The Effect of Natural Fractures on the Mechanical Behavior of Limestone Pillars: A Synthetic Rock Mass Approach Application

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The Effect of Natural Fractures on the Mechanical Behavior of Limestone Pillars: A Synthetic Rock Mass Approach Application

Mustafa Can Suner

Thesis submitted to the Benjamin M. Statler College of Engineering and Mineral Resources at West Virginia University

in partial fulfillment of the requirements for the degree of

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in

Mining Engineering

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ABSTRACT

The Effect of Natural Fractures on the Mechanical Behavior of Limestone Pillars: A Synthetic Rock Mass Approach Application

Suner, Mustafa Can

In general, underground limestone mines have inherently strong rock and experience good ground stability. Also, modern pillar design guidelines developed by National Institute for Occupational Safety and Health (NIOSH) have improved the design of stable layouts for modern limestone mines. However, ground control-related incidents are still an important problem. In underground limestone mines, previously mined sections stay open for the life of the mine which may be many years, and it is possible for travel ways to working faces to pass through these old sections. In a recent massive pillar collapse in an old section of a mine in Pennsylvania (Pa), three miners were injured outside of the mine due to an air blast. Also, frequent reports are indicating pillar sloughing, spalling and roof falls. These incidents highlight the potential safety impact on the miners in underground limestone mines. In the pillar design guidelines published by NIOSH, pillars are mostly examined for the existence of one-large discontinuity crossing completely through the pillar. However, the influence of multiple joint sets and natural fractures on the insitu pillar strength prediction and localized failures of the pillar are not covered by the guidelines. In this thesis, the influence of naturally exiting joint sets and fractures on the mechanical behavior (i.e. strength and failure mechanisms) of underground stone pillars is studied.

In order to investigate pillar mechanics, a systematical methodology is developed based on the novel approach, the Synthetic Rock Mass (SRM) by utilizing the two-dimensional Universal Distinct Element Code (UDEC). In order to form the first component of SRM, the Bonded Particle Model (BPM), the mechanical properties of the standard size laboratory rock specimen scaled up to the upper-limit of the Hoek-and-Brown Scaling Equation. Then, Voronoi-Trigon Discretization
Logic is used to model the intact rock matrix of the stone mine pillars. Later, field data is used to stochastically generate Discrete Fracture Networks (DFNs), and SRM models are established by integrating the BPM and DFNs. Then, rock specimen sizes are increased from laboratory size to field size by sampling the generated DFNs. In the up-scaling operation (i.e. specimens’ size increase), the homogenization process is applied that the estimated strength properties of the pillars by SRM are captured with a new BPM. By doing so, the numerical simulations calibrated against the empirical stone mine pillar strength equation established by NIOSH. Finally, the predicted strength parameters are used to examine the pillar failure mechanics with various width-to-height ratios.

As a result, the study proposes a methodology to explain the pillar strength and failure mechanism with the explicit consideration of naturally existing joint sets in the stone mines which ultimately aims to enhance the pillar design procedures currently used in the United States. Pillar strengths predicted by the SRM approach developed in this thesis are in good agreement with the stone pillar strength equation published by NIOSH. The findings also indicated that the joint systems developed in the pillars are directly affecting the pillar strength. The pillar models having high-strength intact rock properties estimated lower normalized strength than the pillar models having low-strength intact rock properties due to higher joint density in high-strength pillars. It is also supported by the failure cases in the S-Pillar Database that there are no failed pillars in the low-strength categories. Also, in the pillars having a width-to-height ratio of 0.5, tensile failure governs the pillar behavior. On the other hand, combined shear and tensile failure mechanisms are captured for pillars having a width-to-height ratio equal to or greater than 1.0. While shear failure dominates the core of the pillars, tensile failure is observed at the ribs. The numerical simulations revealed that the pillar failure starts in terms of spalling when the average stress on the pillar is around 9 – 10% of the intact rock strength, which is also reported in the literature. The modeling methodology developed in this thesis for simulating the influence of natural fractures and joint sets on stone mine pillar strength will improve underground stone mine worker safety by enabling assessment of the influence of joint sets on pillar stability.
To My Dear Family,

and

To The Greatest Dwarf, Gimli,
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Chapter 1 Introduction

Lamar (1967) defined limestone as a general name for sedimentary rocks having calcium carbonate in their mineralogical formation. Historically, limestone has been an important natural source for construction and other industries. It was used during the construction of the Great Pyramid of Giza, cemented roads in the Roman Empire, and in the restoration of the Great Wall of China. More recently, it was also used to make Portland cement. Now, it is used in road construction, concrete production, lime manufacturing, and agricultural activities.

Texas, Florida, Pennsylvania, Georgia, and North Carolina are the primary stone producers in the United States of America (USA) (Mineral Commodity Summaries, 2021). In the USA, 7.07 billion tons of crushed stone, 70% of which was limestone and dolomite, were produced between 2016 and 2020. In 2020, the total produced crushed stone was around 1.46 billion tons with an economical value of more than $17.8 billion that a slight decrease from 2019 is observed due to the global COVID – 19 pandemic (Mineral Commodity Summaries, 2021). Also, in the mining industry, stone operations provide high-paying jobs to their local communities. Between 2000 and 2019, stone mines employed a total of 73,597 people where 71,738 of them worked in surface mines while the rest worked for underground mines (NIOSH, 2021). Behind the coal mining industry, the stone mining industry is the second largest employer in the overall mining industry.

There are 115 underground stone mines in the USA as of 2019 (NIOSH, 2021). Underground stone mines generally use the room-and-pillar mining method. In this mining method, pillars maintain the global stability and supplement the local stability of rock around individual excavations called rooms (Brady and Brown, 2004). In 2011, NIOSH published modern pillar design guidelines for the US underground stone mines (Esterhuizen et al., 2011). After the development of pillar design guidelines, Esterhuizen et al. (2011) also published the S-Pillar design software, a practical tool frequently used by the underground stone mine industry for designing pillars.

1.1. Problem Statement

The underground stone mines have inherently strong rock mass, and operators use modern pillar guidelines and S-Pillar software published by NIOSH. However, ground control-related incidents are still an important problem in the underground stone mine industry. There were 4 fatalities and
22 non-fatal lost-time injuries due to the fall of ground in the underground stone mines between 2011 and 2019 (NIOSH, 2021). In a recent event, three miners were also injured outside of the mine due to an air blast because of a massive pillar collapse in an old section of a mine in Pennsylvania (Esterhuizen et al., 2019). Also, frequent reports indicate that pillar sloughing, spalling and roof fails still occur in Eastern US stone mines (MSHA, 2011; 2014; 2017). These incidents highlight the potential safety problems with ground control in underground stone mines.

1.2. Objective of Thesis

The underground stone mine design guidelines published by NIOSH (2011) do not account for the influence of more than one joint set and natural fractures on the stability of a pillar. The goal of this research is to improve the safety of stone mine workers by developing a new modeling approach that can simulate the influence of naturally existing fractures on the short-term strength and failure mechanisms of the underground stone mine pillars. Hence, with this study, a universally applicable pillar strength estimation methodology is developed to assist in the design stages of underground mine layouts with the explicit consideration of naturally existing joint sets and natural fractures.

1.3. Statement of Work

In this thesis, the influence of the joint sets and natural fractures on the mechanical behavior of the stone pillars are studied with the utilization of the Synthetic Rock Mass (SRM) approach in the two-dimensional Universal Distinct Element Code (UDEC). The laboratory size specimens having 50 mm width and 100 mm height are modeled with yielding zone elements. Then, the simulated intact rock samples’ properties and specifications are directly employed into larger laboratory specimens discretized with the Voronoi-Trigon Tessellation to build a Bonded Particle Model (BPM). To attribute the strength reduction due to size increase, the Hoek and Brown Scaling Equation (1980) is used to estimate the strength of larger laboratory samples (i.e. 200 mm in width and 400 mm in height). The strength reduction is satisfied with the Voronoi-Trigon Tessellation. Later, field data is used to stochastically generate Discrete Fracture Networks (DFNs). With the integration of BPM and DFNs, the SRM models are established to represent stone rock masses. In this study, the simulated rock mass volume is systematically scaled-up from laboratory size to field size. A multi-stage upscaling methodology with the homogenization process at the interim stages.
is applied to establish rock block strength from laboratory to field sizes. The stone mine pillar strength equation (NIOSH, 2011) is used as an empirical control measure.

The study consisted of four specific tasks: 1) S-Pillar Database Analysis; 2) Synthetic Rock Mass Generation; 3) Rock Block Up-Scaling; 4) Stone Mine Pillar Failure Mechanism Investigation. A detailed explanation of each task is given below:

- **Task 1:** Statistical analysis of the S-Pillar database is carried out to select input parameters that represent the US stone mine rock mass accurately.

- **Task 2:** The SRM model to represent the stone mine pillar in UDEC is generated with two consecutive steps. First, the BPM is generated with the yielding zone materials and the Voronoi-Trigon Discretization Logic. Then, SRM models are generated by integrating the DNFS realizations, which are generated by the statistical results from Task -1, with the BPM.

- **Task 3:** Before employing the up-scaling methodology, two conceptual DFNs are generated and implemented into BPM models to:
  - Investigate the effect of discontinuity sets; trace length/persistency, spatial location, and the orientation on the pillar strength.
  - Observe the effect of failure mechanisms caused by the existence of natural fractures, and inclined or lateral weakness planes in the pillar failure process.

Later, the SRM models are systematically scaled-up from the upper limit of the Hoek and Brown Scaling Equation (1980) to the average field sizes. In the up-scaling operations, a homogenization process is employed to capture numerically predicted SRM model strength properties by the new BPM. This process allows preserving the numerical efficiency of the models. In the homogenization process, while the Voronoi-Trigon Contact’s frictional and cohesive strength components are systematically reduced, the other parameters are held constant. Hence, the pillar strengths are numerically estimated from the laboratory size samples to the field size pillars.

- **Task 4:** The failure mechanism of the pillars in terms of shear, tensile, or a combination of these two failure modes is studied with the help of the calibrated pillar models in Task 3. The failure and strength changes as a function of pillar width-to-height ratio and the influences of the discrete discontinuities are examined. According to these findings, a
practical methodology is established to create a link from laboratory size intact rock specimens to field-size pillars with the consideration of naturally existing joint sets.

1.3. Thesis Outline

The thesis consists of 6 chapters. The chapters are described as:

- Chapter – 1 is the introduction chapter.
- Chapter – 2 introduces the literature by discussing the existing studies on hard-rock pillar mechanics.
- Chapter – 3 explains the methodology used in this thesis to study stone mine pillar mechanics.
- Chapter – 4 presents the S-Pillar Database statistical analysis results.
- Chapter – 5 discusses the modeling techniques utilized in the study and the results. The comparison with the previous research studies and the findings is also concluded.
- Chapter – 6 summarizes the conclusions drawn from the study. The future recommendations for the following studies are outlined in this chapter.
Chapter 2 – Literature Review

In the United States, underground stone mines, as well as the other flat-lying stratiform or lenticular underground hard-rock mines, utilize the room-and-pillar mining method (Brady and Brown, 2004; Esterhuizen et al., 2011). The conceptual drawing of the room-and-pillar mining method is shown in Figure – 2.1. In the room-and-pillar mining method, pillars maintain the global stability of the overlying rock strata and assist the local stability (Brady and Brown, 2004; Esterhuizen et al., 2011). Zipf (2001) indicated that the design of room-and-pillar mine layouts can be achieved using the traditional strength-based pillar design methodologies. First, the estimation of field stresses on the pillar can be calculated by either the tributary area method (Brady and Brown, 2004) or numerical approaches such as LaModel (Heasley, 1997; 1998). Also, the pillar strengths can be estimated by the empirical approaches (i.e. Hedley and Grant (1972)) with the help of the field observations. Finally, the factor of safety can be established by dividing the pillar strength to stress applied to a pillar to calculate the room-and-pillar mining method dimensions. In recent years, Esterhuizen et al. (2011) revealed that the pillar load assessments and/or pillar failure analysis with numerical approaches supported with the field observations and rock mass characterizations improved the room-and-pillar layout designs. Hence, in order to gain a holistic view of the pillar design methodologies, the rock mass classification systems, empirical hard-rock pillar strength equations, and numerical simulations to study pillar’s mechanical behavior are discussed in this chapter.

![Figure 2.1 Elements of Room-and-Pillar Mining Method (Hamrin, 2001)](image-url)
2.1. Rock Mass Classification Systems

There are various rock mass classification systems developed to utilize in the mining operations such as Rock Mass Rating (RMR) by Bieniawski (1973; 1989), Rock Mass Quality Index (Q-Index) by Barton et al. (1974), Basic Geotechnical Description of Rock Masses by International Society for Rock Mechanics (1980), Modified Rock Mass Rating (M-RMR) by Unal et al. (1992) and Unal (1996), and Geological Strength Index (GSI) by Hoek and Brown (1997). In this thesis, RMR and GSI are discussed in detail since they are utilized in this research study.

The RMR system was first constructed by Bieniawski (1973) in South Africa. It has had several modifications over years, and the last version of it was published by Celada et al. (2014). However, the 1989 version is still the most widely used version of the system. RMR accounts for six different rock mass parameters: 1) Strength of intact rock, 2) rock quality designation, 3) spacing of discontinuities, 4) condition of discontinuities, 5) groundwater conditions, and 6) orientation of discontinuities. Strength of intact rock rating ranges from 0 for very weak, \( UCS < 1 \text{ MPa} \), to 15 for very strong, \( UCS > 250 \text{ MPa} \), rock matrix. The second parameter, Rock Quality Designation (RQD) was developed by Deere (1968). It quantifies the rock mass quality by computing the ratio of the length of broken core pieces, longer than 10 cm, to the total core length. The application of the RQD procedure is explained by a figure taken from Deere and Deere (1989) as follows (Figure – 2.2):

![RQD Procedure](image)

\[
RQD = \frac{\sum(\text{Length of Core Pieces})}{\text{Total Core Length}} \times 100\%
\]

\[
\text{Length of Core Pieces} = 38 + 17 + 20 + 43 = 118
\]

\[
RQD = \frac{118}{200} \times 100\% = 59\% (FAIR)
\]

*Figure 2.2 RQD Procedure (After Deere and Deere, 1989)*

Discontinuity is a term used in rock mechanics to define fractures: faults, joints, weak bedding planes, shears and contacts (Brady and Brown, 2004). The density of the weakness planes within
the rock influences the mechanical response of a rock mass. As the spacing of joints increases, both the discontinuity rating and the integrity of rock mass increase (i.e. if the spacing is larger than 2 m, the rating is 20). Condition of discontinuities is described by their persistence, roughness, aperture distances, weathering, and filling material. The orientation of discontinuities relative to the mining/tunnel direction also influences the rock mass stability. Bieniawski (1989) categorized the groundwater condition into three as: (i) inflow in the rock structure per 10 m lengths, (ii) joint water pressure ratio to major principle stress ($\sigma_1$), and (iii) general conditions of the rock mass. For completely dry conditions, the groundwater condition rating is 15. Table – 2.1 shows the whole schematic of the RMR system.

The Geological Strength Index (GSI) was primarily developed for jointed rock masses in general underground excavations. It was constructed by Hoek and Brown (1994) along with their empirical strength criteria for rock masses (1980). GSI is easier to use than other rock mass classification systems since it depends more on the visual assessment of the rock masses than other methods. The two main parameters in the GSI system can be listed as: 1) The area or volume of rock block which results from the joint sets, and 2) the condition of discontinuity surfaces. By its nature, the GSI system does not account for intact rock strength rating as does RMR, but it also does not account for the effect of discontinuities on the rating twice like RMR does with the RQD rating and joint spacing rating. Figure – 2.3 exhibits the GSI Table.

Cai et al. (2004) proposed a method to quantify rock structure in GSI by block volumes (Equation – 1) and surface condition in GSI by a joint condition factor (Equation – 2) based on the Palmstorm’s “joint condition factor” (1994). The block volumes are calculated as follows:

$$V_b = \frac{s_1 \cdot s_2 \cdot \ldots \cdot s_i \cdot \sin(\gamma_1) \cdot \sin(\gamma_2) \cdot \ldots \cdot \sin(\gamma_i)}{\sin(\gamma_1) \cdot \sin(\gamma_2) \cdot \ldots \cdot \sin(\gamma_i)}$$  \hspace{1cm} (1)

where $V_b, s_i$ and $\gamma_i$ are the block volume, joint spacing, and the angle between joint sets. The joint condition factor is defined as:

$$J_c = (J_W \cdot J_s)/J_A$$ \hspace{1cm} (2)

where $J_c, J_s$ and $J_A$ are the joint condition factor, joint large-scale waviness, joint small-scale smoothness, and the joint alteration factor. The associated values can be selected from Table – 2.2. Figure – 2.4 shows the updated GSI chart to calculate the GSI rating as a function of block volume, and the joint condition factor.
### Table 2-1 Rock Mass Classification System (After Bieniawski, 1989)

<table>
<thead>
<tr>
<th>A</th>
<th>Classification Parameters and Their Ratings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><strong>Strength of Intact Rock Material</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Point-Load Index</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Uniaxial Comp. Strength</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Rating</strong></td>
</tr>
<tr>
<td>2</td>
<td><strong>RQD</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Rating</strong></td>
</tr>
<tr>
<td>3</td>
<td><strong>Discontinuity Spacing</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Rating</strong></td>
</tr>
<tr>
<td>4</td>
<td><strong>Condition of Discontinuities</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Rating</strong></td>
</tr>
<tr>
<td>5</td>
<td><strong>Groundwater</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Inflow per 10 m Tunnel Length (l/m)</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Joint Water Press / Major Principal</strong></td>
</tr>
<tr>
<td></td>
<td><strong>General Conditions</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Rating</strong></td>
</tr>
<tr>
<td>B</td>
<td><strong>Rating Adjustment for Discontinuity Orientations (See F)</strong></td>
</tr>
<tr>
<td>C</td>
<td><strong>Rock Mass Classes Determined from Total Ratings</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Class Number</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Description</strong></td>
</tr>
<tr>
<td>D</td>
<td><strong>Meaning of Rock Classes</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Class Number</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Average Stand-up Time</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Cohesion of Rock Mass (kPa)</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Friction Angle of Rock Mass (°)</strong></td>
</tr>
<tr>
<td>E</td>
<td><strong>Guidelines for Classification of Discontinuity Condition</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Discontinuity Length (Persistency)</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Rating</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Separation (Aperture)</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Rating</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Roughness</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Rating</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Infilling (Gauge)</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Rating</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Weathering</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Rating</strong></td>
</tr>
<tr>
<td>F</td>
<td><strong>Effect of Discontinuity Strike and Dip Orientation in Tunneling</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Strike Perpendicular to Tunnel Axis</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Strike Parallel to Tunnel Axis</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Drive with Dip 45° - 90°</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Drive with Dip 20° - 45°</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Dip 45° - 90°</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Dip 20° - 45°</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Drive Against Dip 45° - 90°</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Drive against Dip 20° - 45°</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Dip 0° - 20° - Irrespective of Strike °</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Dip 20° - 45°</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Dip 45° - 90°</strong></td>
</tr>
</tbody>
</table>

---

8
Figure 2.3 Geological Strength Index System (Hoek and Marinos, 2000)
Table 2-2 Terms with Descriptions to Calculate Joint Condition Factor (Directly taken from Cai et al. (2004) which are adopted from Palmstorm (1994) for Waviness, Smoothness and Joint Alteration Terms and Barton et al. (1974) for Joint Alteration Terms)

<table>
<thead>
<tr>
<th>Waviness Terms</th>
<th>Undulation</th>
<th>Rating for Waviness</th>
<th>Description</th>
<th>Rating for Waviness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interlocking (Large-Scale)</td>
<td></td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stepped</td>
<td>2.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Large Undulation</td>
<td>&gt; 3%</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small to Moderate Undulation</td>
<td>0.3 - 3%</td>
<td>1.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Planar</td>
<td>&lt; 0.3%</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Smoothness Terms</th>
<th>Description</th>
<th>Rating for Waviness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Rough</td>
<td>Near vertical steps and ridges occur with interlocking effect on the joint surface</td>
<td>3</td>
</tr>
<tr>
<td>Rough</td>
<td>Some ridge and side-angle are evident; asperities are clearly visible; discontinuity surface feels very abrasive (rougher than sandpaper grade 30)</td>
<td>2</td>
</tr>
<tr>
<td>Slightly Rough</td>
<td>Asperities on the discontinuity surfaces are distinguishable and can be felt (like sandpaper grade 30 - 300)</td>
<td>1.5</td>
</tr>
<tr>
<td>Smooth</td>
<td>Surface appear smooth and feel so to touch (Smoother than sandpaper grade 300)</td>
<td>1</td>
</tr>
<tr>
<td>Polished</td>
<td>Visual evidence of polishing exists. This is often seen in coating of chlorite and specially talc</td>
<td>0.75</td>
</tr>
<tr>
<td>Slickensided</td>
<td>Polished and striated surface that results from sliding along a fault surface or other movement surface</td>
<td>0.6-1.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Alteration Terms</th>
<th>Description</th>
<th>Rating for Alteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock Wall Contact</td>
<td>Clear Joints</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Healed or welded Joints (Unweathered)</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>Fresh Rock Walls (Unweathered)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Alteration of joint wall, slightly to moderately weathered</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Alteration of joint wall highly weathered</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Coating or thin Filling</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sand, silt, calcite, etc.</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Clay, chlorite, talc, etc.</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Compacted clay Materials</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Soft Clay Materials</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Swelling Clay Materials</td>
<td>8</td>
</tr>
<tr>
<td>Filled Joints with partial contact between the rock wall surfaces</td>
<td>Filling material exhibits swelling properties</td>
<td>8 - 12</td>
</tr>
</tbody>
</table>
Figure 2.4 Quantification of GSI Chart (Cai et al., 2004)
2.2. Empirical Hard-Rock Mine Pillar Strength Equations

Pillar strength is defined as the maximum resistance of a pillar to axial compression, and it is related to both pillar volume and shape (Salamon and Munro, 1967; Brady and Brown, 2004; Esterhuizen et al., 2011). The general form of the pillar strength equations was defined by Peng (2008):

\[
Pillar\ Strength = K \cdot \left( A + B \cdot \frac{w^a}{h^b} \right)
\]

where

- \( K \) is the constant to reduce the intact rock UCS to field size rock,
- \( A, a, B \) and \( b \) are the empirically derived pillar size and shape constants,
- \( w \) is the width of the pillar,
- \( h \) is the height of the pillar.

Hedley and Grant (1972) published one of the first hard-rock pillar strength equations widely accepted and applied by the industry. They surveyed approximately 28 different pillar and rib failures at the Elliot Lake Uranium mines and derived Equation – 4. Later, von Kimmelmann et al. (1984) back-calculated pillar strength in underground nickel and copper mines using the displacement-discontinuity variation of boundary element method, NFOLD (Golder Associates, 1977) (Equation – 5). Based on field observations and back-analyses of fractured pillars, Krauland and Soder (1987) established a linear empirical equation from back-analyses of limestone mine pillars (Equation – 6). Later, Potvin et al. (1989) proposed an empirical strength equation (Equation – 7) to aid in the design stage of the open-stope pillars by calibrating the proposed solution against the Canadian open-stope mines. Sjoberg (1992) studied 9 limestone/skarn pillars to establish a linear empirical equation (Equation – 8). Table – 2.3 lists the normalized empirical equations (i.e. divided by UCS).

Lunder and Pakalnis (1997) established a hybrid strength equation that combines the confined core theory and the shape effect of the Pillars (Equation – 9). In their study, they used pillar failure cases from Westmin Resources, Hudyma (1988), von Kimmelman (1984), and Hedley and Grant (1972).
Table 2-3 Empirically Derived Equations for Estimation Hard-Rock Mine Pillar Strengths

<table>
<thead>
<tr>
<th>Previous Studies</th>
<th>Strength Equation</th>
<th>UCS, MPa</th>
<th>Equation Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hedley and Grant (1972)</td>
<td>$0.58 \cdot UCS \cdot \frac{w^{0.5}}{h^{0.75}}$</td>
<td>230</td>
<td>(4)</td>
</tr>
<tr>
<td>Von Kimmelmann et al. (1984)</td>
<td>$0.69 \cdot UCS \cdot \frac{w^{0.46}}{h^{0.66}}$</td>
<td>94</td>
<td>(5)</td>
</tr>
<tr>
<td>Krauland and Soder (1987)</td>
<td>$0.35 \cdot UCS \cdot \left(0.778 + 0.222 \cdot \frac{w}{h}\right)$</td>
<td>100</td>
<td>(6)</td>
</tr>
<tr>
<td>Potvin et al. (1989)</td>
<td>$0.42 \cdot \sigma_c \cdot \frac{w}{h}$</td>
<td>-</td>
<td>(7)</td>
</tr>
<tr>
<td>Sjoberg (1992)</td>
<td>$0.31 \cdot UCS \cdot \left(0.778 + 0.222 \cdot \frac{w}{h}\right)$</td>
<td>240</td>
<td>(8)</td>
</tr>
</tbody>
</table>

$Pillar\ Strength = (K \cdot UCS) \cdot (C_1 + C_2 \cdot \kappa)$ \hspace{1cm} (9)$

where:
- $K$ is the rock mass strength factor,
- UCS is the uniaxial compressive strength of rock,
- $C_1$ and $C_2$ empirically derived constants,
- $\kappa$ is the mine pillar friction term which can be calculated from Equation – 10.

$$K = \tan \left[ \left( \frac{1 - C_{pav}}{1 + C_{pav}} \right) \right]$$ \hspace{1cm} (10)$

where the $C_{pav}$ is average pillar confinement can be calculated by Equation – 11:

$$C_{pav} = 0.46 \cdot \left[ \log \left( \frac{w}{h} + 0.75 \right) \right]^{1.4 \cdot \frac{h}{w}}$$ \hspace{1cm} (11)
Finally, with the investigation of Westmin Resources, Hudyma (1988), von Kimmelman (1984), and Hedley and Grant (1972) pillar databases, Lunder and Pakalnis (1997) derived the constants $K$ and $C_{pav}$ in Equation – 10 and – 11, respectively. Later, they substituted the constants into Equation – 9:

$$\text{Pillar Strength} = (0.44 \cdot UCS) \cdot (0.68 + 0.52 \cdot K)$$  \hspace{1cm} (12)

Roberts et al. (2007) developed empirical pillar design guidelines in lead mines of the Viburnum Trend in Southeastern Missouri from back-analysis of pillar damage with Displacement-Discontinuity Analysis (DDA). They modeled the strength of hard-rock pillars with different rib and core cell elements to simulate the effect of confinement on pillar strength (Figure – 2.5). This approach is essentially the same approach used in LaModel to simulate coal pillar strength (Heasley et al., 2010). Later, Esterhuizen et al. (2011) used this empirical information to derive the base strength equation used in S-Pillar.

![Figure 2.5 Elements Type to Examine Strength of Pillar with Confinement Theory (Roberts et al., 2007)](image)

The empirical pillar strength equations mentioned above are site-specific. When applying those equations in another site, region, or even in the same mine, great care must be taken. Geological formations, discontinuities such as joints or faults can greatly differ from one site to another. Hence, to have more information about the pillar strength estimation methodologies and the pillar failure mechanisms, researchers utilized variations of numerical simulation codes. These numerical simulation codes together with the geomechanical data might help to investigate and understand the influence of geological structures on pillar mechanics. The next section summarizes available numerical simulation codes and numerical hard rock pillar mechanics studies.

**2.3. Numerical Simulation Techniques on Pillar Mechanics**

The primary goal of a numerical simulation in mining geomechanics is to approximate the behavior of geomaterials due to mining by a set of differential equations’ solutions in a given
region of space and time, together with the pre-determined boundary conditions (LeVeque, 2007). There are three material mechanics methods available to simulate geomaterials: 1) continuum, 2) distinct/discrete fracture (i.e. dis-continuum), and 3) hybrid. The most common continuum methods used in geomechanics are Finite Element Methods (FEM), Finite Difference Method (FDM), and Boundary Element Method (BEM). FEM is the widely used variation of continuum methods that the algebraic equation matrix is established for the global model by assembling the local function approximations hold in the finite element zones (Singiresu, 2018). On the other hand, FDM has an identical purpose with the FEM but the algebraic equations are placed into every derivative in the set of governing equations (Itasca, 2020). The BEM is another method that originated from the early 19th century with the known name of boundary integral equation method (Katsikadelis, 2016) and in the BEM, the partial differential equations are represented with the integral equations on the boundary surface, and the solution is obtained by the integral equations (Farahmand and Li, 1986).

In the fractured rock mass, sliding and separation of blocks along the discrete discontinuity planes influence the rock mass behavior. With the continuum approach, it is possible to model few non-intersecting fractures. However, moderately and heavily fractured rock mass cannot be approximated with continuum formulation explicitly. Hence, dis-continuum methods should be used to simulate the explicit influence of discontinuities on rock mass responses. The most common dis-continuum modeling methods can be listed as Distinct Element Method (DEM) and Discrete Fracture Network (DFN) method. DEM is developed by Cundall (1971) as a two-dimensional representation of jointed rock mass. The method is later improved to capture mechanisms of particle flow, granular material, and crack developments (Itasca, 2020). In the method, both explicit and implicit time-marching schemes are available to better capture material characteristics. Universal Distinct Element Code (UDEC in two-dimension, 3DEC in three-dimension) and Particle Flow Code (PFC2D in two-dimension, PFC3D in three-dimension) developed by Itasca Consultants are commonly used in the mining geomechanics. The DFN method is the extension of DEM that the fractured rock masses represented with the population of individual fractures with the associated statistical probability distributions derived from the field observations (Lavoine et al., 2020).
Lastly, the hybrid methods, combining two or more approaches, are used to study rock engineering problems (Jing and Hudson, 2002). The combinations can be BEM/FEM (Zienkiewicz et al., 1977; Brady and Wassyng, 1981), DEM/FEM (Elmo and Stead, 2010), and BEM/DEM (Lorig and Brady, 1982 and 1984; Lorig et al., 1986). The next section introduces the commonly utilized empirical failure criterion is the numerical simulations to model failure of rock material.

2.3.1. Empirical Failure Criterions

In mining geomechanics, Mohr-Coulomb is probably the most widely used failure criterion. The failure envelope for this criterion corresponds to a shear yield function (Equation – 13) with the tension cut-off (i.e. tension yield function, Equation – 14) (Itasca, 2020). If the normal stress turns to be tensile, the Mohr-Coulomb Failure Criterion loses its validity that the minor principal stress cannot exceed the tensile strength (Equation – 15).

\[ f_s = \sigma_1 - \sigma_3 \cdot N_\phi + 2 \cdot c \cdot \sqrt{N_\phi} \text{, where } N_\phi = \frac{1 + \sin(\phi)}{1 - \sin(\phi)} \]  \hspace{1cm} (13)

\[ f_t = \sigma_3 - \sigma_t \]  \hspace{1cm} (14)

\[ \sigma_{t_{\text{max}}} = \frac{c}{\tan(\phi)} \]  \hspace{1cm} (15)

In 1980, Hoek and Brown introduced an empirical failure criterion for rock masses (Equation – 16). They indicated that realistic rock mass strength should include the transition from intact rock material to a jointed rock mass. Figure – 2.6 illustrates this transition concept. As the sample size of rock block increases from laboratory scale to field scale, joint density within the representative rock mass value also increases.

\[ \sigma_1 = \sigma_3 + \sigma_{cl} \cdot \left( m_b \cdot \frac{\sigma_3}{\sigma_{cl}} + s \right)^a \]  \hspace{1cm} (16)

where:

- \( \sigma_1 \) is the major principle stress,
- \( \sigma_3 \) is the minor principal stress,
- \( \sigma_{cl} \) is the uniaxial compressive strength (UCS) of the intact rock,
- \( m_b \) is the rock material constant that for intact rock it becomes equal to \( m_i \) which is intact rock material constant,
- $s$ is the rock material constant that for intact rock, it becomes 1.00,
- $a$ is the rock material constant that for intact rock, it becomes 0.5.

Parameters, $m_b$, $s$, and $a$ can be derived for the specific rock mass conditions with the help of the Geological Strength Index (GSI) value, and the disturbance factor, $D$ from Equations – 17, – 18, and – 19, respectively.

\[
m_b = m_i \cdot e^{\frac{GSI-100}{28-14D}}
\]  
\[
s = e^{\frac{GSI-10}{9-3D}}
\]  
\[
a = \frac{1}{2} + \frac{1}{6} \cdot \left( e^{\frac{GSI}{15}} - e^{\frac{-20}{3}} \right)
\]

**Figure 2.6 Hoek and Brown Jointing Degree on the Rock Blocks (modified after Hoek and Brown, 2018)**

In 2014, Hoek and Martin included the tensile cut-off value that can be expressed as a ratio of intact rock UCS to tensile strength, $\sigma_{ci}/|\sigma_c|$. Previously, Fairhurst (1964) proposed this criterion based on the Griffith Failure Theory. Then, Hoek and Brown derived Equation – 20 from the regression analysis of laboratory triaxial compressive strength tests and analysis published by
several researchers (Lau and Gorski, 1992; Hoek, 1965; Bobich, 2005; Ramsey and Chester, 2004; Gerogiannopoulos and Brown 1978).

\[
\frac{\sigma_{ci}}{\sigma_t} = 0.81 \cdot m_i + 7 \tag{20}
\]

Later, Equation – 21 and – 22 are derived to compute Mohr-Coulomb equivalent friction (\( \phi \)) and cohesion (\( c \)) parameters from the Hoek and Brown Strength criteria.

\[
\phi = \frac{6 \cdot a \cdot m_b \cdot (s + m_b \cdot \sigma_{3n})^{a-1}}{2 \cdot (1 + a) \cdot (2 + a) + 6 \cdot a \cdot m_b \cdot (s + m_b \cdot \sigma_{3n})^{a-1}} \tag{21}
\]

\[
c = \sigma_{ci} \cdot \frac{[(1 + 2 \cdot a) \cdot s + (1 - a) \cdot m_b \cdot \sigma_{3n}] \cdot (s + m_b \cdot \sigma_{3n})^{a-1}}{(1 + a) \cdot (2 + a) \cdot \sqrt{1 + \frac{6 \cdot a \cdot m_b \cdot \sigma_{3n})^{a-1}}}} \tag{22}
\]

Where, \( \sigma_{3n} = \sigma_{3_{max}}/\sigma_{ci} \) and \( \sigma_t < \sigma_3 < \sigma_{3_{max}} \).

Equation – 21 and – 22 can be inserted in Equation – 23 to compute Mohr-Coulomb equivalent parameters for the Hoek and Brown strength criterion. When the Mohr-Coulomb equivalent parameters are used, tensile cut-off must also be used.

\[
\sigma_1 = \frac{2 \cdot c \cdot cos (\phi)}{1 - sin (\phi)} + \frac{1 + sin (\phi)}{1 - sin (\phi)} \cdot \sigma_3 \tag{23}
\]

### 2.3.2. Hard-Rock Pillar Strength Estimations with Numerical Simulations

Martin and Maybee (2000) utilized two-dimensional finite element analyses to model hard-rock pillars with the various GSI values and the conventional Hoek and Brown parameters. The results showed that the pillar strengths predicted by Hoek and Brown parameters do not agree with the published studies in the pillar stability graphs. The main reason is attributed that Hoek and Brown's failure criterion over-predicts the strength of hard-rock pillars when the conventional Hoek and Brown parameters are utilized. It is also indicated that the cohesion loss process is governing the failure of the hard-rock pillars. Later, with the Hoek and Brown brittle parameters (i.e. \( m_b = 0, s = 0.11 \)), two-dimensional elastic analyses revealed that the pillar strength predictions are in good agreement with the empirical equations. The stability graph by Martin and Maybee (2000) can be seen in Figure – 2.7.
In the US, Esterhuizen et al. (2006, 2007, 2011) used a continuum modeling approach to study stone mine pillar mechanics. He utilized the bilinear rock strength constitutive model based on the Mohr-Coulomb failure criteria using FLAC3D numerical code. Later, they calibrated their numerical simulations against Lunder and Pakalnis’s (1997) empirical strength equation (Equation – 12) by adjusting the cohesive strength components of the models. Later, by changing the RMR of the pillars implicitly, they carried out sensitivity analyses and found a positive correlation between RMR and the strength of the pillars. Also, the ubiquitous joint constitutive model is utilized by Esterhuizen et al. (2006) to implicitly incorporate the effect of discontinuities on the strength of slender pillars. Esterhuizen et al. (2011) indicated that for a slender pillar (i.e. width-to-height ratio is equal to 0.5), the strength reduction factor due to the large discontinuity in the pillar can be more than 80%. On the other hand, as the width-to-height ratio of the pillar increases, the influence of discontinuity on the strength reduction factor diminishes to about 10 – 15%. Figure – 2.8 shows the strength reduction factor as a function of the large discontinuity dip angle and the pillar width-to-height ratio.
Esterhuizen et al. (2011) collected operational and pillar performance information from 34 different stone mines in the Eastern and Midwestern US. They classified a total of 18 cases of individual pillars as failed and assessed each of the failed pillars visually. Later, by completing a comprehensive numerical modeling work to understand pillar behavior within the consideration of brittle rock spalling, large and angular discontinuities, weak bedding bands, floor benching, and length of pillar, they established a base pillar strength equation governing stone mines in the US (Equation 24).

Figure 2.8 Strength Reduction Rates with the Dip of Large Discontinuities in the Limestone Pillars with Different W/H (Esterhuizen et al., 2011)

\[
Pillar \text{ Strength Equation Modified for Stone Mines} = k \cdot \frac{W^{0.3}}{H^{0.59}}\tag{24}
\]

where \(k\) is the rock strength parameter that is calculated as \(0.65 \cdot UCS\) (Esterhuizen et al., 2011). Esterhuizen et al. (2011) also studied the large and angular discontinuity effects on the pillar strength. Large and angular discontinuity is defined as a joint system passing through the pillar from floor to roof with different dip angles. Later, they introduced the ‘large discontinuity factor’ (LDF) (Equation – 25) in addition to the base equation (Equation – 24) to reduce the strength of pillars with large and angular joints in their matrix. The large discontinuity factor consists of two different parameters that discontinuity dip factor and fracture frequency constants which can be seen in Table – 2.4 and Table – 2.5, respectively. Finally, Esterhuizen et al. (2011) published Equation – 26 to estimate stone mine pillar strength in the Eastern and Midwestern US.
\[ LDF = 1 - DDF \cdot FF \]  

(25)

\[ Pillar\ Strength\ Equation\ Modified\ for\ Stone\ Mines = 0.65 \cdot UCS \cdot LDF \cdot \frac{w^{0.3}}{h^{0.59}} \]  

(26)

Table 2-4 2 Discontinuity Dip Factor for Individual Pillar Having passing through Joints (Esterhuizen et al., 2011)

<table>
<thead>
<tr>
<th>Discontinuity Dip, °</th>
<th>Pillar Width-to-Height Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&lt; 0.5</td>
</tr>
<tr>
<td>30</td>
<td>0.15</td>
</tr>
<tr>
<td>40</td>
<td>0.23</td>
</tr>
<tr>
<td>50</td>
<td>0.61</td>
</tr>
<tr>
<td>60</td>
<td>0.94</td>
</tr>
<tr>
<td>70</td>
<td>0.83</td>
</tr>
<tr>
<td>80</td>
<td>0.53</td>
</tr>
<tr>
<td>90</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Table 2-5 Frequency Factor for Individual Pillar Having passing Through Joints (Esterhuizen et al., 2011)

<table>
<thead>
<tr>
<th>Fracture Frequency of Large Discontinuities per Pillar</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>&gt; 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Factor</td>
<td>0</td>
<td>0.1</td>
<td>0.18</td>
<td>0.26</td>
<td>0.39</td>
<td>0.63</td>
<td>0.86</td>
<td>0.95</td>
<td>1</td>
</tr>
</tbody>
</table>

Esterhuizen and Murphy (2011) also programmed S-Pillar software to help the US underground stone operators. NIOSH (2011) recommended that pillar safety factor must be at least 1.8 and the width-to-height ratio of the pillar must be larger than 0.8. The constructed factor of safety chart for the underground stone mine pillars can be seen in Figure – 2.9.

Kaiser and Kim (2008) introduced the S-shaped failure criterion to capture the brittle failure of intact rock and rock masses in low and high confinement environments. The S-shaped failure criterion indicates that the shape of failure envelopes for brittle rocks is confinement dependent, and there are three-linear failure regions: 1) low confinement zone, 2) transitional zone; 3) high confinement zone (Kaiser and Kim, 2014).
Figure 2.9 Factor of Safety Chart with Different W/H Ratios (S-Pillar, 2020)

Kaiser et al. (2011) introduced the modified S-shaped failure criterion based on the Hoek and Brown failure criterion. The modified version considers two regions: 1) the low confinement region (i.e. $\sigma_3 < \sigma_{ci}/10$) where the failure envelope is similar to the Hoek and Brown failure envelope; 2) high confinement region (i.e. $\sigma_3 > \sigma_{ci}/10$) where the degradation of the rock mass strength is limited. However, they could not interpret a clear transitional zone. They assumed transition from low to high confinement regions happens when the minimum principal stress is equal to the 10% of the rock mass UCS (Kaiser et al., 2011). Figure – 2.13 compares the modified Hoek and Brown (Figure – 2.10a) and S-shaped (Figure – 2.10b) failure envelopes with various GSI values.

Parallel to Martin and Maybee (2000), Kaiser et al. (2011) drawn similar conclusions around the hypothesis of “rock mass strength under the confinement is higher than derived from standard approaches”. Kaiser et al. (2011) used both elastic and plastic models. For the elastic analyses, they utilized BEM of Examine2D (Rocscience, 2010) to find stresses on two different pillars with the width-to-height ratio of 0.5 and 2.0. The results exhibited that the slender pillar has higher field stresses with near-zero confinement. However, the pillar having the width-to-height ratio of 2.0 has confinement around 35 MPa. Later, the plastic modeling showed that if the pillar is modeled with the modified S-shaped failure criterion, when the width-to-height ratio is equal and greater than 1.5 and 2.0, the stress-strain behavior is experiencing the strain-hardening (Figure –
2.11). Kaiser et al. (2011) claimed that the general Hoek and Brown failure criterion is conservative and pillars have more potential to bear loads.

![Figure 2.10 Comparison of (a) Hoek and Brown Empirical Failure Criteria and (b) Modified S-shaped Criteria with Various GSI values (Kaiser et al., 2011)](image1)

Figure 2.10 Comparison of (a) Hoek and Brown Empirical Failure Criteria and (b) Modified S-shaped Criteria with Various GSI values (Kaiser et al., 2011)

![Figure 2.11 Stress-Strain Behavior Comparison with Hoek and Brown and Modified S-shaped Failure Criteria (Kaiser et al., 2011)](image2)

Figure 2.11 Stress-Strain Behavior Comparison with Hoek and Brown and Modified S-shaped Failure Criteria (Kaiser et al., 2011)

Even though the mentioned continuum numerical modeling approaches improved the understanding of pillar mechanics, they have the same disadvantage with empirically estimated pillar strength methodologies, inability to simulate discrete discontinuities explicitly. In this respect, the synthetic rock mass (SRM) approach was developed to assess the rock mass strength by explicitly modeling the influence of discontinuities (Ivars et al., 2007, 2008, 2011; Pierce et al.,
Later, the SRM approach became a widely accepted approach to model naturallyjointed rock masses with the generation of intact rock with the bonded particle models (BPM) in PFC, and implementation of the DFNs approach. Many researchers followed similar procedures with different numerical codes to study rock mass or pillar mechanical behaviors (Elmo and Stead, 2010; Zhang et al., 2014; Bahrani and Kaiser, 2015; Farahmand et al., 2015; Stavrou and Vazaious, 2017; Wang and Cai, 2020). The conceptual explanation of the SRM application can be seen in Figure – 2.12 with the utilization of UDEC.

Figure 2.12 Conceptual Visualization of the SRM Approach: (a) The Calibrated Bonded Particle Model against Lab Results, (b) Generated DFN Model from the Field Surveys, (c) Synthetic Rock Mass with the Integration of BPM and DFNs

Cundall et al. (2008) simulated jointed rock mass strengths to account for the size effect of the rock mass strength using the PFC3D and SRM approach. They generated the rock mass model with PFC3D and DFNs. They sampled the higher dimensioned SRM model into smaller dimensions (Figure – 2.13). Then, they simulated uniaxial tests to predict the strengths of sampled rock blocks. They demonstrated that the SRM approach can be used to simulate size effects seen in the practice. However, Cundall et al. (2008) noted that great caution must be taken since the large variability in the strength of blocks is expected due to the spatial variation in the DFNs.
Elmo and Stead (2010) used a hybrid approach of the finite-discrete element method (FDEM) to capture progressive failure of pillars with natural fractures. The failure of pillars with the natural fractures is studied as a function of joint orientation and fracture length to estimate pillar strength with the application of DFNs. Three failure modes are obtained as a function of DFNs’ dip angle: 1) splitting; 2) shearing; 3) rotating the fractured blocks along the discontinuity planes. Furthermore, Elmo and Stead (2010) carried out series of numerical tests on the mine pillars by changing the tested rock volumes. Then, they attributed the uncertainty in predicting the pillar strength to the influence of the natural fractures. Pillar strength as a function of pillar size simulated by Elmo and Stead (2010) can be seen in Figure – 2.14. They proposed a systematical method to perform rock mass characterization with the SRM approach of hybrid FDEM.

![Figure 2.13 Sampled SRM into Smaller Rock Blocks from Higher Dimensions (A) to Lowers (D) (Adopted after Cundall et al., 2008)](image)

![Figure 2.14 Scale Effects on the Pillar Strength (Adopted after Elmo and Stead, 2010)](image)
While the numerical simulations are utilized to assess the strength of pillars and rock masses, they also enable researchers to investigate pillar failure mechanisms. The following section is introducing the pillar failure mechanisms captured by the field observations and numerical modeling techniques.

2.4. Pillar Failure Mechanics

Brady and Brown (2004) categorized the failure modes of pillars into five different mechanisms/modes: 1) the spalling along the edge of the pillars (Figure – 2.15a); 2) the shearing of a large discontinuity in the pillar (Figure – 2.15b); 3) the internal splitting as a tensile failure due to soft partings in the pillar (Figure – 2.15c); 4) the toppling and sliding like failure due to the weakness planes inside the pillar (Figure – 2.15d and – 2.15e). Details on the observed failure modes are discussed as stress and structurally controlled failures in the following sections.

![Failure Modes of Pillars](image)

*Figure 2.15 Failure Modes of Pillars (Brady and Brown, 2004)*

2.4.1. Stress-Controlled Failure

The failure mechanism of the hard-rock pillars is generally more brittle than ductile. The brittle failure mechanism manifests itself in the field as a crushing and spalling of pillar ribs because of the lack of confinement in this zone (Lunder and Pakalnis, 1997; Hajiabdulmajid et al., 2000; Martin and Maybee. 2000; Kaiser et al., 2011; Renani and Martin, 2018). Spalling is defined as the splitting of the pillar ribs parallel to the direction of major principle stress (Stacey, 1981; Elmo and Stead, 2010; Esterhuizen et al., 2011). Esterhuizen et al. (2011) indicated that this failure behavior is a progressive mechanism starting in the pillar corners and ending up within the core of the pillar.

Roberts et al. (2007) published following qualitative rating system to categorize pillar damage (Figure 2.16): 1) there is no stress-induced fracturing developing around the pillar; 2) the minimum amount of spalling in the corners are developed; 3) signs of spalling can be seen with the development of fractures; 4) the hour-glass shape is visible that the fractures opening and spalling
occurs with the fractures having greater length than half of the pillar height; 5) the hour-glass shape is well-developed with the massive spalling in the corners; 6) the pillar is totally failed with the extreme hour-glass shape and massive block loss.

Figure 2.16 Stages of Pillar Loading: 1) Intact Pillar; 2) Minimum Amount of Spalling in The Corners; 3) Signs of Spalling with Fractures Having Length up to Half of Pillar Height; 4) Initial Signs of Hour-Glassing Shape with Spallings on the Corners; 5) Well-Developed Hour-Glass Shape with Open Fractures and Massive Spalling; 6) Failed pillar with Extreme Neckening and Massive Block Loss

Esterhuizen et al. (2011) demonstrated that the tensile failure mainly governs the failure of slender pillars and a combination of tensile and shear failure mechanism governs the failure of square and squat pillars (the width-to-height ratio is greater than 1.5) (Figure – 2.17). Also, Esterhuizen et al. (2011) indicated that the pillar rib spalling is observed when the average pillar stress is 10% of the intact rock UCS.

Figure 2.17 Failure Mechanisms in the Pillar with Different W/H Ratios (Esterhuizen et al., 2011)
2.4.2. Structurally-Controlled Failure

The rock masses where the geological structures naturally exist can control the failure of a pillar. These geological structures act as failure planes within the rock bodies that the rock blocks can slide, split or rotate according to the kinematical equilibriums (Nordlund et al., 1995; Elmo and Stead, 2010). Nordlund et al. (1995) studied the failure mechanism of a pillar with a large discontinuity by establishing the kinematical relations including joint plane dip angle ($\theta$), joint plane friction angle ($\phi_j$) and the pillar contacts’ friction angle ($\phi_c$) (Figure – 2.18). They indicated that the kinematical equilibrium cannot be satisfied if the total of joint plane friction and the pillar contact angle is less than the dip angle of the fracture plane. However, if the summation is greater than the dip angle or the joint friction angle, the failure can be observed in both intact rock and along the joint plane. Also, when the dip angle of the discontinuity is higher than the critical joint dip angle (i.e. the angle defining the sliding on the joint plane or intact rock failure), the expected failure mechanism is the rotation of blocks and separation of pillar contacts from the host rock.

![Figure 2.18 Pillar Model Explaining the Used Angle Terms (Nordlund et al., 1995)](image)

Apart from the failure criteria for the jointed system, the fundamental concern of the researchers is to understand how the parameters such as joint density (i.e. fracture frequency), joint dip angle, joint persistency affect the pillar strength. Researchers demonstrated that U-Shaped strength changes on the pillar strength exist if the dip angle of joint systematically increases (Iannacchione, 1999; Iannacchione et al., 2002; Esterhuizen, 1999; Esterhuizen et al., 2008; Elmo and Stead, 2010; Esterhuizen et al., 2011; Zhang et al., 2014). Figure – 2.8 is one of the examples of how joint dip angle affects the pillar strength. Also, the effect of persistency, in other words, the length of the discontinuities along the pillar, is discussed by Elmo and Stead (2010) and Zhang et al.
(2014) that the inverse relationship is established between the length of fractures and pillar strength.

From the field observations and numerical simulations, Esterhuizen et al. (2011) concluded that slender type pillars (the width-to-height ratio is 0.5) are severely affected by the discontinuities. They observed that 65% of the surveyed mines have the major discontinuity sets and 7 out of 18 failed pillar cases were found to be caused by the large, angular discontinuities. Figure – 2.19 represents a structurally controlled failure at one of the limestone mines surveyed by Esterhuizen et al. (2011). The red downward triangle indicates the failure plane and the loss of the volume in the pillar.

*Figure 2.19 Pillar Rib Loss due to Large, Angular Discontinuity (Esterhuizen et al., 2011)*
Chapter 3 – Methodology

In the fractured rock mass, discrete discontinuities might influence rock mass behavior slightly or significantly depending on the density of discontinuities within the representative rock mass volume. In this thesis, the influence of naturally existing joint sets on the limestone mine pillar mechanics is studied by the SRM approach with the BPM and DFNs. The following sections explain the methodology in detail.

3.1. Standard Size Laboratory Specimen Calibration

In the study, Hoek and Brown strength criteria (Equation – 16) is used to derive Mohr-Coulomb equivalent friction and cohesive strength parameters (Equation – 21 and Equation – 22). Then, 0.05 m wide laboratory size rock specimen with the width-to-height ratio of 0.5 is calibrated with the parametric studies to simulate intact rock stress-strain behavior in the numerical code.

3.2. Generation of Bonded Particle Model with Voronoi-Trigon Discretization

BPM logic is implemented using UDEC to represent rock matrix as an assembly of Voronoi-Trigon Blocks where the explicit fracturing of the intact rock will be simulated along the trigon block boundaries. During the generation of BPM, size of the rock block is increased from standard laboratory scale by following the scaling equation of Hoek and Brown (1980) (Equation - 27). It should be noted that this equation is valid for sample sizes ranging from 10 mm to 200 mm.

\[
Rock Block Strength = \sigma_{50} \cdot \left( \frac{50}{\text{width of target rock block}} \right)^{0.18}
\]  

(27)

In the BPM, the strength reduction from Equation – 27 is satisfied by calibrating the micro-properties (i.e. contact strength properties such as friction or cohesion) of the Voronoi-Trigon Tessellation. The simulated macro-properties (i.e. target strength properties), such as friction and cohesive strength, are captured with a series of test configurations constructed in UDEC: 1) uniaxial compressive strength test; 2) triaxial or confined uniaxial compressive strength test; 3) Brazilian indirect tensile strength test. Voronoi-Trigon contact properties are named as micro-properties in the following sections of the thesis; micro-cohesion indicates Voronoi-Trigon contacts’ cohesive strength component while macro-cohesion is the cohesion of the rock sample. Calibration procedures proposed by Potyondy and Cundall (2004), Christianson et al. (2006),
Kazerani and Zhao (2010), Gao and Stead (2013), and Ghazvinian et al. (2014) are considered during this study, and the following procedure is constructed.

### 3.2.1. Trigon Contact Property Calibration Procedure

1. **Grain Size Determination:** The first step is the selection of the grain size of the numerical models. According to the International Society for Rock Mechanics and Rock Engineering Suggested Methods (2007), in laboratory testing, the ratio of 10:1 must be satisfied with the diameter of the specimen and the largest grain size in the rock specimen. Previous studies also supported taking this ratio as a reference for selecting element size in numerical simulations (Vardar et al., 2019). Hence, the largest Voronoi-Trigon Block edge size is set to be smaller or equal to 10% of the simulated rock mass sample width.

2. **Elastic Property Calibration:** The second step is to capture the macro elastic properties. In BPMs, the macro-elastic properties of the simulated rock material (i.e. Young’s Modulus, $E$ and Poisson’s Ratio, $\nu$), are controlled by Voronoi-Trigon Block contact normal and shear stiffnesses (Potyondy and Cundal, 2004; Kazerani and Zhao, 2010; Ghazvinian et al., 2014). Normal and shear stiffness parameters must also be selected to ensure numerical stability. Equation – 28 recommended in UDEC User Manuals (2019) is used to compute contact stiffnesses.

$$Joint \ Normal \ Stiffness, \ k_n = n \cdot \left[ \frac{K + \frac{4}{3} \cdot G}{\Delta z_{min}} \right]; n \in [0,10]\quad (28)$$

The values of $K$, $G$, and $\Delta z_{min}$ is the bulk and shear modulus, and the smallest length of the adjoining zones. The calculation of bulk and shear modulus is followed by Equations – 29 and – 30.

$$K, \ Bulk \ Modulus = \frac{E}{3 \cdot (1 - 2 \cdot \nu)} \quad (29)$$

$$G, \ Shear \ Modulus = \frac{E}{2 \cdot (1 + \nu)} \quad (30)$$

To simulate accurate Poisson’s Ratio of a rock mass sample, the shear to normal stiffness ratio of the Voronoi-Trigon contact must be calibrated. Kazerani and Zhao (2010) indicated that the stiffness ratio should be equal to the ratio of shear modulus to Young’s modulus.
of the simulated rock material to capture rock specimen’s Poisson’s Ratio when the rigid blocks are utilized. Next, Young’s Modulus can be calibrated by adjusting contact normal stiffness magnitude while keeping the stiffness ratio constant.

3. **Tensile Strength:** The third step of the calibration procedure is to employ the correct micro-tensile strength property. The calibration of the macro-tensile strength response of the rock material is achieved by adjusting the micro-tensile strength while keeping the other micro-properties constant in the Brazilian test configuration.

4. **Frictional Strength:** The fourth step is to calibrate the micro-friction angle of the Voronoi-Trigon contacts. The macro-friction angle of a rock sample is captured by adjusting the micro-friction value. Other micro-properties are held constant. During the calibration, triaxial compressive strength test boundary conditions are applied to the simulated sample. The confinement levels should be in the range of $0 < \sigma_3 < \sigma_1/10$ to capture low- and high-confinement environments as discussed by Diederichs et al. (2004).

5. **Cohesive Strength:** The next step of the calibration process is to adjust the micro-cohesion of the Voronoi-Trigon contacts to simulate the macro-cohesion of the rock material accurately. The same procedure must be followed with the micro-friction calibration. Also, the micro-cohesion to micro-tension strength ratio should be adjusted to simulate the brittleness of the material and to the crack initiation thresholds realistically. This ratio affects the peak strength of the specimen as shown by Ghazvinian et al. (2014).

6. **Full Stress-Strain Behavior:** Stress-strain behavior and final macro properties of the simulated rock mass sample should be verified as a final step of the calibration process.

To ensure the consistency of the model response with changing model geometries, micro-, and macro-properties, an automated post-processing scheme is also constructed to calculate the critical model response parameters. The post-processing scheme is identical to the one used by Christianson et al. (2006) to simulate the mechanical response of lithophyhsal tuff. The calculations steps can be found as follows:

1. **Peak Strength** ($UCS, MPa$): It is the value of maximum axial stress on the simulated stress-strain curve.

2. **Young’s Modulus** ($E, GPa$): It is calculated at 50% of the peak strength, by dividing the axial stress with axial strain.
3. **Poisson’s Ratio** ($\nu$): It is calculated at 50% of the peak strength by Equation – 31.

$$\nu = \frac{\left(1 - \frac{volumetric\ strain}{axial\ strain}\right)}{\left(2 - \frac{volumetric\ strain}{axial\ strain}\right)}$$ (31)

4. **Angle of Internal Friction** ($\phi$): The coefficient of friction is the slope of major and minor principle stress graph ($\sigma_1$ vs $\sigma_3$) constructed from the simulated triaxial compressive strength tests. Equation – 32 is used to calculate the internal friction angle.

$$\phi = \arcsin\left(\frac{\frac{\sigma_1}{\sigma_3} - 1}{\frac{\sigma_1}{\sigma_3} + 1}\right) \cdot \frac{180}{\pi}$$ (32)

5. **Cohesion** ($c, MPa$): The cohesion of the model is calculated from Equation – 33.

$$c = UCS \cdot \frac{1 - \sin (\phi)}{2 \cdot \cos (\phi)}$$ (33)

3.3. **Discrete Fracture Networks Approach**

In this section, a transitional phase from BPM to SRM is discussed with the implementation of DFNs generated from the stone mine field survey database. DFN generation requires a stochastic approach, and knowledge in statistical analysis to prevent biases that would result in an inaccurate representation of the rock mass. Brady and Brown (2004) categorized the geological mapping methods to eliminate the biases: 1) Spot Mapping; 2) Lineal Mapping; 3) Areal Mapping. They also proposed a systematical method to carry out the technique of scanline survey to count discontinuities. Fortunately, recent developments in sensor technologies, new data collection systems (i.e. laser scanners and photogrammetry), and data analysis software allow researchers to gather a large amount of data quickly and accurately and reduce the potential biases. Slaker (2015), Vazaious et al. (2014) and Monsalve et al. (2019) spent great effort to use these laser scanning and photogrammetry methods to map geological structures, characterize the rock mass, monitor rock mass deformation, and develop DFNs for advanced distinct model analyses. Hence, in the
following section, to understand the nature of DFNs, the basic terms are explained together with the DFNs generation procedure.

3.3.1. Features of Discrete Fracture Networks

The term fracture or joint is defined as a thin natural planar crack that is larger than the grain size of a rock (Brady and Brown, 2004; Suppe, 2005). Unlike the faults, joints do not experience any displacement at their planes. Brady and Brown (2004) indicated that the parallel joints are called joint sets while the intersection of more than one joint sets forms a joint system. In the modeling perspective, a single fracture or the joint is represented as a line segment and a planar disc depending on the dimension of the problem, and, representation of numerous fractures in a systematic way is called as DFNs (Itasca, 2019).

With the field surveying, either in conventional or advanced methods, the lower and upper boundary of the parameters of discontinuity sets can be estimated, and the distribution of the field discontinuity parameters can be fit to appropriate statistical models. Then these statistical models can be used to construct stochastic DFNs (Itasca, 2019). DFNs of a rock mass is represented by various parameters: fracture density size, size/trace length, orientation, and position. The basic density parameters are listed by Itasca (2019) as follows:

1. **Fracture Frequency** \( (P_{10}, \frac{1}{m}) \): It is defined as the number of fractures that intercepts the unit length of a scanline.

2. **Fracture Mass Density on Outcrop** \( (P_{21}, \frac{m}{m^2}) \): Total fracture length per unit area of the sampling location.

3. **Fracture Mass Density** \( (d_m, P_{32}, \frac{m^2}{m^3}) \): It is the total fracture surface area per unit volume of the sampling medium.

4. **Fracture Center Density** \( (d_c, P_{30}, \frac{1}{m^3}) \): It is the definition of the number of fracture centers per the unit volume of the rock medium.

Fracture size density, the combination of fracture trace length, and density is an important parameter affecting the DFNs model generations. It controls the number of fractures per unit volume \( (n(l)) \) that their size should be in the range of \([l, l + dl]\), where \( l \) and \( dl \) are the size and size increment of the fractures, respectively. The general knowledge indicates that fracture size
density distribution is expressed as a negative-power law distribution functions in most of the field cases as can be seen in Equation – 34 (Priest and Hudson, 1981; Elmo, 2006; Vazaious et al., 2017, Monsalve et al., 2019).

\[ n(l) = \alpha \cdot l^{-a} \]  

(34)

where \( a \) is the scaling exponent and \( \alpha \) is the density term of the model. The scaling exponent is defined by the ratio of smaller trace length to the larger trace length of the discontinuities among the discontinuity system. If the scaling exponent increases, the proportion of the small fractures increases relative to the larger dimensioned fractures. This behavior is explained with the conceptual models. From Figure – 3.1a to – 3.1d, the scaling exponent is assigned to the values of 2, 3, 4, and 10. The domination of large fractures can be observed when the scaling exponent is small in Figure – 3.1.

Figure 3.1 Domination of Large and Small Size Fractures as a Function of Scaling Exponent: a) Scaling Exponent is 2; b) Scaling Exponent is 3; c) Scaling Exponent is 4; d) Scaling Exponent is 100

The orientation of a discontinuity set influences the mechanical behavior of a rock mass. The orientation is represented by the dip angle and dip direction of the discontinuity. Dip angle is the maximum declination of the discontinuity plane with respect to the horizontal axis while the dip
direction is the measured orientation of the discontinuities clockwise from the true north (Brady and Brown, 2004). Visual representation of the dip angle and the dip direction is shown in Figure – 3.2.

![Figure 3.2 Representation of Dip and Dip Angle (Brady and Brown, 2004)](image)

**3.3.2. Discrete Fracture Network Generation Steps**

Steps for generation of a DFNs realization, in a two-dimensional representation of rock masses, are listed below:

1. The input parameters must be derived with the associated distributions to stochastically generate DFNs. Input parameters can be listed as: 1) fracture frequency, $P_{10}$; 2) trace length; 3) orientation, and 4) position.

2. The DFNs generation scheme is generally a random Poisson model, and the user-defined biases could be introduced and it could lead to the misrepresentation of the rock mass. Hence, validation is required. To validate and to successfully generate DFNs, either GSI table visualizations should be used or deterministic DFNs should be generated to restrict the randomization. Also, the realistic representation of the rock mass can be achieved with the numerous DFNs realizations to generate a database.

3. The database of realizations (for modeling purposes) must be simplified to reduce the uncertainties by pruning or combining the relatively closely spaced fractures as one fracture.

4. Every DFNs, in the database, should be implemented into the pillar geometry to calculate the distribution of block areas resulted from stochastic DFNs realizations.
5. The last step of the DFNs generation is to validate the realizations. The distribution of the block areas obtained in Step – 4 should be used to back-calculate the GSI value from the table published by Cai et al. (2004). Later, the back-calculated GSI value from the modeling should be compared against the initial GSI value determined in Step – 2. This comparison should be done for every single DFNs realizations. If a good fit cannot be found with the existing DFNs realizations, the number of realizations must be increased until the good match is found.

The next chapter discussed the sampling methodology from DFNs to study rock mass behavior with the SRM approach.

3.4. Synthetic Rock Mass Approach: Sampling Process from the Discrete Fracture Networks

Ivars et al. (2011) suggested that grid size should be the same for both laboratory and field-scale models to eliminate the influence of zone size on the model response. Unfortunately, using the same size discretization in laboratory specimen (i.e. 200 mm x 400 mm) and field size limestone pillars (i.e. 15 m x 30 m) found to be impractical due to excessive computer memory requirement. A rational procedure to scale simulated samples from laboratory to field size, a homogenization process, is introduced. This homogenization process can be thought as representative elementary volume (REV). The stress-strain response of the jointed pillar model is simulated with a new joint-free BPM with a larger particle size to study scale effects. The main advantage of utilizing this methodology is the reduced modeling time when simulating specimens from laboratory size to field size.

During the development of SRM, the DFNs are generated from the S-Pillar Database. From the stochastically generated DFNs model, the multi-stage upscaling methodology with the homogenization process at the interim stages is applied to establish rock block strength from laboratory to field. Figure – 3.3 visualizes the up-scaling process. Figure – 3.3a is the first BPM (\( w = 0.2; W/H = 0.5 \)) calibrated to simulate limestone rocks’ mechanical behavior while Figure – 3.3b is the first SRM model, where model width is 0.3 m (\( W/H = 0.5 \)), constructed by the integration of the BPM and DFNs. Models represented in Figure – 3.3a, 3.3b, and 3.3c are identical to each other in terms of particle size and mechanical properties. The model represented in Figure – 3.3d is, a new BPM with larger particle sizes that has the same stress-strain response as the model represented in Figure – 3.3c (\( w = 0.5; W/H = 0.5 \)). The new 0.5 m wide BPM is established.
with the reduced Trigon Contact strength parameters and particle size of 0.05 m (Figure – 3.3d) to capture calculated strength value from 0.5 m wide SRM model having 0.02 m particle size (Figure – 3.3c). Later, homogenized BPM properties are directly implemented to model having the width of 0.75 m (Figure – 3.3e) and 1.25 m (Figure – 3.3f) with the W/H of 0.5. Hence, the up-scaling process is carried up to 15.0 m wide pillars.

![Figure 3.3 The representation of multi-stage up-scaling operation: a) BPM having 0.2 m width; b) SRM having 0.3 m width; c) SRM having 0.5 m width; d) Homogenized BPM having width of 0.5 m; e) SRM having width of 0.75 m; f) SRM having width of 1.25 m; g) Homogenized BPM having width of 1.25 m]

The four-stage methodology discussed above is applied to numerically assess the field size limestone mine pillars’ strength and failure mechanism. The flow chart for this methodology can be seen in Figure – 3.4.
Figure 3.4 Flow Chart of Proposed Methodology
Chapter 4 – S-Pillar Database Analyses

In the United States, S-Pillar software is developed to assist in the design of stable pillars for room-and-pillar workings in underground limestone mines (Esterhuizen et al., 2011). Esterhuizen et al. (2011) collected operational information and pillar performances from 34 different limestone mines in the Eastern and Midwestern US. The collected information, later, was used to construct the S-Pillar Database containing the following information: 1) Mechanical properties of limestone mine rocks (i.e. Young’s Modulus, Poisson’s Ratio); 2) Geotechnical field properties to characterize limestone rock masses. In this chapter, statistical analysis of the S-Pillar database is carried out to derive input parameters for the numerical simulations.

4.1. Classification of the S-Pillar Database

In the S-Pillar Database, there is a total of 187 rock samples tested uniaxially with the constant width-to-height ratio of 0.5, and there is a total of 13 different limestone geological rock formations classified into three different categories according to intact rock strength: 1) low; 2) medium; 3) high. In Table – 4.1, the strength classification of limestone mine rocks with respect to their formation is summarized (Esterhuizen et al., 2011) while Figure – 4.1 visualizes the intact rock strength distribution for each strength group.

Table 4-1 Strength Classification of Limestone Mine Rocks with their Formations (Esterhuizen et al., 2011)

<table>
<thead>
<tr>
<th>Strength Group</th>
<th>Mean Strength, MPa</th>
<th>Strength Range, MPa</th>
<th>Limestone Formation Name Abr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower</td>
<td>88.45</td>
<td>44 – 144</td>
<td>Burlington, Salem, Galena-Plattsville</td>
</tr>
<tr>
<td>Medium</td>
<td>135.31</td>
<td>82 – 208</td>
<td>Chickamauga, Camp Nelson, Greenbrier, Monteagle, Newman (Upper), Platting, North Vennon, Vanport</td>
</tr>
<tr>
<td>High</td>
<td>214.21</td>
<td>152 – 302</td>
<td>Loyalhanna, Tyrone</td>
</tr>
</tbody>
</table>

The associated mechanical intact properties and the geotechnical parameters at rock mass scale later analyzed using One – Way Analysis of Variance (ANOVA). The main objective of carrying out this analysis is to reduce the number of parameters that are going to be used in the numerical simulations.
simulations. Each intact rock strength categories’ mechanical properties and the geotechnical parameters are compared to each other to understand whether they can be represented with one mean value or not. When the mean values of the two pair groups are found to be significant to each other, the paired groups are combined and one final mean value with the associated distribution is established. Figure – 4.2, – 4.3. and – 4.4 illustrate the distributions of Young’s Modulus, Poisson’s Ratio, and specific gravity, respectively.

*Figure 4.1 Distribution of Rock Strengths in the Categorical Groups*

*Figure 4.2 Young’s Modulus Raw Distribution*
The general and null hypothesis for the ANOVA are constructed as defined in Equation – 35. ANOVA assumptions are: 1) residual values of the errors ($e_i$) should have normally distributed and 2) all the populations should have equal variance ($\sigma^2$) (i.e. homogeneity). The corresponding null and alternative hypotheses for the ANOVA assumptions are identified in Equation – 36 and Equation – 37.

\[ N \rightarrow H_0: \mu_1 = \mu_2 = \mu_3 \quad \text{and} \quad A \rightarrow H_1: \mu_1 \neq \mu_2 \neq \mu_3 \quad (35) \]

\[ N_1 \rightarrow H_{01}: e_i \text{ Distributed Normally} \quad \text{and} \quad A_1 \quad \rightarrow H_{11}: e_i \text{ is NOT Distributed Normally} \quad (36) \]
\[ N_2 \rightarrow H_{02}: \sigma_1^2 = \sigma_2^2 = \sigma_3^2 \quad \text{and} \quad A_2 \rightarrow H_{22}: \sigma_1^2 \neq \sigma_2^2 \neq \sigma_3^2 \]  

(37)

JASP Version 0.13.1 developed by JASP Team (2020) is used to test ANOVA. First, one-way ANOVA model is fitted through the software to obtain p-values. The alpha (\(\alpha\)) level is selected as 0.05. The assumption of ANOVA is verified for homogeneity by Levene’s Test and normally distributed residual errors. In the cases where both assumptions are satisfied, the global F-Test is carried out to conclude if the test is significant or not. When the F-Test results concluded as significant, post-hoc analysis using Tukey and Bonferroni Comparison Test is applied to identify which group differs from others. If one of the ANOVA assumptions failed, a non-parametric ANOVA test with Kruskal-Wallis Test is carried out. Finally, Dunn’s Post-Hoc Comparison Test is applied to understand the different groups among others.

4.1.1. Analyses

Table – 4.2 shows the initial conclusions drawn from the S-Pillar Database with the ANOVA test. The assumptions of ANOVA are verified. While Table – 4.3 summarizes Levene’s Test results for homogeneity assumption, the normality assumption is visually examined with the Q-Q plots shown in Figure – 4.5.

<table>
<thead>
<tr>
<th>Mechanical Properties</th>
<th>ANOVA Results</th>
<th>F-Value</th>
<th>P-Value</th>
<th>Null or Alternative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniaxial Compressive Strength</td>
<td></td>
<td>33.375</td>
<td>&lt;0.001</td>
<td>Alternative Hypothesis</td>
</tr>
<tr>
<td>Young's Modulus</td>
<td></td>
<td>5.167</td>
<td>0.01</td>
<td>Alternative Hypothesis</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td></td>
<td>5.01</td>
<td>0.012</td>
<td>Alternative Hypothesis</td>
</tr>
<tr>
<td>Specific Gravity</td>
<td></td>
<td>6.049</td>
<td>0.005</td>
<td>Alternative Hypothesis</td>
</tr>
</tbody>
</table>

Table 4-3 Test Results for Homogeneity

<table>
<thead>
<tr>
<th>Mechanical Properties</th>
<th>Levene's Results</th>
<th>F-Value</th>
<th>P-Value</th>
<th>Null or Alternative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniaxial Compressive Strength</td>
<td></td>
<td>1.378</td>
<td>0.264</td>
<td>Null Hypothesis</td>
</tr>
<tr>
<td>Young's Modulus</td>
<td></td>
<td>0.012</td>
<td>0.969</td>
<td>Null Hypothesis</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td></td>
<td>2.448</td>
<td>0.1</td>
<td>Null Hypothesis</td>
</tr>
<tr>
<td>Specific Gravity</td>
<td></td>
<td>2.71</td>
<td>0.079</td>
<td>Null Hypothesis</td>
</tr>
</tbody>
</table>
From the graphical examination, Poisson’s Ratio and the specific gravity do not meet the normality assumption that non-parametric analysis is carried out with the help of the Kruskal-Willas Test. Results verified that the mean value for Poisson’s Ratio and specific gravity of the strength groups are different than each other (Table – 4.4).

Table 4-4 Kruskal-Willas Test Results for Models do not Meet the Second Assumption of ANOVA

<table>
<thead>
<tr>
<th>Mechanical Properties</th>
<th>Test Results</th>
<th>H-Statistic</th>
<th>P-Value</th>
<th>Null or Alternative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poisson’s Ratio</td>
<td></td>
<td>6.543</td>
<td>0.038</td>
<td>Alternative Hypothesis</td>
</tr>
<tr>
<td>Specific Gravity</td>
<td></td>
<td>12.902</td>
<td>0.002</td>
<td>Alternative Hypothesis</td>
</tr>
</tbody>
</table>

Later, to understand which strength groups’ mean properties are different than each other, the paired comparison is carried out. For Poisson’s ratio and specific gravity, Dunn’s Post-Hoc
Comparison is applied while the Standard Post-Hoc Comparisons are utilized for UCS and Young’s Modulus. Table – 4.5 summarizes the test results obtained from the comparison tests.

Table 4-5 Post-Hoc Comparisons between Strength Groups

<table>
<thead>
<tr>
<th>Mechanical Properties</th>
<th>Strength Groups</th>
<th>$P_{tukey}$</th>
<th>$P_{bonf}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Uniaxial Compressive Strength</strong></td>
<td>Low-Medium</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>Low-High</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td></td>
<td>Medium-High</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td><strong>Young's Modulus</strong></td>
<td>Low-Medium</td>
<td>0.156</td>
<td>0.199</td>
</tr>
<tr>
<td></td>
<td>Low-High</td>
<td>0.007</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>Medium-High</td>
<td>0.129</td>
<td>0.162</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mechanical Properties</th>
<th>Strength Groups</th>
<th>$P_{bonf}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Poisson's Ratio</strong></td>
<td>Low-Medium</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>Low-High</td>
<td>0.034</td>
</tr>
<tr>
<td></td>
<td>Medium-High</td>
<td>0.032</td>
</tr>
<tr>
<td><strong>Specific Gravity</strong></td>
<td>Low-Medium</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>Low-High</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td></td>
<td>Medium-High</td>
<td>0.202</td>
</tr>
</tbody>
</table>

4.1.2. Interpretation

The findings on each category and their parameters can be listed as follows:

- The ANOVA test assumptions for UCS are verified. According to Levene’s Test ($p-value = 0.264$), the homogeneity assumption is satisfied. From Figure – 4.5, the normality assumption is also met. Hence, there is a significant difference among each strength group in terms of UCS with $F = 33.375$, and $p-value < 0.001$. Standard post-hoc testing revealed that there are significant differences between pairs of categories.
- One-Way ANOVA Models for Young’s Modulus revealed that there is at least one mean value for one specific category different from others. Levene’s test showed that models satisfied the homogeneity assumption and Figure – 4.6 shows the normality assumption is
also met. The standard post-hoc comparisons test indicated that there is a significant difference between pairs.

- The models are fitted for Poisson’s Ratio that according to Levene’s Test, the homogeneity assumptions is met. However, the normality assumption cannot be achieved (Figure 4.7). Then, Kruskal-Wallis Test is applied where the results indicated that the ANOVA can proceed with Dunn’s Post-Hoc Comparison Test, and it is concluded that there is again a significant difference between pairs of groups.

- Finally, One – Way ANOVA is utilized for specific gravity. The global results indicated that there is at least one mean value that similar to the other ones. Levene’s test showed that the homogeneity assumption is verified with \( p – value = 0.079 \). Unfortunately, Figure – 4.8 revealed that the normality assumption is not satisfied. Then, with the application of the Kruskal-Wallis Test, it is revealed that the comparison can be done with Dunn’s Post-Hoc Comparison Test. It is found that there is a significant difference between mean values of low – medium and low – high strength intact rock groups while there is not a significant difference between medium – high strength rock groups with the \( p – values \) of 0.009, < 0.001 and 0.202.

4.1.3. Conclusions
One – Way ANOVA models are used to compare associated means of UCS, Young’s Modulus, Poisson’s Ratio and specific gravity within three different strength categories: 1) low, 2) medium, and 3) high. From these analyses, the following conclusions are drawn:

1. For the UCS parameter, the means of the three categories must be different than each other and the same classification as Esterhuizen et al. (2011) defined should be used.
2. For the Young’s Modulus, there must be three different mean values to represent the elastic property of intact rock specimens.
3. For Poisson’s Ratio, the low and medium strength rocks can share the same mean value while the high strength rocks must be studied with individual parameters.
4. For the specific gravity, high and medium strength rocks can share the same value while the low strength rocks need to have an individual parameter.

In terms of numerical simulations to capture laboratory size intact rock specimen’s behavior, three different calibration properties are required. The final mean values with the standard deviation of
these parameters can be seen in Table – 4.6 while Figure – 4.6 and Figure – 4.7 show the distributions for Poisson’s Ratio and specific gravity, respectively.

Table 4-6 Derived Reduced Input Parameters for Numerical Simulations for Each Strength Categories at S-Pillar Database

<table>
<thead>
<tr>
<th>Categories</th>
<th>Parameters</th>
<th>UCS, MPa (σ)</th>
<th>Young’s Modulus, GPa (σ)</th>
<th>Poisson’s Ratio (σ)</th>
<th>Specific Gravity (σ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Strength</td>
<td></td>
<td>88.45 (31.1)</td>
<td>46.56 (10.91)</td>
<td>0.12 (0.04)</td>
<td>2569 (130)</td>
</tr>
<tr>
<td>Medium Strength</td>
<td></td>
<td>135.31 (27.77)</td>
<td>54.49 (10.23)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>High Strength</td>
<td></td>
<td>214.21 (42.04)</td>
<td>62.84 (10.10)</td>
<td>0.17 (0.05)</td>
<td>2667 (62)</td>
</tr>
</tbody>
</table>

Figure 4.6 Distribution of Derived Poisson’s Ratio for Numerical Simulations for Each Strength Categories at S-Pillar Database

Figure 4.7 Distribution of Derived Specific Gravity for Numerical Simulations for Each Strength Categories at S-Pillar Database
4.2. Rock Mass Characterization

In the second part of the statistical analysis of the S-Pillar Database, required parameters to characterize underground limestone rock masses are compared. There is a total of 92 different field observations recorded from 34 different mines. The observations are logged on the “field data collection guidelines” which can be seen in Figure – 4.8. The field surveys are carried on two different perpendicular directions that two sides of the pillars are recorded as Side – A and Side – B (G.S. Esterhuizen, personal communication, September 12, 2019). The representative schematic of the field survey data collection on the pillar sides is illustrated in Figure – 4.9. Also, during the field surveys, 3 – meters wide and 2 – meters high window is used to count the vertical and horizontal (i.e. bedding planes) discontinuities and to characterize the discontinuity conditions. An example of this window from a field survey can be seen in Figure – 4.10.

![Figure 4.8 Field Data Collection Guideline](image)

![Figure 4.9 A Representative Pillar Model to Show the Field Survey Directions (The shaded areas just indicates the sides)](image)
During the S-Pillar Database analysis, the frequency of discontinuities, their associated distributions, joint conditions groundwater conditions, and intact rock strengths are used to calculated corresponding rock mass ratings with RMR$_{89}$ and GSI.

4.2.1. Rock Mass Rating Calculation

4.2.1.1. Intact Rock Strength Rating

The intact rock strength rating is calculated based on the rock strength categories established in Table – 4.5. Lowson and Bieniawski (2013) published the continuous rating graphs for intact rock strength rating, RQD, and combined rating for RQD and joint spacing. It is recognized by Lowson and Bieniawski (2013) that some users have been applying the rating system as the ratings are discrete although in practice ratings should be continuous. Hence, by sampling from the original rating graphs, new graphs are constructed again with the Lagrange Polynomial Interpolation to further advances in back- and forward-calculation of ratings. In the left of Figure – 4.11 original graph for intact rock strength rating can be seen while the graph in the right of Figure 4.11 shows the constructed graph.

By utilizing the graph in Figure – 4.11, the intact rock strength rating is calculated for S-Pillar Database. The descriptive statistics and the distribution of ratings can be seen in Table – 4.7 and Figure – 4.12, respectively.
Figure 4.11 Rating for Intact Rock Strength. Left: Original Graph Taken from Lowson and Bieniawski (2013). Right: Constructed Graph by Lagrange Polynomial Interpolation to Digitalize

Table 4-7 Descriptive Statistics of Intact Rock Strength Rating Calculations for Each Strength Category at S-Pillar Database

<table>
<thead>
<tr>
<th>Categories</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Strength</td>
<td>7.65</td>
<td>1.75</td>
<td>5.07</td>
<td>10.68</td>
</tr>
<tr>
<td>Medium Strength</td>
<td>10.21</td>
<td>1.35</td>
<td>7.35</td>
<td>13.44</td>
</tr>
<tr>
<td>High Strength</td>
<td>13.5</td>
<td>1.31</td>
<td>10.09</td>
<td>15</td>
</tr>
</tbody>
</table>

Figure 4.12 Distribution of Intact Rock Strength Rating Calculations for Each Strength Category at S-Pillar Database
4.2.1.2. RQD and Joint Spacing Rating

In this thesis, RQD and joint spacing are computed as combined ratings (Lowson and Bieniawski, 2013). Lowson and Bieniawski (2013) indicated that for the viability of the applications, it is hard to determine RQD from the tunnel or mining face since only the outcrop of the fractures can be observed. They recommended the usage of ‘fracture frequency’. Graph of combined rating is sampled, and the same graph is constructed again with Lagrange Polynomial Interpolation. The original and replicated graphs can be seen in Figure – 4.13 at left and right, respectively.

To estimate the discontinuity rating of different rock groups in the S-Pillar database, the following descriptive statistics are computed: 1) Table – 4.8 shows the joint frequencies per meter long scanline for both vertical and horizontal discontinuities, 2) Table – 4.9 shows vertical and horizontal trace length measurements, and 3) Figure – 4.14 exhibits the vertical joint frequency and horizontal joint frequency.

Table 4-8 Descriptive Statistics of Vertical and Horizontal Joint Frequency per 1-meter Scanline Calculations for Each Strength Category at S-Pillar Database (V for Vertical, H for Horizontal)
Table 4-9 Descriptive Statistics of Vertical and Horizontal Joint Trace Lengths Calculations for Each Strength Category at S-Pillar Database (All Units are in meter; V for Vertical, H for Horizontal)

<table>
<thead>
<tr>
<th>Categories</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>V</td>
<td>H</td>
<td>V</td>
<td>H</td>
</tr>
<tr>
<td>Low Strength</td>
<td>1.18</td>
<td>6.46</td>
<td>0.22</td>
<td>4.294</td>
</tr>
<tr>
<td>Medium Strength</td>
<td>1.52</td>
<td>5.32</td>
<td>0.9</td>
<td>4.39</td>
</tr>
<tr>
<td>High Strength</td>
<td>1.78</td>
<td>4.04</td>
<td>1.35</td>
<td>4.25</td>
</tr>
</tbody>
</table>

Figure 4.14 Distribution of Vertical and Horizontal Joint’s Frequency Calculations for Each Strength Category at S-Pillar Database on the Left and Right, respectively

The combined rating for RQD and joint spacing is calculated for each strength group. The descriptive statistics and distributions after calculations can be viewed in Table – 4.10 and Figure – 4.15, respectively. It is important to note that low-strength rock masses have the lowest intact rock strength but their combined rating is the highest.

Table 4-10 Descriptive Statistics of Combined Rating for RQD and Joint Spacing Calculations for Each Strength Category at S-Pillar Database

<table>
<thead>
<tr>
<th>Categories</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Strength</td>
<td>33.74</td>
<td>1.46</td>
<td>31.27</td>
<td>36.49</td>
</tr>
<tr>
<td>Medium Strength</td>
<td>31.29</td>
<td>2.83</td>
<td>22.42</td>
<td>36.21</td>
</tr>
<tr>
<td>High Strength</td>
<td>30.53</td>
<td>2.94</td>
<td>21.99</td>
<td>35.83</td>
</tr>
</tbody>
</table>
4.2.1.3. Joint Conditions Rating

S-Pillar Database joint condition ratings’ descriptive statistics and associated distributions can be seen in Table – 4.11 and Figure – 4.16, respectively.

*Table 4-11 Descriptive Statistics of Joint Condition Rating Calculations for Each Strength Category at S-Pillar Database*

<table>
<thead>
<tr>
<th>Categories</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Strength</td>
<td>26.84</td>
<td>2.49</td>
<td>19.25</td>
<td>29</td>
</tr>
<tr>
<td>Medium Strength</td>
<td>25.43</td>
<td>2.35</td>
<td>19.25</td>
<td>28.5</td>
</tr>
<tr>
<td>High Strength</td>
<td>25.46</td>
<td>2.27</td>
<td>18.5</td>
<td>29</td>
</tr>
</tbody>
</table>

*Figure 4.15 Distributions of Combined Rating for RQD and Joint Spacing Calculations for Each Strength Category at S-Pillar Database*

*Figure 4.16 Distribution of Joint Condition Rating Calculations for Each Category at S-Pillar Database*
4.2.1.4. Groundwater Rating

The S-Pillar database analysis revealed that the underground limestone mine environments are mostly in dry conditions. The statistics for groundwater rating can be seen in Table – 4.12.

Table 4-12 Descriptive Statistics of Groundwater Rating Calculations for Each Strength Category at S-Pillar Database

<table>
<thead>
<tr>
<th>Categories</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Strength</td>
<td>11.09</td>
<td>3.75</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>Medium Strength</td>
<td>14.23</td>
<td>2.14</td>
<td>7.5</td>
<td>15</td>
</tr>
<tr>
<td>High Strength</td>
<td>12.3</td>
<td>3.9</td>
<td>5</td>
<td>15</td>
</tr>
</tbody>
</table>

4.2.2. Conclusion

As a final step, RMR$_{89}$ is calculated. To numerically assess the GSI value, Equation – 38 is used (Hoek and Brown, 1997). The calculated RMR$_{89}$ and GSI statistics can be seen in Table – 4.13. Figure – 4.17 shows the distribution of RMR$_{89}$ and GSI values for each rock strength category in S-Pillar Database.

\[ RMR_{89} = GSI + 5 \]  (38)

Table 4-13 Descriptive Statistics for RMR89 Calculations for Each Strength Category at S-Pillar Database (square brackets indicates the GSI value)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Strength</td>
<td>79.33 [74.33]</td>
<td>4.58</td>
<td>68.69 [63.69]</td>
<td>88.14 [83.14]</td>
</tr>
<tr>
<td>Medium Strength</td>
<td>81.17 [76.17]</td>
<td>4.25</td>
<td>69.58 [64.58]</td>
<td>89.92 [84.92]</td>
</tr>
<tr>
<td>High Strength</td>
<td>81.79 [76.79]</td>
<td>5.21</td>
<td>73.32 [68.32]</td>
<td>91.51 [86.51]</td>
</tr>
</tbody>
</table>

Figure 4.17 Distribution of RMR89 and GSI Values for Each Strength Category at S-Pillar Database on the Left and Right, respectively
4.3. Mining Operational Measures
In S-Pillar Database, operational parameters are also included (Table – 4.14). It is found that 33% of the pillars are benched. Esterhuizen et al. (2011) reported that 18 individual pillars are classified as completely failed. The main causes of pillar failures are due to angular discontinuities and the stress elevation caused by benching (39% of each). 61% of all pillar failure cases are reported as structurally controlled failures. There is not any case where a pillar from a low strength rock mass was failed. The reported pillar failure cases are only observed in the medium and high strength rock masses. Table – 4.15 summarizes the failed pillar dimensions as height, width, and width-to-height ratio.

<table>
<thead>
<tr>
<th>Pillar Statistics</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height</td>
<td>11.1</td>
<td>6.17</td>
<td>4.82</td>
<td>37.98</td>
</tr>
<tr>
<td>Width</td>
<td>13.06</td>
<td>3.42</td>
<td>4.57</td>
<td>24.49</td>
</tr>
<tr>
<td>Width-to-Height Ratio</td>
<td>1.41</td>
<td>0.61</td>
<td>0.29</td>
<td>3.52</td>
</tr>
</tbody>
</table>

Table 4-14 Descriptive Statistics of Mining Operational Measures of S-Pillar Database (After Esterhuizen et al., 2011)

<table>
<thead>
<tr>
<th>Pillar Statistics</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height</td>
<td>9.93</td>
<td>2.97</td>
<td>3.7</td>
<td>15.2</td>
</tr>
<tr>
<td>Width</td>
<td>15.82</td>
<td>6.38</td>
<td>7.3</td>
<td>27.4</td>
</tr>
<tr>
<td>Width-to-Height Ratio</td>
<td>0.69</td>
<td>0.29</td>
<td>0.44</td>
<td>1.75</td>
</tr>
</tbody>
</table>

Table 4-15 Descriptive Statistics of Failed Pillar Dimensions of S-Pillar Database (After Esterhuizen et al., 2011)

4.4. Conclusions
In this chapter, statistical analyses of the S-Pillar database are conducted. To employ in the calibration of the numerical simulations, intact rock material properties are established. Also, the input parameters to generate DFNs realizations are derived from field surveillance data. Together with the input parameters derivation, the rock mass characterization of the US underground stone mine rock masses are carried out and quantified with the RMR89 and GSI values. Hence, established input parameters are derived to employ the methodology described in Chapter – 3.
Chapter 5 – Numerical Simulations

Two-dimensional Universal Distinct Element Code, UDEC, is utilized in this study. UDEC is frequently used to simulate rock mass with joint sets, and fracture networks in explicit solution schemes with the time-stepping algorithm (Itasca, 2020). Also, UDEC enables monitoring of large displacements at contacts that can represent natural fractures or tessellation algorithms to simulate intact rock structure. The rock can be modeled as either deformable elastic, elasto-plastic, or rigid blocks. It is found that UDEC is appropriate software to investigate the stone mine pillar mechanical behavior by explicit consideration of naturally existing joint sets.

5.1. Modeling

Figure 5.1 visualizes three possible geometry generation methods that can be utilized in the UDEC to simulate vertical loading of laboratory size intact rock specimen or field size pillar. In the first method (Figure 5.1a), horizontal and vertical construction joints are introduced to divide a massive block into domains to generate the final geometry. In the second method, vertical construction joints are only extended up to the pillar height (see Figure 5.1b). In the last method ‘block config cell’ command in the UDEC is used to generate three separate blocks with the desired shape and dimensions, then the bottom and upper blocks are connected with the intrinsic command of ‘block join-contact’ (J. Hazzard, personal communication, July, 9, 2020) (Figure 5.1c). It is found during this study that the third method is the most suitable geometry generation method because Voronoi-Trigon Tessellation and the DFNs cutting through the rock material is simplified by this method. Indirect tensile strength test model geometry can also be constructed using the third method (Figure 5.1c). However, for the Brazilian Tensile Strength Test configuration, the best results are obtained from the second method and applied for generating indirect tensile strength test geometry (Figure 5.1d).

The axial loading is simulated with the displacement boundary condition. The constant velocity of 0.01 \( m/s \) is selected as a constant loading rate (Kazernai and Zhao, 2010; Preston et al., 2013; Stavrou and Murphy, 2018; Vardar et al., 2019). The UDEC time-stepping algorithm constrains the velocities and the accelerations by adjusting the time-stepping value automatically (Itasca, 2019). Kazerani and Zhao (2010) explained that the UDEC algorithm adjusts 0.01 \( m/s \) velocity rate to \( 10^{-6} \ mm/step \), and in order to move upper platen 1 \( mm \), at least 1 – \( \text{million} \) step is
required. Therefore, the quasi-static state of the model is reserved until the end of the simulations with this loading rate, and the numerical oscillations are prevented.

Figure 5.1 Model Geometry Constructions: a) Massive Block Divided into Smaller Blocks with Horizontal and Vertical Going Through Joints; b) Massive Block Divided into Smaller Blocks with Horizontal Going through Construction Joints and Vertical Construction Cracks; c) Construction the Geometry from Three Blocks via Connecting Them; d) Constructing the Brazilian Tensile Test Geometry from Three Blocks via Connecting Them
Embedded programming language FLACish (FISH) is utilized to capture stress-strain behavior during the simulations. The total stress is computed in every step by using two different FISH routines: 1) stresses are obtained from the zone elements; 2) stress is calculated from the bottom boundary reaction forces. Therefore, it is ensured that the total stress is computed accurately.

Tensile strength is computed using Equation - 39 (Gao and Stead, 2013; Stavrou and Murphy, 2018). Maximum load during the failure is queried from the bottom boundary reaction forces.

\[
Tensile \text{Strength} = \frac{2 \cdot F_{\text{max}}}{\pi \cdot w \cdot t}
\]  

(39)

where \(F_{\text{max}}, w\) and \(t\) are the maximum loads at failure, the width of the sample, and the thickness of the sample (1.0 in two-dimensional simulations). Figure – 5.2 shows the boundary conditions and final model geometries. History points (data collection points) are also shown in Figure – 5.2a for compression and for Figure – 5.2b for indirect tension tests.

![Figure 5.2 Final Model Geometries and Boundary Conditions: a) Uniaxial Compressive Strength Test; b) Brazilian Tensile Strength Test](image)

The axial strain is calculated by collecting the axial displacement at the upper and bottom platen/pillar contacts. For lateral strain calculation, displacements are sampled from five points near the center rib of the specimen and averaged to calculate lateral expansion. Since the models are two-dimensional, the volumetric strain is estimated from the area calculation. Figure – 5.3 shows the steps to calculate volumetric strain in the numerical simulations.
5.2. Application of Proposed Methodology

5.2.1. Standard Size Laboratory Specimen Modeling

In the proposed up-scaling methodology, the minimum rock specimen having 50 mm width and 100 mm height is numerically calibrated against the laboratory test results. The input parameters derived in Chapter – 4.1.3 (Table – 4.6) are captured with the plastic zone elements via the Mohr-Coulomb Failure Criterion. Due to the lack of a triaxial compressive test in the S-Pillar Database, Equations – 21 and – 22 are implemented, and to calculate tensile strength, Equation – 20 is introduced. However, in order to estimate the Mohr-Coulomb equivalent parameters, it is required to estimate $m_i$, $m_b$, $s$, $a$, and $D$. The recommended values by Itasca (2020) can be found in Table – 5.1. The failure envelopes for Hoek and Brown, and Mohr-Coulomb equivalent with tensile cut-off are shown in Figure – 5.4, – 5.5, and – 5.6 for low, medium, and high strength categories. Also, Table – 5.2 shows the target or macro properties of simulated rock sample.

| Table 5-1 Utilized Hoek and Brown Failure Criterion Parameters (Itasca, 2020) |
|---|---|---|---|---|
| $m_i$ | $m_b$ | $s$ | $a$ | $D$ |
| 7    | 7    | 1   | 0.5 | 0   |
Figure 5.4 Low Strength Rocks Failure Hoek and Brown Failure Envelope and Mohr-Coulomb Equivalent Envelope with the Tensile Cut-off

Figure 5.5 Medium Strength Rocks Failure Hoek and Brown Failure Envelope and Mohr-Coulomb Equivalent Envelope with the Tensile Cut-off

Figure 5.6 High Strength Rocks Failure Hoek and Brown Failure Envelope and Mohr-Coulomb Equivalent Envelope with the Tensile Cut-off
Table 5-2 Calculated Mohr-Coulomb Equivalent Parameters from Hoek and Brown Failure Criterion

<table>
<thead>
<tr>
<th>MC Parameters</th>
<th>Strength Groups</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohesion, MPa</td>
<td></td>
<td>19.55</td>
<td>29.89</td>
<td>47.33</td>
</tr>
<tr>
<td>Friction, °</td>
<td></td>
<td>38.9</td>
<td>38.9</td>
<td>38.9</td>
</tr>
<tr>
<td>Tension, MPa</td>
<td></td>
<td>6.98</td>
<td>10.68</td>
<td>16.9</td>
</tr>
</tbody>
</table>

Later, these target properties are calibrated with the plastic zone elements. An iterative process is followed to calibrate elastic-perfectly plastic zone elements. The zone element properties and captured responses are summarized in Table – 5.3. The stress-strain behavior of the intact rock samples with various confinement rates can be seen in Figure – 5.7, – 5.8, and – 5.9 for low, medium, and high strength rocks. Also, unconfined stress-strain behavior and the tensile strength test results are illustrated in Figures – 5.10 and – 5.11.

Table 5-3 Utilized Zone Properties and Capture Model Responses

<table>
<thead>
<tr>
<th>Plastic Zone</th>
<th>Cohesion, MPa</th>
<th>Friction, °</th>
<th>Tensile, MPa</th>
<th>Young's Modulus, GPa</th>
<th>Poisson's Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS MS HS</td>
<td>20.5 31 50.3</td>
<td>39 40.3 39.3</td>
<td>7.26 10.6 16.5</td>
<td>48.5 58 66</td>
<td>0.12 0.12 0.17</td>
</tr>
<tr>
<td>LS MS HS</td>
<td>20.87 31.21 50.92</td>
<td>39.7 40 39.13</td>
<td>7.2 10.6 16.5</td>
<td>46.58 54.44 64.8</td>
<td>0.12 0.12 0.17</td>
</tr>
</tbody>
</table>

Figure 5.7 Low Strength Rocks Stress-Strain Behavior for Unconfined and Confined Uniaxial Loading Tests
Figure 5.8 Medium Strength Rocks Stress-Strain Behavior for Unconfined and Confined Uniaxial Loading Tests

Figure 5.9 High Strength Rocks Stress-Strain Behavior for Unconfined and Confined Uniaxial Loading Tests

Figure 5.10 Stress-Strain Behavior of All Strength Groups for Standard Size Laboratory Specimen

Figure 5.11 Brazilian Tensile Strength Test Results for Standard Size Laboratory Specimens
5.2.2. Bonded Particle Model Generation

Bonded particle models (also known as bonded block models, bonded grain models) are introduced in two- and three-dimensional particle flow codes (PFC2D and PFC3D) by Potyondy and Cundall (2004). The mechanical behavior of rock is captured in the numerical models by simulating intact rock matrix as dense packages of circular particles and failure as fracture propagation, simulated by slipping and sliding of the contacts. The application of the BPM is not limited to PFC, numerous researchers utilized this technique with the Voronoi Tessellation algorithms in UDEC or 3DEC (Christianson et al., 2006; Kazerani and Zhao, 2010; Preston et al., 2013; Ghazvinian et al., 2014; Stavrou and Murphy, 2018; Vardar et al., 2019). Voronoi Tessellation is the technique in which a rock matrix is simulated as the assembly of granular minerals.

There are two different tessellation algorithms available to simulate intact rock failure: 1) Conventional Voronoi and 2) Voronoi-Trigon. The Voronoi method employs randomly sized polygonal blocks within an intact portion of the medium to generate a bonded block models where the actual failure takes place at the boundaries of the blocks. Christianson et al. (2006) used the UDEC Voronoi model to simulate the mechanical behavior of lithophysal tuff. Numerical simulations revealed that the porous behavior of lithophysal tuff can be captured by Voronoi Tessellation logic. Later, Kazerani and Zhao (2010) employed UDEC-Voronoi models in rigid blocks to simulate brittle behavior in both strong and weak rock. In 3DEC, Ghazvinian et al. (2014) also studied the crack damage development in brittle rock mediums with the cohesion weakening – friction mobilization approach, which is introduced by Martin (1997) and Hajiabdolmajid et al. (2002). The Voronoi-Trigon Tessellation or named as “modified-Voronoi Logic” by Gao and Stead (2014) is the second method. In this method, trigon-shaped blocks are constructed via the Delaunay triangles resulting from the Voronoi polygons. Gao and Stead (2014) proposed the Voronoi-Trigon Tessellation model and listed the advantages of it as: 1) to capture realistic fracture propagation, 2) to correctly estimate the friction component of the rock materials, 3) to reduce the mesh-dependencies with the finer-tessellations and 4) to reduce block interlocking compared to conventional Voronoi Tessellation models. Voronoi-Trigon Tessellation method is selected to simulate both laboratory size rock sample and field size pillar mechanical behavior. Figure – 5.12 shows the difference between polygonal Voronoi and Voronoi-Trigon Tessellation. Figure – 5.12a exhibits the single polygon generated with the conventional Voronoi block. However, Figure – 5.12b visualizes the generated Voronoi-Trigon blocks based on the conventional Voronoi
polygons. Figure – 13 illustrates the general random discretization of a rock sample with Voronoi-Trigon Tessellation and Voronoi Tessellation.

Figure 5.12 The Blocks Created by Voronoi and Voronoi-Trigon Tessellation Logics: a) A pentagonal shape continuous block generated via Voronoi Logic, b) Five different blocks are generated from the Voronoi-Trigon Tessellation Logic

Figure 5.13 Random Discretization: a) Voronoi-Trigon Tessellation; b) Voronoi Tessellation

5.2.2.1. Equation of Motions in UDEC

In this study, UDEC is used to model rock matrix as an assembly of discrete blocks. Newton's second law defines the motion of the blocks in UDEC (Itasca, 2020). Since deformable blocks are utilized for this study, the resultant movements and displacements of each block are calculated from the gridpoints of the triangular finite-strain elements. Newton’s second law of motion can be written for a single mass as:

$$\frac{d\dot{u}}{dt} = \frac{F}{m} \quad (40)$$

where $\dot{u}$, $t$ and $m$ are velocities, time, and mass. The central difference scheme for the left-hand side of Equation – 40 at an arbitrary time of $t$ can be found as:
\[
\frac{d\dot{u}}{dt} = \left(\dot{u}^{(t+\Delta t)} - \dot{u}^{(t-\Delta t)}\right) / \Delta t \quad (41)
\]

The substitution of Equation – 41 into Equation – 40 results as:

\[
\dot{u}^{(t+\Delta t)} = \dot{u}^{(t-\Delta t)} + \frac{F^{(t)}}{m} \cdot \Delta t \quad (42)
\]

With velocities stored at the half-timestep point, it is possible to express displacements as:

\[
u^{(t+\Delta t)} = u^{(t)} + \dot{u}^{(t+\Delta t)} \cdot \Delta t \quad (43)
\]

When two-dimensional blocks involved in the solution, the velocities can be calculated as subject to several forces as well as gravity:

\[
\dot{u}_i^{(t+\frac{\Delta t}{2})} = \dot{u}_i^{(t-\frac{\Delta t}{2})} + \left(\frac{\Sigma F_i(t)}{m_i} + g_i\right) \cdot \Delta t \quad (44)
\]

\[
\dot{\theta}^{(t+\frac{\Delta t}{2})} = \dot{\theta}^{(t-\frac{\Delta t}{2})} + \left(\frac{\Sigma M_i(t)}{I_i}\right) \cdot \Delta t \quad (45)
\]

where \(\dot{\theta}, I, \Sigma M, u_i\) and \(g_i\) are the angular velocity of the block about centroid, the moment of inertia of block, total moment acting on the block, velocity components of block centroid, and components of gravitational acceleration. The indice, \(i\), noted in Equation – 44 and – 45 denote components in a Cartesian coordinate frame. The new block locations can be found from:

\[
x_i^{(t+\Delta t)} = x_i^{(t)} + u_i^{(t+\frac{\Delta t}{2})} \cdot \Delta t \quad (46)
\]

\[
\theta^{(t+\Delta t)} = \theta^{(t)} + \dot{\theta}^{(t+\frac{\Delta t}{2})} \cdot \Delta t \quad (47)
\]

where \(\theta\) and \(x_i\) rotation of block about centroid and coordinates of block centroid. However, it must be noted that the rotations are not stored. In fact, the incremental rotations are used as an update rule to position the block vertices. Consequently, the algorithm mentioned above can be summarized as follows. The instantaneous new block positions generate new contact forces in each timestep. Linear and angular accelerations of individual blocks are calculated from resultant forces.
and moments. Integration over increments in time results in the block velocities and displacements. The above-summarized steps are repeated until the state of equilibrium or one continuing failure results (Itasca, 2020).

5.2.2.2. Contact Constitutive Relations

In the contacts, at the direction perpendicular to the contact plane, stress-displacement relation is assumed linear and governed by the normal stiffness \((k_n)\):

\[
\Delta \sigma_n = -k_n \cdot \Delta u_n
\]  

(48)

where \(\Delta \sigma_n\) and \(\Delta u_n\) are the effective normal stress increment and the normal displacement increment. The effective normal stress of the contacts must not exceed the tensile strength (i.e. if \(\sigma_n < -T\), then \(\sigma_n = 0\)). In the block contacts where the direction is parallel to its plane, the shear stress \((\tau)\) is governed by the cohesive \((C)\) and frictional \((\phi)\) strength components:

\[
|\tau_s| \leq C + \sigma_n \cdot \tan \tan (\phi) = \tau_{max}
\]  

(49)

The shear stress is also controlled with the shear stress increment and elastic component of the incremental shear displacement:

\[
\Delta \tau_s = -k_s \cdot u_s^e
\]  

(50)

where \(u_s^e\) is the elastic component of the incremental shear displacement. The relationship between shear stress and maximum shear stress can be explained as:

\[
|\tau_s| \geq \tau_{max} \rightarrow \tau_s = \text{sign}(\Delta u_s) \cdot \tau_{max}
\]  

(51)

The visual representation of the contact constitutive model is shown in Figure – 5.14. The micro-properties of Voronoi-Trigon blocks and their geometries govern the rock medium's mechanical behavior in the macro-scale (Kazerani and Zhao, 2014). However, in order to capture the realistic material response from the Voronoi-Trigon Tessellation, a calibration of the micro-properties is necessary, which can be considered as one of the disadvantages. It is an iterative process that needs to be carried out with great care. In the following section, the macro-properties (i.e. target parameters to be captured by the micro-properties of Voronoi-Trigon Contacts) of the intact rock materials for three different strength groups (i.e. low, medium, and high) are introduced. Later, the micro-properties are calibrated against the target properties.
5.2.2.3. Micro-Property Calibration Procedure

To account for strength reduction due to size increment, Hoek and Brown Scaling Equation (1980) (Equation – 27) is employed. The reason for employing Equation – 27 is to increase the rock block volume to the largest accountable homogenous intact rock specimen. The target uniaxial compressive strength properties of each rock category are computed (Table – 5.4). Figure – 5.15 visualizes the intact rock material up-scaling process using Equation – 27.

Table 5-4 UCS Degradation Resulted from Equation - 27

<table>
<thead>
<tr>
<th></th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>UCS at 50 mm in width, MPa</td>
<td>88.45</td>
<td>135.31</td>
<td>214.21</td>
</tr>
<tr>
<td>UCS at 200 mm in width, MPa</td>
<td>68.92</td>
<td>105.43</td>
<td>166.90</td>
</tr>
</tbody>
</table>

Mohr- Coulomb Equivalent parameters and the tensile strength are computed using Equation – 20, - 21, and – 22 for the parameters listed in Table – 5.1. The Hoek and Brown and Mohr-Coulomb Equivalent failure envelopes with tension cut-off can be seen in Figures – 5.16, – 5.17, and – 5.18 for low, medium, and high strength rocks in the S-Pillar database. Table – 5.4 lists friction, cohesion, and tensile strength values for three strength rock groups. These properties are the macro-properties of the simulated rock sample. Since the BPM approach is used, micro-properties have to be calibrated to simulate macro-properties of the rock materials listed in Table – 5.4.
Figure 5.15 Intact Rock Up-Scaling Equation and Application in this Study (After Hoek and Brown, 1980)

Figure 5.16 Hoek and Brown and Equivalent Mohr-Coulomb Failure Envelope together with Tension Cut-off for Low Strength BPM
The micro-property calibration procedures published by Potyondy and Cundall (2004), Christianson et al. (2006), Kazerani and Zhao (2010), Gao and Stead (2013), and Ghazvinian et al. (2014) are followed to capture macro-properties listed in Table – 5.5.
Table 5-5 Derived Mohr-Coulomb Equivalent Parameters and Tensile Strengths

<table>
<thead>
<tr>
<th>Mohr-Coulomb Equivalent Parameters</th>
<th>Low Strength</th>
<th>Medium Strength</th>
<th>High Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohesion, MPa</td>
<td>15.25</td>
<td>23.20</td>
<td>36.88</td>
</tr>
<tr>
<td>Friction, MPa</td>
<td>38.91</td>
<td>38.91</td>
<td>38.91</td>
</tr>
<tr>
<td>Tension, MPa</td>
<td>5.45</td>
<td>8.32</td>
<td>13.17</td>
</tr>
</tbody>
</table>

a. Grain Size Determination

The grain size, or element size, is not an independent parameter that can be changed arbitrarily (Potyondy and Cundall, 2004). Potyondy and Cundall (2004) showed that the particle size controls the model resolution and it is directly related to the fracture toughness of the intact rock material in the PFC. Hence, in the multi-scale approaches, it is necessary to satisfy consistency in particle size (Ivars et al., 2011). Gao and Stead (2014) indicate that the influence of the fracture pattern can be eliminated in the smaller grain sizes such as 8.5% to 5% of the width of the sample in their research. In order to understand the effect of particle size on both uniaxial compressive strength and elastic responses of the model, the Voronoi-Trigon maximum edge size is increased from 5% to 25% of the specimen width. Figure – 5.19 shows the uniaxial compressive strength response as a function of particle size while Figure – 5.20 shows the macro-elastic properties of simulated rock samples.

Figure 5.19 Response of Model Uniaxial Compressive Strength as the Trigon Edge Size Changes
Figure 5.20 Response of Model Elastic Properties as the Trigon Edge Size Changes

Figure – 5.19 shows that there is no correlation between particle size and modeled strength. Voronoi-Trigon block size changes the failure patterns and increases the uncertainty in predicting the rock block strength. Particle size increase resulted in a reduction in Poisson’s Ratio, and an increase in Young’s modulus. As the size of the trigon blocks increased, Young’s Modulus of the blocks starts to govern the material behavior, and the influence of Voronoi-Trigon contact properties on macro model response diminishes. To satisfy the goal of this study, a 1/10 ratio between the rock specimen and the average particle size is assumed as suggested by ISRM (2007) and employed by Vardar et al. (2019) during the BPM generation processes.

b. Elastic Property Analysis

Two different parametric studies are carried out to examine the macro-response of Young’s Modulus and Poisson’s Ratio. To visualize the influence of joint stiffness ratio (joint shear stiffness divided by normal stiffness) on the model response, the stiffness ratio is increased from 0.30 to 0.50 where normal stiffness is kept constant. Later, another parametric study is carried out to understand the effect of normal stiffness on the model elastic response by changing the normal stiffness from 28,000 $GPa/m$ to 60,000 $GPa/m$, and keeping the stiffness ratio constant at 0.45. The elastic responses on the macro-scale of the rock material can be observed in Figure – 5.21 and Figure – 5.22.
As it was discussed by Diederichs (2000) and Potyondy and Cundall (2004), Poisson’s Ratio is directly affected by the joint stiffness ratio. In UDEC, Kazerani and Zhao (2010) showed that Poisson’s Ratio changes as a function of stiffness ratio, and an increase in this ratio results in a decrease of Poisson’s Ratio. Furthermore, in the constant stiffness ratio, they found that Poisson’s Ratio remains constant if rigid blocks are used. Ghazvinian et al. (2014) also verified that this phenomenon is mostly true for the 3DEC, except that the changes in Poisson’s Ratio are a function of the stiffness ratio: An increase in the stiffness ratio results in a decrease in Poisson’s Ratio and
increase in Young’s Modulus. Therefore, the same behavior is observed in this study (Figure – 5.22) also published by Ghazvinian et al. (2014).

c. Tensile Strength Analysis

The influence of micro-tensile strength on macro-tensile strength is studied in the Brazilian Tensile Test Configuration by changing micro-tensile strength from 1 MPa to 20 MPa. Figure – 5.23 shows the macro-tensile strength response. An increase of micro-tensile strength resulted in increase of macro-tensile strength. The S-Shaped behavior is attributed to the fact that in the low (i.e. from 1 to 5 MPa) and high (i.e. after 10 MPa) tensile strengths, the material behavior is controlled by the plastic zone elements.

![Graph showing the relationship between Contact Tensile Strength and Model Tensile Strength](image)

*Figure 5.23 Response of Brazilian Tensile Strength Test Model with the Increase of Contact Tensile Strength*

d. Frictional Strength Analysis

The influence of the micro-frictional strength component of Voronoi-Trigon on macro response is studied with series of triaxial compressive strength tests with the confinement values of 5 MPa, 7.5 MPa, and 10 MPa. The micro-friction angle of Voronoi-Trigon contacts increased from 10° to 30°. Macro-cohesion and macro-friction responses as a function of micro-friction angle can be seen in Figure – 5.24. It is found that as the micro-friction increases macro-friction angle also increases. However, macro-cohesion tends to remain the same. The previous research
results published by Kazerani and Zhao (2010) in rigid blocks and Gao and Stead (2014) in elastic blocks found similar results with this research.

![Graph showing the response of Model Cohesion and Friction with the Change of Contact Friction](image)

**Figure 5.24 Response of Model Cohesion and Friction with the Change of Contact Friction**

e. Cohesive Strength Analysis

The influence of micro-cohesion on the macro-response is studied. The triaxial compressive strength test with the varying confining stresses is applied to the models while the micro-cohesion is changed from 10 MPa to 30 MPa. The macro-friction and macro-cohesion response can be seen in Figure – 5.25. It is observed that as the micro-cohesion increases, the macro-cohesion increases, and macro-friction decreases and converges to a constant value. This behavior is also observed by other researchers but the reduction in the simulated rock sample friction angle is not observed as sharp as obtained here (i.e. Kazerani and Zhao (2014) observed the macro-friction change is about 5% for the rigid blocks). The zone elements cohesive strength components became dominants after 20 MPa for this case that the model response controlled by the plastic zone element properties causing a sharp decrease in the simulated rock sample friction angle.
Figure 5.25 Response of Model Cohesion and Friction with the Change of Contact Cohesion

f. Micro-Cohesion to Micro-Tension Ratio, $c/T$, Analysis

Ghazvinian et al. (2014) discussed that the micro-cohesion to micro-tensile strength ratio ($c/T$) is also an important measure to control model brittleness and to observe crack initiation threshold. In order to discover the effect of this ratio in the UDEC, a FISH routine is written to collect joint states from the Voronoi-Trigon contacts (i.e. tensile, shear, and total failure). For simplicity, the crack initiation threshold is directly attributed to the starting point of the tensile contact failures and the crack damage is attributed to the failed shear contacts as Gao and Stead (2014) utilized in their research. Also, Ghazvinian et al (2014) and Diederichs (2003) indicated that fractures opened in the crack initiation threshold are controlled by the tensile strength of the contacts. Hence, this is supporting the idea of relating tensile failure count with the crack initiation threshold. In the simulations, identical micro-cohesion is introduced while the ratio of the $c/T$ is changed with four different numbers (2, 3, 4, and 10). Figure – 5.26 shows the holistic view of how the $c/T$ ratio affects the crack initiation and crack damage thresholds. Figure – 5.26 visualizes the cumulative tensile and shear crack behavior together with the stress-strain behavior of the models while blue and pink solid colors indicate the crack initiation and crack damage thresholds on the stress-strain curve. Figure – 5.26b reveals the crack initiation and damage as a function of normalized strength and Figure – 5.26c is the zoomed-in version of the graphs in Figure – 5.26b to understand how the tensile and shear cracks start to mobilize. It is found that as the $c/T$ ratio increases the model brittleness increases. It implies that for the high $c/T$ ratios, the tensile and shear crack initiations
start as low as 0.017 and 0.54 of the model’s peak-strength. In other words, a dramatic decrease in both crack initiation and damage threshold is observed when the $c/T$ ratio is increased. Figure 5.27 exhibits the change in crack initiation and the damage with various $c/T$ ratios.

Figure 5.26 Model Responses as a Function of the $c/T$ Ratio: a) The Normalized Strength, Cumulative Tensile and Cumulative Shear Cracks against Axial Strain are Shown; b) The Cumulative Tensile and Cumulative Shear Cracks against Normalized Strength are Shown; c) The Zoomed in version of (b) to Capture Crack Starting Points

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5.2.2.4. Conclusions of Parametric Study on Voronoi-Trigon’s Micro-Properties

Several conclusions are drawn from the parametric study:

- As the Voronoi-Trigon particle size increases, while a negative relation is captured with Poisson’s Ratio, a positive relation is observed with Young’s Modulus.
- As the joint shear to normal stiffness ratio increases, Young’s Modulus increases, and Poisson’s Ratio decreases.
- As the joint stiffness ratio is held constant and the joint normal stiffness value increases, Young’s Modulus increases, and Poisson’s Ratio decreases.
- In the low (i.e. from 1 to 5 MPa) and high (i.e. higher than 10 MPa) values of the micro-tensile strength, the macro-tensile strength is controlled by the plastic zone elements.
- While the micro-friction angle increases gradually, the macro-friction angle increases too but the macro-cohesion tends to be constant.
- As the micro-cohesion increases, the macro-cohesion increases while the macro-friction decreases.
- As the ratio of micro-cohesion to micro-tensile strength component of Voronoi-Trigon Blocks increases, crack initiation and crack damage threshold are lowered so that tensile cracking starts to dominate all the material behavior.

The conclusions listed above are found to be in good agreement with the literature. After having the holistic view on the micro-properties effect on the macro-properties, the strength groups (i.e.
low, medium, and high) are calibrated. The stress-strain behavior of each strength group can be observed in Figure – 5.28 and Figure – 5.29. Table – 5.6 also shows the calibrated properties and captured model responses.

*Figure 5.28 Axial Stress-Strain Behavior of Low, Medium and High Strength Groups’ BPM*

*Figure 5.29 Brazilian Tensile Strength Behavior of Low, Medium and High Strength Groups’ BPM*
Table 5-6 Calibrated Trigon Contact Parameters and Captured Model Responses

<table>
<thead>
<tr>
<th>Trigon Contacts</th>
<th>Strength Group</th>
<th>Model Responses</th>
<th>Strength Group</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>Medium</td>
<td>High</td>
</tr>
<tr>
<td>Cohesion, MPa</td>
<td>14.5</td>
<td>22.5</td>
<td>36.2</td>
</tr>
<tr>
<td>Friction, °</td>
<td>35</td>
<td>38.6</td>
<td>38</td>
</tr>
<tr>
<td>Tension, MPa</td>
<td>5.8</td>
<td>10.3</td>
<td>15.8</td>
</tr>
<tr>
<td>Shear Stiffness, GPa/m</td>
<td>80,000</td>
<td>85,000</td>
<td>90,000</td>
</tr>
<tr>
<td>Normal Stiffness, GPa/m</td>
<td>32,000</td>
<td>42,500</td>
<td>40,500</td>
</tr>
<tr>
<td>Stiffness Ratio</td>
<td>0.4</td>
<td>0.5</td>
<td>0.45</td>
</tr>
</tbody>
</table>

5.2.3. Failure Mechanism of a Rock Sample with Single Angular Discontinuity

The peak strength of a rock sample having a pre-existing joint is discussed by Hoek and Brown (1980). Elmo (2006) defined the shear strength of a joint as:

$$\tau_{joint} = \sigma_n \cdot \tan (\phi_{joint})$$  \hspace{1cm} (52)

where $\sigma_n$ and $\phi_{joint}$ are the normal stress acting on the joint plane and the joint friction angle while Jaeger and Cook (1979) also defined the fracture shear strength as:

$$\tau_{joint} = c + \sigma_n \cdot \tan (\phi_{joint})$$  \hspace{1cm} (53)

where $c$ is the cohesive strength of the shear plane.

The kinematical equilibrium suggests that if the joint dip angle, $\theta$ is equal to or less than the joint plane friction angle, the intact rock failure dominates the rock behavior. However, there is also a possibility for shear failure along the joint plane. If the joint dip angle is higher than the joint friction angle, pure shear failure along the joint plane happens with some possible intact rock failures (Elmo, 2006). In this study, pillar contacts between upper and bottom platens are connected to each other; therefore, the following kinematical equilibrium equations adopted from Elmo (2006) as:

$$\theta \leq \phi_{joint}$$  \hspace{1cm} (54)

$$\theta > \phi_{joint}$$  \hspace{1cm} (55)
Micro-properties for low-strength limestone rocks listed in Table – 5.5 are used to carry out this study. A single joint with a pre-defined dip angle, ranging from 0° to 90°, is inserted into the intact rock sample, and uniaxial test boundary conditions are applied. Figure – 30 illustrates a single joint with different dip angles used in this study. Three different confinements are applied to each model to observe the effect of confinement on the jointed rock samples. The friction angle and cohesion of the joint are set to 30° and 0.1 MPa. Cohesion value was assigned to prevent the noise in the simulations during the unfavorable joint orientation (i.e. $\theta = 60°$) The U-Shaped behavior observed from UDEC results, Figure – 5.31a, is similar to the response computed with an analytical model (Figure – 5.31b).

![Figure 5.30 Single Going through Joints with 10-Different Dipping Angle and the Model Geometry](image1)

**Figure 5.30 Single Going through Joints with 10-Different Dipping Angle and the Model Geometry**

![Figure 5.31 Single Going through Joint Effect on Intact Rock: a) UDEC Calculated Strength at Different Confinement Levels; b) Theoretical Behavior of Intact Rocks with Single Going through Joints (Elmo, 2006)](image2)

**Figure 5.31 Single Going through Joint Effect on Intact Rock: a) UDEC Calculated Strength at Different Confinement Levels; b) Theoretical Behavior of Intact Rocks with Single Going through Joints (Elmo, 2006)**
5.3. Discrete Fracture Network Generation

In the DFNs generation, the input parameters utilized in the numerical simulations are derived from the field observations (Esterhuizen et al., 2011) as summarized in Table – 4.9 and – 4.10. Mean GSI values of the S-Pillar database vary between 74 and 76 (Table – 4.13). The area of interest for this study on the GSI chart marked on Figure – 5.32. Stone mine rock masses, within the range of the S-Pillar database, can be represented as blocky. The red line is the mean values for each rock strength category (i.e. low, medium, and strong) while the orange line represents the boundary for the one standard deviation from the mean value.

![Figure 5.32 S-Pillar Database GSI Representation in GSI Table](image)
During the field surveys, dip angles of the discontinuities were not recorded by Esterhuizen et al. (2011) except for the angular and large discontinuities. However, they reported that 81° is the mean dip angle for the joints observed in the underground limestone mines. Also, it is indicated that 18% of the discontinuities have a dip angle less than 70°. Hence, it is decided to define two different joint sets nearly perpendicular to each other having 81° and 0° dip angles for vertical and horizontal joints, respectively. The step-by-step DFNs generation is listed below:

1. The mean values and standard deviations with the associated statistical distributions of fracture frequency ($P_{10}$), trace length, orientation, and position are found (Table – 4.9 and – 4.10). The trace length of the discontinuities follows the log-normal distribution while the orientation and position are assumed as uniform. The trace length distributions of each strength group can be seen in Figures – 5.33, and – 5.34 for vertical and horizontal joints.

![Figure 5.33 Vertical Joint Length Distributions](image)

![Figure 5.34 Horizontal Joint/Bedding Plane Length Distributions](image)
2. Initially, the GSI table is used to generate DFNs realizations stochastically from the different rock masses listed in the S-Pillar database. Unfortunately, in the literature, there are not reliable recommendations on the required number of realizations (Vazaios, et al., 2017). Palleske (2014) discussed that 10 DFNs realizations could be enough to represent rock mass variability. Hence, an arbitrary number of 15 is selected for this study as the number of realizations. Each DFNs realization is generated with 100 m in width and height rock mass domain to eliminate boundary effects.

3. DFNs realizations are simplified with the intrinsic UDEC command ‘fracture combine’ to decrease the uncertainties caused in the realization steps. The main reason for this simplification is to avoid ultra-fine meshing. Relatively-low-distanced two fractures are represented with one fracture at the end of this process.

4. Rock mass domain (100 m in width and 100 m in height) is sampled with 15 m in width and 30 m in height boxes to generated pillar geometry with representative DFNs structures. Then, the intact rock block areas, between the joint sets, are calculated for each realization. The block areas are converted into block volumes with the unit-length assumption to back-calculate GSI from the numerical models. The generated block volume databases for each strength category can be viewed in Figure – 5.35.

![Figure 5.35 Block Area Distributions for Each Strength Group](image)

5. The joint condition factors for the S-Pillar database are calculated using Equation – 56. The equation is established by Palmstorm (1994).
\[ J_C = J_W \cdot J_S / J_A \]  

(56)

where \( J_C, J_W, J_S \) and \( J_A \) are the joint condition factor, joint large-scale waviness (in meters), joint small-scale smoothness (i.e. roughness), and joint alteration factor. Since there is not any information provided in the S-Pillar database about the large-scale joint waviness, it is assumed as 1 and 3 to generate minimum and maximum joint condition factors. Figure – 5.36 and – 5.37 visualize the minimum and maximum joint condition factors calculated according to Equation – 46.

![Figure 5.36 Distribution of Minimum Joint Condition Factor for Each Strength Groups](image)

![Figure 5.37 Distribution of Maximum Joint Condition Factors for Each Strength Groups](image)

6. The calculated block volumes and the joint condition factors are used to back-calculate GSI values based on the quantification established by Cai et al. (2004). For each realization, the mean, standard deviation, and median values of block volumes are calculated. Later,
on the GSI chart, the database boundaries are drawn to visualize the range of associated GSI values simulated in the numerical models. When a good match on the GSI value is achieved between single DFNs realization and the database, the DFNs realization is utilized in the numerical simulations to form Synthetic Rock Mass.

Three different DFNs realizations are generated. Figure – 5.38a, – 5.38b, and – 5.38c show the back-calculated GSI values for low, medium, and high strength rocks together with their DFNs realizations which are used in the SRM generation step. In the following section, the influence of discontinuities on the pillar strength is studied conceptually; then, the application of SRM to the underground limestone mines is carried out.
Figure 5.38 Discrete Fracture Networks Generated to Utilize in Numerical Simulations: a) Low Strength, b) Medium Strength, c) High Strength
5.4. Synthetic Rock Mass Approach

In this section, Synthetic Rock Mass studies are carried out with the integration of BPM and DFNs. First, conceptual DFNs models are generated with different input parameters, and the influences of joint dip angle and joint trace length on the pillar strength are studied. Then, the application of the proposed rock block up-scaling methodology is carried out to derive field size underground limestone pillar strength.

5.4.1. Parametric Studies

As it was discussed in the above sections, underground stone mine rock mass generally contains two joint sets perpendicular to each other. Two different conceptual DFNs realizations consisting of two perpendicular joint sets are generated to be consistent with the field observations. The joint frequency number($P_{10}$) is set to 1.0 for all conceptual realizations. However, the joint trace length is varied using a log-normal distribution with mean values of 0.5 m and 1.0 m, consistent with field observations. The normal distribution for vertical and horizontal joints orientations, with 90° and 0° dip angles are assumed. Also, the uniformly distributed position assumption is applied in the DFNs generation scheme. Later, the fractures are rotated along their center in the clockwise direction with 20° degree increments. At the last rotation increment (i.e. 80° − 170°), 10° is applied until the horizontally oriented joint oriented vertically and vice versa. The representation of fracture rotations can be seen in Figure – 5.39.

![Figure 5.39 Representation of Generated Fractures: a) 0°-90°; b) 20°-110°; c) 40°-130°; d) 60°-150°; e) 80°-170°; f) 90°-180°](image)
The simulated pillar has a 7.8 m width and 15.6 m in height dimensions. The intact rock failure also simulated with the Voronoi-Trigon Tessellation having an average block length of 0.3 m. In order to satisfy the consistency with the proposed methodology, plastic properties are attributed to the Trigon Blocks. Input parameters are listed in Table – 5.7. Also, 30° friction angle (Esterhuizen et al., 2011; Vardar et al., 2019) and 0.1 MPa cohesion are assigned to DFNs as contact strength properties. Pillars are axially loaded until failure.

<table>
<thead>
<tr>
<th>Table 5-7 Properties Utilized in Conceptual Pillar Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Property</td>
</tr>
<tr>
<td>---------------------------</td>
</tr>
<tr>
<td>Cohesion, MPa</td>
</tr>
<tr>
<td>Friction, MPa</td>
</tr>
<tr>
<td>Tension, MPa</td>
</tr>
<tr>
<td>Young’s Modulus</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
</tr>
<tr>
<td>Joint Normal Stiffness, GPa</td>
</tr>
<tr>
<td>Joint Shear Stiffness, GPa</td>
</tr>
</tbody>
</table>

The influence of two joint sets on the pillar strength can be seen in Figure – 5.40. The maximum pillar strengths are observed when the joint pair orientations are 0° – 90° and 90° – 180° (Figure – 5.40a and Figure – 5.40f, respectively). The pillar strength decreases while the orientation of the fractures changes from 0° – 90° to 40° – 130°. Between the orientations of 60° – 150° and 90° – 180°, the pillar strength starts to increase. The U-shaped behavior on the pillar strength is also documented by Elmo and Stead (2010), Esterhuizen et al. (2011), and Zhang et al. (2015). However, the results published by Esterhuizen et al. (2011) indicated that the minimum pillar strength is achieved when the joint dip angle is equal to 60°. The main difference between that study and this one is the number of joint sets. In this study, as it is discussed, two perpendicular joint sets are introduced but Esterhuizen et al. (2011) utilized a single joint. In addition to the joint orientation effect on the pillar strength, the influence of joint trace length on pillar strength is also studied. Figure – 5.41 summarizes that longer trace length results in lower strength which is found to be in good agreement with Elmo and Stead (2010).
Figure 5.40 The Effect of Discontinuity Dip Angle and Trace Length

Figure – 5.41 reveals the maximum principle stresses developed in the pillars. When the dip angles of the fracture pairs are 0° – 90°, the intact rock splitting controls the pillar behavior that vertical segmentation can be seen in Figure – 5.41a. In Figure – 5.41b, similar to Figure – 5.41a, the axial splitting of the Voronoi-Trigon blocks governs the pillar behavior. However, when the orientations of the fracture pairs (i.e. 40° – 130°) exceeded the fracture friction angle (i.e. 30°), the shear failure starts to dominate the pillar behavior. This behavior is captured in Figure – 5.41c that a shear plane is developed from the upper-right corner of the pillar to the core of the pillar. On the other hand, in Figure – 5.41d, a combined failure of shear and tensile are observed (orientation of fracture pairs is 60° – 130°). The shear plane extends from the left-upper corner of the pillar to the right-lower, and the axial splitting in the rib of the pillar is captured. In Figure – 5.41e and Figure – 5.41f, the tensile failure mechanism dominates the pillar behavior but potential shear failure planes are also captured.

In order to further advance on the joint dip angle influence on pillar strength, the angle between two joint sets is systematically reduced from 90° to 10° (Figure – 5.42). Results indicated that as the angle between two joint sets decreases, pillar strength decreases too (Figure – 5.43).
Figure 5.41 Failure Mode Development in the Pillars: a) Dip Angle 0° – 90° Intact Rock Failure in Tensile Fashion Governs the Behavior; b) Dip Angle 20° – 110° Intact Rock Failure in Tensile Fashion Governs the Behavior; c) Dip Angle 40° – 130° Intact Rock Failure Starts to Replace with Shear Failure; d) Dip Angle 60° – 130° Complete Shear Failure Plane Development with some Tensile Failure in the Corner Elements; e) Dip Angle 80° – 170° The Combination of Intact Rock Failure in Tensile and Shear Failure; f) Dip Angle 90° – 180° The Combination of Intact Rock Failure in Tensile and Shear Failure

Figure 5.42 Angle Reduction Between two Different Joint Sets: a) Dip Angle 10°-80°; b) Dip Angle 20°-70°; c) Dip Angle 30°-60°; d) Dip Angle 40°-50°

Figure 5.43 Strength Reduction Rates as a Function of Angle Difference between Two Joint Sets
5.4.2. Rock Block Up-Scaling
In the proposed study, after calibrating the plastic zone elements, and Voronoi-Trigon Contacts, and generating DFN realizations to represent limestone rock masses, the last step is to establish Synthetic Rock Mass models to calculate pillar strengths from the numerical simulations.

5.4.2.1. Up-Scaling Operation
The homogenization process adopted in this study is explained in Chapter – 3 of this thesis. Figure – 5.44, - 5.45, and – 5.46 show the change of normalized pillar strength as a function of pillar width for low, medium, and high strength rock categories. Normalized strength calculated as the model estimated strength over UCS. In addition to the S-Pillar pillar strength equation represented with orange line and dots, power strength equations of Hedley and Grant (1972) with a green line and Von Kimmelmann (1984) with magenta color, are also presented in the figures. The numerically estimated pillar strength and the strength reduction trend as a function of size increment in each strength group are found to be in good agreement with the literature.

![Figure 5.44 Low Strength Up-Scaling Behavior Captured by UDEC among the Literature](image)

*Figure 5.44 Low Strength Up-Scaling Behavior Captured by UDEC among the Literature*
5.4.2.2. Pillar Strength Prediction with the Various Width-to-Height Ratios

In Chapter 4.3, the analysis of the S-Pillar database indicated that pillar width-to-height ratios are changing from 0.29 to 3.52 with a mean value of 1.41. Hence, it is decided to model each limestone intact rock strength group with four different width-to-height ratios of 0.5, 1.0, 1.5, and 2.0. Input properties are listed in Table 5.8.
Table 5-8 Utilized Voronoi-Trig Micro-Properties in Different Models and Voronoi-Trig
Block Properties

<table>
<thead>
<tr>
<th></th>
<th>Voronoi-Trig Micro-Properties</th>
<th>Voronoi-Trig Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>Medium</td>
</tr>
<tr>
<td></td>
<td>7.8 m</td>
<td>15 m</td>
</tr>
<tr>
<td>Cohesion, MPa</td>
<td>9.8</td>
<td>18.5</td>
</tr>
<tr>
<td>Friction, MPa</td>
<td>8.4</td>
<td>17</td>
</tr>
<tr>
<td>Tension, MPa</td>
<td>5.8</td>
<td>10.3</td>
</tr>
<tr>
<td>Joint Normal Stiffness, GPa</td>
<td>80,000</td>
<td>85,000</td>
</tr>
<tr>
<td>Joint Shear Stiffness, GPa</td>
<td>32,000</td>
<td>42,500</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Young's Modulus, GPa</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Figures from 5.47 to 5.52 Summarizes the UDEC estimated pillar strengths for various width-to-height ratios. Empirical equations are also included in these graphs. Pillar strengths are normalized with UCS to eliminate biases arising from the intact rock strength differences. Good agreement is achieved for the pillar strength prediction by using the proposed methodology in the UDEC. The low-strength rock groups are shown in Figure – 5.47 (pillar width is 7.8 m) and Figure – 5.48 (pillar width is 15.0 m). UDEC predicted strengths higher than the empirical equations. In the S-Pillar database, there is not any failed pillar case reported for low-strength rock masses. However, during the derivation of the pillar strength equation for stone mines (Equation – 24), average intact rock strength of all strength categories is used. During the statistical analyses of the S-Pillar database, it is observed that combined RQD and joint spacing rating is the highest in the low strength rocks when compared to other strength groups. Esterhuizen (personal communications, March 26, 2021) indicated that the stone mines having the high-strength intact rock are suffering from structurally controlled failure. Also, it is supported by Esterhuizen et al. (2019) that the Loyalhanna formation (a high-strength limestone rock formation) has well-developed joint sets due to folding in the Appalachian Plateau causing to form joint sets and fault.

It is observed that as the pillar intact rock strength increases from low to high (Figures from – 5.47 to 5.52), the difference between UDEC predicted pillar strength and empirical strength equations.
decreases. Especially, when the pillar with the high strength intact rocks is analyzed, a good correlation is achieved by numerical simulations against the stone mine pillar strength equation.

Figure 5.47 Low Strength Limestone Rocks with Various Width-to-Height Ratio – Width 7.8125 m in Pillar Stability Charts: a) Power Strength Estimation Equations; b) Linear Strength Estimation Equations

Figure 5.48 Low Strength Limestone Rocks with Various Width-to-Height Ratio - Width 15.0 m in Pillar Stability Charts: a) Power Strength Estimation Equations; b) Linear Strength Estimation Equations
Figure 5.49 Medium Strength Limestone Rocks with Various Width-to-Height Ratio – Width 7.8125 m in Pillar Stability Charts: a) Power Strength Estimation Equations; b) Linear Strength Estimation Equations

Figure 5.50 Medium Strength Limestone Rocks with Various Width-to-Height Ratio – Width 15.0 m in Pillar Stability Charts: a) Power Strength Estimation Equations; b) Linear Strength Estimation Equations
When the pillar width-to-height ratio is equal or greater than 1.5, a sudden increase in the pillar strength, predicted by UDEC, is observed for most of the strength groups. The confinement developed along the width of the pillars is queried to investigate the mechanics behind this observation. Figure – 5.53 visualizes the confinement along the 15 m wide and low strength group pillar. The queried confinements are normalized by the intact rock strength. The core of the pillars, with the width-to-height ratios of 1.5 and 2.0, experienced confinement rates 4 to 6 times higher than pillars with the width-to-height ratios of 0.5 and 1.0. This kind of steeper region is observed by both Martin and Maybee (2000) when the conventional Hoek and Brown brittle parameters are
utilized, and Elmo and Stead (2010) in their numerical simulation results. In addition, Kaiser et al. (2011) indicated that squat pillars may have higher load carrying capacity due to confinement-dependent stress development in the pillar core. Hence, the rapid strength increase in the higher width-to-height ratios is attributed to the confinement.

Figure 5.53 Confinement Development in the Low Strength Pillar Having 15.0 m Width

Also, it is achieved that strengths predicted for 15 m wide pillars are higher than for 7.8 m wide pillars when the width-to-height ratio is 1.5. To understand the mechanism leading to this behavior, the pillar confinements are calculated along the pillar center with 2 − meters high rectangle. Then, confinement rates are averaged and normalized with respect to UCS. In Figure – 5.54, the lateral axis defines the confinement location numbers along the pillar center (i.e. 0 means the pillar center, −1 means 2.5 meters left from the pillar center) while the vertical axis represents the normalized confinement. The observations showed that confinement at the core of the larger dimensioned pillars (15.0 m in width) is higher than pillars having the width of 7.8 m. Figure – 5.54 visualizes the confinement rates in the pillar cores. Hence, the numerically high prediction of wider pillar strength is attributed to observation on the greater confinement rates.
5.4.3. Pillar Failure Mechanism

5.4.3.1. Failure Modes

Esterhuizen et al. (2011) reported that pillars with the width-to-height ratio of 0.5 experience the tensile failure mechanism in the form of axial splitting. On the other hand, squat pillars are failing through a combination of shear and tensile failure. The tensile failure in the form of splitting is taking place at the outer elements of the pillars in the direction perpendicular to the maximum principal stress while the shear failure governs the pillar behavior in the core.

In the following four figures (Figure – 5.55 to 5.58), the maximum principal stress contours are plotted with stress tensors. Figure – 5.55 shows the slender pillar failure mechanism (width is 15.0 m), all the elements are already yielded. On the other hand, square pillars are shown in Figure – 5.56. The transition from slender to the square pillar is clear that all elements are not yielded in the square pillars. Indeed, the stress tensors are intensified in the core of the square pillars indicating the elements can bear the load. The pillars having the width-to-height ratio of 1.5 are visualized in Figure – 5.57. The emergence of hour-glass shape is observed. Finally, Figure – 5.66 exhibits the pillar having a width-to-height ratio of 2.0. Pillar ribs are already failed and the well-developed hour-glass shape with the open fractures (attributed to the 0 maximum principal stresses at the pillar rib). Similar observations are captured by other researchers (Martin and Maybee, 2000; Roberts et al., 2007; Elmo and Stead, 2010; Esterhuizen et al., 2011).
Figure 5.55 Width-to-Height Ratio 0.5 – Pillar Failure at the End of the Simulations: a) Low Strength, b) Medium Strength, c) High Strength

Figure 5.56 Width-to-Height Ratio 1.0 – Pillar Failure at the End of the Simulations: a) Low Strength, b) Medium Strength, c) High Strength

Figure 5.57 Width-to-Height Ratio 1.5 – Pillar Failure at the End of the Simulations: a) Low Strength, b) Medium Strength, c) High Strength
In order to observe the failure modes as a function of the width-to-height ratio of the pillars, the joint states are also investigated. The following four figures (Figure – 5.59 to 5.62) are constructed for low, medium, and high strength limestone pillars with various width-to-height ratios. Figure – 5.59 visualizes the failure modes in the slender pillars. The blue lines indicate the tensile failure in the models. In all pillars in Figure – 5.59, the segmentation in the direction parallel to the joint planes is observed indicating that the axial splitting is mobilized. In Figure – 5.60, square pillars are visualized. A combination of two failure modes is observed that the yellow lines, indicating the shear failure, are concentrated on the core of the pillar. On the contrary, blue lines concentrate on the pillar ribs. The failure modes of pillars having the width-to-height ratio of 1.5 are shown in Figure – 5.61. In the core of the pillars, the shear failure planes are well-developed as it is observed with the green lines. Similar to the pillar having the width-to-height ratio of 1.0, the tensile failure is observed in the outer elements. Finally, when the width-to-height ratio of the pillar is equal to 2.0, the complete hour-glass shape can be identified by the yellow and green lines at the core while the tensile failure mode governs the pillar rib’s behavior (Figure – 5.62). Hence, the failure modes as a function of the pillar width-to-height ratio are confirmed against the literature.
**Figure 5.59** Width-to-Height Ratio 0.5 – Joint States at the End of the Simulations: a) Low Strength, b) Medium Strength, c) High Strength

**Figure 5.60** Width-to-Height Ratio 1.0 – Joint States at the End of the Simulations: a) Low Strength, b) Medium Strength, c) High Strength

**Figure 5.61** Width-to-Height Ratio 1.5 – Joint States at the End of the Simulations: a) Low Strength, b) Medium Strength, c) High Strength
5.4.3.2. Progressive Failure of Pillars

In the simulations, to understand how the failure is evolving with the loading stages, two pillars having the width-to-height ratio of 0.5 and 1.0 are inspected. The pillars having the low intact rock strength can be seen in Figure – 5.63 and Figure – 5.64. From top to bottom, the different loading stages are established with the instantaneous snapshots of maximum principal stress contours and the joint plane states together with the stress-strain behavior of the pillars.

In the first stage of Figure – 5.63, the pillar has the 8 MPa load, approximately 9% of the UCS. The tensile failure mode is already started to develop at the corner elements. Also, in the upper left side of the pillar, while the other elements maximum principle stress developed, the development of stress does not exist in that particular region indicated with the black circle. In the second stage, the tensile cracks become more visible that some preliminary block detachment in the left side of the pillar start. The block detachment happens and tensile failure extends through the pillar core during the third stage, and in the fourth stage, the pillar shows more spalling. At the fifth stage, the pillar reaches its peak strength. Even though it sustains the loads, tensile failure is dominating the pillar behavior. At the final stage, the tensile failure governs all the pillar elements.

For the pillar having the width-to-height ratio of 1.0, when the pillar load is approximately at 10% of the pillar intact rock UCS, the preliminary failure starts at the corners (first stage of Figure – 5.64). In the second stage, shearing along the joint planes is observed that governs the pillar behavior. In the third and fourth stage, a transition from the pre-failure stage to the failure stage is captured. While the outer elements are experiencing tensile failure, they start to extend towards the core. The diagonal green lines, representing the shear failure in the joint state visualizations, indicate the shear failure is fully developed in the core. In the fifth stage of Figure – 5.64, the emergence of the hour-glass shape is clear that both maximum principle strength contours and

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*Figure 5.62 Width-to-Height Ratio 2.0 – the Joint States at the End of the Simulations: a) Low Strength, b) Medium Strength, c) High Strength*
joint states indicate. Later, the complete block detachments, spalling in the pillar rib elements, are observed with the domination of tensile failure in the final stage. The shear failure still governs the core of the pillar but tensile failure is also experienced by core elements. The findings in this study and the pillar loading stages revealed by Roberts et al. (2007) based on the field observations support each other.

5.5. Conclusions

In this chapter, the proposed up-scaling methodology is applied to the US underground stone mine pillars. Two main procedures for Voronoi-Trigon micro-property calibration with the plastic zone elements and the DFNs generation are established after careful literature review. The strength of intact rock having the pre-defined fracture with various dip angles is predicted and a similar response is captured with the analytically computed intact rock response. Later, the pillar strengths are successfully predicted with the up-scaling operations by the numerical simulations. The empirical pillar strength equations are used as a control measure to validate the numerical simulations. The various pillar width-to-height ratios are employed both to assess pillar strength and to observe failure mechanisms as a function of the pillar width-to-height ratio.
Figure 5.63 Loading Stages for Pillar Having the Width-to-Height Ratio of 0.5
Figure 5.64 Loading Stages for Pillar Having the Width-to-Height Ratio of 1.0
Chapter 6 Conclusions and Future Studies

6.1. Summary
In underground stone mining operations, the room-and-pillar mining method is the primary mining method, and pillars maintain global stability and provide support for the immediate roof (Brady and Brown, 2004). In order to improve underground stone mining layouts, NIOSH (2011) established pillar and roof support design guidelines. Unfortunately, pillar design guidelines do not cover the influence of more than one joint sets or naturally existing joint sets in the pillar strength estimation. In addition, underground stone mining operations are still experiencing ground control-related instabilities, and structurally controlled massive pillar failures (Esterhuizen et al., 2019). Therefore, the primary objective of this thesis is to develop a methodology to explain the short-term strength and failure mechanisms of the underground stone mine pillars with consideration of naturally existing fractures along the pillars and to improve health and safety in underground stone mining operations. In this thesis, a coupled SRM and DFNs methodology is developed to estimate pillar strengths from laboratory scale rock specimens and to assess the failure mechanism of the pillars with consideration of naturally existing joint sets. In order to achieve the research objective, four research tasks were performed as:

1. Statistical analyses on the S-Pillar database were carried out to derive input parameters to be utilized in the numerical simulations.

2. A procedure for Bonded Particle Model (BPM) generation with the Voronoi-Trigon Tessellation was established. The BPMs generated with the Voronoi-Trigon Tessellation revealed that the intact rock failure is modeled accurately that the parametrical studies on the BPMs indicated that the results are similar to other published studies (Potyondy and Cundall, 2004; Kazerani, and Zhao, 2010; Gao and Stead 2013; and Ghazvinian et al., 2014) that indicates an accurate Voronoi-Trigon micro-property calibration procedure was established.

3. A Discrete Fracture Networks (DFNs) generation procedure was proposed in this study to represent rock masses in numerical simulations using a back-calculation of Geological Strength Index (GSI) based on the quantified table established by Cai et al. (2004). It was found that the proposed procedure is working well to generate DFNs with the embedded DFN module in UDEC.
4. A coupled, practical methodology between Synthetic Rock Mass (SRM) and DFNs was established. The methodology was constructed to estimate in-situ pillar strength from the laboratory size intact rock specimen’s strength. It was achieved with the rock block up-scaling operation that the size of the laboratory size rock specimens was systematically increased until the field size average pillar dimensions. In the interim stages, a homogenization process was implemented to capture jointed rock specimen response with the non-jointed, homogenized new BPM that field-scale pillar strengths were estimated in a meaningful time frame. After completing the up-scaling operations, pillar models were generated with the various width-to-height ratios to numerically calculate the pillar strength and to examine failure mechanism and progressive failure development in the pillars.

With the completion of these four tasks, in-situ size pillar strength parameters were successfully estimated and the pillar failure mechanisms were simulated realistically. A good agreement between UDEC estimated strength and similar observations on the pillar failure mechanisms, and the literature was found. Hence, the research study established a practical methodology to estimate pillar strength.

6.2. Conclusions

This research proposed a systematical methodology to explicitly simulate the influence of naturally existing joints sets and fractures on pillar strength. It was observed during this study that the strength of the pillar models with low intact UCS (i.e. 89 MPa) was over-estimated relative to the empirical S-Pillar equation. In addition to this, it was also revealed that the strength of the pillar models with the high intact rock strength (i.e. 214 MPa) was in good agreement with the empirical equation. In other words, normalized strength of the pillars with low intact rock strength was higher than the pillars with high intact rock strength. This observation is supported by the fact that there are no pillar failure cases in the low-strength rock masses. Indeed, all the failure cases in S-pillar database are in medium and high-strength rock masses. The S-Pillar database analyses also further supported these findings that combined RQD and joint spacing rating, and the joint condition rating in the RMR calculations was calculated higher for the low strength rock masses. Therefore, explicit consideration of the joint sets indicated that the strength reduction is higher in the pillars with higher joint density.
Unlike the empirical approaches in pillar strength estimations, the developed methodology also provided an opportunity to capture failure mechanisms and the progressive failure development of the field scale pillars. As it is observed by numerous researchers (Martin and Maybee, 2000; Roberts et al., 2007; Elmo and Stead, 2010; Esterhuizen et al., 2011), the pillars having the width-to-height ratio of 0.5 experienced axial splitting as the main governing failure mechanisms. In the higher width-to-height ratios (i.e. 1.0, 1.5, and 2.0), tensile failure was observed at the pillar ribs. Due to higher confinement in the core of the pillars shear failures are observed in the center of the pillars. As the width-to-height ratio increases, the emergence of the pillar hour-glass shape becomes clearer. An analysis of progressive failure development within the pillar models indicated that the failure is starting at around 9 – 10% of the pillars’ UCS and that the early signs of the block detachments develop for both pillars having the width-to-height ratio of 0.5 and 1.0. These findings on the pillar failure mechanisms were also observed by Esterhuizen et al. (2011) in actual mine workings.

Overall, the study explained the short-term pillar strength and the failure mechanisms with explicit consideration of the naturally existing joint sets. In order to increase the health and safety in the underground stone mines, a systematical methodology is developed, and the developed rock block up-scaling methodology with the homogenization process is found to successfully estimate the pillar strengths. The uncertainty in pillar strength estimation as the dimension increases was revealed that the well-known phenomena of strength decrease with the dimension increase is numerically proved with the usage of empirical stone pillar strength equation as a control measure. The developed methodology brings a unique and universal solution to be implemented in various regions, locations, and mines, which are not limited to only underground stone mines, to aid pillar design methodologies via estimating the pillar strengths and identifying the pillar failure mechanisms from pillar joint properties.

6.2. Suggestions for Future Studies

The further extension of the research should be performed on the explicit consideration of large-angular through-going discontinuities in the stone mine pillars. In this study, the pillar strength equation modified for stone mines (Esterhuizen et al., 2011) is utilized in its base form that large discontinuities are not studied yet with the developed methodology. The effect of more than one large discontinuity on the pillar strength and failure development is still under investigation that
the unique development of the structural geology parameters may adversely affect the general stability of the underground mines.

Secondly, there is great uncertainty in predicting the spatial location of the discrete discontinuities that the spalling limit, the crack initiation limit, and the driving stress ratio to the failure can differ from mine to mine, even pillar to pillar. Hence, these factors in the pillar stabilities are primary questions to be answered in the first place. In Figure – 6.1, three different pillar models are shown. Three different pillar models are sampled from the different locations of the same DFNs. The location of the critically located fractures is indicated with the black circles. After simulations are concluded, the blocks are deleted with the constant displacements (i.e. 0.05 m) for each model as an assumption. Then, the depth of the failure is measured. The increase in density of the horizontal joint sets results in a higher depth of failure in the pillars. Hence, in order to prevent the fall of ground incidents (i.e. rib failures), the pillar spalling limit can be established as a function of joint density and the spatial locations of discontinuities in future studies.

There is not any accepted methodology to estimate the long-term time-dependent strength of the underground stone mine pillars. In the underground stone mines, it is normal for travel ways to working faces to pass through the old sections, which puts the abandoned pillars next to active workings. The creep behavior due to the stress corrosion should be studied with the crack length of the fracture developments, confinement rate, and the rock toughness to explain the stress-corrosion-based strength degradation as proposed by Damjanac and Fairhurst (2010), and Damjanac et al. (2012). The potential research on the creep behavior and the strength reduction of the stone mine pillars can be utilized in order to improve the design layouts and health and safety concerns in the underground stone mines by establishing the long-term strength of the stone mine pillars and time-to-failure predictions.
Figure 6.1 Pillar Depth of Failure with the Sampling of the Three Different Location in the Same DFNs
References


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