672132	3664287	3786701	3794916	3664287	1.80%
30-20-75	4050938	4407066	4331100	4263142	4152522	4238119	4336481	4152522	2.51%
30-20-95	5726530	7780929	8300102	8114864	7999346	8071604	7908528	7780929	35.88%
35-15-35	4456670	4568304	4623489	4578428	4475921	4665918	4551509	4475921	0.43%
35-15-55	4639128	4833868	4765403	4817371	4833912	4843301	4843082	4765403	2.72%
35-15-75	6301723	6397501	6496238	6349183	6380077	6390791	6350896	6349183	0.75%
35-15-95	6670264	10611932	11166777	11392651	10630882	10786901	10603120	10603120	58.96%
40-07-75	7405793	7581276	7508405	7608138	7530671	7599048	7592428	7508405	1.39%
40-09-95	7667719	11210421	10736737	10429446	9355239	10624813	11254917	9355239	22.01%
40-10-65	7265559	7321819	7344992	7286973	7405811	7291789	7286973	7286973	0.29%
50-10-65	10513029	10696816	10513029	10637184	10656900	10654528	10715283	10513029	0.00%
50-10-75	11217503	12177476	12018025	12439906	12413982	12042092	11967764	11967764	6.69%
50-10-95	12845598	18274578	18422922	16217825	19025566	18052809	16129920	16129920	25.57%

Table 5.5: Results with $N(t) = 40\%$ of the size of the neighborhood

Problem Instances	Best solution in literature	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Best Results	% Deviation
20-15-35	1471896	1559884	1556917	1643634	1528175	1600962	1550655	1528175	3.82%
20-15-55	1723638	1748942	1794024	1768575	1760779	1765301	1757978	1748942	1.47%
20-15-75	1967315	2083803	2063520	2134400	2028103	2514987	2108292	2028103	3.09%
30-06-95	5160920	7151151	7104924	7102726	7432501	7286580	7263728	7102726	37.63%
30-07-75	4383923	4587834	4657508	4652654	4676684	4782071	4641935	4587834	4.65%
30-08-55	3501695	3591805	3532566	3501695	3552573	3501695	3532566	3501695	0.00%
30-10-65	3620959	3891258	3863265	3893366	3840977	3935145	3871526	3840977	6.08%
30-20-35	3633644	3530892	3781576	3730253	3765343	3737191	3739933	3530892	-2.83%
30-20-55	3599602	3764621	4389471	3930419	3862648	3745105	3754261	3745105	4.04%
30-20-75	4050938	4133686	4319506	4504684	4407599	4130034	4255918	4130034	1.95%
30-20-95	5726530	8251191	8055889	7871059	8069134	7909580	7573947	7573947	32.26%
35-15-35	4456670	4498330	4549263	4511450	4635500	4475921	4584288	4475921	0.43%
35-15-55	4639128	4854988	4848034	4749599	4845739	4959720	4738547	4738547	2.14%
35-15-75	6301723	6392902	6464192	6559522	6318744	6403600	6452743	6318744	0.27%
35-15-95	6670264	10861275	10499851	11314476	10488040	10298816	10504337	10298816	54.40%
40-07-75	7405793	7663150	7545165	7639618	7629281	7537946	7473559	7473559	0.92%
40-09-95	7667719	10821908	9755471	10391745	9882145	11111991	10012516	9755471	27.23%
40-10-65	7265559	7321429	7265559	7298103	7325376	7265559	7308532	7265559	0.00%
50-10-65	10513029	10672779	10674227	10614690	10744411	10513029	10609765	10513029	0.00%
50-10-75	11217503	12149323	11861248	12262942	11870200	11626712	12423208	11626712	3.65%
50-10-95	12845598	17584924	17049092	16681197	16569118	17014096	17380906	16569118	28.99%

Table 5.6: Results with $N(t) = 50\%$ of the size of the neighborhood

Problem Instances	Best solution in literature	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Best Results	% Deviation
20-15-35	1471896	1558679	1540827	1688644	1577173	1506076	1568709	1506076	2.32%
20-15-55	1723638	1748802	1750956	1749239	1723638	1723638	1723638	1723638	0.00%
20-15-75	1967315	2512615	2136576	2069258	2037924	2046022	2089578	2037924	3.59%
30-06-95	5160920	7115392	7380273	7039637	7473350	6737629	7271187	6737629	30.55%
30-07-75	4383923	4721240	4712002	5193019	4792308	4645271	4918105	4645271	5.96%
30-08-55	3501695	3532566	3717046	3574190	3501695	3501695	3585269	3501695	0.00%
30-10-65	3620959	4132036	3938842	3675880	4061888	3980126	3832514	3675880	1.52%
30-20-35	3633644	3735999	3744394	3697497	3507855	3775495	3379359	3379359	-7.00%
30-20-55	3599602	3681638	3739926	3815328	3794765	3748363	3726853	3681638	2.28%
30-20-75	4050938	4206006	4275035	4344092	4208611	4356155	4445755	4206006	3.83%
30-20-95	5726530	7909580	7780961	8060757	7703063	7854754	7919961	7703063	34.52%
35-15-35	4456670	4518355	4488838	4514053	4494650	4540174	4466588	4466588	0.22%
35-15-55	4639128	4708530	4840727	4657037	4860235	4895690	4863835	4657037	0.39%
35-15-75	6301723	6487853	6458198	6316674	6475040	6301723	6401514	6301723	0.00%
35-15-95	6670264	10995700	10152100	10937121	10657443	11124050	10600533	10152100	52.20%
40-07-75	7405793	7624973	7592188	7481964	7508644	7456976	7520676	7456976	0.69%
40-09-95	7667719	10624137	11771971	11016515	10954067	10891649	10275821	10275821	34.01%
40-10-65	7265559	7308514	7288263	7265559	7305381	7378020	7325376	7265559	0.00%
50-10-65	10513029	10614690	10617874	10614494	10513029	10617874	10616746	10513029	0.00%
50-10-75	11217503	12115902	12302901	12065104	12139666	11832519	12469026	11832519	5.48%
50-10-95	12845598	17997798	16139493	17915442	17107898	17621964	16884109	16139493	25.64%

The percent deviations obtained in the five tables above are shown in a bar chart. See Table 5.7 below. Notice that $N(t) = 50\%$ of the size of the neighborhood performed better for most of the

problems. For example, consider the problem instance 35-15-75. The percent deviations are 3.83%, 1.44%, 0.75%, 0.27%, and 0.00% where $N(t) = 10\%$, 20%, 30%, 40% and 50% of the size of the neighborhood, respectively. But the percent deviations for the problems with high space capacity utilization are still very high. For example, for the problem instance 35-15-95, the percent deviations are ranged from 52.20% to 58.96%, which are very high values.

Table 5.7: Bar Chart with Percent Deviation for Different values of $N(t)$

The value of the cooling ratio, α is now increased to 0.9999 and the minimum allowable temperature, T_{min} is lowered to 0.0001 so that we can explore more solutions and chance of getting better solutions increases. All parameters used for the proposed SA heuristics are summarized below.

Initial temperature, $T(0)$: the probability of accepting a non-improving neighboring solution with a cost of 10% above the cost of the initial solution is 90%. See section 4.7.3.1 for the formula and explanation for calculating $T(0)$ where here $x = 10$.

Cooling ratio, $\alpha = 0.9999$

Number of iterations at each temperature $N(t) = 50\%$ of the size of the neighborhood

Minimum allowable temperature, $T_{min} = 0.0001$

These parameter settings are used to test the performances of the proposed SA heuristics for both neighborhood structures (i.e., drop/add and pairwise exchange). The results are presented and discussed below in *Section 5.4.1*.

5.3.2 Parameters in Tabu Search (TS) Heuristics

1. Tabu size or tabu length

For the proposed TS heuristics with both drop/add and pairwise exchange neighborhood structures, the tabu length (*tabusize*) is set to the exact values used in McKendall and Li (2017).

McKendall and Li (2017) obtained the best feasible solutions for the GQAP while using the tabu length by multiplying the size of the neighborhood by 0.005 to 0.02. Smaller values of *tabusize* tend to result in poor solutions, since the TS heuristic either spends too much time repeating the same moves, or may get trapped at a local optima (i.e. cycling). On the other hand, using a large value for *tabusize* results in restricting the solution space too much whereby many potentially good moves may be overlooked, resulting in poor solutions. The proposed TS heuristics with only drop/add neighborhood structure used *tabusize* equal to 0.02 and 0.03 times the size of the neighborhood.

2. Computational time

The stopping criterion for the proposed TS heuristic is computational time. The maximum computational time (see *Table 5.8* and *5.10*) for each problem from SA heuristic is set as the computational time for the respective problems.

These parameter settings are used to test the performances of the proposed TS heuristics. The results are presented and discussed below, in *Section 5.4.2*.

5.4 Results

As stated previously, the proposed TS and SA heuristics used the dataset from Cordeau *et. al.* (2006) to test the performances of the proposed heuristics. The results are presented below.

5.4.1 Simulated Annealing (SA) Heuristics

The proposed SA heuristics was implemented using the parameter settings mentioned in *Section 5.3.1*. The results of the SA heuristics with drop/add neighborhood structure is given in *Table 5.8*.

Table 5.8: Results from SA Heuristics with Drop/Add Neighborhood

Problem Instances	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average		Best		Average Runtime (seconds)	Maximum Runtime (seconds)
						Cost	% Deviation	Cost	% Deviation		
20-15-35	1506076	1539678	1509123	1511402	1511402	1515536.2	2.96%	1506076	2.32%	3831.8	4194.7
20-15-55	1723638	1723638	1723638	1723638	1723638	1723638	0.00%	1723638	0.00%	4092.7	4457.5
20-15-75	1987133	1988689	1988689	1976791	1989271	1986114.6	0.96%	1976791	0.48%	4266.9	4565.4
30-06-95	6416228	6219342	6476590	6513339	6416329	6408365.6	24.17%	6219342	20.51%	1154.6	1212.5
30-07-75	4540060	4640070	4600906	4595501	4400593	4555426	3.91%	4400593	0.38%	2047.1	2287.2
30-08-55	3501695	3501695	3501695	3501695	3501695	3501695	0.00%	3501695	0.00%	2707.6	2962.6
30-10-65	3663198	3746644	3736251	3640684	3713822	3700119.8	2.19%	3640684	0.54%	3557.4	3767.8
30-20-35	3510015	3380252	3451254	3380252	3468225	3437999.6	-5.38%	3380252	-6.97%	8924.0	9672.0
30-20-55	3593105	3626509	3622258	3649512	3660919	3630460.6	0.86%	3593105	-0.18%	9414.6	10907.9
30-20-75	4050938	4116011	4215750	4115879	4107614	4121238.4	1.74%	4050938	0.00%	10405.2	11363.0
30-20-95	7573110	7575493	7573110	7575493	7573110	7574063.2	32.26%	7573110	32.25%	16476.4	17004.0
35-15-35	4456670	4457348	4459912	4464070	4456670	4458934	0.05%	4456670	0.00%	7024.2	7714.4
35-15-55	4713054	4722172	4673657	4639128	4670370	4683676.2	0.96%	4639128	0.00%	7436.5	7974.1
35-15-75	6319838	6319838	6319838	6301723	6332985	6318844.4	0.27%	6301723	0.00%	7846.6	8936.8
35-15-95	9089146	9917835	9089146	10108923	10000498	9641109.6	44.54%	9089146	36.26%	8632.8	10332.0
40-07-75	7466532	7458588	7478567	7469028	7428176	7460178.2	0.73%	7428176	0.30%	2797.8	2956.6
40-09-95	9189719	9619006	9985473	9499848	9546826	9568174.4	24.79%	9189719	19.85%	3658.7	3915.8
40-10-65	7265559	7286973	7274192	7285016	7265559	7275459.8	0.14%	7265559	0.00%	4976.9	5214.0
50-10-65	10513029	10513029	10513029	10513029	10513029	10513029	0.00%	10513029	0.00%	6312.7	6713.1
50-10-75	11967764	11935565	12096401	11482321	11637986	11824007	5.41%	11482321	2.36%	6032.5	6340.1
50-10-95	14278379	15335045	14964326	14115461	14455373	14629717	13.89%	14115461	9.89%	3831.8	6058.7

Table 5.9 gives a bar chart of the results presented in Tables 5.7 & 5.8 (i.e., with a new column which represents the percent deviations with the new parameter settings (see “Final Results”). For example, consider problem instance 30-10-65. With the previous parameter setting the best result was 1.52% deviation (see Table 5.6). With the new parameter setting 0.54% deviation (see Table 5.8) is obtained. Consider another problem instance 35-15-55. With the previous parameter setting the best result was 0.39% deviation (see Table 5.6). With the new parameter setting 0.0% deviation is obtained *i.e.* the solution is equal to the best solution found in literature. Also, notice that the deviations for the problems with high space capacity utilization have decreased and it produced better results for other problems too. For example, the problem instance 50-10-95 is a problem with high space capacity utilization of 95. With the previous parameter setting the best result was 22.62% deviation (see Table 5.2). With the new parameter setting 9.89% deviation (see Table 5.8) is obtained.

Table 5.9: Bar Chart with Percent Deviations

The same values of the parameter settings are used for the SA with both neighborhood structures. The results are shown in *Table 5.10*. Results for three (3) test problems are better than the results presented in literature. The three problem instances are 20-15-75, 30-20-35 and 30-20-55. The percent reductions are -0.72%, -7.00%, and -0.18%, respectively (*i.e.*, the costs for these problems are 0.72%, 7.00%, and 0.18% less than the best found solution for the above mentioned problems). Also, 14 results are exactly equal to the best results found in literature. The other 4 results are worse than the best found results, but the percent deviations for these problems are less than 1% (*i.e.*, the costs are less than 1% above the best found results).

Table 5.10: Results from SA Heuristics with Both Neighborhoods

Problem Instances	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average		Best		Average Runtime (seconds)	Maximum Runtime (seconds)
						Cost	% Deviation	Cost	% Deviation		
20-15-35	1471896	1471896	1471896	1471896	1471896	1471896	0.00%	1471896	0.00%	4161.6	4289.1
20-15-55	1723638	1723638	1723638	1723638	1723638	1723638	0.00%	1723638	0.00%	4470.4	4680.3
20-15-75	1953188	1953188	1953188	1953188	1953188	1953188	-0.72%	1953188	-0.72%	4494.3	4595.7
30-06-95	5160920	5160920	5160920	5160920	5160920	5160920	0.00%	5160920	0.00%	1576.3	1597.8
30-07-75	4383923	4383923	4383923	4383923	4383923	4383923	0.00%	4383923	0.00%	2238.1	2406.7
30-08-55	3501695	3501695	3501695	3501695	3501695	3501695	0.00%	3501695	0.00%	2771.6	2969.4
30-10-65	3654908	3620959	3620959	3620959	3620959	3627748.8	0.19%	3620959	0.00%	3758.9	3957.7
30-20-35	3379359	3379359	3379359	3379359	3379359	3379359	-7.00%	3379359	-7.00%	9835.7	10190.0
30-20-55	3593105	3593105	3593105	3593105	3593105	3593105	-0.18%	3593105	-0.18%	10460.1	10910.0
30-20-75	4050938	4050938	4050938	4104358	4104358	4072306	0.53%	4050938	0.00%	11526.5	12212.5
30-20-95	5759808	5961222	5804873	5961222	5759808	5849386.6	2.15%	5759808	0.58%	16138.3	16631.2
35-15-35	4457348	4457348	4456670	4457348	4457348	4457212.4	0.01%	4456670	0.00%	7720.9	7963.4
35-15-55	4639128	4639128	4639128	4639128	4639128	4639128	0.00%	4639128	0.00%	8053.0	8371.3
35-15-75	6319838	6316674	6301723	6316674	6316674	6314316.6	0.20%	6301723	0.00%	9176.7	11094.6
35-15-95	6811310	6777590	6713937	6751402	6751402	6761128.2	1.36%	6713937	0.65%	10005.3	11005.5
40-07-75	7405793	7432555	7405793	7432555	7432555	7421850.2	0.22%	7405793	0.00%	3075.8	3180.7
40-09-95	7732515	7783476	7769026	7773594	7773594	7766441	1.29%	7732515	0.85%	4274.2	4625.3
40-10-65	7265559	7265559	7265559	7265559	7265559	7265559	0.00%	7265559	0.00%	5520.4	5765.2
50-10-65	10513029	10513029	10513029	10513029	10513029	10513029	0.00%	10513029	0.00%	7105.5	7296.1
50-10-75	11251072	11275061	11259251	11275061	11251072	11262303.4	0.40%	11251072	0.30%	6774.2	6928.5
50-10-95	12864406	12845598	12845598	12845598	12845598	12849359.6	0.03%	12845598	0.00%	6607.8	6921.1

5.4.2 Tabu Search (TS) Heuristics

Tabu Search (TS) heuristics with drop/add neighborhood structure is considered with two different set of tabu sizes (mentioned in *Section 5.3.2*). *Tables 5.11a and 5.11b* present the results with the *tabu size* equal to the size of the neighborhood (*NBHD*) times 0.02 and 0.03 ($0.02NBHD$ and $0.03NBHD$), respectively. The computation time is equal to the maximum runtime we obtained in SA heuristics with drop/add neighborhood structure. See *Table 5.8* for computation times.

Table 5.11a: Results from TS Heuristics with Drop/Add Neighborhood (tabu size = $0.02NBHD$)

Problem Instances	Best solution in literature	Cost	% Deviation	Tabu size	Runtime (seconds)
20-15-35	1471896	1555456	5.68%	6	4194.7
20-15-55	1723638	1761175	2.18%	6	4457.5
20-15-75	1967315	2635767	33.98%	6	4565.4
30-06-95	5160920	7311111	41.66%	3	1212.5
30-07-75	4383923	4964696	13.25%	4	2287.2
30-08-55	3501695	3628917	3.63%	4	2962.6
30-10-65	3620959	4264490	17.77%	5	3767.8
30-20-35	3633644	3753614	3.30%	11	9672.0
30-20-55	3599602	4067925	13.01%	11	10907.9
30-20-75	4050938	4759033	17.48%	11	11363.0
30-20-95	5726530	8197698	43.15%	11	17004.0
35-15-35	4456670	4710133	5.69%	10	7714.4
35-15-55	4639128	5046883	8.79%	10	7974.1
35-15-75	6301723	6772916	7.48%	10	8936.8
35-15-95	6670264	10638463	59.49%	10	10332.0
40-07-75	7405793	8930083	20.58%	5	2956.6
40-09-95	7667719	11240530	46.60%	6	3915.8
40-10-65	7265559	7343987	1.08%	7	5214.0
50-10-65	10513029	11025649	4.88%	9	6713.1
50-10-75	11217503	13209132	17.75%	9	6340.1
50-10-95	12845598	15962204	24.26%	9	6058.7

Table 5.11b: Results from TS Heuristics with Drop/Add Neighborhood (tabusize = 0.03NBHD)

Problem Instances	Best solution in literature	Cost	% Deviation	Tabu size	Runtime (seconds)
20-15-35	1471896	1555456	5.68%	8	4194.7
20-15-55	1723638	1761175	2.18%	8	4457.5
20-15-75	1967315	2635767	33.98%	8	4565.4
30-06-95	5160920	7164409	38.82%	5	1212.5
30-07-75	4383923	4964696	13.25%	5	2287.2
30-08-55	3501695	3585428	2.39%	6	2962.6
30-10-65	3620959	4264490	17.77%	8	3767.8
30-20-35	3633644	3768876	3.72%	17	9672.0
30-20-55	3599602	3834865	6.54%	17	10907.9
30-20-75	4050938	4756691	17.42%	17	11363.0
30-20-95	5726530	8197698	43.15%	17	17004.0
35-15-35	4456670	4710133	5.69%	15	7714.4
35-15-55	4639128	5046883	8.79%	15	7974.1
35-15-75	6301723	6772916	7.48%	15	8936.8
35-15-95	6670264	10638463	59.49%	15	10332.0
40-07-75	7405793	8250469	11.41%	7	2956.6
40-09-95	7667719	11240530	46.60%	10	3915.8
40-10-65	7265559	7343987	1.08%	11	5214.0
50-10-65	10513029	10921687	3.89%	14	6713.1
50-10-75	11217503	13136023	17.10%	14	6340.1
50-10-95	12845598	15962204	24.26%	14	6058.7

If we compare the solutions obtained from these two different tabu sizes, we can see *Table 5.11b* produced either same or better results compared to *Table 5.11a*. It can be seen more clearly in the bar chart presented in *Table 5.12*.

Table 5.12: Results Obtained from TS Heuristics with Drop/Add where tabusize = 0.02NBHD or 0.03NBHD

Results obtained from the TS heuristics with both drop/add and pairwise exchange neighborhood structures are given in Table 5.13. It obtained two results equal to the best found solution in literature. The computation time is equal to the maximum runtime we obtained in SA heuristics with both neighborhood structures. See Table 5.10 for computation times.

Table 5.13: Results from TS Heuristics with Both Neighborhoods

Problem Instances	Best solution in literature	Cost	% Deviation	Tabu size	Runtime (seconds)
20-15-35	1471896	1500497	1.94%	8	4289.1
20-15-55	1723638	1723638	0.00%	8	4680.3
20-15-75	1967315	2155374	9.56%	7	4595.7
30-06-95	5160920	5222897	1.20%	7	1597.8
30-07-75	4383923	4519174	3.09%	7	2406.7
30-08-55	3501695	3578548	2.19%	7	2969.4
30-10-65	3620959	3867245	6.80%	11	3957.7
30-20-35	3633644	3853792	6.06%	12	10190.0
30-20-55	3599602	3798453	5.52%	12	10910.0
30-20-75	4050938	4417311	9.04%	11	12212.5
30-20-95	5726530	6436452	12.40%	12	16631.2
35-15-35	4456670	4543691	1.95%	12	7963.4
35-15-55	4639128	4793722	3.33%	10	8371.3
35-15-75	6301723	6432304	2.07%	11	11094.6
35-15-95	6670264	7387460	10.75%	12	11005.5
40-07-75	7405793	7405793	0.00%	11	3180.7
40-09-95	7667719	8238651	7.45%	8	4625.3
40-10-65	7265559	7308532	0.59%	10	5765.2
50-10-65	10513029	10588213	0.72%	10	7296.1
50-10-75	11217503	11783796	5.05%	8	6928.5
50-10-95	12845598	13922455	8.38%	10	6921.1

5.4 Comparison of Performances

The results obtained from the proposed heuristics are summarized in *Table 5.14*. Consider the bolded results (best solutions found), notice that the SA heuristic with both drop/add and pairwise exchange neighborhood structures performed better compared to the other ones.

Overall, when comparing the objective function values (or Cost), the proposed SA heuristics performed better than the proposed TS heuristics (see *Table 5.15*). Both TS and SA heuristics with only drop/add neighborhood performed poorly for the problems with high space capacity utilization. The percent deviations are very high for these problems (see *Table 5.15*). For example, consider the problem instance 30-06-95. The percent deviations while using both drop/add and pairwise exchange neighborhood are 0.00% (SA) and 1.20% (TS) whereas these deviations while using only drop/add neighborhood are 20.51% (SA) and 38.82% (TS).

Table 5.14: Summary of the Results

Problem Instances	SA (Drop/Add)		SA (Both Neighborhood)		TS (Drop/Add)		TS (Both Neighborhood)	
	Cost	% Deviation	Cost	% Deviation	Cost	% Deviation	Cost	% Deviation
20-15-35	1506076	2.32%	1471896	0.00%	1555456	5.68%	1500497	1.94%
20-15-55	1723638	0.00%	1723638	0.00%	1761175	2.18%	1723638	0.00%
20-15-75	1976791	0.48%	1953188	-0.72%	2635767	33.98%	2155374	9.56%
30-06-95	6219342	20.51%	5160920	0.00%	7164409	38.82%	5222897	1.20%
30-07-75	4400593	0.38%	4383923	0.00%	4964696	13.25%	4519174	3.09%
30-08-55	3501695	0.00%	3501695	0.00%	3585428	2.39%	3578548	2.19%
30-10-65	3640684	0.54%	3620959	0.00%	4264490	17.77%	3867245	6.80%
30-20-35	3380252	-6.97%	3379359	-7.00%	3768876	3.72%	3853792	6.06%
30-20-55	3593105	-0.18%	3593105	-0.18%	3834865	6.54%	3798453	5.52%
30-20-75	4050938	0.00%	4050938	0.00%	4756691	17.42%	4417311	9.04%
30-20-95	7573110	32.25%	5759808	0.58%	8197698	43.15%	6436452	12.40%
35-15-35	4456670	0.00%	4456670	0.00%	4710133	5.69%	4543691	1.95%
35-15-55	4639128	0.00%	4639128	0.00%	5046883	8.79%	4793722	3.33%
35-15-75	6301723	0.00%	6301723	0.00%	6772916	7.48%	6432304	2.07%
35-15-95	9089146	36.26%	6713937	0.65%	10638463	59.49%	7387460	10.75%
40-07-75	7428176	0.30%	7405793	0.00%	8250469	11.41%	7405793	0.00%
40-09-95	9189719	19.85%	7732515	0.85%	11240530	46.60%	8238651	7.45%
40-10-65	7265559	0.00%	7265559	0.00%	7343987	1.08%	7308532	0.59%
50-10-65	10513029	0.00%	10513029	0.00%	10921687	3.89%	10588213	0.72%
50-10-75	11482321	2.36%	11251072	0.30%	13136023	17.10%	11783796	5.05%
50-10-95	14115461	9.89%	12845598	0.00%	15962204	24.26%	13922455	8.38%

Table 5.15: Comparison of Results for the Proposed Heuristics

Table 5.16: Comparison of Percent Deviations for the Proposed Heuristics

In summary, the proposed SA heuristic with both drop/add and pairwise exchange neighborhood structures outperformed the other proposed heuristics with respect to solution quality. However, the SA heuristics were run five (5) times and the maximum computation time among the five runs were used as a stopping criterion for the TS heuristic. As a result, SA heuristics were run for roughly five times the computation time of the TS heuristics. It is believed that the TS heuristics did not perform well because of it being completely deterministic. Unlike in McKendall and Li (2017) the authors added diversification to their TS heuristic by generating a set of diverse initial solutions.

CHAPTER 6

CONCLUSION

6.1 Summary of Research

In this research, the generalized quadratic assignment problem (GQAP) is considered and a simulated annealing (SA) and a tabu search (TS) heuristic are presented to solve the GQAP. Two types of neighborhood structures are considered for each heuristic. Either a drop/add or drop/add with pairwise exchange neighborhood structure is used.

The proposed meta-heuristics are tested on a set of 21 test problems given by Cordeau *et. al.* (2006). The proposed SA heuristics performed better than the proposed TS heuristics. The SA heuristics obtained results better than those presented in the literature for three of the test problems. On the other hand, the TS heuristics did not perform well for the problems with high space capacity utilization. Overall, the proposed SA heuristics performed better than the proposed TS heuristics with respect to the solution quality. As stated previously, it is believed that the proposed TS heuristics did not perform well because of it being completely deterministic. Unlike in McKendall and Li (2017) the authors added diversification to their TS heuristic by generating a set of diverse initial solutions.

6.2 Recommendations for Future Research

The following recommendations are given for future research.

1. Consider the material handling costs between the machines assigned to the same location.
2. Consider solving the GQAP by using a hybrid technique that combines deterministic and stochastic heuristics (e.g. TS and Genetic Algorithm or TS and SA) to obtain better results.
3. Consider developing different construction algorithms to construct good initial solutions.

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